Unification of flavor SU(3) analyses of heavy Hadron weak decays

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Abstract Analyses of heavy mesons and baryons hadronic charmless decays using the flavor SU(3) symmetry can be formulated in two different forms. One is to construct the SU(3) irreducible representation amplitude by decomposing effective Hamiltonian, and the other is to draw the topological diagrams. In the flavor SU(3) limit, we study various $B/D \rightarrow PP, VP, VV, B_c \rightarrow DP/DV$ decays, and two-body nonleptonic decays of beauty/charm baryons, and demonstrate that when all terms are included these two ways of analyzing the decay amplitudes are completely equivalent. Furthermore we clarify some confusions in drawing topological diagrams using different ways of describing beauty/charm baryons.

1 Introduction

Weak decays of heavy mesons and baryons carrying a bottom and/or a charm quark are of great interests and have been studied extensively on both experimental and theoretical sides. These decays provide useful information about the strong and electroweak interactions in the standard model (SM). Rare decays are ideal to look for new physics effects beyond SM, and recent measurements of lepton flavor universality have shown notable deviations from the standard model (see Ref. [1] for a recent brief review on the anomalies in B decays). Quite a number of physical observables like branching fractions, CP asymmetries and polarizations have been precisely measured by experiments [2–4]. On the other hand, due to our limited understanding of QCD at low energy regions, theoretical calculations of decay amplitudes are not well understood. Most of the current calculations rely on the factorization methods. Among them, many available

studies are conducted at leading power in $1/m_b$, while recent analyses of semileptonic and radiative processes have indicated the importance of next-to-leading power corrections [5, 6].

Apart from factorization approaches, the flavor SU(3) symmetry is a powerful tool frequently used in two-body and three-body heavy meson decays [7–20]. Although flavor SU(3) symmetry is approximate, yet it still provides very useful information about the decays. Since the SU(3) invariant amplitudes can be determined by fitting the data, the SU(3) analysis bridges experimental data and the dynamical approaches.

Among different realizations of carrying out SU(3) analysis for decay amplitudes there are two popular methods. One of them is topological diagram amplitude (TDA) method, where decay amplitudes are represented by connecting quark lines flows in different ways and then relate them by SU(3) symmetry, and another way is to construct the SU(3) irreducible representation amplitude (IRA) by decomposing effective Hamiltonian. The TDA approach gives a better understanding of decay dynamics especially in the bottom hadron decays, where the power expansion of $1/m_b$ can be properly conducted so that each TDA amplitude can be related with matrix elements of SCET operators [21]. On the other hand, the IRA approach shows a convenient connection with the SU(3) symmetry. These two methods look very different in formulations, one may wonder whether they will obtain the same results. This equivalence for some considered decays has been discussed in Refs. [7, 16, 20], in particular for charmed meson decays, Ref. [20] has shown the equivalence in the presence of SU(3) symmetry breaking effects. In Ref. [22], two of us have explored two-body B/D meson decays, $B \rightarrow PP$ and $D \rightarrow PP$ and pointed out that in the exact SU(3) symmetry limit, the two methods are consistent when all contributions are included. However this equivalence is nontrivial: a few amplitudes are suppressed and thus were not

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included in some TDA analysis; among the known diagrammatic amplitudes, one of them is not SU(3) independent, and should be absorbed into other amplitudes. Actually, in charm meson decays, the fact that one of the known amplitudes, T , A , C , E , is not independent has already been noticed in Ref. [20].

In this work, we extend our analysis to several other types of two body decays of B/D and B_c mesons and also beauty/charm baryons to show the equivalence of the TDA and IRA methods. We will also work in the exact flavor SU(3) limit throughout this work. For two-body decays of beauty/charm baryons, we clarify some subtleties including the description of baryon representation, one or two indices for $\bar{3}$, in relation to TDA. As we will show, it is easy to determine the independent amplitudes in IRA while TDA gives some redundancy. A few amplitudes are not independent and therefore should be absorbed into other amplitudes. Despite this disadvantage, the topological nature of TDA is still helpful for understanding the internal dynamics underlying in b decays in a more intuitive way.

The rest of this paper is arranged as follows. In Sect. 2, we briefly summarize the SU(3) properties of various inputs. In Sect. 3, we give the results for $B \rightarrow PP$ in the TDA and IRA methods to set up the notation. Then we provide results for $B \rightarrow PV$, VV and discuss some points specialized to these decays. In Sect. 4, we give the results for $D \rightarrow PP$, VV , PV in the TDA and IRA methods. In Sect. 5, we carry out a similar analysis for $B_c \rightarrow DP$, DV . In Sects. 6 and 7, we respectively discuss beauty and charm baryon decays into an octet baryon and an octet pseudo-scalar meson, while for beauty baryon decays we also consider the final states containing a decuplet baryon. The expanded amplitudes and relations given these sections are useful for a global analysis when enough data is available in the future. In Sect. 8 we summarize our results. In the “Appendix”, we give the relations for different parametrizations in TDA and IRA methods for bottom and charmed baryon decays.

2 SU(3) properties of Hamiltonian and Hadron states

2.1 Hadron multiplets

Several classes of heavy hadron, containing at heavy quark b or c , will be considered in this work. The involved processes include decays of heavy SU(3) triplet mesons B and D into PP , PV , VV , and the B_c meson into DP , DV . For heavy baryons, the decay processes include a heavy anti-triplet $T_{c\bar{3}}$ or a $T_{b\bar{3}}$ decays into a baryon in the decuplet T_{10} plus a light meson, and decay into a baryon in the octet T_8 plus a light meson. We display the hadron SU(3) properties and their component fields in this section.

The B_c meson contains no light quark and it is a singlet. Heavy mesons containing one heavy quark

$$\begin{aligned}(B_i) &= (B^-(b\bar{u}), \bar{B}^0(b\bar{d}), \bar{B}_s^0(b\bar{s})), \\(D_i) &= (D^0(c\bar{u}), D^+(c\bar{d}), D_s^+(c\bar{s})),\end{aligned}\quad (1)$$

are flavor SU(3) anti-triplets.

Light pseudoscalar P and vector V mesons are mixtures of octets and singlets so that each of them contain nine hadrons:

$$\begin{aligned}P &= \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_8}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & \frac{\eta_1}{\sqrt{3}} - 2\frac{\eta_8}{\sqrt{6}} \end{pmatrix}, \\V &= \begin{pmatrix} \frac{\rho^{0+\omega}}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho^{0+\omega}}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix},\end{aligned}\quad (2)$$

where ω and ϕ mix in an ideal form. The η and η' are mixtures of η_8 and η_1 with the mixing angle θ :

$$\begin{aligned}\eta &= \cos\theta\eta_8 + \sin\theta\eta_1, \\ \eta' &= -\sin\theta\eta_8 + \cos\theta\eta_1.\end{aligned}\quad (3)$$

Since η_8 and η_1 are not physical states, optionally one can choose the η_q and η_s basis for the η mixing, which are defined so that the pseudoscalar octets P has the same form of parametrization as vector octets V :

$$P = \begin{pmatrix} \frac{\pi^0+\eta_q}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0+\eta_q}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_s \end{pmatrix}.\quad (4)$$

with

$$\eta_8 = \frac{1}{\sqrt{3}}\eta_q - \sqrt{\frac{2}{3}}\eta_s, \quad \eta_1 = \sqrt{\frac{2}{3}}\eta_q + \frac{1}{\sqrt{3}}\eta_s\quad (5)$$

An advantage of the parametrization in Eq. (4) is that there is a one-to-one correspondence between the decay amplitudes of channels with vector final state and that of channels with pseudoscalar final state in the SU(3) limit.

A charmed or a bottom baryons with two light quarks can form an anti-triplet or sextet. Most members of the sextet can decay through strong interaction or electromagnetic interactions. The only exceptions are Ω_b and Ω_c [24]. We will concentrate on anti-triplet weak decays. For the anti-triplet bottom and charmed baryons, we have the following matrix expressions:

$$(T_{c\bar{3}}^{ij}) = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix},$$

$$(T_{\mathbf{b}\bar{\mathbf{3}}}^{ij}) = \begin{pmatrix} 0 & \Lambda_b^0 & \Xi_b^0 \\ -\Lambda_b^0 & 0 & \Xi_b^- \\ -\Xi_b^0 & -\Xi_b^- & 0 \end{pmatrix}. \quad (6)$$

One can also contract the above matrix with the anti-symmetric tensor ϵ_{ijk} ($\epsilon_{123} = +1$) to have $T_{\bar{\mathbf{3}},i} = \epsilon_{ijk} T_{\bar{\mathbf{3}}}^{jk}$ with

$$((T_{\bar{\mathbf{3}}})_i) = (\Xi_c^0 - \Xi_c^+ \Lambda_c^+), \quad ((T_{\mathbf{b}\bar{\mathbf{3}}})_i) = (\Xi_b^- - \Xi_b^0 \Lambda_b^0). \quad (7)$$

The lowest-lying baryon octet is given by:

$$((T_8)_i^j) = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix}. \quad (8)$$

One can also contract the above with ϵ_{ijk} to have $(T_8)_{ijk} \equiv \epsilon_{ijn}(T_8)_k^n$.

The light baryon decuplet is given as:

$$\begin{aligned} T_{10}^{111} &= \Delta^{++}, \quad T_{10}^{112} = T_{10}^{121} = T_{10}^{211} = \frac{1}{\sqrt{3}}\Delta^+, \\ T_{10}^{222} &= \Delta^-, \quad T_{10}^{122} = T_{10}^{212} = T_{10}^{221} = \frac{1}{\sqrt{3}}\Delta^0, \\ T_{10}^{113} &= T_{10}^{131} = T_{10}^{311} = \frac{1}{\sqrt{3}}\Sigma'^+, \\ T_{10}^{223} &= T_{10}^{232} = T_{10}^{322} = \frac{1}{\sqrt{3}}\Sigma'^-, \\ T_{10}^{123} &= T_{10}^{132} = T_{10}^{213} = T_{10}^{231} = T_{10}^{312} = T_{10}^{321} = \frac{1}{\sqrt{6}}\Sigma'^0, \\ T_{10}^{133} &= T_{10}^{313} = T_{10}^{331} = \frac{1}{\sqrt{3}}\Xi'^0, \\ T_{10}^{233} &= T_{10}^{323} = T_{10}^{332} = \frac{1}{\sqrt{3}}\Xi'^-, \quad T_{10}^{333} = \Omega^-. \end{aligned} \quad (9)$$

2.2 SU(3) properties of effective Hamiltonian

Effective Hamiltonian for charmless b decays

In the SM weak decays of charmless b decays are induced by the following electroweak effective Hamiltonian [25–27]:

$$\mathcal{H}_{eff}^b = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{uq}^* [C_1 O_1 + C_2 O_2] - V_{tb}V_{tq}^* \sum_{i=3}^{10} C_i O_i \right\} + \text{h.c.} \quad (10)$$

Here G_F is the Fermi constant, and the V_{uq} and V_{tq} are CKM matrix elements. The O_i is a four-quark operator with C_i as its Wilson coefficient. The explicit forms of O_i s are given as follows:

$$O_1 = (\bar{q}^i u^j)_{V-A} (\bar{u}^j b^i)_{V-A},$$

$$O_2 = (\bar{q}u)_{V-A} (\bar{u}b)_{V-A},$$

$$O_3 = (\bar{q}b)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A},$$

$$O_4 = (\bar{q}^i b^j)_{V-A} \sum_{q'} (\bar{q}'^j q'^i)_{V-A},$$

$$O_5 = (\bar{q}b)_{V-A} \sum_{q'} (\bar{q}' q')_{V+A},$$

$$O_6 = (\bar{q}^i b^j)_{V-A} \sum_{q'} (\bar{q}'^j q'^i)_{V+A},$$

$$O_7 = \frac{3}{2} (\bar{q}b)_{V-A} \sum_{q'} e_{q'} (\bar{q}' q')_{V+A},$$

$$O_8 = \frac{3}{2} (\bar{q}^i b^j)_{V-A} \sum_{q'} e_{q'} (\bar{q}'^j q'^i)_{V+A},$$

$$O_9 = \frac{3}{2} (\bar{q}b)_{V-A} \sum_{q'} e_{q'} (\bar{q}' q')_{V-A},$$

$$O_{10} = \frac{3}{2} (\bar{q}^i b^j)_{V-A} \sum_{q'} e_{q'} (\bar{q}'^j q'^i)_{V-A}. \quad (11)$$

$q = d, s$ and $q' = u, d, s$. Here the $V - A$ and $V + A$ corresponds a left-handed $\gamma_\mu(1 - \gamma_5)$ and a right-handed current $\gamma_\mu(1 + \gamma_5)$ respectively.

In the SU(3) group for light flavors, tree operators $O_{1,2}$ and electroweak penguin operators O_{7-10} can be decomposed in terms of a vector H_3^i , a traceless tensor antisymmetric in upper indices, $(H_6)_k^{[ij]}$, and a traceless tensor symmetric in upper indices, $(H_{15})_k^{[ij]}$. For the $\Delta S = 0(b \rightarrow d)$ decays, the non-zero components of the effective Hamiltonian are [8, 11, 12]:

$$\begin{aligned} (H_3)^2 &= 1, \quad (H_6)_1^{12} = -(H_6)_1^{21} = (H_6)_3^{23} = -(H_6)_3^{32} = 1, \\ 2(H_{15})_1^{12} &= 2(H_{15})_1^{21} = -3(H_{15})_2^{22} \\ &= -6(H_{15})_3^{23} = -6(H_{15})_3^{32} = 6. \end{aligned} \quad (12)$$

For the $\Delta S = -1(b \rightarrow s)$ decays the nonzero entries in the H_3, H_6, H_{15} can be obtained from Eq. (12) with the exchange $2 \leftrightarrow 3$ corresponding to the $d \leftrightarrow s$ exchange.

QCD penguin operators O_{3-6} behave as the $\bar{\mathbf{3}}$ representation. For the magnetic moment operators, the color magnetic moment operator $O_{8g} = (g_s m_b / 4\pi) \bar{s} \sigma^{\mu\nu} T^a G_{\mu\nu}^a (1 + \gamma_5) b$ is an SU(3) triplet, while the electromagnetic moment operator $O_{7\gamma} = \frac{em_b}{4\pi} \bar{s} \sigma^{\mu\nu} F_{\mu\nu} (1 + \gamma_5) b$ can be effectively incorporated into the O_{7-10} . Thus both of them are not included in Eq. (10) and the above decomposition is complete.

The irreducible representation amplitude (IRA) method of describing related decays is to decompose effective Hamiltonian according to the above mentioned representations and construct the amplitudes accordingly. On the other hand the topological diagrams (TDA) method is to take the effective Hamiltonian with two light anti-quarks and a light quark

H_k^{ij} to represent $\bar{q}u\bar{u}b$ with $i = \bar{u}$, $k = u$ and $j = \bar{q}$ (omitting the Lorentz indices), and then contract the indices with initial and final hadron states. In this way the decays are represented by diagrams following the quark line flows. Note that in the TDA method, the indices i and j ordering matters which are neither symmetry nor anti-symmetric. They are not traceless neither.

The effective Hamiltonian have both tree and loop contributions. When strong penguin and electroweak penguin are all included the tree and loop contributions have $\bar{3}$, 6 and $\bar{15}$ representations. The independent amplitudes have the same numbers, except that one can make one of the tree or penguin amplitude real and the rest all in principle complex. Using the unitarity property of the CKM matrix $V_{ub}V_{uq}^* + V_{cb}V_{cq}^* + V_{tb}V_{tq}^* = 0$, one can also rewrite c loop induced penguin contributions into amplitude proportional to $V_{ub}V_{uq}^*$ and $V_{tb}V_{tq}^*$,

$$\mathcal{A} = V_{ub}V_{uq}^*\mathcal{A}_u + V_{tb}V_{tq}^*\mathcal{A}_t. \quad (13)$$

For simplicity, we refer to $\mathcal{A}_u$ as “tree” amplitude since it is dominated by tree contributions with modifications from u and c loop contributions. $\mathcal{A}_t$ is “penguin” amplitude with c and t loop contributions. It is necessary to stress that not all contributions in $\mathcal{A}_u$ are tree diagrams in topology, and the same for $\mathcal{A}_t$.

In Ref. [22], two of us have shown that both $\mathcal{A}_u$ and $\mathcal{A}_t$ have similar form of amplitudes with $SU(3)$ representations. Thus in our later discussions we will concentrate on the $\mathcal{A}_u$ amplitudes. One can easily obtain the $\mathcal{A}_t$ amplitudes by just changing the amplitude labels.

Effective electroweak Hamiltonian for c hadronic decays

For hadronic decays of charmed hadrons, the effective Hamiltonian with $\Delta C = 1$ is given as:

$$\begin{aligned} \mathcal{H}_{eff}^c = \frac{G_F}{\sqrt{2}} \{ & V_{cs}V_{ud}^*[C_1O_1^{sd} + C_2O_2^{sd}] \\ & + V_{cd}V_{ud}^*[C_1O_1^{dd} + C_2O_2^{dd}] \\ & + V_{cs}V_{us}^*[C_1O_1^{ss} + C_2O_2^{ss}] \\ & + V_{cd}V_{us}^*[C_1O_1^{ds} + C_2O_2^{ds}] \}, \end{aligned} \quad (14)$$

where we have neglected the highly suppressed penguin contributions, and

$$\begin{aligned} O_1^{sd} &= [\bar{s}^i\gamma_\mu(1-\gamma_5)c^j][\bar{u}^i\gamma^\mu(1-\gamma_5)d^j], \\ O_2^{sd} &= [\bar{s}\gamma_\mu(1-\gamma_5)c][\bar{u}\gamma^\mu(1-\gamma_5)d], \end{aligned} \quad (15)$$

while other operators can be obtained by replacing the d, s quark fields. Tree operators transform under the flavor $SU(3)$ symmetry as $\bar{3} \otimes 3 \otimes \bar{3} = \bar{3} \oplus \bar{3} \oplus 6 \oplus \bar{15}$.

For the Cabibbo allowed $c \rightarrow su\bar{d}$ transition, we have amplitudes proportional to $V_{cs}V_{ud}^*$ and the Hamiltonians are:

$$(H_6)_2^{31} = -(H_6)_2^{13} = 1, \quad (H_{\bar{15}})_2^{31} = (H_{\bar{15}})_2^{13} = 1. \quad (16)$$

For the doubly Cabibbo suppressed $c \rightarrow du\bar{s}$ transition, we have amplitudes to be proportional to $V_{cd}V_{us}^*$ and the Hamiltonians are:

$$(H_6)_3^{21} = -(H_6)_3^{12} = 1, \quad (H_{\bar{15}})_3^{21} = (H_{\bar{15}})_3^{12} = 1. \quad (17)$$

For decays proportional to $V_{cs}V_{us}^*$, we have:

$$(H_6)_3^{31} = -(H_6)_3^{13} = 1, \quad (H_{\bar{15}})_3^{31} = (H_{\bar{15}})_3^{13} = 1, \quad (18)$$

and for decays proportional to $V_{cd}V_{ud}^*$, we have:

$$(H_6)_2^{12} = -(H_6)_2^{21} = 1, \quad (H_{\bar{15}})_2^{12} = (H_{\bar{15}})_2^{21} = -1. \quad (19)$$

For singly Cabibbo suppressed decays, $c \rightarrow u\bar{d}d$ and $c \rightarrow u\bar{s}s$ transitions have approximately equal magnitudes but opposite signs: $V_{cd}V_{ud}^* = -V_{cs}V_{us}^* - V_{cb}V_{ub}^* \approx -V_{cs}V_{us}^*$ (with 10^{-3} deviation). As a result, the contributions from the $\bar{3}$ representation vanish, and one has the nonzero components contributed only by 6 and $\bar{15}$ representations.

For the singly Cabibbo-suppressed transition, there are also loop contributions proportional to $V_{cb}V_{ub}^*$. Such loop contributions are small so that we will concentrate on the dominant amplitude proportional to $V_{cs}V_{us}^*$. However, one can include these contributions by adding a 3 representation in the Hamiltonian.

Again, we use the above $SU(3)$ decompositions for IRA analysis and use the effective Hamiltonian H_k^{ij} with $i = \bar{s}$, $j = \bar{u}$ and $k = q$ for TDA analysis to trace the quark line flows.

3 Charmless two-body B decays

3.1 $B \rightarrow PP$ decays

Let us start with the $B \rightarrow PP$ decays. The generic amplitude is decomposed according to CKM matrix elements:

$$\begin{aligned} \mathcal{A} &= V_{ub}V_{uq}^*\mathcal{A}_u^{IRA} + V_{tb}V_{tq}^*\mathcal{A}_t^{IRA} \\ &= V_{ub}V_{uq}^*\mathcal{A}_u^{TDA} + V_{tb}V_{tq}^*\mathcal{A}_t^{TDA}, \end{aligned} \quad (20)$$

where the amplitudes expressed by IRA and TDA should be equivalent. Although in fact this equivalence is concrete and obvious, as we argued in [22], it has not been thoroughly discussed in the literature.

To obtain IRA, one takes various representations in Eq. (12) and contracts all indices in B_i and light meson P_j^i with various combinations:

$$\begin{aligned} \mathcal{A}_u^{IRA} &= A_3^T B_i(H_3)^i P_k^j P_j^k + C_3^T B_i(H_3)^k P_j^i P_j^k \\ &\quad + B_3^T B_i(H_3)^i P_k^k P_j^j + D_3^T B_i(H_3)^j P_j^i P_j^k \\ &\quad + A_6^T B_i(H_6)_k^{[ij]} P_j^l P_l^k + C_6^T B_i(H_6)_k^{[jl]} P_j^i P_l^k \\ &\quad + B_6^T B_i(H_6)_k^{[ij]} P_j^k P_l^l \end{aligned}$$

Fig. 1 Topological diagrams for the amplitudes with CKM factor $V_{ub}V_{uq}^*$ in $B \rightarrow PP$ and $B \rightarrow VV$ decays. We work in the effective field theory at m_b scale, and thus there exists four-quark interaction operators as shown in the above. The quark flavors corresponding to

these operators are explicitly given. In these diagrams, when the $\bar{u}u$ annihilate, two or more gluons are needed to create one pair of quarks with flavor u, d, s , which are denoted by the unspecified lines

$$+A_{15}^T B_i (H_{15}^*)_{jk} P_j^l P_k^l + C_{15}^T B_i (H_{15}^*)_{jk} P_j^i P_k^l + B_{15}^T B_i (H_{15}^*)_{jk} P_j^k P_l^l. \quad (21)$$

For the TDA decomposition, one can classify the different topologies of diagram as [18]:

- (i) T , denoting the color-allowed tree amplitude with W emission;
- (ii) C , denoting the color-suppressed tree diagram;
- (iii) E denoting the W -exchange diagram;
- (iv) P , corresponding to the QCD penguin contributions;
- (v) S , being the flavor singlet QCD penguin;
- (vi) A , annihilation diagrams.

With each coefficient denoted by the above topologies, one has the TDA decomposition as:

$$\begin{aligned} A_u^{TDA} = & T B_i H_k^{jl} P_j^i P_l^k + C B_i H_k^{lj} P_j^i P_l^k \\ & + A B_i H_j^{il} P_k^j P_l^k + E B_i H_j^{li} P_k^j P_l^k \\ & + S^u B_i H_l^{lj} P_j^i P_k^k + P^u B_i H_l^{lk} P_j^i P_k^j \\ & + P_A^u B_i H_l^{li} P_k^j P_j^k + S_S^u B_i H_l^{li} P_j^j P_k^k \\ & + E_S^u B_i H_l^{ji} P_j^l P_k^k + A_S^u B_i H_l^{ij} P_j^l P_k^k. \end{aligned} \quad (22)$$

According to this decomposition, topological diagrams for $B \rightarrow PP$ decays can be found in Fig. 1. Since we work in the effective field theory at m_b scale, as shown in this figure there exists four-quark interaction operators. In these diagrams, the quark flavors corresponding to these operators are explicitly given. When the $\bar{u}u$ annihilate, two or more gluons are needed to create one pair of quarks with flavor u, d, s , which are denoted by the unspecified lines. Apart

from the ordinary T , C , A , E , we have also included the other SU(3) irreducible amplitudes, most of which come from loop diagrams, and/or the flavor singlet diagram.

Expanding Eqs. (21,22), one obtains $B \rightarrow PP$ amplitudes in Table 1, where the IRA amplitudes are consistent with Ref. [12] and an earlier work [23]. Since we have decomposed the effective Hamiltonian into irreducible representations, one may expect that there are 10 independent amplitudes for $\mathcal{A}_u$ and similarly 10 amplitudes for $\mathcal{A}_t$. A careful examination shows that the A_6^T can be absorbed into B_6^T and C_6^T with a redefinition:

$$C_6^{T'} = C_6^T - A_6^T, \quad B_6^{T'} = B_6^T + A_6^T. \quad (23)$$

This combination can also be found explicitly from Table 1. After eliminating the redundant amplitude, there are actually only 18 ($\mathcal{A}_u$ and $\mathcal{A}_t$ contribute 9 each) SU(3) independent amplitudes. Since one overall phase can be chosen free, there are 35 independent real independent parameters. If one consider $\eta_1 - \eta_8$ mixing, the mixing angle θ should also be introduced.

We list all TDA amplitudes in Table 1. It is necessary to point out that the last six diagrams in Fig. 1 are omitted in some SU(3) TDA analysis [16,18], while other amplitudes are consistent with Refs. [16,18]. However only by including them the complete equivalence of IRA and TDA can be established. One of the 10 TDA amplitudes must be redundant. Such redundancy can be understood through the following relations between the IRA and TDA amplitudes:

$$\begin{aligned} T + E &= 4A_{15}^T + 2C_6^{T'} + 4C_{15}^T, \\ C - E &= -4A_{15}^T - 2C_6^{T'} + 4C_{15}^T, \\ A + E &= 8A_{15}^T, \\ P^u - E &= -5A_{15}^T + C_3^T - C_6^{T'} - C_{15}^T, \\ P_A^u + \frac{E}{2} &= A_3^T + A_{15}^T, \\ E_S^u + E &= 4A_{15}^T - 2B_6^{T'} + 4B_{15}^T, \\ A_S^u - E &= -4A_{15}^T + 2B_6^{T'} + 4B_{15}^T, \\ S_S^u - \frac{E}{2} &= -2A_{15}^T + B_3^T + B_6^{T'} - B_{15}^T, \\ S^u + E &= 4A_{15}^T - B_6^{T'} - B_{15}^T + C_6^{T'} - C_{15}^T + D_3^T. \end{aligned} \quad (24)$$

We have adopted the choice in which E is always in companion with another amplitude. It is also possible to replace the role of E by one of the amplitudes A , C or even T . One can also reversely obtain:

$$\begin{aligned} A_3^T &= -\frac{A}{8} + \frac{3E}{8} + P_A^u, \\ B_3^T &= S_S^u + \frac{3E_S^u - A_S^u}{8}, \\ C_3^T &= \frac{1}{8}(3A - C - E + 3T) + P^u, \end{aligned}$$

$$\begin{aligned} D_3^T &= S^u + \frac{1}{8}(3C - E_S^u + 3A_S^u - T), \\ B_6^{T'} &= \frac{1}{4}(A - E + A_S^u - E_S^u), \\ C_6^{T'} &= \frac{1}{4}(-A - C + E + T), \\ A_{15}^T &= \frac{A + E}{8}, \\ B_{15}^T &= \frac{A_S^u + E_S^u}{8}, \\ C_{15}^T &= \frac{C + T}{8}. \end{aligned} \quad (25)$$

Similar analysis for the $\mathcal{A}_t$ contributions can be obtained with the replacement for the IRA:

$$A_i^T \rightarrow A_i^P, \quad B_i^T \rightarrow B_i^P, \quad C_i^T \rightarrow C_i^P, \quad D_i^T \rightarrow D_i^P. \quad (26)$$

while for TDA, we have:

$$\begin{aligned} T &\rightarrow P_T, \quad C \rightarrow P_C, \quad A \rightarrow P_{TA}, \quad P^u \rightarrow P, \quad E \rightarrow P_{TE}, \\ P_A^u &\rightarrow P_A, \quad E_S^u \rightarrow P_{AS}, \quad A_S^u \rightarrow P_{ES}, \quad S_S^u \rightarrow P_{SS}, \quad S^u \rightarrow S. \end{aligned} \quad (27)$$

It should be pointed out that the amplitudes generated $Q_{7,8,9,10}$ operators have the form $(2/3)\bar{u}u - (1/3)(\bar{d}d + \bar{s}s)$ and these amplitudes can be again be as a sum of a tree-like operator $\bar{u}u$ and a penguin-like operator $-(1/3)(\bar{u}u + \bar{d}d + \bar{s}s)$. Penguin-like contributions have been absorbed into P , P_A , S , P_{SS} , while tree-like amplitudes are denoted as P_T , P_C , P_{TA} , P_{TE} , P_{ES} . It should be noticed that the tree-like amplitudes P_T and P_C are also denoted as P_{EW} , $P_{EW,C}$ in some references.

