The studies on $Z \rightarrow \Upsilon(1S) + g + g$ at the next-to-leading-order QCD accuracy

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Received: 10 February 2020 / Accepted: 24 March 2020 / Published online: 8 April 2020
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Abstract In this paper, we carry out the next-to-leading-order (NLO) studies on $Z \rightarrow \Upsilon(1S) + g + g$ via the color-singlet (CS) $b\bar{b}$ state. We find the newly calculated NLO QCD corrections to this process can significantly influence its leading-order (LO) results and greatly improve the dependence on the renormalization scale. By including the considerable feeddown contributions, the branching ratio $\mathcal{B}_{Z \rightarrow \Upsilon(1S)+g+g}$ is predicted to be $(0.56\text{--}0.95) \times 10^{-6}$, which can reach up to 19–31% of the LO predictions given by the CS dominant process $Z \rightarrow \Upsilon(1S) + b + \bar{b}$. Moreover, $Z \rightarrow \Upsilon(1S) + g + g$ also seriously affects the CS predictions on the $\Upsilon(1S)$ energy distributions, especially when z is relatively small. In summary, for the inclusive $\Upsilon(1S)$ productions in Z decay, besides $Z \rightarrow \Upsilon(1S) + b + \bar{b}$, the gluon radiation process $Z \rightarrow \Upsilon(1S) + g + g$ can provide indispensable contributions as well.

1 Introduction

The L3 group at LEP has released the measurement on the total decay width of $Z \rightarrow \Upsilon(1S) + X$ [1]

$$\mathcal{B}_{Z \rightarrow \Upsilon(1S)+X} < 4.4 \times 10^{-5}.$$

The leading-order (LO) color-singlet (CS) predictions obtained by calculating the CS dominant process $Z \rightarrow \Upsilon(1S) + b + \bar{b}$ are only at the 10^{-6} order [2–4]. Subsequently Li et al. [4] accomplish the next-to-leading-order (NLO) QCD corrections to this $b\bar{b}$ pair associated channel, pointing out the higher-order terms can give rise to a 24% enhancement to the total decay width.

Reviewing the inclusive J/ψ productions via Z decay [2–15], besides the $\alpha\alpha_s^2$ -order process $Z \rightarrow J/\psi + c + \bar{c}$ that serves as the leading role in the CS predictions, the electromagnetic processes $Z \rightarrow f\bar{f}\gamma^*$ with $\gamma^* \rightarrow J/\psi$ ($f = l, u, d, s, c, b$) and the gluon fragmentation processes

$Z \rightarrow f_q\bar{f}_q g^*$ with $g^* \rightarrow J/\psi gg$ ($f_q = u, d, s, c, b$) can also provide nonnegligible contributions. Meanwhile, due to the suppression by $\frac{m_c^2}{m_Z^2}$ [3], the total decay width of the other $\alpha\alpha_s^2$ -order process $Z \rightarrow J/\psi + g + g$ is only about two orders of magnitudes smaller than that of $Z \rightarrow J/\psi + c + \bar{c}$. As for the Υ productions, the situations become just the opposite. The relative significances of the electromagnetic and gluon fragmentation processes are much less important than the J/ψ case, since the value of m_b , larger than m_c , highly suppresses the denominator of the propagators $\gamma^*(\rightarrow \Upsilon)$ and $g^*(\rightarrow \Upsilon gg)$. However, for $Z \rightarrow \Upsilon + g + g$, the suppression effect stated above ($\frac{m_c^2}{m_Z^2}$) will be largely weakened by replacing m_c with m_b , subsequently rendering this process indispensable in comparison with $Z \rightarrow \Upsilon + b + \bar{b}$ [2]. Moreover, the Υ energy distributions in $Z \rightarrow \Upsilon + g + g$ and $Z \rightarrow \Upsilon + b + \bar{b}$ may be thoroughly different. This can be understood by the fact that the former process is seriously suppressed by the factor $\frac{M_\Upsilon^2}{E_\Upsilon^2}$ for large z [11, 16, 17], so the value of z concerning the maximum $\frac{d\Gamma}{dz}$ should be small; however, as a result of the b quark fragmentation, the dominant contributions of $Z \rightarrow \Upsilon + b + \bar{b}$ exist in the region of large z . From these points of view, the process $Z \rightarrow \Upsilon + g + g$ would be crucial for Z decaying to the inclusive Υ , deserving a separate investigation.

