Energy dependence of the inelasticity in $pp/p\bar{p}$ collisions from experimental information on charged particle multiplicity distributions

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Abstract The dependence of the inelasticity in terms of the center of mass energy is studied in the eikonal formalism, which provides connection between elastic and inelastic channels. Due to the absence of inelasticity experimental datasets, the present analysis is based on experimental information available on the full phase space multiplicity distribution covering a large range of energy, namely $30 < \sqrt{s} \leq 1800$ GeV. Our results indicate that the decrease of inelasticity is consequence of minijets production from semihard interactions arising from the scattering of gluons carrying only a very small fractions of the momenta from their parent protons. Alternative methods of estimating the inelasticity are discussed and predictions to the LHC energies are presented.

1 Introduction

In $p + p(\bar{p})$ collisions at center of mass energy, $\sqrt{s}$, the effective energy left behind by the two leading protons, or correspondingly the inelasticity K [1–4], is an essential concept because it defines the energy effectively used for producing n new secondary particles. That in turn, determines the dynamics of the interaction in high-energy hadronic and nuclear collisions. The inelasticity varies from event to event, so that one has to introduce an inelasticity distribution $\chi(K, s)$ normalized by [5]

$$\int_0^1 \chi(K, s) dK = 1. \quad (1)$$

Experimental data on K are very limited and the form of its distribution function, $\chi(K, s)$, has not yet been established. It is known as the only experimental information available on

$\chi(K, s)$ is from pp interactions at $\sqrt{s} = 16.5$ GeV, which exhibits a maximum at ~ 0.5 [6]. At the ISR energies the mean inelasticity is approximately constant with $\langle K \rangle \sim 0.5$ [7].

The energy dependence of the inelasticity is an important problem which has been subject of discussions [1, 8–13]. As example, comparing $p + p(\bar{p})$ with e^-e^+ collisions the $\sqrt{s}$ dependence of the inelasticity in $p + p(\bar{p})$ collisions was calculated in [1] for three different assumptions on the parameters involved in the analysis and the results were compared with the theoretical study from [7].

Although, as mentioned, experimental information on $K(s)$ is limited, the probability for producing n charged particles in final states $P_n(s)$, or simply multiplicity distribution, is strictly connected with the inelasticity concept [12, 13]. Thus, we can study $P_n(s)$ features in order to derive informations on the $K(s)$ behavior, since there are experimental informations available in the full space phase for $P_n(s)$ covering the interval of $30 < \sqrt{s} \leq 1800$ GeV [14–16].

With that in mind, we have studied the relation between $P_n(s)$ and $K(s)$ in the framework of a *phenomenological procedure* related to $P_n(s)$ [17, 18], as well as a formula connecting the inelasticity to the imaginary part of the eikonal function in the impact parameter b space, $\chi_I(s, b)$, which was obtained in [13]. However, in the analysis done in [13] the $P_n(s)$ dataset studied was restrict to collision energies of 52.6, 200, 546 and 900 GeV, and only a limited success was reached in describing the $P_n(s)$ data at 200 and 900 GeV. Here, however, we treat the full phase space $P_n(s)$ and $K(s)$ at $\sqrt{s} = 30.4, 44.5, 52.6, 62.2, 300, 546, 1000$ and 1800 GeV [14–16].

Since in our studies $K(s, b) \propto \chi_I(s, b)$, in [19] we have updated the eikonal formalism of the aforementioned *phenomenological procedure* in order to describe, in a connected

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way, $p + p(\bar{p})$ observables in both elastic and inelastic channels through the unitarity condition of the S-Matrix in impact parameter space. All the parameters of the eikonal function, $\chi_{pp}^{\bar{p}p}(s, b)$, were determined carrying out a global fit to all high energy forward pp and $\bar{p}p$ scattering data above $\sqrt{s} = 10$ GeV, namely the total cross section, $\sigma_{tot}^{pp, \bar{p}p}$, the ratio of the real to imaginary part of the forward scattering amplitude $\rho^{pp, \bar{p}p}$, the elastic differential scattering cross sections $d\sigma_{el}^{\bar{p}p}/dt$ at $\sqrt{s} = 546$ GeV and $\sqrt{s} = 1.8$ TeV as well as the TOTEM datum on σ_{tot}^{pp} at $\sqrt{s} = 7$ TeV. The results obtained in [19] were compared with the correspondent experimental information and also with the full phase space P_n and the H_q moments, yielding successful descriptions of all experimental data. In [20] the *phenomenological procedure* from [19] was applied to investigate the $\sqrt{s}$ dependence of the parton-parton inelastic cross sections, parton-parton inelastic overlap functions and the C_q moments in proton interactions from $\sqrt{s} = 10$ to 14.000 GeV, providing also predictions for the ratio $\sigma_{el}(s)/\sigma_{tot}(s)$ as a function of the $\sqrt{s}$, in agreement with the experimental data. Therefore, the success in that global description of elastic and inelastic hadronic observables, over wide interval of $\sqrt{s}$ [19,20], motivated us to investigate the problem of the $\sqrt{s}$ dependence of the $K(s)$ from $P_n(s)$ studies.

