Light-cone sum rules analysis of $\Xi_{QQ'} \rightarrow \Sigma_{Q'}$ weak decays

Xiao-Hui Hu^{1,a}, Yu-Ji Shi^{2,b}

¹ INPAC, SKLPPC, MOE KLPPC, School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China

² Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, 53115 Bonn, Germany

Received: 24 October 2019 / Accepted: 8 January 2020 / Published online: 24 January 2020

© The Author(s) 2020

Abstract As a continuation of our previous work, we investigate the weak decays of doubly-heavy baryons $\Xi_{QQ'}$ into sextet $\Sigma_{Q'}$ with light-cone sum rules. We calculate the form factors for these decays with the parallel light-cone distribution amplitudes of $\Sigma_{Q'}$. Numerical results of these form factors are used to predict the decay widths and branching ratios of the corresponding semi-leptonic processes. Parametric uncertainties and theoretical analyses are also given in detail. We find that the decay widths of Ξ_{cc} and Ξ_{bc} decays are several orders of magnitude larger than those of Ξ_{bb} decays.

1 Introduction

The naive quark model for mesons and baryons is very successful in explaining most hadrons observed on the experimental side. Despite of these successes, there has been an important renaissance in the hadron spectroscopy study in the past decades. On the one hand, a number of unexpected hadrons were observed most of which defy the standard quark-anti-quark interpretation for mesonic states, and three-quark scenario for baryonic states. These observations of hadron exotics have triggered tremendous theoretical studies, but no conclusive results are obtained yet. On the other side, not all predicted particles by the quark model are well established on the experimental side. Among the latter category, doubly-charmed baryon is an intriguing example that has been discussed in the recent years. In 2017, the LHCb collaboration announced the observation of the ground state doubly-charmed baryon Ξ_{cc}^{++} whose mass is given as [1]

$$m_{\Xi_{cc}^{++}} = (3621.40 \pm 0.72 \pm 0.27 \pm 0.14) \text{ MeV}. \quad (1)$$

This newly observed particle was reconstructed from the decay channel $\Lambda_c^+ K^- \pi^+ \pi^+$, which had been predicted in Ref. [2]. One year later LHCb announced their measurement on Ξ_{cc}^{++} lifetime [3] as well as the confirmation of Ξ_{cc}^{++} in the $\Xi_c^+ \pi^+$ final state [4]. Undoubtedly, experimentalists will continue to search for other heavier particles included in the doubly-heavy baryon spectroscopy [5–7]. Table 1 lists various doubly heavy baryons as well as their corresponding quantum numbers. On the other hand, the great progress in experiments also arouses theoretical analyses, together with which a deeper understanding of hadron spectrum might be achieved in future. To date there have been some theoretical studies which aim to understand the dynamic and spectroscopy properties of the doubly-heavy baryon states [8–36], however, a comprehensive description of the decay mechanism of doubly heavy baryons is not established yet.

One of the ideal platforms for studying doubly heavy baryon is its semi-leptonic decay, where a doubly heavy baryon transforms to a singly heavy baryon through a weak current. All the QCD dynamics in such processes are packed in a single transition matrix element which is suitable for a pure theoretical study. On the other hand, unlike the singly heavy baryon decays such as $\Lambda_b \rightarrow p$, the doubly to singly heavy baryon transitions contain a heavy quark in the final state, which means that the final state is most likely a low energy state. So the heavy quark symmetry, as a static property of heavy hadrons, may still emerge in such processes. Thus the transition matrix element is expected to be dominated by the dynamics describing how the light degrees of freedom response to the change of the heavy quark velocity. In our previous work [37], we have given this estimation according to our phenomenological study on the semi-leptonic weak decays of $\Xi_{QQ'} \rightarrow \Lambda_{Q'}$ within the framework of light cone sum rules (LCSR). In that work, the final states $\Lambda_{Q'}$ belong to SU(3) antitriplets $\bar{3}$. According to the flavor SU(3) symmetry, the singly heavy baryons are classified by irreducible SU(3) representations $3 \otimes 3 = 6 \oplus \bar{3}$. Thus besides antitriplets, sextet baryons including Σ_Q , Ξ'_Q

^a e-mail: huxiaohui@sjtu.edu.cn

^b e-mail: shiyuji92@126.com

Table 1 Quantum numbers and quark content for the ground state of doubly heavy baryons. The S_h^π denotes the spin of the heavy quark system. The light quark q corresponds to u, d quark

