Eur. Phys. J. C (2020) 80:140
https://doi.org/10.1140/epjc/s10052-020-7699-y

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Study on $\Xi_{cc} \to \Xi_c$ and $\Xi_{cc} \to \Xi_c'$ weak decays in the light-front quark model

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Received: 15 December 2019 / Accepted: 30 January 2020 / Published online: 17 February 2020

Abstract In this work we study the weak decays of $\Xi_{cc} \to \Xi_c$ and $\Xi_{cc} \to \Xi_c'$ in the light-front quark model. Generally, a naive, but reasonable conjecture suggests that the cc subsystem in Ξ_{cc} (us pair in $\Xi_c^{(\prime)}$) stands as a diquark with definite spin and color assignments. During the concerned processes, the diquark of the initial state is not a spectator, namely must be broken. A Racah transformation would decompose the original $(cc)q$ into a combination of $c(cq)$ components. Thus we may deal with the decaying c quark alone while keeping the (cq) subsystem as a spectator. With the re-arrangement of the inner structure we calculate the form factors numerically and then obtain the rates of semi-leptonic decays and non-leptonic decays, which will be measured in the future.

1 Introduction

In 2017, the LHCb collaboration observed a doubly charmed baryon Ξ_{cc}^{++} [1] with mass $3621.40\pm0.72\pm0.27\pm0.14$ MeV which indeed was long-expected by physicists of high energy physics. In terms of the constituent quark model there should exist 40 baryon-states of $J^P = 1/2$ and 35 $J^P = 3/2$ states which are composed of the five flavors u, d, s, c and b. Most of the light baryons and several heavy baryons with only one heavy quark (b or c) have been observed experimentally. Thus the doubly heavy baryons would be the goal of experimental search. One of such states Ξ_{cc}^{++} was reported by the SELEX [2,3] collaboration with a mass about 3520 MeV, however, it was not confirmed by other collaborations. Recently Ξ_{cc}^{++} has been measured by the LHCb collaboration in the four-body final state $\Lambda_c K^-\pi^+\pi^+$ [1] and later Ξ_{cc}^{++} was observed via the $\Xi^+\pi^+$ portal [4].

Theoretically, the structure about the series of Ξ_{cc} has not been fully investigated yet (in this paper we only consider Ξ_{cc}^{++} and the result can be generalized to Ξ_{cc}^{+}). For example, its life time and decay rates into several main channels are not well measured yet while on the theoretical aspect a reliable approach to study the decays of doubly heavy baryons still lacks. Therefore, all attempts to deeply investigate Ξ_{cc} from different angles should be valuable. By this study we may determine the inner structure of Ξ_{cc} and its decay behaviors by the light front quark model, consequently the results can be tested and the gained knowledge would be helpful for designing new experiments for searching other baryons with two heavy quarks.

Since the mass of Ξ_{cc} is smaller than the production threshold of Λ_c and D, it only decays via weak interaction. Apparently Ξ_{cc} should favorably decay into products involving a single-charmed baryon. In order to evaluate its decay rates we first need to know its inner structure. A naive and reasonable conjecture suggests that the subsystem of the two c quarks composes a diquark as a color source for the light quark [5,6].

In the most works about single charmed-baryons, the two light quarks can be regarded as a light diquark [7,8]. Thus while dealing with weak decays of such baryons, the diquarks with two light quarks can be safely regarded as spectators which do not undergo any changes during the transitions. The spectator scenario indeed greatly alleviates the theoretical difficulties.

On the contrary, in the decay process of Ξ_{cc}, one of the two c quarks in the initial diquark would transit into a lighter quark by emitting gauge bosons and the newly emerged quark s in the final state will be bound with the u quark to form a light diquark subsystem of color anti-triplet us via a recombination from $(cu)s$ into $(us)c$. Namely the picture is that the old diquark is broken and a new diquark emerges during the transition. Anyhow, the diquark [no matter the original (cc) or the final (us)] can no longer be treated as a specta-

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tor. Therefore the simple quark-diquark picture could not be directly applied to this decay process.

In this paper we will extend the light-front quark model to study the weak decays of Ξ_{cc} where three-body vertex function was obtained in our previous works. The light-front quark model (LFQM) is a relativistic quark model which has been applied to study transitions among mesons and the results agree with the data within reasonable error tolerance [9–25]. We also studied the weak decays of Λ_b and Σ_b in the heavy-quark-light-diquark picture of baryon [26–29] in this model and our results [26–29] are consistent with those given in literatures. Thus we have a certain confidence that the extension of the light-front quark model to baryon cases is also successful to the leading order at least [26–33].

In Ref. [30] we construct the three-body vertex function which is applied to the decay of heavy baryons. Now we try extending the approach to the concerned process. The transition process can be divided into two steps: first the old diquark of cc is broken and a subsystem of cu serves as a spectator during the transition, and then in the finally produced Ξ_c, the subsystem of cu would be broken again and a proper structure of $c(us)$ is reformed via the QCD interaction. As a matter of fact, it is easy to realize as we rearrange the (cc) diquark-(u) quark structure into a combination of the cu (diquark-like subsystem)-c (quark) structures by a Racah transformation. During the transition the (cu) diquark-like subsystem can be regarded as a spectator. Namely, one c quark transits into an s quark but the other c quark and the u quark are not touched approximately. Then for the second step we also need a Racah transformation to rearrange the $(cu)s$ structure into the $(us)c$ system.

In Refs. [31,32] the authors used the quark-diquark picture to explore the weak decays of doubly charmed baryon. It should be emphasized, the scenario in this paper where three individual quarks are concerned is different from the picture with one-diquark and one quark adopted by the authors of Refs. [31,32].

Indeed, since in Ξ_{cc} the u-quark has a relative momentum with respect to the diquark cc (the distance between two c quarks is small), thus after the recombination, in the subsystem uc, between the two constituents u and c, there exists a relative momentum. Therefore, rigorously speaking, the subsystem of uc is a diquark-like subsystem. In our work, we have to take into account the momenta carried by all the individual quarks which would undergo some changes during the transition. It is stressed again that in this work, we treat the combination involving one c quark and a u quark as a diquark-like effective subsystem. In other words, cc and us in Ξ_{cc} and $\Xi_c^{(\prime)}$ possess definite spin and color quantum numbers, so we can transform physical subsystem (diquarks) into other effective subsystems. However, since the subsystem cu is not a diquark, the inner degree of freedom could not be ignored.

This paper is organized as follows: after the introduction, in Sect. 2 we write up the transition amplitude for $\Xi_{cc} \to \Xi_c^{(\prime)}$ in the light-front quark model and give the resultant form factors, then we present our numerical results for $\Xi_{cc} \to \Xi_c^{(\prime)}$ along with all necessary input parameters in Sect. 3. Section 4 is devoted to our conclusion and discussions.

2 $\Xi_{cc} \to \Xi_c$ in the light-front quark model

2.1 The vertex functions of Ξ_{cc} and Ξ_c

In our previous works [26–29], we employed the quark–diquark picture to study the baryon transitions, where the diquark has definite spin and serves as a spectator approximately during the transition process. However in the process Ξ_{cc} and Ξ_c the picture is no longer valid. For a generally accepted consideration the two charm quarks in Ξ_{cc} compose a diquark which stands as a color source for the light u quark which is moving around the diquark with a certain relative momentum. The relative orbital angular momentum between the two c quarks is 0, i.e. the cc pair is in an S wave, due to the symmetry requirement the spin of the cc pair must be 1. In Ref. [8] the us-diquark in Ξ_c is a scalar diquark whereas in Ξ_c' it is a vector. In the decay process of Ξ_{cc} the cc diquark must be physically broken and one of the two charm quarks transits into an s quark via weak interaction and a light cu subsystem is formed which becomes a spectator for the decay process. That means neither the diquarks cc in the initial state, nor the us in the final state is spectator. To realize the transition, we mathematically reorder the quark structure of $(cc)u$ into a sum of $\sum_i c(cu)_i$ where the sum runs over all possible configurations (spin etc.) via a Racah transformation. Because of existence of relative momenta among the quarks, in this work we explore the baryon transition in the three-quark picture where the three quarks are individual subjects and possess their own momenta.

