Eur. Phys. J. C (2020) 80:144
https://doi.org/10.1140/epjc/s10052-020-7720-5

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

The role of transverse momentum on the nuclear valence and sea quark distribution functions

H. Nematollahi[a]

Faculty of physics, Shahid Bahonar University of Kerman, Kerman, Iran

Received: 22 November 2019 / Accepted: 6 February 2020 / Published online: 18 February 2020

Abstract We investigate the transverse momentum dependence of valence and sea quark distribution functions of light asymmetric nuclei (^3He and ^7Li). To this end, we first calculate the valence and sea distributions of these nuclei applying a parametrization method in which the parton distribution functions (PDFs) of nucleus are related to those of free nucleon via a weight function that contains the nuclear modifications. Then we obtain the unpolarized transverse momentum dependent (TMD) PDFs of the nucleus using the covariant parton model (CPM) approach. We also compute the valence and sea quark distributions ratios of ^3He and ^7Li to those of deuteron and present the results with respect to x (Bjorken variable) at fixed values of transverse momentum. It is found that these ratios shift to the larger values of x by increasing the transverse momentum value as expected and they are not transverse momentum dependent in large x region.

1 Introduction

The investigation of the parton distribution functions of the nucleons and nuclei has been performed by analysing the deep inelastic scattering (DIS) processes of different targets. These processes have provided the detailed knowledge about internal structure of the nucleons and nuclei.

The European Muon Collaboration (EMC) group found the difference between the bound state nucleon structure and that of free one, for the first time in 1983 [1]. They observed that a significant effect appeared in the nucleon structure while changing the target from deuteron to the heavy nuclei which is the so called EMC effect [2–4]. In fact the EMC effect originates from an essential difference between the PDFs of free nucleons and those of the bounded ones in nucleus. Several theoretical models, such as [5,6]: the pion excess model [7–12], the deconfinement model [7,13], the quark exchange model (QEM) [14,15], the cluster model [7,16,17] and the rescaling model [7,18–21], have been suggested to study the nuclear PDFs and describe the EMC effect, which has been measured [6] in the Drell-Yan process [7,22–26], charged lepton-nucleus scattering [27–32] and neutrino-nucleus scattering processes [33–38]. We have already probed the structure of light nuclei applying the chiral quark exchange model [39] and also investigated the role of nuclear corrections on the structure function and the EMC-ratio of deuteron [40].

The usual PDFs do not have transverse momentum dependence. They describe the probability for finding a parton with longitudinal momentum fraction x of the parent hadron integrated over the transverse momentum of the parton. Since the transverse momentum of parton is not negligible especially at small values of x and the description of some experimental observations is not possible without three-dimensional (3D) picture of the hadron, the transverse momentum dependent PDFs (TMDs) [41–49] have appeared in recent years. TMDs, which contain some information on partons with specific transverse momenta, open a new way to obtain a 3D picture of the parton structure of hadrons [50]. TMDs can be achieved by measuring the transverse momentum of hadron which is produced in semi-inclusive DIS. We have already calculated the unpolarized TMD distributions of light quarks and gluon for light nuclei using the modified chiral quark exchange model in our previous work [51]. On the other hand, the transverse momentum dependence of the PDFs of nucleons and nuclei has been studied via another set of distributions called unintegrated parton distribution functions (UPDFs). The UPDFs of a number of nuclei have been probed in Ref. [52,53] based on an approach first given by Kimber, Martin and Ryskin (KMR) [54] and then extended by Martin, Ryskin and Watt (MRW) [55].

In this article we study the TMD valence and sea quark distribution functions of ^3He and ^7Li nuclei. For this purpose we first calculate the PDFs of these nuclei using a global

[a] e-mail: hnematollahi@uk.ac.ir (corresponding author)

analysis of the experimental data which has been performed by Khanpour and Atashbar Tehrani [56]. In their method a weight function, which contains the nuclear modifications, relates the PDFs of free nucleons to those of bounded ones in nucleus [56–65]. Then we obtain the unpolarized TMD distributions of ^3He and ^7Li in the covariant parton model framework [66–76]. The base of this model is the 3D picture of parton momentum with rotational symmetry in the nucleon rest frame.

The organization of this paper is as follows: in Sect. 2 we provide the description of the parametrization formalism applied to obtain the nuclear PDFs using the free nucleon PDFs and taking into account the nuclear modifications via a weight function. We provide a brief explanation for calculating the unpolarized TMD nuclear PDFs based on the covariant parton model in Sect. 3. We present our results in Sect. 4 and give our conclusions in Sect. 5.

2 Nuclear PDFs parametrization formalism

In this section we obtain the valence and sea quark distribution functions of nuclei applying the parametrization method of Ref. [56]. At the first step of this method, the PDFs of nucleus are considered as the PDFs of free proton multiplied by a weight function which originates from nuclear modifications [56–65]. So the initial valence quark distribution of bounded proton in the nucleus A at Q_0^2 scale, $f_{v_i}^A\left(x, Q_0^2\right)$, can be written as:

$$f_{v_i}^A\left(x, Q_0^2\right) = W_{v_i}(x, A, Z) f_{v_i}\left(x, Q_0^2\right), \tag{1}$$

where $f_{v_i}\left(x, Q_0^2\right)$ denotes the valence quark distribution of free proton with flavor i and $W_{v_i}(x, A, Z)$ is the valence nuclear modification. The sea quark distribution of the bounded proton with flavor i, $f_{s_i}^A\left(x, Q_0^2\right)$, is given as:

$$f_{s_i}^A\left(x, Q_0^2\right) = W_s(x, A, Z) f_{s_i}\left(x, Q_0^2\right). \tag{2}$$

Z and A denote the atomic and mass number of the nucleus, respectively. It should be pointed that in above equations the large-x ($x > 1$) nuclear valence and sea quark distributions are neglected [62–64,77–83].

We use JR09 set for valence and sea quark distribution functions of free proton at $Q_0^2 = 2\ \mathrm{GeV}^2$ [56,84]:

$$\begin{aligned}
xu_v\left(x, Q_0^2\right) &= 3.2350x^{0.6710}(1-x)^{3.9293} \\
&\quad \times(1 - 0.5302x^{0.5} + 3.9029x), \\
xd_v\left(x, Q_0^2\right) &= 13.058x^{1.0701}(1-x)^{6.2177} \\
&\quad \times(1 - 2.5830x^{0.5} + 3.8965x), \\
x(\bar{d}+\bar{u}) &= 0.4250x^{-0.1098}(1-x)^{10.34} \\
&\quad \times(1 - 3.0946x^{0.5} + 11.613x), \\
x(\bar{d}-\bar{u}) &= 8.1558x^{1.328}(1-x)^{21.043} \\
&\quad \times(1 - 7.6334x^{0.5} + 20.054x),
\end{aligned} \tag{3}$$

it is assumed that $xs = x\bar{s} = \frac{1}{4}x(\bar{d}+\bar{u})$ [56].