3.1.1 Impact of the new TDA amplitudes

The new TDA amplitudes in Fig. 1 may play an important role in understanding CP violation (CPV) phenomena. Without the new TDA amplitudes, some decays only have terms proportional to $V_{tq}^* V_{tb}$, such as $\bar{B}^0 \rightarrow K^0 \bar{K}^0$ and $\bar{B}_s^0 \rightarrow K^0 \bar{K}^0$. For instance, in Ref. [18], the amplitudes for $\bar{B}^0 \rightarrow K^0 \bar{K}^0$ read:

$$A(\bar{B}^0 \rightarrow K^0 \bar{K}^0) = V_{tb} V_{td}^* \left(P - \frac{1}{2} P_{EW}^C + 2P_A \right). \quad (28)$$

This would imply the CP violating asymmetry is *identically zero*. However, as we have shown, these two decays receive contributions from the $P^u + 2P_A^u$ multiplied by $V_{uq}^* V_{ub}$:

$$\begin{aligned} A(\bar{B}^0 \rightarrow K^0 \bar{K}^0) &= V_{ub} V_{ud}^* (P^u + 2P_A^u) \\ &\quad + V_{tb} V_{td}^* (P + 2P_A^u). \end{aligned} \quad (29)$$

Table 1 Decay amplitudes for two-body $B \rightarrow PP$ decays. Only the amplitudes with CKM factor $V_{ub}V_{uq}^*$ are shown in this table and the following ones. “Penguin” amplitudes with $V_{tb}V_{tq}^*$ can be obtained with the replacement Eq. (26) and Eq. (27)

$b \rightarrow d$	IRA	TDA
$B^- \rightarrow \pi^0 \pi^-$	$4\sqrt{2}C_{15}^T$	$(C + T)/\sqrt{2}$
$B^- \rightarrow \pi^- \eta_q$	$\sqrt{2}(A_6^T + 3A_{15}^T + B_6^T + 3B_{15}^T + C_3^T + 2C_{15}^T + D_3^T)$	$(2A_s^u + 2A + C + 2P^u + 2S^u + T)/\sqrt{2}$
$B^- \rightarrow \pi^- \eta_s$	$B_6^T + 3B_{15}^T + C_6^T - C_{15}^T + D_3^T$	$A_s^u + S^u$
$B^- \rightarrow K^0 K^-$	$A_6^T + 3A_{15}^T + C_3^T - C_6^T - C_{15}^T$	$A + P^u$
$\bar{B}^0 \rightarrow \pi^+ \pi^-$	$2A_3^T - A_6^T + A_{15}^T + C_3^T + C_6^T + 3C_{15}^T$	$2P_A^u + E + P^u + T$
$\bar{B}^0 \rightarrow \pi^0 \pi^0$	$2A_3^T - A_6^T + A_{15}^T + C_3^T + C_6^T - 5C_{15}^T$	$2P_A^u - C + E + P^u$
$\bar{B}^0 \rightarrow \pi^0 \eta_q$	$-A_6^T + 5A_{15}^T - B_6^T + 5B_{15}^T - C_3^T + 2C_{15}^T - D_3^T$	$E_s^u + E - P^u - S^u$
$\bar{B}^0 \rightarrow \pi^0 \eta_s$	$-(B_6^T - 5B_{15}^T + C_6^T - C_{15}^T + D_3^T)/\sqrt{2}$	$(E_s^u - S^u)/\sqrt{2}$
$\bar{B}^0 \rightarrow K^+ K^-$	$2(A_3^T + A_{15}^T)$	$2P_A^u + E$
$\bar{B}^0 \rightarrow K^0 \bar{K}^0$	$2A_3^T + A_6^T - 3A_{15}^T + C_3^T - C_6^T - C_{15}^T$	$2P_A^u + P^u$
$\bar{B}^0 \rightarrow \eta_q \eta_q$	$2A_3^T - A_6^T + A_{15}^T + 4B_3^T - 2B_6^T + 2B_{15}^T + C_3^T - C_6^T + C_{15}^T + 2D_3^T$	$2P_A^u + C + 2E_s^u + E + P^u + 2S^u + 4S_s^u$
$\bar{B}^0 \rightarrow \eta_q \eta_s$	$(4B_3^T + B_6^T - B_{15}^T + C_6^T - C_{15}^T + D_3^T)/\sqrt{2}$	$(E_s^u + S^u + 4S_s^u)/\sqrt{2}$
$\bar{B}^0 \rightarrow \eta_s \eta_s$	$2(A_3^T + A_6^T - A_{15}^T + B_3^T + B_6^T - B_{15}^T)$	$2(P_A^u + S_s^u)$
$\bar{B}_s^0 \rightarrow \pi^0 K^0$	$(A_6^T + A_{15}^T - C_3^T - C_6^T + 5C_{15}^T)/\sqrt{2}$	$(C - P^u)/\sqrt{2}$
$\bar{B}_s^0 \rightarrow \pi^- K^+$	$-A_6^T - A_{15}^T + C_3^T + C_6^T + 3C_{15}^T$	$P^u + T$
$\bar{B}_s^0 \rightarrow K^0 \eta_q$	$-(A_6^T + A_{15}^T + 2B_6^T + 2B_{15}^T - C_3^T + C_6^T - C_{15}^T - 2D_3^T)/\sqrt{2}$	$(C + P^u + 2S^u)/\sqrt{2}$
$\bar{B}_s^0 \rightarrow K^0 \eta_s$	$-A_6^T - A_{15}^T - B_6^T - B_{15}^T + C_3^T - 2C_{15}^T + D_3^T$	$P^u + S^u$
$b \rightarrow s$	IRA	TDA
$B^- \rightarrow \pi^0 K^-$	$(A_6^T + 3A_{15}^T + C_3^T - C_6^T + 7C_{15}^T)/\sqrt{2}$	$(A + C + P^u + T)/\sqrt{2}$
$B^- \rightarrow \pi^- \bar{K}^0$	$A_6^T + 3A_{15}^T + C_3^T - C_6^T - C_{15}^T$	$A + P^u$
$B^- \rightarrow K^- \eta_q$	$(A_6^T + 3A_{15}^T + 2B_6^T + 6B_{15}^T + C_3^T + C_6^T + 5C_{15}^T + 2D_3^T)/\sqrt{2}$	$(2A_s^u + A + C + P^u + 2S^u + T)/\sqrt{2}$
$B^- \rightarrow K^- \eta_s$	$A_6^T + 3A_{15}^T + B_6^T + 3B_{15}^T + C_3^T - 2C_{15}^T + D_3^T$	$A_s^u + A + P^u + S^u$
$\bar{B}^0 \rightarrow \pi^+ K^-$	$-A_6^T - A_{15}^T + C_3^T + C_6^T + 3C_{15}^T$	$P^u + T$
$\bar{B}^0 \rightarrow \pi^0 \bar{K}^0$	$(A_6^T + A_{15}^T - C_3^T - C_6^T + 5C_{15}^T)/\sqrt{2}$	$(C - P^u)/\sqrt{2}$
$\bar{B}^0 \rightarrow \bar{K}^0 \eta_q$	$-(A_6^T + A_{15}^T + 2B_6^T + 2B_{15}^T - C_3^T + C_6^T - C_{15}^T - 2D_3^T)/\sqrt{2}$	$(C + P^u + 2S^u)/\sqrt{2}$
$\bar{B}^0 \rightarrow \bar{K}^0 \eta_s$	$-A_6^T - A_{15}^T - B_6^T - B_{15}^T + C_3^T - 2C_{15}^T + D_3^T$	$P^u + S^u$
$\bar{B}_s^0 \rightarrow \pi^+ \pi^-$	$2(A_3^T + A_{15}^T)$	$2P_A^u + E$
$\bar{B}_s^0 \rightarrow \pi^0 \pi^0$	$2(A_3^T + A_{15}^T)$	$2(P_A^u + (E)/2)$
$\bar{B}_s^0 \rightarrow \pi^0 \eta_q$	$-2(A_6^T - 2A_{15}^T + B_6^T - 2B_{15}^T)$	$E_s^u + E$
$\bar{B}_s^0 \rightarrow \pi^0 \eta_s$	$-\sqrt{2}(B_6^T - 2B_{15}^T + C_6^T - 2C_{15}^T)$	$(C + E_s^u)/\sqrt{2}$
$\bar{B}_s^0 \rightarrow K^+ K^-$	$2A_3^T - A_6^T + A_{15}^T + C_3^T + C_6^T + 3C_{15}^T$	$2P_A^u + E + P^u + T$
$\bar{B}_s^0 \rightarrow K^0 \bar{K}^0$	$2A_3^T + A_6^T - 3A_{15}^T + C_3^T - C_6^T - C_{15}^T$	$2P_A^u + P^u$
$\bar{B}_s^0 \rightarrow \eta_q \eta_q$	$2(A_3^T + A_{15}^T + 2(B_3^T + B_{15}^T))$	$2(P_A^u + E_s^u + (E)/2 + 2S_s^u)$
$\bar{B}_s^0 \rightarrow \eta_q \eta_s$	$\sqrt{2}(2B_3^T - B_{15}^T + C_{15}^T + D_3^T)$	$(C + E_s^u + 2S^u + 4S_s^u)/\sqrt{2}$
$\bar{B}_s^0 \rightarrow \eta_s \eta_s$	$A_3^T - 2A_{15}^T + B_3^T - 2B_{15}^T + C_3^T - 2C_{15}^T + D_3^T$	$P_A^u + P^u + S^u + S_s^u$

Therefore a non-vanishing direct CP asymmetry is obtained, as noticed in many references for instance Refs. [28–30]. This would certainly affect the search for new physics in a precise CP violation measurement.

Most new TDA amplitudes in Fig. 1 arise from higher order loop corrections, and thus they are likely small in magnitude. However, sometimes they can not be completely neglected. In Ref. [12], the authors have performed a fit of $B \rightarrow PP$ decays in the IRA framework. Depending on different choices of data, four cases were considered in their analysis [12]. Here for illustration, we give their results in case 4:

$$\begin{aligned} |C_3^T| &= -0.211 \pm 0.027, \quad \delta_3^T = (-140 \pm 6)^\circ, \\ |B_{15}^T| &= -0.038 \pm 0.016, \quad \delta_{B_{15}^T} = (78 \pm 48)^\circ, \end{aligned} \quad (30)$$

where the magnitudes and strong phases are defined relative to the amplitude C_3^P . From Eq. (25), one can find that the C_3^T is a mixture of T , C and others, while the B_{15}^T equals $(E_S^u + A_S^u)/8$. The fitted results in Eq. (30) indicate, compared to C_3^T , the B_{15}^T could reach 20% in magnitude, and more notably, the strong phases are sizably different. The fact that the B_{15}^T , namely E_S^u and A_S^u , have non-negligible contributions supports our call for a complete analysis.

3.1.2 Comparison with QCDF amplitudes

The topological amplitudes in $B \rightarrow PP$ decays can be compared to the QCDF amplitude in Ref. [31]. Such a comparison requests two remarks. Firstly, in our decomposition, we adopt the CKM matrix elements $V_{ub}V_{uq}^*$ and $V_{tb}V_{tq}^*$, while Ref. [31] used $V_{ub}V_{uq}^*$ and $V_{cb}V_{cq}^*$. The unitarity of CKM matrix guarantees the equivalence of the two approaches. So we will directly compare the “tree” $\mathcal{A}_u$ and “penguin” $\mathcal{A}_t$ amplitudes, though some of them might be recombined in order to have the same CKM factors. Secondly, we have decomposed one part of the electroweak penguin into the QCD penguin as shown in Sect. 2, and we will do so for QCDF amplitudes too.

We have the following correspondence between the SU(3) TDA amplitudes and the QCDF amplitudes for “tree” amplitudes:

$$\begin{aligned} T &\rightarrow \alpha_1, \quad P^u \rightarrow \alpha_4^u + \beta_3^u, \\ C &\rightarrow \alpha_2, \quad S^u \rightarrow \alpha_3^u + \beta_{S3}^u, \quad A \rightarrow \beta_2, \\ E &\rightarrow \beta_1, \quad P_A^u \rightarrow \beta_4^u, \quad A_S^u \rightarrow \beta_{S2}^u, \\ E_S^u &\rightarrow \beta_{S1}^u, \quad S_S^u \rightarrow \beta_{S4}^u. \end{aligned} \quad (31)$$

where the notations α_i and β_i are from Ref. [31]. The correspondence for “penguin” ones is given as:

$$\begin{aligned} P_T &\rightarrow \alpha_{4,EW}^c, \quad P \rightarrow \alpha_4^c + \beta_3^c, \\ P_C &\rightarrow \alpha_{3,EW}^c, \quad S \rightarrow \alpha_3^c + \beta_{S3}^c, \end{aligned}$$

$$\begin{aligned} P_{TA} &\rightarrow \beta_{3,EW}^c, \\ P_{TE} &\rightarrow \beta_{4,EW}^c, \quad P_A \rightarrow \beta_4^c, \\ P_{ES} &\rightarrow \beta_{S3,EW}^c, \quad P_{AS} \rightarrow \beta_{S4,EW}^c, \quad P_{SS} \rightarrow \beta_{S4}^c. \end{aligned} \quad (32)$$

3.1.3 U-Spin relations

Some decay channels shown in Table 1 with $\Delta S = 0$ and $\Delta S = 1$ are related by U-spin, the $d \leftrightarrow s$ exchange symmetry. The relations will be discussed explicitly in the following. These pairs of channels include: $B^- \rightarrow K^0 K^-$ and $B^- \rightarrow \pi^- \bar{K}^0$; $\bar{B}^0 \rightarrow \pi^+ \pi^-$ and $\bar{B}_s^0 \rightarrow K^+ K^-$; $\bar{B}^0 \rightarrow K^0 \bar{K}^0$ and $\bar{B}_s^0 \rightarrow K^0 \bar{K}^0$; $\bar{B}^0 \rightarrow K^- K^+$ and $\bar{B}_s^0 \rightarrow \pi^+ \pi^-$; $\bar{B}_s^0 \rightarrow \pi^0 K^0$ and $\bar{B}^0 \rightarrow \pi^0 \bar{K}^0$; $\bar{B}_s^0 \rightarrow \pi^- K^+$ and $\bar{B}^0 \rightarrow \pi^+ K^-$; $\bar{B}_s^0 \rightarrow K^0 \pi^0$ and $\bar{B}^0 \rightarrow \pi^0 \bar{K}^0$.

In the past years, there have been extensive examinations on the U-spin symmetry. One of the interesting features of these U-spin pairs is that there are CP violating relation among them. Here we consider two U-spin related decays with the same “tree” $\mathcal{A}_u$ and “penguin” $\mathcal{A}_t$:

$$\begin{aligned} A(B_i \rightarrow PP, \Delta S = 0) &= V_{ub}V_{ud}^* \mathcal{A}_u + V_{tb}V_{td}^* \mathcal{A}_t, \\ A(B_i \rightarrow PP, \Delta S = 1) &= V_{ub}V_{us}^* \mathcal{A}_u + V_{tb}V_{ts}^* \mathcal{A}_t. \end{aligned} \quad (33)$$

Through the relation $\text{Im}(V_{ub}V_{ud}^*V_{tb}^*V_{td}) = -\text{Im}(V_{ub}V_{us}^*V_{tb}^*V_{ts})$, one can obtain the CP violating rate difference $\Delta(B_i \rightarrow PP, \Delta S) = \Gamma(\Delta S) - \bar{\Gamma}(\Delta S)$ [9,10,32]

$$\Delta(B_i \rightarrow PP, \Delta S = 0) = -\Delta(B_j \rightarrow PP, \Delta S = 1). \quad (34)$$

This leads to a relation between branching ratio and CP asymmetry $A_{CP}^i(\Delta S) = \Delta(B_i \rightarrow PP, \Delta S)/\mathcal{B}(B_i \rightarrow PP)$:

$$\frac{A_{CP}^i(\Delta S = 0)}{A_{CP}^j(\Delta S = 1)} = -\frac{\tau_j \mathcal{B}(\Delta S = 1)}{\tau_i \mathcal{B}(\Delta S = 0)}. \quad (35)$$

Here $\mathcal{B}(B_i \rightarrow PP)$ is the branching ratio of $B_i \rightarrow PP$ and τ_i is the lifetime of B_i .

One of the most prominent example is the case of the U-spin pair $\bar{B}_s^0 \rightarrow \pi^- K^+$ and $\bar{B}^0 \rightarrow \pi^+ K^-$. Their CP asymmetry have been studied in Ref. [33]. Here we will comment on the experimental situation for this case and introduce a parameter r_c to account for the deviation from SU(3) sym-

¹ For decay modes to be discussed in the following, there are non-trivial Clebsch-Gordon coefficients, such that Eq. (33) is modified as

$$\begin{aligned} A(\Delta S = 0) &= r(V_{ub}V_{ud}^* \mathcal{A}_u + V_{tb}V_{td}^* \mathcal{A}_t), \\ A(\Delta S = 1) &= V_{ub}V_{us}^* \mathcal{A}_u + V_{tb}V_{ts}^* \mathcal{A}_t. \end{aligned}$$

The relation in Eq. (35) is changed to:

$$\frac{A_{CP}^i(\Delta S = 0)}{A_{CP}^j(\Delta S = 1)} = -r^2 \frac{\tau_j \mathcal{B}(\Delta S = 1)}{\tau_i \mathcal{B}(\Delta S = 0)}.$$

metry.

$$\frac{A_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-)}{A_{CP}(\bar{B}_s^0 \rightarrow \pi^- K^+)} + r_c \frac{\tau_B \mathcal{B}(\bar{B}_s^0 \rightarrow \pi^- K^+)}{\tau_{B_s} \mathcal{B}(\bar{B}^0 \rightarrow \pi^+ K^-)} = 0. \quad (36)$$

In the SU(3) symmetry limit $r_c = 1$.

Using the experimental data from PDG [3,4]:

$$\begin{aligned} \mathcal{B}(\bar{B}_s^0 \rightarrow \pi^- K^+) &= (5.7 \pm 0.6) \times 10^{-6}, \\ A_{CP}(\bar{B}_s^0 \rightarrow \pi^- K^+) &= (0.26 \pm 0.04), \\ \mathcal{B}(\bar{B}^0 \rightarrow \pi^+ K^-) &= (19.6 \pm 0.5) \times 10^{-6}, \\ A_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-) &= -0.082 \pm 0.006, \end{aligned} \quad (37)$$

one finds:

$$r_c = 1.084 \pm 0.219 \quad (38)$$

where all errors have been added in quadrature. The resulting r_c value indicates that the U-spin symmetry is well in the case of this decay pair. The exploration in more decay pairs is helpful for further investigation on this symmetry.

Similar U-spin relations existing in other decays will be studied in the following sections. We will comment on them when specific decay channels are be discussed.

3.2 $B \rightarrow VV$ decays

Decay amplitudes for $B \rightarrow VV$ channels can be obtained similarly by replacing the pseudo-scalar multiplet P by the vector multiplet V in Eq. (21) and in Eq. (22).

- Since we have chosen the same parametrization for pseudoscalar and vector mesons, the expanded amplitudes for the $B \rightarrow VV$ channels can be obtained directly from the $B \rightarrow PP$.
- There are three sets of amplitudes for $B \rightarrow VV$ decays, corresponding to different polarizations. For convenience, one can choose the helicity amplitudes A_0, A_+, A_- defined as:

$$\begin{aligned} \mathcal{A} &= S_1 \epsilon_{V_1}^* \cdot \epsilon_{V_2}^* + S_2 \frac{1}{m_B^2} \epsilon_{V_1}^* \cdot p_B \epsilon_{V_2}^* \cdot p_B \\ &\quad - i S_3 \epsilon_{\mu\nu\rho\sigma} p_{V_1}^\mu p_{V_2}^\nu \epsilon_{V_1}^{*\rho} \epsilon_{V_2}^{*\sigma}, \end{aligned} \quad (39)$$

with $\epsilon_{0123} = 1$, and

$$A_0 = \frac{m_B^2}{2m_{V_1} m_{V_2}} \left(S_1 + \frac{S_2}{2} \right), \quad A_\pm = S_1 \mp S_3. \quad (40)$$

Thus there are in total $3 \times 9 = 27$ complex amplitudes for both tree and penguin, where “9” is the number of the polarization combination of final two vectors. These amplitudes correspond to $2 \times 54 - 1 = 107$

real parameters in theory. Two phases can not be measured through direct measurements of individual B and $\bar{B}$ decays, but one of the two can be obtained through the time-dependent analysis.

- In principle, all these 107 parameters could be determined through the angular distribution studies in experiment. Each $B \rightarrow V(\rightarrow P_1 P_2) V(\rightarrow P_3 P_4)$ channel can provide 10 observables. The angular distribution is given as:

$$\begin{aligned} \frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} &\propto |A_0|^2 \cos^2\theta_1 \cos^2\theta_2 \\ &\quad + \frac{1}{4} \sin^2\theta_1 \sin^2\theta_2 (|A_+|^2 + |A_-|^2) \\ &\quad + \frac{1}{2} \sin^2\theta_1 \sin^2\theta_2 \text{Re}(e^{2i\phi} A_+ A_-^*) \\ &\quad - \cos\theta_1 \sin\theta_1 \cos\theta_2 \sin\theta_2 [\text{Re}(e^{-i\phi} A_0 A_+^* \\ &\quad + \text{Re}(e^{i\phi} A_0 A_-^*)]. \end{aligned}$$

Here θ_1 (θ_2) is defined by the flight direction of P_1 (P_3) in the rest frame of V_1 (V_2) and the flight direction of V_1 (V_2) in the B meson rest frame. ϕ is the relative angle between the two decay planes.

- Unfortunately, due to the large amount of input parameters, it is a formidable task to perform a global fit, and in particular only limited data is available [3]. A realistic analysis at this stage will pick up only a limited amount of amplitudes. In this direction, the weak annihilations and hard scattering amplitudes were extracted by fitting relevant data in Ref. [34], while the authors in Ref. [35] have performed a factorization-assisted TDA analysis. This allows one to remove some suppressed amplitudes at the leading order approximation. In Ref. [36], the authors have adopted the dynamical analysis in the SCET and performed a flavor SU(3) fit of $B \rightarrow VV$ decays. On the other hand, recent dynamical improvements exist in Refs. [37,38] using the perturbative QCD approach and Ref. [39] in QCDF.

3.3 $B \rightarrow VP$ decays

We now study the $B \rightarrow VP$ decays, whose amplitudes can be obtained by replacing one of the P in Eq. (21) and in Eq. (22) by V to obtain the IRA and TDA amplitudes. There are two ways to replace one of the P , therefore the amplitudes will be doubled compared with $B \rightarrow PP$. We have IRA and TDA for $B \rightarrow VP$ decays as follows:

$$\begin{aligned} \mathcal{A}_u^{IRA} &= A_3^T B_i(H_3)^i P_k^j V_j^k + C_3^{T1} B_i(H_3)^k P_j^i V_k^j \\ &\quad + C_3^{T2} B_i(H_3)^k V_j^i P_k^j + B_3^T B_i(H_3)^i P_k^j V_j^k \\ &\quad + D_3^{T1} B_i(H_3)^j P_j^i V_k^k + D_3^{T2} B_i(H_3)^j V_j^i P_k^k \end{aligned}$$

$$\begin{aligned}
& +A_6^{T1} B_i(H_6)_k^{[ij]} P_j^l V_l^k + A_6^{T2} B_i(H_6)_k^{[ij]} V_j^l P_l^k \\
& +C_6^{T1} B_i(H_6)_k^{[jl]} P_j^i V_l^k + C_6^{T2} B_i(H_6)_k^{[jl]} V_j^i P_l^k \\
& +B_6^{T1} B_i(H_6)_k^{[ij]} P_j^k V_l^l + B_6^{T2} B_i(H_6)_k^{[ij]} V_j^k P_l^l \\
& +A_{15}^{T1} B_i(H_{15})_k^{[ij]} P_j^l V_l^k + A_{15}^{T2} B_i(H_{15})_k^{[ij]} V_j^l P_l^k \\
& +C_{15}^{T1} B_i(H_{15})_k^{[jk]} P_j^i V_l^l + C_{15}^{T2} B_i(H_{15})_k^{[jk]} V_j^i P_l^l \\
& +B_{15}^{T1} B_i(H_{15})_k^{[ij]} P_j^k V_l^l + B_{15}^{T2} B_i(H_{15})_k^{[ij]} V_j^k P_l^l,
\end{aligned} \quad (41)$$

$$\begin{aligned}
\mathcal{A}_u^{TDA} = & T_1 B_i H_k^{jl} P_j^i V_l^k + T_2 B_i H_k^{jl} V_j^i P_l^k \\
& +C_1 B_i H_k^{lj} P_j^i V_l^k + C_2 B_i H_k^{lj} V_j^i P_l^k \\
& +A_1 B_i H_j^{il} P_k^j V_l^k + A_2 B_i H_j^{il} V_k^j P_l^k \\
& +E_1 B_i H_j^{li} P_k^j V_l^k + E_2 B_i H_j^{li} V_k^j P_l^k \\
& +S^{u1} B_i H_l^{lj} P_j^i V_k^k + S^{u2} B_i H_l^{lj} V_j^i P_k^k \\
& +P^{u1} B_i H_l^{lk} P_j^j V_k^k + P^{u2} B_i H_l^{lk} V_j^j P_k^k \\
& +P_A^u B_i H_l^{li} P_k^j V_j^k + S_S^u B_i H_l^{li} P_j^j V_k^k \\
& +E_S^{u1} B_i H_l^{ji} P_j^l V_k^k + E_S^{u2} B_i H_l^{ji} V_j^l P_k^k \\
& +A_S^{u1} B_i H_l^{ij} P_j^k V_k^k + A_S^{u2} B_i H_l^{ij} V_j^k P_k^k.
\end{aligned} \quad (42)$$

The expanded amplitudes are given in Table 2 and Tab. 3. Relations between the two sets of amplitudes are derived as:

$$\begin{aligned}
A_3^T &= -\frac{1}{8}(A_1 + A_2 - 3E_1 - 3E_2) + P_A^u, \\
B_3^T &= S_S^u + \frac{1}{8}(3E_S^{u1} + 3E_S^{u2} - A_S^{u1} - A_S^{u2}) \\
C_3^{T1} &= \frac{1}{8}(3T_1 - C_1 + 3A_1 - E_1) + P^{u1}, \\
C_3^{T2} &= \frac{1}{8}(3T_2 - C_2 + 3A_2 - E_2) + P^{u2} \\
D_3^{T1} &= \frac{1}{8}(3C_1 - T_1 - E_S^{u1} + 3A_S^{u1}) + S^{u1}, \\
D_3^{T2} &= \frac{1}{8}(3C_2 - T_2 - E_S^{u2} + 3A_S^{u2}) + S^{u2} \\
A_6^{T1} &= \frac{1}{4}(A_2 - E_2), \quad A_6^{T2} = \frac{1}{4}(A_1 - E_1), \\
C_6^{T1} &= \frac{1}{4}(T_1 - C_1), \quad C_6^{T2} = \frac{1}{4}(T_2 - C_2) \\
A_{15}^{T1} &= \frac{1}{8}(A_2 + E_2), \quad A_{15}^{T2} = \frac{1}{8}(A_1 + E_1), \\
C_{15}^{T1} &= \frac{1}{8}(T_1 + C_1), \quad C_{15}^{T2} = \frac{1}{8}(T_2 + C_2), \\
B_6^{T1} &= \frac{1}{4}(A_S^{u1} - E_S^{u1}), \quad B_6^{T2} = \frac{1}{4}(A_S^{u2} - E_S^{u2}), \\
B_{15}^{T1} &= \frac{1}{8}(E_S^{u1} + A_S^{u1}), \quad B_{15}^{T2} = \frac{1}{8}(E_S^{u2} + A_S^{u2}).
\end{aligned} \quad (43)$$

The inverse relations are solved as:

$$A_1 = 4A_{15}^{T2} + 2A_6^{T2}, \quad A_2 = 2(2A_{15}^{T1} + A_6^{T1}),$$

$$\begin{aligned}
T_1 &= 2(2C_{15}^{T1} + C_6^{T1}), \quad T_2 = 2(2C_{15}^{T2} + C_6^{T2}), \\
C_1 &= 2(2C_{15}^{T1} - C_6^{T1}), \quad C_2 = 2(2C_{15}^{T2} - C_6^{T2}), \\
E_1 &= 2(2A_{15}^{T2} - A_6^{T2}), \quad E_2 = 2(2A_{15}^{T1} - A_6^{T1}), \\
A_S^{u1} &= 2(2B_{15}^{T1} + B_6^{T1}), \quad A_S^{u2} = 2(2B_{15}^{T2} + B_6^{T2}), \\
E_S^{u1} &= 2(2B_{15}^{T1} - B_6^{T1}), \quad E_S^{u2} = 2(2B_{15}^{T2} - B_6^{T2}), \\
P_A^u &= -A_{15}^{T1} + A_6^{T1} - A_{15}^{T2} + A_6^{T2} + A_3^T, \\
S_S^u &= -B_{15}^{T1} + B_6^{T1} - B_{15}^{T2} + B_6^{T2} + B_3^T, \\
P^{u1} &= -A_{15}^{T2} - A_6^{T2} - C_{15}^{T1} + C_3^{T1} - C_6^{T1}, \\
P^{u2} &= -A_{15}^{T1} - A_6^{T1} - C_{15}^{T2} + C_3^{T2} - C_6^{T2}, \\
S^{u1} &= -B_{15}^{T1} - B_6^{T1} - C_{15}^{T1} + C_6^{T1} + D_3^{T1}, \\
S^{u2} &= -B_{15}^{T2} - B_6^{T2} - C_{15}^{T2} + C_6^{T2} + D_3^{T2}.
\end{aligned} \quad (44)$$

Unlike the $B \rightarrow PP$ and $B \rightarrow VV$ case, we are not able to find any redundant amplitude. Thus in total, we have 18 complex amplitudes for “tree” and “penguin”, respectively. It corresponds to $2 \times 36 - 1 = 71$ real parameters in theory. A fit with all parameters is not available again, and most of the current analyses have made approximations by neglecting some suppressed amplitudes [18, 19, 40, 41].

Decay amplitudes for $B \rightarrow VP$ decays in Eqs. (42) and (43) are organized in a symmetric way. Taking the $B^- \rightarrow \rho^- \bar{K}^0$ and $B^- \rightarrow \bar{K}^* \pi^-$ as an example, the IRA amplitudes are related with the replacement $A_6^{T1} \leftrightarrow A_6^{T2}$, $A_{15}^{T1} \leftrightarrow A_{15}^{T2}$, $C_3^{T1} \leftrightarrow C_3^{T2}$, $C_6^{T1} \leftrightarrow C_6^{T2}$, and $C_{15}^{T1} \leftrightarrow C_{15}^{T2}$, while for the TDA amplitudes, the correspondence is: $A_1 \leftrightarrow A_2$ and $P^{u1} \leftrightarrow P^{u2}$. Other relations can be found in a similar way.

The $B \rightarrow VP$ channels related by the U-spin include: $B^- \rightarrow \bar{K}^* \pi^-$ and $B^- \rightarrow K^* \bar{K}^-$; $B^- \rightarrow \rho^- \bar{K}^0$ and $B^- \rightarrow K^* \bar{K}^0$; $\bar{B}_s^0 \rightarrow K^* \pi^-$ and $\bar{B}_s^0 \rightarrow \rho^+ \pi^-$; $\bar{B}_s^0 \rightarrow K^* \pi^+$ and $\bar{B}_s^0 \rightarrow \rho^- \pi^+$; $\bar{B}_s^0 \rightarrow \rho^+ \pi^-$ and $\bar{B}_s^0 \rightarrow K^* \pi^-$; $\bar{B}_s^0 \rightarrow \bar{K}^* \bar{K}^0$ and $\bar{B}_s^0 \rightarrow K^* \bar{K}^0$; $\bar{B}_s^0 \rightarrow K^* \bar{K}^0$ and $\bar{B}_s^0 \rightarrow \rho^- \pi^+$ and $\bar{B}_s^0 \rightarrow K^* \pi^-$; $\bar{B}_s^0 \rightarrow K^* \pi^+$ and $\bar{B}_s^0 \rightarrow \rho^- \pi^+$; $\bar{B}_s^0 \rightarrow \rho^+ \pi^-$ and $\bar{B}_s^0 \rightarrow K^* \pi^-$. However on the experimental side, there are not enough measurements to examine these relations, in particular the CPV in B_s sector has received less consideration. We expect the situation will be improved when a large amount of data is available at LHCb, and Belle-II.