In analog to Refs. [30,34,35] the vertex functions of Ξ_{cc} and Ξ_c with total spin $S = 1/2$ and momentum P are

$$
\begin{aligned}
&|\Xi_{cc}(P,S,S_z)\rangle \\
&\quad = \int \{d^3\tilde{p}_1\}\{d^3\tilde{p}_2\}\{d^3\tilde{p}_3\}\, 2(2\pi)^3\delta^3(\tilde{P}-\tilde{p}_1-\tilde{p}_2-\tilde{p}_3) \\
&\qquad \times \sum_{\lambda_1,\lambda_2,\lambda_3} \Psi^{SS_z}(\tilde{p}_i,\lambda_i)\mathcal{C}^{\alpha\beta\gamma}\mathcal{F}_{ccu} \\
&\qquad |\, c_\alpha(p_1,\lambda_1)c_\beta(p_2,\lambda_2)u_\gamma(p_3,\lambda_3)\rangle, \qquad (1) \\
&|\Xi_c^{(\prime)}(P,S,S_z)\rangle \\
&\quad = \int \{d^3\tilde{p}_1\}\{d^3\tilde{p}_2\}\{d^3\tilde{p}_3\}\, 2(2\pi)^3\delta^3(\tilde{P}-\tilde{p}_1-\tilde{p}_2-\tilde{p}_3)
\end{aligned}
$$

$$\times \sum_{\lambda_1,\lambda_2,\lambda_3} \Psi^{(\prime)SS_z}(\tilde{p}_i,\lambda_i)\mathcal{C}^{\alpha\beta\gamma}\mathcal{F}_{csu}$$
$$| s_\alpha(p_1,\lambda_1)c_\beta(p_2,\lambda_2)u_\gamma(p_3,\lambda_3)\rangle. \tag{2}$$

As the spectator approximation cannot be directly applied, dealing with the process seems more complicated. In fact the c quark which does not undergo a transition and the u quark play the same role in the transition of $\Xi_{cc} \to \Xi_c$, i.e. they are approximately a spectator and their combination can be regarded as an effective subsystem. Actually the cc and us are physical subsystems for Ξ_{cc} and Ξ_c respectively since they possess definite spin-color quantum number. Baryon is a three-body-system, the total spin can be obtained by different coupling steps and Racah coefficient can transform one to others. By the aforementioned rearrangement of quark flavors the physical states $(cc)u$ and $c(us)$ are written into sums over effective forms $c(cu)$ and $s(cu)$ for Ξ_{cc} and Ξ_c respectively. The detailed transformations are [31]

$$[c^1c^2]_1[u] = \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{3}}{2}[c^2][c^1u]_0 + \frac{1}{2}[c^2][c^1u]_1 - \frac{\sqrt{3}}{2}[c^1][c^2u]_0 + \frac{1}{2}[c^1][c^2u]_1\right) \tag{3}$$

$$[su]_0[c] = -\frac{1}{2}[s][cu]_0 + \frac{\sqrt{3}}{2}[s][cu]_1 \tag{4}$$

$$[su]_1[c] = \frac{\sqrt{3}}{2}[s][cu]_0 + \frac{1}{2}[s][cu]_1 \tag{5}$$

and then

$$\Psi^{SS_z}_{ccu}(\tilde{p}_i,\lambda_i) = \sqrt{2}\left[-\frac{\sqrt{3}}{2}\Psi^{SS_z}_0(\tilde{p}_i,\lambda_i) + \frac{1}{2}\Psi^{SS_z}_1(\tilde{p}_i,\lambda_i)\right],$$
$$\Psi^{SS_z}_{csu}(\tilde{p}_i,\lambda_i) = -\frac{1}{2}\Psi^{SS_z}_0(\tilde{p}_i,\lambda_i) + \frac{\sqrt{3}}{2}\Psi^{SS_z}_1(\tilde{p}_i,\lambda_i),$$
$$\Psi'^{SS_z}_{csu}(\tilde{p}_i,\lambda_i) = \frac{\sqrt{3}}{2}\Psi^{SS_z}_0(\tilde{p}_i,\lambda_i) + \frac{1}{2}\Psi^{SS_z}_1(\tilde{p}_i,\lambda_i),$$

with [36]

$$\Psi^{SS_z}_0(\tilde{p}_i,\lambda_i) = A_0\bar{u}(p_3,\lambda_3) \times \left[\left(\bar{\not{P}} + M_0\right)\gamma_5\right]v(p_2,\lambda_2)\bar{u}(p_1,\lambda_1)u(\bar{P},S)\varphi(x_i,k_{i\perp}), \tag{6}$$

$$\Psi^{SS_z}_1(\tilde{p}_i,\lambda_i) = A_1\bar{u}(p_3,\lambda_3) \times \left[\left(\bar{\not{P}} + M_0\right)\gamma_{\perp\alpha}\right]v(p_2,\lambda_2)\bar{u}(p_1,\lambda_1) \gamma_{\perp\alpha}\gamma_5 u(\bar{P},S)\varphi(x_i,k_{i\perp}), \tag{7}$$

where p_1 is the momentum of the c quark which participates in the transition, p_2 , p_3 are the momenta of the spectator quarks c and u, and $\lambda_1,\lambda_2,\lambda_3$ are the helicities of the constituents.

Under the normalization of the state $|\Xi_{cc}\rangle$ (or $|\Xi^{(\prime)}_c\rangle$) ,

$$\langle \Xi_{cc}(P',S',S'_z)|\Xi_{cc}(P,S,S_z)\rangle = 2(2\pi)^3P^+\delta^3(\tilde{P}'-\tilde{P})\delta_{S'S}\delta_{S'_zS_z}. \tag{8}$$

and

$$\int\left(\prod_{i=1}^{3}\frac{dx_id^2k_{i\perp}}{2(2\pi)^3}\right)2(2\pi)^3\delta\left(1-\sum x_i\right)\delta^2\left(\sum k_{i\perp}\right)\varphi^*(x_i,k_{i\perp})\varphi(x_i,k_{i\perp}) = 1. \tag{9}$$

With a simple manipulation, one can obtain [30]

$$A_0 = \frac{1}{4\sqrt{P^+(M_0m_1+p_1\cdot\bar{P})(m_2m_3M_0^2+m_3M_0p_2\cdot\bar{P}+m_2M_0p_3\cdot\bar{P}+p_2\cdot\bar{P}p_3\cdot\bar{P})}} = \frac{1}{4\sqrt{P^+M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)}}. \tag{10}$$

$$A_1 = \frac{1}{4\sqrt{3P^+(M_0m_1+p_1\cdot\bar{P})(M_0m_2+p_2\cdot\bar{P})(M_0m_3+p_3\cdot\bar{P})}} = \frac{1}{4\sqrt{3P^+M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)}}. \tag{11}$$

The spatial wave function is

$$\varphi(x_i,k_{i\perp}) = \frac{e_1e_2e_3}{x_1x_2x_3M_0}\varphi\left(\overrightarrow{k}_1,\beta_1\right)\varphi\left(\frac{\overrightarrow{k}_2-\overrightarrow{k}_3}{2},\beta_{23}\right) \tag{12}$$

with $\varphi(\overrightarrow{k},\beta) = 4\left(\frac{\pi}{\beta^2}\right)^{3/4}\exp\left(\frac{-k_z^2-k_\perp^2}{2\beta^2}\right)$.