In the framework of Ref. [56], a functional form is considered for the weight function as the nuclear modification [56,62–64]. This form can be written as:

$$\begin{aligned}
W_{v_i}(x, A, Z) &= \left[\frac{a_{v_i}(A, Z) + b_v(A)x + c_v(A)x^2 + d_v(A)x^3}{(1-x)^{\beta_v}}\right] \\
&\quad \times\left(1 - \frac{1}{A^\alpha}\right) + 1,
\end{aligned} \tag{4}$$

$$\begin{aligned}
W_s(x, A, Z) &= \left[\frac{a_s(A, Z) + b_s(A)x + c_s(A)x^2 + d_s(A)x^3}{(1-x)^{\beta_s}}\right] \\
&\quad \times\left(1 - \frac{1}{A^\alpha}\right) + 1,
\end{aligned} \tag{5}$$

for valence and sea weight functions, respectively. The coefficients of above equations which are dependent on A and Z are calculated using a global χ^2 analysis of experimental data in Ref. [56]. The parameter α is fixed to $\alpha = 1/3$ and β_i's parameters for valence and sea quarks are fixed to $\beta_v = 0.4$ and $\beta_s = 0.1$, respectively [56]. We extract the coefficients of Eqs. (4) and (5) from Ref. [56]. These coefficients are given in Table1 as the functions of A.

In order to compute the valence parameters $a_{v_i}(A, Z)$, the conservation constraints for atomic and mass number of nucleus are considered [56,57,62,63,85]:

$$\begin{aligned}
Z &= \int \frac{A}{3}(2u_v^A - d_v^A)(x, Q_0^2)dx, \\
3 &= \int (u_v^A + d_v^A)(x, Q_0^2)dx,
\end{aligned} \tag{6}$$

so the valence parameters are expressed in terms of four integral values, I_1, I_2, I_3 and I_4, as [56,57]:

$$\begin{aligned}
a_{u_v}(A, Z) &= -\frac{ZI_1 + NI_2}{ZI_3 + NI_4}, \\
a_{d_v}(A, Z) &= -\frac{ZI_2 + NI_1}{ZI_4 + NI_3},
\end{aligned} \tag{7}$$

in which $N = A - Z$ denotes the number of neutrons in the nucleus. The numerical values of I's in above equation are as follows [56]: $I_1 = 0.0890676$, $I_2 = 0.0537472$, $I_3 = 2.1693$, $I_4 = 1.06856$.

Considering Eqs. (1–5), and the isospin symmetry between proton and neutron, the valence quark distribution functions of nucleus A per nucleon are obtained as [56–65]:

$$u_v^A(x, Q_0^2) = W_{u_v}(x, A, Z)\left[\frac{Z}{A}u_v(x, Q_0^2) + \frac{N}{A}d_v(x, Q_0^2)\right],$$

Table 1 The input parameters of valence and sea weight functions, Eqs. (4) and (5), as the functions of A at $Q_0^2 = 2$ GeV^2 extracted from Ref. [56]

a_v	Eq. (7)	a_s	$-0.14364 \pm 8.938466 \times 10^{-3} A^{0.149757 \pm 1.3456148 \times 10^{-2}}$
b_v	$1.98347 \pm 0.1705875 A^{-0.0791784 \pm 1.19181 \times 10^{-2}}$	b_s	$3.1188 \pm 0.2080143 A^{0.159521 \pm 1.4907795 \times 10^{-2}}$
c_v	$-6.46451 \pm 0.3582447 A^{-0.038812 \pm 1.36899 \times 10^{-2}}$	c_s	$-15.5991 \pm 1.1211789 A^{0.183694 \pm 1.8131510 \times 10^{-2}}$
d_v	$4.90165 \pm 0.3045687 A^{0.00900608 \pm 1.81409 \times 10^{-2}}$	d_s	$18.7266 \pm 2.2757606 A^{0.255328 \pm 2.9314540 \times 10^{-2}}$

$$d_v^A(x, Q_0^2) = W_{d_v}(x, A, Z)\left[\frac{Z}{A} d_v(x, Q_0^2) + \frac{N}{A} u_v(x, Q_0^2)\right], \tag{8}$$

and the per-nucleon nuclear sea quark distribution functions are written as [56–65]:

$$\begin{aligned}
\bar{u}^A\left(x, Q_0^2\right) &= W_s(x, A, Z) \\
&\quad \left[\frac{Z}{A}\bar{u}\left(x, Q_0^2\right) + \frac{N}{A}\bar{d}\left(x, Q_0^2\right)\right], \\
\bar{d}^A(x, Q_0^2) &= W_s(x, A, Z) \\
&\quad \left[\frac{Z}{A}\bar{d}\left(x, Q_0^2\right) + \frac{N}{A}\bar{u}\left(x, Q_0^2\right)\right], \\
s^A(x, Q_0^2) &= \bar{s}^A(x, Q_0^2) = W_s(x, A, Z) s(x, Q_0^2).
\end{aligned} \tag{9}$$

It should be pointed that from Eqs. (8) and (9) for symmetric nuclei, ($Z = N$), such as deuteron $u_v^A = d_v^A$ and $\bar{u}^A = \bar{d}^A$.

Now we are able to calculate the valence and sea PDFs of ^3He and ^7Li, asymmetric nuclei, using Eqs. (8) and (9).

3 Transverse momentum dependence of nuclear PDFs

In this section we calculate the unpolarized transverse momentum dependent valence and sea quark distributions of ^3He and ^7Li using the covariant parton model approach [66–76]. In this approach the unpolarized TMD distribution functions are related to integrated ones via a simple derivation. In CPM the unpolarized TMD distribution of quark, $q(x, p_T)$, is given as [67,68,71,72]:

$$q(x, p_T) = Mx \int G^q(p_0)\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{dp_1}{p_0}. \tag{10}$$

Here, M and p denote the nucleon mass and the quark momentum, respectively. $G^q(p_0)$ is the generalized quark distribution. This distribution is only dependent on p_0 due to rotational symmetry in the nucleon rest frame [67,68,71,72].