Baryon	Quark content	S_h^π	J^P	Baryon	Quark content	S_h^π	J^P
Ξ_{cc}	$\{cc\}q$	1^+	$1/2^+$	Ξ_{bb}	$\{bb\}q$	1^+	$1/2^+$
Ξ_{cc}^*	$\{cc\}q$	1^+	$3/2^+$	Ξ_{bb}^*	$\{bb\}q$	1^+	$3/2^+$
Ω_{cc}	$\{cc\}s$	1^+	$1/2^+$	Ω_{bb}	$\{bb\}s$	1^+	$1/2^+$
Ω_{cc}^*	$\{cc\}s$	1^+	$3/2^+$	Ω_{bb}^*	$\{bb\}s$	1^+	$3/2^+$
Ξ'_{bc}	$\{bc\}q$	0^+	$1/2^+$	Ω'_{bc}	$\{bc\}s$	0^+	$1/2^+$
Ξ_{bc}	$\{bc\}q$	1^+	$1/2^+$	Ω_{bc}	$\{bc\}s$	1^+	$1/2^+$
Ξ_{bc}^*	$\{bc\}q$	1^+	$3/2^+$	Ω_{bc}^*	$\{bc\}s$	1^+	$3/2^+$

and Ω_Q can also be final states of $\Xi_{QQ'}$ decays. In this paper we analyze the transition $\Xi_{QQ'} \rightarrow \Sigma_{Q'}$ by LCSR, where the u, d quark mass of the final state can be neglected so that the corresponding LCDAs are well defined. It can be expected that this work will provide a powerful guide for future experiments on doubly heavy baryon decays.

The transition matrix element of $\Xi_{QQ'} \rightarrow \Sigma_{Q'}$ is parametrized by six form factors. In previous literatures [8, 24, 39], these form factors were calculated by light-front quark model (LFQM) or QCD sum rules (QCDSR). By LFQM, the two spectator quarks are treated as a diquark, which can simplify the calculation of the transition matrix element [24, 28]. However, since the dynamics in the diquark system are smeared, the diquark approximation may introduce some systematic uncertainties. In the QCDSR approach, to evaluate the transition matrix element one needs to introduce a three-point correlation function. This correlation function is calculated by operator product expansion (OPE), while the truncated OPE may lead to the potential irregularities. In addition, such calculation is complex especially when high dimensional operator condensations are included. Most of them must heavily depend on numerical evaluations, where large systematic uncertainties are also inevitable [40]. In this work, to avoid the above problems, we apply LCSR to analyze doubly-heavy baryon transition form factors. In the framework of LCSR, the non-perturbative dynamics of the quarks and gluons in the baryons are described by the light-cone distribution amplitudes (LCDAs). While only a two-point correlation function is needed for calculating these form factors. We hope our study basing on LCSR can help people to understand the heavy baryon LCDAs, and examine their justifiability.

This paper is arranged as follows. In Sect. 2, we parametrize the matrix elements of transitions $\Xi_{QQ'} \rightarrow \Sigma_{Q'}$ with form factors f_i and g_i . Then we introduce the LCSR approach for calculating these form factors. The numerical results are presented in Sect. 3, including the results of form factors, the predictions of decay widths and branching ratios of doubly heavy baryon semi-leptonic decays. Section 4 is a brief summary of this work.

2 Transition form factors in light-cone sum rules

2.1 Form factors

The transition $\Xi_{QQ'} \rightarrow \Sigma_{Q'}$ matrix elements induced by $(V - A)^\mu$ current are expressed in terms of six form factors, three for vector current while the other three for axial-vector current

$$\begin{aligned}
 &\langle \Sigma_{Q'}(p_\Sigma, s_\Sigma) | (V - A)^\mu | \Xi_{QQ'}(p_\Xi, s_\Xi) \rangle \\
 &= \bar{u}_\Sigma(p_\Sigma, s_\Sigma) \\
 &\quad \times \left[\gamma^\mu f_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{m_\Xi} f_2(q^2) + \frac{q^\mu}{m_\Xi} f_3(q^2) \right] u_\Xi(p_\Xi, s_\Xi) \\
 &\quad - \bar{u}_\Sigma(p_\Sigma, s_\Sigma) \\
 &\quad \times \left[\gamma^\mu g_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{m_\Xi} g_2(q^2) + \frac{q^\mu}{m_\Xi} g_3(q^2) \right] \gamma_5 u_\Xi(p_\Xi, s_\Xi),
 \end{aligned} \quad (2)$$

where $q^\mu = p_\Xi^\mu - p_\Sigma^\mu$ is transferring momentum.