2.2 Calculating the form factors of $\Xi_{cc} \to \Xi_c$ and $\Xi_{cc} \to \Xi'_c$ in LFQM

The leading order Feynman diagram responsible for the weak decay $\Xi_{cc} \to \Xi_c$ is shown in Fig. 1. Following the procedures given in Refs. [26,28,34,35] the transition matrix element can be computed with the vertex functions

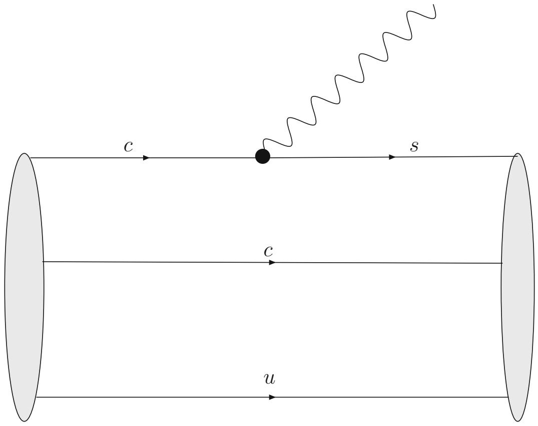

Fig. 1 The Feynman diagram for $\Xi_{cc} \to \Xi_c$ transitions, where • denotes $V - A$ current vertex

of $|\ \Xi_{cc}(P, S, S_z)\rangle$ and $|\ \Xi_c^{(}P', S', S'_z)\rangle$. The cu subsystem stands as a spectator, i.e. its spin configuration does not change during the transition, so the transition matrix element can be divided into two parts:

$$\begin{aligned}&\langle \Xi_c(P', S'_z) \mid \bar{s}\gamma^\mu(1-\gamma_5)c \mid \Xi_{cc}(P, S_z)\rangle \\ &= \frac{\sqrt{6}}{4}\langle \Xi_c(P', S'_z) \mid \bar{s}\gamma^\mu(1-\gamma_5)c \mid \Xi_{cc}(P, S_z)\rangle_0 \\ &\quad + \frac{\sqrt{6}}{4}\langle \Xi_c(P', S'_z) \mid \bar{s}\gamma^\mu(1-\gamma_5)c \mid \Xi_{cc}(P, S_z)\rangle_1 \quad (13)\end{aligned}$$

with

$$\begin{aligned}&\langle \Xi_c(P', S'_z) \mid \bar{s}\gamma^\mu(1-\gamma_5)c \mid \Xi_{cc}(P, S_z)\rangle_0 \\ &= \int \frac{\{d^3\tilde{p}_2\}\{d^3\tilde{p}_3\}\phi^*_{\Xi_c}(x', k'_\perp)\phi_{\Xi_{cc}}(x, k_\perp) Tr[(\bar{\not{P}}' - M'_0)\gamma_5(\not{p}_2 + m_2)(\bar{\not{P}} + M_0)\gamma_5(\not{p}_3 - m_3)]}{16\sqrt{p_1^+ p_1'^+ \bar{P}^+ \bar{P}'^+ M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)(m'_1+e'_1)(m'_2+e'_2)(m'_3+e'_3)}} \\ &\quad \times \bar{u}(\bar{P}', S'_z)(\not{p}'_1 + m'_1)\gamma^\mu(1-\gamma_5)(\not{p}_1 + m_1)u(\bar{P}, S_z), \quad (14)\end{aligned}$$

and

$$\begin{aligned}&\langle \Xi_c(P', S'_z) \mid \bar{s}\gamma^\mu(1-\gamma_5)c \mid \Xi_{cc}(P, S_z)\rangle_1 \\ &= \frac{\int\{d^3\tilde{p}_2\}\{d^3\tilde{p}_3\}\phi^*_{\Xi_c}(x', k'_\perp)\phi_{\Xi_{cc}}(x, k_\perp) Tr[\gamma_\perp^\alpha(\bar{\not{P}}' + M'_0)\gamma_5(\not{p}_2 + m_2)(\bar{\not{P}} + M_0)\gamma_5\gamma_\perp^\beta(\not{p}_3 - m_3)]}{48\sqrt{p_1^+ p_1'^+ \bar{P}^+ \bar{P}'^+ M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)(m'_1+e'_1)(m'_2+e'_2)(m'_3+e'_3)}} \\ &\quad \times \bar{u}(\bar{P}', S'_z)\gamma_{\perp\alpha}\gamma_5(\not{p}'_1 + m'_1)\gamma^\mu(1-\gamma_5)(\not{p}_1 + m_1)\gamma_{\perp\beta}\gamma_5 u(\bar{P}, S_z), \quad (15)\end{aligned}$$

where

$$\begin{aligned}&m_1 = m_c, \quad m'_1 = m_s, \qquad m_2 = m_c, \quad m_3 = m_u, \\ &\gamma_{\perp\alpha} = \gamma_\alpha - \not{P}'P'_\alpha/M'^2, \quad \gamma_{\perp\beta} = \gamma_\beta - \not{P}P_\beta/M^2 \quad (16)\end{aligned}$$

with P (P') is the four-momentum of Ξ_{cc} (Ξ_c) and M (M') is the mass of Ξ_{cc} (Ξ_c). Setting $\tilde{p}_1 = \tilde{p}'_1 + \tilde{q}$, $\tilde{p}_2 = \tilde{p}'_2$ and $\tilde{p}_3 = \tilde{p}'_3$ we have

$$\begin{aligned}x'_{1,2,3} &= x_{1,2,3}, \quad k'_{1\perp} = k_{1\perp} - (1-x_1)q_\perp, \quad k'_{2\perp} \\ &= k_{2\perp} + x_2 q_\perp, \quad k'_{3\perp} = k_{3\perp} + x_3 q_\perp. \quad (17)\end{aligned}$$

The form factors for the weak transition $\Xi_{cc} \to \Xi_c$ are defined in the standard way as

$$\begin{aligned}&\langle \Xi_c(P', S', S'_z) \mid \bar{s}\gamma_\mu(1-\gamma_5)c \mid \Xi_{cc}(P, S, S_z)\rangle \\ &= \bar{u}_{\Xi_c}(P', S'_z)\left[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M_{\Xi_{cc}}} f_2(q^2)\right. \\ &\quad \left. + \frac{q_\mu}{M_{\Xi_{cc}}} f_3(q^2)\right] u_{\Xi_{cc}}(P, S_z) \\ &\quad - \bar{u}_{\Xi_c}(P', S'_z)\left[\gamma_\mu g_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M_{\Xi_c}} g_2(q^2)\right. \\ &\quad \left. + \frac{q_\mu}{M_{\Xi_{cc}}} g_3(q^2)\right]\gamma_5 u_{\Xi_{cc}}(P, S_z). \quad (18)\end{aligned}$$

where $q \equiv P - P'$. For $\langle \Xi_c(P', S', S'_z) \mid \bar{s}\gamma_\mu(1-\gamma_5)c \mid \Xi_{cc}(P, S, S_z)\rangle_0$ and $\langle \Xi_c(P', S', S'_z) \mid \bar{s}\gamma_\mu(1-\gamma_5)c \mid \Xi_{cc}(P, S, S_z)\rangle_1$ the form factors are denoted to f_i^s, g_i^s and f_i^v, g_i^v, so we have

$$\begin{aligned}f_1 &= \frac{\sqrt{6}}{4}f_1^s + \frac{\sqrt{6}}{4}f_1^v, \quad g_1 = \frac{\sqrt{6}}{4}g_1^s + \frac{\sqrt{6}}{4}g_1^v, \\ f_2 &= \frac{\sqrt{6}}{4}f_2^s + \frac{\sqrt{6}}{4}f_2^v, \quad g_2 = \frac{\sqrt{6}}{4}g_2^s + \frac{\sqrt{6}}{4}g_2^v. \quad (19)\end{aligned}$$