The main purpose of this paper is to apply the *phenomenological procedure* formalism in full phase space $P_n(s)$ from [19], also applied in [20], in order to study the energy dependence of the inelasticity based on the experimental information from $p + p(\bar{p})$ multiplicity distributions, since experimental data on $K(s)$ are very limited.

The paper is organized as follows: in the next section we discuss the main ideas associated with the *phenomenological procedure* as well as their inputs. In Sect. 3, we apply the theoretical formalism computing the inelasticity as a function of b at fixed $\sqrt{s}$, discussing the results. Inelasticity predictions to the LHC energies are made. The concluding remarks are the content of the Sect. 4.

2 Phenomenological procedure

2.1 The $P_n(s)$ model

The multiplicity distribution is defined at $\sqrt{s}$ in terms of the topological cross section, σ_n , and the inelastic cross section, σ_{in} , by the formula

$$P_n(s) = \frac{\sigma_n(s)}{\sum_n \sigma_n(s)} = \frac{\sigma_n(s)}{\sigma_{in}(s)}. \quad (2)$$

In the impact parameter formalism a normalized $P_n(s)$ may be constructed by summing contributions coming from $p + p(\bar{p})$ collisions taking place at fixed b and $\sqrt{s}$. Thus $P_n(s)$ is written as

$$P_n(s) = \frac{\sigma_n(s)}{\sigma_{in}(s)} = \frac{\int d^2b [1 - e^{-2\chi_I(s,b)}] \left[\frac{\sigma_n(s,b)}{\sigma_{in}(s,b)} \right]}{\int d^2b [1 - e^{-2\chi_I(s,b)}]} \quad (3)$$

where the $\sigma_n(s)$ is decomposed into contributions from each impact parameter b , and $\sigma_{in}(s,b) = G_{in}(s,b) = [1 - e^{-2\chi_I(s,b)}]$ is the weight function, called inelastic overlap function. As in its original formulation [17,18] the quantity in brackets scales in KNO sense and we can rewritten the last Eq. as

$$P_n(s) = \frac{\int d^2b \frac{[1 - e^{-2\chi_I(s,b)}]}{\langle n(s,b) \rangle} \left[\langle n(s,b) \rangle \frac{\sigma_n(s,b)}{\sigma_{in}(s,b)} \right]}{\int d^2b [1 - e^{-2\chi_I(s,b)}]}, \quad (4)$$

where $\langle n(s,b) \rangle$ is the average number of particles produced at b and $\sqrt{s}$ and its factorizes as [18]

$$\langle n(s,b) \rangle = \langle N(s) \rangle f(s,b). \quad (5)$$

In this equation $\langle N(s) \rangle$ is the average multiplicity at $\sqrt{s}$ and $f(s,b)$ is called multiplicity function. Similarly to KNO, it is introduced the elementary multiplicity distribution related to microscopic processes

$$\psi \left(\frac{n}{\langle n(s,b) \rangle} \right) = \langle n(s,b) \rangle \frac{\sigma_n(s,b)}{\sigma_{in}(s,b)}. \quad (6)$$

As in previous works [13,18–23], we have assumed that the particles created at $\sqrt{s}$ and b follows the KNO form of the Negative Binomial distribution, or Gamma distribution, normalized to 2

$$\psi \left(\frac{n}{\langle n(s,b) \rangle} \right) = 2 \frac{k^k}{\Gamma(k)} \left[\frac{n}{\langle n(s,b) \rangle} \right]^{k-1} e^{-k \left[\frac{n}{\langle n(s,b) \rangle} \right]} \quad (7)$$

which is characterized by the k parameter and Γ represents the usual gamma function. Its choose was motivated by the fact that this distribution arises as the dominant part of the solution of the equation for three gluon branching process in the very large n limit [24,25]. This branching equation, which takes into account only gluon bremsstrahlung process, gives the main contribution at high energies since semihard gluons dominate the parton-parton cross sections. Thus, with the Eqs. (5) and (6), the Eq. (4) becomes

$$P_n(s) = \frac{\int d^2b \frac{[1 - e^{-2\chi_I(s,b)}]}{f(s,b)} \left[\psi \left(\frac{n}{\langle N(s) \rangle f(s,b)} \right) \right]}{\langle N(s) \rangle \int d^2b [1 - e^{-2\chi_I(s,b)}]}. \quad (8)$$