To simplify the extraction of form factors, another parameterizing scheme can be applied to the same matrix element, which is more applicable in the frame work of LCSR.

$$\begin{aligned}
 &\langle \Sigma_{Q'}(p_\Sigma, s_\Sigma) | (V - A)^\mu | \Xi_{QQ'}(p_\Xi, s_\Xi) \rangle \\
 &= \bar{u}_\Sigma(p_\Sigma, s_\Sigma) \\
 &\quad \times \left[F_1(q^2) \gamma^\mu + F_2(q^2) p_\Sigma^\mu + F_3(q^2) p_\Xi^\mu \right] u_\Xi(p_\Xi, s_\Xi) \\
 &\quad - \bar{u}_\Sigma(p_\Sigma, s_\Sigma) \\
 &\quad \times \left[G_1(q^2) \gamma^\mu + G_2(q^2) p_\Sigma^\mu + G_3(q^2) p_\Xi^\mu \right] \gamma_5 u_\Xi(p_\Xi, s_\Xi).
 \end{aligned} \quad (3)$$

The form factors F_i and G_i in the above equation have the following relations with the f_i and g_i defined in Eq. (2)

$$\begin{aligned}
 f_1(q^2) &= F_1(q^2) + \frac{1}{2}(m_\Xi + m_\Sigma)(F_2(q^2) + F_3(q^2)), \\
 f_2(q^2) &= \frac{1}{2}m_\Xi(F_2(q^2) + F_3(q^2)),
 \end{aligned}$$

$$\begin{aligned}
f_3(q^2) &= \frac{1}{2}m_\Xi(F_3(q^2) - F_2(q^2)), \\
g_1(q^2) &= G_1(q^2) - \frac{1}{2}(m_\Xi - m_\Sigma)(G_2(q^2) + G_3(q^2)), \\
g_2(q^2) &= \frac{1}{2}m_\Xi(G_2(q^2) + G_3(q^2)), \\
g_3(q^2) &= \frac{1}{2}m_\Xi(G_3(q^2) - G_2(q^2)). \quad (4)
\end{aligned}$$

2.2 Light-cone distribution amplitudes of Σ_Q

The light-cone distribution functions of sextet baryons with spin-parities $J^P = 1/2^+$ were provided in Ref. [38], where the LCDAs are calculated by QCDSR in the heavy quark mass limit. In Ref. [38], the sextet LCDAs are classified by the total spin polarization of the two light quarks. If the polarization vector is parallel to the light-cone plane, four parallel LCDAs are defined by four parallel currents respectively

$$\begin{aligned}
&\frac{\bar{v}^\mu}{v_+} \langle 0 | [q_1^T(t_1) C \not{n} q_2(t_2)] Q_\gamma(0) | \Sigma_Q(v) \rangle \\
&= \frac{1}{\sqrt{3}} \psi_\parallel^n(t_1, t_2) f^{(1)} \epsilon_\parallel^\mu u_\gamma, \\
&\frac{i \bar{v}^\mu}{2} \langle 0 | [q_1^T(t_1) C \sigma_{\alpha\beta} q_2(t_2)] Q_\gamma(0) \bar{n}^\alpha n^\beta | \Sigma_Q(v) \rangle \\
&= \frac{1}{\sqrt{3}} \psi_\parallel^{n\bar{n}}(t_1, t_2) f^{(2)} \epsilon_\parallel^\mu u_\gamma, \\
&\bar{v}^\mu \langle 0 | [q_1^T(t_1) C q_2(t_2)] Q_\gamma(0) | \Sigma_Q(v) \rangle \\
&= \frac{1}{\sqrt{3}} \psi_\parallel^1(t_1, t_2) f^{(2)} \epsilon_\parallel^\mu u_\gamma, \\
&-v_+ \bar{v}^\mu \langle 0 | [q_1^T(t_1) C \bar{n} q_2(t_2)] Q_\gamma(0) | \Sigma_Q(v) \rangle \\
&= \frac{1}{\sqrt{3}} \psi_\parallel^{\bar{n}}(t_1, t_2) f^{(1)} \epsilon_\parallel^\mu u_\gamma. \quad (5)
\end{aligned}$$

Here γ is a Dirac spinor index. n and $\bar{n}$ are the two light-cone vectors, while $\bar{v}^\mu = \frac{1}{2}(\frac{n^\mu}{v_+} - v_+ \bar{n}^\mu)$. t_i are the distances between the i th light quark and the origin along the direction of n . The space-time coordinates of the light quarks can be expressed as $t_i n^\mu$. The four-velocity of Σ_Q is defined by light-cone coordinates $v^\mu = \frac{1}{2}(\frac{n^\mu}{v_+} + v_+ \bar{n}^\mu)$, where $v_+ = 1$ since we choose the rest frame of Σ_Q in this work. When q_1 and q_2 are the same, an extra factor $\sqrt{2}$ should times on the right hand side of Eq. (5). Note that Q should be effective heavy quark field satisfying $\not{Q} = Q$. However, at the leading order we will not distinguish it from the original field. $\{\psi_\parallel^n, \psi_\parallel^{n\bar{n}}, \psi_\parallel^1, \psi_\parallel^{\bar{n}}\}$ denote the four parallel LCDAs with different twists, which correspond to $\{\psi_2, \psi_3^\sigma, \psi_3^s, \psi_4\}$.