In our earlier paper [30] f_i^s, g_i^s, f_i^v and g_i^v are presented as

$$\begin{aligned}f_1^s &= \int \frac{dx_2 d^2k_{2\perp}^2}{2(2\pi)^3}\frac{dx_3 d^2k_{3\perp}^2}{2(2\pi)^3} \\ &\quad \times \frac{\mathrm{Tr}\left[(\bar{\not{P}}' - M'_0)\gamma_5(\not{p}_2 + m_2)(\bar{\not{P}} + M_0)\gamma_5(\not{p}_3 - m_3)\right]}{\sqrt{M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)(m'_1+e'_1)(m'_2+e'_2)(m'_3+e'_3)}}\end{aligned}$$

$$
\begin{aligned}
&\times\frac{\phi^*_{\Xi_c}(x',k'_\perp)\phi_{\Xi_{cc}}(x,k_\perp)}{16\sqrt{x_1x'_1}}\\
&\times\frac{\mathrm{Tr}\left[(\bar{\not{P}}+M_0)\gamma^+(\bar{\not{P}}'+M'_0)(\not{p}'_1+m'_1)\gamma^+(\not{p}_1+m_1)\right]}{8P^+P'^+},\\
\frac{f_2^s}{M_{\Xi_{cc}}}&=\frac{-i}{q^i_\perp}\int\frac{dx_2d^2k_{2\perp}^2}{2(2\pi)^3}\frac{dx_3d^2k_{3\perp}^2}{2(2\pi)^3}\\
&\times\frac{\mathrm{Tr}\left[(\bar{\not{P}}'-M'_0)\gamma_5(\not{p}_2+m_2)(\bar{\not{P}}+M_0)\gamma_5(\not{p}_3-m_3)\right]}{\sqrt{M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)(m'_1+e'_1)(m'_2+e'_2)(m'_3+e'_3)}}\\
&\times\frac{\phi^*_{\Xi_c}(x',k'_\perp)\phi_{\Xi_{cc}}(x,k_\perp)}{16\sqrt{x_1x'_1}}\\
&\times\frac{\mathrm{Tr}\left[(\bar{\not{P}}+M_0)\sigma^{i+}(\bar{\not{P}}'+M'_0)(\not{p}'_1+m'_1)\gamma^+(\not{p}_1+m_1)\right]}{8P^+P'^+},\\
g_1^s&=\int\frac{dx_2d^2k_{2\perp}^2}{2(2\pi)^3}\frac{dx_3d^2k_{3\perp}^2}{2(2\pi)^3}\\
&\times\frac{\mathrm{Tr}\left[(\bar{\not{P}}'-M'_0)\gamma_5(\not{p}_2+m_2)(\bar{\not{P}}+M_0)\gamma_5(\not{p}_3-m_3)\right]}{\sqrt{M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)(m'_1+e'_1)(m'_2+e'_2)(m'_3+e'_3)}}\\
&\times\frac{\phi^*_{\Xi_c}(x',k'_\perp)\phi_{\Xi_{cc}}(x,k_\perp)}{16\sqrt{x_1x'_1}}\\
&\times\frac{\mathrm{Tr}\left[(\bar{\not{P}}+M_0)\gamma^+\gamma_5(\bar{\not{P}}'+M'_0)(\not{p}'_1+m'_1)\gamma^+\gamma_5(\not{p}_1+m_1)\right]}{8P^+P'^+},\\
\frac{g_2^s}{M_{\Xi_{cc}}}&=\frac{i}{q^i_\perp}\int\frac{dx_2d^2k_{2\perp}^2}{2(2\pi)^3}\frac{dx_3d^2k_{3\perp}^2}{2(2\pi)^3}\\
&\times\frac{\mathrm{Tr}\left[(\bar{\not{P}}'-M'_0)\gamma_5(\not{p}_2+m_2)(\bar{\not{P}}+M_0)\gamma_5(\not{p}_3-m_3)\right]}{\sqrt{M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)(m'_1+e'_1)(m'_2+e'_2)(m'_3+e'_3)}}\\
&\times\frac{\phi^*_{\Xi_c}(x',k'_\perp)\phi_{\Xi_{cc}}(x,k_\perp)}{16\sqrt{x_1x'_1}}\\
&\times\frac{\mathrm{Tr}\left[(\bar{\not{P}}+M_0)\sigma^{i+}\gamma_5(\bar{\not{P}}'+M'_0)(\not{p}'_1+m'_1)\gamma^+\gamma_5(\not{p}_1+m_1)\right]}{8P^+P'^+},\\
f_1^v&=\int\frac{dx_2d^2k_{2\perp}^2}{2(2\pi)^3}\frac{dx_3d^2k_{3\perp}^2}{2(2\pi)^3}\\
&\times\frac{\mathrm{Tr}\left[\gamma_\perp^\alpha(\bar{\not{P}}'+M'_0)\gamma_5(\not{p}_2+m_2)(\bar{\not{P}}+M_0)\gamma_5\gamma_\perp^\beta(\not{p}_3-m_3)\right]}{\sqrt{M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)(m'_1+e'_1)(m'_2+e'_2)(m'_3+e'_3)}}\\
&\times\frac{\phi^*_{\Xi_c}(x',k'_\perp)\phi_{\Xi_{cc}}(x,k_\perp)}{48\sqrt{x_1x'_1}}\\
&\times\frac{\mathrm{Tr}\left[(\bar{\not{P}}+M_0)\gamma^+(\bar{\not{P}}'+M'_0)\gamma_{\perp\alpha}\gamma_5(\not{p}'_1+m'_1)\gamma^+(\not{p}_1+m_1)\gamma_{\perp\beta}\gamma_5\right]}{8P^+P'^+},\\
\frac{f_2^v}{M_{\Xi_{cc}}}&=\frac{-i}{q^i_\perp}\int\frac{dx_2d^2k_{2\perp}^2}{2(2\pi)^3}\frac{dx_3d^2k_{3\perp}^2}{2(2\pi)^3}\\
&\times\frac{\mathrm{Tr}\left[\gamma_\perp^\alpha(\bar{\not{P}}'+M'_0)\gamma_5(\not{p}_2+m_2)(\bar{\not{P}}+M_0)\gamma_5\gamma_\perp^\beta(\not{p}_3-m_3)\right]}{\sqrt{M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)(m'_1+e'_1)(m'_2+e'_2)(m'_3+e'_3)}}\\
&\times\frac{\phi^*_{\Xi_c}(x',k'_\perp)\phi_{\Xi_{cc}}(x,k_\perp)}{48\sqrt{x_1x'_1}}\\
&\frac{\mathrm{Tr}\left[(\bar{\not{P}}-M_0)\sigma^{i+}(\bar{\not{P}}'-M'_0)\gamma_{\perp\alpha}\gamma_5(\not{p}'_1+m'_1)\gamma^+(\not{p}_1+m_1)\gamma_{\perp\beta}\gamma_5\right]}{8P^+P'^+},\\
g_1^v&=\int\frac{dx_2d^2k_{2\perp}^2}{2(2\pi)^3}\frac{dx_3d^2k_{3\perp}^2}{2(2\pi)^3}\\
&\times\frac{\mathrm{Tr}\left[\gamma_\perp^\alpha(\bar{\not{P}}'+M'_0)\gamma_5(\not{p}_2+m_2)(\bar{\not{P}}+M_0)\gamma_5\gamma_\perp^\beta(\not{p}_3-m_3)\right]}{\sqrt{M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)(m'_1+e'_1)(m'_2+e'_2)(m'_3+e'_3)}}\\
&\times\frac{\phi^*_{\Xi_c}(x',k'_\perp)\phi_{\Xi_{cc}}(x,k_\perp)}{48\sqrt{x_1x'_1}}\\
&\times\frac{\mathrm{Tr}\left[(\bar{\not{P}}-M_0)\gamma^+\gamma_5(\bar{\not{P}}'-M'_0)\gamma_{\perp\alpha}\gamma_5(\not{p}'_1+m'_1)\gamma^+(\not{p}_1+m_1)\gamma_{\perp\beta}\gamma_5\right]}{8P^+P'^+},\\
\frac{g_2^v}{M_{\Xi_{cc}}}&=\frac{i}{q^i_\perp}\int\frac{dx_2d^2k_{2\perp}^2}{2(2\pi)^3}\frac{dx_3d^2k_{3\perp}^2}{2(2\pi)^3}\\
&\times\frac{\mathrm{Tr}\left[\gamma_\perp^\alpha(\bar{\not{P}}'+M'_0)\gamma_5(\not{p}_2+m_2)(\bar{\not{P}}+M_0)\gamma_5\gamma_\perp^\beta(\not{p}_3-m_3)\right]}{\sqrt{M_0^3(m_1+e_1)(m_2+e_2)(m_3+e_3)(m'_1+e'_1)(m'_2+e'_2)(m'_3+e'_3)}}\\
&\times\frac{\phi^*_{\Xi_c}(x',k'_\perp)\phi_{\Xi_{cc}}(x,k_\perp)}{48\sqrt{x_1x'_1}}\\
&\times\frac{\mathrm{Tr}\left[(\bar{\not{P}}-M_0)\sigma^{i+}\gamma_5(\bar{\not{P}}'-M'_0)\gamma_{\perp\alpha}\gamma_5(\not{p}'_1+m'_1)\gamma^+(\not{p}_1+m_1)\gamma_{\perp\beta}\gamma_5\right]}{8P^+P'^+}.
\end{aligned}
\tag{20}
$$