By rewriting the δ-function in Eq. (10) [67,68,72],

$$\delta\left(\frac{p_0 + p_1}{M} - x\right) dp_1 = \frac{\delta(p_1 - \tilde{p}_1)dp_1}{x/p_0}, \tag{11}$$

this equation can be modified as [67,68,72]:

$$q(x, p_T) = M \int G^q(p_0)\delta(p_1 - \tilde{p}_1)dp_1 = MG^q(\tilde{p}_0), \tag{12}$$

in which

$$\begin{aligned}
\tilde{p}_1 &= \frac{1}{2}Mx\left[1 - \frac{p_T^2}{M^2x^2}\right], \\
\tilde{p}_0 &= \frac{1}{2}Mx\left[1 + \frac{p_T^2}{M^2x^2}\right].
\end{aligned} \tag{13}$$

p_T denotes the quark transverse momentum.

By considering the following identity [67,68,72,75]:

$$\frac{d}{dx}\left[\frac{q(x)}{x}\right] = -\pi M^3 G^q\left(\frac{Mx}{2}\right), \tag{14}$$

and defining ξ variable, in which longitudinal and transverse momenta of quark are related to each other, as [66–76]:

$$\xi(x, p_T) = x\left(1 + \frac{p_T^2}{M^2x^2}\right), \tag{15}$$

the unpolarized TMD quark distribution can be written as [67–69,72]:

$$q(x, p_T) = -\frac{1}{\pi M^2}\frac{d}{dy}\left[\frac{q(y)}{y}\right]_{y=\xi}\theta\left[x(1-x)M^2 - p_T^2\right], \tag{16}$$

where $q(x)$ is the integrated quark distribution function. The θ-function appears in Eq. (16) due to the constraint $p_T \leq M^2x(1-x)$ which is obtained for massless quarks in covariant parton model [68,69,76]. This function is used in the notation of Ref. [69].

Now we can calculate the transverse momentum dependence of the unpolarized valence and sea quark distribution functions of ^3He and ^7Li nuclei applying Eq. (16) which predicts the dependences of the unpolarized TMD quark distribution, $q(x, p_T)$, on both x and p_T via the x dependence of corresponding integrated quark distribution function $q(x)$ [67].

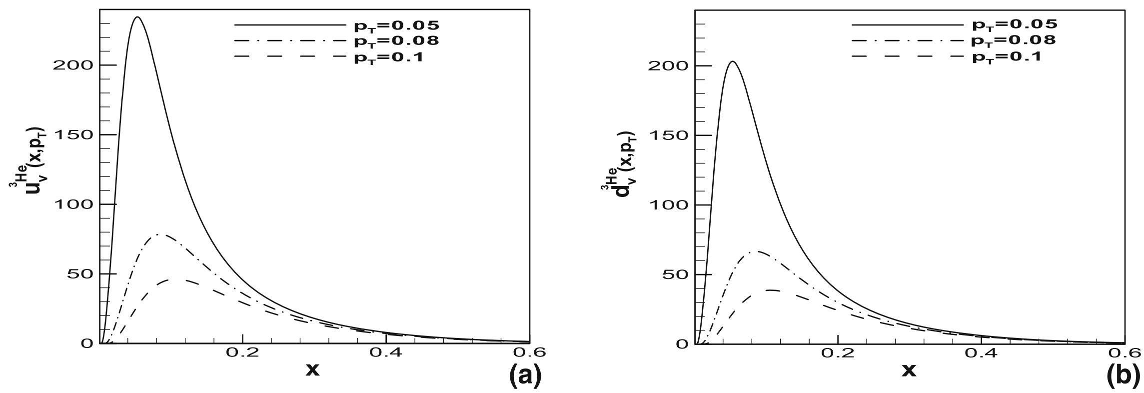

Fig. 1 The TMD valence quark distributions of ^3He nucleus: **a** $u_v(x, p_T)$, **b** $d_v(x, p_T)$ with respect to x at $p_T = 0.05$ GeV, 0.08 GeV and 0.1 GeV

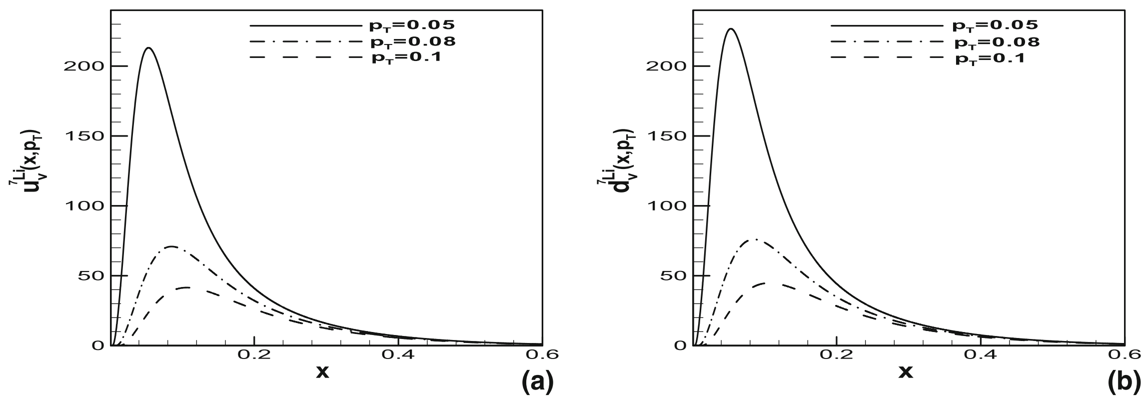

Fig. 2 The TMD valence quark distributions of ^7Li nucleus: **a** $u_v(x, p_T)$, **b** $d_v(x, p_T)$ with respect to x at $p_T = 0.05$ GeV, 0.08 GeV and 0.1 GeV

