FCNC transitions of Λ_b to neutron in Bethe–Salpeter equation approach

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Abstract In a covariant quark–diquark model, we investigate the rare decay of $\Lambda_b \rightarrow nl^+l^-$ and $\Lambda_b \rightarrow n\gamma$ in the Bethe–Salpeter equation approach. In this model the baryons are treated as bound states of a constituent quark and a diquark interacting via a gluon exchange and the linear confinement. We find that the ratio of form factors R is varies from -0.90 to -0.25 and the branching ratios $Br(\Lambda_b \rightarrow nl^+l^-) \times 10^8$ are $6.79 (l = e)$, $4.08 (l = \mu)$, $2.9 (l = \tau)$ and the branching ratio $Br(\Lambda_b \rightarrow n\gamma) \times 10^7 = 3.69$ in central values of parameters.

1 Introduction

The flavor changing neutral current (FCNC) decays of b -quark such as $b \rightarrow s\gamma(l^+l^-)$ can provide constrains on new physics, give essential information about the quark structure of heavy baryons and give more model-independent information such as CKM matrix elements. Significant experimental progresses about rare decays of the Λ_b baryon have been achieved at LHCb [1–5]. The rare decay $\Lambda_b \rightarrow \Lambda\mu^+\mu^-$ first observed by CDF collaboration in 2011 [6]. The radiative decay $\Lambda_b \rightarrow \Lambda\gamma$ was observed at LHCb in 2019 [2]. There have been also many theoretical works on the rare decays Λ_b induced by $b \rightarrow s$ transition [7–31]. Ref. [32] gave the branching ratios $Br(\Lambda_b \rightarrow nl^+l^-) \times 10^8 = 3.19 \pm 0.46 (l = e)$, $3.76 \pm 0.42 (l = \mu)$, $1.65 \pm 0.19 (l = \tau)$ in the context of light cone QCD sum rules (LCSR). The form factors (FFs) of $\Lambda_b(\Lambda_b^*) \rightarrow Nl^+l^-$ were given in Ref. [34] in LCSR and taking into account the contribution of Λ_b^* the branching ratios $Br(\Lambda_b \rightarrow Nl^+l^-) \times 10^8 = 8 \pm 2 (l = e)$, $7 \pm 2 (l = \mu)$, $2 \pm 0.4 (l = \tau)$ were obtained. Ref. [35] gave the branching ratios $Br(\Lambda_b \rightarrow n\mu^+\mu^-) \times 10^8 = 3.75 \pm 0.38$

and $Br(\Lambda_b \rightarrow n\gamma) \times 10^7 = 3.7$ in the relativistic quark–diquark picture in the QCD-motivated interquark potential model. Ref. [36] studied the $\Lambda_b \rightarrow N^*l^+l^-$ decay in LCSR and gave the branching ratios $Br(\Lambda_b \rightarrow N^*l^+l^-) \times 10^8 = 4.62 \pm 1.85 (l = e)$, $4.25 \pm 1.5 (l = \mu)$, and $0.25 \pm 0.09 (l = \tau)$. Ref. [33] gave analysed CP-violation in polarized $b \rightarrow dl^+l^-$. With the experiment development, the transition $\Lambda_b \rightarrow n$ will be detected in the near future, so it is necessary to study $\Lambda_b \rightarrow n$ theoretically.

In this work, we will calculate the FFs of $\Lambda_b \rightarrow n$ in the Bethe–Salpeter equation approach in a covariant quark–diquark model. This model has been used to study nucleon electromagnetic form factors and N - Δ transition form factors [42–44]. In the previous works, heavy baryon properties have been studied extensively in this model [45–50, 52, 53]. The possible existence of diquark within baryons has been studied for a long time [37, 38, 41]. The negative neutron mean square charge radius can be explained by diquark model, which cannot be explained in pure SU(6) quark model [38].

In our model, Λ_b can be regarded as a bound state of two particles: one is a heavy quark b and the other is a scalar diquark (ud). Using the SU(6) wave function of baryons, we can get the neutron wave function in the quark–diquark model [39, 40].

$$n^\uparrow = \frac{1}{3\sqrt{3}}[3d^\uparrow(du)_{00} + d^\uparrow(du)_{10} - \sqrt{2}d^\downarrow(du)_{11} - \sqrt{2}u^\uparrow(dd)_{10} + 2u^\downarrow(dd)_{11}],$$

where the arrow $\uparrow$ ($\downarrow$) indicates the spin direction. Therefore, in our model only the $d^\uparrow(ud)_{00}/\sqrt{3}$ component of the neutron contributes to $\Lambda_b \rightarrow n$ since Λ_b has the structure $b(ud)_{00}$.