Now, to define $f(s,b)$ in terms of the imaginary eikonal $\chi_I(s,b)$ we have assumed that

1. the fraction of $\sqrt{s}$, which is deposited by the two leading protons for particle production in a collision at b , represented by $\sqrt{s'}$, is proportional to $\chi_I(s,b)$:

$$\sqrt{s'} = \beta(s) \chi_I(s, b), \quad (9)$$

where $\beta(s)$ is a function to be defined.

2. The average number of produced particles depends on the $\sqrt{s'}$ at each b value in a power law form

$$\langle n(s, b) \rangle = \gamma \left(\frac{s'}{s_0} \right)^{\zeta(s)}, \quad (10)$$

where $s'_0 = 1 \text{ GeV}^2$. Substituting the Eq. (9) into (10) we obtain the energy and impact parameter dependence of $\langle n \rangle$

$$\langle n(s, b) \rangle = \frac{\gamma [\beta(s) \chi_I(s, b)]^{2\zeta(s)}}{(s'_0)^{\zeta(s)}}. \quad (11)$$

The γ parameter and the $\zeta(s)$ function will be discussed in the next subsection.

The physical motivation of the Eq. (9) is that the eikonal may be interpreted as an overlap, on the impact parameter plane, of two colliding matter distributions [26]. Physically, the Eq. (9) corresponds to the effective energy for particle production, then we can write $\sqrt{s'} \equiv E_{eff}$.

The Eq. (10) deserves a more detailed comment: a power law dependence of the multiplicity on the energy emerged in the context of statistical and hydrodynamical models. It also was successfully applied in the context of the parton model, either connecting KNO and Bjorken scaling or treating the violation of the KNO scaling and can also arise from a simple picture of branching decay producing a tree structure (see [18] and references therein). In [27] the authors reproduced the power like energy behavior of the mean multiplicity in the hadronic multiparticle production model with antishadowing, which provided estimated values of the average multiplicity over a large energy interval, in good agreement with the data and predicting multiplicities at the LHC energies. Based on the gluon saturation scenario (Color Glass Condensate approach), in [28], the authors showed that the power law energy dependence of charged hadron multiplicity leads to a very good description of the LHC experimental data in both, pp ($s^{0.11}$) and AA (nucleus–nucleus) ($s^{0.145}$) collisions, including the ALICE data in Pb + Pb collisions at 2.76 TeV and showed that this different energy dependence can be explained by inclusion of a strong angular-ordering in the gluon decay cascade. A power law behavior is characteristic of several analyses of experimental data on hadronic interactions and also several theoretical approaches. Thus, at the present stage of our studies, the power law for the multiplicity seems a hypothesis reasonable.

Matching the Eqs. (5), (9) and (10) we have

$$f(s, b) = \frac{\gamma}{\langle N(s) \rangle} \left[\frac{\beta(s)}{\sqrt{s'_0}} \right]^{2\zeta(s)} [\chi_I(s, b)]^{2\zeta(s)} \quad (12)$$

and defining $\xi(s)$ in the last Eq. as

$$\xi(s) \equiv \frac{\gamma}{\langle N(s) \rangle} \left[\frac{\beta(s)}{\sqrt{s'_0}} \right]^{2\zeta(s)} \quad (13)$$

the Eq. (12) can be written as

$$f(s, b) = \xi(s) [\chi_I(s, b)]^{2\zeta(s)}. \quad (14)$$

In turn, substituting the Eq. (14) into Eq. (8) results in

$$P_n(s) = \frac{\int d^2b \frac{[1 - e^{-2\chi_I(s, b)}]}{\xi(s) [\chi_I(s, b)]^{2\zeta(s)}} \left[\psi \left(\frac{n}{\langle N(s) \rangle \xi(s) [\chi_I(s, b)]^{2\zeta(s)}} \right) \right]}{\langle N(s) \rangle \int d^2b [1 - e^{-2\chi_I(s, b)}]}, \quad (15)$$

with $\xi(s)$ determined by the usual normalization conditions on the charged $P_n(s)$ ($\int P_n dn = \int P_n n dn = 2$), explicitly we have obtained [18]

$$\xi(s) = \frac{\int d^2b [1 - e^{-2\chi_I(s, b)}]}{\int d^2b [1 - e^{-2\chi_I(s, b)}] [\chi_I(s, b)]^{2\zeta(s)}}. \quad (16)$$

The formalism permits the calculation of the $P_n(s)$, Eq. (15), once an eikonal parametrization is assumed and appropriate values to the parameters k and $\zeta(s)$ are adjusted in order to provide reliable results concerning calculations of strongly interacting processes, as discussed in the next subsection.