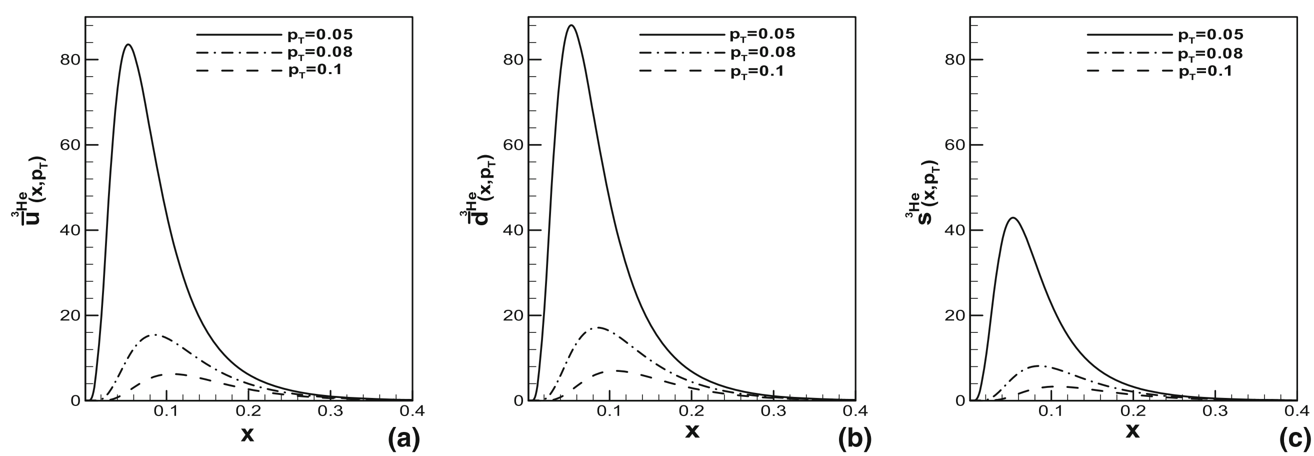

Fig. 3 The TMD sea quak distributions of ^3He, **a** $\bar{u}(x, p_T)$, **b** $\bar{d}(x, p_T)$, **c** $s(x, p_T)$, with respect to x at $p_T = 0.05$ GeV, 0.08 GeV and 0.1 GeV

4 Results and discussion

In this article, we first calculate the valence and sea quark distribution functions of light asymmetric nuclei, ^3He and ^7Li, applying the parametrization method described in Sect. 2 and then compute the transverse momentum dependence of these distributions using Eq. (16) at $Q_0^2 = 2$ GeV^2.

In Figs. 1 and 2, we present our results for the unpolarized TMD valence quark distributions of ^3He and ^7Li nuclei with respect to x at three values of transverse momentum, $p_T = 0.05$ GeV, 0.08 GeV and 0.1 GeV, respectively. We also depict the TMD sea quark distributions of these light nuclei, $\bar{u}(x, p_T)$, $\bar{d}(x, p_T)$ and $s(x, p_T)$, at $p_T =$ 0.05 GeV, 0.08 GeV and 0.1 GeV in Figs. 3 and 4. From

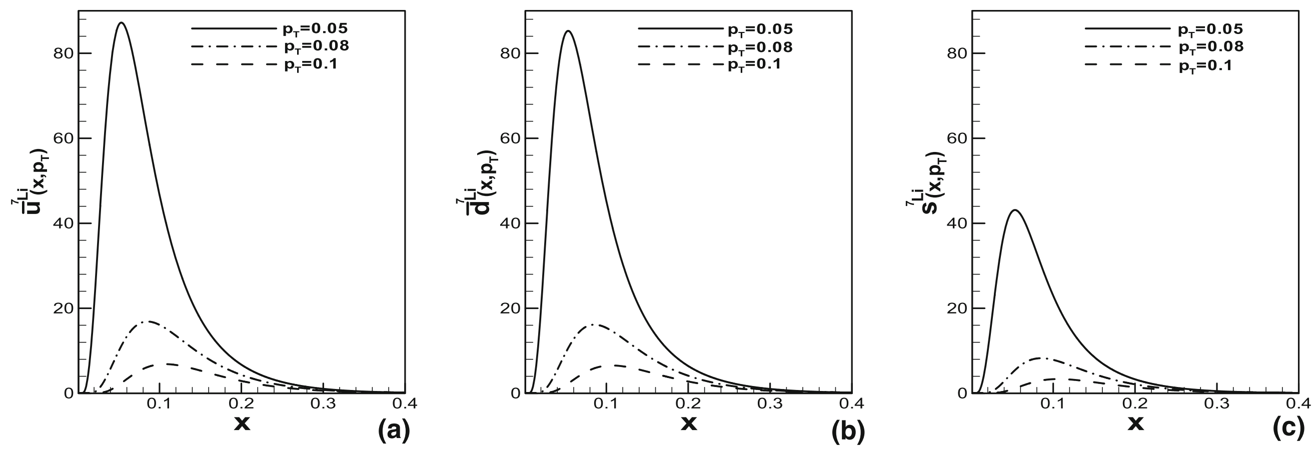

Fig. 4 The TMD sea quak distributions of ^7Li, **a** $\bar{u}(x, p_T)$, **b** $\bar{d}(x, p_T)$ **c** $s(x, p_T)$, with respect to x at $p_T = 0.05$ GeV, 0.08 GeV and 0.1 GeV

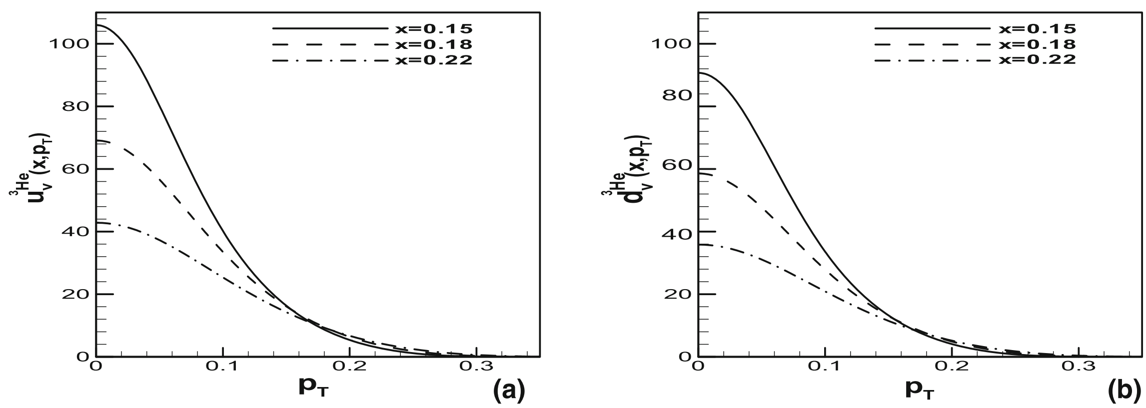

Fig. 5 The TMD valence **a** u and **b** d quark distributions of ^3He with respect to p_T at $x = 0.15$, 0.18 and 0.22

