Fully open-flavor tetraquark states $bc\bar{q}\bar{s}$ and $sc\bar{q}\bar{b}$ with $J^P = 0^+, 1^+$

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Abstract We have studied the masses for fully open-flavor tetraquark states $bc\bar{q}\bar{s}$ and $sc\bar{q}\bar{b}$ ($q = u, d$) with quantum numbers $J^P = 0^+, 1^+$. We systematically construct all diquark–antiquark interpolating currents and calculate the two-point correlation functions and spectral densities in the framework of QCD sum rule method. Our calculations show that the masses are about 7.1–7.2 GeV for the $bc\bar{q}\bar{s}$ tetraquark states and 7.0–7.1 GeV for the $sc\bar{q}\bar{b}$ tetraquarks. The masses of $bc\bar{q}\bar{s}$ tetraquarks are below the thresholds of $\bar{B}_s D$ and $\bar{B}_s^* D$ final states for the scalar and axial-vector channels respectively. The $sc\bar{q}\bar{b}$ tetraquark states with $J^P = 1^+$ lie below the $B_c^+ K^*$ and $B_s^* D$ thresholds. Such low masses for these possible tetraquark states indicate that they can only decay via weak interaction and thus are very narrow and stable.

1 Introduction

In the conventional quark model [1, 2], hadrons generally have two kinds of structures: a meson consisting of a quark and an antiquark, and a baryon consisting of three quarks. However, quantum chromodynamics (QCD) allows the existence of hadrons different from the above two structures, such as the tetraquarks, hadronic molecules, pentaquarks, hybrids and so on [3–6].

A compact tetraquark is composed of a diquark and an antidiquark, bounding by the color force among quarks and antiquarks. The light tetraquarks have been widely studied via different theoretical methods [7–10]. For the heavy quark sector, the hidden-charm/bottom $Q\bar{Q}q\bar{q}$ tetraquarks have been extensively investigated to interpret some observed XYZ states in various methods, such as the constituent quark models [11–13], meson exchange and scattering methods [14–16], QCD sum rules [17–20], chromomagnetic interaction models [21, 22], etc. The doubly heavy tetraquark

states $QQ\bar{q}\bar{q}$ have been studied to investigate the stability of tetraquarks [23, 24]. In Refs. [25–27], the open-flavor heavy $bc\bar{q}\bar{q}$ tetraquark states have also been investigated, the results suggest that their masses may lie below the corresponding two-meson thresholds. In addition, such tetraquarks cannot decay via the annihilation channels and thus they will be very stable with narrow widths.

Comparing with the above several tetraquark configurations, the fully open-flavor tetraquarks $bc\bar{s}\bar{q}$ and $sc\bar{q}\bar{b}$ are more exotic since they contain four valence quarks with totally different flavors. However, the studies of these tetraquarks have drawn much less interest to date. In Ref. [28], the non-strange fully open-flavor tetraquark states $uc\bar{d}\bar{b}$ and $dc\bar{u}\bar{b}$ have been studied in the method of QCD sum rules. In Ref. [21], the authors studied the masses of $qs\bar{c}\bar{b}$ and $qc\bar{s}\bar{b}$ tetraquarks by using the color-magnetic interaction with the flavor symmetry breaking corrections. Their results show that the masses of $qs\bar{c}\bar{b}$ and $qc\bar{s}\bar{b}$ tetraquark states are about 7.1 GeV and 7.2 GeV, which are lower than the corresponding two-meson S-wave thresholds. In the heavy quark symmetry, the mass of $bc\bar{q}\bar{s}$ tetraquark states with $J^P = 1^+$ was also evaluated to be around 7445 MeV [29], which is about 163 MeV above the $D\bar{B}_s^*$ threshold and thus allows such decay channel via strong interaction. The above conflicting results from different phenomenological models are inspiring more theoretical studies for the existence of these fully open-flavor tetraquark states. In this paper, we shall study the mass spectra of the fully open-flavor $bc\bar{q}\bar{s}$ and $sc\bar{q}\bar{b}$ tetraquarks in the method of QCD sum rules [30, 31].

This paper is organized as follows. In Sect. 2, we construct the interpolating tetraquark currents of the $bc\bar{q}\bar{s}$ and $sc\bar{q}\bar{b}$ systems with $J^P = 0^+, 1^+$, respectively. In Sect. 3, we evaluate the correlation functions and spectral densities for these interpolating currents. The spectral densities will be listed in the appendix because of their complicated form. We extract the masses for these tetraquarks by performing

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the QCD sum rule analyses in Sect. 4. The last section is a brief summary.

2 Interpolating currents for the $bc\bar{q}\bar{s}$ and $sc\bar{q}\bar{b}$ tetraquark systems

In this section, we construct the interpolating currents for $bc\bar{q}\bar{s}$ and $sc\bar{q}\bar{b}$ tetraquarks with $J^P = 0^+, 1^+$. In general, there are five independent diquark fields, $q_a^T C \gamma_5 q_b$, $q_a^T C q_b$, $q_a^T C \gamma_\mu \gamma_5 q_b$, $q_a^T C \gamma_\mu q_b$, and $q_a^T C \sigma_{\mu\nu} q_b$, where q stands for quark field, a, b are the color indices, C denotes the charge conjugate operator, and T represents the transpose of the quark fields. The $q_a^T C \gamma_5 q_b$ ($J^P = 0^+$) and $q_a^T C \gamma_\mu q_b$ ($J^P = 1^+$) are S-wave operators with $L = 0$ while $q_a^T C q_b$ ($J^P = 0^-$) and $q_a^T C \gamma_\mu \gamma_5 q_b$ ($J^P = 1^-$) are P-wave operators with $L = 1$. The diquark $q_a^T C \sigma_{\mu\nu} q_b$ contains both S-wave ($J^P = 1^+$) and P-wave ($J^P = 1^-$) pieces according to its different components.

To study the lowest lying $bc\bar{q}\bar{s}$ and $sc\bar{q}\bar{b}$ tetraquark states, we use only S-wave diquarks and corresponding antidiquark fields to construct the tetraquark interpolating currents with quantum numbers $J^P = 0^+, 1^+$. For the $bc\bar{q}\bar{s}$ system, the scalar currents with $J^P = 0^+$ can be written as

$$\begin{aligned} J_1 &= b_a^T C \gamma_5 c_b \left(\bar{q}_a \gamma_5 C \bar{s}_b^T + \bar{q}_b \gamma_5 C \bar{s}_a^T \right), \\ J_2 &= b_a^T C \gamma_5 c_b \left(\bar{q}_a \gamma_5 C \bar{s}_b^T - \bar{q}_b \gamma_5 C \bar{s}_a^T \right), \\ J_3 &= b_a^T C \gamma_\mu c_b \left(\bar{q}_a \gamma^\mu C \bar{s}_b^T + \bar{q}_b \gamma^\mu C \bar{s}_a^T \right), \\ J_4 &= b_a^T C \gamma_\mu c_b \left(\bar{q}_a \gamma^\mu C \bar{s}_b^T - \bar{q}_b \gamma^\mu C \bar{s}_a^T \right), \end{aligned} \quad (1)$$

in which q is light quark field (up or down). The color structure for the currents J_1 and J_3 are symmetric $[\mathbf{6}_c]_{bc} \otimes [\mathbf{6}_c]_{\bar{q}\bar{s}}$, while for the J_2 and J_4 are antisymmetric $[\mathbf{3}_c]_{bc} \otimes [\mathbf{3}_c]_{\bar{q}\bar{s}}$. The axial-vector currents with $J^P = 1^+$ can be written as

$$\begin{aligned} J_{1\mu} &= b_a^T C \gamma_\mu c_b \left(\bar{q}_a \gamma_5 C \bar{s}_b^T + \bar{q}_b \gamma_5 C \bar{s}_a^T \right), \\ J_{2\mu} &= b_a^T C \gamma_\mu c_b \left(\bar{q}_a \gamma_5 C \bar{s}_b^T - \bar{q}_b \gamma_5 C \bar{s}_a^T \right), \\ J_{3\mu} &= b_a^T C \gamma_5 c_b \left(\bar{q}_a \gamma^\mu C \bar{s}_b^T + \bar{q}_b \gamma^\mu C \bar{s}_a^T \right), \\ J_{4\mu} &= b_a^T C \gamma_5 c_b \left(\bar{q}_a \gamma^\mu C \bar{s}_b^T - \bar{q}_b \gamma^\mu C \bar{s}_a^T \right), \end{aligned} \quad (2)$$

where the currents $J_{1\mu}$ and $J_{3\mu}$ are color symmetric while the $J_{2\mu}$ and $J_{4\mu}$ are color antisymmetric.

For the $sc\bar{q}\bar{b}$ system, the currents with $J^P = 0^+$ are

$$\begin{aligned} \eta_1 &= s_a^T C \gamma_5 c_b \left(\bar{q}_a \gamma_5 C \bar{b}_b^T + \bar{q}_b \gamma_5 C \bar{b}_a^T \right), \\ \eta_2 &= s_a^T C \gamma_5 c_b \left(\bar{q}_a \gamma_5 C \bar{b}_b^T - \bar{q}_b \gamma_5 C \bar{b}_a^T \right), \end{aligned}$$

$$\begin{aligned} \eta_3 &= s_a^T C \gamma_\mu c_b \left(\bar{q}_a \gamma^\mu C \bar{b}_b^T + \bar{q}_b \gamma^\mu C \bar{b}_a^T \right), \\ \eta_4 &= s_a^T C \gamma_\mu c_b \left(\bar{q}_a \gamma^\mu C \bar{b}_b^T - \bar{q}_b \gamma^\mu C \bar{b}_a^T \right), \end{aligned} \quad (3)$$

where the currents η_1 and η_3 are color symmetric with $[\mathbf{6}_c]_{sc} \otimes [\mathbf{6}_c]_{\bar{q}\bar{b}}$, while the η_2 and η_4 are color antisymmetric with $[\mathbf{3}_c]_{sc} \otimes [\mathbf{3}_c]_{\bar{q}\bar{b}}$. The currents with $J^P = 1^+$ are

$$\begin{aligned} \eta_{1\mu} &= s_a^T C \gamma_\mu c_b \left(\bar{q}_a \gamma_5 C \bar{b}_b^T + \bar{q}_b \gamma_5 C \bar{b}_a^T \right), \\ \eta_{2\mu} &= s_a^T C \gamma_\mu c_b \left(\bar{q}_a \gamma_5 C \bar{b}_b^T - \bar{q}_b \gamma_5 C \bar{b}_a^T \right), \\ \eta_{3\mu} &= s_a^T C \gamma_5 c_b \left(\bar{q}_a \gamma^\mu C \bar{b}_b^T + \bar{q}_b \gamma^\mu C \bar{b}_a^T \right), \\ \eta_{4\mu} &= s_a^T C \gamma_5 c_b \left(\bar{q}_a \gamma^\mu C \bar{b}_b^T - \bar{q}_b \gamma^\mu C \bar{b}_a^T \right), \end{aligned} \quad (4)$$

in which the currents $\eta_{1\mu}$ and $\eta_{3\mu}$ are color symmetric while the $\eta_{2\mu}$ and $\eta_{4\mu}$ are color antisymmetric.