For the transition $\langle\Xi'_c(P',S',S'_z)\mid\bar{s}\gamma_\mu(1-\gamma_5)c\mid\Xi_{cc}(P,S,S_z)\rangle$ the form factors are also defined as in Eq. (18). Here we just add " $'$ " on f_1, f_2, g_1 and g_2 in order to distinguish the quantities for $\Xi_{cc}\to\Xi_c$ and those for $\Xi_{cc}\to\Xi'_c$. They are

$$
\begin{aligned}
f'_1&=-\frac{3\sqrt{2}}{4}f_1^s+\frac{\sqrt{2}}{4}f_1^v,\; g'_1=-\frac{3\sqrt{2}}{4}g_1^s+\frac{\sqrt{2}}{4}g_1^v,\\
f'_2&=-\frac{3\sqrt{2}}{4}f_2^s+\frac{\sqrt{2}}{4}f_2^v,\; g'_2=-\frac{3\sqrt{2}}{4}g_2^s+\frac{\sqrt{2}}{4}g_2^v.
\end{aligned}
\tag{21}
$$

In the calculation one also needs to use $\phi^*_{\Xi'_c}(x',k'_\perp)$ to replace $\phi^*_{\Xi_c}(x',k'_\perp)$.

3 Numerical results

3.1 The form factors for $\Xi_{cc}\to\Xi_c$ and $\Xi_{cc}\to\Xi'_c$

Before we start to evaluate those form factors numerically the parameters in the concerned model are needed to be determined. The masses of quarks given in Ref. [37] are collected in Table 1. The masses of Ξ_c and Ξ'_c are taken from [38]. Indeed, we know very little about the parameters β_1 and β_{23} in the wave function of the initial baryon and β'_1 and β'_{23} in that of the final baryon. Generally the reciprocal of β is related to the electrical radius of two constituents. Since the strong interaction between q and $q^{(')}$ is a half of that between $q\bar{q}^{(')}$, for a Coulomb-like potential one can expect the electrical radius of $qq^{(')}$ to be $1/\sqrt{2}$ times that of $q\bar{q}^{(')}$ i.e. $\beta_{qq^{(')}}\approx\sqrt{2}\beta_{q\bar{q}^{(')}}$. In Ref. [39] considering the binding energy the authors obtained the same results. In our early paper for a compact $qq^{(')}$ system we find $\beta_{qq^{(')}}=2.9\beta_{q\bar{q}^{(')}}$ i.e the electrical radius of $qq^{(')}$ to be $1/2.9$ times that of $q\bar{q}^{(')}$. In terms of the knowledge the estimate is $\beta_{c[cu]}\approx 2.9\beta_{c\bar{c}}$, $\beta_{s[cu]}\approx\sqrt{2}\beta_{c\bar{s}}$, $\beta_{[cu]}\approx\sqrt{2}\beta_{c\bar{u}}$ where $\beta_{c\bar{c}}$, $\beta_{c\bar{u}}$ and $\beta_{c\bar{s}}$ were obtained for the meson case [37]. With these parameters we calculate the form factors and make theoretical predictions on the transition rates.

Since these form factors $f_i^{s(v)}$ $(i=1,2)$ and $g_i^{s(v)}$ $(i=1,2)$ are evaluated in the frame $q^+=0$ i.e. $q^2=-q_\perp^2\leq 0$ (the

Table 1 The quark mass and the parameter β (in units of GeV)

m_c	m_s	m_u	$\beta_{c[cu]}$	$\beta_{s[cu]}$	$\beta_{[cu]}$
1.5	0.5	0.25	1.898	0.760	0.656

Table 2 The form factors given in the three-parameter form

F	$F(0)$	a	b
f_1^s	0.586	0.640	−0.194
f_2^s	−0.484	1.23	−0.222
g_1^s	0.420	−0.0142	0.0748
g_2^s	0.228	1.02	−0.101
f_1^v	0.610	1.18	−0.0492
f_2^v	0.463	1.32	−0.0642
g_1^v	−0.140	−0.501	0.274
g_2^v	0.0673	0.00936	0.327

space-like region) one needs to extrapolate them into the time-like region. In Ref. [35] a three-parameter form was employed

$$F(q^2) = \frac{F(0)}{\left(1 - \frac{q^2}{M^2_{\Xi_{cc}}}\right)\left[1 - a\left(\frac{q^2}{M^2_{\Xi_{cc}}}\right) + b\left(\frac{q^2}{M^2_{\Xi_{cc}}}\right)^2\right]}, \tag{22}$$

where $F(q^2)$ denotes the form factors $f_i^{s(v)}$ and $g_i^{s(v)}$. Using the form factors in the space-like region we may calculate numerically the parameters a, b and $F(0)$ in the un-physical region; namely fix $F(q^2 \leq 0)$. As discussed in previous section, these form factors are extended into the physical region with $q^2 \geq 0$ through Eq. (22). The fitted values of a, b and $F(0)$ in the form factors f_1, f_1, g_1 and g_2 are presented in Table 2. The dependence of the form factors on q^2 is depicted in Fig. 2.

Table 3 The form factors given in polynomial form

F	$F(0)$	a'	b'	c'
f_1^s	0.586	1.57	1.59	0.704
f_2^s	−0.484	2.06	2.42	1.17
g_1^s	0.420	0.983	0.692	0.258
g_2^s	0.228	1.90	2.07	0.960
f_1^v	0.610	2.04	2.27	1.06
f_2^v	0.463	2.14	2.49	1.19
g_1^v	−0.140	0.422	0.0931	0.00632
g_2^v	0.0673	0.925	0.245	−0.0862

Since the form factors f_1^s, f_1^s, f_1^v and f_2^v rise too quickly after $q^2 > 6$ GeV which are very different from the results in other $\frac{1}{2} \to \frac{1}{2}$ transitions [7,8,26,30,35], we suggest to use a polynomial to parameterize these form factors

$$F(q^2) = F(0)\left[1 + a'\left(\frac{q^2}{M^2_{\Xi_{cc}}}\right) + b'\left(\frac{q^2}{M^2_{\Xi_{cc}}}\right)^2 + c'\left(\frac{q^2}{M^2_{\Xi_{cc}}}\right)^3\right]. \tag{23}$$

The fitted values of a', b', c' and $F(0)$ in the form factors are presented in Table 3. The dependence of the form factors on q^2 is depicted in Fig. 3. The figures of these form factors are apparently more smooth which are in analog to those for $\frac{1}{2} \to \frac{1}{2}$ transition [7,8,26,30,35]. With the form factors which are polynomials of powers of $\frac{q^2}{M^2_{\Xi_{cc}}}$, we estimate the decay rates of semi-leptonic and no-leptonic decays.