Seeing that all the existing studies on $Z \rightarrow \Upsilon(1S) + g + g$ are only accurate to the first order in α_s , to investigate the effects of the higher-order terms, in this paper we will for the first time carry out the NLO QCD corrections to this process. In general, for the inclusive $\Upsilon(1S)$ productions, the feeddown via the excited states can provide nonnegligible contributions. Therefore, in addition to the direct productions, we will take the feeddown effects via $\Upsilon(2, 3S)$ and $\chi_{bJ}(1, 2, 3P)$ ($J = 0, 1, 2$) into account as well.

The rest of the paper is organized as follows: in Sect. 2, we give a description on the calculation formalism. In Sect. 3,

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the phenomenological results and discussions are presented. Section 4 is reserved for a summary.

2 Calculation formalism

Based on the nonrelativistic quantum chromodynamics [18], the decay width of $Z \rightarrow \Upsilon(\chi_{bJ}) + g + g$ ¹ can be factorized as

$$\Gamma = \hat{\Gamma}_{Z \rightarrow b\bar{b}[n]+g+g} \langle \mathcal{O}^{\Upsilon(\chi_{bJ})}(n) \rangle, \quad (1)$$

where $\hat{\Gamma}_{Z \rightarrow b\bar{b}[n]+g+g}$ is the perturbative calculable short distance coefficients, representing the production of a configuration of the $b\bar{b}[n]$ intermediate state. The universal nonperturbative long distance matrix elements (LDMEs) $\langle \mathcal{O}^{\Upsilon(\chi_{bJ})}(n) \rangle$ stands for the probability of $b\bar{b}[n]$ into $\Upsilon(\chi_{bJ})$. In our studies, we only concentrate on the CS contributions, so $n = {}^3S_1^{[1]}$ for Υ , and $n = {}^3P_J^{[1]}$ ($J = 0, 1, 2$) for χ_{bJ} . The procedures for dealing with the soft singularities involved in $Z \rightarrow b\bar{b}[{}^3P_J^{[1]}] + g + g$ ($J = 0, 1, 2$) have been described in detail in our previous paper [19, 20], so here we just give a brief presentation on the $b\bar{b}[{}^3S_1^{[1]}]$ related calculations.

The NLO short distance coefficients for $n = {}^3S_1^{[1]}$ can be written as

$$\hat{\Gamma}_{Z \rightarrow b\bar{b}[{}^3S_1^{[1]}]+g+g}^{NLO} = \hat{\Gamma}_{\text{Born}} + \hat{\Gamma}_{\text{Virtual}} + \hat{\Gamma}_{\text{Real}}, \quad (2)$$

where

$$\begin{aligned} \hat{\Gamma}_{\text{Virtual}} &= \hat{\Gamma}_{\text{Loop}} + \hat{\Gamma}_{\text{CT}}, \\ \hat{\Gamma}_{\text{Real}} &= \hat{\Gamma}_{\text{S}} + \hat{\Gamma}_{\text{HC}} + \hat{\Gamma}_{\text{HC}\bar{C}}. \end{aligned} \quad (3)$$

$\hat{\Gamma}_{\text{Virtual}}$ is the virtual corrections, consisting of the contributions from the one-loop diagrams ($\hat{\Gamma}_{\text{Loop}}$) and the counter-terms ($\hat{\Gamma}_{\text{CT}}$). $\hat{\Gamma}_{\text{Real}}$ stands for the real corrections, which include the soft terms ($\hat{\Gamma}_{\text{S}}$), hard-collinear terms ($\hat{\Gamma}_{\text{HC}}$), and hard-noncollinear terms ($\hat{\Gamma}_{\text{HC}\bar{C}}$). For $\hat{\Gamma}_{\text{Real}}$, three processes are involved:

$$\begin{aligned} Z &\rightarrow b\bar{b}[{}^3S_1^{[1]}] + g + g + g, \\ Z &\rightarrow b\bar{b}[{}^3S_1^{[1]}] + g + q + \bar{q} \quad (q = u, d, s), \\ Z &\rightarrow b\bar{b}[{}^3S_1^{[1]}] + g + u_g + \bar{u}_g \quad (\text{ghost}). \end{aligned} \quad (4)$$

There are 177 Feynman diagrams in total, including 6 diagrams for $\hat{\Gamma}_{\text{Born}}$, 111 diagrams for $\hat{\Gamma}_{\text{Virtual}}$ (30 counter-terms, 12 self-energy, 30 3-points, 27 4-points, 12 5-points), and 60 diagrams for $\hat{\Gamma}_{\text{Real}}$ (42 ggg_v , 6 ggg_{av} , 6 $gq\bar{q}$, and 6 $gu_g\bar{u}_g$), as representatively shown in Fig. 1. ggg_v and ggg_{av} denote the

vector and axial-vector parts of $Z \rightarrow b\bar{b}[{}^3S_1^{[1]}] + g + g + g$, respectively.