This paper is organized as follows. In Sect. 2, we will establish the BS equation for $q(ud)_{00}$ system ($q = b, d$). In Sect. 3 we will derive the FFs for $\Lambda_b \rightarrow n$ in the BS equation approach. In Sect. 4 the numerical results for the decay FFs

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of $\Lambda_b \rightarrow nl^+l^-$ will be given. Finally, the summary and discussion will be given in Sect. 5.

2 BS equation for $Q(ud)_{00}$ system

Following our previous work as is shown in Fig. 1, the BS equation of the $q(ud)_{00}$ system in momentum space satisfies the following homogeneous integral equation [45–50, 52]:

$$\chi_P(p) = iS_F(p_1) \int \frac{d^4p}{(2\pi)^4} [I \otimes IV_1(p, q) + \gamma_\mu \otimes (p_2 + q_2)^\mu V_2(p, q)] \chi_P(q) S_D(p_2), \quad (1)$$

where $S_F(p_1)$ and $S_D(p_2)$ are propagators of the q quark and the (ud) scalar diquark, respectively, $p_1 = \lambda_1 P + p$ and $p_2 = \lambda_2 P - p$ correspond to the momenta of the quark and the diquark, respectively. P is the momentum of the baryon. V_1 and V_2 are the scalar confinement and one-gluon-exchange terms in the kernel, respectively. Generally, the $q(ud)_{00}$ system needs two scalar functions to describe its BS wave function [45, 46, 49].

$$\chi_P(p) = (f_1(p_t^2) + \not{p}_t f_2(p_t^2)) u(P), \quad (2)$$

where f_i ($i = 1, 2$) are the Lorentz-scalar functions of p_t^2 , $u(P)$ is the spinor of the baryons, p_t is the transverse projection of the relative momenta along the momentum P , $p_t^\mu = p^\mu - (v \cdot p)v^\mu$, and $p_l = \lambda_2 M - v \cdot p$ (where we defined $v^\mu = P^\mu/M$). We use M , m , and m_D to represent the masses of the baryons, the q -quark and the (ud) diquark, respectively.

According to the potential model, V_1 and V_2 have the following forms in the covariant instantaneous approximation ($p_l = q_l$) [47, 48, 52, 53]:

$$\begin{aligned} \tilde{V}_1(p_t - q_t) &= \frac{8\pi\kappa}{[(p_t - q_t)^2 + \mu^2]^2} \\ &- (2\pi)^2 \delta^3(p_t - q_t) \int \frac{d^3k}{(2\pi)^3} \frac{8\pi\kappa}{(k^2 + \mu^2)^2}, \end{aligned} \quad (3)$$

where q_t is the transverse projection of the relative momenta along the momentum P and defined as $q_t^\mu = q^\mu - (v \cdot q)v^\mu$, $q_l = \lambda_2 M - v \cdot q$. The second term of $\tilde{V}_1$ is introduced to avoid infrared divergence at the point $p_t = q_t$, and μ is a small parameter to avoid the divergence in numerical calculations.

$$\tilde{V}_2(p_t - q_t) = -\frac{16\pi}{3} \frac{\alpha_{eff}^2 Q_0^2}{[(p_t - q_t)^2 + \mu^2][(p_t - q_t)^2 + Q_0^2]}, \quad (4)$$

It was found that $Q_0^2 = 3.2 \text{ GeV}^2$ can lead to consistent results with the experimental data by analyzing the electromagnetic FFs of the proton [7]. The parameters κ and

α_{eff} are related to the scalar confinement and the one-gluon-exchange diagram, respectively.

The quark and diquark propagators can be written as the follows:

$$S_F(p_1) = i\not{p} \left[\frac{\Lambda_q^+}{M - p_l - \omega_q + i\epsilon} + \frac{\Lambda_q^-}{M - p_l + \omega - i\epsilon} \right], \quad (5)$$

$$S_D(p_2) = \frac{i}{2\omega_D} \left[\frac{1}{p_l - \omega_D + i\epsilon} - \frac{1}{p_l + \omega_D - i\epsilon} \right], \quad (6)$$

where $\omega_q = \sqrt{m^2 - p_t^2}$ and $\omega_D = \sqrt{m_D^2 - p_t^2}$. $\Lambda^\pm$ are the projection operators which have the following relations:

$$2\omega_q \Lambda_q^\pm = \omega_q \pm \not{p}_t + m, \quad (7)$$

$$\Lambda_q^\pm \Lambda_q^\pm = \Lambda_q^\pm, \quad (8)$$

$$\Lambda_q^\pm \Lambda_q^\mp = 0. \quad (9)$$

Following our previous work, in order more precisely calculate the FFs of $\Lambda_b \rightarrow n$, we can take $E_0 = -0.14 \text{ GeV}$ (where $E_0 = M - m - m_D$ is the binding energy) and κ to be about $0.05 \pm 0.005 \text{ GeV}^3$ for $\Lambda_b \rightarrow n$ [51]. Defining $\tilde{f}_{1(2)} = \int \frac{d^3p_l}{2\pi} f_{1(2)}$, and using the covariant instantaneous approximation, $p_l = q_l$, the scalar BS wave functions satisfy the following coupled integral equation:

$$\begin{aligned} \tilde{f}_1(p_t) &= \int \frac{d^3q_t}{(2\pi)^3} M_{11}(p_t, q_t) \tilde{f}_1(q_t) \\ &+ M_{12}(p_t, q_t) \tilde{f}_2(q_t), \end{aligned} \quad (10)$$

$$\begin{aligned} \tilde{f}_2(p_t) &= \int \frac{d^3q_t}{(2\pi)^3} M_{21}(p_t, q_t) \tilde{f}_1(q_t) \\ &+ M_{22}(p_t, q_t) \tilde{f}_2(q_t), \end{aligned} \quad (11)$$

where

$$\begin{aligned} M_{11}(p_t, q_t) &= \frac{(\omega_q + m)(\tilde{V}_1 + 2\omega_D \tilde{V}_2) - p_t \cdot (p_t + q_t) \tilde{V}_2}{4\omega_D \omega_q (-M + \omega_D + \omega_q)} \\ &- \frac{(\omega_q - m)(\tilde{V}_1 - 2\omega_D \tilde{V}_2) + p_t \cdot (p_t + q_t) \tilde{V}_2}{4\omega_D \omega_c (M + \omega_D + \omega_q)}, \end{aligned} \quad (12)$$

$$\begin{aligned} M_{12}(p_t, q_t) &= \frac{-(\omega_q + m)(q_t + p_t) \cdot q_t \tilde{V}_2 + p_t \cdot q_t (\tilde{V}_1 - 2\omega_D \tilde{V}_2)}{4\omega_D \omega_c (-M + \omega_D + \omega_c)} \\ &- \frac{(m - \omega_q)(q_t + p_t) \cdot q_t \tilde{V}_2 - p_t \cdot q_t (\tilde{V}_1 + 2\omega_D \tilde{V}_2)}{4\omega_D \omega_q (M + \omega_D + \omega_q)}, \end{aligned} \quad (13)$$

$$\begin{aligned} M_{21}(p_t, q_t) &= \frac{(\tilde{V}_1 + 2\omega_D \tilde{V}_2) - (-\omega_q + m) \frac{(p_t + q_t) \cdot p_t}{p_t^2} \tilde{V}_2}{4\omega_D \omega_q (-M + \omega_D + \omega_q)} \\ &- \frac{(\tilde{V}_1 - 2\omega_D \tilde{V}_2) + (\omega_q + m) \frac{(p_t + q_t) \cdot p_t}{p_t^2} \tilde{V}_2}{4\omega_D \omega_q (M + \omega_D + \omega_q)}, \end{aligned} \quad (14)$$

$$M_{22}(p_t, q_t)$$

Fig. 1 The BS equation for the $q(ud)_{00}$ ($q = b, d$) system in momentum space (K is the interaction kernel)

$$= \frac{(m - \omega_q)(\tilde{V}_1 + 2\omega_D \tilde{V}_2) \frac{p_t \cdot q_t}{p_t^2} - (q_t^2 + p_t \cdot q_t) \tilde{V}_2}{4\omega_D \omega_q (-M + \omega_D + \omega_q)} - \frac{(m + \omega_q)(-\tilde{V}_1 - 2\omega_D \tilde{V}_2) \frac{p_t \cdot q_t}{p_t^2} + (q_t^2 + p_t \cdot q_t) \tilde{V}_2}{4\omega_D \omega_q (M + \omega_D + \omega_q)}. \quad (15)$$

When $\frac{1}{m} \rightarrow 0$ [47], the quark propagator can be written as following,

$$S_F(p_1) = i \frac{1 + \not{p}}{2(E_0 + m_D - p_l + i\epsilon)}, \quad (16)$$

considering the Dirac equation for Λ_b we have

$$\phi(p) = - \frac{i}{(E_0 + m_D - p_l + i\epsilon)(p_l^2 - \omega_D^2)} \times \int \frac{d^4 q}{(2\pi)^4} (\tilde{V}_1 + 2p_l \tilde{V}_2) \phi(q), \quad (17)$$

where the BS wave function of Λ_b was given in the previous work [47] and has the form $\chi_P(v) = \phi(p)u_{\Lambda_b}(v, s)$ with $\phi(p)$ being the scalar BS wave function.