3 QCD sum rules

In this section, we investigate the two-point correlation functions of the above scalar and axial-vector interpolating currents. For the scalar currents, the correlation function can be written as

$$\Pi(p^2) = i \int d^4x e^{ip \cdot x} \langle 0 | T [J(x) J^\dagger(0)] | 0 \rangle, \quad (5)$$

and for the axial-vector current

$$\Pi_{\mu\nu}(p^2) = i \int d^4x e^{ip \cdot x} \langle 0 | T [J_\mu(x) J_\nu^\dagger(0)] | 0 \rangle. \quad (6)$$

The correlation function $\Pi_{\mu\nu}(p^2)$ in Eq. (6) can be expressed as

$$\Pi_{\mu\nu}(p^2) = \left(\frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} \right) \Pi_1(p^2) + \frac{p_\mu p_\nu}{p^2} \Pi_0(p^2), \quad (7)$$

where $\Pi_0(p^2)$ and $\Pi_1(p^2)$ are the scalar and vector current polarization functions related to the spin-0 and spin-1 intermediate states, respectively.

At the hadron level, the correlation function can be written through the dispersion relation

$$\begin{aligned} \Pi(p^2) &= \frac{(p^2)^N}{\pi} \int_{(m_b+m_c)^2}^{\infty} \frac{\text{Im } \Pi(s)}{s^N (s - p^2 - i\epsilon)} ds \\ &\quad + \sum_{n=0}^{N-1} b_n (p^2)^n, \end{aligned} \quad (8)$$

in which b_n is the subtraction constant. In QCD sum rules, the imaginary part of the correlation function is defined as the spectral function

$$\begin{aligned}\rho(s) &= \frac{1}{\pi} \text{Im}\Pi(s) \\ &= f_H^2 \delta(s - m_H^2) + \text{QCD continuum and higher states},\end{aligned}\quad (9)$$

where the “pole plus continuum parametrization” is adopted. The parameters f_H and m_H are the coupling constant and mass of the lowest-lying hadronic resonance H respectively

$$\begin{aligned}\langle 0|J|H\rangle &= f_H, \\ \langle 0|J_\mu|H\rangle &= f_H \epsilon_\mu\end{aligned}\quad (10)$$

with the polarization vector ϵ_μ .

At the quark-gluon level, we can evaluate the correlation function $\Pi(p^2)$ and spectral density $\rho(s)$ using the method of operator product expansion (OPE). To calculate the Wilson coefficients, we use the light quark propagator in coordinate space and heavy quark propagator in momentum space

$$\begin{aligned}iS_q^{ab}(x) &= \frac{i\delta^{ab}}{2\pi^2 x^4} \hat{x} + \frac{i}{32\pi^2} \frac{\lambda_{ab}^n}{2} g_s G_{\mu\nu}^n \\ &\quad \times \frac{1}{x^2} (\sigma^{\mu\nu} \hat{x} + \hat{x} \sigma^{\mu\nu}) - \frac{\delta^{ab} x^2}{12} \\ &\quad \times \langle \bar{q} g_s \sigma \cdot G q \rangle - \frac{m_q \delta^{ab}}{4\pi^2 x^2} \\ &\quad + \frac{i\delta^{ab} m_q \langle \bar{q} q \rangle}{48} \hat{x} - \frac{i m_q \langle \bar{q} g_s \sigma \cdot G q \rangle \delta^{ab} x^2 \hat{x}}{1152}, \\ iS_Q^{ab}(p) &= \frac{i\delta^{ab}}{\hat{p} - m_Q} + \frac{i}{4} g_s \frac{\lambda_{ab}^n}{2} G_{\mu\nu}^n \\ &\quad \frac{\sigma^{\mu\nu} (\hat{p} + m_Q) + (\hat{p} + m_Q) \sigma^{\mu\nu}}{12} \\ &\quad + \frac{i\delta^{ab}}{12} \langle g_s^2 G G \rangle m_Q \frac{p^2 + m_Q \hat{p}}{(p^2 - m_Q^2)^4},\end{aligned}\quad (11)$$

where q represents u , d or s quark and Q represents c or b quark. The superscripts a, b are color indices and $\hat{x} = x^\mu \gamma_\mu$, $\hat{p} = p^\mu \gamma_\mu$. In this work, we evaluate the Wilson coefficients up to dimension eight condensates at the leading order in α_s . To improve the convergence of the OPE series and suppress the contributions from continuum and higher states region, one can perform the Borel transformation to the correlation function in both hadron and quark-gluon levels. The QCD sum rules are then established as

$$\mathcal{L}_k(s_0, M_B^2) = f_H^2 m_H^{2k} e^{-m_H^2/M_B^2} = \int_{(m_b+m_c)^2}^{s_0} ds e^{-s/M_B^2} \rho(s) s^k, \quad (12)$$

where M_B is the Borel mass introduced via the Borel transformation and s_0 the continuum threshold. The lowest-lying hadron mass can be thus extracted via the following expression

$$m_H(s_0, M_B^2) = \sqrt{\frac{\mathcal{L}_1(s_0, M_B^2)}{\mathcal{L}_0(s_0, M_B^2)}}. \quad (13)$$

4 Numerical analysis

In this section, we perform the QCD sum rule analyses for the $bc\bar{q}\bar{s}$ and $sc\bar{q}\bar{b}$ tetraquarks. We use the following values of quark masses and condensates [27, 32–35]

$$\begin{aligned}m_u(2\text{GeV}) &= (2.2_{-0.4}^{+0.5})\text{MeV}, \\ m_d(2\text{GeV}) &= (4.7_{-0.3}^{+0.5})\text{MeV}, \\ m_q(2\text{GeV}) &= (3.5_{-0.2}^{+0.5})\text{MeV}, \\ m_s(2\text{GeV}) &= (95_{-3}^{+9})\text{MeV}, \\ m_c(m_c) &= (1.275_{-0.035}^{+0.025})\text{GeV}, \\ m_b(m_b) &= (4.18_{-0.03}^{+0.04})\text{GeV}, \\ \langle \bar{q} q \rangle &= -(0.23 \pm 0.03)^3 \text{GeV}^3, \\ \langle \bar{q} g_s \sigma \cdot G q \rangle &= -M_0^2 \langle \bar{q} q \rangle, \\ M_0^2 &= (0.8 \pm 0.2) \text{GeV}^2, \\ \langle \bar{s} s \rangle / \langle \bar{q} q \rangle &= 0.8 \pm 0.1, \\ \langle g_s^2 G G \rangle &= (0.48 \pm 0.14) \text{GeV}^4,\end{aligned}\quad (14)$$

in which the masses of u, d, s are the current quark masses in the $\overline{MS}$ scheme at a scale $\mu = 2$ GeV. We consider the scale dependence of the charm and bottom quark masses at the leading order

$$\begin{aligned}m_c(\mu) &= \bar{m}_c \left(\frac{\alpha_s(\mu)}{\alpha_s(\bar{m}_c)} \right)^{12/25}, \\ m_b(\mu) &= \bar{m}_b \left(\frac{\alpha_s(\mu)}{\alpha_s(\bar{m}_b)} \right)^{12/23},\end{aligned}\quad (15)$$

where

$$\alpha_s(\mu) = \frac{\alpha_s(M_\tau)}{1 + \frac{25\alpha_s(M_\tau)}{12\pi} \log\left(\frac{\mu^2}{M_\tau^2}\right)}, \quad \alpha_s(M_\tau) = 0.33 \quad (16)$$

is determined by evolution from the τ mass using PDG values.

To obtain a stable sum rule, the working regions should also be determined, i.e., the continuum threshold s_0 and the Borel mass M_B^2 . The threshold s_0 can be fixed by minimizing the variation of the hadronic masses m_H with the Borel mass

Fig. 1 OPE convergence for the current J_1 in the $J^P = 0^+ bc\bar{q}\bar{s}$ system

M_B^2 . The Borel mass M_B^2 can be obtained by requiring the OPE convergence, which results in the lower bound of M_B^2 , and a sufficient pole contribution, which results in the upper bound of M_B^2 . Specific details of these procedures will be shown later. The pole contribution is defined as

$$\text{PC} \left(s_0, M_B^2 \right) = \frac{\mathcal{L}_0(s_0, M_B^2)}{\mathcal{L}_0(\infty, M_B^2)}, \quad (17)$$

in which $\mathcal{L}_0$ is defined in Eq. (12).

4.1 $bc\bar{q}\bar{s}$ systems

We firstly perform the QCD sum rule analyses for $bc\bar{q}\bar{s}$ tetraquarks. The spectral densities for the interpolating currents in Eqs. (1)–(2) are calculated and listed in the appendix A. For any interpolating current in the $bc\bar{q}\bar{s}$ system, contributions from the quark condensate $\langle \bar{q}q \rangle$ and quark-gluon mixed condensate $\langle \bar{q}g_s\sigma \cdot Gq \rangle$ are numerically small since they are proportional to the quark mass m_q and m_s . The dominant nonperturbative contribution to the correlation function comes from the four-quark condensate $\langle \bar{q}q \rangle \langle \bar{s}s \rangle$. In Fig. 1, we take the scalar interpolating current J_1 as an example to present the contributions to correlation function from the perturbative and various condensate terms. To extract the output parameters, the Borel mass M_B^2 should be large enough to guarantee the convergence of OPE series. Here, we require that the four-quark condensate contribution be less than one-fifth of the perturbative term. In Fig. 1, we can see that the convergence of OPE series can be ensured while $M_B^2 \geq 5.4 \text{ GeV}^2$.