To isolate the ultraviolet (UV) and infrared (IR) divergences, we adopt the dimensional regularization with $D = 4 - 2\epsilon$. The on-mass-shell (OS) scheme is employed to set the renormalization constants for the heavy quark mass (Z_m), heavy quark field (Z_2), and gluon field (Z_3). The modified minimal-subtraction ($\overline{MS}$) scheme is used for the QCD gauge coupling (Z_g). The renormalization constants are [19–21]

$$\begin{aligned} \delta Z_m^{OS} &= -3C_F \frac{\alpha_s N_\epsilon}{4\pi} \left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln \frac{4\pi\mu_r^2}{m_b^2} + \frac{4}{3} \right], \\ \delta Z_2^{OS} &= -C_F \frac{\alpha_s N_\epsilon}{4\pi} \left[\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} - 3\gamma_E + 3\ln \frac{4\pi\mu_r^2}{m_b^2} + 4 \right], \\ \delta Z_3^{OS} &= \frac{\alpha_s N_\epsilon}{4\pi} \left[(\beta'_0 - 2C_A) \left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right) - \frac{4}{3} T_F \left(\frac{1}{\epsilon_{UV}} - \gamma_E + \ln \frac{4\pi\mu_r^2}{m_c^2} \right) - \frac{4}{3} T_F \left(\frac{1}{\epsilon_{UV}} - \gamma_E + \ln \frac{4\pi\mu_r^2}{m_b^2} \right) \right], \\ \delta Z_g^{\overline{MS}} &= -\frac{\beta_0}{2} \frac{\alpha_s N_\epsilon}{4\pi} \left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln(4\pi) \right], \end{aligned} \quad (5)$$

where γ_E is the Euler constant, $N_\epsilon = \Gamma[1 - \epsilon]/(4\pi\mu_r^2/(4m_b^2))^\epsilon$, $\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$ is the one-loop coefficient of the β -function, and $\beta'_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_{lf}$. $n_f (= 5)$ and $n_{lf} (= n_f - 2)$ are the number of active quark flavors and light quark flavors, respectively. In SU(3), the color factors are given by $T_F = \frac{1}{2}$, $C_F = \frac{4}{3}$, and $C_A = 3$. The two-cutoff slicing strategy is utilized to subtract the IR divergences in Γ_{Real} [22].

The package MALT@FDC that has been employed to perform the NLO QCD corrections to several heavy quarkonium related processes [19, 20, 23–25] is used to deal with $\hat{\Gamma}_{\text{Virtual}}$, $\hat{\Gamma}_{\text{S}}$, and $\hat{\Gamma}_{\text{HC}}$. The FDC package [26] serves as the agent for calculating the hard-noncollinear part $\hat{\Gamma}_{\text{HC}\bar{C}}$. Both the cancellation of the $\epsilon^{-2(-1)}$ -order divergences and the independence on the cutoff parameters $\delta_{s,c}$ have been checked carefully, as shown in Figs. 2 and 3.

As a crosscheck, taking the same input parameters, we have reproduced the NLO results of $\sigma_{e^+e^- \rightarrow J/\psi + g + g}$ in Refs. [27, 28].

3 Phenomenological results

In our calculations, the input parameters are taken as

$$\begin{aligned} \alpha &= 1/128, \quad m_b = 4.9 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \\ m_Z &= 91.1876 \text{ GeV}, \quad m_{q/\bar{q}} = 0 \quad (q = u, d, s), \\ \sin^2(\theta_W) &= 0.23116. \end{aligned} \quad (6)$$

¹ For the final gluons associated $\Upsilon(\chi_{bJ})$ productions in Z decay via the CS $b\bar{b}$ states, due to the color conservation, the process $Z \rightarrow b\bar{b}[{}^3S_1^{[1]}] + g$ is forbidden, so the lowest order process in α_s is $Z \rightarrow b\bar{b}[{}^3S_1^{[1]}, {}^3P_J^{[1]}] + g + g$.