The physical picture of the $P_n(s)$ is discussed in detail in [19] and asserts that the full phase space $P_n(s)$ is constructed by summing contributions from parton–parton collisions occurring at each value of b , with the formation of strings that subsequently fragments into hadrons. The idea of string formation for multiparticle production is similar to the Lund model [29].

2.2 QCD-inspired eikonal model, k , $\zeta(s)$ and γ parameters

We adopted the QCD-inspired eikonal model referred as Dynamical Gluon Mass (DGM) model [30], which incorporates soft and semihard processes using a formulation compatible with analyticity and unitarity principles. The eikonal function is written in terms of even and odd eikonal parts, connected by crossing symmetry and this combination leads [30, 31]:

$$\chi_{pp}^{\bar{p}p}(s, b) = \chi^+(s, b) \pm \chi^-(s, b). \quad (17)$$

The even eikonal is written as the sum of quark–quark, quark–gluon and gluon–gluon contributions

$$\chi^+(s, b) = \chi_{qq}(s, b) + \chi_{qg}(s, b) + \chi_{gg}(s, b). \quad (18)$$

$$\chi^+(s, b) = i[\sigma_{qq}(s) W(b; \mu_{qq}) + \sigma_{qg}(s) W(b; \mu_{qg}) + \sigma_{gg}(s) W(b; \mu_{gg})]. \quad (19)$$

where $W(b; \mu_{ij}) = \mu^5 b^3 K_3(\mu_{ij} b) / 96\pi$ is the overlap density for the partons at b , $\sigma_{ij}(s)$ are the elementary subprocess

cross sections of colliding quarks and gluons ($i, j = q, g$) and $K_3(x)$ is the modified Bessel function of second kind. The eikonal functions $\chi_{qq}(s, b)$ and $\chi_{qg}(s, b)$ are needed to describe the lower energy forward data and are parametrized with inputs from Regge phenomenology (for details see [30]).

It is important to note that the term $\chi_{gg}(s, b)$ gives the main contribution to the asymptotic behavior of the $p + p(\bar{p})$ total cross sections and its energy dependence comes from gluon–gluon cross section

$$\sigma_{gg}(s) = C_{gg}(s) \int_{4m_g^2/s}^1 d\tau F_{gg}(\tau) \hat{\sigma}(\hat{s}), \quad (20)$$

where $\tau = x_1 x_2 = \hat{s}/s$, $F_{gg}(\tau) = \int_{\tau}^1 \frac{dx}{x} g(x)g(\frac{\tau}{x})$ is the convoluted structure function for a pair gluon–gluon, $\hat{\sigma}(\hat{s})$ is the total cross section for the subprocess $gg \rightarrow gg$ and C_{gg} is a free parameter [19,20].

Relating to the term $\chi^-(s, b)$, Eq. (17), the role of the odd eikonal is to account the difference between pp and $p\bar{p}$ channels at low energies and it is written as

$$\chi^-(s, b) = C^- \sum \frac{m_g}{\sqrt{s}} e^{i\pi/4} W(b; \mu^-), \quad (21)$$

where $m_g = 364 \pm 26$ MeV is an infrared mass scale [32] and C^- a fitted constant. All the DGM model parameters used in this work were determined in [19] carrying out a global fit to all high energy forward $p + p(\bar{p})$ scattering data above $\sqrt{s} = 10$ GeV, namely the total cross section, $\sigma_{tot}^{pp, p\bar{p}}$, the ratio of the real to imaginary part of the forward scattering amplitude, $\rho^{pp, p\bar{p}}$, the elastic differential scattering cross sections, $d\sigma^{pp}/dt$, at $\sqrt{s} = 546$ GeV and $\sqrt{s} = 1.8$ TeV as well as the TOTEM datum on σ_{tot}^{pp} at 7 TeV. The χ^2/DOF for the global fit was 0.98 for 320 degrees of freedom. The values of the fitted parameters and the results of the fits to $\sigma_{tot}^{pp, p\bar{p}}$, $\rho^{pp, p\bar{p}}$ and $d\sigma^{pp}/dt$ are presented and discussed in [19]. Thus, all free parameters of the DGM model were completely determined from elastic channel fits.