To get the upper bound of M_B^2 , we need to fix the value of s_0 at first. As mentioned before, the output hadron mass m_H should be irrelevant to M_B^2 . In Fig. 2, we show the variations of m_H with the threshold s_0 and Borel mass M_B^2 . It is shown that the variation of m_H with M_B^2 minimizes around $s_0 \sim 59 \text{ GeV}^2$, which will result in the working region $57 \leq s_0 \leq$

61 GeV^2 for the scalar current J_1 . Using this value of s_0 , the upper bound of M_B^2 can be obtained by requiring the pole contribution be larger than 30%. Finally, the working region of the Borel parameter for the scalar current J_1 can be determined to be $5.2 \leq M_B^2 \leq 6.0 \text{ GeV}^2$. We show the Borel curves in these regions in Fig. 2 and extract the hadron mass to be $m_H = 7.24 \pm 0.13 \text{ GeV}$. The errors come from the continuum threshold s_0 , condensates $\langle \bar{q}q \rangle$ and $\langle \bar{q}g_s\sigma \cdot Gq \rangle$, the heavy quark masses m_b and m_c . The errors from the Borel mass and the gluon condensate are small enough to be neglected.

For all other interpolating currents in Eqs. (1)–(2), we perform similar analyses and obtain the suitable working regions for the threshold s_0 , Borel mass M_B^2 , output hadron masses, pole contributions. We collect the numerical results in Table 1 for the scalar $bc\bar{q}\bar{s}$ tetraquarks and Table 2 for the axial-vector $bc\bar{q}\bar{s}$ tetraquarks. The last columns are the S -wave two-meson $\bar{B}_s D$ and $\bar{B}_s^* D$ thresholds for these possible tetraquark states. It is shown that both the scalar and axial-vector $bc\bar{q}\bar{s}$ tetraquarks lie below the corresponding two-meson thresholds, implying their stabilities against the strong interaction.

4.2 $sc\bar{q}\bar{b}$ systems

For the $sc\bar{q}\bar{b}$ systems, we calculate and list the correlation functions and spectral densities in the appendix A for all interpolating currents in Eqs. (3)–(5). Comparing to the $bc\bar{q}\bar{s}$ system, the OPE series behaviors are very different as shown in Fig. 3 (for the scalar current η_1), where the contributions from the quark condensate $\langle \bar{q}q \rangle$ and quark-gluon mixed condensate $\langle \bar{q}g_s\sigma \cdot Gq \rangle$ are much larger than the four-quark condensate $\langle \bar{q}q \rangle^2$. Such difference is due to the existence of $m_Q \langle \bar{q}q \rangle$ and $m_Q \langle \bar{q}g_s\sigma \cdot Gq \rangle$ in the $sc\bar{q}\bar{b}$ system. These terms are proportional to the heavy quark mass, which are absent in the OPE series of the $bc\bar{q}\bar{s}$ system.

For the interpolating current η_1 with $J^P = 0^+$, we require that the quark condensate contributions be smaller than one-third of the perturbative term to obtain the lower bound on M_B while the pole contribution be larger than 10% to yield the upper bound on M_B . The working regions of the parameters are then determined as $53 \leq s_0 \leq 57 \text{ GeV}^2$ and $7.6 \leq M_B^2 \leq 7.9 \text{ GeV}^2$. We show the variations of m_H with threshold s_0 and Borel mass M_B^2 in Fig. 4. Accordingly, the hadron mass can be extracted in these parameter regions. For all interpolating currents in the $sc\bar{q}\bar{b}$ systems, we can only establish reliable mass sum rules for $\eta_1, \eta_3, \eta_4, \eta_{1\mu}$ and $\eta_{3\mu}$. We list the numerical results for the threshold s_0 , Borel mass M_B^2 , output masses, pole contributions and the two-meson thresholds in Table 3 for the scalar $sc\bar{q}\bar{b}$ system and Table 4 for the axial-vector channel. In Table 3, the extracted masses for the scalar $sc\bar{q}\bar{b}$ tetraquarks are slightly below the $B_s D$ threshold while higher than the $B_c^+ K$ threshold. However, the

Fig. 2 Variation of m_H with s_0 and M_B^2 corresponding to the current J_1 in the $J^P = 0^+$ $bc\bar{q}\bar{s}$ system

Table 1 The continuum threshold, Borel window, hadron mass and pole contribution for the $bc\bar{q}\bar{s}$ system with $J^P = 0^+$

Current	s_0 (GeV ²)	M_B^2 (GeV ²)	m_H (GeV)	PC (%)	Two-meson threshold (GeV)
J_1	59 ± 2	$5.2 \sim 6.0$	7.24 ± 0.13	35.4	
J_2	57 ± 2	$5.2 \sim 5.6$	7.14 ± 0.12	30.8	7.24
J_3	58 ± 2	$5.3 \sim 5.8$	7.18 ± 0.12	32.2	$(\bar{B}_s D)$
J_4	58 ± 2	$5.3 \sim 5.7$	7.20 ± 0.14	33.0	

Table 2 The continuum threshold, Borel window, hadron mass and pole contribution for the $bc\bar{q}\bar{s}$ system with $J^P = 1^+$

Current	s_0 (GeV ²)	M_B^2 (GeV ²)	m_H (GeV)	PC (%)	Two-meson threshold (GeV)
$J_{1\mu}$	58 ± 2	$5.3 \sim 5.8$	7.23 ± 0.13	34.4	
$J_{2\mu}$	57 ± 2	$5.3 \sim 6.0$	7.14 ± 0.12	34.0	7.28
$J_{3\mu}$	58 ± 2	$5.4 \sim 5.9$	7.17 ± 0.14	34.6	$(\bar{B}_s^* D)$
$J_{4\mu}$	59 ± 2	$5.4 \sim 5.9$	7.23 ± 0.13	35.2

Fig. 3 The OPE behavior for the current η_1 in the $J^P = 0^+$ $sc\bar{q}\bar{b}$ system

numerical results in Table 4 show that the axial-vector $sc\bar{q}\bar{b}$ tetraquarks lie below both the $B_s^* D$ and $B_c^+ K^*$ thresholds.

5 Conclusion

We have investigated the mass spectra for the fully open-flavored $bc\bar{q}\bar{s}$ and $sc\bar{q}\bar{b}$ tetraquark states in the framework of QCD sum rules. We construct the interpolating tetraquark currents with $J^P = 0^+$ and 1^+ and calculate their two-point correlation functions and spectral densities up to dimension eight condensates at the leading order of α_s .

For the $bc\bar{q}\bar{s}$ tetraquark states, we find that the quark condensate $\langle \bar{q}q \rangle$ and quark-gluon mixed condensate $\langle \bar{q}g_s \sigma \cdot Gq \rangle$ are proportional to the light quark mass and thus numerically small. The dominant nonperturbative contribution to the correlation function comes from the four-quark condensate $\langle \bar{q}q \rangle \langle \bar{s}s \rangle$. The OPE series are very different for the $sc\bar{q}\bar{b}$ tetraquark systems, where the quark condensate and quark-gluon mixed condensate provide more important contribution than the four-quark condensate. Such difference leads

Fig. 4 Variation of m_H with s_0 and M_B^2 corresponding to the current η_1 in the $J^P = 0^+$ $sc\bar{q}\bar{b}$ system

Table 3 The continuum threshold, Borel window, mass and pole contribution for the $sc\bar{q}\bar{b}$ system with $J^P = 0^+$

Current	s_0 (GeV ²)	M_B^2 (GeV ²)	m_H (GeV)	PC (%)	Two-meson threshold (GeV)
η_1	55 ± 2	$7.6 \sim 7.9$	7.10 ± 0.11	10.8	
η_2	—	—	—	—	$6.77 (B_c^+ K)$
η_3	54 ± 2	$6.4 \sim 7.3$	7.01 ± 0.12	12.8	$7.24 (B_s D)$
η_4	56 ± 2	$6.4 \sim 7.7$	7.13 ± 0.12	14.8	

Table 4 The continuum threshold, Borel window, mass and pole contribution for the $sc\bar{q}\bar{b}$ system with $J^P = 1^+$

Current	s_0 (GeV ²)	M_B^2 (GeV ²)	m_H (GeV)	PC (%)	Two-meson threshold (GeV)
$\eta_{1\mu}$	55 ± 2	$7.7 \sim 7.9$	7.09 ± 0.12	11.2	
$\eta_{2\mu}$	—	—	—	—	$7.17 (B_c^+ K^*)$
$\eta_{3\mu}$	54 ± 2	$7.6 \sim 7.8$	7.03 ± 0.12	10.5	$7.28 (B_s^* D)$
$\eta_{4\mu}$	—	—	—	—	

to distinct behavior of the mass sum rules between the $bc\bar{q}\bar{s}$ and $sc\bar{q}\bar{b}$ tetraquark systems.

After the numerical analyses, we extract the masses around 7.1 – 7.2 GeV for both the scalar and axial-vector $bc\bar{q}\bar{s}$ tetraquark states while 7.0 – 7.1 GeV for the $sc\bar{q}\bar{b}$ tetraquarks. These results show that the masses of the $bc\bar{q}\bar{s}$ tetraquark states are below the $\bar{B}_s D$ and $\bar{B}_s^* D$ two-meson S-wave thresholds, which are consistent with the results from the color-magnetic interaction model [21]. For the axial-vector $sc\bar{q}\bar{b}$ tetraquarks, their masses are also lower than the two-meson thresholds of $B_c^+ K^*$ and $B_s^* D$ modes. Such results indicate that the two-meson strong decay modes are kinematically forbidden for these possible tetraquark states. They can only decay via the weak interaction if they do exist. For the scalar $sc\bar{q}\bar{b}$ tetraquark state, their decay to the $B_s D$ final states is also forbidden, but the $B_c^+ K$ decay mode is allowed due to their slightly higher masses. However, such

decay will be difficult since the low production rate of the B_c^+ meson.

The $sc\bar{q}\bar{b}$ tetraquarks could be searched for in high energy pp collisions, while LHCb has collected an impressive amount of B_c events [37]. Similar partonic production process will be considered for the $bc\bar{q}\bar{s}$ tetraquark, which is as a fragmentation product of a jet initiated by a bc -diquark. These fully open-flavor tetraquarks may also be found in the heavy ion collisions, where plenty of heavy quarks are produced [38].

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Appendix A: The spectral densities

In this appendix, we list the spectral densities evaluated for the $bc\bar{q}\bar{s}$ and $sc\bar{q}\bar{b}$ tetraquark systems with $J^P = 0^+$ and 1^+ . The spectral density includes the perturbative term, quark condensate, gluon condensate, quark-gluon mixed condensate, four-quark condensate and dimension eight condensate

$$\rho(s) = \rho^0(s) + \rho^3(s) + \rho^4(s) + \rho^5(s) + \rho^6(s) + \rho^8(s), \quad (\text{A.1})$$

in which the superscripts stand for the dimension of various condensates.