Fig. 1 Representative Feynman diagrams for the NLO QCD corrections to $Z \rightarrow b\bar{b}[{}^3S_1^{(1)}] + g + g$. The superscripts “q” and “ u_g ” denote the light quarks (u, d, s) and the ghost particles, respectively

Fig. 2 Cancellation of the ϵ^{-2} - and ϵ^{-1} -order divergences. The superscripts “(2)” and “(1)” denote the ϵ^{-2} - and ϵ^{-1} -order terms, respectively

Fig. 3 The verification of the independence on the cutoff parameters δ_s and δ_c . The superscript “(0)” denotes the ϵ^0 -order terms. In the left diagram, $\delta_c = 2 \times 10^{-7}$. In the right diagram, $\delta_s = 1 \times 10^{-3}$

Table 1 The total decay widths of $Z \rightarrow \Upsilon(1S) + g + g$ (in units of keV). K denotes $\Gamma_{DR}^{NLO} / \Gamma_{DR}^{LO}$. “NLO” represents the sum of the contribution of the LO terms and that of the QCD corrections. The superscripts “DR” and “FD” denote the direct and feeddown contributions, respectively

μ_r	m_b (GeV)	Γ_{DR}^{LO}	Γ_{DR}^{NLO}	K	$\Gamma_{FD}^{\chi_{bJ}(1,2,3P)}$	$\Gamma_{FD}^{\Upsilon(2,3S)}$	Γ_{Total}
$2m_b$	4.7	1.94	1.83	0.94	0.20	0.34	2.37
	4.9	1.75	1.66	0.95	0.17	0.30	2.13
	5.1	1.59	1.51	0.95	0.15	0.28	1.94
m_Z	4.7	0.82	1.33	1.62	0.10	0.24	1.67
	4.9	0.76	1.22	1.61	0.09	0.22	1.53
	5.1	0.70	1.12	1.60	0.08	0.20	1.40

Fig. 4 The total decay widths of $Z \rightarrow \Upsilon(1S) + g + g$ as a function of the renormalization scale μ_r with $m_b = 4.9$ GeV. “NLO” represents the sum of the contribution of the LO terms and that of the QCD corrections. The superscripts “DR” and “FD” denote the direct and feeddown contributions, respectively

To determine $\langle \mathcal{O}^{\Upsilon(nS)}(\mathcal{P}_1^{[1]}) \rangle$ and $\langle \mathcal{O}^{\chi_{bJ}(mP)}(\mathcal{P}_J^{[1]}) \rangle$, we employ the relations to the radial wave functions at the origin ($n, m = 1, 2, 3$)

$$\frac{\langle \mathcal{O}^{\Upsilon(nS)}(\mathcal{P}_1^{[1]}) \rangle}{6N_c} = \frac{1}{4\pi} |R_{\Upsilon(nS)}(0)|^2,$$

$$\frac{\langle \mathcal{O}^{\chi_{bJ}(mP)}(\mathcal{P}_J^{[1]}) \rangle}{2N_c} = (2J+1) \frac{3}{4\pi} |R'_{\chi_{bJ}(mP)}(0)|^2, \quad (7)$$

where $|R_{\Upsilon(nS)}(0)|^2$ and $|R'_{\chi_{bJ}(mP)}(0)|^2$ read [29]

$$\begin{aligned} |R_{\Upsilon(1S)}(0)|^2 &= 6.477 \text{ GeV}^3, & |R_{\Upsilon(2S)}(0)|^2 &= 3.234 \text{ GeV}^3, \\ |R_{\Upsilon(3S)}(0)|^2 &= 2.474 \text{ GeV}^3, \\ |R'_{\chi_{b1}(1P)}(0)|^2 &= 1.417 \text{ GeV}^5, & |R'_{\chi_{b2}(2P)}(0)|^2 &= 1.653 \text{ GeV}^5, \\ |R'_{\chi_{b3}(3P)}(0)|^2 &= 1.794 \text{ GeV}^5. \end{aligned} \quad (8)$$

The branching ratios of $\chi_{bJ}(mP) \rightarrow \Upsilon(nS)$, $\Upsilon(nS) \rightarrow \chi_{bJ}(mP)$, $\Upsilon(3S) \rightarrow \Upsilon(2S)$, $\Upsilon(3S) \rightarrow \Upsilon(1S)$, and $\Upsilon(2S) \rightarrow \Upsilon(1S)$ can be found in Refs. [30–32].

The total decay widths of $Z \rightarrow \Upsilon(1S) + g + g$ are listed in Table 1. To demonstrate the dependence on the renormalization scale μ_r , the results for $\mu_r = 2m_b$ and $\mu_r = m_Z$ are

presented simultaneously. From the data in this table, one can observe the following.