As shown in Ref. [38], if the polarization vector is transversal to the light-cone plane, one needs to introduce other four transversal LCDAs by four transversal currents. However, due to the fact that the form factors are only functions of q^2 , one can choose any polarization configuration of the sextet baryon to extract the form factors. In

fact, as argued by Ref. [38], $\psi_\parallel$ and $\psi_\perp$ are not independent with each other, they are related by the parameter A which will be shown in Table 2. When $A = 1/2$, they are the same. Thus in this work, we only use the parallel LCDAs defined in Eq. (5) by choosing the light part polarization vector of the sextet baryon being parallel to the light-cone plane. With the four parallel LCDAs, the matrix element $\epsilon_{abc} \langle \Sigma_Q(v) | \bar{q}_{1k}^a(t_1) \bar{q}_{2i}^b(t_2) \bar{Q}_\gamma^c(0) | 0 \rangle$ is expressed as

$$\begin{aligned}
&\epsilon_{abc} \langle \Sigma_Q(v) | \bar{q}_{1k}^a(t_1) \bar{q}_{2i}^b(t_2) \bar{Q}_\gamma^c(0) | 0 \rangle \\
&= \frac{1}{8} v_+ \psi_\parallel^{n*}(t_1, t_2) f^{(1)} (\bar{u}_\Sigma \bar{\psi} \gamma_5)_\gamma (C^{-1} \not{n})_{ki} \\
&\quad - \frac{1}{8} \psi_\parallel^{n\bar{n}*}(t_1, t_2) f^{(2)} (\bar{u}_\Sigma \bar{\psi} \gamma_5)_\gamma (C^{-1} i \sigma^{\alpha\beta})_{ki} \bar{n}_\alpha n_\beta \\
&\quad + \frac{1}{4} \psi_\parallel^{1*}(t_1, t_2) f^{(2)} (\bar{u}_\Sigma \bar{\psi} \gamma_5)_\gamma (C^{-1})_{ki} \\
&\quad - \frac{1}{8 v_+} \psi_\parallel^{\bar{n}*}(t_1, t_2) f^{(1)} (\bar{u}_\Sigma \bar{\psi} \gamma_5)_\gamma (C^{-1} \not{n})_{kl}, \quad (6)
\end{aligned}$$

where the color indexes a, b, c are explicitly summed over. The Fourier transformation of the LCDAs are

$$\psi(x_1, x_2) = \int_0^\infty d\omega_1 d\omega_2 e^{-i\omega_1 t_1} e^{-i\omega_2 t_2} \psi(\omega_1, \omega_2), \quad (7)$$

where ω_1 and ω_2 are the momentum of the two light quarks, which are along the light-cone direction. The total diquark momentum has the same direction with magnitude $\omega = \omega_1 + \omega_2$. Note that $x_1 = t_1 n, x_2 = t_2 n$

$$\psi(t_1, t_2) = \int_0^\infty d\omega d\omega_2 e^{-i\omega t_1} e^{-i\omega_2(t_2-t_1)} \psi(\omega_1, \omega_2), \quad (8)$$

$$\psi(0, t_2) = \int_0^\infty d\omega \omega \int_0^1 du e^{-i\bar{u}\omega v \cdot x_2} \psi(\omega, u), \quad (9)$$

where $\omega_2 = (1-u)\omega = \bar{u}\omega$. t_i are expressed in Lorentz invariant form $t_i = v \cdot x_i$. Although Ref. [38] only provides LCDAs of bottom baryons, according to the argument given in Ref. [37], one can safely apply these LCDAs for charm baryons in heavy quark limit. In this work both Σ_b and Σ_c are described by the same LCDAs given in Ref. [38], which are expressed as

$$\begin{aligned}
\psi_2(\omega, u) &= \omega^2 \bar{u} u \sum_{n=0}^2 \frac{a_n}{\epsilon_n^4} \frac{C_n^{3/2}(2u-1)}{|C_n^{3/2}|^2} e^{-\omega/\epsilon_n}, \\
\psi_4(\omega, u) &= \sum_{n=0}^2 \frac{a_n}{\epsilon_n^2} \frac{C_n^{1/2}(2u-1)}{|C_n^{1/2}|^2} e^{-\omega/\epsilon_n}, \\
\psi_3^{\sigma,s}(\omega, u) &= \frac{\omega}{2} \sum_{n=0}^2 \frac{a_n}{\epsilon_n^3} \frac{C_n^{1/2}(2u-1)}{|C_n^{1/2}|^2} e^{-\omega/\epsilon_n}, \quad (10)
\end{aligned}$$

with

$$|C_n^\lambda|^2 = \int_0^1 [C_n^\lambda(2u-1)]^2, \quad (11)$$

Table 2 Parameters for the parallel LCDAs of Σ_b in Eqs. (10). A replacement $A \rightarrow 1 - A$ is made for transversal LCDAs [38]. 2, 3 σ , 3s and 4 are twist notations