4 $D \rightarrow PP, VV, PV$ decays

Using the effective Hamiltonian in Eqs. (16) and (17), one can easily obtain the $SU(3)$ decay amplitudes in a similar fashion as that for $B \rightarrow PP, VV, PV$. For D decays we will only discuss tree contributions which are written as $\mathcal{A} = V_{cs/d} V_{ud/s}^* \mathcal{A}_u^{IRA, TDA}$. The penguin amplitudes are

Table 2 $B \rightarrow VP$ decays induced by the $b \rightarrow d$ transition

Channel	IRA	TDA
$B^- \rightarrow \rho^0 \pi^-$	$(A_6^{T1} + 3A_{15}^{T1} - A_6^{T2} - 3A_{15}^{T2} - C_3^{T1} - C_6^{T1} + 5C_{15}^{T1} + C_3^{T2} + C_6^{T2} + 3C_{15}^{T2})/\sqrt{2}$	$(-A_1 + A_2 + C_1 + P^{u2} - P^{u1} + T_2)/\sqrt{2}$
$B^- \rightarrow \rho^- \pi^0$	$(-A_6^{T1} - 3A_{15}^{T1} + A_6^{T2} + 3A_{15}^{T2} + C_3^{T1} + C_6^{T1} + 3C_{15}^{T1} - C_3^{T2} - C_6^{T2} + 5C_{15}^{T2})/\sqrt{2}$	$(A_1 - A_2 + C_2 - P^{u2} + P^{u1} + T_1)/\sqrt{2}$
$B^- \rightarrow \rho^- \eta_q$	$(1/\sqrt{2})(A_6^{T1} + 3A_{15}^{T1} + A_6^{T2} + 3A_{15}^{T2} + 2B_6^{T2} + 6B_{15}^{T2} + C_3^{T1} + C_6^{T1} + 3C_{15}^{T1} + C_3^{T2} - C_6^{T2} + C_{15}^{T2} + 2D_3^{T2})$	$(2A_S^{u2} + A_1 + A_2 + C_2 + P^{u2} + P^{u1} + 2S^{u2} + T_1)/\sqrt{2}$
$B^- \rightarrow \rho^- \eta_s$	$B_6^{T2} + 3B_{15}^{T2} + C_6^{T2} - C_{15}^{T2} + D_3^{T2}$	$A_S^{u2} + S^{u2}$
$B^- \rightarrow K^{*0} K^-$	$A_6^{T2} + 3A_{15}^{T2} + C_3^{T1} - C_6^{T1} - C_{15}^{T1}$	$A_1 + P^{u1}$
$B^- \rightarrow K^{*-} K^0$	$A_6^{T1} + 3A_{15}^{T1} + C_3^{T2} - C_6^{T2} - C_{15}^{T2}$	$A_2 + P^{u2}$
$B^- \rightarrow \omega \pi^-$	$(1/\sqrt{2})(A_6^{T1} + 3A_{15}^{T1} + A_6^{T2} + 3A_{15}^{T2} + 2B_6^{T1} + 6B_{15}^{T1} + C_3^{T1} - C_6^{T1} + C_{15}^{T1} + C_3^{T2} + C_6^{T2} + 3C_{15}^{T2} + 2D_3^{T1})$	$(2A_S^{u1} + A_1 + A_2 + C_1 + P^{u2} + P^{u1} + 2S^{u1} + T_2)/\sqrt{2}$
$B^- \rightarrow \phi \pi^-$	$B_6^{T1} + 3B_{15}^{T1} + C_6^{T1} - C_{15}^{T1} + D_3^{T1}$	$A_S^{u1} + S^{u1}$
$\bar{B}^0 \rightarrow \rho^+ \pi^-$	$A_3^T - 2A_{15}^{T1} - A_6^{T2} + 3A_{15}^{T2} + C_3^{T2} + C_6^{T2} + 3C_{15}^{T2}$	$P_A^u + E_1 + P^{u2} + T_2$
$\bar{B}^0 \rightarrow \rho^0 \pi^0$	$(1/2)(2A_3^T - A_6^{T1} + A_{15}^{T1} - A_6^{T2} + A_{15}^{T2} + C_3^{T1} + C_6^{T1} - 5C_{15}^{T1} + C_3^{T2} + C_6^{T2} - 5C_{15}^{T2})$	$(1/2)(2P_A^u - C_1 - C_2 + E_1 + E_2 + P^{u2} + P^{u1})$
$\bar{B}^0 \rightarrow \rho^0 \eta_q$	$(1/2)(-A_6^{T1} + 5A_{15}^{T1} - A_6^{T2} + 5A_{15}^{T2} - 2B_6^{T2} + 10B_{15}^{T2} - C_3^{T1} - C_6^{T1} + 5C_{15}^{T1} - C_3^{T2} + C_6^{T2} - C_{15}^{T2} - 2D_3^{T2})$	$(1/2)(C_1 - C_2 + 2E_S^{u2} + E_1 + E_2 - P^{u2} - P^{u1} - 2S^{u2})$
$\bar{B}^0 \rightarrow \rho^0 \eta_s$	$-(B_6^{T2} - 5B_{15}^{T2} + C_6^{T2} - C_{15}^{T2} + D_3^{T2})/\sqrt{2}$	$(E_S^{u2} - S^{u2})/\sqrt{2}$
$\bar{B}^0 \rightarrow \rho^- \pi^+$	$A_3^T - A_6^{T1} + 3A_{15}^{T1} - 2A_{15}^{T2} + C_3^{T1} + C_6^{T1} + 3C_{15}^{T1}$	$P_A^u + E_2 + P^{u1} + T_1$
$\bar{B}^0 \rightarrow K^{*+} K^-$	$A_3^T + A_6^{T1} - A_{15}^{T1} - A_6^{T2} + 3A_{15}^{T2}$	$P_A^u + E_1$
$\bar{B}^0 \rightarrow K^{*0} \bar{K}^0$	$A_3^T + A_6^{T1} - A_{15}^{T1} - 2A_{15}^{T2} + C_3^{T1} - C_6^{T1} - C_{15}^{T1}$	$P_A^u + P^{u1}$
$\bar{B}^0 \rightarrow \bar{K}^{*0} K^0$	$A_3^T - 2A_{15}^{T1} + A_6^{T2} - A_{15}^{T2} + C_3^{T2} - C_6^{T2} - C_{15}^{T2}$	$P_A^u + P^{u2}$
$\bar{B}^0 \rightarrow K^{*-} K^+$	$A_3^T - A_6^{T1} + 3A_{15}^{T1} + A_6^{T2} - A_{15}^{T2}$	$P_A^u + E_2$
$\bar{B}^0 \rightarrow \omega \pi^0$	$(1/2)(-A_6^{T1} + 5A_{15}^{T1} - A_6^{T2} + 5A_{15}^{T2} - 2B_6^{T1} + 10B_{15}^{T1} - C_3^{T1} + C_6^{T1} - C_{15}^{T1} - C_3^{T2} - C_6^{T2} + 5C_{15}^{T2} - 2D_3^{T1})$	$(1/2)(-C_1 + C_2 + 2E_S^{u1} + E_1 + E_2 - P^{u2} - P^{u1} - 2S^{u1})$
$\bar{B}^0 \rightarrow \omega \eta_q$	$(1/2)(2A_3^T - A_6^{T1} + A_{15}^{T1} - A_6^{T2} + A_{15}^{T2} + 4B_3^T - 2B_6^{T1} + 2B_{15}^{T1} - 2B_6^{T2} + 2B_{15}^{T2} + C_3^{T1} - C_6^{T1} + C_{15}^{T1} + C_3^{T2} - C_6^{T2} + C_{15}^{T2} + 2D_3^{T1} + 2D_3^{T2})$	$(1/2)(2P_A^u + C_1 + C_2 + 2E_S^{u1} + 2E_S^{u2} + E_1 + E_2 + P^{u2} + P^{u1} + 2S^{u2} + 2S^{u1} + 4S_S^u)$
$\bar{B}^0 \rightarrow \omega \eta_s$	$(2B_3^T + 2B_6^{T1} - 2B_{15}^{T1} - B_6^{T2} + B_{15}^{T2} + C_6^{T2} - C_{15}^{T2} + D_3^{T2})/\sqrt{2}$	$(E_S^{u2} + S^{u2} + 2S_S^u)/\sqrt{2}$
$\bar{B}^0 \rightarrow \phi \pi^0$	$-(B_6^{T1} - 5B_{15}^{T1} + C_6^{T1} - C_{15}^{T1} + D_3^{T1})/\sqrt{2}$	$(E_S^{u1} - S^{u1})/\sqrt{2}$
$\bar{B}^0 \rightarrow \phi \eta_q$	$(2B_3^T - B_6^{T1} + B_{15}^{T1} + 2B_6^{T2} - 2B_{15}^{T2} + C_6^{T1} - C_{15}^{T1} + D_3^{T1})/\sqrt{2}$	$(E_S^{u1} + S^{u1} + 2S_S^u)/\sqrt{2}$
$\bar{B}^0 \rightarrow \phi \eta_s$	$A_3^T + A_6^{T1} - A_{15}^{T1} + A_6^{T2} - A_{15}^{T2} + B_3^T + B_6^{T1} - B_{15}^{T1} + B_6^{T2} - B_{15}^{T2}$	$P_A^u + S_S^u$
$\bar{B}_s^0 \rightarrow \rho^0 K^0$	$(A_6^{T2} + A_{15}^{T2} - C_3^{T1} - C_6^{T1} + 5C_{15}^{T1})/\sqrt{2}$	$(C_1 - P^{u1})/\sqrt{2}$
$\bar{B}_s^0 \rightarrow \rho^- K^+$	$-A_6^{T2} - A_{15}^{T2} + C_3^{T1} + C_6^{T1} + 3C_{15}^{T1}$	$P^{u1} + T_1$
$\bar{B}_s^0 \rightarrow K^{*+} \pi^-$	$-A_6^{T1} - A_{15}^{T1} + C_3^{T2} + C_6^{T2} + 3C_{15}^{T2}$	$P^{u2} + T_2$

Table 2 continued

Channel	IRA	TDA
$\bar{B}_s^0 \rightarrow K^{*0} \pi^0$	$(A_6^{T1} + A_{15}^{T1} - C_3^{T2} - C_6^{T2} + 5C_{15}^{T2})/\sqrt{2}$	$(C_2 - P^{u2})/\sqrt{2}$
$\bar{B}_s^0 \rightarrow K^{*0} \eta_q$	$-(A_6^{T1} + A_{15}^{T1} + 2B_6^{T2} + 2B_{15}^{T2} - C_3^{T2} + C_6^{T2} - C_{15}^{T2} - 2D_3^{T2})/\sqrt{2}$	$(C_2 + P^{u2} + 2S^{u2})/\sqrt{2}$
$\bar{B}_s^0 \rightarrow K^{*0} \eta_s$	$-A_6^{T2} - A_{15}^{T2} - B_6^{T2} - B_{15}^{T2} + C_3^{T1} - C_6^{T1} - C_{15}^{T1} + C_6^{T2} - C_{15}^{T2} + D_3^{T2}$	$P^{u1} + S^{u2}$
$\bar{B}_s^0 \rightarrow \omega K^0$	$-(A_6^{T2} + A_{15}^{T2} + 2B_6^{T1} + 2B_{15}^{T1} - C_3^{T1} + C_6^{T1} - C_{15}^{T1} - 2D_3^{T1})/\sqrt{2}$	$(C_1 + P^{u1} + 2S^{u1})/\sqrt{2}$
$\bar{B}_s^0 \rightarrow \phi K^0$	$-A_6^{T1} - A_{15}^{T1} - B_6^{T1} - B_{15}^{T1} + C_6^{T1} - C_{15}^{T1} + C_3^{T2} - C_6^{T2} - C_{15}^{T2} + D_3^{T1}$	$P^{u2} + S^{u1}$

suppressed and thus neglected in this work, however it is necessary to stress penguin amplitudes are mandatory for the CP violation. The SU(3) amplitudes for $D \rightarrow PP$ are parametrized as

$$\begin{aligned} \mathcal{A}_u^{IRA} = & A_6^T D_i(H_6)_k^{[ij]} P_j^l P_l^k + C_6^T D_i(H_6)_k^{[jl]} P_j^i P_l^k \\ & + B_6^T D_i(H_6)_k^{[ij]} P_j^k P_l^l \\ & + A_{15}^T D_i(H_{15})_k^{[ij]} P_j^l P_l^k + C_{15}^T D_i(H_{15})_l^{\{jk\}} P_j^i P_l^k \\ & + B_{15}^T D_i(H_{15})_k^{[ij]} P_j^k P_l^l, \end{aligned} \quad (45)$$

$$\begin{aligned} \mathcal{A}_u^{TDA} = & T D_i H_k^{jl} P_j^i P_l^k + C D_i H_k^{lj} P_j^i P_l^k \\ & + A D_i H_j^{il} P_k^j P_l^k + E D_i H_j^{ij} P_k^j P_l^k \\ & + E_S^u D_i H_l^{ji} P_j^l P_k^k + A_S^u D_i H_l^{ij} P_j^l P_k^k. \end{aligned} \quad (46)$$

The expanded amplitudes are given in Table 4. The amplitudes A_6^T can be incorporated in $B_6^{T'}$ and $C_6^{T'}$, and then we have five independent amplitudes for $D \rightarrow PP$:

$$\begin{aligned} A_{15}^T = \frac{A + E}{2}, \quad B_{15}^T = \frac{A_S^u + E_S^u}{2}, \quad C_{15}^T = \frac{T + C}{2}, \\ B_6^{T'} = \frac{A_S^u - E_S^u + A - E}{2}, \quad C_6^{T'} = \frac{T - C - A + E}{2}, \end{aligned} \quad (47)$$

with the inverse relation:

$$\begin{aligned} T + E = A_{15}^T + C_6^{T'} + C_{15}^T, \\ C - E = -A_{15}^T - C_6^{T'} + C_{15}^T, \quad A + E = 2A_{15}^T, \\ A_S^u - E = -A_{15}^T + B_6^{T'} + B_{15}^T, \\ E_S^u + E = A_{15}^T - B_6^{T'} + B_{15}^T. \end{aligned} \quad (48)$$

Since one amplitude is redundant, fits with all six complex amplitudes should not be resolved in principle. This has been indicated by the strong correlation of parameters in the fits in Ref. [42].

Again for $D \rightarrow VV$ decays there are three sets of amplitudes similar as the $D \rightarrow PP$, and thus we have 15 independent amplitudes in total (Fig. 2).

The IRA and TDA for $D \rightarrow VP$ decays are given as:

$$\begin{aligned} \mathcal{A}_u^{IRA} = & A_6^{T1} D_i(H_6)_k^{[ij]} P_j^l V_l^k + A_6^{T2} D_i(H_6)_k^{[ij]} V_j^l P_l^k \\ & + C_6^{T1} D_i(H_6)_k^{[jl]} P_j^l V_l^k + C_6^{T2} D_i(H_6)_k^{[jl]} V_j^l P_l^k \\ & + B_6^{T1} D_i(H_6)_k^{[ij]} P_j^k V_l^l + B_6^{T2} D_i(H_6)_k^{[ij]} V_j^k P_l^l \\ & + A_{15}^{T1} D_i(H_{15})_k^{[ij]} P_j^l V_l^k + A_{15}^{T2} D_i(H_{15})_k^{[ij]} V_j^l P_l^k \\ & + C_{15}^{T1} D_i(H_{15})_l^{\{jk\}} P_j^i V_l^k + C_{15}^{T2} D_i(H_{15})_l^{\{jk\}} V_j^i P_l^k \\ & + B_{15}^{T1} D_i(H_{15})_k^{[ij]} P_j^k V_l^l + B_{15}^{T2} D_i(H_{15})_k^{[ij]} V_j^k P_l^l, \end{aligned} \quad (49)$$

$$\begin{aligned} \mathcal{A}_u^{TDA} = & T_1 D_i H_k^{jl} P_j^i V_l^k + T_2 D_i H_k^{jl} V_j^i P_l^k \\ & + C_1 D_i H_k^{lj} P_j^i V_l^k + C_2 D_i H_k^{lj} V_j^i P_l^k \\ & + A_1 D_i H_j^{il} P_k^j V_l^k + A_2 D_i H_j^{il} V_k^j P_l^k \\ & + E_1 D_i H_j^{li} P_k^j V_l^k + E_2 D_i H_j^{li} V_k^j P_l^k \\ & + E_S^{u1} D_i H_l^{ji} P_j^l V_k^k + E_S^{u2} D_i H_l^{ji} V_j^l P_k^k \\ & + A_S^{u1} D_i H_l^{ij} P_j^l V_k^k + A_S^{u2} D_i H_l^{ij} V_j^l P_k^k. \end{aligned} \quad (50)$$

The above amplitudes are also expanded in a symmetric way. The expanded amplitudes are collected in Tables 5, 6 and 7 for the different transitions. Relations between the two sets of amplitudes are derived as:

$$\begin{aligned} A_6^{T1} = \frac{1}{2}(A_2 - E_2), \quad A_6^{T2} = \frac{1}{2}(A_1 - E_1), \\ B_6^{T1} = \frac{1}{2}(A_S^{u1} - E_S^{u1}), \quad B_6^{T2} = \frac{1}{2}(A_S^{u2} - E_S^{u2}) \\ C_6^{T1} = \frac{1}{2}(T_1 - C_1), \quad C_6^{T2} = \frac{1}{2}(T_2 - C_2), \\ A_{15}^{T1} = \frac{1}{2}(A_2 + E_2), \quad A_{15}^{T2} = \frac{1}{2}(A_1 + E_1) \\ B_{15}^{T1} = \frac{1}{2}(E_S^{u1} + A_S^{u1}), \quad B_{15}^{T2} = \frac{1}{2}(E_S^{u2} + A_S^{u2}), \\ C_{15}^{T1} = \frac{1}{2}(T_1 + C_1), \quad C_{15}^{T2} = \frac{1}{2}(T_2 + C_2). \end{aligned} \quad (51)$$

Table 3 $B \rightarrow VP$ decays induced by the $b \rightarrow s$ transition

Channel	IRA	TDA
$B^- \rightarrow \rho^0 K^-$	$(A_6^{T_1} + 3A_{15}^{T_1} - 2C_6^{T_1} + 4C_{15}^{T_1} + C_3^{T_2} + C_6^{T_2} + 3C_{15}^{T_2})/\sqrt{2}$	$(A_2 + C_1 + P^{u2} + T_2)/\sqrt{2}$
$B^- \rightarrow \rho^- \bar{K}^0$	$A_6^{T_1} + 3A_{15}^{T_1} + C_3^{T_2} - C_6^{T_2} - C_{15}^{T_2}$	$A_2 + P^{u2}$
$B^- \rightarrow \bar{K}^{*0} \pi^-$	$A_6^{T_2} + 3A_{15}^{T_2} + C_3^{T_1} - C_6^{T_1} - C_{15}^{T_1}$	$A_1 + P^{u1}$
$B^- \rightarrow K^{*-0} \pi^0$	$(A_6^{T_2} + 3A_{15}^{T_2} + C_3^{T_1} + C_6^{T_1} + 3C_{15}^{T_1} - 2C_6^{T_2} + 4C_{15}^{T_2})/\sqrt{2}$	$(A_1 + C_2 + P^{u1} + T_1)/\sqrt{2}$
$B^- \rightarrow K^{*-} \eta_q$	$(A_6^{T_2} + 3A_{15}^{T_2} + 2B_6^{T_2} + 6B_{15}^{T_2} + C_3^{T_1} + C_6^{T_1} + 3C_{15}^{T_1} + 2C_{15}^{T_2} + 2D_3^{T_2})/\sqrt{2}$	$(2A_S^{u2} + A_1 + C_2 + P^{u1} + 2S^{u2} + T_1)/\sqrt{2}$
$B^- \rightarrow K^{*-} \eta_s$	$A_6^{T_1} + 3A_{15}^{T_1} + B_6^{T_2} + 3B_{15}^{T_2} + C_3^{T_2} - 2C_{15}^{T_2} + D_3^{T_2}$	$A_S^{u2} + A_2 + P^{u2} + S^{u2}$
$B^- \rightarrow \omega K^-$	$(A_6^{T_1} + 3A_{15}^{T_1} + 2B_6^{T_1} + 6B_{15}^{T_1} + 2C_{15}^{T_1} + C_3^{T_2} + C_6^{T_2} + 3C_{15}^{T_2} + 2D_3^{T_1})/\sqrt{2}$	$(2A_S^{u1} + A_2 + C_1 + P^{u2} + 2S^{u1} + T_2)/\sqrt{2}$
$B^- \rightarrow \phi K^-$	$A_6^{T_2} + 3A_{15}^{T_2} + B_6^{T_1} + 3B_{15}^{T_1} + C_3^{T_1} - 2C_{15}^{T_1} + D_3^{T_1}$	$A_S^{u1} + A_1 + P^{u1} + S^{u1}$
$\bar{B}^0 \rightarrow \rho^+ K^-$	$-A_6^{T_1} - A_{15}^{T_1} + C_3^{T_2} + C_6^{T_2} + 3C_{15}^{T_2}$	$P^{u2} + T_2$
$\bar{B}^0 \rightarrow \rho^0 \bar{K}^0$	$(A_6^{T_1} + A_{15}^{T_1} - 2C_6^{T_1} + 4C_{15}^{T_1} - C_3^{T_2} + C_6^{T_2} + C_{15}^{T_2})/\sqrt{2}$	$(C_1 - P^{u2})/\sqrt{2}$
$\bar{B}^0 \rightarrow \bar{K}^{*0} \pi^0$	$(A_6^{T_2} + A_{15}^{T_2} - C_3^{T_1} + C_6^{T_1} + C_{15}^{T_1} - 2C_6^{T_2} + 4C_{15}^{T_2})/\sqrt{2}$	$(C_2 - P^{u1})/\sqrt{2}$
$\bar{B}^0 \rightarrow \bar{K}^{*0} \eta_q$	$-(A_6^{T_2} + A_{15}^{T_2} + 2B_6^{T_2} + 2B_{15}^{T_2} - C_3^{T_1} + C_6^{T_1} + C_{15}^{T_1} - 2C_{15}^{T_2} - 2D_3^{T_2})/\sqrt{2}$	$(C_2 + P^{u1} + 2S^{u2})/\sqrt{2}$
$\bar{B}^0 \rightarrow \bar{K}^{*0} \eta_s$	$-A_6^{T_1} - A_{15}^{T_1} - B_6^{T_2} - B_{15}^{T_2} + C_3^{T_2} - 2C_{15}^{T_2} + D_3^{T_2}$	$P^{u2} + S^{u2}$
$\bar{B}^0 \rightarrow K^{*-} \pi^+$	$-A_6^{T_2} - A_{15}^{T_2} + C_3^{T_1} + C_6^{T_1} + 3C_{15}^{T_1}$	$P^{u1} + T_1$
$\bar{B}^0 \rightarrow \omega \bar{K}^0$	$-(A_6^{T_1} + A_{15}^{T_1} + 2B_6^{T_1} + 2B_{15}^{T_1} - 2C_{15}^{T_1} - C_3^{T_2} + C_6^{T_2} + C_{15}^{T_2} - 2D_3^{T_1})/\sqrt{2}$	$(C_1 + P^{u2} + 2S^{u1})/\sqrt{2}$
$\bar{B}^0 \rightarrow \phi \bar{K}^0$	$-A_6^{T_2} - A_{15}^{T_2} - B_6^{T_1} - B_{15}^{T_1} + C_3^{T_1} - 2C_{15}^{T_1} + D_3^{T_1}$	$P^{u1} + S^{u1}$
$\bar{B}_s^0 \rightarrow \rho^+ \pi^-$	$A_3^T + A_6^{T_1} - A_{15}^{T_1} - A_6^{T_2} + 3A_{15}^{T_2}$	$P_A^u + E_1$
$\bar{B}_s^0 \rightarrow \rho^0 \pi^0$	$A_3^T + A_{15}^{T_1} + A_{15}^{T_2}$	$(1/2)(2P_A^u + E_1 + E_2)$
$\bar{B}_s^0 \rightarrow \rho^0 \eta_q$	$2(A_{15}^{T_1} + A_{15}^{T_2} - B_6^{T_2} + 2B_{15}^{T_2}) - A_6^{T_1} - A_6^{T_2}$	$(1/2)(2E_S^{u2} + E_1 + E_2)$
$\bar{B}_s^0 \rightarrow \rho^0 \eta_s$	$-\sqrt{2}(B_6^{T_2} - 2B_{15}^{T_2} + C_6^{T_1} - 2C_{15}^{T_1})$	$(C_1 + E_S^{u2})/\sqrt{2}$
$\bar{B}_s^0 \rightarrow \rho^- \pi^+$	$A_3^T - A_6^{T_1} + 3A_{15}^{T_1} + A_6^{T_2} - A_{15}^{T_2}$	$P_A^u + E_2$
$\bar{B}_s^0 \rightarrow K^{*+} K^-$	$A_3^T - 2A_{15}^{T_1} - A_6^{T_2} + 3A_{15}^{T_2} + C_3^{T_2} + C_6^{T_2} + 3C_{15}^{T_2}$	$P_A^u + E_1 + P^{u2} + T_2$
$\bar{B}_s^0 \rightarrow K^{*0} \bar{K}^0$	$A_3^T - 2A_{15}^{T_1} + A_6^{T_2} - A_{15}^{T_2} + C_3^{T_2} - C_6^{T_2} - C_{15}^{T_2}$	$P_A^u + P^{u2}$
$\bar{B}_s^0 \rightarrow \bar{K}^{*0} K^0$	$A_3^T + A_6^{T_1} - A_{15}^{T_1} - 2A_{15}^{T_2} + C_3^{T_1} - C_6^{T_1} - C_{15}^{T_1}$	$P_A^u + P^{u1}$
$\bar{B}_s^0 \rightarrow K^{*-} K^+$	$A_3^T - A_6^{T_1} + 3A_{15}^{T_1} - 2A_{15}^{T_2} + C_3^{T_1} + C_6^{T_1} + 3C_{15}^{T_1}$	$P_A^u + E_2 + P^{u1} + T_1$
$\bar{B}_s^0 \rightarrow \omega \pi^0$	$2(A_{15}^{T_1} + A_{15}^{T_2} - B_6^{T_2} + 2B_{15}^{T_2}) - A_6^{T_1} - A_6^{T_2}$	$(1/2)(2E_S^{u1} + E_1 + E_2)$
$\bar{B}_s^0 \rightarrow \omega \eta_q$	$A_3^T + A_{15}^{T_1} + A_{15}^{T_2} + 2(B_3^T + B_{15}^{T_1} + B_{15}^{T_2})$	$P_A^u + E_S^{u1} + E_S^{u2} + (E_1)/2 + (E_2)/2 + 2S_S^u$
$\bar{B}_s^0 \rightarrow \omega \eta_s$	$\sqrt{2}(B_3^T - 2B_{15}^{T_1} + B_{15}^{T_2} + C_{15}^{T_1} + D_3^{T_1})$	$(C_1 + E_S^{u2} + 2(S^{u1} + S_S^u))/\sqrt{2}$
$\bar{B}_s^0 \rightarrow \phi \pi^0$	$-\sqrt{2}(B_6^{T_1} - 2B_{15}^{T_1} + C_6^{T_2} - 2C_{15}^{T_2})$	$(C_2 + E_S^{u1})/\sqrt{2}$
$\bar{B}_s^0 \rightarrow \phi \eta_q$	$\sqrt{2}(B_3^T + B_{15}^{T_1} - 2B_{15}^{T_2} + C_{15}^{T_2} + D_3^{T_2})$	$(C_2 + E_S^{u1} + 2(S^{u2} + S_S^u))/\sqrt{2}$
$\bar{B}_s^0 \rightarrow \phi \eta_s$	$A_3^T - 2A_{15}^{T_1} - 2A_{15}^{T_2} + B_3^T - 2B_{15}^{T_1} - 2B_{15}^{T_2} + C_3^{T_1} - 2C_{15}^{T_1} + C_3^{T_2} - 2C_{15}^{T_2} + D_3^{T_1} + D_3^{T_2}$	$P_A^u + P^{u2} + P^{u1} + S^{u2} + S^{u1} + S_S^u$

Table 4 Decay amplitudes for two-body $D \rightarrow PP$ decays. Decay amplitudes for two-body $D \rightarrow PP$ decays. The CKM factor should be multiplied: $V_{cs} V_{ud}^*$ for Cabibbo-Allowed decays; $V_{cs} V_{us}^*$ for singly Cabibbo-suppressed modes and $V_{cd} V_{us}^*$ for doubly Cabibbo-suppressed modes

$V_{cs} V_{ud}^*$	IRA	TDA
$D^0 \rightarrow \pi^+ K^-$	$-A_6^T + A_{15}^T + C_6^T + C_{15}^T$	$E + T$
$D^0 \rightarrow \pi^0 \bar{K}^0$	$(A_6^T - A_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(C - E)/\sqrt{2}$
$D^0 \rightarrow \bar{K}^0 \eta_q$	$(-A_6^T + A_{15}^T - 2B_6^T + 2B_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(C + 2E_S^u + E)/\sqrt{2}$
$D^0 \rightarrow \bar{K}^0 \eta_s$	$-A_6^T + A_{15}^T - B_6^T + B_{15}^T$	$E_S^u + E$
$D^+ \rightarrow \pi^+ \bar{K}^0$	$2C_{15}^T$	$C + T$
$D_s^+ \rightarrow \pi^+ \eta_q$	$\sqrt{2} (A_6^T + A_{15}^T + B_6^T + B_{15}^T)$	$\sqrt{2} (A_S^u + A)$
$D_s^+ \rightarrow \pi^+ \eta_s$	$B_6^T + B_{15}^T + C_6^T + C_{15}^T$	$A_S^u + T$
$D_s^+ \rightarrow K^+ \bar{K}^0$	$A_6^T + A_{15}^T - C_6^T + C_{15}^T$	$A + C$
$V_{cs} V_{us}^*$	IRA	TDA
$D^0 \rightarrow \pi^+ \pi^-$	$A_6^T - A_{15}^T - C_6^T - C_{15}^T$	$-E - T$
$D^0 \rightarrow \pi^0 \pi^0$	$A_6^T - A_{15}^T - C_6^T + C_{15}^T$	$C - E$
$D^0 \rightarrow \pi^0 \eta_q$	$-A_6^T + A_{15}^T - B_6^T + B_{15}^T$	$E_S^u + E$
$D^0 \rightarrow \pi^0 \eta_s$	$(-B_6^T + B_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(C + E_S^u)/\sqrt{2}$
$D^0 \rightarrow K^+ K^-$	$-A_6^T + A_{15}^T + C_6^T + C_{15}^T$	$E + T$
$D^0 \rightarrow \eta_q \eta_q$	$A_6^T - A_{15}^T + 2B_6^T - 2B_{15}^T + C_6^T - C_{15}^T$	$-C - 2E_S^u - E$
$D^0 \rightarrow \eta_q \eta_s$	$(-B_6^T + B_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(C + E_S^u)/\sqrt{2}$
$D^0 \rightarrow \eta_s \eta_s$	$-A_6^T + A_{15}^T - B_6^T + B_{15}^T$	$E_S^u + E$
$D^+ \rightarrow \pi^+ \pi^0$	$\sqrt{2} C_{15}^T$	$(C + T)/\sqrt{2}$
$D^+ \rightarrow \pi^+ \eta_q$	$-\sqrt{2} (A_6^T + A_{15}^T + B_6^T + B_{15}^T + C_{15}^T)$	$-(2A_S^u + 2A + C + T)/\sqrt{2}$
$D^+ \rightarrow \pi^+ \eta_s$	$-B_6^T - B_{15}^T - C_6^T + C_{15}^T$	$C - A_S^u$
$D^+ \rightarrow K^+ \bar{K}^0$	$-A_6^T - A_{15}^T + C_6^T + C_{15}^T$	$T - A$
$D_s^+ \rightarrow \pi^+ K^0$	$A_6^T + A_{15}^T - C_6^T - C_{15}^T$	$A - T$
$D_s^+ \rightarrow \pi^0 K^+$	$(A_6^T + A_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(A + C)/\sqrt{2}$
$D_s^+ \rightarrow K^+ \eta_q$	$(A_6^T + A_{15}^T + 2B_6^T + 2B_{15}^T + C_6^T - C_{15}^T)/\sqrt{2}$	$(2A_S^u + A - C)/\sqrt{2}$
$D_s^+ \rightarrow K^+ \eta_s$	$A_6^T + A_{15}^T + B_6^T + B_{15}^T + 2C_{15}^T$	$A_S^u + A + C + T$
$V_{cd} V_{us}^*$	IRA	TDA
$D^0 \rightarrow \pi^0 K^0$	$(A_6^T - A_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(C - E)/\sqrt{2}$
$D^0 \rightarrow \pi^- K^+$	$-A_6^T + A_{15}^T + C_6^T + C_{15}^T$	$E + T$
$D^0 \rightarrow K^0 \eta_q$	$(-A_6^T + A_{15}^T - 2B_6^T + 2B_{15}^T - C_6^T + C_{15}^T)/\sqrt{2}$	$(C + 2E_S^u + E)/\sqrt{2}$
$D^0 \rightarrow K^0 \eta_s$	$-A_6^T + A_{15}^T - B_6^T + B_{15}^T$	$E_S^u + E$
$D^+ \rightarrow \pi^+ K^0$	$A_6^T + A_{15}^T - C_6^T + C_{15}^T$	$A + C$
$D^+ \rightarrow \pi^0 K^+$	$(A_6^T + A_{15}^T - C_6^T - C_{15}^T)/\sqrt{2}$	$(A - T)/\sqrt{2}$
$D^+ \rightarrow K^+ \eta_q$	$(A_6^T + A_{15}^T + 2B_6^T + 2B_{15}^T + C_6^T + C_{15}^T)/\sqrt{2}$	$(2A_S^u + A + T)/\sqrt{2}$
$D^+ \rightarrow K^+ \eta_s$	$A_6^T + A_{15}^T + B_6^T + B_{15}^T$	$A_S^u + A$
$D_s^+ \rightarrow K^+ K^0$	$2C_{15}^T$	$C + T$

Fig. 2 Topological diagrams for tree amplitudes in Cabibbo-allowed $D \rightarrow PP$ and $D \rightarrow VV$ decays. The penguin amplitudes are suppressed by both CKM matrix elements and Wilson coefficients, and thus neglected in this work, however it is necessary to point out penguin amplitudes are mandatory to study the CP violation. The s can be replaced by d and $\bar{d}$ can be replaced by $\bar{s}$ to obtain the singly-Cabibbo suppressed and doubly-Cabibbo suppressed decays

The inverse relations are solved as:

$$\begin{aligned}
 A_1 &= A_{15}^{T2} + A_6^{T2}, & A_2 &= A_{15}^{T1} + A_6^{T1}, \\
 T_1 &= C_{15}^{T1} + C_6^{T1}, & T_2 &= C_{15}^{T2} + C_6^{T2}, \\
 C_1 &= C_{15}^{T1} - C_6^{T1}, & C_2 &= C_{15}^{T2} - C_6^{T2}, \\
 E_1 &= A_{15}^{T2} - A_6^{T2}, & E_2 &= A_{15}^{T1} - A_6^{T1}, \\
 A_S^{u1} &= B_{15}^{T1} + B_6^{T1}, & A_S^{u2} &= B_{15}^{T2} + B_6^{T2}, \\
 E_S^{u1} &= B_{15}^{T1} - B_6^{T1}, & E_S^{u2} &= B_{15}^{T2} - B_6^{T2}.
 \end{aligned} \quad (52)$$

It is interesting to explore the useful relations for decay widths from the amplitudes listed in Tables 5, 6 and 7. For Cabibbo Allowed channels, we find $\Gamma(D_s^+ \rightarrow \rho^+ \pi^0) = \Gamma(D_s^+ \rightarrow \rho^0 \pi^+)$. For singly Cabibbo suppressed channels, one has:

$$\begin{aligned}
 \Gamma(D^0 \rightarrow \rho^+ \pi^-) &= \Gamma(D^0 \rightarrow K^{*+} K^-), \\
 \Gamma(D^0 \rightarrow \rho^- \pi^+) &= \Gamma(D^0 \rightarrow K^{*-} K^+), \\
 \Gamma(D^+ \rightarrow K^{*+} \bar{K}^0) &= \Gamma(D_s^+ \rightarrow \rho^+ K^0), \\
 \Gamma(D^+ \rightarrow \bar{K}^{*0} K^+) &= \Gamma(D_s^+ \rightarrow K^{*0} \pi^+), \\
 \Gamma(D^0 \rightarrow \bar{K}^{*0} K^0) &= \Gamma(D^0 \rightarrow K^{*0} \bar{K}^0).
 \end{aligned} \quad (53)$$

We refer the reader to Refs. [42–45] for some explorations of the implications on decay rates and CP asymmetries, and Refs. [46,47] for the experimental analyses.