Generally, the BS wave function can be normalized under the condition of the covariant instantaneous approximation [53]:

$$i\delta_{j_1 j_2}^{i_1 i_2} \int \frac{d^4 q d^4 p}{(2\pi)^8} \bar{\chi}_P(p, s) \left[\frac{\partial}{\partial P_0} I_p(p, q)^{i_1 i_2 j_2 j_1} \right] \chi_P(q, s') = \delta_{ss'}, \quad (18)$$

where $i_{1(2)}$ and $j_{1(2)}$ represent the color indices of the quark and the diquark, respectively, $s^{(l)}$ is the spin index of the baryon, $I_p(p, q)^{i_1 i_2 j_2 j_1}$ is the inverse of the four-point propagator written as follows

$$I_p(p, q)^{i_1 i_2 j_2 j_1} = \delta^{i_1 j_1} \delta^{i_2 j_2} (2\pi)^4 \delta^4(p - q) S_F^{-1} \times (p_1) S_D^{-1}(p_2). \quad (19)$$

3 Matrix element of $\Lambda_b \rightarrow nl^+l^-$ and $\Lambda_b \rightarrow n\gamma$ decays

In the standard model, the $\Lambda_b \rightarrow nl^+l^-$ transition is described by $b \rightarrow dl^+l^-$ at the quark level. The effective Hamiltonian describing the electroweak penguin and weak

box diagrams related to this transition is given by

$$\mathcal{H}(b \rightarrow dl^+l^-) = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{td}^* \left[C_9^{eff} \bar{d} \gamma_\mu (1 - \gamma_5) b \bar{l} \gamma^\mu l - i C_7^{eff} \bar{d} \frac{2m_b \sigma_{\mu\nu} q^\mu}{q^2} (1 + \gamma_5) b \bar{l} \gamma^\mu l + C_{10} \bar{d} \gamma_\mu (1 - \gamma_5) b \bar{l} \gamma^\mu \gamma_5 l \right], \quad (20)$$

where G_F is the Fermi coupling constant, α is the fine structure constant at Z mass scale, ϵ^ν is the polarization vector of photon, respectively. q is the total momentum of the lepton pair and C_i^{eff} ($i = 7, 9, 10$) are the Wilson coefficients, $C_7^{eff} = -0.313$, $C_9^{eff} = 4.334$, $C_{10} = -4.669$ [56–58]. The amplitude is obtained by sandwiching the effective Hamiltonian between the initial and final states. The matrix element for $\Lambda_b \rightarrow n$ can be parameterized in terms of the FFs as the following:

$$\begin{aligned} \langle n(P') | \bar{d} \gamma_\mu b | \Lambda_b(P) \rangle &= \bar{u}_n(P') \\ &\times (g_1 \gamma^\mu + i g_2 \sigma_{\mu\nu} p^\nu + g_3 p_\mu) u_{\Lambda_b}(P), \\ \langle n(P') | \bar{d} \gamma_\mu \gamma_5 b | \Lambda_b(P) \rangle &= \bar{u}_n(P') \\ &\times (t_1 \gamma^\mu + i t_2 \sigma_{\mu\nu} p^\nu + t_3 p^\mu) \gamma_5 u_{\Lambda_b}(P), \\ \langle n(P') | \bar{d} i \sigma^{\mu\nu} q^\nu b | \Lambda_b(P) \rangle &= \bar{u}_n(P') \\ &\times (s_1 \gamma^\mu + i s_2 \sigma_{\mu\nu} q^\nu + s_3 q^\mu) u_{\Lambda_b}(P), \\ \langle n(P') | \bar{d} i \sigma^{\mu\nu} \gamma_5 q^\nu b | \Lambda_b(P) \rangle &= \bar{u}_n(P') \\ &\times (d_1 \gamma^\mu + i d_2 \sigma_{\mu\nu} q^\nu + d_3 q^\mu) \gamma_5 u_{\Lambda_b}(P), \end{aligned} \quad (21)$$

where P' and P are the momenta of the neutron and Λ_b respectively, $q = P - P'$, u_n and u_{Λ_b} are the spinors of the initial and final baryons respectively, g_i , t_i , s_i , and d_i ($i = 1, 2$ and 3) are the transition FFs which are Lorentz scalar functions of q^2 . When working in the limit $m_b \rightarrow \infty$, the number of independent FFs is reduced to 2. The $\Lambda_b \rightarrow n$ matrix element with an arbitrary matrix Γ is given by