Twist	a_0	a_1	a_2	ε_0 [GeV]	ε_1 [GeV]	ε_2 [GeV]
Σ_b						
2	1	—	$\frac{6.4A}{A+0.44}$	$\frac{1.4A+0.6}{A+5.7}$	—	$\frac{0.32A}{A-0.17}$
3s	1	—	$\frac{0.12A-0.08}{A-1.4}$	$\frac{0.56A-0.77}{A-2.6}$	—	$\frac{0.25A-0.16}{A+0.41}$
3 σ	—	1	—	—	$\frac{0.35A-0.43}{A-1.2}$	—
4	1	—	$\frac{-0.07A-0.05}{A+0.34}$	$\frac{0.65A+0.22}{A+1}$	—	$\frac{5.5A+3.8}{A+29}$

where $C_0^\lambda(x) = 1$, $C_1^\lambda(x) = 2\lambda x$ and $C_2^\lambda(x) = 2\lambda(1+\lambda)x^2 - \lambda$. The parameters in Eq. (10) are collected in Table 2. The parameter A is chosen to be around 1/2 and also make sure that ε_i s are non-negative. In this work, we simply choose it to be $A = 1/2$.

2.3 Light-cone sum rules framework

According to the framework of LCSR, to deal with the transition defined in Eq. (3), one needs to construct a two point correlation function

$$\begin{aligned} \Pi_\mu(p_\Sigma, q) \\ = i \int d^4x e^{iq \cdot x} \langle \Sigma_{Q'}(p_\Sigma) | T \{ J_\mu^{V-A}(x) \bar{J}_{\Xi_{QQ'}}(0) \} | 0 \rangle. \end{aligned} \quad (12)$$

The two currents J^{V-A} , $J_{\Xi_{QQ'}}$ are $V - A$ current and the $\Xi_{QQ'}$ interpolating current respectively

$$J_\mu^{V-A}(x) = \bar{q}_e \gamma_\mu (1 - \gamma_5) Q_e, \quad (13)$$

while for $Q = Q' = b, c$

$$J_{\Xi_{QQ}} = \epsilon_{abc} (Q_a^T C \gamma^\nu Q_b) \gamma_\nu \gamma_5 q_c', \quad (14)$$

for $Q = b, Q' = c$

$$J_{\Xi_{bc}} = \frac{1}{\sqrt{2}} \epsilon_{abc} (b_a^T C \gamma^\nu c_b + c_a^T C \gamma^\nu b_b) \gamma_\nu \gamma_5 q_c'. \quad (15)$$

In the frame work of LCSR, the correlation function Eq. (12) is calculated both at hadron level and QCD level. Then the results at the two levels can be matched according to quark-hadron duality. At hadron level, by inserting a complete set of baryon states including both positive and negative parity states between J^{V-A} and $J_{\Xi_{QQ'}}$, and using the definition of $\Xi_{QQ'}^{P+}$ and $\Xi_{QQ'}^{P-}$ decay constants f_Ξ^+ and f_Ξ^-

$$\begin{aligned} \langle \Xi_{QQ'}^{P+}(p_\Xi, s) | \bar{J}_{\Xi_{QQ'}}(0) | 0 \rangle &= f_\Xi^+ \bar{u}_\Xi(p_\Xi, s) \\ \langle \Xi_{QQ'}^{P-}(p_\Xi, s) | \bar{J}_{\Xi_{QQ'}}(0) | 0 \rangle &= -i\gamma_5 f_\Xi^- \bar{u}_\Xi(p_\Xi, s), \end{aligned} \quad (16)$$