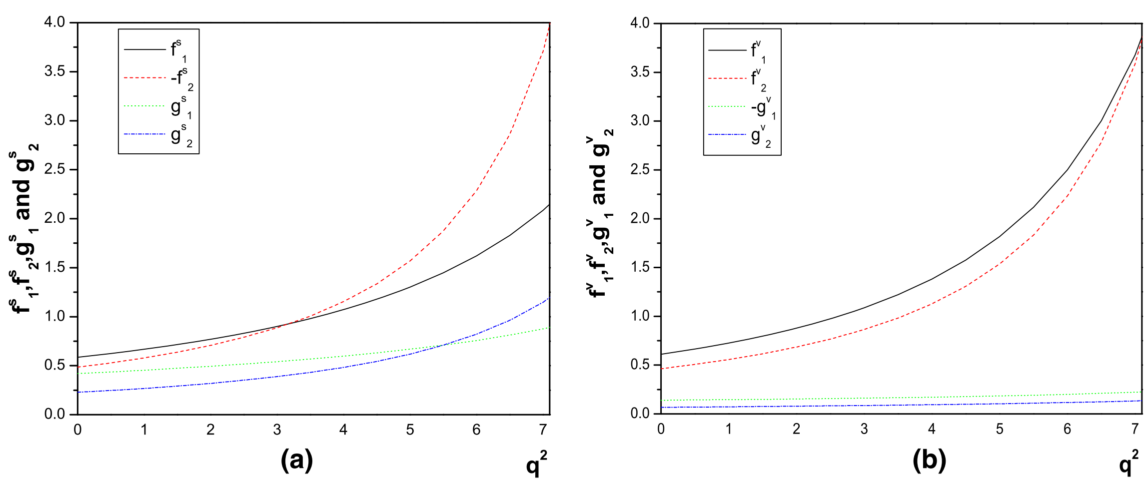

Fig. 2 The dependence of form factors f_1^s, f_2^s, g_1^s and g_2^s in a three-parameter form on q^2 (**a**) and the dependence of the form factors f_1^v, f_2^v, g_1^v and g_2^v on q^2 (**b**)

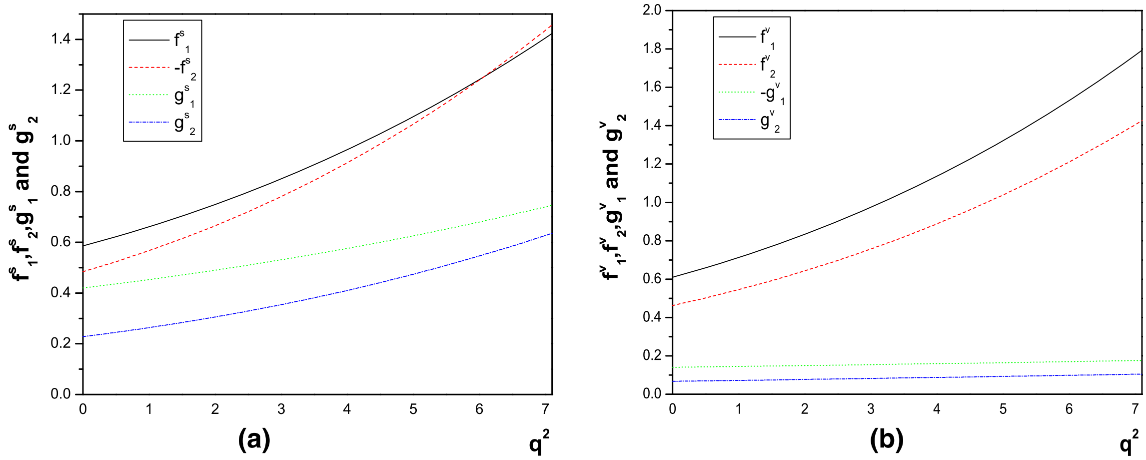

Fig. 3 The dependence of form factors f_1^s, f_2^s, g_1^s and g_2^s in polynomial form on q^2 (**a**) and The dependence of form factors f_1^v, f_2^v, g_1^v and g_2^v on q^2 (**b**)

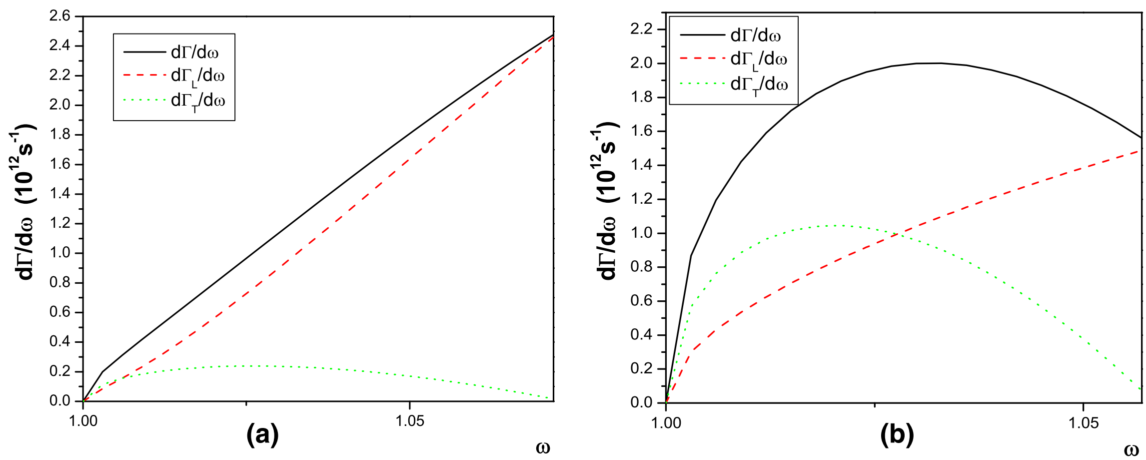

Fig. 4 Differential decay rates $d\Gamma/d\omega$ for the decay $\Xi_{cc} \to \Xi_c l \bar{\nu}_l$ (**a**) and $\Xi'_{cc} \to \Xi_c l \bar{\nu}_l$ (**b**)

Table 4 The width (in unit 10^{12} s^{-1}) of $\Xi_{cc} \to \Xi_c l \bar{\nu}_l$ (left) and $\Xi_{cc} \to \Xi'_c l \bar{\nu}_l$ (right)

	Γ	R	Γ	R
This work	0.100 ± 0.015	7.14 ± 0.61	0.0995 ± 0.0091	1.34 ± 0.07
Ref. [31]	0.173	9.99	0.193	1.42
Ref. [40]	0.092 ± 0.014	22 ± 8	0.032 ± 0.006	1.1 ± 0.2
Ref. [41]	0.106	–	0.147	–

3.2 Semi-leptonic decays of $\Xi_{cc} \to \Xi_c + l\bar{\nu}_l$ and $\Xi_{cc} \to \Xi'_c + l\bar{\nu}_l$

Using the form factors obtained in last subsection, we evaluate the rate of $\Xi_{cc} \to \Xi_c l \bar{\nu}_l$ and $\Xi_{cc} \to \Xi'_c l \bar{\nu}_l$. The differential decay widths $d\Gamma/d\omega$ ($\omega = \frac{P \cdot P'}{MM'}$) are depicted in Fig. 4. Our predictions on the total decay widths and the ratio of the longitudinal to transverse decay rates R are all listed in Table 4. Deliberately letting the quark masses and all βs fluctuate up to 5%, one can estimate possible theoretical uncertainties of the numerical results.

In Ref. [31] the authors employ three-parameter parametrization scheme to fix these form factors and their predictions on $\Gamma(\Xi_{cc} \to \Xi_c l \bar{\nu}_l)$ and $\Gamma(\Xi_{cc} \to \Xi'_c) l \bar{\nu}_l$ are almost twice larger than our results presented in Table 4. Our estimate on the ratio of longitudinal to transverse decay rates R for $\Xi_{cc} \to \Xi'_c l \bar{\nu}_l$ is close to 1.42 given in [31] but that of $\Xi_{cc} \to \Xi_c l \bar{\nu}_l$ slightly deviates from theirs. One also notices that the predictions on $\Gamma(\Xi_{cc} \to \Xi_c l \bar{\nu}_l)$ are close to each

Table 5 Our predictions on Widths (in unit $10^{10}\ \mathrm{s}^{-1}$) and up-down asymmetry of non-leptonic decays $\Xi_{cc} \to \Xi_c^{(\prime)} M$

Mode	Width	Up-down asymmetry	Mode	Width	Up-down asymmetry
$\Xi_{cc} \to \Xi_c \pi$	13.6 ± 1.8	-0.441 ± 0.009	$\Xi_{cc} \to \Xi_c' \pi$	7.68 ± 0.92	-0.982 ± 0.005
$\Xi_{cc} \to \Xi_c \rho$	11.0 ± 1.5	-0.429 ± 0.016	$\Xi_{cc} \to \Xi_c' \rho$	13.9 ± 1.2	-0.111 ± 0.034
$\Xi_{cc} \to \Xi_c K$	1.03 ± 0.14	-0.402 ± 0.008	$\Xi_{cc} \to \Xi_c' K$	0.492 ± 0.059	-0.998 ± 0.002
$\Xi_{cc} \to \Xi_c K^*$	0.414 ± 0.055	-0.422 ± 0.021	$\Xi_{cc} \to \Xi_c' K^*$	0.623 ± 0.052	-0.014 ± 0.030