It is necessary to notice that since the QCD scale is comparable to charm quark mass, finite quark mass difference in s and d quarks may lead to sizable SU(3) symmetry breaking effect. A notable example to explore SU(3) symmetry breaking effects is the $D^0 \rightarrow K^0 \bar{K}^0$. This channel has vanishing branching fraction if exact SU(3) symmetry holds and

the $\bar{3}$ contribution proportional to $V_{cb} V_{ub}^*$ is neglected, which can also be seen from Table 4. The experimental data for branching ratios of $D^0 \rightarrow K^0 \bar{K}^0$ and $D^0 \rightarrow K^+ K^-$ are given as [3,4]:

$$\begin{aligned}
 \mathcal{B}(D^0 \rightarrow K^+ K^-) &= (3.97 \pm 0.07) \times 10^{-3}, \\
 \mathcal{B}(D^0 \rightarrow K^0 \bar{K}^0) &= (3.40 \pm 0.12) \times 10^{-4}.
 \end{aligned} \quad (54)$$

The decay amplitude of $D^0 \rightarrow K^+ K^-$ is $E + T$, where T is color-allowed and could be estimated using the factorization framework. As $|V_{cb} V_{ub}^* / V_{cs} V_{sd}^*| < 1.3 \times 10^{-3}$, it is unlikely that the above non-zero branching ratio is caused by the $\bar{3}$ penguin contribution. The above data shows that the SU(3) symmetry breaking effects in some channels can be as large as 30% at the amplitude level. Though the above estimate is channel-dependent, it indicates that symmetry breaking effects must carefully treated in D meson and charmed baryon decays in a systematic way [20,42,44,45,48,49].

5 $B_c \rightarrow DP, DV$ decays

The effective Hamiltonian for b quark decays can induce $B_c \rightarrow DP, DV$ transitions. The corresponding topological diagrams are given in Fig. 3. The IRA and TDA for $B_c \rightarrow DP$ decays are given as:

$$\begin{aligned}
 \mathcal{A}_u^{IRA} &= A_3^T B_c D_i H_3^i P_j^j + B_3^T B_c D_i H_3^i P_j^j \\
 &\quad + A_6^T B_c D_i (H_6)_{ij}^{[ik]} P_k^j + A_{15}^T B_c D_i (H_{15})_{ij}^{[ik]} P_k^j,
 \end{aligned} \quad (55)$$

$$\begin{aligned}
 \mathcal{A}_u^{TDA} &= S^u B_c D_i H_l^{li} P_j^j + P^u B_c D_i H_l^{lj} P_j^j \\
 &\quad + T B_c D_i H_l^{ik} P_k^l + C B_c D_i H_l^{ki} P_k^l.
 \end{aligned} \quad (56)$$

Table 5 Decay amplitudes for two-body Cabibbo-Allowed $D \rightarrow VP$ decays

Channel	IRA	TDA
$D^0 \rightarrow \rho^+ K^-$	$-A_6^{T1} + A_{15}^{T1} + C_6^{T1} + C_{15}^{T1}$	$T_1 + E_2$
$D^0 \rightarrow \rho^0 \bar{K}^0$	$(A_6^{T1} - A_{15}^{T1} - C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$(C_2 - E_2)/\sqrt{2}$
$D^0 \rightarrow \bar{K}^{*0} \pi^0$	$(A_6^{T2} - A_{15}^{T2} - C_6^{T1} + C_{15}^{T1})/\sqrt{2}$	$(C_1 - E_1)/\sqrt{2}$
$D^0 \rightarrow \bar{K}^{*0} \eta_q$	$(-A_6^{T2} + A_{15}^{T2} - 2B_6^{T2} + 2B_{15}^{T2} - C_6^{T1} + C_{15}^{T1})/\sqrt{2}$	$(C_1 + 2E_S^{u2} + E_1)/\sqrt{2}$
$D^0 \rightarrow \bar{K}^{*0} \eta_s$	$-A_6^{T1} + A_{15}^{T1} - B_6^{T2} + B_{15}^{T2}$	$E_S^{u2} + E_2$
$D^0 \rightarrow K^{*-} \pi^+$	$-A_6^{T2} + A_{15}^{T2} + C_6^{T2} + C_{15}^{T2}$	$T_2 + E_1$
$D^0 \rightarrow \omega \bar{K}^0$	$(-A_6^{T1} + A_{15}^{T1} - 2B_6^{T1} + 2B_{15}^{T1} - C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$(C_2 + 2E_S^{u1} + E_2)/\sqrt{2}$
$D^0 \rightarrow \phi \bar{K}^0$	$-A_6^{T2} + A_{15}^{T2} - B_6^{T1} + B_{15}^{T1}$	$E_S^{u1} + E_1$
$D^+ \rightarrow \rho^+ \bar{K}^0$	$C_6^{T1} + C_{15}^{T1} - C_6^{T2} + C_{15}^{T2}$	$C_2 + T_1$
$D^+ \rightarrow \bar{K}^{*0} \pi^+$	$-C_6^{T1} + C_{15}^{T1} + C_6^{T2} + C_{15}^{T2}$	$C_1 + T_2$
$D_s^+ \rightarrow \rho^+ \pi^0$	$(A_6^{T1} + A_{15}^{T1} - A_6^{T2} - A_{15}^{T2})/\sqrt{2}$	$(A_2 - A_1)/\sqrt{2}$
$D_s^+ \rightarrow \rho^+ \eta_q$	$(A_6^{T1} + A_{15}^{T1} + A_6^{T2} + A_{15}^{T2} + 2(B_6^{T2} + B_{15}^{T2}))/\sqrt{2}$	$(2A_S^{u2} + A_1 + A_2)/\sqrt{2}$
$D_s^+ \rightarrow \rho^+ \eta_s$	$B_6^{T2} + B_{15}^{T2} + C_6^{T1} + C_{15}^{T1}$	$A_S^{u2} + T_1$
$D_s^+ \rightarrow \rho^0 \pi^+$	$(-A_6^{T1} - A_{15}^{T1} + A_6^{T2} + A_{15}^{T2})/\sqrt{2}$	$(A_1 - A_2)/\sqrt{2}$
$D_s^+ \rightarrow K^{*+} \bar{K}^0$	$A_6^{T2} + A_{15}^{T2} - C_6^{T2} + C_{15}^{T2}$	$A_1 + C_2$
$D_s^+ \rightarrow \bar{K}^{*0} K^+$	$A_6^{T1} + A_{15}^{T1} - C_6^{T1} + C_{15}^{T1}$	$A_2 + C_1$
$D_s^+ \rightarrow \omega \pi^+$	$(A_6^{T1} + A_{15}^{T1} + A_6^{T2} + A_{15}^{T2} + 2(B_6^{T1} + B_{15}^{T1}))/\sqrt{2}$	$(2A_S^{u1} + A_1 + A_2)/\sqrt{2}$
$D_s^+ \rightarrow \phi \pi^+$	$B_6^{T1} + B_{15}^{T1} + C_6^{T2} + C_{15}^{T2}$	$A_S^{u1} + T_2$

The expanded amplitudes can be found in Table 8. Relations between the two sets of amplitudes are given as:

$$\begin{aligned}
 A_3^T &= S^u - \frac{1}{8}T + \frac{3}{8}C, & B_3^T &= P^u + \frac{3}{8}T - \frac{1}{8}C, \\
 A_6^T &= \frac{1}{4}T - \frac{1}{4}C, & A_{15}^T &= \frac{1}{8}T + \frac{1}{8}C.
 \end{aligned} \quad (57)$$

“Penguin” amplitudes are obtained similarly:

$$\begin{aligned}
 A_{3,6,15}^T &\rightarrow A_{3,6,15}^P, & B_3^T &\rightarrow B_3^P, & S^u &\rightarrow S, \\
 P^u &\rightarrow P, & T &\rightarrow P_T, & C &\rightarrow P_C.
 \end{aligned} \quad (58)$$

Including the “penguins”, one has 8 complex amplitudes in total.

Again decay amplitudes for $B_c \rightarrow DV$ can be obtained by replacing the pseudoscalars by their vector counterparts. The U-spin related channels include: $B_c^- \rightarrow \bar{D}^0 K^-$ and $B_c^- \rightarrow \bar{D}^0 \pi^-$; $B_c^- \rightarrow D^- \bar{K}^0$ and $B_c^- \rightarrow D_s^- K^0$; $B_c^- \rightarrow \bar{D}^0 K^{*-}$ and $B_c^- \rightarrow \bar{D}^0 \rho^-$; $B_c^- \rightarrow D^- \bar{K}^{*0}$ and $B_c^- \rightarrow D_s^- \bar{K}^{*0}$.

In Ref. [50], the LHCb collaboration has measured the product:

$$\frac{f(B_c)}{f(B^+)} \times \mathcal{B}(B_c^+ \rightarrow D^0 K^+) = (9.3_{-2.5}^{+2.8} \pm 0.6) \times 10^{-7}, \quad (59)$$

where the $f(B_c)$ and $f(B^+)$ are the production rates of B_c^+ and B^+ , respectively. With the measured ratio [51]:

$$\frac{f(B_c)}{f(B^+)} \sim 0.004 - 0.012, \quad (60)$$

one can obtain an estimated branching fraction:

$$\mathcal{B}(B_c^+ \rightarrow D^0 K^+) \sim 7.8 \times 10^{-5} - 2.3 \times 10^{-4}. \quad (61)$$

On theoretical side, model-dependent analyses give 1.3×10^{-7} [52], and 6.6×10^{-5} [53], while a phenomenological study implies the $\mathcal{B}(B_c^+ \rightarrow D^0 K^+) \sim [4.4 - 9] \times 10^{-5}$ [54]. Since this transition is induced by $b \rightarrow s$, the large branching fraction may imply a large penguin amplitude P . Such a scenario can be tested by measuring the corresponding $(B_c^+ \rightarrow D^+ K^0)$, which has the same penguin amplitude. Model-dependent calculations of other B_c decays can be found in Refs. [55–58]. Some recent SU(3) analyses of two-body B_c decays can be found in Refs. [59,60]. Compared to these studies, we have included all penguin amplitudes.

For the B_c^- meson, the charm quark can also decays, with the final state BP or BV [61]. Since the heavy bottom quark plays as a spectator, the decay modes are simpler. For example, for Cabibbo-allowed decay modes, there are only two channels: $B_c^- \rightarrow \pi^- \bar{B}_s^0$ and $B_c^- \rightarrow \rho^- \bar{B}_s^0$. Thus we expect that the SU(3) symmetry will not provide much information in these decays.

Table 6 Decay amplitudes for two-body Singly Cabibbo-Suppressed $D \rightarrow VP$ decays

Channel	IRA	TDA
$D^0 \rightarrow \rho^+ \pi^-$	$A_6^{T1} - A_{15}^{T1} - C_6^{T1} - C_{15}^{T1}$	$-T_1 - E_2$
$D^0 \rightarrow \rho^0 \pi^0$	$(1/2)(A_6^{T1} - A_{15}^{T1} + A_6^{T2} - A_{15}^{T2} - C_6^{T1} + C_{15}^{T1} - C_6^{T2} + C_{15}^{T2})$	$(1/2)(C_1 + C_2 - E_1 - E_2)$
$D^0 \rightarrow \rho^0 \eta_q$	$(1/2)(-A_6^{T1} + A_{15}^{T1} - A_6^{T2} + A_{15}^{T2} - 2B_6^{T2} + 2B_{15}^{T2} - C_6^{T1} + C_{15}^{T1} + C_6^{T2} - C_{15}^{T2})$	$(1/2)(C_1 - C_2 + 2E_s^{u2} + E_1 + E_2)$
$D^0 \rightarrow \rho^0 \eta_s$	$(-B_6^{T2} + B_{15}^{T2} - C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$(C_2 + E_s^{u2})/\sqrt{2}$
$D^0 \rightarrow \rho^- \pi^+$	$A_6^{T2} - A_{15}^{T2} - C_6^{T2} - C_{15}^{T2}$	$-T_2 - E_1$
$D^0 \rightarrow K^{*+} K^-$	$-A_6^{T1} + A_{15}^{T1} + C_6^{T1} + C_{15}^{T1}$	$T_1 + E_2$
$D^0 \rightarrow K^{*0} \bar{K}^0$	$-A_6^{T1} + A_{15}^{T1} + A_6^{T2} - A_{15}^{T2}$	$E_2 - E_1$
$D^0 \rightarrow \bar{K}^{*0} K^0$	$A_6^{T1} - A_{15}^{T1} - A_6^{T2} + A_{15}^{T2}$	$E_1 - E_2$
$D^0 \rightarrow K^{*-} K^+$	$-A_6^{T2} + A_{15}^{T2} + C_6^{T2} + C_{15}^{T2}$	$T_2 + E_1$
$D^0 \rightarrow \omega \pi^0$	$(1/2)(-A_6^{T1} + A_{15}^{T1} - A_6^{T2} + A_{15}^{T2} - 2B_6^{T1} + 2B_{15}^{T1} + C_6^{T1} - C_{15}^{T1} - C_6^{T2} + C_{15}^{T2})$	$(1/2)(-C_1 + C_2 + 2E_s^{u1} + E_1 + E_2)$
$D^0 \rightarrow \omega \eta_q$	$(1/2)(A_6^{T1} - A_{15}^{T1} + A_6^{T2} - A_{15}^{T2} + 2B_6^{T1} - 2B_{15}^{T1} + 2B_6^{T2} - 2B_{15}^{T2} + C_6^{T1} - C_{15}^{T1} + C_6^{T2} - C_{15}^{T2})$	$(1/2)(-C_1 - C_2 - 2E_s^{u1} - 2E_s^{u2} - E_1 - E_2)$
$D^0 \rightarrow \omega \eta_s$	$(-2B_6^{T1} + 2B_{15}^{T1} + B_6^{T2} - B_{15}^{T2} - C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$(C_2 + 2E_s^{u1} - E_s^{u2})/\sqrt{2}$
$D^0 \rightarrow \phi \pi^0$	$(-B_6^{T1} + B_{15}^{T1} - C_6^{T1} + C_{15}^{T1})/\sqrt{2}$	$(C_1 + E_s^{u1})/\sqrt{2}$
$D^0 \rightarrow \phi \eta_q$	$(B_6^{T1} - B_{15}^{T1} - 2B_6^{T2} + 2B_{15}^{T2} - C_6^{T1} + C_{15}^{T1})/\sqrt{2}$	$(C_1 - E_s^{u1} + 2E_s^{u2})/\sqrt{2}$
$D^0 \rightarrow \phi \eta_s$	$-A_6^{T1} + A_{15}^{T1} - A_6^{T2} + A_{15}^{T2} - B_6^{T1} + B_{15}^{T1} - B_6^{T2} + B_{15}^{T2}$	$E_s^{u1} + E_s^{u2} + E_1 + E_2$
$D^+ \rightarrow \rho^+ \pi^0$	$(-A_6^{T1} - A_{15}^{T1} + A_6^{T2} + A_{15}^{T2} + C_6^{T1} + C_{15}^{T1} - C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$(A_1 - A_2 + C_2 + T_1)/\sqrt{2}$
$D^+ \rightarrow \rho^+ \eta_q$	$(-A_6^{T1} + A_{15}^{T1} + A_6^{T2} + A_{15}^{T2} + 2B_6^{T2} + 2B_{15}^{T2} + C_6^{T1} + C_{15}^{T1} - C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$-(2A_s^{u2} + A_1 + A_2 + C_2 + T_1)/\sqrt{2}$
$D^+ \rightarrow \rho^+ \eta_s$	$-B_6^{T2} - B_{15}^{T2} - C_6^{T2} + C_{15}^{T2}$	$C_2 - A_s^{u2}$
$D^+ \rightarrow \rho^0 \pi^+$	$(A_6^{T1} + A_{15}^{T1} - A_6^{T2} - A_{15}^{T2} - C_6^{T1} + C_{15}^{T1} + C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$(-A_1 + A_2 + C_1 + T_2)/\sqrt{2}$
$D^+ \rightarrow K^{*+} \bar{K}^0$	$-A_6^{T2} - A_{15}^{T2} + C_6^{T1} + C_{15}^{T1}$	$T_1 - A_1$
$D^+ \rightarrow \bar{K}^{*0} K^+$	$-A_6^{T1} - A_{15}^{T1} + C_6^{T2} + C_{15}^{T2}$	$T_2 - A_2$
$D^+ \rightarrow \omega \pi^+$	$(-A_6^{T1} + A_{15}^{T1} + A_6^{T2} + A_{15}^{T2} + 2B_6^{T1} + 2B_{15}^{T1} - C_6^{T1} + C_{15}^{T1} + C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$-(2A_s^{u1} + A_1 + A_2 + C_1 + T_2)/\sqrt{2}$
$D^+ \rightarrow \phi \pi^+$	$-B_6^{T1} - B_{15}^{T1} - C_6^{T1} + C_{15}^{T1}$	$C_1 - A_s^{u1}$
$D_s^+ \rightarrow \rho^+ K^0$	$A_6^{T2} + A_{15}^{T2} - C_6^{T1} - C_{15}^{T1}$	$A_1 - T_1$
$D_s^+ \rightarrow \rho^0 K^+$	$(A_6^{T2} + A_{15}^{T2} - C_6^{T1} + C_{15}^{T1})/\sqrt{2}$	$(A_1 + C_1)/\sqrt{2}$
$D_s^+ \rightarrow K^{*+} \pi^0$	$(A_6^{T1} + A_{15}^{T1} - C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$(A_2 + C_2)/\sqrt{2}$
$D_s^+ \rightarrow K^{*+} \eta_q$	$(A_6^{T1} + A_{15}^{T1} + 2B_6^{T2} + 2B_{15}^{T2} + C_6^{T2} - C_{15}^{T2})/\sqrt{2}$	$(2A_s^{u2} + A_2 - C_2)/\sqrt{2}$
$D_s^+ \rightarrow K^{*+} \eta_s$	$A_6^{T2} + A_{15}^{T2} + B_6^{T2} + B_{15}^{T2} + C_6^{T1} + C_{15}^{T1} - C_6^{T2} + C_{15}^{T2}$	$A_s^{u2} + A_1 + C_2 + T_1$
$D_s^+ \rightarrow K^{*0} \pi^+$	$A_6^{T1} + A_{15}^{T1} - C_6^{T2} - C_{15}^{T2}$	$A_2 - T_2$
$D_s^+ \rightarrow \omega K^+$	$(A_6^{T2} + A_{15}^{T2} + 2B_6^{T1} + 2B_{15}^{T1} + C_6^{T1} - C_{15}^{T1})/\sqrt{2}$	$(2A_s^{u1} + A_1 - C_1)/\sqrt{2}$
$D_s^+ \rightarrow \phi K^+$	$A_6^{T1} + A_{15}^{T1} + B_6^{T1} + B_{15}^{T1} - C_6^{T1} + C_{15}^{T1} + C_6^{T2} + C_{15}^{T2}$	$A_s^{u1} + A_2 + C_1 + T_2$

It is necessary to point out that the charmless two-body B_c decays are purely annihilation, and the typical branching fractions are below the order 10^{-6} [62–64]. Since there are not too many channels, it is less useful to apply the flavor SU(3) symmetry to these modes.

6 Antitriplet bottom Baryon decay into a Baryon and a Meson

In this and next sections, we discuss weak decays of baryons with a heavy b and c quark. Charmed or bottom baryons with two light quarks can form an anti-triplet or a sextet. Most members of the sextet can decay via strong interactions or electromagnetic interactions. The only exceptions are Ω_b and

Table 7 Decay amplitudes for two-body Doubly Cabibbo-Suppressed $D \rightarrow VP$ decays

Channel	IRA	TDA
$D^0 \rightarrow \rho^0 K^0$	$(A_6^{T2} - A_{15}^{T2} - C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$(C_2 - E_1)/\sqrt{2}$
$D^0 \rightarrow \rho^- K^+$	$-A_6^{T2} + A_{15}^{T2} + C_6^{T2} + C_{15}^{T2}$	$T_2 + E_1$
$D^0 \rightarrow K^{*+} \pi^-$	$-A_6^{T1} + A_{15}^{T1} + C_6^{T1} + C_{15}^{T1}$	$T_1 + E_2$
$D^0 \rightarrow K^{*0} \pi^0$	$(A_6^{T1} - A_{15}^{T1} - C_6^{T1} + C_{15}^{T1})/\sqrt{2}$	$(C_1 - E_2)/\sqrt{2}$
$D^0 \rightarrow K^{*0} \eta_q$	$(-A_6^{T1} + A_{15}^{T1} - 2B_6^{T2} + 2B_{15}^{T2} - C_6^{T1} + C_{15}^{T1})/\sqrt{2}$	$(C_1 + 2E_S^{u2} + E_2)/\sqrt{2}$
$D^0 \rightarrow K^{*0} \eta_s$	$-A_6^{T2} + A_{15}^{T2} - B_6^{T2} + B_{15}^{T2}$	$E_S^{u2} + E_1$
$D^0 \rightarrow \omega K^0$	$(-A_6^{T2} + A_{15}^{T2} - 2B_6^{T1} + 2B_{15}^{T1} - C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$(C_2 + 2E_S^{u1} + E_1)/\sqrt{2}$
$D^0 \rightarrow \phi K^0$	$-A_6^{T1} + A_{15}^{T1} - B_6^{T1} + B_{15}^{T1}$	$E_S^{u1} + E_2$
$D^+ \rightarrow \rho^+ K^0$	$A_6^{T2} + A_{15}^{T2} - C_6^{T2} + C_{15}^{T2}$	$A_1 + C_2$
$D^+ \rightarrow \rho^0 K^+$	$(A_6^{T2} + A_{15}^{T2} - C_6^{T2} - C_{15}^{T2})/\sqrt{2}$	$(A_1 - T_2)/\sqrt{2}$
$D^+ \rightarrow K^{*+} \pi^0$	$(A_6^{T1} + A_{15}^{T1} - C_6^{T1} - C_{15}^{T1})/\sqrt{2}$	$(A_2 - T_1)/\sqrt{2}$
$D^+ \rightarrow K^{*+} \eta_q$	$(A_6^{T1} + A_{15}^{T1} + 2B_6^{T2} + 2B_{15}^{T2} + C_6^{T1} + C_{15}^{T1})/\sqrt{2}$	$(2A_S^{u2} + A_2 + T_1)/\sqrt{2}$
$D^+ \rightarrow K^{*+} \eta_s$	$A_6^{T2} + A_{15}^{T2} + B_6^{T2} + B_{15}^{T2}$	$A_S^{u2} + A_1$
$D^+ \rightarrow K^{*0} \pi^+$	$A_6^{T1} + A_{15}^{T1} - C_6^{T1} + C_{15}^{T1}$	$A_2 + C_1$
$D^+ \rightarrow \omega K^+$	$(A_6^{T2} + A_{15}^{T2} + 2B_6^{T1} + 2B_{15}^{T1} + C_6^{T2} + C_{15}^{T2})/\sqrt{2}$	$(2A_S^{u1} + A_1 + T_2)/\sqrt{2}$
$D^+ \rightarrow \phi K^+$	$A_6^{T1} + A_{15}^{T1} + B_6^{T1} + B_{15}^{T1}$	$A_S^{u1} + A_2$
$D_s^+ \rightarrow K^{*+} K^0$	$C_6^{T1} + C_{15}^{T1} - C_6^{T2} + C_{15}^{T2}$	$C_2 + T_1$
$D_s^+ \rightarrow K^{*0} K^+$	$-C_6^{T1} + C_{15}^{T1} + C_6^{T2} + C_{15}^{T2}$	$C_1 + T_2$

Ω_c . In the following we will concentrate on the anti-triplet baryons, whose weak decays are induced by the effective Hamiltonian H_{eff}^b and H_{eff}^c .

6.1 $T_b : (\Lambda_b, \Xi_b^0, \Xi_b^-)$ decay into a decuplet baryon T_{10} and a light meson

The IRA amplitudes for the T_b decays into a decuplet baryon and a light meson can be parametrized as:

$$\begin{aligned}
 \mathcal{A}_u^{IRA} = & A_3^T T_{b\bar{3}}^{[ij]} H_3^k (\bar{T}_{10})_{ikl} P_j^l \\
 & + A_6^T T_{b\bar{3}}^{[ij]} (H_6)_j^{[kl]} (\bar{T}_{10})_{ikm} P_l^m \\
 & + A_{15}^T T_{b\bar{3}}^{[ij]} (H_{15})_j^{[kl]} (\bar{T}_{10})_{ikm} P_l^m \\
 & + B_{15}^T T_{b\bar{3}}^{[ij]} (H_{15})_m^{[kl]} (\bar{T}_{10})_{ikl} P_j^m \\
 & + C_{15}^T T_{b\bar{3}}^{[ij]} (H_{15})_j^{[kl]} (\bar{T}_{10})_{ikl} P_m^m \\
 & + D_{15}^T T_{b\bar{3}}^{[ij]} (H_{15})_j^{[kl]} (\bar{T}_{10})_{klm} P_i^m. \quad (62)
 \end{aligned}$$

The TDA amplitudes are shown in Fig. 4 with the parametrization:

$$\begin{aligned}
 \mathcal{A}_u^{TDA} = & a_1 T_{b\bar{3}}^{[ij]} H_m^{mk} (\bar{T}_{10})_{ikl} P_j^l + b_1 T_{b\bar{3}}^{[ij]} H_j^{kl} (\bar{T}_{10})_{ikm} P_l^m \\
 & + b_2 T_{b\bar{3}}^{[ij]} H_j^{lk} (\bar{T}_{10})_{ikm} P_l^m \\
 & + b_3 T_{b\bar{3}}^{[ij]} H_m^{kl} (\bar{T}_{10})_{ikl} P_j^m + b_4 T_{b\bar{3}}^{[ij]} H_j^{kl} (\bar{T}_{10})_{ikl} P_m^m \\
 & + b_5 T_{b\bar{3}}^{[ij]} H_j^{kl} (\bar{T}_{10})_{klm} P_i^m. \quad (63)
 \end{aligned}$$

We find relations between the two sets of amplitudes as:

$$\begin{aligned}
 a_1 &= A_3^T + A_6^T - A_{15}^T - 2B_{15}^T + 2D_{15}^T, \\
 b_1 &= 4A_{15}^T + 2A_6^T, \\
 b_2 &= 4A_{15}^T - 2A_6^T, \quad b_3 = 8B_{15}^T, \\
 b_4 &= 8C_{15}^T, \quad b_5 = 8D_{15}^T. \quad (64)
 \end{aligned}$$

The expanded amplitudes for individual decay modes can be found in Table 9.

A few remarks are given in order.