Now, we see from Eqs. (15) and (7) that the only free parameters in the $P_n(s)$ analysis are k and $\zeta(s)$. With respect to k , assuming the Gamma distribution, Eq. (7), experimental data on e^+e^- annihilation were fitted obtaining $k = 10.775 \pm 0.064$ ($\chi^2/N_{DF} = 2.61$) [18]. By using the DGM eikonal model parametrization, fixing the value of $k = 10.775$ and assuming $\zeta(s)$ as the single fitting parameter, $p + p(\bar{p})$ experimental data of $P_n(s)$ in full phase space in the interval $30.4 \text{ GeV} \leq \sqrt{s} \leq 1800 \text{ GeV}$ were fitted by the Eq. (15) [19], yielding the $\zeta(s)$ values summarized in Table 1, together with the values of $\xi(s)$ computed from Eq. (16). The $\langle N(s) \rangle$ values were obtained from experimental data [14–16], Table 1. The $\zeta(s)$ energy dependence can be described in a consistent way through the function [19]

$$\zeta(s) = 0.189 + 0.00197 [\ln(s)]^{1.536}. \quad (22)$$

Table 1 Results reproduced from Ref. [19], where the P_n phenomenological procedure was applied

$\sqrt{s}$ GeV	$\zeta(s)$	χ^2/N_{DF}	$\xi(s)$	$\langle N(s) \rangle$
30.4	0.239 ± 0.011	0.588	1.642	9.43
44.5	0.240 ± 0.011	0.306	1.643	10.86
52.6	0.239 ± 0.009	0.765	1.639	11.55
62.2	0.231 ± 0.008	1.717	1.613	12.25
300	0.263 ± 0.003	0.608	1.589	24.47
546	0.305 ± 0.004	0.300	1.599	29.53
1000	0.288 ± 0.005	1.469	1.508	38.46
1800	0.315 ± 0.002	0.782	1.468	44.82
7000	0.352		1.308	81.79
14000	0.372		1.209	108

Fig. 1 Top panel: Comparison of the theoretical, Eqs. (15) and (16), and experimental results in full phase space $P_n(s)$ at 30.4 GeV. Data points from [14]. The another panel shows prediction of $K(s)$, Eq. (28), by using the parameters obtained from $P_n(s)$ analysis done in [19]

This procedure in fact does provided an excellent description of the $P_n(s)$ data at high multiplicities, avoiding the introduction of more free parameters. The $P_n(s)$ plots from Ref. [19] are reproduced in this work, as shown at the top panels in Figs. 1, 2, 3, 4, 5, 6, 7 and 8. All the $P_n(s)$ results are in good agreement with the experimental points [14–16], the values of χ^2/N_{DF} are presented in Table 1. Theoretical predictions in full phase space $P_n(s)$ at the LHC energies of $\sqrt{s} = 7$ and 14 TeV are shown in Fig. 9.

Fig. 2 Same as Fig. 1 but at 44.5 GeV

Fig. 4 Same as Fig. 1 but at 62.2 GeV

Fig. 3 Same as Fig. 1 but at 52.6 GeV

Fig. 5 Same as Fig. 1 but at 300 GeV. Data points from [16]

Fig. 6 Same as Fig. 1 but at 546 GeV. Data points from [16]

Fig. 8 Same as Fig. 1 but at 1800 GeV. Data points from [16]

Fig. 7 Same as Fig. 1 but at 1000 GeV. Data points from [16]

With respect to γ parameter, it is unnecessary to calculate the $P_n(s)$ since it is absorbed into the definition of the function $\xi(s)$, Eq. (13) and, in turn, $\xi(s)$ is calculated by using the Eq. (16). However, we cannot calculate $K(s, b)$ until its values are known (see Eq. (28) below) and, in this formalism, we cannot estimate the γ value directly from $p + p(\bar{p})$ data. This parameter was introduced in the $P_n(s)$ *phenomenological procedure* by Eq. (10) on the hypothesis that the average number of produced particles depends on the effective energy for particle production through a power law. In order to have a reliable estimate of γ , from a strongly interacting system, we considered the experimental data on e^-e^+ annihilation as a possible source of information concerning parton-parton interaction in $p + p(\bar{p})$ collisions and adopted the results from Ref. [21], where average multiplicity data in e^+e^- annihilations, covering the interval $10 \leq (\sqrt{s})_{e^+e^-} \leq 200$ GeV, were fitted by Eq. (10), yielding the values of $\gamma = 3.36$ and $\zeta_{(e^+e^-)} = 0.200$, with $\chi^2/N_{DF} = 0.94$. In e^-e^+ annihilation probably one $q\bar{q}$ pair has triggered the multitude of the final particles and, despite the fact that in $p + p(\bar{p})$ more channels should contribute, this approximation seems reasonable because when the average multiplicity increases, the relevance of the original parton may decrease [18].