Table 6 Widths (in unit $10^{10}\ \mathrm{s}^{-1}$) of non-leptonic decays $\Xi_{cc} \to \Xi_c^{(\prime)} M$ in references

Mode	[31]	[40]	Mode	[31]	[40]	[41]
$\Xi_{cc} \to \Xi_c \pi$	23.9	12.0 ± 1.7	$\Xi_{cc} \to \Xi_c' \pi$	16.7	3.64 ± 0.76	11.9
$\Xi_{cc} \to \Xi_c \rho$	46.0	24.3 ± 3.1	$\Xi_{cc} \to \Xi_c' \rho$	62.6	9.72 ± 1.98	62.9
$\Xi_{cc} \to \Xi_c K$	1.99	0.972 ± 0.152	$\Xi_{cc} \to \Xi_c' K$	1.14	0.334 ± 0.061	–
$\Xi_{cc} \to \Xi_c K^*$	1.81	0.972 ± 0.152	$\Xi_{cc} \to \Xi_c' K^*$	2.84	0.349 ± 0.05	–

others in different approaches (not include the prediction in [31]) but those on $\Gamma(\Xi_{cc} \to \Xi_c' l \bar{\nu}_l)$ deviate from each others a little bit.

3.3 Non-leptonic decays of $\Xi_{cc} \to \Xi_c + M$ and $\Xi_{cc} \to \Xi_c' + M$

On the theoretical aspect, calculating the concerned quantities of the non-leptonic decays seems to be more complicated than for semi-leptonic processes. Our theoretical framework is based on the factorization assumption, namely the hadronic transition matrix element is factorized into a product of two independent hadronic matrix elements of currents,

$$\begin{aligned} &\langle \Xi_c^{(\prime)}(P', S_z')M \mid \mathcal{H} \mid \Xi_{cc}(P, S_z)\rangle \\ &= \frac{G_F V_{cs} V_{qq'}^*}{\sqrt{2}} \langle M \mid \bar{q}'\gamma^\mu(1-\gamma_5)q \mid 0\rangle \\ &\quad \times \left\langle \Xi_c^{(\prime)}(P', S_z') \mid \bar{s}\gamma^\mu(1-\gamma_5)\, c \mid \Xi_{cc}(P, S_z)\right\rangle, \end{aligned} \tag{24}$$

where the term $\langle M \mid \bar{q}'\gamma^\mu(1-\gamma_5)q \mid 0\rangle$ is determined by a decay constant and the transition $\Xi_{cc} \to \Xi_c^{(\prime)}$ is evaluated in the previous sections. Since the decay $\Xi_{cc} \to \Xi_c^{(\prime)} + M$ is the so-called color-favored portal, the factorization should be a plausible approximation. The results on these non-leptonic decays can be checked in the coming measurements and the validity degree of the obtained form factors in the doubly charmed baryon would be further examined.

From the results shown in Table 5, we find $\Xi_{cc} \to \Xi_c^{(\prime)}\pi$ and $\Xi_{cc} \to \Xi_c^{(\prime)}\rho$ are the main two-body decay channels for Ξ_{cc}. Especially $\Gamma(\Xi_{cc} \to \Xi_c' \rho)$ is close to $\Gamma(\Xi_{cc} \to \Xi_c \pi)$ which should also be observed in LHCb soon. The predictions in other approaches are listed in Table 6. The theoretical predictions on the widths calculated in Ref. [31] are two or three times larger than ours. The results on $\Gamma(\Xi_{cc} \to \Xi_c' \pi)$ and $\Gamma(\Xi_{cc} \to \Xi_c' K)$ in Ref. [40] are close to ours but there exists still a discrepancy for other channels. This should also be tested in the future more precise measurements.

4 Conclusions and discussions

In this paper we calculate the transition rate of $\Xi_{cc} \to \Xi_c^{(\prime)}$ in the light front quark model. For the baryons Ξ_{cc} and $\Xi_c^{(\prime)}$ we employ a three-quark picture instead of the quark-diquark one for calculating the transition rates. Generally, two charm quarks constitute a diquark which joins the light quark to constitute the baryon and this widely accepted scenario determines the wave function of Ξ_{cc}. Because two c quarks are identical heavy flavor particles in a color anti-triplet, it must be a vector boson, whereas, in $\Xi_c^{(\prime)}$ the light us pair is seen as a diquark. In the concerned process, the diquark in the initial state is different from that in the final state, so that the cc diquark is no longer a spectator and the diquark picture cannot be directly applied in this case. However in the process the charm quark which does not undergo a transition and the u quark are approximatively spectators when higher order QCD effects are neglected, so the cu pair can be regarded as an effective subsystem. Baryon is a three-body-system whose total spin can be obtained through different schemes just in analog to the L–S coupling and J–J coupling in the quantum mechanics. Making a Racah transformation we can convert one configuration into another. The Racah coefficients of such transformation determines the correlation between the two configuration $(cc)u$ and $c(cu)$. However, one is noted that the subsystem of (cu) is not a diquark in a rigorous meaning and between the two constituents there exists a relative momentum. Thus in the vertex function of the three-body system there exists an inner degree of freedom for the *cu* subsystem which just manifests by the relative momentum.

We calculate the form factors for the transitions $\Xi_{cc} \to \Xi_c$ and $\Xi_{cc} \to \Xi_c^{'}$ in the space-like region. When we extrapolate them to physical region we find three-parameter form is not a good choice, so that we suggest to parameterize these form factors in terms of polynomials. Using these form factors we calculate the rates of semileptonic decays $\Xi_{cc} \to \Xi_c l \nu_l$ and $\Xi_{cc} \to \Xi_c^{'} l \nu_l$. We find that $\Gamma(\Xi_{cc} \to \Xi_c l \bar{\nu}_l)$ is very close to $\Gamma(\Xi_{cc} \to \Xi_c^{\prime} l \bar{\nu}_l)$ but our results on $\Gamma(\Xi_{cc} \to \Xi_c l \bar{\nu}_l)$ and $\Gamma(\Xi_{cc} \to \Xi_c^{\prime} l \bar{\nu}_l)$ are about a half of those in Ref. [31]. The ratio of the longitudinal to transverse decay rates R for $\Xi_{cc} \to \Xi_c l \bar{\nu}_l$ and $\Xi_{cc} \to \Xi_c^{\prime} l \bar{\nu}_l$ are roughly consistent with the predictions of [31] . With the same theoretical framework, we also evaluate the rates of several non-leptonic decays. Our numerical results indicate that the channel $\Xi_{cc} \to \Xi_c \pi$ has the largest branching ratio for the transition $\Xi_{cc} \to \Xi_c$; instead, the channel $\Xi_{cc} \to \Xi_c^{\prime} \rho$ is the main channel for the transition $\Xi_{cc} \to \Xi_c^{\prime}$. The predictions of Ref. [31] are two or three times larger than ours since in the two approaches the different pictures about the inner structure are adopted. We urge the experimentalists to make more accurate measurements on the channels, and the data would tell us which approach is closer to the reality. Definitely, the theoretical studies on the double-heavy baryons are helpful for getting a better understanding about the quark model and the non-perturbative QCD effects. Especially, the scenarios adopted for investigating the double-charm baryons can be generalized to study the baryons with *bb* and *bc* components which will be measured by the LHCb collaboration and other collaborations in the near future.

Acknowledgements This work is supported by the National Natural Science Foundation of China (NNSFC) under the contract Nos. 11375128, 11675082, 11735010 and 11975165. We thank Prof. Wei Wang for helpful discussions.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: Since our manuscript is a theoretical paper all results are included in it.]

Funded by SCOAP[3].