- As the two light quarks in the initial state are antisymmetric in the flavor space while they are symmetric in the final state. An overlap of wave functions of the initial and final baryons is zero [65], which lead to vanishing decay amplitudes unless hard scattering interactions occur [66]. In other words, there is no “factorizable” contribution in the transition. In addition, all diagrams in Fig. 4 are suppressed by powers of $1/N_c$ compared to the $T_b \rightarrow T_8 P$. This will indicate that branching fractions for these decays are likely smaller than the relevant B decays and $T_b \rightarrow T_8 P$ decays, where T_8 represents the octet baryon.
- For the $T_b \rightarrow T_{10} P$, one can construct the amplitudes with the spinors, and a general form is:

$$\mathcal{A} = p_{T_b, \mu} \bar{u}^\mu (p_{T_{10}}) (A + B \gamma_5) u(p_{T_b}), \quad (65)$$

Fig. 3 Feynman diagrams for tree amplitudes in the $B_c \rightarrow DP$, DV decays

Table 8 Decay amplitudes for $B_c \rightarrow DP$ decays

$b \rightarrow d$	IRA	TDA	$b \rightarrow s$	IRA	TDA
$B_c^- \rightarrow \bar{D}^0 \pi^-$	$A_6^T + 3A_{15}^T + B_3^T$	$P^u + T$	$B_c^- \rightarrow \bar{D}^0 K^-$	$A_6^T + 3A_{15}^T + B_3^T$	$P^u + T$
$B_c^- \rightarrow D^- \pi^0$	$(-A_6^T + 5A_{15}^T - B_3^T)/\sqrt{2}$	$(C - P^u)/\sqrt{2}$	$B_c^- \rightarrow D^- \bar{K}^0$	$-A_6^T - A_{15}^T + B_3^T$	P^u
$B_c^- \rightarrow D^- \eta_q$	$(2A_3^T - A_6^T + A_{15}^T + B_3^T)/\sqrt{2}$	$(C + P^u + 2S^u)/\sqrt{2}$	$B_c^- \rightarrow D_s^- \pi^0$	$\sqrt{2}(2A_{15}^T - A_6^T)$	$C/\sqrt{2}$
$B_c^- \rightarrow D^- \eta_s$	$A_3^T + A_6^T - A_{15}^T$	S^u	$B_c^- \rightarrow D_s^- \eta_q$	$\sqrt{2}(A_3^T + A_{15}^T)$	$(C + 2S^u)/\sqrt{2}$
$B_c^- \rightarrow D_s^- K^0$	$-A_6^T - A_{15}^T + B_3^T$	P^u	$B_c^- \rightarrow D_s^- \eta_s$	$A_3^T - 2A_{15}^T + B_3^T$	$P^u + S^u$

Fig. 4 Topology diagrams for the bottom baryon decays into a decuplet baryon and a light meson. We have explicitly specified the quark flavors for the four-quark interaction vertex, while the unspecified quarks can be u , d , s

Table 9 Decay amplitudes for $T_b \rightarrow T_{10}P$ decays. Only those amplitudes proportional to $V_{ub}V_{uq}^*$ are shown, while “penguin” amplitudes proportional to $V_{tb}V_{tq}^*$ are similar

$b \rightarrow d$	IRA	TDA
$\Lambda_b^0 \rightarrow \Delta^+\pi^-$	$(A_3^T - A_6^T - 5A_{15}^T + 6B_{15}^T - 6D_{15}^T)/\sqrt{3}$	$(a_1 - b_1 + b_3 - b_5)/\sqrt{3}$
$\Lambda_b^0 \rightarrow \Delta^0\pi^0$	$\sqrt{2/3}(-A_3^T + A_6^T + A_{15}^T - 2B_{15}^T + 2D_{15}^T)$	$(-2a_1 + b_1 - b_2 - b_3 + b_5)/\sqrt{6}$
$\Lambda_b^0 \rightarrow \Delta^0\eta_q$	$-4\sqrt{2/3}(A_{15}^T + B_{15}^T + 2C_{15}^T + D_{15}^T)$	$-(b_1 + b_2 + b_3 + 2b_4 + b_5)/\sqrt{6}$
$\Lambda_b^0 \rightarrow \Delta^0\eta_s$	$-(8C_{15}^T)/\sqrt{3}$	$-b_4/\sqrt{3}$
$\Lambda_b^0 \rightarrow \Delta^-\pi^+$	$-A_3^T + A_6^T - 3A_{15}^T + 2B_{15}^T - 2D_{15}^T$	$-a_1 - b_2$
$\Lambda_b^0 \rightarrow \Sigma'^0 K^0$	$-(-A_3^T + A_6^T + 5A_{15}^T + 2B_{15}^T + 6D_{15}^T)/\sqrt{6}$	$(a_1 - b_1 - b_5)/\sqrt{6}$
$\Lambda_b^0 \rightarrow \Sigma'^- K^+$	$(-A_3^T + A_6^T - 3A_{15}^T + 2B_{15}^T - 2D_{15}^T)/\sqrt{3}$	$-(a_1 + b_2)/\sqrt{3}$
$\Xi_b^0 \rightarrow \Delta^+ K^-$	$(A_3^T + A_6^T - A_{15}^T + 6B_{15}^T - 6D_{15}^T)/\sqrt{3}$	$(a_1 + b_3 - b_5)/\sqrt{3}$
$\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0$	$(A_3^T + A_6^T - A_{15}^T - 2B_{15}^T - 6D_{15}^T)/\sqrt{3}$	$(a_1 - b_5)/\sqrt{3}$
$\Xi_b^0 \rightarrow \Sigma'^+ \pi^-$	$-(2(A_6^T + 2A_{15}^T))/\sqrt{3}$	$-b_1/\sqrt{3}$
$\Xi_b^0 \rightarrow \Sigma'^0 \pi^0$	$(-A_3^T + 3A_6^T + A_{15}^T - 6B_{15}^T - 2D_{15}^T)/2\sqrt{3}$	$-(a_1 - b_1 + b_2 + b_3)/2\sqrt{3}$
$\Xi_b^0 \rightarrow \Sigma'^0 \eta_q$	$-(A_3^T + A_6^T + 7A_{15}^T + 6B_{15}^T + 16C_{15}^T + 2D_{15}^T)/2\sqrt{3}$	$-(a_1 + b_1 + b_2 + b_3 + 2b_4)/2\sqrt{3}$
$\Xi_b^0 \rightarrow \Sigma'^0 \eta_s$	$(A_3^T + A_6^T - A_{15}^T - 2B_{15}^T - 8C_{15}^T - 6D_{15}^T)/\sqrt{6}$	$(a_1 - b_4 - b_5)/\sqrt{6}$
$\Xi_b^0 \rightarrow \Sigma'^- \pi^+$	$(-A_3^T + A_6^T - 3A_{15}^T + 2B_{15}^T - 2D_{15}^T)/\sqrt{3}$	$-(a_1 + b_2)/\sqrt{3}$
$\Xi_b^0 \rightarrow \Xi'^0 K^0$	$-(2(A_6^T + 2A_{15}^T))/\sqrt{3}$	$-b_1/\sqrt{3}$
$\Xi_b^0 \rightarrow \Xi'^- K^+$	$(-A_3^T + A_6^T - 3A_{15}^T + 2B_{15}^T - 2D_{15}^T)/\sqrt{3}$	$-(a_1 + b_2)/\sqrt{3}$
$\Xi_b^- \rightarrow \Delta^0 K^-$	$(A_3^T + A_6^T - A_{15}^T + 6B_{15}^T + 2D_{15}^T)/\sqrt{3}$	$(a_1 + b_3)/\sqrt{3}$
$\Xi_b^- \rightarrow \Delta^- \bar{K}^0$	$A_3^T + A_6^T - A_{15}^T - 2B_{15}^T + 2D_{15}^T$	a_1
$\Xi_b^- \rightarrow \Sigma'^0 \pi^-$	$(-A_3^T - A_6^T + A_{15}^T - 6B_{15}^T - 2D_{15}^T)/\sqrt{6}$	$-(a_1 + b_3)/\sqrt{6}$
$\Xi_b^- \rightarrow \Sigma'^- \pi^0$	$(A_3^T + A_6^T - A_{15}^T - 2B_{15}^T + 2D_{15}^T)/\sqrt{6}$	$a_1/\sqrt{6}$
$\Xi_b^- \rightarrow \Sigma'^- \eta_q$	$(-A_3^T - A_6^T + A_{15}^T + 2B_{15}^T - 2D_{15}^T)/\sqrt{6}$	$-a_1/\sqrt{6}$
$\Xi_b^- \rightarrow \Sigma'^- \eta_s$	$(A_3^T + A_6^T - A_{15}^T - 2B_{15}^T + 2D_{15}^T)/\sqrt{3}$	$a_1/\sqrt{3}$
$\Xi_b^- \rightarrow \Xi'^- K^0$	$(-A_3^T - A_6^T + A_{15}^T + 2B_{15}^T - 2D_{15}^T)/\sqrt{3}$	$-a_1/\sqrt{3}$
$b \rightarrow s$	IRA	TDA
$\Lambda_b^0 \rightarrow \Delta^+ K^-$	$-(2(A_6^T + 2A_{15}^T))/\sqrt{3}$	$-b_1/\sqrt{3}$
$\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^0$	$-(2(A_6^T + 2A_{15}^T))/\sqrt{3}$	$-b_1/\sqrt{3}$
$\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-$	$(A_3^T + A_6^T - A_{15}^T + 6B_{15}^T - 6D_{15}^T)/\sqrt{3}$	$(a_1 + b_3 - b_5)/\sqrt{3}$
$\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0$	$-(A_3^T + A_{15}^T + 2B_{15}^T - 2D_{15}^T)/\sqrt{3}$	$-(2a_1 + b_2 + b_3 - b_5)/2\sqrt{3}$
$\Lambda_b^0 \rightarrow \Sigma'^0 \eta_q$	$(A_6^T - 2A_{15}^T - 4(B_{15}^T + 2C_{15}^T + D_{15}^T))/\sqrt{3}$	$-(b_2 + b_3 + 2b_4 + b_5)/2\sqrt{3}$
$\Lambda_b^0 \rightarrow \Sigma'^0 \eta_s$	$-\sqrt{2/3}(A_6^T + 2A_{15}^T + 4C_{15}^T)$	$-(b_1 + b_4)/\sqrt{6}$
$\Lambda_b^0 \rightarrow \Sigma'^- \pi^+$	$(-A_3^T + A_6^T - 3A_{15}^T + 2B_{15}^T - 2D_{15}^T)/\sqrt{3}$	$-(a_1 + b_2)/\sqrt{3}$
$\Lambda_b^0 \rightarrow \Xi'^0 K^0$	$(A_3^T + A_6^T - A_{15}^T - 2B_{15}^T - 6D_{15}^T)/\sqrt{3}$	$(a_1 - b_5)/\sqrt{3}$
$\Lambda_b^0 \rightarrow \Xi'^- K^+$	$(-A_3^T + A_6^T - 3A_{15}^T + 2B_{15}^T - 2D_{15}^T)/\sqrt{3}$	$-(a_1 + b_2)/\sqrt{3}$
$\Xi_b^0 \rightarrow \Sigma'^+ K^-$	$(A_3^T - A_6^T - 5A_{15}^T + 6B_{15}^T - 6D_{15}^T)/\sqrt{3}$	$(a_1 - b_1 + b_3 - b_5)/\sqrt{3}$
$\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0$	$(-A_3^T + A_6^T + 5A_{15}^T + 2B_{15}^T + 6D_{15}^T)/\sqrt{6}$	$(a_1 - b_1 - b_5)/\sqrt{6}$
$\Xi_b^0 \rightarrow \Xi'^0 \pi^0$	$(-A_3^T - A_6^T + 3A_{15}^T + 6B_{15}^T + 2D_{15}^T)/\sqrt{6}$	$-(a_1 + b_2 + b_3)/\sqrt{6}$
$\Xi_b^0 \rightarrow \Xi'^0 \eta_q$	$(-A_3^T - A_6^T + 3A_{15}^T + 6B_{15}^T + 16C_{15}^T + 2D_{15}^T)/\sqrt{6}$	$-(a_1 + b_2 + b_3 + 2b_4)/\sqrt{6}$
$\Xi_b^0 \rightarrow \Xi'^0 \eta_s$	$(-A_3^T + A_6^T + 5A_{15}^T + 2B_{15}^T + 8C_{15}^T + 6D_{15}^T)/\sqrt{3}$	$(a_1 - b_1 - b_4 - b_5)/\sqrt{3}$
$\Xi_b^0 \rightarrow \Xi'^- \pi^+$	$(-A_3^T + A_6^T - 3A_{15}^T + 2B_{15}^T - 2D_{15}^T)/\sqrt{3}$	$-(a_1 + b_2)/\sqrt{3}$
$\Xi_b^0 \rightarrow \Omega^- K^+$	$-A_3^T + A_6^T - 3A_{15}^T + 2B_{15}^T - 2D_{15}^T$	$-a_1 - b_2$
$\Xi_b^- \rightarrow \Sigma'^0 K^-$	$(A_3^T + A_6^T - A_{15}^T + 6B_{15}^T + 2D_{15}^T)/\sqrt{6}$	$(a_1 + b_3)/\sqrt{6}$

Table 9 continued

$b \rightarrow s$	IRA	TDA
$\Xi_b^- \rightarrow \Sigma'^- \bar{K}^0$	$(A_3^T + A_6^T - A_{15}^T - 2B_{15}^T + 2D_{15}^T)/\sqrt{3}$	$a_1/\sqrt{3}$
$\Xi_b^- \rightarrow \Xi'^0 \pi^-$	$(-A_3^T - A_6^T + A_{15}^T - 6B_{15}^T - 2D_{15}^T)/\sqrt{3}$	$-(a_1 + b_3)/\sqrt{3}$
$\Xi_b^- \rightarrow \Xi'^- \pi^0$	$(A_3^T + A_6^T - A_{15}^T - 2B_{15}^T + 2D_{15}^T)/\sqrt{6}$	$a_1/\sqrt{6}$
$\Xi_b^- \rightarrow \Xi'^- \eta_q$	$(-A_3^T - A_6^T + A_{15}^T + 2B_{15}^T - 2D_{15}^T)/\sqrt{6}$	$-a_1/\sqrt{6}$
$\Xi_b^- \rightarrow \Xi'^- \eta_s$	$(A_3^T + A_6^T - A_{15}^T - 2B_{15}^T + 2D_{15}^T)/\sqrt{3}$	$a_1/\sqrt{3}$
$\Xi_b^- \rightarrow \Omega'^- K^0$	$-A_3^T - A_6^T + A_{15}^T + 2B_{15}^T - 2D_{15}^T$	$-a_1$

where A and B are two nonperturbative coefficients containing the CKM factors, and have the same flavor structure with $\mathcal{A}_{u,t}$. Thus in total, one has $6 \times 2 \times 2 = 24$ complex amplitudes in theory.

- Since the initial baryon and final baryons can be polarized, it is convenient to express the decays with helicity amplitudes:

$$\mathcal{A}(S_{in} \rightarrow S_{f1} S_{f2}), \quad (66)$$

where S_{in} and S_{f1}, S_{f2} are polarizations of initial and final states. The two sets of helicity amplitudes for $T_b \rightarrow T_{10} P$ can be derived using the parametrization in Eq. (65):

$$\mathcal{A}\left(\frac{1}{2} \rightarrow \frac{1}{2} 0\right) = \sqrt{\frac{2}{3}} \frac{m_{T_b}}{m_{T_{10}}} p_{cm} N_{T_{10}} N_{T_b} \left(A - B \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right), \quad (67)$$

$$\mathcal{A}\left(-\frac{1}{2} \rightarrow -\frac{1}{2} 0\right) = \sqrt{\frac{2}{3}} \frac{m_{T_b}}{m_{T_{10}}} p_{cm} N_{T_{10}} N_{T_b} \left(A + B \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right). \quad (68)$$

Here $E_{T_{10}}$ and p_{cm} are the energy and 3-momentum magnitude of T_{10} in the rest frame of T_b . $N_{T_{10}}$ and N_{T_b} are normalization factors of T_{10} and T_b spinors:

$$\begin{aligned} p_{cm} &= \frac{1}{2m_{T_b}} \sqrt{(m_{T_b}^2 - (m_{T_{10}} + m_P)^2)(m_{T_b}^2 - (m_{T_{10}} - m_P)^2)}, \\ E_{T_{10}} &= \frac{m_{T_{10}}^2 + m_{T_b}^2 - m_P^2}{2m_{T_b}}, \\ N_{T_{10}} &= \sqrt{\frac{(m_{T_{10}} + m_{T_b})^2 - m_P^2}{2m_{T_b}}}, \quad N_{T_b} = \sqrt{2m_{T_b}}. \end{aligned} \quad (69)$$

- For $T_b \rightarrow T_{10} V$, one can construct the amplitudes with the spinors and polarization vector:²

$$\begin{aligned} \mathcal{A} &= \epsilon^* \cdot p_{T_b} p_{T_b, \mu} \bar{u}^\mu(p_{T_{10}}) (A' + B' \gamma_5) u(p_{T_b}) \\ &\quad + \epsilon^{*\nu} p_{T_b, \mu} \bar{u}^\mu(p_{T_{10}}) (C' \gamma_\nu + D' \gamma_\nu \gamma_5) u(p_{T_b}) \\ &\quad + \epsilon_\mu^* \bar{u}^\mu(p_{T_{10}}) (E' + F' \gamma_5) u(p_{T_b}). \end{aligned} \quad (70)$$

There are six different polarization configurations. The helicity amplitudes are given as:

$$\begin{aligned} \mathcal{A}\left(\frac{1}{2} \rightarrow \frac{1}{2} 0\right) &= \sqrt{\frac{2}{3}} \frac{m_{T_b}}{m_{T_{10}}} p_{cm} N_{T_{10}} N_{T_b} \\ &\quad \times \left[-\frac{m_{T_b} p_{cm}}{m_V} \left(A' - B' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right) \right. \\ &\quad + C' \frac{p_{cm}}{m_V} \left(\frac{m_{T_b} - E_{T_{10}}}{E_{T_{10}} + m_{T_{10}}} - 1 \right) \\ &\quad + D' \left(\frac{m_{T_b} - E_{T_{10}}}{m_V} - \frac{p_{cm}^2}{m_V (E_{T_{10}} + m_{T_{10}})} \right) \\ &\quad + \left(-\frac{p_{cm}}{m_V m_{T_b}} + \frac{E_{T_{10}} (m_{T_b} - E_{T_{10}})}{m_V m_{T_b} p_{cm}} \right) \\ &\quad \left. \times \left(E' - F' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right) \right], \end{aligned}$$

$$\begin{aligned} \mathcal{A}\left(-\frac{1}{2} \rightarrow -\frac{1}{2} 0\right) &= \sqrt{\frac{2}{3}} \frac{m_{T_b}}{m_{T_{10}}} p_{cm} N_{T_{10}} N_{T_b} \\ &\quad \times \left[-\frac{m_{T_b} p_{cm}}{m_V} \left(A' + B' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right) \right. \\ &\quad + C' \frac{p_{cm}}{m_V} \left(\frac{m_{T_b} - E_{T_{10}}}{E_{T_{10}} + m_{T_{10}}} - 1 \right) \\ &\quad - D' \left(\frac{m_{T_b} - E_{T_{10}}}{m_V} - \frac{p_{cm}^2}{m_V (E_{T_{10}} + m_{T_{10}})} \right) \\ &\quad + \left(-\frac{p_{cm}}{m_V m_{T_b}} + \frac{E_{T_{10}} (m_{T_b} - E_{T_{10}})}{m_V m_{T_b} p_{cm}} \right) \\ &\quad \left. \times \left(E' + F' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right) \right], \end{aligned}$$

$$\mathcal{A}\left(\frac{1}{2} \rightarrow -\frac{1}{2} 1\right) = \frac{1}{\sqrt{3}} N_{T_b} N_{T_{10}}$$

² One may expect a term which looks like $\epsilon_{\mu\nu\alpha\beta} \epsilon^{*\mu} \bar{u}^\nu(p_{T_{10}}) (G' \sigma^{\alpha\beta} + H' \sigma^{\alpha\beta} \gamma_5) u(p_b)$. Actually such term can be absorbed into the term $\epsilon_\mu^* \bar{u}^\mu(p_{T_{10}}) (E' + F' \gamma_5) u(p_b)$ by using the fact that the spinor-vector $u^\mu(p_{T_{10}})$, as a irreducible representation of $1/2 \otimes 1$, must satisfy $\gamma_\mu u^\mu(p_{T_{10}}) = 0$.

$$\begin{aligned}
& \times \left[\frac{2p_{cm}m_{T_b}}{m_{T_{10}}} \left(D' - C' \frac{p}{E_{T_{10}} + m_{T_{10}}} \right) \right. \\
& \quad \left. + \left(E' - F' \frac{p}{E_{T_{10}} + m_{T_{10}}} \right) \right], \\
\mathcal{A} \left(-\frac{1}{2} \rightarrow \frac{1}{2} - 1 \right) &= \frac{1}{\sqrt{3}} N_{T_b} N_{T_{10}} \\
& \times \left[-\frac{2p_{cm}m_{T_b}}{m_{T_{10}}} \left(D' + C' \frac{p}{E_{T_{10}} + m_{T_{10}}} \right) \right. \\
& \quad \left. + \left(E' + F' \frac{p}{E_{T_{10}} + m_{T_{10}}} \right) \right], \\
\mathcal{A} \left(\frac{1}{2} \rightarrow \frac{3}{2} - 1 \right) &= N_{T_b} N_{T_{10}} \left(E' - F' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right), \\
\mathcal{A} \left(-\frac{1}{2} \rightarrow -\frac{3}{2} 1 \right) &= N_{T_b} N_{T_{10}} \left(E' + F' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right). \quad (71)
\end{aligned}$$

The definitions of $E_{T_{10}}$, p_{cm} , $N_{T_{10}}$ and N_{T_b} are the same as Eq. (69) except replacing m_P by m_V . Again all these amplitudes can be determined from the angular distributions of the four-body decays $T_b \rightarrow T_{10}(\rightarrow T_8 P_1)V(\rightarrow P_2 P_3)$.

- Branching fractions for T_b decays into a proton with three charged pion/kaons are found at the order 10^{-5} in Ref. [67]. A plausible scenario is that the $T_b \rightarrow T_{10}V$ contribute significantly to the T_b decaying into a proton and three charged light mesons. If this is true, we expect that with more data in future, a detailed analysis will determine the decay widths of $T_b \rightarrow T_{10}V$. Then the flavor SU(3) symmetry can be examined, and meanwhile it will also shed light on the CP and T violation in baryonic transitions by using the triplet product asymmetries [68, 69].
- Through the results in Table 9, we can find the relations both for decays into $T_8 P$ and $T_8 V$. Here only the channels with one vector octet in final states can be listed (72), (73). For channels with one pseudoscalar in final states the relations are almost the same, obtained by replacing the vector multiplets V by the pseudo-scalar multiplets P . However, η_q and η_s are unphysical states so that the decay width relations involving them should be removed. For $b \rightarrow d$ transitions, one has:

$$\begin{aligned}
\Gamma(\Lambda_b^0 \rightarrow \Delta^- \rho^+) &= 3\Gamma(\Lambda_b^0 \rightarrow \Sigma'^- K^{*+}), \\
\Gamma(\Xi_b^- \rightarrow \Delta^- \bar{K}^{*0}) &= 6\Gamma(\Xi_b^- \rightarrow \Sigma'^- \omega), \\
\Gamma(\Lambda_b^0 \rightarrow \Delta^- \rho^+) &= 3\Gamma(\Xi_b^0 \rightarrow \Sigma'^- \rho^+), \\
\Gamma(\Xi_b^- \rightarrow \Delta^- \bar{K}^{*0}) &= 3\Gamma(\Xi_b^- \rightarrow \Sigma'^- \phi), \\
\Gamma(\Lambda_b^0 \rightarrow \Delta^- \rho^+) &= 3\Gamma(\Xi_b^0 \rightarrow \Xi'^- K^{*+}), \\
\Gamma(\Xi_b^- \rightarrow \Delta^- \bar{K}^{*0}) &= 3\Gamma(\Xi_b^- \rightarrow \Xi'^- K^{*0}), \\
\Gamma(\Lambda_b^0 \rightarrow \Sigma'^- K^{*+}) &= \Gamma(\Xi_b^0 \rightarrow \Sigma'^- \rho^+), \\
\Gamma(\Xi_b^- \rightarrow \Sigma'^- \rho^0) &= \Gamma(\Xi_b^- \rightarrow \Sigma'^- \omega), \\
\Gamma(\Lambda_b^0 \rightarrow \Sigma'^- K^{*+}) &= \Gamma(\Xi_b^0 \rightarrow \Xi'^- K^{*+}),
\end{aligned}$$

$$\begin{aligned}
\Gamma(\Xi_b^- \rightarrow \Sigma'^- \rho^0) &= \frac{1}{2}\Gamma(\Xi_b^- \rightarrow \Sigma'^- \phi), \\
\Gamma(\Xi_b^0 \rightarrow \Sigma'^+ \rho^-) &= \Gamma(\Xi_b^0 \rightarrow \Xi'^0 K^{*0}), \\
\Gamma(\Xi_b^- \rightarrow \Sigma'^- \rho^0) &= \frac{1}{2}\Gamma(\Xi_b^- \rightarrow \Xi'^- K^{*0}), \\
\Gamma(\Xi_b^0 \rightarrow \Sigma'^- \rho^+) &= \Gamma(\Xi_b^0 \rightarrow \Xi'^- K^{*+}), \\
\Gamma(\Xi_b^- \rightarrow \Sigma'^- \omega) &= \frac{1}{2}\Gamma(\Xi_b^- \rightarrow \Sigma'^- \phi), \\
\Gamma(\Xi_b^- \rightarrow \Delta^0 K^{*-}) &= 2\Gamma(\Xi_b^- \rightarrow \Sigma'^0 \rho^-), \\
\Gamma(\Xi_b^- \rightarrow \Sigma'^- \omega) &= \frac{1}{2}\Gamma(\Xi_b^- \rightarrow \Xi'^- K^{*0}), \\
\Gamma(\Xi_b^- \rightarrow \Delta^- \bar{K}^{*0}) &= 6\Gamma(\Xi_b^- \rightarrow \Sigma'^- \rho^0), \\
\Gamma(\Xi_b^- \rightarrow \Sigma'^- \phi) &= \Gamma(\Xi_b^- \rightarrow \Xi'^- K^{*0}). \quad (72)
\end{aligned}$$

For $b \rightarrow s$ transition, we have the relations for decay widths:

$$\begin{aligned}
\Gamma(\Lambda_b^0 \rightarrow \Delta^+ K^{*-}) &= \Gamma(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^{*0}), \\
\Gamma(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^{*0}) &= 2\Gamma(\Xi_b^- \rightarrow \Xi'^- \omega), \\
\Gamma(\Lambda_b^0 \rightarrow \Sigma'^- \rho^+) &= \Gamma(\Lambda_b^0 \rightarrow \Xi'^- K^{*+}), \\
\Gamma(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^{*0}) &= \Gamma(\Xi_b^- \rightarrow \Xi'^- \phi), \\
\Gamma(\Lambda_b^0 \rightarrow \Sigma'^- \rho^+) &= \Gamma(\Xi_b^0 \rightarrow \Xi'^- \rho^+), \\
\Gamma(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^{*0}) &= \frac{1}{3}\Gamma(\Xi_b^- \rightarrow \Omega^- K^{*0}), \\
\Gamma(\Lambda_b^0 \rightarrow \Sigma'^- \rho^+) &= \frac{1}{3}\Gamma(\Xi_b^0 \rightarrow \Omega^- K^{*+}), \\
\Gamma(\Xi_b^- \rightarrow \Xi'^- \rho^0) &= \Gamma(\Xi_b^- \rightarrow \Xi'^- \omega), \\
\Gamma(\Lambda_b^0 \rightarrow \Xi'^- K^{*+}) &= \Gamma(\Xi_b^0 \rightarrow \Xi'^- \rho^+), \\
\Gamma(\Xi_b^- \rightarrow \Xi'^- \rho^0) &= \frac{1}{2}\Gamma(\Xi_b^- \rightarrow \Xi'^- \phi), \\
\Gamma(\Lambda_b^0 \rightarrow \Xi'^- K^{*+}) &= \frac{1}{3}\Gamma(\Xi_b^0 \rightarrow \Omega^- K^{*+}), \\
\Gamma(\Xi_b^- \rightarrow \Xi'^- \rho^0) &= \frac{1}{6}\Gamma(\Xi_b^- \rightarrow \Omega^- K^{*0}), \\
\Gamma(\Xi_b^0 \rightarrow \Xi'^- \rho^+) &= \frac{1}{3}\Gamma(\Xi_b^0 \rightarrow \Omega^- K^{*+}), \\
\Gamma(\Xi_b^- \rightarrow \Xi'^- \omega) &= \frac{1}{2}\Gamma(\Xi_b^- \rightarrow \Xi'^- \phi), \\
\Gamma(\Xi_b^- \rightarrow \Sigma'^0 K^{*-}) &= \frac{1}{2}\Gamma(\Xi_b^- \rightarrow \Xi'^0 \rho^-), \\
\Gamma(\Xi_b^- \rightarrow \Xi'^- \omega) &= \frac{1}{6}\Gamma(\Xi_b^- \rightarrow \Omega^- K^{*0}), \\
\Gamma(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^{*0}) &= 2\Gamma(\Xi_b^- \rightarrow \Xi'^- \rho^0), \\
\Gamma(\Xi_b^- \rightarrow \Xi'^- \phi) &= \frac{1}{3}\Gamma(\Xi_b^- \rightarrow \Omega^- K^{*0}). \quad (73)
\end{aligned}$$

- As discussed in the previous section, charmless $b \rightarrow d$ and $b \rightarrow s$ transitions can be connected by U-spin. In Table 10, we collect the $T_b \rightarrow T_{10}V$ decay pairs related

Table 10 U-spin relations for $T_b \rightarrow T_{10}V$. If the final state contains a light pseudoscalar meson, the U-spin relations can be obtained similarly except that η_q and η_s mix

$b \rightarrow d$	$b \rightarrow s$	r	$b \rightarrow d$	$b \rightarrow s$	r
$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \rho^-)$	$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ K^{*-})$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \omega)$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^{*0})$	$-\frac{1}{\sqrt{2}}$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^- \rho^+)$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \rho^+)$	$\sqrt{3}$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \omega)$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \rho^0)$	-1
$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^- \rho^+)$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^- K^{*+})$	$\sqrt{3}$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \omega)$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \omega)$	1
$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^- \rho^+)$	$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \rho^+)$	$\sqrt{3}$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \omega)$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \phi)$	$-\frac{1}{\sqrt{2}}$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^- \rho^+)$	$\mathcal{A}(\Xi_b^0 \rightarrow \Omega^- K^{*+})$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \omega)$	$\mathcal{A}(\Xi_b^- \rightarrow \Omega^- K^{*0})$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 K^{*0})$	$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^{*0})$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \phi)$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^{*0})$	1
$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- K^{*+})$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \rho^+)$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \phi)$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \rho^0)$	$\sqrt{2}$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- K^{*+})$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^- K^{*+})$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \phi)$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \omega)$	$-\sqrt{2}$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- K^{*+})$	$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \rho^+)$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \phi)$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \phi)$	1
$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- K^{*+})$	$\mathcal{A}(\Xi_b^0 \rightarrow \Omega^- K^{*+})$	$\frac{1}{\sqrt{3}}$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \phi)$	$\mathcal{A}(\Xi_b^- \rightarrow \Omega^- K^{*0})$	$-\frac{1}{\sqrt{3}}$
$\mathcal{A}(\Xi_b^0 \rightarrow \Delta^+ K^{*-})$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \rho^-)$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- K^{*0})$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^{*0})$	-1
$\mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^{*0})$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^0 K^{*0})$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- K^{*0})$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \rho^0)$	$-\sqrt{2}$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ \rho^-)$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^{*-})$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- K^{*0})$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \omega)$	$\sqrt{2}$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ \rho^-)$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^{*0})$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- K^{*0})$	$-1 \mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \phi)$	-1
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \rho^+)$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \rho^+)$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- K^{*0})$	$\mathcal{A}(\Xi_b^- \rightarrow \Omega^- K^{*0})$	$\frac{1}{\sqrt{3}}$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \rho^+)$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^- K^{*+})$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 \rho^-)$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 K^{*-})$	-1
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \rho^+)$	$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \rho^+)$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 \rho^-)$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^0 \rho^-)$	$\frac{1}{\sqrt{2}}$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \rho^+)$	$\mathcal{A}(\Xi_b^0 \rightarrow \Omega^- K^{*+})$	$\frac{1}{\sqrt{3}}$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \rho^0)$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^{*0})$	$\frac{1}{\sqrt{2}}$
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 K^{*0})$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^{*-})$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \rho^0)$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \rho^0)$	1
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 K^{*0})$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^{*0})$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \rho^0)$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \omega)$	-1
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- K^{*+})$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \rho^+)$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \rho^0)$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \phi)$	$\frac{1}{\sqrt{2}}$
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- K^{*+})$	$\mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^- K^{*+})$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \rho^0)$	$\mathcal{A}(\Xi_b^- \rightarrow \Omega^- K^{*0})$	$-\frac{1}{\sqrt{6}}$
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- K^{*+})$	$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \rho^+)$	1	$\mathcal{A}(\Xi_b^- \rightarrow \Delta^- \bar{K}^{*0})$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^{*0})$	$\sqrt{3}$
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- K^{*+})$	$\mathcal{A}(\Xi_b^0 \rightarrow \Omega^- K^{*+})$	$\frac{1}{\sqrt{3}}$	$\mathcal{A}(\Xi_b^- \rightarrow \Delta^- \bar{K}^{*0})$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \rho^0)$	$\sqrt{6}$
$\mathcal{A}(\Xi_b^- \rightarrow \Delta^0 K^{*-})$	$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 K^{*-})$	$\sqrt{2}$	$\mathcal{A}(\Xi_b^- \rightarrow \Delta^- \bar{K}^{*0})$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \omega)$	$-\sqrt{6}$
$\mathcal{A}(\Xi_b^- \rightarrow \Delta^0 K^{*-})$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^0 \rho^-)$	-1	$\mathcal{A}(\Xi_b^- \rightarrow \Delta^- \bar{K}^{*0})$	$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \phi)$	$\sqrt{3}$
			$\mathcal{A}(\Xi_b^- \rightarrow \Delta^- \bar{K}^{*0})$	$\mathcal{A}(\Xi_b^- \rightarrow \Omega^- K^{*0})$	-1

by U -spin, while results for the final state with a light pseudoscalar meson can be obtained similarly. CP asymmetries for these pairs satisfy relation in Eq. (35). Inspired from B decay data [3, 4], we expect CP asymmetries for these decays are at the order 10%. Experimental measurements of these relations are important to test flavor $SU(3)$ symmetry and the CKM description of CP violation in SM.