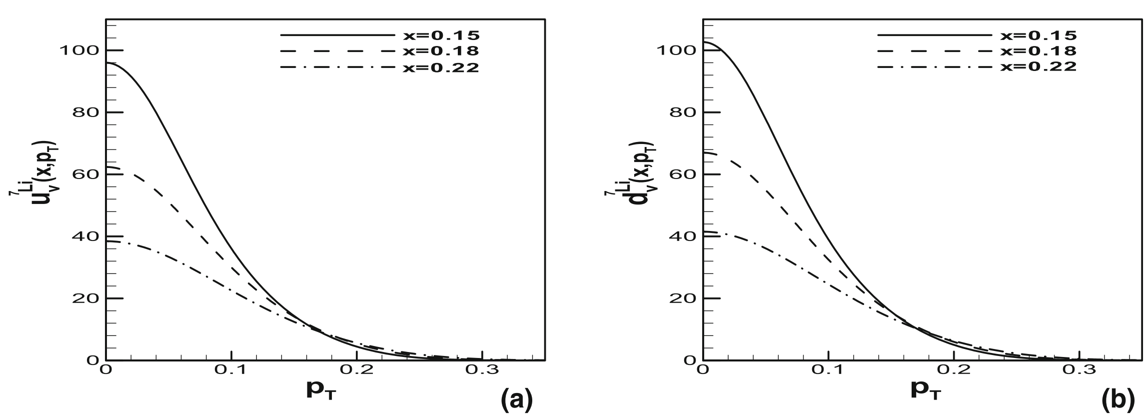

Fig. 6 The TMD valence **a** u and **b** d quark distributions of ^7Li with respect to p_T at $x = 0.15$, 0.18 and 0.22

Figs. 1, 2, 3, 4, it is found that the width of x-distributions is dependent on the value of p_T and the probability of finding the valence and sea quarks decreases by increasing the p_T value as expected [5,51,68]. The latter property appears more obviously in TMD sea quark distributions. Furthermore, both valence and sea distributions have the shifts to the larger values of x by increasing the p_T value.

In Figs. 5, 6, 7, 8, we present the unpolarized TMD valence and sea quark distribution functions of ^3He and ^7Li nuclei with respect to transverse momentum at three x values ($x = 0.15$, 0.18 and 0.22). It is found that these p_T-distributions of valence and sea quarks at fixed x value are very close to the Gaussian distributions and their width change by varying the value of x [5,51,68].

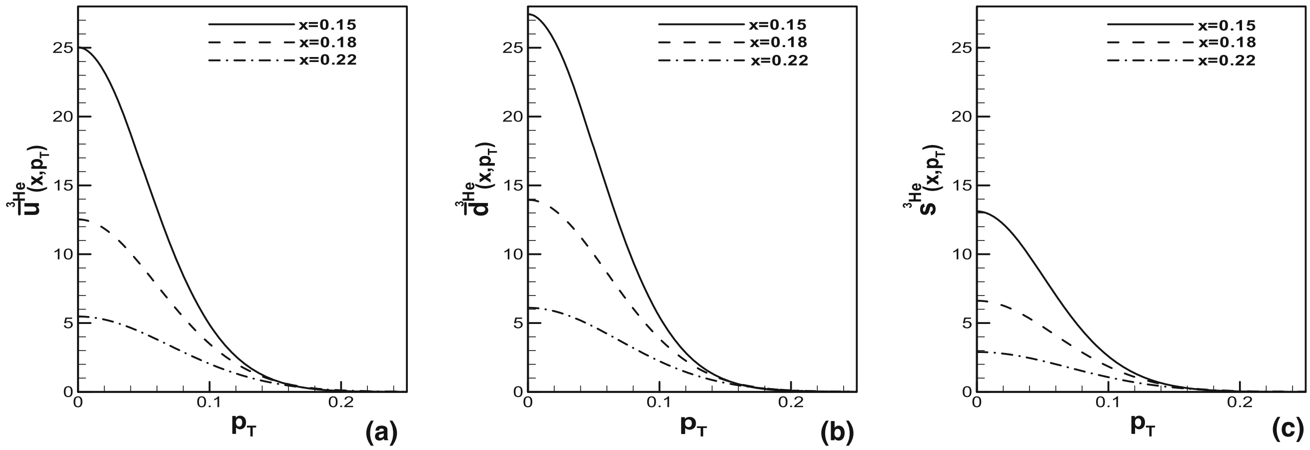

Fig. 7 The sea quak p_T-distributions of ^3He, **a** $\bar{u}$, **b** $\bar{d}$ and **c** s, at $x = 0.15, 0.18$ and 0.22

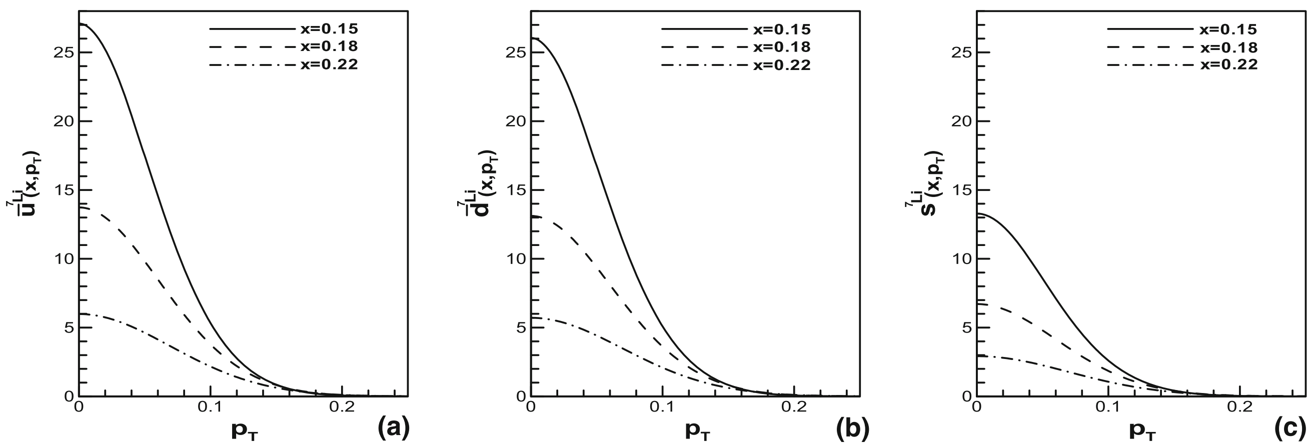

Fig. 8 The p_T-distribution of **a** $\bar{u}$, **b** $\bar{d}$ and **c** s at $x = 0.15, 0.18$ and 0.22 for ^7Li nucleus

Considering Figs. 1, 2, 3, 4, 5, 6, 7, 8, we can say that our results for TMD valence and sea distributions of ^3He and ^7Li nuclei show acceptable general properties expected for unpolarized TMD distribution functions [51,67,68,86–92].