It is important to note that the impact parameter dependence of the inelasticity for some collision energies studied

in this paper also was studied in [13], where the obtained inelasticity values are much larger than the values found in this work. The different values assigned to the gamma parameter in the Eq. (28), in each analysis, is the main reason for this difference. In [13] it was used the value $\gamma = 2.09$ obtained in [18] where average multiplicity data in e^-e^+ annihilations, in the interval $5.1 \leq (\sqrt{s})_{e^+e^-} \leq 183$ GeV, were fitted by Eq. (10) giving $\gamma = 2.09$ and $\zeta_{(e^+e^-)} = 0.258$ with $\chi^2/N_{DF} = 8.89$. As explained before, here we have adopted $\gamma = 3.36$ in reason of a better χ^2/N_{DF} value than those obtained from $\gamma = 2.09$. At an example level, at $\sqrt{s} = 52.6$ GeV and $b = 0$ the corresponding values of the parameters are $\xi(s) = 1.639$, $\langle N(s) \rangle = 11.55$, $\zeta(s) = 0.239$, $\chi_I(s, 0) = 1.305$ and $\gamma = 3.36$. By using them in the Eq. (28) result $K(s, 0) \approx 0.48$. By changing only the value of γ to 2.09 we obtain $K(s, 0) \approx 1.25$, which is clearly wrong.

3 Energy dependence of the inelasticity and discussions

In $p + p(\bar{p})$ collisions at $\sqrt{s}$ the effective energy for particle production, E_{eff} , is the energy left behind by two leading protons and, using four-vector, it may be written [7]

$$E_{eff} = \sqrt{s} - (E_{leading,1} + E_{leading,2}), \quad (23)$$

or

$$E_{eff} = \sqrt{s} - 2 E_{leading}, \quad (24)$$

in the case of symmetric events [1,7] and, for quantitative estimation of the inelasticity, we have used the definition [4]

$$K = E_{eff}/\sqrt{s}, \quad (25)$$

($0 \leq K \leq 1$). We see from Eq. (9) that $\sqrt{s'} = E_{eff}$ [13], and hence we can rewrite the last Eq. in the form

$$K(s, b) = \frac{\beta(s) \chi_I(s, b)}{2\sqrt{s}}. \quad (26)$$

The factor 2 is due the fact that the $P_n(s)$ data are normalized to 2. In turn, the $\beta(s)$ function is related with $\xi(s)$ by Eq. (13), explicitly we have

$$\beta(s) = \left[\frac{\xi(s) \langle N(s) \rangle (s'_0)^{\zeta(s)}}{\gamma} \right]^{\frac{1}{2\zeta(s)}}. \quad (27)$$

Using the Eq. (27) we can rewrite the Eq. (26) in the form

$$K(s, b) = \left[\frac{\xi(s) \langle N(s) \rangle (s'_0)^{\zeta(s)}}{\gamma} \right]^{\frac{1}{2\zeta(s)}} \frac{\chi_I(s, b)}{2\sqrt{s}}. \quad (28)$$

With respect to last expression, the DMG eikonal function $\chi_I(s, b)$ is completely determined from only elastic channel data analysis (subsection II.B), $\xi(s)$ is determined by the

Table 2 Averaged impact parameter, $\langle b \rangle$, at the ISR energies and the corresponding inelasticity values

$\sqrt{s}$ GeV	$\langle b \rangle$	$K(s, \langle b \rangle)$
30.4	0.83	0.15
44.5	0.78	0.16
52.6	0.77	0.16
62.2	0.83	0.15
$\langle K \rangle_{ISR} \sim 0.16$		

normalized condition given by the Eq. (16), the $\langle N(s) \rangle$ values are obtained from experiments and the $\zeta(s)$ values were obtained by full phase space $P_n(s)$ fits [19] and parametrized by the Eq. (22). Thus, by fixing the value of $\gamma = 3.36$, as discussed in the Subsection II.B, we have calculated the $K(s, b)$ as a function of the impact parameter b and the results are displayed in Figs. 1, 2, 3, 4, 5, 6, 7 and 8. The inelasticity behavior is essentially the same at the energies of $\sqrt{s} = 62.2$ and 44.5 GeV, Fig. 4. The same occurs at 1000 and 546 GeV, Fig. 7. It seems consequence from the fact that at 62.2 and 1000 GeV the theoretical P_n does not fits satisfactorily the experimental points in the tail of the distributions.