- (i) For the direct productions, when $\mu_r = 2m_b$, the QCD corrections diminish the LO results by about 5%, and cause a 60% enhancement for $\mu_r = m_Z$. In addition, incorporating these higher-order terms notably weaken the dependence on μ_r . As is illustrated in Fig. 4, the line referring to “ DR_{NLO} ” decreases much more slowly than that for “ DR_{LO} ” with the increase of μ_r .
- (ii) The decay of $\Upsilon(2, 3S)$ and $\chi_{bJ}(1, 2, 3P)$ can raise the predictions of the direct productions by about 30%, manifestly indicating the feeddown significance in the $\Upsilon(1S)$ production.
- (iii) The dependence on the mass of the b quark is mild, e.g., varying m_b by ± 0.2 GeV from the central value of 4.9 GeV just results in a 10% variation of the total decay width.

Summing up the direct and feeddown contributions, we finally obtain

$$\mathcal{B}_{Z \rightarrow \Upsilon(1S) + g + g} = (0.56 - 0.95) \times 10^{-6}, \quad (9)$$

where the theoretical uncertainty is induced by the choices of the values of μ_r ($2m_b \sim m_Z$) and m_b (4.9–5.1 GeV).

In Fig. 5, the $\Upsilon(1S)$ energy distributions in $Z \rightarrow \Upsilon(1S) + g + g$ are presented with z defined as $\frac{2E_{\Upsilon(1S)}}{m_Z}$. For the direct productions, when μ_r is equal to $2m_b$, the QCD corrections are positive for $z < 0.52$, and negative in the remaining scope of z ; however, for $\mu_r = m_Z$, these corrections keep always positive for all the available z values. With μ_r being relatively small, the higher-order terms can significantly enhance the differential decay width. Taking $z = 0.3$ as an example,

$$\begin{aligned} \left(\frac{d\Gamma}{dz} \right)_{NLO} / \left(\frac{d\Gamma}{dz} \right)_{LO} &= 1.23 \text{ for } \mu_r = 2m_b, \\ \left(\frac{d\Gamma}{dz} \right)_{NLO} / \left(\frac{d\Gamma}{dz} \right)_{LO} &= 1.81 \text{ for } \mu_r = m_Z, \end{aligned} \quad (10)$$

where “NLO” represents the sum of the contribution of the LO terms and that of the QCD corrections. Including the

Fig. 5 The $\Upsilon(1S)$ energy distributions in $Z \rightarrow \Upsilon(1S) + g + g$. $z = \frac{2E_{\Upsilon(1S)}}{m_Z}$ and $m_b = 4.9$ GeV. “NLO” represents the sum of the contribution of the LO terms and that of the QCD corrections. $\mu_r = 2m_b$

in the left diagram, and $\mu_r = m_Z$ in the right diagram. The superscripts “DR” and “FD” denote the direct and feeddown contributions, respectively

Fig. 6 The differential decay widths. $z = \frac{2E_{\Upsilon(1S)}}{m_Z}$ and $m_b = 4.9$ GeV. $\mu_r = 2m_b$ in the left diagram, and $\mu_r = m_Z$ in the right diagram. The superscripts “ Υgg ” and “ $\Upsilon b\bar{b}$ ” denote the contributions via $Z \rightarrow \Upsilon(1S) + g + g$ and $Z \rightarrow \Upsilon(1S) + b + \bar{b}$, respectively

feeddown contributions can further increase $\frac{d\Gamma}{dz}$ by about 20–30% for most values of z . Note that, phenomenological and theoretical arguments suggest that the NRQCD factorization holds only when the quarkonium is produced at relatively large momentum, thus in Fig. 5 the predicted z distributions for very small z values (close to the left endpoint) may not hold.

At last, we simply compare the contributions of $Z \rightarrow \Upsilon(1S) + g + g$ with that of the dominant channel $Z \rightarrow \Upsilon(1S) + b + \bar{b}$. With the help of the *FDC* package, taking the parameters in Eq. (6), we have

$$\begin{aligned}\Gamma_{Z \rightarrow \Upsilon(1S)+b+\bar{b}}^{LO, \mu_r=2m_b} &= 11.5 \text{ keV}, \\ \Gamma_{Z \rightarrow \Upsilon(1S)+b+\bar{b}}^{LO, \mu_r=m_Z} &= 4.98 \text{ keV}.\end{aligned}\quad (11)$$