In a further step of our calculations, we first obtain the TMD valence and sea quark distribution functions of deuteron nucleus, d, and then define the following TMD valence and sea ratios for the nucleus A as:

$$\mathcal{R}^A_{q_v}(x, p_T) = \frac{q^A_v(x, p_T)}{q^d_v(x, p_T)},$$
$$\mathcal{R}^A_{sea}(x, p_T) = \frac{q^A_{sea}(x, p_T)}{q^d_{sea}(x, p_T)}, \quad (17)$$

in which q^A_{sea} is the total sea quark distribution of the nucleus A:

$$q^A_{sea} = \bar{u}^A + \bar{d}^A + s^A + \bar{s}^A, \quad (18)$$

$q^d_v(x, p_T)$ and $q^d_{sea}(x, p_T)$ denote the TMD valence and total sea quark distribution functions of deuteron, respectively. We should point that deuteron is a weakly bound nucleus with small nuclear effects. Hence, by considering isospin invariance, it can be approximately viewed as an isoscalar nucleon [78,80,82,93,94]. For this reason we define nuclear valence and sea ratios with respect to deuteron to indicate the EMC effect in the nucleus A [78,80,82,93,94].

In Figs. 9 and 10 we display the results of $\mathcal{R}^A_{q_v}(x, p_T)$, $\mathcal{R}^A_{u_v}(x, p_T)$ and $\mathcal{R}^A_{d_v}(x, p_T)$, for ^3He and ^7Li nuclei, respectively. These ratios are plotted with respect to x at $p_T =$ 0.05 GeV, 0.08 GeV and 0.1 GeV.

As can be seen in Figs. 9 and 10, the TMD valence ratios have the shifts to the larger values of x by increasing the p_T value. Furthermore, These ratios have equivalent properties in large x region. It means that the valence ratios are not transverse momentum dependent at large values of x. The properties of valence ratios of ^3He are equivalent at different values of transverse momentum in $x > 0.3$ region and those of ^7Li do not have p_T dependence within $x > 0.2$. We should also point that while the valence quark distribution functions of ^3He and ^7Li show the same general properties

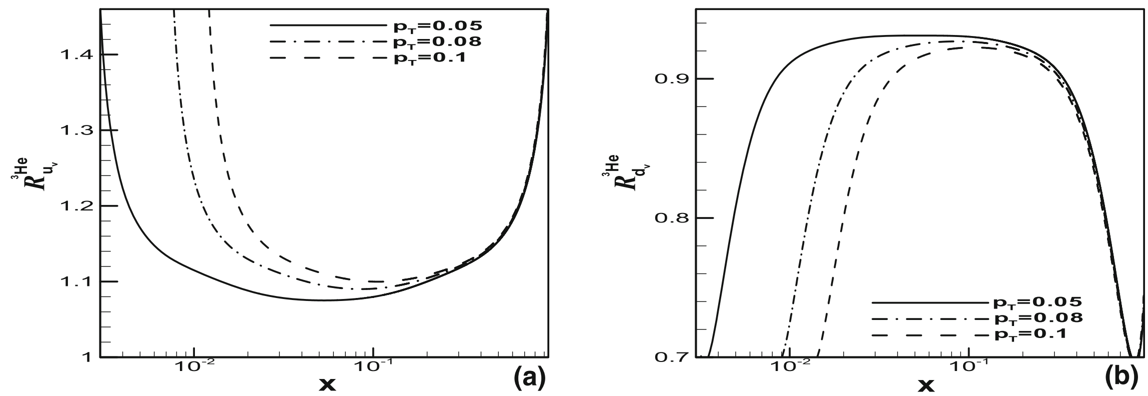

Fig. 9 The ratios of TMD valence quark, **a** u_v and **b** d_v, distributions of ^3He and deuteron with respect to x at three values of p_T

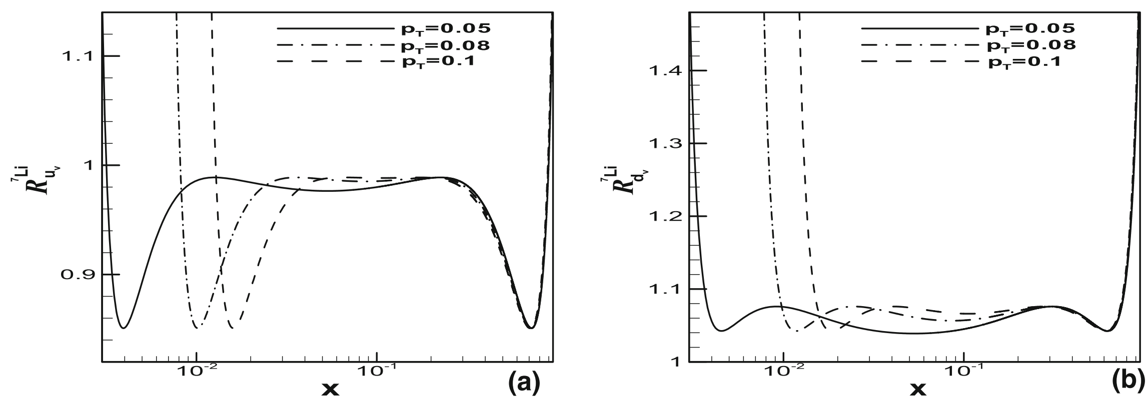

Fig. 10 The TMD valence quark distribution ratios of ^7Li, **a** $\mathcal{R}_{u_v}^{^7Li}(x, p_T)$ and **b** $\mathcal{R}_{d_v}^{^7Li}(x, p_T)$, with respect to x at three values of p_T