After inserting the positive and negative parity hadron states respectively, the correlation function induced by the vector current $\bar{q} \gamma^\mu Q$ has the following forms

$$\begin{aligned} \Pi_{\mu,V}^{hadron}(p_\Sigma, q)^+ \\ = -\frac{f_\Xi^+}{(q+p_\Sigma)^2 - m_\Xi^2} \bar{u}_\Sigma(p_\Sigma) [F_1^+(q^2) \gamma_\mu + F_2^+(q^2) p_{\Sigma\mu} \\ + F_3^+(q^2) p_{\Xi\mu}] (q + \not{p}_\Sigma + m_\Xi) + \cdots \\ = -\frac{f_\Xi^+}{(q+p_\Sigma)^2 - m_\Xi^2} \bar{u}_\Sigma(p_\Sigma) \\ \times [F_1^+(q^2) (m_\Xi - m_\Sigma) \gamma_\mu + (m_\Xi + m_\Sigma) F_3^+(q^2) q_\mu \\ + [(m_\Sigma^2 + m_\Xi m_\Sigma) (F_2^+(q^2) + F_3^+(q^2)) + 2m_\Sigma F_1^+(q^2)] v_\mu \\ + F_1^+(q^2) \gamma_\mu \not{q} + m_\Sigma (F_2^+(q^2) + F_3^+(q^2)) v_\mu \not{q} \\ + F_3^+(q^2) q_\mu \not{q}] + \cdots, \end{aligned} \quad (17)$$

$$\begin{aligned} \Pi_{\mu,V}^{hadron}(p_\Sigma, q)^- \\ = -\frac{f_\Xi^-}{(q+p_\Sigma)^2 - m_\Xi^2} \bar{u}_\Sigma(p_\Sigma) [F_1^-(q^2) \gamma_\mu + F_2^-(q^2) p_{\Sigma\mu} \\ + F_3^-(q^2) p_{\Xi\mu}] (-q - \not{p}_\Sigma + m_\Xi) + \cdots \\ = -\frac{f_\Xi^-}{(q+p_\Sigma)^2 - m_\Xi^2} \bar{u}_\Sigma(p_\Sigma) \\ \times [F_1^-(q^2) (m_\Xi + m_\Sigma) \gamma_\mu + (m_\Xi - m_\Sigma) F_3^-(q^2) q_\mu \\ + [(-m_\Sigma^2 + m_\Xi m_\Sigma) (F_2^-(q^2) + F_3^-(q^2)) - 2m_\Sigma F_1^-(q^2)] v_\mu \\ - F_1^-(q^2) \gamma_\mu \not{q} - m_\Sigma (F_2^-(q^2) + F_3^-(q^2)) v_\mu \not{q} \\ - F_3^-(q^2) q_\mu \not{q}] + \cdots, \end{aligned} \quad (18)$$

where the ellipses stand for the contribution from continuum spectra ρ^h above the threshold s_{th} , which has the integral form

$$\int_{s_{th}}^\infty ds \frac{\rho^h(s, q^2)}{s - p_\Sigma^2}. \quad (19)$$

The total hadron level correlation function is contributed both from Eqs. (17) and (18)

$$\Pi_{\mu,V}^{hadron}(p_\Sigma, q) = \Pi_{\mu,V}^{hadron}(p_\Sigma, q)^+ + \Pi_{\mu,V}^{hadron}(p_\Sigma, q)^-. \quad (20)$$

The correlation function induced by the axial-vector current $\bar{q}\gamma^\mu\gamma^5 Q$ can be calculated by the same procedure. Thus in the following calculations we will mainly focus on the extraction of vector form factors f_i while the axial-vector form factors g_i can be extracted analogously.

At QCD level, the correlation function is calculated by OPE and the use of Eq. (6), the $J_\mu^V(x)$ induced correlation function can be expressed as

$$\begin{aligned} \Pi_{\mu,V}^{QCD}(p_\Sigma, q) &= \frac{i}{4} \int d^4x \int_0^\infty d\omega \int_0^1 du e^{i(q+\bar{u}\omega v)\cdot x} \\ &\times \left\{ v_+ \psi_\parallel^{n*}(\omega, u) f^{(1)} \bar{u}_\Sigma [\bar{\psi} \gamma_5 \gamma^\nu C S^Q(x)^T C^T \gamma_\mu \bar{\psi} \gamma_\nu \gamma_5] \right. \\ &- \psi_\parallel^{\bar{n}*}(\omega, u) f^{(2)} \bar{u}_\Sigma [\bar{\psi} \gamma_5 \gamma^\nu C S^Q(x)^T C^T \gamma_\mu i \sigma^{\alpha\beta} \gamma_\nu \gamma_5] \bar{n}_\alpha n_\beta \\ &- 2 \psi_\parallel^{1*}(\omega, u) f^{(2)} \bar{u}_\Sigma [\bar{\psi} \gamma_5 \gamma^\nu C S^Q(x)^T C^T \gamma_\mu \gamma_\nu \gamma_5] \\ &\left. - \frac{1}{v_+} \psi_\parallel^{\bar{n}*}(\omega, u) f^{(1)} \bar{u}_\Sigma [\bar{\psi} \gamma_5 \gamma^\nu C S^Q(x)^T C^T \gamma_\mu \not{x} \gamma_\nu \gamma_5] \right\}, \quad (21) \end{aligned}$$

where $S^Q(x)$ is the QCD free heavy quark propagator. $\bar{v}$ and the light-cone vectors $n, \bar{n}$ in Eq. (21) should be expressed in Lorentz covariant forms

$$\begin{aligned} n_\mu &= \frac{1}{v \cdot x} x_\mu, \quad \bar{n}_\mu = 2v_\mu - \frac{1}{v \cdot x} x_\mu, \\ \bar{v}_\mu &= \frac{x_\mu}{v \cdot x} - v_\mu. \end{aligned} \quad (22)$$