6.2 $T_b(\Lambda_b, \Xi_b^0, \Xi_b^-)$ decay into an octet baryon and a meson

If the final state contains a baryon octet, the topological diagrams are shown in Fig. 5 where ten diagrams can be found.

However unlike the decuplet baryon, the octet baryon is not fully symmetric or antisymmetric in flavor space. Thus each of the diagrams can provide more than one amplitudes. In total, one can have 26 independent TDA amplitudes:

$$\begin{aligned}
 \mathcal{A}_u^{TDA} = & \bar{a}_1 T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{ijk} P_l^m + \bar{a}_2 T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{ijl} P_k^m \\
 & + \bar{a}_3 T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{jkl} P_m^m + \bar{a}_4 T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{jkm} P_l^m \\
 & + \bar{a}_5 T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{jlk} P_m^m + \bar{a}_6 T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{jmk} P_l^m \\
 & + \bar{a}_7 T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{jlm} P_k^m + \bar{a}_8 T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{jml} P_k^m \\
 & + \bar{a}_9 T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{klj} P_m^m + \bar{a}_{10} T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{kmj} P_l^m \\
 & + \bar{a}_{11} T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{lmj} P_k^m + \bar{a}_{12} T_{b3}^{[ij]} H_i^{kl} (\bar{T}_8)_{klm} P_j^m
 \end{aligned}$$

Fig. 5 Topology diagrams for the bottom baryon decays into an octet baryon and a light meson. Since the octet baryon is not fully symmetric or antisymmetric in flavor space, there are more than one amplitudes

corresponding to one topological diagram. Actually the ten topological diagrams correspond to 26 amplitudes shown in Eq.(74). As in Fig. 4, the unspecified quark flavors can be u, d, s

$$\begin{aligned}
 & +\bar{a}_{13}T_{b\bar{3}}^{[ij]}H_i^{kl}(\bar{T}_8)_{klm}P_j^m + \bar{a}_{14}T_{b\bar{3}}^{[ij]}H_i^{kl}(\bar{T}_8)_{lmk}P_j^m \\
 & +\bar{a}_{15}T_{b\bar{3}}^{[ij]}H_m^{kl}(\bar{T}_8)_{ikj}P_l^m + \bar{a}_{16}T_{b\bar{3}}^{[ij]}H_m^{kl}(\bar{T}_8)_{ilj}P_k^m \\
 & +\bar{a}_{17}T_{b\bar{3}}^{[ij]}H_m^{kl}(\bar{T}_8)_{ikl}P_j^m + \bar{a}_{18}T_{b\bar{3}}^{[ij]}H_m^{kl}(\bar{T}_8)_{ilk}P_j^m \\
 & +\bar{a}_{19}T_{b\bar{3}}^{[ij]}H_m^{kl}(\bar{T}_8)_{klj}P_i^m \\
 & +\bar{b}_1T_{b\bar{3}}^{[ij]}H_l^{lk}(\bar{T}_8)_{ijk}P_m^m + \bar{b}_2T_{b\bar{3}}^{[ij]}H_l^{lk}(\bar{T}_8)_{ijm}P_k^m \\
 & +\bar{b}_3T_{b\bar{3}}^{[ij]}H_l^{lk}(\bar{T}_8)_{ikj}P_m^m + \bar{b}_4T_{b\bar{3}}^{[ij]}H_l^{lk}(\bar{T}_8)_{imj}P_k^m \\
 & +\bar{b}_5T_{b\bar{3}}^{[ij]}H_l^{lk}(\bar{T}_8)_{ikm}P_j^m + \bar{b}_6T_{b\bar{3}}^{[ij]}H_l^{lk}(\bar{T}_8)_{imk}P_j^m \\
 & +\bar{b}_7T_{b\bar{3}}^{[ij]}H_l^{lk}(\bar{T}_8)_{kmi}P_j^m.
 \end{aligned} \quad (74)$$

In the IRA approach, one can construct 14 amplitudes:

$$\begin{aligned}
 \mathcal{A}_u^{IRA} = & A_3^T(T_{b\bar{3}})_i H_3^j(\bar{T}_8)_j^i P_k^k + B_3^T(T_{b\bar{3}})_i H_3^j(\bar{T}_8)_k^i P_j^k \\
 & + C_3^T(T_{b\bar{3}})_i H_3^i(\bar{T}_8)_l^k P_k^l + D_3^T(T_{b\bar{3}})_i H_3^j(\bar{T}_8)_j^k P_k^i \\
 & + A_6^T(T_{b\bar{3}})_i (H_6)_j^{[ik]}(\bar{T}_8)_k^j P_l^l
 \end{aligned}$$

$$\begin{aligned}
 & + B_6^T(T_{b\bar{3}})_i (H_6)_j^{[ik]}(\bar{T}_8)_k^l P_l^j \\
 & + C_6^T(T_{b\bar{3}})_i (H_6)_j^{[ik]}(\bar{T}_8)_l^j P_k^l \\
 & + E_6^T(T_{b\bar{3}})_i (H_6)_l^{[jk]}(\bar{T}_8)_j^i P_k^l \\
 & + D_6^T(T_{b\bar{3}})_i (H_6)_l^{[jk]}(\bar{T}_8)_l^j P_k^i \\
 & + A_{15}^T(T_{b\bar{3}})_i (H_{15})_j^{[ik]}(\bar{T}_8)_k^j P_l^l \\
 & + B_{15}^T(T_{b\bar{3}})_i (H_{15})_j^{[ik]}(\bar{T}_8)_k^j P_l^j \\
 & + C_{15}^T(T_{b\bar{3}})_i (H_{15})_j^{[ik]}(\bar{T}_8)_l^j P_k^l \\
 & + E_{15}^T(T_{b\bar{3}})_i (H_{15})_l^{[jk]}(\bar{T}_8)_j^i P_k^l \\
 & + D_{15}^T(T_{b\bar{3}})_i (H_{15})_l^{[jk]}(\bar{T}_8)_l^j P_k^i.
 \end{aligned} \quad (75)$$

It should be noticed that in the above hadrons have been written in different forms for the same SU(3) multiplet. For illustration, we use the heavy bottom anti-triplet as the example.

Since it is an anti-triplet, it is most straightforward to use the $(T_{b\bar{3}})_i$ to represent this particle multiplet, as in IRA approach. The advantage of this form is its compactness, however, with this form it is not easy to understand the quark flows in the decays. Instead there are two anti-symmetric light quarks in the anti-triplet heavy bottom baryon, and thus it is viable to use $(T_{b\bar{3}})^{ij}$ (with ij anti-symmetric) to denote this multiplet. The second form contains two SU(3) indices, less compact, but it can reflect the quark flows in the decays. Thus the second form is suitable for drawing Feynman diagrams, and thus adopted in the above TDA amplitude. These two forms are equal, and one can establish relations between the SU(3) parameters corresponding to these two forms. The explicit discussions in IRA approach are given in “Appendix A”.

The 14 IRA amplitudes and 26 TDA amplitudes are related as follows:

$$\begin{aligned}
 A_3^T &= \frac{1}{8} (-2\bar{a}_1 + 6\bar{a}_2 - 5\bar{a}_3 - \bar{a}_5 + \bar{a}_6 - 3\bar{a}_8 + 4\bar{a}_9 + \bar{a}_{10} \\
 &\quad - 3\bar{a}_{11} + 2\bar{a}_{13} + 2\bar{a}_{14} - \bar{a}_{15} + 3\bar{a}_{16} + 3\bar{a}_{17} - \bar{a}_{18} + 4\bar{a}_{19}) \\
 &\quad + 2\bar{b}_1 + \bar{b}_3 + \bar{b}_6 + \bar{b}_7, \\
 B_3^T &= \frac{1}{8} (6\bar{a}_1 - 2\bar{a}_2 - 5\bar{a}_4 - 3\bar{a}_6 - \bar{a}_7 + \bar{a}_8 + 2\bar{a}_{10} + 2\bar{a}_{11} \\
 &\quad + 4\bar{a}_{12} + \bar{a}_{13} - 3\bar{a}_{14} + 3\bar{a}_{15} - \bar{a}_{16} - \bar{a}_{17} + 3\bar{a}_{18} - 4\bar{a}_{19}) \\
 &\quad + (2\bar{b}_2 + \bar{b}_4 + \bar{b}_5 - \bar{b}_7), \\
 C_3^T &= \frac{1}{8} (-\bar{a}_4 + 3\bar{a}_7 + \bar{a}_{10} - 3\bar{a}_{11} - 4\bar{a}_{12} - \bar{a}_{13} + 3\bar{a}_{14} \\
 &\quad + \bar{a}_{17} - 3\bar{a}_{18} + 4\bar{a}_{19} - 8\bar{b}_5) + \bar{b}_7, \\
 D_3^T &= \frac{1}{8} (-4(\bar{a}_{19} + 2(\bar{b}_6 + \bar{b}_7)) - \bar{a}_6 \\
 &\quad + 3\bar{a}_8 - \bar{a}_{10} + 3\bar{a}_{11} - 2\bar{a}_{13} - 2\bar{a}_{14} - 3\bar{a}_{17} + \bar{a}_{18}), \\
 A_6^T &= \frac{1}{4} (\bar{a}_3 - \bar{a}_5 - 2\bar{a}_9 - \bar{a}_{13} + \bar{a}_{14}), \\
 B_6^T &= \frac{1}{4} (\bar{a}_{13} - \bar{a}_{14} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19}), \\
 C_6^T &= \frac{1}{4} (\bar{a}_4 - \bar{a}_7 - \bar{a}_{10} + \bar{a}_{11} - 2\bar{a}_{12}), \\
 D_6^T &= \frac{1}{4} (\bar{a}_6 - \bar{a}_8 + \bar{a}_{10} - \bar{a}_{11} - \bar{a}_{13} + \bar{a}_{14}), \\
 E_6^T &= \frac{1}{4} (2\bar{a}_1 - 2\bar{a}_2 - \bar{a}_6 + \bar{a}_8 - \bar{a}_{10} + \bar{a}_{11} + \bar{a}_{13} \\
 &\quad - \bar{a}_{14} + \bar{a}_{15} - \bar{a}_{16} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19}), \\
 A_{15}^T &= \frac{1}{8} (\bar{a}_3 + \bar{a}_5 - \bar{a}_{13} - \bar{a}_{14}), \\
 B_{15}^T &= \frac{1}{8} (\bar{a}_{13} + \bar{a}_{14} - \bar{a}_{17} - \bar{a}_{18}), \\
 C_{15}^T &= \frac{1}{8} (\bar{a}_4 + \bar{a}_7 - \bar{a}_{10} - \bar{a}_{11}), \\
 D_{15}^T &= \frac{1}{8} (\bar{a}_6 + \bar{a}_8 + \bar{a}_{10} + \bar{a}_{11} + \bar{a}_{13} + \bar{a}_{14}), \\
 E_{15}^T &= \frac{1}{8} (2\bar{a}_1 + 2\bar{a}_2 - \bar{a}_6 - \bar{a}_8 - \bar{a}_{10} - \bar{a}_{11} \\
 &\quad - \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{15} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{18}). \quad (76)
 \end{aligned}$$

However, even after such reduction, there still exists one independent degree of freedom among the 14 IRA ampli-

tudes. The redundant amplitude can be made explicit with the redefinitions:

$$\begin{aligned}
 A_6^{T'} &= A_6^T + B_6^T, \quad B_6^{T'} = B_6^T - C_6^T, \\
 C_6^{T'} &= C_6^T - E_6^T, \quad D_6^{T'} = C_6^T + D_6^T. \quad (77)
 \end{aligned}$$

In addition, this redundancy can be understood more explicitly. In this work as well as the previous work Ref. [22] we use the irreducible representation operators for IRA as $(H_{6/\bar{15}})^{ij}_k$. Actually there exists a simpler H_6 representation introduced by Ref. [70], where H_6 has only two lower indexes $(H_6)_{ij}$. With the use of $(H_6)_{ij}$ we do have only 13 IRA amplitudes. However, Since the IRA operators $(H_{6/\bar{15}})^{ij}_k$ have the same index structure as the TDA operators. They make the derivation of IRA/TDA correspondence more directly so we will keep the use of them.

The expanded amplitudes can be found in Table 11 for the $b \rightarrow d$ transition and Table 12 for the $b \rightarrow s$ transition, respectively. Again if the final state is a vector meson, the amplitudes can be derived similarly.

A few remarks are given in order.

- At a first sight, the diagrammatic approach, as depicted in Fig. 5, is more intuitive, however there are more TDA amplitudes than the corresponding diagrams. For an octet baryon in the final state, there are three light quarks. In the same diagram, the symmetry of the quarks in flavor space could be different. For example, in the third diagram of Fig. 5, the u quark and another light quark could be flavor anti-symmetric, or the two unspecified quarks could be flavor anti-symmetric. These different combinations will lead to different TDA amplitudes. Thus it is very hard to determine the independent amplitudes in this approach, which will introduce subtleties to the global fit in the diagrammatic approach. The mismatch between Feynman diagrams and TDA amplitudes will not happen for the decays into decuplet baryons, since all three quarks are symmetric in flavor space.
- Without including the polarization, one can see from the IRA approach, there exist 13 independent complex amplitudes with CKM factor $V_{ub}V_{uq}^*$ and another 13 amplitudes accompanied by $V_{tb}V_{tq}^*$.
- Two polarization configurations exist for decays into a pseudoscalar meson, while there are four possibilities for decays into a vector meson.
- The U-spin related decay pairs are given in Table 13, which completely fits with the results given by Ref. [73]. Here only the case for $T_b \rightarrow B_8 P$ is listed. Since no unphysical states η_q and η_s exist in Table 13. The U-spin pairs for $T_b \rightarrow B_8 V$ are similar by replacing pseudoscalar octets by vector octets.
- Some theoretical analyses of nonleptonic bottom baryon decays based on either explicit modes or the flavor sym-

Table 11 Decay amplitudes for $T_b \rightarrow T_8 P$ decays governed by the $b \rightarrow d$ transition. Results for vector meson final state are similar

Channel	IRA	TDA
$\Lambda_b^0 \rightarrow \Lambda^0 K^0$	$(-2B_3^T + D_3^T - B_6^T + 2C_6^T + 2E_6^T + 3D_6^T - B_{15}^T + 2C_{15}^T + 2E_{15}^T + 3D_{15}^T)/\sqrt{6}$	$(2\bar{a}_4 + \bar{a}_6 - \bar{a}_{10} - 2\bar{a}_{12} - \bar{a}_{13} + \bar{a}_{14} - 4\bar{b}_2 - 2\bar{b}_4 - 2\bar{b}_5 - \bar{b}_6 + \bar{b}_7)/\sqrt{6}$
$\Lambda_b^0 \rightarrow \Sigma^0 K^0$	$(-D_3^T + B_6^T + D_6^T + B_{15}^T + 5D_{15}^T)/\sqrt{2}$	$(\bar{a}_6 + \bar{a}_{10} + \bar{a}_{13} + \bar{a}_{14} + \bar{b}_6 + \bar{b}_7)/\sqrt{2}$
$\Lambda_b^0 \rightarrow \Sigma^- K^+$	$D_3^T - B_6^T - D_6^T - B_{15}^T + 3D_{15}^T$	$\bar{a}_8 + \bar{a}_{11} - \bar{b}_6 - \bar{b}_7$
$\Lambda_b^0 \rightarrow p \pi^-$	$B_3^T - C_6^T + E_6^T - C_{15}^T + 3E_{15}^T$	$2\bar{a}_1 - \bar{a}_4 - \bar{a}_6 + \bar{a}_{12} - \bar{a}_{14} + \bar{a}_{15} + \bar{a}_{18} - \bar{a}_{19} + 2\bar{b}_2 + \bar{b}_4 + \bar{b}_5 - \bar{b}_7$
$\Lambda_b^0 \rightarrow n \pi^0$	$(-B_3^T + C_6^T - E_6^T + C_{15}^T + 5E_{15}^T)/\sqrt{2}$	$(2\bar{a}_2 + \bar{a}_4 - \bar{a}_8 - \bar{a}_{10} - \bar{a}_{11} - \bar{a}_{12} - \bar{a}_{13} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{19} - 2\bar{b}_2 - \bar{b}_4 - \bar{b}_5 + \bar{b}_7)/\sqrt{2}$
$\Lambda_b^0 \rightarrow n \eta_q$	$(2A_3^T + B_3^T - 2A_6^T - C_6^T - E_6^T - 2A_{15}^T - C_{15}^T + E_{15}^T)/\sqrt{2}$	$(1/\sqrt{2})(2\bar{a}_2 - 2\bar{a}_3 - \bar{a}_4 - \bar{a}_8 + 2\bar{a}_9 + \bar{a}_{10} - \bar{a}_{11} + \bar{a}_{12} + \bar{a}_{13} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{19} + 4\bar{b}_1 + 2\bar{b}_2 + 2\bar{b}_3 + \bar{b}_4 + \bar{b}_5 + 2\bar{b}_6 + \bar{b}_7)$
$\Lambda_b^0 \rightarrow n \eta_s$	$A_3^T + D_3^T - A_6^T - B_6^T + E_6^T + D_6^T - A_{15}^T - B_{15}^T - E_{15}^T - D_{15}^T$	$-\bar{a}_3 + \bar{a}_9 + 2\bar{b}_1 + \bar{b}_3$
$\Xi_b^0 \rightarrow \Lambda^0 \pi^0$	$(B_3^T + D_3^T + B_6^T + C_6^T + E_6^T + 3D_6^T - 5B_{15}^T - 5C_{15}^T - 5E_{15}^T + 3D_{15}^T)/2\sqrt{3}$	$(1/2\sqrt{3})(-2\bar{a}_2 - \bar{a}_4 + \bar{a}_6 - \bar{a}_7 + \bar{a}_8 + 2\bar{a}_{10} + 2\bar{a}_{11} - \bar{a}_{16} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19} + 2\bar{b}_2 + \bar{b}_4 + \bar{b}_5 - \bar{b}_6 - 2\bar{b}_7)$
$\Xi_b^0 \rightarrow \Lambda^0 \eta_q$	$-(2A_3^T + B_3^T + 2C_3^T + D_3^T - 6A_6^T - B_6^T - C_6^T - E_6^T + 3D_6^T + 6A_{15}^T + B_{15}^T + C_{15}^T + E_{15}^T + 3D_{15}^T)/2\sqrt{3}$	$-(1/2\sqrt{3})(2\bar{a}_2 - 2\bar{a}_3 - \bar{a}_4 + 2\bar{a}_5 + \bar{a}_6 + \bar{a}_7 - \bar{a}_8 + 4\bar{a}_9 + 2\bar{a}_{10} - 2\bar{a}_{11} + \bar{a}_{16} + \bar{a}_{17} - \bar{a}_{18} + 2\bar{a}_{19} + 4\bar{b}_1 + 2\bar{b}_2 + 2\bar{b}_3 + \bar{b}_4 - \bar{b}_5 + \bar{b}_6 + 2\bar{b}_7)$
$\Xi_b^0 \rightarrow \Lambda^0 \eta_s$	$-(A_3^T - 2C_3^T - 3A_6^T - 2B_6^T - 2C_6^T + E_6^T + 3A_{15}^T + 2B_{15}^T + 2C_{15}^T - E_{15}^T)/\sqrt{6}$	$(\bar{a}_3 - \bar{a}_5 - 2\bar{a}_9 - 2\bar{a}_{12} - \bar{a}_{13} + \bar{a}_{14} - 2\bar{b}_1 - \bar{b}_3 - 2\bar{b}_5 - \bar{b}_6 + \bar{b}_7)/\sqrt{6}$
$\Xi_b^0 \rightarrow \Sigma^+ \pi^-$	$-B_3^T - C_3^T + B_6^T - E_6^T - 3B_{15}^T + 2C_{15}^T - 3E_{15}^T$	$-2\bar{a}_1 + \bar{a}_4 + \bar{a}_6 - \bar{a}_{15} - 2\bar{b}_2 - \bar{b}_4$
$\Xi_b^0 \rightarrow \Sigma^0 \pi^0$	$(1/2)(-B_3^T - 2C_3^T - D_3^T + B_6^T + C_6^T - E_6^T + D_6^T - B_{15}^T - C_{15}^T + 5E_{15}^T + 5D_{15}^T)$	$(1/2)(2\bar{a}_2 + \bar{a}_4 + \bar{a}_6 - \bar{a}_7 - \bar{a}_8 + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{18} - 2\bar{b}_2 - \bar{b}_4 + \bar{b}_5 + \bar{b}_6)$
$\Xi_b^0 \rightarrow \Sigma^0 \eta_q$	$(1/2)(2A_3^T + B_3^T + D_3^T + 2A_6^T + B_6^T + C_6^T - E_6^T - D_6^T - 10A_{15}^T - 5B_{15}^T - 5C_{15}^T + E_{15}^T - 5D_{15}^T)$	$(1/2)(2\bar{a}_2 - 2\bar{a}_3 - \bar{a}_4 - 2\bar{a}_5 - \bar{a}_6 - \bar{a}_7 - \bar{a}_8 + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{18} + 4\bar{b}_1 + 2\bar{b}_2 + 2\bar{b}_3 + \bar{b}_4 + \bar{b}_5 + \bar{b}_6)$
$\Xi_b^0 \rightarrow \Sigma^0 \eta_s$	$(A_3^T + A_6^T + E_6^T - 5A_{15}^T - E_{15}^T)/\sqrt{2}$	$(-\bar{a}_3 - \bar{a}_5 + \bar{a}_{13} + \bar{a}_{14} + 2\bar{b}_1 + \bar{b}_3 + \bar{b}_6 + \bar{b}_7)/\sqrt{2}$
$\Xi_b^0 \rightarrow \Sigma^- \pi^+$	$-C_3^T - D_3^T + C_6^T + D_6^T + 2B_{15}^T - 3C_{15}^T - 3D_{15}^T$	$-\bar{a}_7 - \bar{a}_8 + \bar{b}_5 + \bar{b}_6$
$\Xi_b^0 \rightarrow p K^-$	$-C_3^T + B_6^T - C_6^T - 3B_{15}^T + C_{15}^T$	$\bar{a}_{12} - \bar{a}_{14} + \bar{a}_{18} - \bar{a}_{19} + \bar{b}_5 - \bar{b}_7$
$\Xi_b^0 \rightarrow n \bar{K}^0$	$-C_3^T - D_3^T - C_6^T - D_6^T + 2B_{15}^T + C_{15}^T + D_{15}^T$	$\bar{a}_{12} + \bar{a}_{13} + \bar{b}_5 + \bar{b}_6$
$\Xi_b^0 \rightarrow \Xi^- K^+$	$-C_3^T - B_6^T + C_6^T + B_{15}^T - 3C_{15}^T$	$-\bar{a}_7 + \bar{a}_{11} + \bar{b}_5 - \bar{b}_7$
$\Xi_b^0 \rightarrow \Xi^0 K^0$	$-B_3^T - C_3^T - B_6^T + E_6^T + B_{15}^T + 2C_{15}^T + E_{15}^T$	$\bar{a}_4 - \bar{a}_{10} - 2\bar{b}_2 - \bar{b}_4$
$\Xi_b^- \rightarrow \Lambda^0 \pi^-$	$(B_3^T + D_3^T + B_6^T + C_6^T + E_6^T + 3D_6^T + 3B_{15}^T + 3C_{15}^T + 3E_{15}^T + 3D_{15}^T)/\sqrt{6}$	$(2\bar{a}_1 + \bar{a}_{15} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19} + 2\bar{b}_2 + \bar{b}_4 + \bar{b}_5 - \bar{b}_6 - 2\bar{b}_7)/\sqrt{6}$
$\Xi_b^- \rightarrow \Sigma^0 \pi^-$	$(B_3^T - D_3^T - B_6^T + C_6^T + E_6^T + D_6^T - 3B_{15}^T + 3C_{15}^T + 3E_{15}^T + 5D_{15}^T)/\sqrt{2}$	$(2\bar{a}_1 + \bar{a}_{15} + \bar{a}_{17} + \bar{a}_{18} + 2\bar{b}_2 + \bar{b}_4 + \bar{b}_5 + \bar{b}_6)/\sqrt{2}$
$\Xi_b^- \rightarrow \Sigma^- \pi^0$	$-(B_3^T - D_3^T - B_6^T + C_6^T + E_6^T + D_6^T - 3B_{15}^T + 3C_{15}^T - 5E_{15}^T - 3D_{15}^T)/\sqrt{2}$	$(2\bar{a}_2 + \bar{a}_{16} - 2\bar{b}_2 - \bar{b}_4 - \bar{b}_5 - \bar{b}_6)/\sqrt{2}$
$\Xi_b^- \rightarrow \Sigma^- \eta_q$	$(2A_3^T + B_3^T + D_3^T + 2A_6^T + B_6^T + C_6^T - E_6^T - D_6^T + 6A_{15}^T + 3B_{15}^T + 3C_{15}^T + E_{15}^T + 3D_{15}^T)/\sqrt{2}$	$(2\bar{a}_2 + \bar{a}_{16} + 4\bar{b}_1 + 2\bar{b}_2 + 2\bar{b}_3 + \bar{b}_4 + \bar{b}_5 + \bar{b}_6)/\sqrt{2}$
$\Xi_b^- \rightarrow \Sigma^- \eta_s$	$A_3^T + A_6^T + E_6^T + 3A_{15}^T - E_{15}^T$	$2\bar{b}_1 + \bar{b}_3 + \bar{b}_6 + \bar{b}_7$
$\Xi_b^- \rightarrow n K^-$	$D_3^T + B_6^T + D_6^T + 3B_{15}^T - D_{15}^T$	$-\bar{a}_{17} - \bar{a}_{19} - \bar{b}_6 - \bar{b}_7$
$\Xi_b^- \rightarrow \Xi^- K^0$	$B_3^T + C_6^T - E_6^T + 3C_{15}^T - E_{15}^T$	$2\bar{b}_2 + \bar{b}_4 + \bar{b}_5 - \bar{b}_7$

Table 12 Decay amplitudes for two-body $T_b \rightarrow T_8 P$ decays induced by the $b \rightarrow s$ transition