1. The spectral densities for the $bc\bar{q}\bar{s}$ tetraquarks

1. spectral densities for J_1 :

$$\begin{aligned} \rho_{J_1}^0(s) &= \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(1-\alpha-\beta)^2(m_b^2\beta + m_c^2\alpha - \alpha\beta s)^3(m_b^2\beta + m_c^2\alpha - 3\alpha\beta s - 2m_b m_c)}{256\pi^6\alpha^3\beta^3}, \\ \rho_{J_1}^3(s) &= [m_q \langle \bar{q}q \rangle + m_s \langle \bar{s}s \rangle] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s - m_b m_c)}{16\pi^4\alpha\beta} \\ &- [m_s \langle \bar{q}q \rangle + m_q \langle \bar{s}s \rangle] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s - m_b m_c)}{8\pi^4\alpha\beta}, \\ \rho_{J_1}^{4a}(s) &= (g_s^2 GG) \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(1-\alpha-\beta)^2}{1536\pi^6} [(2m_b^2\beta + 2m_c^2\alpha - 3\alpha\beta s) \\ &\times \left(\frac{m_b^2}{\alpha^3} + \frac{m_c^2}{\beta^3} \right) \\ &- \frac{m_b m_c}{\alpha\beta} \left(\frac{4m_b^2\beta + 3m_c^2\alpha - 3\alpha\beta s}{\alpha^2} \right. \\ &\left. + \frac{3m_b^2\beta + 4m_c^2\alpha - 3\alpha\beta s}{\beta^2} \right)], \\ \rho_{J_1}^{4b}(s) &= -(g_s^2 GG) \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \end{aligned}$$

$$\begin{aligned} &\times \frac{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)}{1024\pi^6} \left[\frac{(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s - m_b m_c)}{\alpha\beta} \right. \\ &\left. + \frac{(1-\alpha-\beta)^2}{2\alpha^2\beta^2} (m_b^2\beta + m_c^2\alpha - 2\alpha\beta s - 2m_b m_c) \right], \end{aligned}$$

$$\begin{aligned} \rho_{J_1}^5(s) &= -[m_q \langle \bar{s}s \rangle g_s \sigma \cdot Gs + m_s \langle \bar{q}q \rangle g_s \sigma \cdot Gq] \\ &\times \frac{s - (m_b - m_c)^2}{64\pi^4} \sqrt{\left(1 + \frac{m_c^2 - m_b^2}{s}\right)^2 - \frac{4m_b^2}{s}} \\ &+ [m_s \langle \bar{s}s \rangle g_s \sigma \cdot Gs + m_q \langle \bar{q}q \rangle g_s \sigma \cdot Gq] \\ &\times \frac{s - (m_b - m_c)^2}{192\pi^4} \\ &\times \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}}, \\ \rho_{J_1}^6(s) &= [\langle \bar{q}q \rangle \langle \bar{s}s \rangle] \\ &\times \frac{s - (m_b - m_c)^2}{12\pi^2} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}}, \\ \rho_{J_1}^8(s) &= [\langle \bar{q}q \rangle \langle \bar{s}s \rangle g_s \sigma \cdot Gs \\ &+ \langle \bar{s}s \rangle \langle \bar{q}q \rangle g_s \sigma \cdot Gq] \int_0^1 d\alpha \frac{1}{24\pi^2} \\ &\times \left\{ \left[\frac{m_b m_c^3 + m_c^4\alpha + m_b^2 m_c^2(1-\alpha)}{(1-\alpha)^2} \right] \right. \\ &\times \delta' \left[s - \frac{m_b^2\alpha + m_c^2(1-\alpha)}{\alpha(1-\alpha)} \right] \\ &+ \frac{m_b^2(1-\alpha) + 2m_c^2\alpha}{(1-\alpha)} \\ &\times \delta \left[s - \frac{m_b^2(1-\alpha) + m_c^2\alpha}{\alpha(1-\alpha)} \right] \\ &\left. + 2\alpha H \left[s - \frac{m_b^2(1-\alpha) + m_c^2\alpha}{\alpha(1-\alpha)} \right] \right\}, \end{aligned}$$

where

$$\begin{aligned} \alpha_{\min} &= \frac{1}{2} \left[1 + \frac{m_b^2 - m_c^2}{s} \right. \\ &\left. - \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}} \right], \\ \alpha_{\max} &= \frac{1}{2} \left[1 + \frac{m_b^2 - m_c^2}{s} \right. \end{aligned}$$

$$+ \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}},$$

$$\beta_{\min} = \frac{\alpha m_c^2}{\alpha s - m_b^2}, \quad \beta_{\max} = 1 - \alpha, \quad (\text{A.2})$$

δ and H denote the Dirac delta and Heavyside theta function, respectively.

2. Spectral densities for J_2 :

$$\begin{aligned} \rho_{J_2}^0(s) &= \frac{1}{2} \rho_{J_1}^0(s), \quad \rho_{J_2}^3(s) = \frac{1}{2} \rho_{J_1}^3(s), \\ \rho_{J_2}^{4a}(s) &= \frac{1}{2} \rho_{J_1}^{4a}(s), \quad \rho_{J_2}^{4b}(s) = -\rho_{J_1}^{4b}(s), \\ \rho_{J_2}^5(s) &= \frac{1}{2} \rho_{J_1}^5(s), \quad \rho_{J_2}^6(s) = \frac{1}{2} \rho_{J_1}^6(s), \\ \rho_{J_2}^8(s) &= \frac{1}{2} \rho_{J_1}^8(s). \end{aligned}$$

3. Spectral densities for J_3 :

$$\begin{aligned} \rho_{J_3}^0(s) &= \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(1 - \alpha - \beta)^2 (m_b^2 \beta + m_c^2 \alpha - \alpha \beta s)^3 (m_b^2 \beta + m_c^2 \alpha - 3\alpha \beta s - m_b m_c)}{64\pi^6 \alpha^3 \beta^3}, \\ \rho_{J_3}^3(s) &= [m_q \langle \bar{q} q \rangle + m_s \langle \bar{s} s \rangle] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(m_b^2 \beta + m_c^2 \alpha - \alpha \beta s) (2m_b^2 \beta + 2m_c^2 \alpha - 4\alpha \beta s - m_b m_c)}{8\pi^4 \alpha \beta} \\ &- [m_s \langle \bar{q} q \rangle + m_q \langle \bar{s} s \rangle] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(m_b^2 \beta + m_c^2 \alpha - \alpha \beta s) (m_b^2 \beta + m_c^2 \alpha - 2\alpha \beta s - 2m_b m_c)}{4\pi^4 \alpha \beta}, \\ \rho_{J_3}^{4a}(s) &= \langle g_s^2 G G \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1 - \alpha - \beta)^2}{384\pi^6} \\ &\times \left[(2m_b^2 \beta + 2m_c^2 \alpha - 3\alpha \beta s) \left(\frac{m_b^2}{\alpha^3} + \frac{m_c^2}{\beta^3} \right) \right. \\ &\quad \left. - \frac{m_b m_c}{2\alpha \beta} \left(\frac{4m_b^2 \beta + 3m_c^2 \alpha - 3\alpha \beta s}{\alpha^2} \right) \right. \\ &\quad \left. + \frac{3m_b^2 \beta + 4m_c^2 \alpha - 3\alpha \beta s}{\beta^2} \right], \\ \rho_{J_3}^{4b}(s) &= -\langle g_s^2 G G \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(m_b^2 \beta + m_c^2 \alpha - \alpha \beta s)}{512\pi^6} \frac{m_b m_c}{\alpha \beta}, \\ \rho_{J_3}^5(s) &= -[m_q \langle \bar{s} g_s \sigma \cdot G s \rangle + m_s \langle \bar{q} g_s \sigma \cdot G q \rangle] \\ &\times \frac{s - (m_b - m_c)^2 + 2m_b m_c}{32\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}} \\ &+ [m_s \langle \bar{s} g_s \sigma \cdot G s \rangle + m_q \langle \bar{q} g_s \sigma \cdot G q \rangle] \end{aligned}$$

$$\begin{aligned} &\times \frac{s - (m_b - m_c)^2 - m_b m_c}{48\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}}, \\ \rho_{J_3}^6(s) &= [\langle \bar{q} q \rangle \langle \bar{s} s \rangle] \\ &\times \frac{s - (m_b - m_c)^2 + 2m_b m_c}{6\pi^2} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}}, \\ \rho_{J_3}^8(s) &= [\langle \bar{q} q \rangle \langle \bar{s} g_s \sigma \cdot G s \rangle + \langle \bar{s} s \rangle \langle \bar{q} g_s \sigma \cdot G q \rangle] \int_0^1 d\alpha \\ &\times \frac{1}{12\pi^2} \left\{ \left[\frac{2m_b m_c^3 + m_c^4 \alpha + m_b^2 m_c^2 (1 - \alpha)}{(1 - \alpha)^2} \right] \right. \\ &\times \delta' \left[s - \frac{m_b^2 (1 - \alpha) + m_c^2 \alpha}{\alpha (1 - \alpha)} \right] \\ &+ \frac{m_b^2 (1 - \alpha) + 2m_c^2 \alpha}{(1 - \alpha)} \delta \left[s - \frac{m_b^2 (1 - \alpha) + m_c^2 \alpha}{\alpha (1 - \alpha)} \right] \\ &\left. + 2\alpha H \left[s - \frac{m_b^2 (1 - \alpha) + m_c^2 \alpha}{\alpha (1 - \alpha)} \right] \right\}. \end{aligned}$$

4. Spectral densities for J_4 :

$$\begin{aligned} \rho_{J_4}^0(s) &= \frac{1}{2} \rho_{J_3}^0(s), \quad \rho_{J_4}^3(s) = \frac{1}{2} \rho_{J_3}^3(s), \\ \rho_{J_4}^{4a}(s) &= \frac{1}{2} \rho_{J_3}^{4a}(s), \quad \rho_{J_4}^{4b}(s) = -\rho_{J_3}^{4b}(s), \\ \rho_{J_4}^5(s) &= \frac{1}{2} \rho_{J_3}^5(s), \quad \rho_{J_4}^6(s) = \frac{1}{2} \rho_{J_3}^6(s), \\ \rho_{J_4}^8(s) &= \frac{1}{2} \rho_{J_3}^8(s). \end{aligned}$$