Comparing to the results with $m_b = 4.9$ GeV in Table 1, one can obtain

$$\frac{\Gamma_{Z \rightarrow \Upsilon(1S)+g+g}^{LO}}{\Gamma_{Z \rightarrow \Upsilon(1S)+b+\bar{b}}^{LO}} = 19\text{--}31\%, \quad (12)$$

where the uncertainty arises from the variation of μ_r in $[2m_b, m_Z]$. This ratio suggests that, for the total decay width,

the contributions via $Z \rightarrow \Upsilon(1S) + g + g$ is comparable with the magnitude of the QCD corrections to $Z \rightarrow \Upsilon(1S) + b + \bar{b}$. In Fig. 6, the comparison of the $\Upsilon(1S)$ energy distributions in $Z \rightarrow \Upsilon(1S) + g + g$ and $Z \rightarrow \Upsilon(1S) + b + \bar{b}$ are presented, where “ Υgg ” denotes the contributions via the former process up to the NLO accuracy in α_s , including the feeddown effects; “ $\Upsilon b\bar{b}$ ” stands for the direct contributions of the $b\bar{b}$ pair associated process at the LO QCD accuracy. One can find, for relatively small z , that adding the Υgg contributions can increase the $\Upsilon b\bar{b}$ predictions to a surprisingly large extent. Such a remarkable enhancement on $\frac{d\Gamma}{dz}$ is almost the same in size as the QCD corrections to $Z \rightarrow \Upsilon(1S) + b + \bar{b}$. All these points obviously reveal the phenomenological importance of $Z \rightarrow \Upsilon(1S) + g + g$ for Z decaying to the inclusive $\Upsilon(1S)$.

Note that, within the NRQCD framework, for the Υ production in association with final gluon(s) via Z decay, in addition to the CS process $Z \rightarrow b\bar{b}[^3S_1^{[1]}] + g + g$, which we focus on in our present paper, the color-octet (CO) channels $Z \rightarrow b\bar{b}[^1S_0^{[8]}, ^3S_1^{[8]}, ^3P_J^{[8]}] + g$ are also allowed. Generally speaking, according to the velocity-scaling rule of NRQCD, the CO LDMEs are suppressed by a power of v^4 ($v_b^2 \simeq 0.1$),

where v denotes the relative velocity between the constituent quark and anti-quark of the heavy quarkonium. However, the smallness of α_s in $Z \rightarrow b\bar{b}[^1S_0^{[8]}, ^3S_1^{[8]}, ^3P_J^{[8]}] + g$ comparing to $Z \rightarrow b\bar{b}[^3S_1^{[1]}] + g + g$, together with the potential enhancement via the NLO corrections to these CO channels, may partly compensate for the suppression caused by LDMEs, subsequently making the CO contributions nonnegligible. Of course, whether this is indeed the case depends on the future rigorous NLO calculations for these CO production channels.

4 Summary

In this manuscript, we for the first time perform the complete NLO studies on the process $Z \rightarrow \Upsilon(1S) + g + g$ via the CS $b\bar{b}$ states. We find that the impacts of the QCD corrections on the LO results are significant, including both the total and differential decay widths. In addition, these higher-order terms markedly weaken the dependence of the theoretical predictions on μ_r . By incorporating the substantial feeddown effects via $\Upsilon(2, 3S)$ and $\chi_{bJ}(1, 2, 3P)$, $\mathcal{B}_{Z \rightarrow \Upsilon(1S)+g+g}$ is scattered in the range $(0.56-0.95) \times 10^{-6}$. This value is about 19–31% of the LO predictions given by $Z \rightarrow \Upsilon(1S) + b + \bar{b}$, which is responsible for the main contributions in the CS predictions. Moreover, for small $E_{\Upsilon(1S)}$, $Z \rightarrow \Upsilon(1S) + g + g$ has vital influence on the $\Upsilon(1S)$ energy distributions. In view of these points, for Z decaying to the inclusive $\Upsilon(1S)$, in addition to $Z \rightarrow \Upsilon(1S) + b + \bar{b}$, the process $Z \rightarrow \Upsilon(1S) + g + g$ is also phenomenologically important.

Acknowledgements This work is supported in part by the Natural Science Foundation of China under the Grant No. 11647113, and No. 11705034, by the Project for Young Talents Growth of Guizhou Provincial Department of Education under Grant No. KY[2017]135, and by the Project of GuiZhou Provincial Department of Science and Technology under Grant No. QKHJC[2019]1160.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All the numerical results can be repeated according to the formulas given in the paper, so there is not necessary to provide the data.]

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