At the ISR energies the average inelasticity is determined to be about 0.5 [7,33]. Interestingly in the present analysis is that the average inelasticity at the ISR, when $b \sim 0$, yields the same value, specifically: $\langle K \rangle_{ISR} = (0.54 + 0.49 + 0.48 + 0.50)/4 \sim 0.5$, however, the choice of $b \sim 0$ is so arbitrary. Based on the results displayed in Fig. 4 and by using the formulae of the mathematical expectation of the function $K(s, b)$, we have calculated the average impact parameter, $\langle b \rangle$, at each ISR energy and the corresponding $K(s, \langle b \rangle)$ values as well as the new value of $\langle K \rangle_{ISR} \sim 0.16$. The results are summarized in Table 2 and the average inelasticity, thus obtained, do not agree with those from [7,33]. However, we recall that the impact parameter dependence of the inelasticity was not analysed in the framework of the both mentioned works, [7,33].

From Fig. 9, where the plots of $K(s)$ versus b at the energies investigated in this work are presented together, is possible see that the particle production processes tend to be more peripheral ($b > 1$ fm) at the ISR energies of 30.4, 44.5, 52.6 and 62.2 GeV when compared with the results from other energies investigated. In this interval of b the $K(s)$ values, at the ISR, are rather greater than $K(s)$ values at the others energies at fixed value of b . In order to substantiate this statement in Fig. 10 we show the ratios $K(s, b)/K(30.4, 0)$ calculated for different collision energies at the impact parameter values of 1.0, 1.1, 1.25 and 1.5 fm. Based in Fig. 1 we have used $K(30.4, 0) = 0.54$. In fact, the results presented in Fig. 10 are indicatives that the particle production is more peripheral at the mentioned ISR energies than at other energies studied. This behavior of $K(s)$ is compatible with the

Fig. 9 Top panel: Theoretical results in full phase space $P_n(s)$ at 7 and 14 TeV, Eqs. (16) and (17). Experimental data of the $P_n(s)$ at 1800 GeV added to comparison. The another panel shows predictions of $K(s)$, Eq. (28), by using parameters obtained from $P_n(s)$ analysis done in [19]

minijets production, since semihard processes are more central in the impact parameter than purely soft processes and do not use much collision energy [34]. The inelasticity K is proportional to the $\chi_I(s, b)$, Eq. (28), and in the DGM eikonal model the gluon semihard contribution $\chi_{gg}(s, b)$, Eq. (18), dominates at high energies and the rise of the cross sections with $\sqrt{s}$ is consequence of the increasing number of soft gluons populating the colliding particles, increasing, therefore, the probability of perturbative gluon-gluon collisions at small x , which can leads to the appearance of minijets and, as mentioned, do not use much collision energy. This scenario leads to the conclusion that the $K(s)$ decreases as a consequence of the minijet production from semihard soft gluon-gluon interactions when $\sqrt{s}$ increases.

We show in Fig. 11 the energy dependence of the $K(s)$ calculated at $b \approx 0$, Eq. (28), and observe a marked decrease in the inelasticity from ISR to LHC, while at the $\sqrt{s} > 7$ TeV the inelasticity shows a slow decrease. The error bars represent the uncertainties of the parameters γ and ζ propagated to the inelasticity values. The star symbol represents theoretical predictions at the LHC and the solid line is drawn only as guidance for the points. The LHC has measured the multiplicity distributions in a limited pseudorapidity range

Fig. 10 Ratios $K(s, b)/K(30.4, 0)$ calculated for different collision energies and impact parameters values. The value of $K(30.4, 0)$ is 0.54, Fig. 1. The particle production processes tend to be more peripheral at the ISR specific energies of 30.4, 44.5, 52.6 and 62.2 GeV

Fig. 11 Inelasticities calculated at $b \approx 0$ as a function of the center of mass energy, Eq. (28). The results show marked decrease in the inelasticity from ISR to LHC energies. The error bars represent the uncertainties of the parameters γ and ζ propagated to the inelasticity values, the line is drawn only as guidance for the points

[35–40], and for this reason we do not compare our results with those from LHC.

We observe that the structure found around the peak in the $P_n(s)$ data at higher energies, which appears in the region of low multiplicities, has not been considered in the analysis done in [19]. However, the $P_n(s)$ approach used describes

very well the energy dependence of the F -moments and reproduces the H_q versus q oscillations observed in the experimental data and predicted by QCD [19,23].