The correlation function can be written as a convolution of the LCDAs and the perturbation kernel, where diquark momenta ω and momenta fraction u are integrated

$$\begin{aligned} \Pi_{\mu,V}^{QCD}(p_\Sigma, q) &= \frac{1}{4} \int d^4x \int_0^{2s_0} d\omega \int_0^1 du \int \frac{d^4k}{(2\pi)^4} e^{i(q+\bar{u}\omega v-k)\cdot x} \\ &\times \left\{ \psi_2(\omega, u) f^{(1)} \bar{u}_{\Sigma_c} \right. \\ &\times \left[\left(\frac{\not{x}}{v \cdot x} - \not{v} \right) \gamma_5 \gamma^\nu \frac{\not{k} - m_Q}{k^2 - m_Q^2} \gamma_\mu \left(2\not{v} - \frac{\not{x}}{v \cdot x} \right) \gamma_\nu \gamma_5 \right] \\ &- \psi_{3\sigma}(\omega, u) f^{(2)} \bar{u}_{\Sigma_c} \\ &\times \left[\left(\frac{\not{x}}{v \cdot x} - \not{v} \right) \gamma_5 \gamma^\nu \frac{\not{k} - m_Q}{k^2 - m_Q^2} \gamma_\mu i \sigma^{\alpha\beta} \left(2v_\alpha - \frac{x_\alpha}{v \cdot x} \right) \frac{x_\beta}{v \cdot x} \gamma_\nu \gamma_5 \right] \\ &- 2 \psi_{3s}(\omega, u) f^{(2)} \bar{u}_{\Sigma_c} \\ &\times \left[\left(\frac{\not{x}}{v \cdot x} - \not{v} \right) \gamma_5 \gamma^\nu \frac{\not{k} - m_Q}{k^2 - m_Q^2} \gamma_\mu \gamma_\nu \gamma_5 \right] \\ &+ \psi_4(\omega, u) f^{(1)} \bar{u}_{\Sigma_c} \\ &\times \left[\left(\frac{\not{x}}{v \cdot x} - \not{v} \right) \gamma_5 \gamma^\nu \frac{\not{k} - m_Q}{k^2 - m_Q^2} \gamma_\mu \frac{\not{x}}{v \cdot x} \gamma_\nu \gamma_5 \right] \left. \right\}. \quad (23) \end{aligned}$$

After integrating out the space-time coordinate x , one arrive at the explicit form of the correlation function at QCD level