$$\langle n(P') | \bar{d} \Gamma b | \Lambda_b(v) \rangle = \bar{u}_n(P') (F_1(\omega) + F_2(\omega) \not{v}) \Gamma u_{\Lambda_b}(v), \quad (22)$$

where $\Gamma = \gamma_\mu, \gamma_\mu \gamma_5, q^\nu \sigma_{\nu\mu}, q^\nu \sigma_{\nu\mu} \gamma_5$. F_1 and F_2 can be expressed as functions solely of $\omega = v \cdot P'/m_\Lambda$, which is the energy of the neutron in the Λ_b rest frame. The baryons

states can be normalized as follows,

$$\langle n(P') | n(P) \rangle = 2E_n (2\pi)^3 \delta^3(P - P'), \quad (23)$$

$$\langle \Lambda_b(v', P') | \Lambda_b(v, P) \rangle = 2v_0 (2\pi)^3 \delta^3(P - P'). \quad (24)$$

Comparing Eqs. (21) with (22), we obtain the following relations:

$$\begin{aligned} g_1 &= t_1 = s_2 = d_2 = \left(F_1 + \sqrt{r} F_2 \right), \\ g_2 &= t_2 = g_3 = t_3 = \frac{1}{m_{\Lambda_b}} F_2, \\ s_3 &= F_2(\sqrt{r} - 1), \quad d_3 = F_2(\sqrt{r} + 1), \\ s_1 &= d_1 = F_2 m_{\Lambda_b} (1 + r - 2\sqrt{r}\omega), \end{aligned} \quad (25)$$

where $r = m_n^2/m_{\Lambda_b}^2$. On the other hand, the transition matrix for $\Lambda_b \rightarrow n$ can be expressed in terms of the BS wave functions of Λ_b and n ,

$$\begin{aligned} \langle n(P') | \bar{d} \Gamma b | \Lambda_b(P) \rangle \\ = \int \frac{d^4 p}{(2\pi)^4} \bar{\chi}_{P'}^n(p') \Gamma \chi_P^{\Lambda_b}(p) S_D^{-1}(p_2). \end{aligned} \quad (26)$$

where the $\chi_{P'}^n$ and $\chi_P^{\Lambda_b}$ are the BS wave function of neutron and Λ_b respectively.

Define

$$\begin{aligned} \int \frac{d^4 p}{(2\pi)^4} f_1(p') \phi(p) S_D^{-1}(p_2) &= k_1(\omega), \\ \int \frac{d^4 p}{(2\pi)^4} f_2(p') p'_{i\mu} \phi(p) S_D^{-1}(p_2) \\ &= k_2(\omega) v_\mu + k_3(\omega) v'_\mu, \end{aligned} \quad (27)$$

where $v' = P'/m_n$, then we find the following relations when $\omega \neq 1$:

$$\begin{aligned} k_3 &= -\omega k_2, \\ k_2 &= \frac{1}{1 - \omega^2} \int \frac{d^4 p}{(2\pi)^4} f_2(p') p'_{i\mu} \cdot v \phi(p) S_D^{-1}, \\ F_1 &= k_1 - \omega k_2, \\ F_2 &= k_2. \end{aligned} \quad (28)$$

The differential decay rate of $\Lambda_b \rightarrow nl^+l^-$ is obtained as:

$$\begin{aligned} \mathcal{M}(\Lambda_b \rightarrow nl^+l^-) &= \frac{G_F \lambda_t}{2\sqrt{2}\pi} \\ &\times [\bar{l} \gamma_\mu l \{ \bar{u}_n [\gamma_\mu (A_1 + B_1 + (A_1 - B_1) \gamma_5) \\ &+ i\sigma^{\mu\nu} p_\nu (A_2 + B_2 + (A_2 - B_2) \gamma_5)] u_{\Lambda_b} \} \\ &+ \bar{l} \gamma_\mu \gamma_5 l \{ \bar{u}_n [\gamma^\mu (D_1 + E_1 + (D_1 - E_1) \gamma_5) \\ &+ i\sigma^{\mu\nu} p_\nu (D_2 + E_2 + (D_2 - E_2) \gamma_5) \\ &+ p^\mu (D_3 + E_3 + (D_3 - E_3) \gamma_5)] u_{\Lambda_b} \}], \end{aligned} \quad (29)$$

where $\lambda_t = |V_{tb} * V_{td}^*|$, the parameters A_i , B_i and D_j , E_j ($i = 1, 2$ and $j = 1, 2, 3$) are defined as

$$\begin{aligned} A_i &= \frac{1}{2} \left\{ C_9^{eff} (g_i - t_i) - \frac{2C_7^{eff} m_b}{p^2} (d_i + s_i) \right\}, \\ B_i &= \frac{1}{2} \left\{ C_9^{eff} (g_i + t_i) - \frac{2C_7^{eff} m_b}{p^2} (d_i - s_i) \right\}, \\ D_j &= \frac{1}{2} C_{10} (g_j - t_j), \quad E_j = \frac{1}{2} C_{10} (g_j + t_j). \end{aligned} \quad (30)$$