With respect to alternative methods of estimating the inelasticity, in [1], the coefficient of inelasticity in $p + p(\bar{p})$ collisions and its possible $\sqrt{s}$ dependence was estimated by comparing $p + p(\bar{p})$ with e^-e^+ collisions for three different assumptions on the values for both the parameters involved in the analysis, namely n_0 and Δm . The parameter n_0 corresponds to the contribution from the two leading protons to the total multiplicity, while Δm takes the contribution of the masses of the two participating constituent quarks to the centre-of-mass energy into account. Having varied the n_0 and Δm values three different inelasticities were defined. In one of the results the inelasticity decreases from $K \sim (0.55\text{--}0.6)$ at the ISR energies to 0.4 at $\sqrt{s} = 1800$ GeV. The two other results indicated the constant value of $K \sim 0.35$.

Investigating the very high energy pp interactions by cosmic ray data it was shown [8] that the Feynman scaling violation, in the form proposed by Wdowczyk and Wolfendale, leads to continuous decrease of the inelasticity, which was found be consistent with LHC measurements up to 7 TeV, qualitatively in agreement with our results, Fig. 11.

In another work [9] and by using methods of information theory approach, calculations of the inelasticity coefficient and its energy dependence were studied, resulting that the inelasticity remains essentially constant in energy, except for a variation around $K \sim 0.5$ in the range $20 < \sqrt{s} < 1800$ GeV to $p + p(\bar{p})$ data.

The Interacting Gluon Model (IGM) was an approach used in studies about the inelasticities and leading particle spectra in hadronic and nuclear collisions [3,9–11]. In [10] an extended version of the IGM incorporating the production of minijets was applied and, as a result, it was concluded that the inelasticity slowly increases towards some limited value. The inclusion of minijets reversed the trend of decreasing inelasticities found in previous calculations with the IGM.

In subsequent work [11] the authors introduced a hadronization mechanism in the IGM concluding that the minijet production leads to inelasticities increasing with $\sqrt{s}$ and that hadronization process does not change this trend.

Based on the above considerations, one can note that the various approaches are largely in conflict with each other in explaining the energy dependence of the inelasticity, reflecting the subtlety of the theme. Hence, we have based the present study on the experimental information on charged particle multiplicity distributions in $p + p(\bar{p})$ collisions. Thus, we provide a new argument in favor of the hypothesis that the $K(s)$ decreases as a function of the center of mass energy.

4 Concluding remarks

In the absence of sufficient experimental information on the energy dependence of the inelasticity to test the several existing model predictions, we have based our analysis in the connection between $K(s)$ and the full phase space $P_n(s)$ by using a satisfactory modeling to $P_n(s)$ adjusted for the experimental reality over a large range of energy, $30 < \sqrt{s} \leq 1800$ GeV, which is consistent with several QCD prescriptions [19].

In the present approach $K(s, b) \propto \chi_I(s, b)$, Eq. (28), and we have adopted the DGM QCD inspired eikonal model [19,30,31]. The only free parameter in the $P_n(s)$ formalism adjusted to $p + p(\bar{p})$ experimental data is $\zeta(s)$, (Eq. (22)—Table 1), while all the parameters of the eikonal function, $\chi_{pp}^{\bar{p}p}(s, b)$, were determined carrying out a global fit to $\sigma_{tot}^{pp,\bar{p}p}$, $\rho^{pp,\bar{p}p}$ and $d\sigma_{el}^{\bar{p}p}/dt$ data. The results of the fits to $\sigma_{tot}^{pp,\bar{p}p}$, $\rho^{pp,\bar{p}p}$ and $d\sigma_{el}^{\bar{p}p}/dt$ are presented in [19]. Our results predict the average inelasticity to be ~ 0.5 at the ISR energies if calculated at $b \sim 0$, in agreement with that from Refs. [6,7,33] (see Sect. 3).

The term $\chi_{gg}(s, b)$ in the Eq. (18) gives the main contribution to high multiplicities, being the responsible for the rise of the cross sections with $\sqrt{s}$. Thus, we have concluded that minijets from semihard interactions, arising from scattering of gluons carrying only a very small fraction of the momenta of their parent protons, are the responsible for the decrease of the inelasticity as a function of the $\sqrt{s}$.

Results obtained by using alternative methods to estimate the energy dependence of the inelasticity are in conflict with each other. Thus, based on the experimental information on charged particle multiplicity distributions in $p + p(\bar{p})$ collisions we provide new evidence in favor of the hypothesis that the $K(s)$ decreases when $\sqrt{s}$ is increased.

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