5. Spectral densities for $J_{1\mu}$:

$$\begin{aligned} \rho_{J_{1\mu}}^0(s) &= \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(1 - \alpha - \beta)^2 (m_b^2 \beta + m_c^2 \alpha - \alpha \beta s)^3 (m_b^2 \beta + m_c^2 \alpha - 5\alpha \beta s - 4m_b m_c)}{512\pi^6 \alpha^3 \beta^3}, \\ \rho_{J_{1\mu}}^3(s) &= [m_q \langle \bar{q} q \rangle + m_s \langle \bar{s} s \rangle] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(m_b^2 \beta + m_c^2 \alpha - \alpha \beta s) (m_b^2 \beta + m_c^2 \alpha - 3\alpha \beta s - 2m_b m_c)}{32\pi^4 \alpha \beta} \\ &- [m_s \langle \bar{q} q \rangle + m_q \langle \bar{s} s \rangle] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(m_b^2 \beta + m_c^2 \alpha - \alpha \beta s) (m_b^2 \beta + m_c^2 \alpha - 3\alpha \beta s - 2m_b m_c)}{16\pi^4 \alpha \beta}, \\ \rho_{J_{1\mu}}^{4a}(s) &= \langle g_s^2 G G \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(1 - \alpha - \beta)^2}{1536\pi^6} \\ &\times \left[(m_b^2 \beta + m_c^2 \alpha - 2\alpha \beta s) \left(\frac{m_b^2}{\alpha^3} + \frac{m_c^2}{\beta^3} \right) \right. \\ &\quad \left. - \frac{m_b m_c}{\alpha \beta} \left(\frac{4m_b^2 \beta + 3m_c^2 \alpha - 3\alpha \beta s}{\alpha^2} \right) \right. \\ &\quad \left. + \frac{3m_b^2 \beta + 4m_c^2 \alpha - 3\alpha \beta s}{\beta^2} \right], \\ \rho_{J_{1\mu}}^{4b}(s) &= -\langle g_s^2 G G \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\times \frac{(m_b^2 \beta + m_c^2 \alpha - \alpha \beta s)}{2048\pi^6} \end{aligned}$$

$$\times \left[\frac{(m_b^2\beta + m_c^2\alpha - 3\alpha\beta s - 2m_b m_c)}{\alpha\beta} - \frac{(1-\alpha-\beta)^2}{6\alpha^2\beta^2} (3m_b^2\beta + 3m_c^2\alpha - 5\alpha\beta s) \right],$$

$$\begin{aligned} \rho_{J_{1\mu}}^{5a}(s) = & - \left[m_q \langle \bar{s} g_s \sigma \cdot G s \rangle + m_s \langle \bar{q} g_s \sigma \cdot G q \rangle \right] \\ & \times \frac{[(m_b + m_c)^2 + 2s][s - (m_b - m_c)^2]}{192\pi^4 s} \\ & \times \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}} \\ & + \left[m_s \langle \bar{s} g_s \sigma \cdot G s \rangle + m_q \langle \bar{q} g_s \sigma \cdot G q \rangle \right] \\ & \times \frac{[(m_b + m_c)^2 + 2s][s - (m_b - m_c)^2]}{576\pi^4 s} \\ & \times \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}}, \\ \rho_{J_{1\mu}}^6(s) = & \left[\langle \bar{q} q \rangle \langle \bar{s} s \rangle \right] \\ & \times \frac{[(m_b + m_c)^2 + 2s][s - (m_b - m_c)^2]}{36\pi^2 s} \\ & \times \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}}, \\ \rho_{J_{1\mu}}^8(s) = & \left[\langle \bar{q} q \rangle \langle \bar{s} g_s \sigma \cdot G s \rangle + \langle \bar{s} s \rangle \right. \\ & \times \langle \bar{q} g_s \sigma \cdot G q \rangle \left. \right] \int_0^1 d\alpha \frac{1}{24\pi^2} \\ & \times \left\{ \left[\frac{m_b m_c^3 + m_c^4 \alpha + m_b^2 m_c^2 (1-\alpha)}{(1-\alpha)^2} \right] \right. \\ & \times \delta' \left[s - \frac{m_b^2 (1-\alpha) + m_c^2 \alpha}{\alpha(1-\alpha)} \right] \\ & + \frac{m_b^2 (1-\alpha) + m_c^2 \alpha}{(1-\alpha)} \delta \left[s - \frac{m_b^2 (1-\alpha) + m_c^2 \alpha}{\alpha(1-\alpha)} \right] \\ & \left. + \alpha H \left[s - \frac{m_b^2 (1-\alpha) + m_c^2 \alpha}{\alpha(1-\alpha)} \right] \right\}. \end{aligned}$$

6. Spectral densities for $J_{2\mu}$:

$$\begin{aligned} \rho_{J_{2\mu}}^0(s) = & \frac{1}{2} \rho_{J_{1\mu}}^0(s), \quad \rho_{J_{2\mu}}^3(s) = \frac{1}{2} \rho_{J_{1\mu}}^3(s), \\ \rho_{J_{2\mu}}^{4a}(s) = & \frac{1}{2} \rho_{J_{1\mu}}^{4a}(s), \quad \rho_{J_{2\mu}}^{4b}(s) = -\rho_{J_{1\mu}}^{4b}(s), \\ \rho_{J_{2\mu}}^5(s) = & \frac{1}{2} \rho_{J_{1\mu}}^5(s), \quad \rho_{J_{2\mu}}^6(s) = \frac{1}{2} \rho_{J_{1\mu}}^6(s), \\ \rho_{J_{2\mu}}^8(s) = & \frac{1}{2} \rho_{J_{1\mu}}^8(s). \end{aligned}$$

7. Spectral densities for $J_{3\mu}$:

$$\begin{aligned} \rho_{J_{3\mu}}^0(s) = & \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ & \times \frac{(1-\alpha-\beta)^2 (m_b^2\beta + m_c^2\alpha - \alpha\beta s)^3}{1536\pi^6 \alpha^3 \beta^3} \\ & \times \left[6(m_b^2\beta + m_c^2\alpha - 3\alpha\beta s - 2m_b m_c) \right. \\ & \left. - (1-\alpha-\beta)(3m_b^2\beta + 3m_c^2\alpha - 7\alpha\beta s - 4m_b m_c) \right], \\ \rho_{J_{3\mu}}^3(s) = & \left[m_q \langle \bar{q} q \rangle + m_s \langle \bar{s} s \rangle \right] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ & \times \frac{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)}{16\pi^4 \alpha \beta} \left[(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s - m_b m_c) \right. \\ & \left. - \frac{1}{2}(1-\alpha-\beta)(3m_b^2\beta + 3m_c^2\alpha - 5\alpha\beta s - 2m_b m_c) \right] \\ & - \left[m_s \langle \bar{q} q \rangle + m_q \langle \bar{s} s \rangle \right] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ & \times \frac{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)}{8\pi^4 \alpha \beta} \left[(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s - m_b m_c) \right], \\ \rho_{J_{3\mu}}^{4a}(s) = & \langle g_s^2 G G \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ & \times \frac{(1-\alpha-\beta)^2}{4608\pi^6} \left\{ [3(2m_b^2\beta + 2m_c^2\alpha - 3\alpha\beta s) \right. \\ & - (1-\alpha-\beta)(3m_b^2\beta + 3m_c^2\alpha - 4\alpha\beta s)] \left(\frac{m_b^2}{\alpha^3} + \frac{m_c^2}{\beta^3} \right) \right. \\ & - \frac{m_b m_c (\alpha + \beta + 2)}{\alpha \beta} \left(\frac{4m_b^2\beta + 3m_c^2\alpha - 3\alpha\beta s}{\alpha^2} \right. \\ & \left. \left. + \frac{3m_b^2\beta + 4m_c^2\alpha - 3\alpha\beta s}{\beta^2} \right) \right\}, \\ \rho_{J_{3\mu}}^{4b}(s) = & \langle g_s^2 G G \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ & \times \frac{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)}{12288\pi^6} \left\{ \frac{2}{\alpha \beta} [2(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s - m_b m_c) \right. \\ & + (1-\alpha-\beta)(3m_b^2\beta + 3m_c^2\alpha - 5\alpha\beta s - 2m_b m_c)] \\ & - \frac{(1-\alpha-\beta)^2}{\alpha^2 \beta^2} [6(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s - 2m_b m_c) \\ & - (1-\alpha-\beta)(3m_b^2\beta + 3m_c^2\alpha - 5\alpha\beta s - 4m_b m_c)] \right\}, \\ \rho_{J_{3\mu}}^{5a}(s) = & - \left[m_q \langle \bar{s} g_s \sigma \cdot G s \rangle + m_s \langle \bar{q} g_s \sigma \cdot G q \rangle \right] \\ & \times \frac{s - (m_b - m_c)^2}{64\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}} \\ & + \left[m_s \langle \bar{s} g_s \sigma \cdot G s \rangle + m_q \langle \bar{q} g_s \sigma \cdot G q \rangle \right] \\ & \times \frac{s - (m_b - m_c)^2}{192\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}}, \\ \rho_{J_{3\mu}}^{5b}(s) = & \left[m_s \langle \bar{s} g_s \sigma \cdot G s \rangle + m_q \langle \bar{q} g_s \sigma \cdot G q \rangle \right] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ & \times \frac{(3m_b^2\beta + 3m_c^2\alpha - 4\alpha\beta s - m_b m_c)}{96\pi^4}, \end{aligned}$$

$$\begin{aligned}\rho_{J_{3\mu}}^6(s) &= [\langle \bar{q}q \rangle \langle \bar{s}s \rangle] \frac{s - (m_b - m_c)^2}{12\pi^2} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}}, \\ \rho_{J_{3\mu}}^8(s) &= [\langle \bar{q}q \rangle \langle \bar{s}s \sigma \cdot Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}q \sigma \cdot Gq \rangle] \int_0^1 d\alpha \\ &\quad \times \frac{1}{24\pi^2} \left\{ \left[\frac{m_b m_c^3 + m_c^4 \alpha + m_b^2 m_c^2 (1 - \alpha)}{(1 - \alpha)^2} \right] \right. \\ &\quad \times \delta' \left[s - \frac{m_b^2 (1 - \alpha) + m_c^2 \alpha}{\alpha (1 - \alpha)} \right] \\ &\quad + \frac{m_b^2 (1 - \alpha) + 2m_c^2 \alpha}{(1 - \alpha)} \delta \left[s - \frac{m_b^2 (1 - \alpha) + m_c^2 \alpha}{\alpha (1 - \alpha)} \right] \\ &\quad \left. + 2\alpha H \left[s - \frac{m_b^2 (1 - \alpha) + m_c^2 \alpha}{\alpha (1 - \alpha)} \right] \right\}.\end{aligned}$$