Appendix A: Semi-leptonic decays of $\mathcal{B}_1 \to \mathcal{B}_2 l \bar{\nu}_l$

The helicity amplitudes are related to the form factors for $\mathcal{B}_1 \to \mathcal{B}_2 l \bar{\nu}_l$ through the following expressions [42–44]

$$\begin{aligned}
H^V_{\frac{1}{2},0} &= \frac{\sqrt{Q_-}}{\sqrt{q^2}}\left(\left(M_{\mathcal{B}_1}+M_{\mathcal{B}_2}\right) f_1 - \frac{q^2}{M_{\mathcal{B}_1}} f_2\right),\\
H^V_{\frac{1}{2},1} &= \sqrt{2Q_-}\left(-f_1 + \frac{M_{\mathcal{B}_1}+M_{\mathcal{B}_2}}{M_{\mathcal{B}_1}} f_2\right),\\
H^A_{\frac{1}{2},0} &= \frac{\sqrt{Q_+}}{\sqrt{q^2}}\left(\left(M_{\mathcal{B}_1}-M_{\mathcal{B}_2}\right) g_1 + \frac{q^2}{M_{\mathcal{B}_1}} g_2\right),\\
H^A_{\frac{1}{2},1} &= \sqrt{2Q_+}\left(-g_1 - \frac{M_{\mathcal{B}_1}-M_{\mathcal{B}_2}}{M_{\mathcal{B}_1}} g_2\right).
\end{aligned} \tag{A1}$$

where $Q_\pm = 2(P\cdot P' \pm M_{\mathcal{B}_1} M_{\mathcal{B}_2})$ and $M_{\mathcal{B}_1}$ ($M_{\mathcal{B}_2}$) represents $M_{\Xi_{cc}}$ (M_{Ξ_c}). The amplitudes for the negative helicities are obtained in terms of the relation

$$H^{V,A}_{-\lambda',-\lambda_W} = \pm H^{V,A}_{\lambda',\lambda_W}, \tag{A2}$$

where the upper (lower) index corresponds to V(A). The helicity amplitudes are

$$H_{\lambda',\lambda_W} = H^V_{\lambda',\lambda_W} - H^A_{\lambda',\lambda_W}. \tag{A3}$$

The helicities of the W-boson λ_W can be either 0 or 1, which correspond to the longitudinal and transverse polarizations, respectively. The longitudinally (L) and transversely (T) polarized rates are respectively[42–44]

$$\begin{aligned}
\frac{d\Gamma_L}{d\omega} &= \frac{G_F^2 |V_{cb}|^2}{(2\pi)^3}\frac{q^2\, p_c\, M_{\mathcal{B}_2}}{12 M_{\mathcal{B}_1}}\left[|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2\right],\\
\frac{d\Gamma_T}{d\omega} &= \frac{G_F^2 |V_{cb}|^2}{(2\pi)^3}\frac{q^2\, p_c\, M_{\mathcal{B}_2}}{12 M_{\mathcal{B}_1}}\left[|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2\right].
\end{aligned} \tag{A4}$$

where p_c is the momentum of $\mathcal{B}_2$ in the reset frame of $\mathcal{B}_1$.

The ratio of the longitudinal to transverse decay rates R is defined by

$$R = \frac{\Gamma_L}{\Gamma_T} = \frac{\int_1^{\omega_{\max}} d\omega\, q^2\, p_c \left[|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2\right]}{\int_1^{\omega_{\max}} d\omega\, q^2\, p_c \left[|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2\right]}. \tag{A5}$$

Appendix B: $\mathcal{B}_1 \to \mathcal{B}_2 M$

In general, the transition amplitude of $\mathcal{B}_1 \to \mathcal{B}_2 M$ can be written as

$$\begin{aligned}
\mathcal{M}(\mathcal{B}_1 \to \mathcal{B}_2 P) &= \bar{u}_{\mathcal{B}_2}(A + B\gamma_5) u_{\mathcal{B}_1},\\
\mathcal{M}(\mathcal{B}_1 \to \mathcal{B}_2 V) &= \bar{u}_{\mathcal{B}_2}\epsilon^{*\mu}\left[A_1\gamma_\mu\gamma_5 + A_2(p_c)_\mu\gamma_5\right.\\
&\quad \left.+B_1\gamma_\mu + B_2(p_c)_\mu\right] u_{\mathcal{B}_1},
\end{aligned} \tag{B1}$$

where ϵ^μ is the polarization vector of the final vector or axial-vector mesons. Including the effective Wilson coefficient $a_1 = c_1 + c_2/N_c$, the decay amplitudes in the factorization approximation are [7,45]

$$
\begin{aligned}
A &= \lambda f_P (M_{\mathcal{B}_1} - M_{\mathcal{B}_2}) f_1(M^2),\\
B &= \lambda f_P (M_{\mathcal{B}_1} + M_{\mathcal{B}_2}) g_1(M^2),\\
A_1 &= -\lambda f_V M \left[g_1(M^2) + g_2(M^2)\frac{M_{\mathcal{B}_1} - M_{\mathcal{B}_2}}{M_{\mathcal{B}_1}} \right],\\
A_2 &= -2\lambda f_V M \frac{g_2(M^2)}{M_{\mathcal{B}_1}},\\
B_1 &= \lambda f_V M \left[f_1(M^2) - f_2(M^2)\frac{M_{\mathcal{B}_1} + M_{\mathcal{B}_2}}{M_{\mathcal{B}_1}} \right],\\
B_2 &= 2\lambda f_V M \frac{f_2(M^2)}{M_{\mathcal{B}_1}},
\end{aligned}
\tag{B2}
$$

where $\lambda = \frac{G_F}{\sqrt{2}} V_{cs} V^*_{q_1 q_2} a_1$ and M is the meson mass. Replacing P, V by S and A in the above expressions, one can easily obtain similar expressions for scalar and axial-vector mesons.

The decay rates of $\mathcal{B}_1 \to \mathcal{B}_2 P(S)$ and up-down asymmetries are [45]

$$
\begin{aligned}
\Gamma &= \frac{p_c}{8\pi} \left[\frac{(M_{\mathcal{B}_1} + M_{\mathcal{B}_2})^2 - M^2}{M^2_{\mathcal{B}_1}} |A|^2 \right.\\
&\quad \left. + \frac{(M_{\mathcal{B}_1} - M_{\mathcal{B}_2})^2 - m^2}{M^2_{\mathcal{B}_1}} |B|^2 \right],\\
\alpha &= -\frac{2\kappa \mathrm{Re}(A^* B)}{|A|^2 + \kappa^2 |B|^2},
\end{aligned}
\tag{B3}
$$

where p_c is the $\mathcal{B}_2$ momentum in the rest frame of $\mathcal{B}_1$ and m is the mass of pseudoscalar (scalar). For $\mathcal{B}_1 \to \mathcal{B}_2 V(A)$ decays, the decay rate and up-down asymmetries are

$$
\begin{aligned}
\Gamma &= \frac{p_c (E_{\mathcal{B}_2} + M_{\mathcal{B}_2})}{4\pi M_{\mathcal{B}_1}} \left[2\left(|S|^2 + |P_2|^2 \right) \right.\\
&\quad \left. + \frac{\varepsilon^2}{m^2} \left(|S + D|^2 + |P_1|^2 \right) \right],\\
\alpha &= \frac{4m^2 \mathrm{Re}(S^* P_2) + 2\varepsilon^2 \mathrm{Re}(S + D)^* P_1}{2m^2 \left(|S|^2 + |P_2|^2 \right) + \varepsilon^2 \left(|S + D|^2 + |P_1|^2 \right)},
\end{aligned}
\tag{B4}
$$

where ε (m) is energy (mass) of the vector (axial vector) meson, and

$$
\begin{aligned}
S &= -A_1,\\
P_1 &= -\frac{p_c}{\varepsilon} \left(\frac{M_{\mathcal{B}_1} + M_{\mathcal{B}_2}}{E_{\mathcal{B}_2} + M_{\mathcal{B}_2}} B_1 + M_{\mathcal{B}_1} B_2 \right),\\
P_2 &= \frac{p_c}{E_{\mathcal{B}_2} + M_{\mathcal{B}_2}} B_1,\\
D &= -\frac{p_c^2}{\varepsilon (E_{\mathcal{B}_2} + M_{\mathcal{B}_2})} (A_1 - M_{\mathcal{B}_1} A_2).
\end{aligned}
\tag{B5}
$$

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