In the physical region ($4m_l^2 \leq q^2 \leq (m_{\Lambda_b} - m_n)^2$), the decay rate of $\Lambda_b \rightarrow nl^+l^-$ is obtained as

$$\begin{aligned} \frac{d\Gamma(\Lambda_b \rightarrow nl^+l^-)}{dq^2} \\ = \frac{G_F^2 \alpha^2}{2^{13} \pi^5 m_{\Lambda_b}} |V_{tb} V_{td}^*|^2 v_l \sqrt{\lambda(1, r, s)} \mathcal{M}(s), \end{aligned} \quad (31)$$

where $s = 1 + r - 2\sqrt{r}\omega$, $\lambda(1, r, s) = 1 + r^2 + s^2 - 2r - 2s - 2rs$, and $v_l = \sqrt{1 - \frac{4m_l^2}{s m_{\Lambda_b}^2}}$, and the decay amplitude is given as [26]

$$\mathcal{M}(s) = \mathcal{M}_0(s) + \mathcal{M}_2(s), \quad (32)$$

where

$$\begin{aligned} \mathcal{M}_0(s) &= 32m_l^2 m_{\Lambda_b}^4 s(1 + r - s)(|D_3|^2 + |E_3|^2) \\ &+ 64m_l^2 m_{\Lambda_b}^3 (1 - r - s) \text{Re}(D_1^* E_3 + D_3 E_1^*) \\ &+ 64m_{\Lambda_b}^2 \sqrt{r} (6m_l^2 - M_{\Lambda_b}^2 s) \text{Re}(D_1^* E_1) \\ &+ 64m_l^2 m_{\Lambda_b}^3 \sqrt{r} (2m_{\Lambda_b} s \text{Re}(D_3^* E_3) \\ &+ (1 - r + s) \text{Re}(D_1^* D_3 + E_1^* E_3)) \\ &+ 32m_{\Lambda_b}^2 (2m_l^2 + m_{\Lambda_b}^2 s) \\ &\times \left\{ (1 - r + s) m_{\Lambda_b} \sqrt{r} \text{Re}(A_1^* A_2 + B_1^* B_2) \right. \\ &- m_{\Lambda_b} (1 - r - s) \text{Re}(A_1^* B_2 + A_2^* B_1) \\ &- 2\sqrt{r} (\text{Re}(A_1^* B_1) + m_{\Lambda_b}^2 s \text{Re}(A_2^* B_2)) \left. \right\} \\ &+ 8m_{\Lambda_b}^2 \left[4m_l^2 (1 + r - s) + m_{\Lambda_b}^2 ((1 + r)^2 - s^2) \right] \\ &\times (|A_1|^2 + |B_1|^2) \\ &+ 8m_{\Lambda_b}^4 \left\{ 4m_l^2 [\lambda + (1 + r - s)s] \right. \\ &+ m_{\Lambda_b}^2 s [(1 - r)^2 - s^2] \left. \right\} (|A_2|^2 + |B_2|^2) \\ &- 8m_{\Lambda_b}^2 \left\{ 4m_l^2 (1 + r - s) - m_{\Lambda_b} [(1 - r)^2 - s^2] \right\} \\ &\times (|D_1|^2 + |E_1|^2) \\ &+ 8m_{\Lambda_b}^5 s v^2 \left\{ -8m_{\Lambda_b} s \sqrt{r} \text{Re}(D_2^* E_2) \right. \end{aligned}$$

Table 1 The values of α_{seff} for the neutron and Λ_b

κ (GeV ³)	0.045	0.047	0.049	0.051	0.053	0.055
Neutron	0.829	0.811	0.793	0.775	0.758	0.741
Λ_b	0.775	0.777	0.778	0.780	0.782	0.784

$$\begin{aligned}
& +4(1-r+s)\sqrt{r}Re(D_1^*D_2 + E_1^*E_2) \\
& -4(1-r-s)Re(D_1^*E_2 + D_2^*E_1) \\
& +m_{\Lambda_b}[(1-r)^2 - s^2](|D_2|^2 + |E_2|^2) \Big\}, \quad (33)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}(s) = & 8m_{\Lambda_b}^6 s v_l^2 \lambda (|A_2|^2 + |B_2|^2 + |C_2|^2 + |D_2|^2) \\
& - 8m_{\Lambda_b}^4 v_l^2 \lambda (|A_1|^2 + |B_1|^2 + |C_1|^2 + |D_1|^2). \quad (34)
\end{aligned}$$