Channel	IRA	TDA
$\Lambda_b^0 \rightarrow \Lambda^0 \pi^0$	$-(-2(E_6^T + B_{15}^T + C_{15}^T - 2E_{15}^T) + B_6^T + C_6^T)/\sqrt{3}$	$(-4\bar{a}_2 + \bar{a}_7 + 2\bar{a}_8 + \bar{a}_{11} + \bar{a}_{12} + 2\bar{a}_{13} + \bar{a}_{14} - 2\bar{a}_{16} - 2\bar{a}_{17} - \bar{a}_{18} - \bar{a}_{19})/2\sqrt{3}$
$\Lambda_b^0 \rightarrow \Lambda^0 \eta_q$	$(-2A_3^T + C_3^T + 6A_{15}^T + B_{15}^T + C_{15}^T - 2E_{15}^T)/\sqrt{3}$	$-1/2\sqrt{3}(4\bar{a}_2 - 4\bar{a}_3 - 2\bar{a}_5 - \bar{a}_7 - 2\bar{a}_8 + 2\bar{a}_9 - \bar{a}_{11} + \bar{a}_{12} + 2\bar{a}_{13} + \bar{a}_{14} + 2\bar{a}_{16} + 2\bar{a}_{17} + \bar{a}_{18} + \bar{a}_{19} + 8\bar{b}_1 + 4\bar{b}_3 + 2\bar{b}_5 + 4\bar{b}_6 + 2\bar{b}_7)$
$\Lambda_b^0 \rightarrow \Lambda^0 \eta_s$	$-\sqrt{\frac{2}{3}}(A_3^T + B_3^T + C_3^T + D_3^T - 3A_{15}^T - 2B_{15}^T - 2C_{15}^T - 2E_{15}^T - 3D_{15}^T)$	$(2\bar{a}_3 + 2\bar{a}_4 + \bar{a}_5 + \bar{a}_6 - \bar{a}_9 - \bar{a}_{10} - 4\bar{b}_1 - 4\bar{b}_2 - 2\bar{b}_3 - 2\bar{b}_4)/\sqrt{6}$
$\Lambda_b^0 \rightarrow \Sigma^+ \pi^-$	$C_3^T - B_6^T + C_6^T + 3B_{15}^T - C_{15}^T$	$-\bar{a}_{12} + \bar{a}_{14} - \bar{a}_{18} + \bar{a}_{19} - \bar{b}_5 + \bar{b}_7$
$\Lambda_b^0 \rightarrow \Sigma^0 \pi^0$	$C_3^T + B_{15}^T + C_{15}^T$	$1/2(\bar{a}_7 - \bar{a}_{11} - \bar{a}_{12} + \bar{a}_{14} - \bar{a}_{18} + \bar{a}_{19} - 2\bar{b}_5 + 2\bar{b}_7)$
$\Lambda_b^0 \rightarrow \Sigma^0 \eta_q$	$-2A_6^T - B_6^T - C_6^T + 4A_{15}^T + 2B_{15}^T + 2C_{15}^T$	$1/2(2\bar{a}_5 + \bar{a}_7 + 2\bar{a}_9 - \bar{a}_{11} + \bar{a}_{12} - \bar{a}_{14} - \bar{a}_{18} + \bar{a}_{19})$
$\Lambda_b^0 \rightarrow \Sigma^0 \eta_s$	$\sqrt{2}(-A_6^T + D_6^T + 2(A_{15}^T + D_{15}^T))$	$(\bar{a}_5 + \bar{a}_6 + \bar{a}_9 + \bar{a}_{10})/\sqrt{2}$
$\Lambda_b^0 \rightarrow \Sigma^- \pi^+$	$C_3^T + B_6^T - C_6^T - B_{15}^T + 3C_{15}^T$	$\bar{a}_7 - \bar{a}_{11} - \bar{b}_5 + \bar{b}_7$
$\Lambda_b^0 \rightarrow p K^-$	$B_3^T + C_3^T - B_6^T + E_6^T + 3B_{15}^T - 2C_{15}^T + 3E_{15}^T$	$2\bar{a}_1 - \bar{a}_4 - \bar{a}_6 + \bar{a}_{15} + 2\bar{b}_2 + \bar{b}_4$
$\Lambda_b^0 \rightarrow n \bar{K}^0$	$B_3^T + C_3^T + B_6^T - E_6^T - B_{15}^T - 2C_{15}^T - E_{15}^T$	$-\bar{a}_4 + \bar{a}_{10} + 2\bar{b}_2 + \bar{b}_4$
$\Lambda_b^0 \rightarrow \Xi^- K^+$	$C_3^T + D_3^T - C_6^T - D_6^T - 2B_{15}^T + 3C_{15}^T + 3D_{15}^T$	$\bar{a}_7 + \bar{a}_8 - \bar{b}_5 - \bar{b}_6$
$\Lambda_b^0 \rightarrow \Xi^0 K^0$	$C_3^T + D_3^T + C_6^T + D_6^T - 2B_{15}^T - C_{15}^T - D_{15}^T$	$-\bar{a}_{12} - \bar{a}_{13} - \bar{b}_5 - \bar{b}_6$
$\Xi_b^0 \rightarrow \Lambda^0 \bar{K}^0$	$(-B_3^T + 2D_3^T - 2B_6^T + C_6^T + E_6^T - 2B_{15}^T + C_{15}^T + E_{15}^T - 6D_{15}^T)/\sqrt{6}$	$(\bar{a}_4 - \bar{a}_6 - 2\bar{a}_{10} - \bar{a}_{12} - 2\bar{a}_{13} - \bar{a}_{14} - 2\bar{b}_2 - \bar{b}_4 - \bar{b}_5 - 2\bar{b}_6 - \bar{b}_7)/\sqrt{6}$
$\Xi_b^0 \rightarrow \Sigma^+ K^-$	$-B_3^T + C_6^T - E_6^T + C_{15}^T - 3E_{15}^T$	$-2\bar{a}_1 + \bar{a}_4 + \bar{a}_6 - \bar{a}_{12} + \bar{a}_{14} - \bar{a}_{15} - \bar{a}_{18} + \bar{a}_{19} - 2\bar{b}_2 - \bar{b}_4 - \bar{b}_5 + \bar{b}_7$
$\Xi_b^0 \rightarrow \Sigma^0 \bar{K}^0$	$(B_3^T - C_6^T - E_6^T - 2D_6^T - C_{15}^T - E_{15}^T - 4D_{15}^T)/\sqrt{2}$	$-(\bar{a}_4 + \bar{a}_6 - \bar{a}_{12} + \bar{a}_{14} - 2\bar{b}_2 - \bar{b}_4 - \bar{b}_5 + \bar{b}_7)/\sqrt{2}$
$\Xi_b^0 \rightarrow \Xi^- \pi^+$	$-D_3^T + B_6^T + D_6^T + B_{15}^T - 3D_{15}^T$	$-\bar{a}_8 - \bar{a}_{11} + \bar{b}_6 + \bar{b}_7$
$\Xi_b^0 \rightarrow \Xi^0 \pi^0$	$(D_3^T - B_6^T + 2E_6^T + D_6^T - B_{15}^T - 4E_{15}^T - D_{15}^T)/\sqrt{2}$	$-(2\bar{a}_2 - \bar{a}_8 - \bar{a}_{11} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{19} + \bar{b}_6 + \bar{b}_7)/\sqrt{2}$
$\Xi_b^0 \rightarrow \Xi^0 \eta_q$	$(-2A_3^T - D_3^T + 2A_6^T + B_6^T - D_6^T + 2A_{15}^T + B_{15}^T - 2E_{15}^T + D_{15}^T)/\sqrt{2}$	$-(2\bar{a}_2 - 2\bar{a}_3 - \bar{a}_8 + 2\bar{a}_9 - \bar{a}_{11} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{19} + 4\bar{b}_1 + 2\bar{b}_3 + \bar{b}_6 + \bar{b}_7)/\sqrt{2}$
$\Xi_b^0 \rightarrow \Xi^0 \eta_s$	$-A_3^T - B_3^T + A_6^T + C_6^T + A_{15}^T + C_{15}^T + 2E_{15}^T$	$\bar{a}_3 + \bar{a}_4 - \bar{a}_9 - \bar{a}_{10} - \bar{a}_{12} - \bar{a}_{13} - 2\bar{b}_1 - 2\bar{b}_2 - \bar{b}_3 - \bar{b}_4 - \bar{b}_5 - \bar{b}_6$
$\Xi_b^- \rightarrow \Lambda^0 K^-$	$(B_3^T - 2D_3^T - 2B_6^T + C_6^T + E_6^T - 6B_{15}^T + 3C_{15}^T + 3E_{15}^T + 6D_{15}^T)/\sqrt{6}$	$(2\bar{a}_1 + \bar{a}_{15} + 2\bar{a}_{17} + \bar{a}_{18} + \bar{a}_{19} + 2\bar{b}_2 + \bar{b}_4 + \bar{b}_5 + 2\bar{b}_6 + \bar{b}_7)/\sqrt{6}$
$\Xi_b^- \rightarrow \Sigma^0 K^-$	$(B_3^T + C_6^T + E_6^T + 2D_6^T + 3C_{15}^T + 3E_{15}^T + 4D_{15}^T)/\sqrt{2}$	$(2\bar{a}_1 + \bar{a}_{15} + \bar{a}_{18} - \bar{a}_{19} + 2\bar{b}_2 + \bar{b}_4 + \bar{b}_5 - \bar{b}_7)/\sqrt{2}$
$\Xi_b^- \rightarrow \Sigma^- \bar{K}^0$	$B_3^T + C_6^T - E_6^T + 3C_{15}^T - E_{15}^T$	$2\bar{b}_2 + \bar{b}_4 + \bar{b}_5 - \bar{b}_7$
$\Xi_b^- \rightarrow \Xi^- \pi^0$	$(D_3^T + B_6^T - 2E_6^T - D_6^T + 3B_{15}^T + 4E_{15}^T + 3D_{15}^T)/\sqrt{2}$	$(2\bar{a}_2 + \bar{a}_{16} - \bar{b}_6 - \bar{b}_7)/\sqrt{2}$
$\Xi_b^- \rightarrow \Xi^- \eta_q$	$(2A_3^T + D_3^T + 2A_6^T + B_6^T - D_6^T + 6A_{15}^T + 3B_{15}^T + 2E_{15}^T + 3D_{15}^T)/\sqrt{2}$	$(2\bar{a}_2 + \bar{a}_{16} + 4\bar{b}_1 + 2\bar{b}_3 + \bar{b}_6 + \bar{b}_7)/\sqrt{2}$
$\Xi_b^- \rightarrow \Xi^- \eta_s$	$A_3^T + B_3^T + A_6^T + C_6^T + 3A_{15}^T + 3C_{15}^T - 2E_{15}^T$	$2\bar{b}_1 + 2\bar{b}_2 + \bar{b}_3 + \bar{b}_4 + \bar{b}_5 + \bar{b}_6$
$\Xi_b^- \rightarrow \Xi^0 \pi^-$	$D_3^T + B_6^T + D_6^T + 3B_{15}^T - D_{15}^T$	$-\bar{a}_{17} - \bar{a}_{19} - \bar{b}_6 - \bar{b}_7$

metry can be found in Refs. [74–79], while the experimental measurements can be found in Refs. [80–82]. To date, the available measurements of two-body Λ_b branching fractions are [3,4]:

$$\begin{aligned}
 \mathcal{B}(\Lambda_b \rightarrow p \pi^-) &= (4.2 \pm 0.8) \times 10^{-6}, \\
 \mathcal{B}(\Lambda_b \rightarrow p K^-) &= (5.1 \pm 0.9) \times 10^{-6}, \\
 \mathcal{B}(\Lambda_b \rightarrow \Lambda \eta) &= (9_{-5}^{+7}) \times 10^{-6}, \\
 \mathcal{B}(\Lambda_b \rightarrow \Lambda \eta') &< 3.1 \times 10^{-6},
 \end{aligned}$$

$$\mathcal{B}(\Lambda_b \rightarrow p \phi) = (9.2 \pm 2.5) \times 10^{-6}. \quad (78)$$

- The CP asymmetries for $\Lambda_b \rightarrow p \pi^- / p K^-$ [82] have been measured:

$$\begin{aligned}
 A_{CP}^{p\pi^-} &= -0.020 \pm 0.013 \pm 0.019, \\
 A_{CP}^{pK^-} &= -0.035 \pm 0.017 \pm 0.020.
 \end{aligned} \quad (79)$$

Thus measuring the branching fractions and CP asymmetries for $\Xi_b^0 \rightarrow \pi^- \Sigma^+$ and $\Xi_b^0 \rightarrow K^- \Sigma^+$ will help us to understand the U-spin in baryonic decays.

7 Antitriplet charmed Baryon $T_c(\Lambda_c, \Xi_c^+, \Xi_c^0)$ decays

For the charmed baryon decays, the H_3 contributions are vanishingly small, and thus we have 19 amplitudes in TDA:

$$\begin{aligned} \mathcal{A}_u^{TDA} = & \bar{a}_1 T_{c\bar{3}}^{[ij]} H_m^{kl} (\bar{T}_8)_{ijk} P_l^m + \bar{a}_2 T_{c\bar{3}}^{[ij]} H_m^{kl} (\bar{T}_8)_{ijl} P_k^m \\ & + \bar{a}_3 T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{jkl} P_m^m \\ & + \bar{a}_4 T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{jkm} P_l^m \\ & + \bar{a}_5 T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{jlk} P_m^m + \bar{a}_6 T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{jmk} P_l^m \\ & + \bar{a}_7 T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{ilm} P_k^m + \bar{a}_8 T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{jml} P_k^m \\ & + \bar{a}_9 T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{klj} P_m^m + \bar{a}_{10} T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{kmj} P_l^m \\ & + \bar{a}_{11} T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{lmj} P_k^m + \bar{a}_{12} T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{klm} P_j^m \\ & + \bar{a}_{13} T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{kml} P_j^m + \bar{a}_{14} T_{c\bar{3}}^{[ij]} H_i^{kl} (\bar{T}_8)_{lmk} P_j^m \\ & + \bar{a}_{15} T_{c\bar{3}}^{[ij]} H_m^{kl} (\bar{T}_8)_{ikj} P_l^m + \bar{a}_{16} T_{c\bar{3}}^{[ij]} H_m^{kl} (\bar{T}_8)_{ilj} P_k^m \\ & + \bar{a}_{17} T_{c\bar{3}}^{[ij]} H_m^{kl} (\bar{T}_8)_{ikl} P_j^m + \bar{a}_{18} T_{c\bar{3}}^{[ij]} H_m^{kl} (\bar{T}_8)_{ilk} P_j^m \\ & + \bar{a}_{19} T_{c\bar{3}}^{[ij]} H_m^{kl} (\bar{T}_8)_{klj} P_i^m. \end{aligned} \quad (80)$$

The corresponding Feynman diagrams are given in Fig. 6, in which seven Feynman diagrams can be found. The analysis for independent amplitudes are almost the same as that of bottom baryon decays. On the other side, ten IRA amplitudes can be constructed as:

$$\begin{aligned} \mathcal{A}_u^{IRA} = & A_6^T (T_{c\bar{3}})_i (H_6)_j^{[ik]} (\bar{T}_8)_k^j P_l^l \\ & + B_6^T (T_{c\bar{3}})_i (H_6)_j^{[ik]} (\bar{T}_8)_k^l P_l^j \\ & + C_6^T (T_{c\bar{3}})_i (H_6)_j^{[ik]} (\bar{T}_8)_l^j P_k^l \\ & + E_6^T (T_{c\bar{3}})_i (H_6)_l^{[jk]} (\bar{T}_8)_j^i P_k^l \\ & + D_6^T (T_{c\bar{3}})_i (H_6)_l^{[jk]} (\bar{T}_8)_j^l P_k^i \\ & + A_{15}^T (T_{c\bar{3}})_i (H_{15})_j^{[ik]} (\bar{T}_8)_k^j P_l^l \\ & + B_{15}^T (T_{c\bar{3}})_i (H_{15})_j^{[ik]} (\bar{T}_8)_k^l P_l^j \\ & + C_{15}^T (T_{c\bar{3}})_i (H_{15})_j^{[ik]} (\bar{T}_8)_l^j P_k^l \\ & + E_{15}^T (T_{c\bar{3}})_i (H_{15})_l^{[jk]} (\bar{T}_8)_j^i P_k^l \\ & + D_{15}^T (T_{c\bar{3}})_i (H_{15})_l^{[jk]} (\bar{T}_8)_j^l P_k^i. \end{aligned} \quad (81)$$

Only nine of them are independent, and one redundant amplitude can be made explicit with the redefinitions:

$$\begin{aligned} A_6^{T'} &= A_6^T + B_6^T, & B_6^{T'} &= B_6^T - C_6^T, \\ C_6^{T'} &= C_6^T - E_6^T, & D_6^{T'} &= C_6^T + D_6^T, \end{aligned} \quad (82)$$

which is exactly the same as Eq. (77).

After a careful examination, one can also find the relations:

$$\begin{aligned} A_6^T &= \frac{1}{2} (\bar{a}_3 - \bar{a}_5 - 2\bar{a}_9 - \bar{a}_{13} + \bar{a}_{14}), \\ B_6^T &= \frac{1}{2} (\bar{a}_{13} - \bar{a}_{14} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19}), \\ C_6^T &= \frac{1}{2} (\bar{a}_4 - \bar{a}_7 - \bar{a}_{10} + \bar{a}_{11} - 2\bar{a}_{12}), \\ D_6^T &= \frac{1}{2} (\bar{a}_6 - \bar{a}_8 + \bar{a}_{10} - \bar{a}_{11} - \bar{a}_{13} + \bar{a}_{14}), \\ E_6^T &= \frac{1}{2} (2\bar{a}_1 - 2\bar{a}_2 - \bar{a}_6 + \bar{a}_8 - \bar{a}_{10} + \bar{a}_{11} + \bar{a}_{13} \\ &\quad - \bar{a}_{14} + \bar{a}_{15} - \bar{a}_{16} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19}), \\ A_{15}^T &= \frac{1}{2} (\bar{a}_3 + \bar{a}_5 - \bar{a}_{13} - \bar{a}_{14}), \\ B_{15}^T &= \frac{1}{2} (\bar{a}_{13} + \bar{a}_{14} - \bar{a}_{17} - \bar{a}_{18}), \\ C_{15}^T &= \frac{1}{2} (\bar{a}_4 + \bar{a}_7 - \bar{a}_{10} - \bar{a}_{11}), \\ D_{15}^T &= \frac{1}{2} (\bar{a}_6 + \bar{a}_8 + \bar{a}_{10} + \bar{a}_{11} + \bar{a}_{13} + \bar{a}_{14}), \\ E_{15}^T &= \frac{1}{2} (2\bar{a}_1 + 2\bar{a}_2 - \bar{a}_6 - \bar{a}_8 - \bar{a}_{10} - \bar{a}_{11} \\ &\quad - \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{15} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{18}). \end{aligned} \quad (83)$$

Some further remarks are given in order.

- The flavor SU(3) symmetry in charmed baryon decays and the symmetry breaking effects have been extensively explored in Refs. [70,83–90], and we refer the reader to these references for detailed discussions.
- On the experimental side, BESIII collaboration has given the first measurement of decay branching fractions for the W -exchange induced decays [91]:

$$\mathcal{B}(\Lambda_c \rightarrow \Xi^0 K^+) = (5.90 \pm 0.86 \pm 0.39) \times 10^{-3}, \quad (84)$$

$$\mathcal{B}(\Lambda_c \rightarrow \Xi(1530)^0 K^+) = (5.02 \pm 0.99 \pm 0.31) \times 10^{-3}. \quad (85)$$

It indicates that the decays into a decuplet baryon might not be power suppressed compared to those decays into an octet baryon. This introduces a theoretical difficulty to understand the charmed baryon decays.

- One can find some relations between the different channels listed in Tables 14, 15 and 16. For the charmed baryon two-body decay, there is only one relation for decay width:

$$\Gamma(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) = \Gamma(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+). \quad (86)$$

Table 13 U-spin relations for $T_b \rightarrow T_8 P$

$b \rightarrow d$	$b \rightarrow s$	r	$b \rightarrow d$	$b \rightarrow s$	r
$\mathcal{A}(\Xi_b^- \rightarrow K^- n)$	$\mathcal{A}(\Xi_b^- \rightarrow \pi^- \Xi^0)$	+1	$\mathcal{A}(\Xi_b^0 \rightarrow \bar{K}^0 n)$	$\mathcal{A}(\Lambda_b^0 \rightarrow K^0 \Xi^0)$	-1
$\mathcal{A}(\Xi_b^- \rightarrow K^0 \Xi^-)$	$\mathcal{A}(\Xi_b^- \rightarrow \bar{K}^0 \Sigma^-)$	+1	$\mathcal{A}(\Xi_b^0 \rightarrow K^0 \Xi^0)$	$\mathcal{A}(\Lambda_b^0 \rightarrow \bar{K}^0 n)$	-1
$\mathcal{A}(\Xi_b^0 \rightarrow \pi^- \Sigma^+)$	$\mathcal{A}(\Lambda_b^0 \rightarrow K^- p)$	-1	$\mathcal{A}(\Lambda_b^0 \rightarrow \pi^- p)$	$\mathcal{A}(\Xi_b^0 \rightarrow K^- \Sigma^+)$	-1
$\mathcal{A}(\Xi_b^0 \rightarrow \pi^+ \Sigma^-)$	$\mathcal{A}(\Lambda_b^0 \rightarrow K^+ \Xi^-)$	-1	$\mathcal{A}(\Lambda_b^0 \rightarrow K^+ \Sigma^-)$	$\mathcal{A}(\Xi_b^0 \rightarrow \pi^+ \Xi^-)$	-1
$\mathcal{A}(\Xi_b^0 \rightarrow K^- p)$	$\mathcal{A}(\Lambda_b^0 \rightarrow \pi^- \Sigma^+)$	-1	$\mathcal{A}(\Xi_b^0 \rightarrow K^+ \Xi^-)$	$\mathcal{A}(\Lambda_b^0 \rightarrow \pi^+ \Sigma^-)$	+1

Fig. 6 Topology diagrams for the charmed baryon decays into an octet baryon and a light meson. Due to the same reason as that of bottom baryon decays, one topological diagrams correspond to more than one TDA amplitudes. Here the 7 topological diagrams correspond to 19 TDA amplitudes as given in Eq. (80)

This relation fits well with the data in Ref. [3]:

$$\begin{aligned} \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) &= 1.24 \pm 0.10\%, \\ \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+) &= 1.28 \pm 0.07\%. \end{aligned} \quad (87)$$

- In Ref. [70], a global fit was conducted for charmed baryon decays, particularly inspired by the recent BESIII data [71, 72, 91]. In that work the sextet contribution was expressed in a different representation. Relating the four coefficients in [70] with our notations, we have:³

$$\begin{aligned} -A_6^T + D_6^T &= h = (0.105 \pm 0.073) \text{ GeV}^3, \\ -B_6^T + E_6^T &= a_1 = (0.244 \pm 0.006) \text{ GeV}^3, \\ -C_6^T - D_6^T &= a_2 = (0.115 \pm 0.014) \text{ GeV}^3, \end{aligned} \quad (88)$$

³ In Ref. [70], these parameters are denoted as a_1 , a_2 , a_3 , h . Here we add primes in order to distinguish them with the parameters used in this work.

$$E_6^T + D_6^T = a_3 = (0.088 \pm 0.019) \text{ GeV}^3. \quad (89)$$

Such a fit was conducted with the neglect of the H_{15} terms, which might be challenged in interpreting the $\Lambda_c \rightarrow p \pi^0$ [84, 87, 89].

8 Discussions and conclusions

In this work, we have carried out a comprehensive analysis comparing two different realizations of the flavor SU(3) symmetry, the irreducible operator representation amplitude and topological diagram amplitude, to study various bottom/charm meson and baryon decays.

We find that previous analyses in the literature using these two methods do not match consistently in several ways. The TDA approach provides a more intuitive understanding of the decays, however it also suffers from a few subtleties. Using two-body B/D meson decays, we have demonstrated that a

Table 14 Decay amplitudes for two-body Cabibbo-Allowed charmed baryon decays

Channel	IRA	TDA
$\Lambda_c^+ \rightarrow \Lambda^0 \pi^+$	$(B_6^T + C_6^T - 2E_6^T + B_{15}^T + C_{15}^T - 2E_{15}^T)/\sqrt{6}$	$(-4\bar{a}_1 + \bar{a}_4 + 2\bar{a}_6 + \bar{a}_{10} - \bar{a}_{12} + \bar{a}_{13} + 2\bar{a}_{14} - 2\bar{a}_{15} - \bar{a}_{17} - 2\bar{a}_{18} + \bar{a}_{19})/\sqrt{6}$
$\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$	$(-B_6^T + C_6^T - B_{15}^T + C_{15}^T)/\sqrt{2}$	$(\bar{a}_4 - \bar{a}_{10} - \bar{a}_{12} - \bar{a}_{13} + \bar{a}_{17} + \bar{a}_{19})/\sqrt{2}$
$\Lambda_c^+ \rightarrow \Sigma^+ \eta_q$	$(2A_6^T + B_6^T + C_6^T + 2A_{15}^T + B_{15}^T + C_{15}^T)/\sqrt{2}$	$(2\bar{a}_3 + \bar{a}_4 - 2\bar{a}_9 - \bar{a}_{10} - \bar{a}_{12} - \bar{a}_{13} - \bar{a}_{17} - \bar{a}_{19})/\sqrt{2}$
$\Lambda_c^+ \rightarrow \Sigma^+ \eta_s$	$A_6^T - D_6^T + A_{15}^T + D_{15}^T$	$\bar{a}_3 + \bar{a}_8 - \bar{a}_9 + \bar{a}_{11}$
$\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$	$(B_6^T - C_6^T + B_{15}^T - C_{15}^T)/\sqrt{2}$	$(-\bar{a}_4 + \bar{a}_{10} + \bar{a}_{12} + \bar{a}_{13} - \bar{a}_{17} - \bar{a}_{19})/\sqrt{2}$
$\Lambda_c^+ \rightarrow p \bar{K}^0$	$B_6^T - E_6^T + B_{15}^T + E_{15}^T$	$2\bar{a}_2 - \bar{a}_8 - \bar{a}_{11} + \bar{a}_{16}$
$\Lambda_c^+ \rightarrow \Xi^0 K^+$	$C_6^T + D_6^T + C_{15}^T + D_{15}^T$	$\bar{a}_4 + \bar{a}_6 - \bar{a}_{12} + \bar{a}_{14}$
$\Xi_c^+ \rightarrow \Sigma^+ \bar{K}^0$	$E_6^T + D_6^T - E_{15}^T - D_{15}^T$	$-2\bar{a}_2 - \bar{a}_{16} - \bar{a}_{17} - \bar{a}_{19}$
$\Xi_c^+ \rightarrow \Xi^0 \pi^+$	$-E_6^T - D_6^T - E_{15}^T - D_{15}^T$	$-2\bar{a}_1 - \bar{a}_{15} - \bar{a}_{18} + \bar{a}_{19}$
$\Xi_c^0 \rightarrow \Lambda^0 \bar{K}^0$	$(2B_6^T - C_6^T - E_6^T - 2B_{15}^T + C_{15}^T + E_{15}^T)/\sqrt{6}$	$(2\bar{a}_2 + \bar{a}_7 - \bar{a}_8 - 2\bar{a}_{11} + \bar{a}_{12} - \bar{a}_{13} - 2\bar{a}_{14} + \bar{a}_{16} + \bar{a}_{17} + 2\bar{a}_{18} - \bar{a}_{19})/\sqrt{6}$
$\Xi_c^0 \rightarrow \Sigma^+ K^-$	$-C_6^T - D_6^T + C_{15}^T + D_{15}^T$	$\bar{a}_7 + \bar{a}_8 + \bar{a}_{12} + \bar{a}_{13}$
$\Xi_c^0 \rightarrow \Sigma^0 \bar{K}^0$	$(C_6^T - E_6^T - C_{15}^T + E_{15}^T)/\sqrt{2}$	$(2\bar{a}_2 - \bar{a}_7 - \bar{a}_8 - \bar{a}_{12} - \bar{a}_{13} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{19})/\sqrt{2}$
$\Xi_c^0 \rightarrow \Xi^- \pi^+$	$-B_6^T + E_6^T + B_{15}^T + E_{15}^T$	$2\bar{a}_1 - \bar{a}_6 - \bar{a}_{10} + \bar{a}_{15}$
$\Xi_c^0 \rightarrow \Xi^0 \pi^0$	$(B_6^T + D_6^T - B_{15}^T + D_{15}^T)/\sqrt{2}$	$(\bar{a}_6 + \bar{a}_{10} + \bar{a}_{18} - \bar{a}_{19})/\sqrt{2}$
$\Xi_c^0 \rightarrow \Xi^0 \eta_q$	$(-2A_6^T - B_6^T + D_6^T + 2A_{15}^T + B_{15}^T + D_{15}^T)/\sqrt{2}$	$(2\bar{a}_5 + \bar{a}_6 + 2\bar{a}_9 + \bar{a}_{10} - \bar{a}_{18} + \bar{a}_{19})/\sqrt{2}$
$\Xi_c^0 \rightarrow \Xi^0 \eta_s$	$-A_6^T - C_6^T + A_{15}^T + C_{15}^T$	$\bar{a}_5 + \bar{a}_7 + \bar{a}_9 - \bar{a}_{11} + \bar{a}_{12} - \bar{a}_{14}$

Table 15 Decay amplitudes for two-body Singly Cabibbo-Suppressed charmed baryon decays

Channel	IRA	TDA
$\Lambda_c^+ \rightarrow \Lambda^0 K^+$	$(B_6^T - 2C_6^T - 2E_6^T - 3D_6^T + B_{15}^T - 2C_{15}^T - 2E_{15}^T - 3D_{15}^T)/\sqrt{6}$	$(-4\bar{a}_1 + 2\bar{a}_4 + \bar{a}_6 - \bar{a}_{10} - 2\bar{a}_{12} - \bar{a}_{13} + \bar{a}_{14} + 2\bar{a}_{15} + \bar{a}_{17} + 2\bar{a}_{18} - \bar{a}_{19})/\sqrt{6}$
$\Lambda_c^+ \rightarrow \Sigma^+ K^0$	$B_6^T + D_6^T + B_{15}^T - D_{15}^T$	$-\bar{a}_8 - \bar{a}_{11} - \bar{a}_{17} - \bar{a}_{19}$
$\Lambda_c^+ \rightarrow \Sigma^0 K^+$	$(B_6^T + D_6^T + B_{15}^T + D_{15}^T)/\sqrt{2}$	$(\bar{a}_6 + \bar{a}_{10} + \bar{a}_{13} + \bar{a}_{14} - \bar{a}_{17} - \bar{a}_{19})/\sqrt{2}$
$\Lambda_c^+ \rightarrow p \pi^0$	$(C_6^T - E_6^T + C_{15}^T + E_{15}^T)/\sqrt{2}$	$(2\bar{a}_2 + \bar{a}_4 - \bar{a}_8 - \bar{a}_{10} - \bar{a}_{11} - \bar{a}_{12} - \bar{a}_{13} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{19})/\sqrt{2}$
$\Lambda_c^+ \rightarrow p \eta_q$	$(2A_6^T + C_6^T + E_6^T + 2A_{15}^T + C_{15}^T - E_{15}^T)/\sqrt{2}$	$(-2\bar{a}_2 - 2\bar{a}_3 - \bar{a}_4 - \bar{a}_8 + 2\bar{a}_9 + \bar{a}_{10} - \bar{a}_{11} + \bar{a}_{12} + \bar{a}_{13} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{19})/\sqrt{2}$
$\Lambda_c^+ \rightarrow p \eta_s$	$A_6^T + B_6^T - E_6^T - D_6^T + A_{15}^T + B_{15}^T + E_{15}^T + D_{15}^T$	$2\bar{a}_2 + \bar{a}_3 - \bar{a}_9 + \bar{a}_{16}$
$\Lambda_c^+ \rightarrow n \pi^+$	$C_6^T - E_6^T + C_{15}^T - E_{15}^T$	$-2\bar{a}_1 + \bar{a}_4 + \bar{a}_6 - \bar{a}_{12} + \bar{a}_{14} - \bar{a}_{15} - \bar{a}_{18} + \bar{a}_{19}$
$\Xi_c^+ \rightarrow \Lambda^0 \pi^+$	$(B_6^T + C_6^T + E_6^T + 3D_6^T + B_{15}^T + C_{15}^T + E_{15}^T + 3D_{15}^T)/\sqrt{6}$	$(2\bar{a}_1 + \bar{a}_4 + 2\bar{a}_6 + \bar{a}_{10} - \bar{a}_{12} + \bar{a}_{13} + 2\bar{a}_{14} + \bar{a}_{15} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19})/\sqrt{6}$
$\Xi_c^+ \rightarrow \Sigma^+ \pi^0$	$(-B_6^T + C_6^T + E_6^T + D_6^T - B_{15}^T + C_{15}^T - E_{15}^T - D_{15}^T)/\sqrt{2}$	$(-2\bar{a}_2 - \bar{a}_4 + \bar{a}_{10} + \bar{a}_{12} + \bar{a}_{13} + \bar{a}_{16})/\sqrt{2}$
$\Xi_c^+ \rightarrow \Sigma^+ \eta_q$	$(2A_6^T + B_6^T + C_6^T - E_6^T - D_6^T + 2A_{15}^T + B_{15}^T + C_{15}^T + E_{15}^T + D_{15}^T)/\sqrt{2}$	$(2\bar{a}_2 + 2\bar{a}_3 + \bar{a}_4 - 2\bar{a}_9 - \bar{a}_{10} - \bar{a}_{12} - \bar{a}_{13} + \bar{a}_{16})/\sqrt{2}$
$\Xi_c^+ \rightarrow \Sigma^+ \eta_s$	$A_6^T + E_6^T + A_{15}^T - E_{15}^T$	$-2\bar{a}_2 + \bar{a}_3 + \bar{a}_8 - \bar{a}_9 + \bar{a}_{11} - \bar{a}_{16} - \bar{a}_{17} - \bar{a}_{19}$
$\Xi_c^+ \rightarrow \Sigma^0 \pi^+$	$(B_6^T - C_6^T - E_6^T - D_6^T + B_{15}^T - C_{15}^T - E_{15}^T - D_{15}^T)/\sqrt{2}$	$(-2\bar{a}_1 + \bar{a}_4 - \bar{a}_{10} - \bar{a}_{12} - \bar{a}_{13} + \bar{a}_{15} + \bar{a}_{17} + \bar{a}_{18})/\sqrt{2}$
$\Xi_c^+ \rightarrow p \bar{K}^0$	$B_6^T + D_6^T + B_{15}^T - D_{15}^T$	$-\bar{a}_8 - \bar{a}_{11} - \bar{a}_{17} - \bar{a}_{19}$