8. Spectral densities for $J_{4\mu}$:

$$\begin{aligned}\rho_{J_{4\mu}}^0(s) &= \frac{1}{2} \rho_{J_{3\mu}}^0(s), \quad \rho_{J_{4\mu}}^3(s) = \frac{1}{2} \rho_{J_{3\mu}}^3(s), \\ \rho_{J_{4\mu}}^{4a}(s) &= \frac{1}{2} \rho_{J_{3\mu}}^{4a}(s), \quad \rho_{J_{4\mu}}^{4b}(s) = -\rho_{J_{3\mu}}^{4b}(s), \\ \rho_{J_{4\mu}}^{5a}(s) &= \frac{1}{2} \rho_{J_{3\mu}}^{5a}(s), \quad \rho_{J_{4\mu}}^{5b}(s) = \frac{1}{2} \rho_{J_{3\mu}}^{5b}(s), \\ \rho_{J_{4\mu}}^6(s) &= \frac{1}{2} \rho_{J_{3\mu}}^6(s), \quad \rho_{J_{4\mu}}^8(s) = \frac{1}{2} \rho_{J_{3\mu}}^8(s).\end{aligned}$$

2. The spectral densities for $sc\bar{q}\bar{b}$ tetraquarks

1. Spectral densities for η_1 :

$$\begin{aligned}\rho_{\eta_1}^0(s) &= \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \frac{(1 - \alpha - \beta)^2 (m_b^2 \beta + m_c^2 \alpha - \alpha \beta s)^2}{256\pi^6} \\ &\quad \left[\frac{1}{\alpha^3 \beta^3} (m_b^2 \beta + m_c^2 \alpha - \alpha \beta s) \right. \\ &\quad \times (m_b^2 \beta + m_c^2 \alpha - 3\alpha \beta s) - \left(\frac{2m_b m_q}{\alpha^3 \beta^2} \right. \\ &\quad \left. \left. + \frac{2m_c m_s}{\alpha^2 \beta^3} \right) (2m_b^2 \beta + 2m_c^2 \alpha - 5\alpha \beta s) \right], \\ \rho_{\eta_1}^3(s) &= [m_q \langle \bar{q}q \rangle + m_s \langle \bar{s}s \rangle] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \frac{(m_b^2 \beta + m_c^2 \alpha - \alpha \beta s)(m_b^2 \beta + m_c^2 \alpha - 2\alpha \beta s)}{16\pi^4 \alpha \beta} \\ &\quad + [m_s \langle \bar{q}q \rangle + m_q \langle \bar{s}s \rangle] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \frac{(m_b^2 \beta + m_c^2 \alpha - \alpha \beta s)m_b m_c}{8\pi^4 \alpha \beta} \\ &\quad - m_b \langle \bar{q}q \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \frac{(1 - \alpha - \beta)(m_b^2 \beta + m_c^2 \alpha - \alpha \beta s)(m_b^2 \beta + m_c^2 \alpha - 2\alpha \beta s)}{8\pi^4 \alpha^2 \beta} \\ &\quad - m_c \langle \bar{s}s \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta\end{aligned}$$

$$\begin{aligned}&\times \frac{(1 - \alpha - \beta)(m_b^2 \beta + m_c^2 \alpha - \alpha \beta s)(m_b^2 \beta + m_c^2 \alpha - 2\alpha \beta s)}{8\pi^4 \alpha \beta^2}, \\ \rho_{\eta_1}^{4a}(s) &= \langle g_s^2 G G \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \frac{(1 - \alpha - \beta)^2 (2m_b^2 \beta + 2m_c^2 \alpha - 3\alpha \beta s)}{1536\pi^6} \left(\frac{m_b^2}{\alpha^3} + \frac{m_c^2}{\beta^3} \right),\end{aligned}$$

$$\begin{aligned}\rho_{\eta_1}^{4b}(s) &= -\langle g_s^2 G G \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \frac{(1 - \alpha - \beta)(m_b^2 \beta + m_c^2 \alpha - \alpha \beta s)(m_b^2 \beta + m_c^2 \alpha - 2\alpha \beta s)}{1024\pi^6 \alpha \beta} \\ &\quad \times \left(\frac{1}{\alpha} + \frac{1}{\beta} \right),\end{aligned}$$

$$\begin{aligned}\rho_{\eta_1}^{5a}(s) &= m_b \langle \bar{q}g_s \sigma \cdot Gq \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \frac{2m_b^2 \beta + 2m_c^2 \alpha - 3\alpha \beta s}{32\pi^4 \alpha} \\ &\quad + m_c \langle \bar{s}g_s \sigma \cdot Gs \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \frac{2m_b^2 \beta + 2m_c^2 \alpha - 3\alpha \beta s}{32\pi^4 \beta},\end{aligned}$$

$$\begin{aligned}\rho_{\eta_1}^{5b}(s) &= \langle \bar{q}g_s \sigma \cdot Gq \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \left[\frac{m_b(1 - \alpha - \beta)(2m_b^2 \beta + 2m_c^2 \alpha - 3\alpha \beta s)}{64\pi^4 \alpha^2} - \frac{m_b m_c m_s}{64\pi^4 \alpha} \right] \\ &\quad + \langle \bar{s}g_s \sigma \cdot Gs \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\quad \times \left[\frac{m_c(1 - \alpha - \beta)(2m_b^2 \beta + 2m_c^2 \alpha - 3\alpha \beta s)}{64\pi^4 \beta^2} - \frac{m_b m_c m_q}{64\pi^4 \beta} \right],\end{aligned}$$

$$\begin{aligned}\rho_{\eta_1}^{5c}(s) &= [m_q \langle \bar{q}g_s \sigma \cdot Gq \rangle + m_s \langle \bar{s}g_s \sigma \cdot Gs \rangle] \\ &\quad \times \frac{s - m_b^2 - m_c^2}{192\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}} \\ &\quad - [m_q \langle \bar{s}g_s \sigma \cdot Gs \rangle + m_s \langle \bar{q}g_s \sigma \cdot Gq \rangle] \\ &\quad \times \frac{m_b m_c}{32\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}},\end{aligned}$$

$$\begin{aligned}\rho_{\eta_1}^6(s) &= \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{12\pi^2} \left[2m_b m_c + m_b m_s + m_c m_q \right. \\ &\quad \left. + \frac{m_b^2 m_c m_q (s - m_b^2 + m_c^2) + m_b m_c^2 m_s (s + m_b^2 - m_c^2)}{(s - m_b^2 + m_c^2)^2 - 4m_c^2 s} \right] \\ &\quad \times \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s}\right)^2 - \frac{4m_b^2}{s}},\end{aligned}$$

$$\begin{aligned}\rho_{\eta_1}^{8a}(s) &= [\langle \bar{q}q \rangle \langle \bar{s}s \sigma \cdot Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}q \sigma \cdot Gq \rangle] \int_0^1 d\alpha \\ &\quad \times \frac{1}{24\pi^2} \frac{m_b m_c^3}{(1 - \alpha)^2} \delta' \left[s - \frac{m_b^2 (1 - \alpha) + m_c^2 \alpha}{\alpha (1 - \alpha)} \right],\end{aligned}$$

$$\rho_{\eta_1}^{8b}(s) = \langle \bar{q}q \rangle \langle \bar{s}s \sigma \cdot Gs \rangle \int_0^1 d\alpha$$

$$\begin{aligned} & \times \frac{m_b m_c}{48\pi^2(1-\alpha)} \delta \left[s - \frac{m_b^2(1-\alpha) + m_c^2\alpha}{\alpha(1-\alpha)} \right] \\ & + \langle \bar{s}s \rangle \langle \bar{q} g_s \sigma \cdot G q \rangle \int_0^1 d\alpha \\ & \times \frac{m_b m_c}{48\pi^2\alpha} \delta \left[s - \frac{m_b^2(1-\alpha) + m_c^2\alpha}{\alpha(1-\alpha)} \right]. \end{aligned}$$

2. Spectral densities for η_2 :

$$\begin{aligned} \rho_{\eta_2}^0(s) &= \frac{1}{2} \rho_{\eta_1}^0(s), \quad \rho_{\eta_2}^3(s) = \frac{1}{2} \rho_{\eta_1}^3(s), \\ \rho_{\eta_2}^{4a}(s) &= \frac{1}{2} \rho_{\eta_1}^{4a}(s), \quad \rho_{\eta_2}^{4b}(s) = -\rho_{\eta_1}^{4b}(s), \\ \rho_{\eta_2}^{5a}(s) &= \frac{1}{2} \rho_{\eta_1}^{5a}(s), \quad \rho_{\eta_2}^{5b}(s) = -\rho_{\eta_1}^{5b}(s), \\ \rho_{\eta_2}^{5c}(s) &= \frac{1}{2} \rho_{\eta_1}^{5c}(s), \quad \rho_{\eta_2}^6(s) = \frac{1}{2} \rho_{\eta_1}^6(s), \\ \rho_{\eta_2}^{8a}(s) &= \frac{1}{2} \rho_{\eta_1}^{8a}(s), \quad \rho_{\eta_2}^{8b}(s) = -\rho_{\eta_1}^{8b}(s). \end{aligned}$$