Similarly, the Hamiltonian for exclusive rare radiative decay $\Lambda_b \rightarrow n\gamma$ with γ as a real photon is given by

$$\begin{aligned}
\mathcal{H}(b \rightarrow d\gamma) = & -\frac{iG_F e}{4\sqrt{2}\pi^2} V_{tb} V_{td}^* C_7^{eff} \left[m_b \bar{d} \sigma_{\mu\nu} q^\mu (1 + \gamma_5) b \right. \\
& \left. + m_d \bar{d} \sigma_{\mu\nu} q^\mu (1 - \gamma_5) b \right] \epsilon^\nu, \quad (35)
\end{aligned}$$

where ϵ^ν is the polarization vector of the photon. Then, the decay width is given by

$$\begin{aligned}
\Gamma(\Lambda_b \rightarrow n\gamma) = & \frac{\alpha G_F^2 m_b^2 m_{\Lambda_b}^3}{2^6 \pi^4} |V_{tb} V_{td}^*|^2 |C_7^{eff}|^2 \\
& \times [s_2^2(0) + d_2^2(0)] \left(1 - \frac{m_n^2}{m_{\Lambda_b}^2} \right)^3. \quad (36)
\end{aligned}$$

Fig. 2 The BS wave functions for the neutron **Fig. 3** The BS wave function for Λ_b

4 Numerical analysis and discussion

In this section we present a detailed numerical analysis of the rare decay $\Lambda_b \rightarrow nl^+l^-$ and radiative decay $\Lambda_b \rightarrow n\gamma$. In our calculations, we take the masses of baryons as $m_{\Lambda_b} = 5.62$ GeV, $m_n = 0.94$ GeV [59], and the masses of quarks, $m_b = 5.02$ GeV and $m_d = 0.34$ GeV [46, 48, 49]. The variable ω varies from 1 to 3.073, 3.069, 1.89 for e , μ , τ , respectively.

Fig. 4 The values of FFs (the lines become thicker with the increases of κ) and $R(\omega)$ with different values of κ

Solving Eqs. (10), (11) and (17) for the neutron and Λ_b with the parameters we have taken, we get the numerical solutions of BS wave functions. In Table 1, we give the values of α_s with different values of κ for the neutron and Λ_b and in Figs. 2 and 3, we give the BS wave functions for the neutron and Λ_b .

It can be seen from Table 1 that the dependence of α_{seff} for the neutron on κ is obviously stronger than that for Λ_b . From the figures in Figs. 2 and 3, we find that BS wave functions of neutron is very similarly on different κ , the values of $f_1(p_t)$ is about from 0 to 0.14 $f_2(p_t)$ varies about from 0 to 0.06 and $\phi(p_t)$ varies from 0 to 0.17. In Fig. 4, we plot the FFs and $R(\omega) = F_2/F_1$ for different κ . From this figure, we find that $F_1(\omega)$ increases with the increase of κ , but the value of $R(\omega)$ is not sensitive to the change of the value of κ . The value of $R(\omega)$ varies from -0.9 to -0.25 when ω changes from 1 to 3.1.

In the heavy quark limit, assuming the same shape for F_1 and F_2 , the ratio $R = -0.35 \pm 0.04$ (stat) ± 0.04 (syst) was previously measured by the CLEO Collaboration using the experimental data for the semileptonic decay $\Lambda_c \rightarrow \Lambda e^+ \nu_e$ when q^2 changes from m_Λ^2 to $m_{\Lambda_c}^2$ [55]. In the same region, we find that $R(\omega)$ varies from -0.32 to -0.25 in our model. In Ref. [13] $R(\omega)$ varies from -0.42 to -0.83 when q^2 change from 0 to $(M_{\Lambda_b} - M_\Lambda)^2$, and in our model $R(\omega)$ change from -0.25 to -0.75 in the same region. However, in Ref. [14] gives the behaviour $R(q^2) \propto -1/q^2$, which agrees with the pQCD scaling law [54, 60, 61]. Therefore, using the CLEO Collaboration experimental data [55], we can estimate

Fig. 5 The decay widths of $\Lambda_b \rightarrow nl^+ l^-$ (the values of the decay width increase with the increase of κ from 0.045 to 0.055) for the lines with the same color)

that the value of $R(\omega)$ should change from -0.91 ± 0.03 to -0.3 ± 0.03 approximately, which agrees with our result as shown in Fig. 4. From the data in Ref. [32], we find