Table 15 continued

Channel	IRA	TDA
$\Xi_c^+ \rightarrow \Xi^0 K^+$	$C_6^T - E_6^T + C_{15}^T - E_{15}^T$	$-2\bar{a}_1 + \bar{a}_4 + \bar{a}_6 - \bar{a}_{12} + \bar{a}_{14} - \bar{a}_{15} - \bar{a}_{18} + \bar{a}_{19}$
$\Xi_c^0 \rightarrow \Lambda^0 \pi^0$	$-(B_6^T + C_6^T + E_6^T + 3D_6^T - B_{15}^T - C_{15}^T - E_{15}^T + 3D_{15}^T)/2\sqrt{3}$	$(2\bar{a}_2 - 3\bar{a}_6 + \bar{a}_7 - \bar{a}_8 - 3\bar{a}_{10} - 2\bar{a}_{11} + \bar{a}_{12} - \bar{a}_{13} - 2\bar{a}_{14} + \bar{a}_{16} + \bar{a}_{17} - \bar{a}_{18} + 2\bar{a}_{19})/2\sqrt{3}$
$\Xi_c^0 \rightarrow \Lambda^0 \eta_q$	$(6A_6^T + B_6^T + C_6^T + E_6^T - 3D_6^T - 6A_{15}^T - B_{15}^T - C_{15}^T - E_{15}^T - 3D_{15}^T)/2\sqrt{3}$	$-(1/2\sqrt{3})(2\bar{a}_2 + 6\bar{a}_5 + 3\bar{a}_6 + \bar{a}_7 - \bar{a}_8 + 6\bar{a}_9 + 3\bar{a}_{10} - 2\bar{a}_{11} + \bar{a}_{12} - \bar{a}_{13} - 2\bar{a}_{14} + \bar{a}_{16} + \bar{a}_{17} - \bar{a}_{18} + 2\bar{a}_{19})$
$\Xi_c^0 \rightarrow \Lambda^0 \eta_s$	$(3A_6^T + 2B_6^T + 2C_6^T - E_6^T - 3A_{15}^T - 2B_{15}^T - 2C_{15}^T + E_{15}^T)/\sqrt{6}$	$(2\bar{a}_2 - 3\bar{a}_5 - 2\bar{a}_7 - \bar{a}_8 - 3\bar{a}_9 + \bar{a}_{11} - 2\bar{a}_{12} - \bar{a}_{13} + \bar{a}_{14} + \bar{a}_{16} + \bar{a}_{17} + 2\bar{a}_{18} - \bar{a}_{19})/\sqrt{6}$
$\Xi_c^0 \rightarrow \Sigma^+ \pi^-$	$C_6^T + D_6^T - C_{15}^T - D_{15}^T$	$-\bar{a}_7 - \bar{a}_8 - \bar{a}_{12} - \bar{a}_{13}$
$\Xi_c^0 \rightarrow \Sigma^0 \pi^0$	$1/2(B_6^T + C_6^T - E_6^T + D_6^T - B_{15}^T - C_{15}^T + E_{15}^T + D_{15}^T)$	$(1/2)(2\bar{a}_2 + \bar{a}_6 - \bar{a}_7 - \bar{a}_8 + \bar{a}_{10} - \bar{a}_{12} - \bar{a}_{13} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{18})$
$\Xi_c^0 \rightarrow \Sigma^0 \eta_q$	$1/2(-2A_6^T - B_6^T - C_6^T + E_6^T + D_6^T + 2A_{15}^T + B_{15}^T + C_{15}^T - E_{15}^T + D_{15}^T)$	$(1/2)(-2\bar{a}_2 + 2\bar{a}_5 + \bar{a}_6 + \bar{a}_7 + \bar{a}_8 + 2\bar{a}_9 + \bar{a}_{10} + \bar{a}_{12} + \bar{a}_{13} - \bar{a}_{16} - \bar{a}_{17} - \bar{a}_{18})$
$\Xi_c^0 \rightarrow \Sigma^0 \eta_s$	$(-A_6^T - E_6^T + A_{15}^T + E_{15}^T)/\sqrt{2}$	$(2\bar{a}_2 + \bar{a}_5 - \bar{a}_8 + \bar{a}_9 - \bar{a}_{11} - \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{19})/\sqrt{2}$
$\Xi_c^0 \rightarrow \Sigma^- \pi^+$	$B_6^T - E_6^T - B_{15}^T - E_{15}^T$	$-2\bar{a}_1 + \bar{a}_6 + \bar{a}_{10} - \bar{a}_{15}$
$\Xi_c^0 \rightarrow p K^-$	$-C_6^T - D_6^T + C_{15}^T + D_{15}^T$	$\bar{a}_7 + \bar{a}_8 + \bar{a}_{12} + \bar{a}_{13}$
$\Xi_c^0 \rightarrow n \bar{K}^0$	$B_6^T - C_6^T - B_{15}^T + C_{15}^T$	$\bar{a}_7 - \bar{a}_{11} + \bar{a}_{12} - \bar{a}_{14} + \bar{a}_{18} - \bar{a}_{19}$
$\Xi_c^0 \rightarrow \Xi^- K^+$	$-B_6^T + E_6^T + B_{15}^T + E_{15}^T$	$2\bar{a}_1 - \bar{a}_6 - \bar{a}_{10} + \bar{a}_{15}$
$\Xi_c^0 \rightarrow \Xi^0 K^0$	$-B_6^T + C_6^T + B_{15}^T - C_{15}^T$	$-\bar{a}_7 + \bar{a}_{11} - \bar{a}_{12} + \bar{a}_{14} - \bar{a}_{18} + \bar{a}_{19}$

Table 16 Decay amplitudes for two-body Doubly Cabibbo-Suppressed charmed baryon decays

Channel	IRA	TDA
$\Lambda_c^+ \rightarrow p K^0$	$-E_6^T - D_6^T + E_{15}^T + D_{15}^T$	$2\bar{a}_2 + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{19}$
$\Lambda_c^+ \rightarrow n K^+$	$E_6^T + D_6^T + E_{15}^T + D_{15}^T$	$2\bar{a}_1 + \bar{a}_{15} + \bar{a}_{18} - \bar{a}_{19}$
$\Xi_c^+ \rightarrow \Lambda^0 K^+$	$-(B_6^T - 2C_6^T + E_6^T + B_{15}^T - 2C_{15}^T + E_{15}^T)/\sqrt{6}$	$-(2\bar{a}_1 - 2\bar{a}_4 - \bar{a}_6 + \bar{a}_{10} + 2\bar{a}_{12} + \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{15} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19})/\sqrt{6}$
$\Xi_c^+ \rightarrow \Sigma^+ K^0$	$-B_6^T + E_6^T - B_{15}^T - E_{15}^T$	$-2\bar{a}_2 + \bar{a}_8 + \bar{a}_{11} - \bar{a}_{16}$
$\Xi_c^+ \rightarrow \Sigma^0 K^+$	$(-B_6^T + E_6^T - B_{15}^T + E_{15}^T)/\sqrt{2}$	$(2\bar{a}_1 - \bar{a}_6 - \bar{a}_{10} - \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{15} + \bar{a}_{17} + \bar{a}_{18})/\sqrt{2}$
$\Xi_c^+ \rightarrow p \pi^0$	$-(C_6^T + D_6^T + C_{15}^T - D_{15}^T)/\sqrt{2}$	$(-\bar{a}_4 + \bar{a}_8 + \bar{a}_{10} + \bar{a}_{11} + \bar{a}_{12} + \bar{a}_{13})/\sqrt{2}$
$\Xi_c^+ \rightarrow p \eta_q$	$-(2A_6^T + C_6^T - D_6^T + 2A_{15}^T + C_{15}^T + D_{15}^T)/\sqrt{2}$	$(-2\bar{a}_3 - \bar{a}_4 - \bar{a}_8 + 2\bar{a}_9 + \bar{a}_{10} - \bar{a}_{11} + \bar{a}_{12} + \bar{a}_{13})/\sqrt{2}$
$\Xi_c^+ \rightarrow p \eta_s$	$-A_6^T - B_6^T - A_{15}^T - B_{15}^T$	$-\bar{a}_3 + \bar{a}_9 + \bar{a}_{17} + \bar{a}_{19}$
$\Xi_c^+ \rightarrow n \pi^+$	$-C_6^T - D_6^T - C_{15}^T - D_{15}^T$	$-\bar{a}_4 - \bar{a}_6 + \bar{a}_{12} - \bar{a}_{14}$
$\Xi_c^0 \rightarrow \Lambda^0 K^0$	$(-B_6^T + 2C_6^T - E_6^T + B_{15}^T - 2C_{15}^T + E_{15}^T)/\sqrt{6}$	$(2\bar{a}_2 - 2\bar{a}_7 - \bar{a}_8 + \bar{a}_{11} - 2\bar{a}_{12} - \bar{a}_{13} + \bar{a}_{14} + \bar{a}_{16} + \bar{a}_{17} - \bar{a}_{18} + 2\bar{a}_{19})/\sqrt{6}$
$\Xi_c^0 \rightarrow \Sigma^0 K^0$	$(B_6^T - E_6^T - B_{15}^T + E_{15}^T)/\sqrt{2}$	$(2\bar{a}_2 - \bar{a}_8 - \bar{a}_{11} - \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{18})/\sqrt{2}$
$\Xi_c^0 \rightarrow \Sigma^- K^+$	$-B_6^T + E_6^T + B_{15}^T + E_{15}^T$	$2\bar{a}_1 - \bar{a}_6 - \bar{a}_{10} + \bar{a}_{15}$
$\Xi_c^0 \rightarrow p \pi^-$	$-C_6^T - D_6^T + C_{15}^T + D_{15}^T$	$\bar{a}_7 + \bar{a}_8 + \bar{a}_{12} + \bar{a}_{13}$
$\Xi_c^0 \rightarrow n \pi^0$	$(C_6^T + D_6^T - C_{15}^T + D_{15}^T)/\sqrt{2}$	$(\bar{a}_6 - \bar{a}_7 + \bar{a}_{10} + \bar{a}_{11} - \bar{a}_{12} + \bar{a}_{14})/\sqrt{2}$
$\Xi_c^0 \rightarrow n \eta_q$	$(-2A_6^T - C_6^T + D_6^T + 2A_{15}^T + C_{15}^T + D_{15}^T)/\sqrt{2}$	$(2\bar{a}_5 + \bar{a}_6 + \bar{a}_7 + 2\bar{a}_9 + \bar{a}_{10} - \bar{a}_{11} + \bar{a}_{12} - \bar{a}_{14})/\sqrt{2}$
$\Xi_c^0 \rightarrow n \eta_s$	$-A_6^T - B_6^T + A_{15}^T + B_{15}^T$	$\bar{a}_5 + \bar{a}_9 - \bar{a}_{18} + \bar{a}_{19}$

few SU(3) independent amplitudes (the last six diagrams in Fig. 1) are sometimes overlooked in TDA (for instance Refs. [16, 18]). Most of these amplitudes arises from higher order loop corrections, but they are irreducible in the flavor SU(3) space, and thus can not be neglected in principle. Taking these new amplitudes into account, we find a consistent description

in both approaches. In addition, using B and D decays we have found that one of the A , C , E , T amplitudes should be absorbed into others, which has been pointed out in Ref. [20]. For heavy baryon decays, we pointed out though the TDA approach is very intuitive, it suffers the difficulty in providing

the independent amplitudes. On this point, the IRA approach is more helpful.

All results derived in this paper can be used to study the heavy meson and baryon decays in the future when sufficient data become available. Then one can have a better understanding of the role of flavor SU(3) symmetry in heavy meson and baryon decays.

For charm quark decays, we did not include the penguin contributions, which can also be studied in a similar manner. It is also necessary to notice that the flavor SU(3) symmetry has been applied to study weak decays of doubly heavy baryons [92–94], and multi-body Λ_c decays [90]. The equivalence between the TDA and IRA approaches in these decay modes can be studied similarly.

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Appendix A: Relations for bottom antitriplet baryon decay amplitudes

In Sect. 5, we have argued that due to different forms to represent the hadron SU(3) multiplet, the IRA amplitudes can be constructed in different ways. In Eq. 75, the antitriplet baryons and octet baryons are used as $(T_{b\bar{3}})_i$ and $(T_8)^{ij}$. Instead it is possible to use another set of forms, $(T_{b\bar{3}})^{ij}$ and $(T_8)_{ijk}$, which gives the IRA amplitudes with 26 terms:

$$\begin{aligned} \mathcal{A}_u^{IRA} = & \bar{A}_1 T_{b\bar{3}}^{[ij]} H_3^k \epsilon_{ijn} (T_8)_k^n P_l^l + \bar{A}_2 T_{b\bar{3}}^{[ij]} H_3^k \epsilon_{ijn} (T_8)_l^n P_k^l \\ & + \bar{A}_3 T_{b\bar{3}}^{[ij]} H_3^k \epsilon_{ikn} (T_8)_j^n P_k^l + \bar{A}_4 T_{b\bar{3}}^{[ij]} H_3^k \epsilon_{iln} (T_8)_j^n P_k^l \\ & + \bar{A}_5 T_{b\bar{3}}^{[ij]} H_3^k \epsilon_{ikn} (T_8)_l^n P_j^l + \bar{A}_6 T_{b\bar{3}}^{[ij]} H_3^k \epsilon_{iln} (T_8)_k^n P_j^l \\ & + \bar{A}_7 T_{b\bar{3}}^{[ij]} H_3^k \epsilon_{kln} (T_8)_i^n P_j^l \end{aligned}$$

$$\begin{aligned} & + \bar{B}_1 T_{b\bar{3}}^{[ij]} (H_6)_m^{[kl]} \epsilon_{ijn} (T_8)_k^n P_l^m \\ & + \bar{B}_2 T_{b\bar{3}}^{[ij]} (H_6)_m^{[kl]} \epsilon_{ikn} (T_8)_j^n P_l^m \\ & + \bar{B}_3 T_{b\bar{3}}^{[ij]} (H_6)_m^{[kl]} \epsilon_{ikn} (T_8)_l^n P_j^m \\ & + \bar{B}_4 T_{b\bar{3}}^{[ij]} (H_6)_m^{[kl]} \epsilon_{kln} (T_8)_i^n P_j^m \\ & + \bar{B}_5 T_{b\bar{3}}^{[ij]} (H_6)_i^{[kl]} \epsilon_{jkn} (T_8)_m^n P_l^m \\ & + \bar{B}_6 T_{b\bar{3}}^{[ij]} (H_6)_i^{[kl]} \epsilon_{kln} (T_8)_m^n P_j^m \\ & + \bar{B}_7 T_{b\bar{3}}^{[ij]} (H_6)_i^{[kl]} \epsilon_{jkn} (T_8)_l^n P_m^m \\ & + \bar{B}_8 T_{b\bar{3}}^{[ij]} (H_6)_i^{[kl]} \epsilon_{kln} (T_8)_j^n P_m^m \\ & + \bar{B}_9 T_{b\bar{3}}^{[ij]} (H_6)_i^{[kl]} \epsilon_{mjn} (T_8)_k^n P_l^m \\ & + \bar{B}_{10} T_{b\bar{3}}^{[ij]} (H_6)_i^{[kl]} \epsilon_{mkn} (T_8)_j^n P_l^m \\ & + \bar{B}_{11} T_{b\bar{3}}^{[ij]} (H_6)_i^{[kl]} \epsilon_{mkn} (T_8)_l^n P_j^m \\ & + \bar{C}_1 T_{b\bar{3}}^{[ij]} (H_{15})_m^{[kl]} \epsilon_{ijn} (T_8)_k^n P_l^m \\ & + \bar{C}_2 T_{b\bar{3}}^{[ij]} (H_{15})_m^{[kl]} \epsilon_{ikn} (T_8)_j^n P_l^m \\ & + \bar{C}_3 T_{b\bar{3}}^{[ij]} (H_{15})_m^{[kl]} \epsilon_{ikn} (T_8)_l^n P_j^m \\ & + \bar{C}_4 T_{b\bar{3}}^{[ij]} (H_{15})_i^{[kl]} \epsilon_{jkn} (T_8)_m^n P_l^m \\ & + \bar{C}_5 T_{b\bar{3}}^{[ij]} (H_{15})_i^{[kl]} \epsilon_{jkn} (T_8)_l^n P_m^m \\ & + \bar{C}_6 T_{b\bar{3}}^{[ij]} (H_{15})_i^{[kl]} \epsilon_{mjn} (T_8)_k^n P_l^m \\ & + \bar{C}_7 T_{b\bar{3}}^{[ij]} (H_{15})_i^{[kl]} \epsilon_{mkn} (T_8)_j^n P_l^m \\ & + \bar{C}_8 T_{b\bar{3}}^{[ij]} (H_{15})_i^{[kl]} \epsilon_{mkn} (T_8)_l^n P_j^m. \end{aligned} \quad (A1)$$

The relation between the two sets of baryon SU(3) representation is:

$$T_{b\bar{3}}^{[ij]} = \frac{1}{2} \epsilon^{ijk} (T_{b\bar{3}})_k, \quad (T_8)_{ijk} = \epsilon_{ijl} (T_8)_l^k. \quad (A2)$$

These two sets of IRA are related to each other by:

$$\begin{aligned} A_3^T &= 2\bar{A}_1 + \bar{A}_3 + \bar{A}_6 + \bar{A}_7, \quad B_3^T = 2\bar{A}_2 + \bar{A}_4 + \bar{A}_5 - \bar{A}_7, \\ C_3^T &= \bar{A}_7 - \bar{A}_5, \quad D_3^T = -\bar{A}_6 - \bar{A}_7 \\ A_6^T &= \bar{B}_7 + \bar{B}_{11} - 2\bar{B}_8, \quad B_6^T = -(\bar{B}_3 + \bar{B}_{11}) + 2\bar{B}_4, \\ C_6^T &= \bar{B}_5 + \bar{B}_{10} - 2\bar{B}_6, \quad D_6^T = -(\bar{B}_9 + \bar{B}_{10} - \bar{B}_{11}), \\ A_{15}^T &= \bar{C}_5 + \bar{C}_8, \quad B_{15}^T = -(\bar{C}_3 + \bar{C}_8), \\ C_{15}^T &= \bar{C}_4 + \bar{C}_7, \quad D_{15}^T = -(\bar{C}_6 + \bar{C}_8 + \bar{C}_7), \\ E_6^T &= 2\bar{B}_1 + 2\bar{B}_4 + \bar{B}_2 - \bar{B}_3 + \bar{B}_9 + \bar{B}_{10} - \bar{B}_{11}, \\ E_{15}^T &= 2\bar{C}_1 + \bar{C}_2 + \bar{C}_3 + \bar{C}_6 + \bar{C}_7 + \bar{C}_8. \end{aligned} \quad (A3)$$

In addition, the relation between the coefficients of 26 amplitudes in $\mathcal{A}_{T_b \rightarrow PT_8(u)}^{IRA}$ and $\mathcal{A}_{T_b \rightarrow PT_8(u)}^{TDA}$ is:

$$\begin{aligned} \bar{A}_1 &= \frac{1}{8} (-\bar{a}_1 + 3\bar{a}_2 - 3\bar{a}_3 + \bar{a}_5) + \bar{b}_1, \\ \bar{A}_2 &= \frac{1}{8} (3\bar{a}_1 - \bar{a}_2 - 3\bar{a}_4 + \bar{a}_7) + \bar{b}_2, \end{aligned}$$

$$\begin{aligned}
\bar{A}_3 &= \frac{1}{8}(\bar{a}_3 - 3\bar{a}_5 + 4\bar{a}_9) + \bar{b}_3, \\
\bar{A}_4 &= \frac{1}{8}(-3\bar{a}_6 + \bar{a}_8 + 3\bar{a}_{10} - \bar{a}_{11} + 3\bar{a}_{15} - \bar{a}_{16}) + \bar{b}_4, \\
\bar{A}_5 &= \frac{1}{8}(4\bar{a}_{12} - \bar{a}_{17} + 3\bar{a}_{18} + \bar{a}_4 - 3\bar{a}_7) + \bar{b}_5, \\
\bar{A}_6 &= \frac{1}{8}(3\bar{a}_{13} - \bar{a}_{14} + 3\bar{a}_{17} - \bar{a}_{18} + \bar{a}_6 - 3\bar{a}_8) + \bar{b}_6, \\
\bar{A}_7 &= \frac{1}{8}(\bar{a}_{10} - 3\bar{a}_{11} - \bar{a}_{13} + 3\bar{a}_{14} + 4\bar{a}_{19}) + \bar{b}_7, \\
\bar{B}_1 &= \frac{1}{4}(\bar{a}_1 - \bar{a}_2), \quad \bar{C}_1 = \frac{1}{8}(\bar{a}_1 + \bar{a}_2), \\
\bar{B}_2 &= \frac{1}{4}(\bar{a}_{15} - \bar{a}_{16}), \quad \bar{C}_2 = \frac{1}{8}(\bar{a}_{15} + \bar{a}_{16}), \\
\bar{B}_3 &= \frac{1}{4}(\bar{a}_{17} - \bar{a}_{18}), \quad \bar{C}_3 = \frac{1}{8}(\bar{a}_{17} + \bar{a}_{18}), \quad \bar{B}_4 = -\frac{1}{4}\bar{a}_{19}, \\
\bar{B}_5 &= \frac{1}{4}(\bar{a}_4 - \bar{a}_7), \quad \bar{C}_4 = \frac{1}{8}(\bar{a}_4 + \bar{a}_7), \quad \bar{B}_6 = \frac{1}{4}\bar{a}_{12}, \\
\bar{B}_7 &= \frac{1}{4}(\bar{a}_3 - \bar{a}_5), \quad \bar{C}_5 = \frac{1}{8}(\bar{a}_3 + \bar{a}_5), \\
\bar{B}_8 &= \frac{1}{4}\bar{a}_9, \quad \bar{B}_9 = \frac{1}{4}(\bar{a}_8 - \bar{a}_6), \quad \bar{C}_6 = -\frac{1}{8}(\bar{a}_6 + \bar{a}_8), \\
\bar{B}_{10} &= \frac{1}{4}(\bar{a}_{11} - \bar{a}_{10}), \quad \bar{C}_7 = -\frac{1}{8}(\bar{a}_{10} + \bar{a}_{11}), \\
\bar{B}_{11} &= \frac{1}{4}(\bar{a}_{14} - \bar{a}_{13}), \quad \bar{C}_8 = -\frac{1}{8}(\bar{a}_{13} + \bar{a}_{14}). \quad (\text{A4})
\end{aligned}$$

Combination of Eqs. (A3) and (A4) leads to Eq. (76).

The authors of Ref. [95] have given another parametrization of IRA amplitudes, in which they have focused on the flavor non-singlet. Comparing with their results, one finds the following relations:

$$\begin{aligned}
B_3^T &= 2b(\bar{3})_2 + d(\bar{3})_1 - e(\bar{3})_2 + c(\bar{3}), \\
C_3^T &= 2a(\bar{3}) - c(\bar{3}), \\
D_3^T &= 2b(\bar{3})_1 + d(\bar{3})_2 - e(\bar{3})_1 + c(\bar{3}), \\
B_6^T &= 2(a(6)_2 - g(6) - n(6)_1) + d(6)_2 - e(6)_1 - e(6)_2, \\
C_6^T &= 2(a(6)_1 + f(6) - n(6)_2) + d(6)_1, \\
E_6^T &= 2(b(6)_2 - g(6)) - c(6) + d(6)_1 \\
&\quad + d(6)_2 - e(6)_1 - e(6)_2 - g(6), \\
D_6^T &= -2(b(6)_1 + f(6)) + c(6) - d(6)_1 \\
&\quad - d(6)_2 + e(6)_1 + e(6)_2, \\
B_{15}^T &= 2a(\bar{15})_2 + d(\bar{15})_2 - e(\bar{15})_1, \\
C_{15}^T &= 2a(\bar{15})_1 + d(\bar{15})_1 - e(\bar{15})_2, \\
E_{15}^T &= 2b(\bar{15})_2 + c(\bar{15}) + d(\bar{15})_1 \\
&\quad - d(\bar{15})_2 + e(\bar{15})_1 - e(\bar{15})_2, \\
D_{15}^T &= 2b(\bar{15})_1 - c(\bar{15}) - d(\bar{15})_1 \\
&\quad + d(\bar{15})_2 - e(\bar{15})_1 + e(\bar{15})_2. \quad (\text{A5})
\end{aligned}$$

Appendix B: Relations for charmed baryon decays

Similar with the previous section, one can construct another set of IRA for charmed antitriplet baryon decays with 19 amplitudes:

$$\begin{aligned}
\mathcal{A}_u^{IRA} &= \bar{B}_1 T_{c\bar{3}}^{[ij]} (H_6)_m^{kl} \epsilon_{ijn} (T_8)_k^n P_l^m \\
&\quad + \bar{B}_2 T_{c\bar{3}}^{[ij]} (H_6)_m^{kl} \epsilon_{ikn} (T_8)_j^n P_l^m \\
&\quad + \bar{B}_3 T_{c\bar{3}}^{[ij]} (H_6)_m^{kl} \epsilon_{ikn} (T_8)_l^n P_j^m \\
&\quad + \bar{B}_4 T_{c\bar{3}}^{[ij]} (H_6)_m^{kl} \epsilon_{kln} (T_8)_i^n P_j^m \\
&\quad + \bar{B}_5 T_{c\bar{3}}^{[ij]} (H_6)_i^{kl} \epsilon_{jkn} (T_8)_m^n P_l^m \\
&\quad + \bar{B}_6 T_{c\bar{3}}^{[ij]} (H_6)_i^{kl} \epsilon_{kln} (T_8)_m^n P_j^m \\
&\quad + \bar{B}_7 T_{c\bar{3}}^{[ij]} (H_6)_i^{kl} \epsilon_{jkn} (T_8)_l^n P_m^m \\
&\quad + \bar{B}_8 T_{c\bar{3}}^{[ij]} (H_6)_i^{kl} \epsilon_{kln} (T_8)_l^n P_m^m \\
&\quad + \bar{B}_9 T_{c\bar{3}}^{[ij]} (H_6)_i^{kl} \epsilon_{mjn} (T_8)_k^n P_l^m \\
&\quad + \bar{B}_{10} T_{c\bar{3}}^{[ij]} (H_6)_i^{kl} \epsilon_{mkn} (T_8)_j^n P_l^m \\
&\quad + \bar{B}_{11} T_{c\bar{3}}^{[ij]} (H_6)_i^{kl} \epsilon_{mkn} (T_8)_l^n P_j^m \\
&\quad + \bar{C}_1 T_{c\bar{3}}^{[ij]} (H_{15})_m^{kl} \epsilon_{ijn} (T_8)_k^n P_l^m \\
&\quad + \bar{C}_2 T_{c\bar{3}}^{[ij]} (H_{15})_m^{kl} \epsilon_{ikn} (T_8)_j^n P_l^m \\
&\quad + \bar{C}_3 T_{c\bar{3}}^{[ij]} (H_{15})_m^{kl} \epsilon_{ikn} (T_8)_l^n P_j^m \\
&\quad + \bar{C}_4 T_{c\bar{3}}^{[ij]} (H_{15})_i^{kl} \epsilon_{jkn} (T_8)_m^n P_l^m \\
&\quad + \bar{C}_5 T_{c\bar{3}}^{[ij]} (H_{15})_i^{kl} \epsilon_{jkn} (T_8)_l^n P_m^m \\
&\quad + \bar{C}_6 T_{c\bar{3}}^{[ij]} (H_{15})_i^{kl} \epsilon_{mjn} (T_8)_k^n P_l^m \\
&\quad + \bar{C}_7 T_{c\bar{3}}^{[ij]} (H_{15})_i^{kl} \epsilon_{mkn} (T_8)_j^n P_l^m \\
&\quad + \bar{C}_8 T_{c\bar{3}}^{[ij]} (H_{15})_i^{kl} \epsilon_{mkn} (T_8)_l^n P_j^m. \quad (\text{A6})
\end{aligned}$$

The relation between the two sets of IRA is given as:

$$\begin{aligned}
A_6^T &= \frac{1}{2}(\bar{B}_7 + \bar{B}_{11}) - \bar{B}_8, \quad B_6^T = -\frac{1}{2}(\bar{B}_3 + \bar{B}_{11}) + \bar{B}_4, \\
C_6^T &= \frac{1}{2}(\bar{B}_5 + \bar{B}_{10}) - \bar{B}_6, \quad D_6^T = -\frac{1}{2}(\bar{B}_9 + \bar{B}_{10} - \bar{B}_{11}) \\
A_{15}^T &= \frac{1}{2}(\bar{C}_5 + \bar{C}_8), \quad B_{15}^T = -\frac{1}{2}(\bar{C}_3 + \bar{C}_8), \\
C_{15}^T &= \frac{1}{2}(\bar{C}_4 + \bar{C}_7), \quad D_{15}^T = -\frac{1}{2}(\bar{C}_6 + \bar{C}_8 + \bar{C}_7) \\
E_6^T &= \bar{B}_1 + \bar{B}_4 + \frac{1}{2}(\bar{B}_2 - \bar{B}_3 + \bar{B}_9 + \bar{B}_{10} - \bar{B}_{11}), \\
E_{15}^T &= \bar{C}_1 + \frac{1}{2}(\bar{C}_2 + \bar{C}_3 + \bar{C}_6 + \bar{C}_7 + \bar{C}_8). \quad (\text{A7})
\end{aligned}$$

The relation between the new $\mathcal{A}_{T_c \rightarrow PT_8(u)}^{IRA}$ and $\mathcal{A}_{T_c \rightarrow PT_8(u)}^{TDA}$ can be obtained as:

$$\bar{B}_1 = \frac{1}{2}(\bar{a}_1 - \bar{a}_2), \quad \bar{C}_1 = \frac{1}{2}(\bar{a}_1 + \bar{a}_2),$$

$$\begin{aligned}
\bar{B}_2 &= \frac{1}{2}(\bar{a}_{15} - \bar{a}_{16}), & \bar{C}_2 &= \frac{1}{2}(\bar{a}_{15} + \bar{a}_{16}) \\
\bar{B}_3 &= \frac{1}{2}(\bar{a}_{17} - \bar{a}_{18}), & \bar{C}_3 &= \frac{1}{2}(\bar{a}_{17} + \bar{a}_{18}), \\
\bar{B}_4 &= -\frac{1}{2}\bar{a}_{19}, & \bar{B}_5 &= \frac{1}{2}(\bar{a}_4 - \bar{a}_7), & \bar{C}_4 &= \frac{1}{2}(\bar{a}_4 + \bar{a}_7), \\
\bar{B}_6 &= \frac{1}{2}\bar{a}_{12}, & \bar{B}_7 &= \frac{1}{2}(\bar{a}_3 - \bar{a}_5), & \bar{C}_5 &= \frac{1}{2}(\bar{a}_3 + \bar{a}_5), \\
\bar{B}_8 &= \frac{1}{2}\bar{a}_9, & \bar{B}_9 &= \frac{1}{2}(\bar{a}_8 - \bar{a}_6), & \bar{C}_6 &= -\frac{1}{2}(\bar{a}_6 + \bar{a}_8), \\
\bar{B}_{10} &= \frac{1}{2}(\bar{a}_{11} - \bar{a}_{10}), & \bar{C}_7 &= -\frac{1}{2}(\bar{a}_{10} + \bar{a}_{11}), \\
\bar{B}_{11} &= \frac{1}{2}(\bar{a}_{14} - \bar{a}_{13}), & \bar{C}_8 &= -\frac{1}{2}(\bar{a}_{13} + \bar{a}_{14}). \quad (\text{A8})
\end{aligned}$$

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