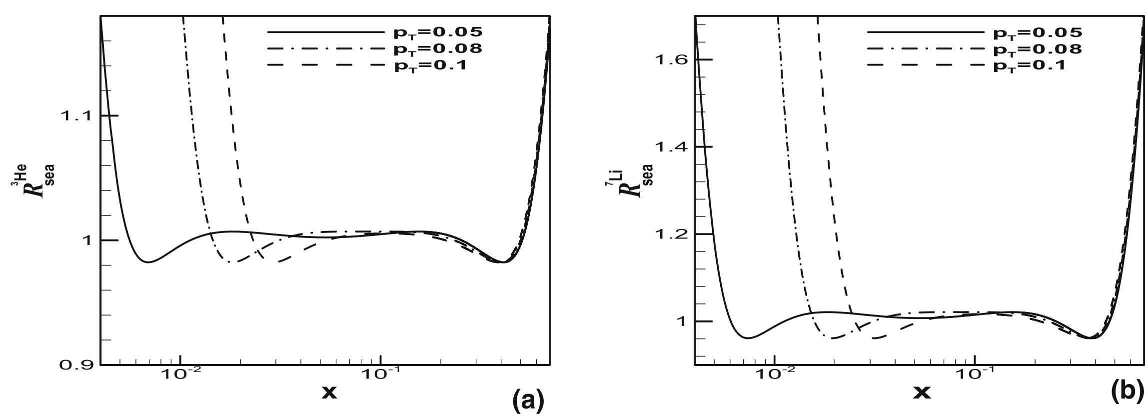

Fig. 11 The ratios of TMD total sea quark distributions of **a** ^3He, **b** ^7Li to those of deuteron with respect to x at three values of p_T

in Figs. 1 and 2, they take different values at each value of x. Therefore, the ratios of these valence distributions to that of deuteron show different behaviour in Figs. 9 and 10. In fact the differences between the valence distributions of ^3He and ^7Li in detail appear more clearly in their ratios with respect to deuteron.

We also depict the results of our calculations for x dependence of $\mathcal{R}_{sea}^{^3He}(x, p_T)$ and $\mathcal{R}_{sea}^{^7Li}(x, p_T)$ at fixed values of p_T in Fig. 11. It is found from this figure that the properties of sea ratios are similar to those of valence ones, i.e. by increasing the p_T value the sea ratios shift to the larger values of x. The total sea quark distributions ratio of ^3He (^7Li) and deuteron is not transverse momentum dependent in $x > 0.3$ ($x > 0.2$) region.

5 Conclusion

In conclusion, we have studied the unpolarized TMD valence and sea quark distribution functions of light asymmetric nuclei. Because of the EMC effect the parton distributions of free nucleons are different from those of bounded ones in nucleus. In order to obtain the PDFs of ^3He and ^7Li, we have used a global analysis method of a weight function which relates the nuclear PDFs to the free nucleon ones and takes the nuclear modifications into account. We should point that in this method the PDFs of bound state nucleons in nucleus are restricted to $0 < x < 1$ due to defining those of free ones in this region [77], while $x > 1$ is valid for the nuclear PDFs.

In the next step we have applied the covariant parton model to obtain the transverse momentum dependence of the nuclear valence and sea quark distributions. The results of the unpolarized TMD distributions for ^3He and ^7Li show convenient general properties at fixed values of x and p_T. We have also calculated the ratio of TMD valence and sea distributions of these nuclei to those of deuteron nucleus. It is found that these ratios are not transverse momentum dependent only in large x region as expected.

We hope to perform our calculations for heavier nuclei and report the results in the future.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: The needed data is given in the text of article.]

Funded by SCOAP^3.