3. Spectral densities for η_3 :

$$\begin{aligned} \rho_{\eta_3}^0(s) &= \frac{\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta}{(1-\alpha-\beta)^2(m_b^2\beta + m_c^2\alpha - \alpha\beta s)} \\ & \times \frac{1}{\alpha^3\beta^3} (m_b^2\beta + m_c^2\alpha - \alpha\beta s) \\ & \times (m_b^2\beta + m_c^2\alpha - 3\alpha\beta s) \\ & - \left(\frac{m_b m_q}{\alpha^3\beta^2} + \frac{m_c m_s}{\alpha^2\beta^3} \right) (2m_b^2\beta + 2m_c^2\alpha - 5\alpha\beta s), \\ \rho_{\eta_3}^3(s) &= \left[m_q \langle \bar{q} q \rangle + m_s \langle \bar{s} s \rangle \right] \frac{\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta}{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)} \\ & + \left[m_s \langle \bar{q} q \rangle + m_q \langle \bar{s} s \rangle \right] \frac{\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta}{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)m_b m_c} \\ & - m_b \langle \bar{q} q \rangle \frac{\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta}{(1-\alpha-\beta)(m_b^2\beta + m_c^2\alpha - \alpha\beta s)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)} \\ & - m_c \langle \bar{s} s \rangle \frac{\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta}{(1-\alpha-\beta)(m_b^2\beta + m_c^2\alpha - \alpha\beta s)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)}, \\ \rho_{\eta_3}^{4a}(s) &= \langle g_s^2 G G \rangle \frac{\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta}{(1-\alpha-\beta)^2(2m_b^2\beta + 2m_c^2\alpha - 3\alpha\beta s)} \left(\frac{m_b^2}{\alpha^3} + \frac{m_c^2}{\beta^3} \right), \\ \rho_{\eta_3}^{4b}(s) &= \langle g_s^2 G G \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \end{aligned}$$

$$\begin{aligned} & \frac{(1-\alpha-\beta)(m_b^2\beta + m_c^2\alpha - \alpha\beta s)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)}{512\pi^6\alpha\beta} \\ & \left(\frac{1}{\alpha} + \frac{1}{\beta} \right), \\ \rho_{\eta_3}^{4c}(s) &= \langle g_s^2 G G \rangle \frac{\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta}{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)} \\ & \frac{1}{512\pi^6\alpha\beta} \\ & \left[1 + \frac{(1-\alpha-\beta)^2}{2\alpha\beta} \right], \\ \rho_{\eta_3}^{5a}(s) &= m_b \langle \bar{q} g_s \sigma \cdot G q \rangle \frac{\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta}{2m_b^2\beta + 2m_c^2\alpha - 3\alpha\beta s} \\ & \frac{1}{16\pi^4\alpha} \\ & + m_c \langle \bar{s} g_s \sigma \cdot G s \rangle \frac{\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta}{2m_b^2\beta + 2m_c^2\alpha - 3\alpha\beta s}, \\ & \frac{1}{16\pi^4\beta}, \end{aligned}$$

$$\begin{aligned} \rho_{\eta_3}^{5b}(s) &= \left[m_q \langle \bar{q} g_s \sigma \cdot G q \rangle + m_s \langle \bar{s} g_s \sigma \cdot G s \rangle \right] \\ & \frac{s - m_b^2 - m_c^2}{48\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s} \right)^2 - \frac{4m_b^2}{s}} \\ & - \left[m_q \langle \bar{s} g_s \sigma \cdot G s \rangle + m_s \langle \bar{q} g_s \sigma \cdot G q \rangle \right] \\ & \frac{m_b m_c}{8\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s} \right)^2 - \frac{4m_b^2}{s}}, \\ \rho_{\eta_3}^6(s) &= \frac{\langle \bar{q} q \rangle \langle \bar{s} s \rangle}{6\pi^2} \left[4m_b m_c + m_b m_s + m_c m_q \right. \\ & + \frac{m_b^2 m_c m_q (s - m_b^2 + m_c^2) + m_b m_c^2 m_s (s + m_b^2 - m_c^2)}{(s - m_b^2 + m_c^2)^2 - 4m_c^2 s} \\ & \times \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s} \right)^2 - \frac{4m_b^2}{s}}, \\ \rho_{\eta_3}^{8a}(s) &= \left[\langle \bar{q} q \rangle \langle \bar{s} g_s \sigma \cdot G s \rangle + \langle \bar{s} s \rangle \langle \bar{q} g_s \sigma \cdot G q \rangle \right] \\ & \int_0^1 d\alpha \frac{m_b m_c^3}{6\pi^2(1-\alpha)^2} \delta' \left[s - \frac{m_b^2(1-\alpha) + m_c^2\alpha}{\alpha(1-\alpha)} \right]. \end{aligned}$$

4. Spectral densities for η_4 :

$$\begin{aligned} \rho_{\eta_4}^0(s) &= \frac{1}{2} \rho_{\eta_3}^0(s), \quad \rho_{\eta_4}^3(s) = \frac{1}{2} \rho_{\eta_3}^3(s), \\ \rho_{\eta_4}^{4a}(s) &= \frac{1}{2} \rho_{\eta_3}^{4a}(s), \quad \rho_{\eta_4}^{4b}(s) = -\rho_{\eta_3}^{4b}(s), \\ \rho_{\eta_4}^{4c}(s) &= -\rho_{\eta_3}^{4c}(s), \quad \rho_{\eta_4}^{5a}(s) = \frac{1}{2} \rho_{\eta_3}^{5a}(s), \\ \rho_{\eta_4}^{5b}(s) &= \frac{1}{2} \rho_{\eta_3}^{5b}(s), \quad \rho_{\eta_4}^6(s) = \frac{1}{2} \rho_{\eta_3}^6(s), \\ \rho_{\eta_4}^{8a}(s) &= \frac{1}{2} \rho_{\eta_3}^{8a}(s). \end{aligned}$$

5. Spectral densities for $\eta_{1\mu}$:

$$\begin{aligned}
\rho_{\eta_{1\mu}}^0(s) &= \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \frac{(1-\alpha-\beta)^2(m_b^2\beta + m_c^2\alpha - \alpha\beta s)^2}{128\pi^6\alpha^2\beta^2} \\
&\quad \left\{ \frac{1}{12\alpha\beta} \left[6(m_b^2\beta + m_c^2\alpha - \alpha\beta s) \right. \right. \\
&\quad \times (m_b^2\beta + m_c^2\alpha - 3\alpha\beta s) - (1-\alpha-\beta)(m_b^2\beta \\
&\quad + m_c^2\alpha - \alpha\beta s)(3m_b^2\beta + 3m_c^2\alpha - 7\alpha\beta s) \\
&\quad - \left[\frac{(2m_b^2\beta + 2m_c^2\alpha - 5\alpha\beta s)m_b m_q}{\alpha} \right. \\
&\quad \left. \left. + \frac{(m_b^2\beta + m_c^2\alpha - 4\alpha\beta s)m_c m_s}{\beta} \right] \right\}, \\
\rho_{\eta_{1\mu}}^3(s) &= \left[m_q \langle \bar{q}q \rangle + m_s \langle \bar{s}s \rangle \right] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \\
&\quad \int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)}{32\pi^4\alpha\beta} [2(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s) \\
&\quad - (1-\alpha-\beta)(3m_b^2\beta + 3m_c^2\alpha - 5\alpha\beta s)] \\
&\quad + \left[m_s \langle \bar{q}q \rangle + m_q \langle \bar{s}s \rangle \right] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \frac{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)m_b m_c}{8\pi^4\alpha\beta} \\
&\quad - m_b \langle \bar{q}q \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \frac{(1-\alpha-\beta)(m_b^2\beta + m_c^2\alpha - \alpha\beta s)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)}{8\pi^4\alpha^2\beta} \\
&\quad - m_c \langle \bar{s}s \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \frac{(1-\alpha-\beta)(m_b^2\beta + m_c^2\alpha - \alpha\beta s)(m_b^2\beta + m_c^2\alpha - 3\alpha\beta s)}{16\pi^4\alpha\beta^2}, \\
\rho_{\eta_{1\mu}}^{4a}(s) &= \langle g_s^2 GG \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \frac{(1-\alpha-\beta)^2}{4608\pi^6} [3(2m_b^2\beta + 2m_c^2\alpha - 3\alpha\beta s) \\
&\quad - (1-\alpha-\beta)(3m_b^2\beta + 3m_c^2\alpha - 4\alpha\beta s)] \left(\frac{m_b^2}{\alpha^3} + \frac{m_c^2}{\beta^3} \right), \\
\rho_{\eta_{1\mu}}^{4b}(s) &= -\langle g_s^2 GG \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \frac{(1-\alpha-\beta)(m_b^2\beta + m_c^2\alpha - \alpha\beta s)}{4096\pi^6\alpha\beta} \\
&\quad \left[\frac{4(3\alpha-\beta)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)}{3\alpha\beta} \right. \\
&\quad \left. - \frac{(3\alpha+\beta)(1-\alpha-\beta)(3m_b^2\beta + 3m_c^2\alpha - 5\alpha\beta s)}{3\alpha\beta} \right], \\
\rho_{\eta_{1\mu}}^{5a}(s) &= \langle \bar{q}g_s\sigma \cdot Gq \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \left[\frac{m_b(2m_b^2\beta + 2m_c^2\alpha - 3\alpha\beta s)}{32\pi^4\alpha} \right.
\end{aligned}$$

$$\begin{aligned}
&\quad + \frac{m_q(3m_b^2\beta + 3m_c^2\alpha - 4\alpha\beta s)}{96\pi^4} \Big] \\
&\quad + \langle \bar{s}g_s\sigma \cdot Gs \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \left[\frac{m_c(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)}{32\pi^4\beta} \right. \\
&\quad \left. + \frac{m_s(3m_b^2\beta + 3m_c^2\alpha - 4\alpha\beta s)}{96\pi^4} \right], \\
\rho_{\eta_{1\mu}}^{5b}(s) &= \langle \bar{s}g_s\sigma \cdot Gs \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\
&\quad \left[\frac{m_c(1-\alpha-\beta)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)}{64\pi^4\beta^2} \right. \\
&\quad \left. - \frac{m_b m_c m_q}{64\pi^4\beta} \right], \\
\rho_{\eta_{1\mu}}^{5c}(s) &= \left[m_q \langle \bar{q}g_s\sigma \cdot Gq \rangle + m_s \langle \bar{s}g_s\sigma \cdot Gs \rangle \right] \\
&\quad \frac{s - m_b^2 - m_c^2}{192\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s} \right)^2 - \frac{4m_b^2}{s}} \\
&\quad - \left[m_q \langle \bar{s}g_s\sigma \cdot Gs \rangle + m_s \langle \bar{q}g_s\sigma \cdot Gq \rangle \right] \\
&\quad \frac{m_b m_c}{32\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s} \right)^2 - \frac{4m_b^2}{s}}, \\
\rho_{\eta_{1\mu}}^6(s) &= \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{12\pi^2} \\
&\quad \left[2m_b m_c + m_b m_s + m_c m_q \right. \\
&\quad + \frac{m_b^2 m_c m_q (s - m_b^2 + m_c^2) + m_b m_c^2 m_s (s + m_b^2 - m_c^2)}{(s - m_b^2 + m_c^2)^2 - 4m_c^2 s} \\
&\quad \left. - \frac{m_c m_q (s + m_b^2 - m_c^2)}{2s} \right] \\
&\quad \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s} \right)^2 - \frac{4m_b^2}{s}}, \\
\rho_{\eta_{1\mu}}^{8a}(s) &= \left[\langle \bar{q}q \rangle \langle \bar{s}g_s\sigma \cdot Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s\sigma \cdot Gq \rangle \right] \\
&\quad \int_0^1 d\alpha \frac{1}{24\pi^2} \frac{m_b m_c^3}{(1-\alpha)^2} \delta' \left[s - \frac{m_b^2(1-\alpha) + m_c^2\alpha}{\alpha(1-\alpha)} \right], \\
\rho_{\eta_{1\mu}}^{8b}(s) &= \langle \bar{q}q \rangle \langle \bar{s}g_s\sigma \cdot Gs \rangle \int_0^1 d\alpha \\
&\quad \frac{m_b m_c}{48\pi^2(1-\alpha)} \delta \left[s - \frac{m_b^2(1-\alpha) + m_c^2\alpha}{\alpha(1-\alpha)} \right].
\end{aligned}$$