Table 2 The values of the branching ratios of $\Lambda_b \rightarrow nl^+l^-$ and $\Lambda_b \rightarrow n\gamma$ and compare with other model

	Present work	LCSR [32]	LQCD [32]	LCSR [34]	Ref. [35]
$Br(\Lambda_b \rightarrow ne^+e^-) \times 10^8$	$6.79^{+8.66}_{-1.82}$	3.79 ± 0.46	3.19 ± 0.32	8 ± 2	3.81
$Br(\Lambda_b \rightarrow n\mu^+\mu^-) \times 10^8$	$4.08^{+5.44}_{-1.19}$	3.76 ± 0.42	3.15 ± 0.29	7 ± 2	3.75
$Br(\Lambda_b \rightarrow n\tau^+\tau^-) \times 10^8$	$2.9^{+3.7}_{-0.78}$	1.65 ± 0.19	1.42 ± 0.13	2 ± 0.4	1.21
$Br(\Lambda_b \rightarrow n\gamma) \times 10^7$	$3.69^{+3.76}_{-1.95}$	—	—	—	3.7

that $R(\omega_{max}) = -2.75$ in LCSR and $R(\omega_{max}) = -2.33$ by fit the data from LQCD [63,64]. From the data with the contribution of Λ_b^* being considered Ref. [34], we find that $R(\omega_{max}) = -3.47$ in LCSR. These results are much larger than experimental data $R(\omega_{max}) = -0.35$ [55] and do not agree with our result.

In Fig. 5, we give the q^2 -dependence of the decay widths of $\Lambda_b \rightarrow nl^+l^-$ ($l = e, \mu, \tau$) for different parameters. For the central values of parameters, we find that the branching ratio are $BR(\Lambda_b \rightarrow nl^+l^-) \times 10^8 = 6.79$ ($l = e$), 4.08 ($l = \mu$), 2.90 ($l = \tau$) and $BR(\Lambda_b \rightarrow n\gamma) \times 10^7 = 3.69$. Our result for the branching ratios of $BR(\Lambda_b \rightarrow nl^+l^-)$ and $BR(\Lambda_b \rightarrow n\gamma)$ are listed in Table 2 together those in other approaches.

Table 2, we can see that the rare semileptonic decay branching fractions are of order 10^{-8} , and the rare radiative decay branching fraction is of order 10^{-7} . In Ref. [32], the authors use the parameters from LCSR [62] and LQCD [63,64] to fit the FFs of $\Lambda_b \rightarrow n$ and gave the branching ratio of $\Lambda_b \rightarrow nl^+l^-$. In Ref. [34], the authors also calculated the FFs of $\Lambda_b \rightarrow n$ in the framework of LCSR, but their results were different. Our results for the branching ratios of $\Lambda_b \rightarrow nl^+l^-$ ($l = e, \tau$) are very similar to those in Ref. [34], our result for $BR(\Lambda_b \rightarrow n\mu^+\mu^-)$ agrees with that in Ref. [32]. Our radiative decay result $BR(\Lambda_b \rightarrow n\gamma)$ agrees with Ref. [35].

5 Summary

In our work, we calculated the FFs between baryons states induced by the rare $b \rightarrow d$ transition in the BS equation approach in a covariant quark–diquark model. In our model, Λ_b is regarded as a bound state of the b -quark and the scalar ud diquark, thus only the $d^\dagger(du)_{00}/\sqrt{3}$ component of the neutron contributes to the FFs. We established the BS equations for the $q(ud)_{00}$ ($q = b, d$) system and derived the FFs for $\Lambda_b \rightarrow n$ in the BS equation approach. We solved the BS equation of $q(ud)_{00}$ ($q = b, d$) system and then we calculated the FFs and R numerically. Using these FFs, we obtained the branching ratios of $\Lambda_b \rightarrow nl^+l^-$ and $\Lambda_b \rightarrow n\gamma$. Comparing with other works we found that our FFs are very different with other model [32,35], but the branching fractions of the semileptonic decay are of the order 10^{-8} and the

radiative branching ratio is of the order 10^{-7} . On the other hand, we do not consider the long distance contributions in our present work because they have a small effect on width of this decay [65,66]. In our future work we will consider such contributions while calculating the FFs in order to predict the branching ratios more precisely.

In the near future, our results can be tested at LHCb. Our model can be used to study the forward–backward asymmetries and CP violation in the rare decays of b baryons to check our FFs.

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Data Availability Statement This manuscript has associated data in a data repository. [Author's comment: We have added two references [65,66] to support our view.]

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