References

1. J.J. Aubert et al., Phys. Lett. B **123**, 275 (1983)
2. J.J. Aubert et al., Nucl. Phys. B **293**, 740 (1987)
3. J. Ashman et al., Phys. Lett. B **202**, 603 (1988)
4. J. Ashman et al., Z. Phys. C **57**, 211 (1993)
5. Y. Zhang, L. Shao, B.-Q. Ma, Nucl. Phys. A **828**, 390 (2009)
6. C. Gong, B.-Q. Ma, Phys. Rev. C **97**, 065207 (2018)
7. R.L. Jaffe, Phys. Rev. Lett. **50**, 228 (1983)
8. C.H.L. Smith, Phys. Lett. B **128**, 107 (1983)
9. M. Ericson, A.W. Thomas, Phys. Lett. B **128**, 112 (1983)
10. B.L. Friman, V.R. Pandharipande, R.B. Wiringa, Phys. Rev. Lett. **51**, 763 (1983)
11. P.J. Sutton, A.D. Martin, R.G. Roberts, W.J. Stirling, Phys. Rev. D **45**, 2349 (1992)
12. E.L. Berger, F. Coester, R.B. Wiringa, Phys. Rev. D **29**, 398 (1984)
13. T. Kawabe, Lett. Nuovo Cimento **44**, 159 (1985)
14. P. Hoodbhoy, R.L. Jaffe, Phys. Rev. D **35**, 113 (1987)
15. P. Hoodbhoy, Nucl. Phys. A **465**, 637 (1987)
16. H.J. Pirner, J.P. Vary, Phys. Rev. Lett. **46**, 1376 (1981)
17. C.E. Carlson, T.J. Havens, Phys. Rev. Lett. **51**, 261 (1983)
18. F.E. Close, R.L. Jaffe, R.G. Roberts, G.G. Ross, Phys. Rev. D **31**, 1004 (1985)
19. F.E. Close, R.G. Roberts, G.G. Ross, Phys. Lett. B **129**, 346 (1983)
20. R.L. Jaffe, F.E. Close, R.G. Roberts, G.G. Ross, Phys. Lett. B **134**, 449 (1984)
21. O. Nachtmann, H.J. Pirner, Z. Phys. C **21**, 277 (1984)
22. P.L. McGaughey, J.M. Moss, J.C. Peng, Ann. Rev. Nucl. Part. Sci. **49**, 217 (1999)
23. T. Chmaj, K.J. Heller, Acta Phys. Pol. B **15**, 473 (1984)
24. T. Chmaj, K.J. Heller, Lett. Nuovo Cimento **42**, 415 (1985)
25. J.D. de Deus, M. Pimenta, J. Varela, Z. Phys. C **26**, 109 (1984)
26. J.D. de Deus, M. Pimenta, J. Varela, Phys. Rev. D **30**, 697 (1984)
27. R.G. Arnold et al., Phys. Rev. Lett. **52**, 727 (1984)
28. J.J. Aubert et al., Nucl. Phys. B **272**, 158 (1986)
29. S. Dasu et al., Phys. Rev. Lett. **60**, 2591 (1988)
30. S. Dasu et al., Phys. Rev. D **49**, 5641 (1994)
31. J. Gomez et al., Phys. Rev. D **49**, 4348 (1994)
32. J. Seely et al., Phys. Rev. Lett. **103**, 202301 (2009)
33. H. Abramowicz et al., Z. Phys. C **25**, 29 (1984)
34. M.A. Parker et al., Nucl. Phys. B **232**, 1 (1984)
35. A.M. Cooper-Sarkar et al., Phys. Lett. B **141**, 133 (1984)
36. V. V. Ammosov et al., Pisma Zh. Eksp. Teor. Fiz. **39**, 327 (1984) [JETP Lett. **39**, 393 (1984)]
37. J. Hanlon et al., Phys. Rev. D **32**, 2441 (1985)
38. J. Guy et al., Z. Phys. C **36**, 337 (1987)
39. H. Nematollahi, M.M. Yazdanpanah, Phys. Rev. C **92**, 015209 (2015)
40. H. Nematollahi, M.M. Yazdanpanah, Eur. Phys. J. A **53**, 222 (2017)
41. J.P. Rarlston, D.E. Soper, Nucl. Phys. B **152**, 109 (1979)
42. D.W. Sivers, Phys. Rev. D **41**, 83 (1990)
43. A. Kotzinian, Nucl. Phys. B **441**, 234 (1995)
44. P.J. Mulders, R.D. Tangerman, Nucl. Phys. B **461**, 197 (1996)
45. D. Boer, P.J. Mulders, Phys. Rev. D **57**, 5780 (1998)
46. P.J. Mulders, J. Rodrigues, Phys. Rev. D **63**, 094021 (2001)
47. A.V. Belitsky, X. Ji, F. Yuan, Phys. Rev. D **69**, 074014 (2004)
48. A. Bacchetta et al., JHEP **02**, 093 (2007)
49. S. Noguera, S. Scopetta, JHEP **11**, 102 (2015)
50. A. Bacchetta, Eur. Phys. J. A **52**, 163 (2016)
51. H. Nematollahi, M.M. Yazdanpanah, J. Phys. G **44**, 075005 (2017)
52. M. Modarres, A. Hadian, Phys. Rev. D **98**, 076001 (2018)
53. M. Modarres, A. Hadian, Nucl. Phys. A **983**, 118 (2019)
54. M.A. Kimber, A.D. Martin, M.G. Ryskin, Phys. Rev. D **63**, 114027 (2001)
55. A.D. Martin, M.G. Ryskin, G. Watt, Eur. Phys. J. C **66**, 163 (2010)
56. H. Khanpour and S. Atashbar Tehrani , Phys. Rev. D **93**, 014026 (2016)
57. S. Atashbar Tehrani, Phys. Rev. C **86**, 064301 (2012)
58. K.J. Eskola, H. Paukkunen, C.A. Salgado, JHEP **07**, 102 (2008)
59. K.J. Eskola, H. Paukkunen, C.A. Salgado, JHEP **04**, 065 (2009)
60. K.J. Eskola, Nucl. Phys. A **910–911**, 163 (2013)
61. K.J. Eskola, P. Paakkinen, H. Paukkunen, C.A. Salgado, Eur. Phys. J. C **77**, 163 (2017)
62. M. Hirai, S. Kumano, M. Miyama, Phys. Rev. D **64**, 034003 (2001)
63. M. Hirai, S. Kumano, T.-H. Nagai, Phys. Rev. C **70**, 044905 (2004)
64. M. Hirai, S. Kumano, T.-H. Nagai, Phys. Rev. C **76**, 065207 (2007)
65. H. Paukkunen, Nucl. Phys. A **926**, 24 (2014)
66. P. Zavada, Phys. Rev. D **85**, 037501 (2012)

67. A.V. Efremov, P. Schweitzer, O.V. Teryaev, P. Zavada, Phys. Rev. D **83**, 054025 (2011)
68. P. Zavada, Phys. Rev. D **83**, 014022 (2011)
69. U. D'Alesio, E. Leader, F. Murgia, Phys. Rev. D **81**, 036010 (2010)
70. A. V. Efremov, P. Schweitzer, O. V. Teryaev, P. Zavada, Proc. Sci. DIS2010, 253 (2010). arXiv:1008.3827
71. A.V. Efremov, P. Schweitzer, O.V. Teryaev, P. Zavada, Phys. Rev. D **80**, 014021 (2009)
72. P. Zavada, Eur. Phys. J. C **52**, 121 (2007)
73. A.V. Efremov, O.V. Teryaev, P. Zavada, Phys. Rev. D **70**, 054018 (2004)
74. P. Zavada, Phys. Rev. D **67**, 014019 (2003)
75. P. Zavada, Phys. Rev. D **65**, 054040 (2002)
76. P. Zavada, Phys. Rev. D **55**, 4290 (1997)
77. D. de Florian, R. Sassot, Phys. Rev. D **69**, 074028 (2004)
78. D. de Florian, M. Stratmann, Phys. Rev. D **85**, 074028 (2012)
79. L. Frankfurt, M. Strikman, Phys. Rev. C **82**, 065203 (2010)
80. L. Frankfurt, M. Strikman, Int. J. Mod. Phys. E **21**, 1230002 (2012)
81. I. Schienbein et al., Phys. Rev. D **77**, 054013 (2008)
82. G. Piller, W. Weise, Phys. Rep. **330**, 1–94 (2000)
83. I.C. Cloët et al., J. Phys. G **46**, 093001 (2019)
84. P. Jimenez-Delgado, E. Reya, Phys. Rev. D **79**, 074023 (2009)
85. L.L. Frankfurt, M.I. Strikman, S. Liuti, Phys. Rev. Lett. **65**, 1725 (1990)
86. H. Avakian et al., Phys. Rev. D **81**, 074035 (2010)
87. S. Boffi et al., Phys. Rev. D **79**, 094012 (2009)
88. M. Wakamatsu, Phys. Rev. D **79**, 094028 (2009)
89. A. Bacchetta, F. Conti, M. Radici, Phys. Rev. D **78**, 074010 (2008)
90. L.P. Gamberg, G.R. Goldstein, M. Schlegel, Phys. Rev. D **77**, 094016 (2008)
91. C. Lorce, B. Pasquini, P. Schweitzer, JHEP **01**, 103 (2015)
92. P. Schweitzer, M. Strikman, C. Weiss, JHEP **01**, 163 (2013)
93. R. Wang, X. Chen, Q. Fu, Nucl. Phys. B **920**, 1 (2017)
94. K.A. Griffioen et al., Phys. Rev. C **92**, 015211 (2015)