6. Spectral densities for $\eta_{2\mu}$:

$$\begin{aligned}
\rho_{\eta_{2\mu}}^0(s) &= \frac{1}{2} \rho_{\eta_{1\mu}}^0(s), \quad \rho_{\eta_{2\mu}}^3(s) = \frac{1}{2} \rho_{\eta_{1\mu}}^3(s), \\
\rho_{\eta_{2\mu}}^{4a}(s) &= \frac{1}{2} \rho_{\eta_{1\mu}}^{4a}(s), \quad \rho_{\eta_{2\mu}}^{4b}(s) = -\rho_{\eta_{1\mu}}^{4b}(s), \\
\rho_{\eta_{2\mu}}^{5a}(s) &= \frac{1}{2} \rho_{\eta_{1\mu}}^{5a}(s), \quad \rho_{\eta_{2\mu}}^{5b}(s) = -\rho_{\eta_{1\mu}}^{5b}(s), \\
\rho_{\eta_{2\mu}}^{5c}(s) &= \frac{1}{2} \rho_{\eta_{1\mu}}^{5c}(s), \quad \rho_{\eta_{2\mu}}^6(s) = \frac{1}{2} \rho_{\eta_{1\mu}}^6(s),
\end{aligned}$$

$$\rho_{\eta_{2\mu}}^{8a}(s) = \frac{1}{2} \rho_{\eta_{1\mu}}^{8a}(s), \rho_{\eta_{2\mu}}^{8b}(s) = -\rho_{\eta_{1\mu}}^{8b}(s).$$

7. Spectral densities for $\eta_{3\mu}$;

$$\begin{aligned} \rho_{\eta_{3\mu}}^0(s) &= \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\frac{(1-\alpha-\beta)^2(m_b^2\beta + m_c^2\alpha - \alpha\beta s)^2}{128\pi^6\alpha^2\beta^2} \\ &\left\{ \frac{1}{12\alpha\beta} \left[6(m_b^2\beta + m_c^2\alpha - \alpha\beta s) \right. \right. \\ &\quad \times (m_b^2\beta + m_c^2\alpha - 3\alpha\beta s) - (1-\alpha-\beta)(m_b^2\beta \\ &\quad + m_c^2\alpha - \alpha\beta s)(3m_b^2\beta + 3m_c^2\alpha - 7\alpha\beta s) \left. \right] \\ &\quad - \left[\frac{(m_b^2\beta + m_c^2\alpha - 4\alpha\beta s)m_b m_q}{\alpha} \right. \\ &\quad \left. \left. + \frac{(2m_b^2\beta + 2m_c^2\alpha - 5\alpha\beta s)m_c m_s}{\beta} \right] \right\}, \\ \rho_{\eta_{3\mu}}^3(s) &= \left[m_q \langle \bar{q}q \rangle + m_s \langle \bar{s}s \rangle \right] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \\ &\int_{\beta_{\min}}^{\beta_{\max}} d\beta \frac{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)}{32\pi^4\alpha\beta} [2(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s) \\ &\quad - (1-\alpha-\beta)(3m_b^2\beta + 3m_c^2\alpha - 5\alpha\beta s)] \\ &+ \left[m_s \langle \bar{q}q \rangle + m_q \langle \bar{s}s \rangle \right] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\frac{(m_b^2\beta + m_c^2\alpha - \alpha\beta s)m_b m_c}{8\pi^4\alpha\beta} \\ &- m_b \langle \bar{q}q \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\frac{(1-\alpha-\beta)(m_b^2\beta + m_c^2\alpha - \alpha\beta s)(m_b^2\beta + m_c^2\alpha - 3\alpha\beta s)}{16\pi^4\alpha^2\beta^2} \\ &- m_c \langle \bar{s}s \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\frac{(1-\alpha-\beta)(m_b^2\beta + m_c^2\alpha - \alpha\beta s)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)}{8\pi^4\alpha\beta^2}, \\ \rho_{\eta_{3\mu}}^{4a}(s) &= \langle g_s^2 GG \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\frac{(1-\alpha-\beta)^2}{4608\pi^6} [3(2m_b^2\beta + 2m_c^2\alpha - 3\alpha\beta s) \\ &\quad - (1-\alpha-\beta)(3m_b^2\beta + 3m_c^2\alpha - 4\alpha\beta s)] \\ &\left(\frac{m_b^2}{\alpha^3} + \frac{m_c^2}{\beta^3} \right), \end{aligned}$$

$$\begin{aligned} \rho_{\eta_{3\mu}}^{4b}(s) &= -\langle g_s^2 GG \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\frac{(1-\alpha-\beta)(m_b^2\beta + m_c^2\alpha - \alpha\beta s)}{4096\pi^6\alpha\beta} \\ &\left[\frac{4(3\beta - \alpha)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)}{3\alpha\beta} \right. \\ &\quad \left. - \frac{(\alpha + 3\beta)(1-\alpha-\beta)(3m_b^2\beta + 3m_c^2\alpha - 5\alpha\beta s)}{3\alpha\beta} \right], \end{aligned}$$

$$\begin{aligned} \rho_{\eta_{3\mu}}^{5a}(s) &= \langle \bar{q}g_s\sigma \cdot Gq \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \\ &\int_{\beta_{\min}}^{\beta_{\max}} d\beta \left[\frac{m_b(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)}{32\pi^4\alpha} \right. \\ &\quad \left. + \frac{m_q(3m_b^2\beta + 3m_c^2\alpha - 4\alpha\beta s)}{96\pi^4} \right] \\ &+ \langle \bar{s}g_s\sigma \cdot Gs \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\left[\frac{m_c(2m_b^2\beta + 2m_c^2\alpha - 3\alpha\beta s)}{32\pi^4\beta} \right. \\ &\quad \left. + \frac{m_s(3m_b^2\beta + 3m_c^2\alpha - 4\alpha\beta s)}{96\pi^4} \right], \\ \rho_{\eta_{3\mu}}^{5b}(s) &= \langle \bar{q}g_s\sigma \cdot Gq \rangle \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta \\ &\left[\frac{m_b(1-\alpha-\beta)(m_b^2\beta + m_c^2\alpha - 2\alpha\beta s)}{64\pi^4\alpha^2} - \frac{m_b m_c m_s}{64\pi^4\alpha} \right], \\ \rho_{\eta_{3\mu}}^{5c}(s) &= \left[m_q \langle \bar{q}g_s\sigma \cdot Gq \rangle + m_s \langle \bar{s}g_s\sigma \cdot Gs \rangle \right] \\ &\frac{s - m_b^2 - m_c^2}{192\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s} \right)^2 - \frac{4m_b^2}{s}} \\ &- \left[m_q \langle \bar{s}g_s\sigma \cdot Gs \rangle + m_s \langle \bar{q}g_s\sigma \cdot Gq \rangle \right] \\ &\frac{m_b m_c}{32\pi^4} \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s} \right)^2 - \frac{4m_b^2}{s}}, \\ \rho_{\eta_{3\mu}}^6(s) &= \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{12\pi^2} \\ &\left[2m_b m_c + m_b m_s + m_c m_q \right. \\ &\quad \left. + \frac{m_b^2 m_c m_q (s - m_b^2 + m_c^2) + m_b m_c^2 m_s (s + m_b^2 - m_c^2)}{(s - m_b^2 + m_c^2)^2 - 4m_c^2 s} \right. \\ &\quad \left. - \frac{m_b m_s (s - m_b^2 + m_c^2)}{2s} \right] \sqrt{\left(1 + \frac{m_b^2 - m_c^2}{s} \right)^2 - \frac{4m_b^2}{s}}, \end{aligned}$$

$$\begin{aligned} \rho_{\eta_{3\mu}}^{8a}(s) &= \left[\langle \bar{q}q \rangle \langle \bar{s}g_s\sigma \cdot Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s\sigma \cdot Gq \rangle \right] \int_0^1 d\alpha \\ &\frac{1}{24\pi^2} \frac{m_b m_c^3}{(1-\alpha)^2} \delta' \left[s - \frac{m_b^2(1-\alpha) + m_c^2\alpha}{\alpha(1-\alpha)} \right], \\ \rho_{\eta_{3\mu}}^{8b}(s) &= \langle \bar{s}s \rangle \langle \bar{q}g_s\sigma \cdot Gq \rangle \int_0^1 d\alpha \\ &\frac{m_b m_c}{48\pi^2\alpha} \delta \left[s - \frac{m_b^2(1-\alpha) + m_c^2\alpha}{\alpha(1-\alpha)} \right]. \end{aligned}$$

8. Spectral densities for $\eta_{4\mu}$;

$$\begin{aligned} \rho_{\eta_{4\mu}}^0(s) &= \frac{1}{2} \rho_{\eta_{3\mu}}^0(s), \rho_{\eta_{4\mu}}^3(s) = \frac{1}{2} \rho_{\eta_{3\mu}}^3(s), \\ \rho_{\eta_{4\mu}}^{4a}(s) &= \frac{1}{2} \rho_{\eta_{3\mu}}^{4a}(s), \rho_{\eta_{4\mu}}^{4b}(s) = -\rho_{\eta_{3\mu}}^{4b}(s), \\ \rho_{\eta_{4\mu}}^{5a}(s) &= \frac{1}{2} \rho_{\eta_{3\mu}}^{5a}(s), \rho_{\eta_{4\mu}}^{5b}(s) = -\rho_{\eta_{3\mu}}^{5b}(s), \end{aligned}$$

$$\rho_{\eta_{4\mu}}^{5c}(s) = \frac{1}{2}\rho_{\eta_{3\mu}}^{5c}(s), \rho_{\eta_{4\mu}}^6(s) = \frac{1}{2}\rho_{\eta_{3\mu}}^6(s),$$

$$\rho_{\eta_{4\mu}}^{8a}(s) = \frac{1}{2}\rho_{\eta_{3\mu}}^{8a}(s), \rho_{\eta_{4\mu}}^{8b}(s) = -\rho_{\eta_{3\mu}}^{8b}(s).$$

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