$$\begin{aligned} \Pi_{\mu,V}^{QCD}((p_\Sigma + q)^2, q^2) &= \int_0^{2s_0} d\omega \int_0^1 du f^{(1)} \left\{ -\frac{1}{2} \bar{u}^2 \hat{\psi}_\parallel^{n*}(\omega, u) \right. \\ &\times \left[\frac{8\bar{u}_\Sigma N_1}{[(q + \bar{u}\omega v)^2 - m_Q^2]^3} - \frac{2\bar{u}_\Sigma N_2}{[(q + \bar{u}\omega v)^2 - m_Q^2]^2} \right] \\ &+ \frac{1}{2} \bar{u} \tilde{\psi}_\parallel^{n*}(\omega, u) \\ &\times \left[-\frac{2\bar{u}_\Sigma N_3}{[(q + \bar{u}\omega v)^2 - m_Q^2]^2} - \frac{2\bar{u}_\Sigma \gamma_\mu}{(q + \bar{u}\omega v)^2 - m_Q^2} \right] \\ &+ \omega \psi_\parallel^{n*}(\omega, u) \frac{\bar{u}_\Sigma N_4}{(q + \bar{u}\omega v)^2 - m_Q^2} \left. \right\} \\ &+ \int_0^{2s_0} d\omega \int_0^1 du f^{(2)} \left\{ \frac{1}{2} \bar{u}^2 \hat{\psi}_\parallel^{\bar{n}*}(\omega, u) \right. \\ &\times \left[\frac{8\bar{u}_\Sigma N_5}{[(q + \bar{u}\omega v)^2 - m_Q^2]^3} - \frac{2\bar{u}_\Sigma N_6}{[(q + \bar{u}\omega v)^2 - m_Q^2]^2} \right] \\ &+ \bar{u} \tilde{\psi}_\parallel^{\bar{n}*}(\omega, u) \\ &\times \left[-\frac{\bar{u}_\Sigma [q, \not{v}](2q_\mu + 2\bar{u}\omega v_\mu - \gamma_\mu m_Q)}{[(q + \bar{u}\omega v)^2 - m_Q^2]^2} \right. \\ &\left. + \frac{\bar{u}_\Sigma [\gamma_\mu, \not{v}]}{(q + \bar{u}\omega v)^2 - m_Q^2} \right] \left. \right\} \\ &+ \int_0^{2s_0} d\omega \int_0^1 du f^{(2)} \left\{ \bar{u} \tilde{\psi}_1^{1*}(\omega, u) \right. \\ &\times \left[-\frac{2\bar{u}_\Sigma N_7}{[(q + \bar{u}\omega v)^2 - m_Q^2]^2} + \frac{2\bar{u}_\Sigma \gamma_\mu}{(q + \bar{u}\omega v)^2 - m_Q^2} \right] \\ &+ \omega \psi_\parallel^{1*}(\omega, u) \frac{\bar{u}_\Sigma (2q_\mu + 2\bar{u}\omega v_\mu - \gamma_\mu m_Q)}{(q + \bar{u}\omega v)^2 - m_Q^2} \left. \right\} \\ &+ \int_0^{2s_0} d\omega \int_0^1 du f^{(1)} \left\{ -\frac{1}{2} \bar{u}^2 \hat{\psi}_\parallel^{\bar{n}*}(\omega, u) \right. \\ &\times \left[\frac{8\bar{u}_\Sigma N_8}{[(q + \bar{u}\omega v)^2 - m_Q^2]^3} + \frac{4\bar{u}_\Sigma N_9}{[(q + \bar{u}\omega v)^2 - m_Q^2]^2} \right] \\ &+ \frac{1}{2} \bar{u} \tilde{\psi}_\parallel^{\bar{n}*}(\omega, u) \\ &\times \left[\frac{2\bar{u}_\Sigma N_{10}}{[(q + \bar{u}\omega v)^2 - m_Q^2]^2} + \frac{2\bar{u}_\Sigma \gamma_\mu}{(q + \bar{u}\omega v)^2 - m_Q^2} \right] \left. \right\}, \quad (24) \end{aligned}$$

where

$$\begin{aligned} N_1 &= (q + \bar{u}\omega v)^2 \gamma_\mu (q + \bar{u}\omega \not{v}) \\ &- 2m_Q(q_\mu + \bar{u}\omega v_\mu)(q + \bar{u}\omega \not{v}), \end{aligned}$$

$$\begin{aligned}
N_2 &= 6\gamma_\mu(\not{q} + \bar{u}\omega\not{v}) - 2m_Q\gamma_\mu, \\
N_3 &= (\not{q} + \bar{u}\omega\not{v})\gamma_\mu(\not{q} + \bar{u}\omega\not{v}) \\
&\quad + 4m_Q v_\mu(\not{q} + \bar{u}\omega\not{v}) + 2m_Q(q_\mu + \bar{u}\omega v_\mu), \\
N_4 &= \gamma_\mu(\not{q} + \bar{u}\omega\not{v}) + 2m_Q v_\mu, \\
N_5 &= (\not{q} + \bar{u}\omega\not{v})[\not{q}, \not{v}](2q_\mu + 2\bar{u}\omega v_\mu - \gamma_\mu m_Q), \\
N_6 &= \gamma_\rho[\gamma^\rho, \not{v}](2q_\mu + 2\bar{u}\omega v_\mu - \gamma_\mu m_Q + 2\gamma_\mu[\not{q}, \not{v}]) \\
&\quad + 2(\not{q} + \bar{u}\omega\not{v})[\gamma_\mu, \not{v}], \\
N_7 &= (\not{q} + \bar{u}\omega\not{v})(2q_\mu + 2\bar{u}\omega v_\mu - \gamma_\mu m_Q), \\
N_8 &= (q + \bar{u}\omega v)^2 \gamma_\mu(\not{q} + \bar{u}\omega\not{v}) \\
&\quad - 2m_Q(q_\mu + \bar{u}\omega v_\mu)(\not{q} + \bar{u}\omega\not{v}), \\
N_9 &= (\not{q} + \bar{u}\omega\not{v})\gamma_\mu - 2\gamma_\mu(\not{q} + \bar{u}\omega\not{v}) \\
&\quad - 2(q_\mu + \bar{u}\omega v_\mu) + m_Q\gamma_\mu, \\
N_{10} &= (\not{q} + \bar{u}\omega\not{v})\gamma_\mu(\not{q} + \bar{u}\omega\not{v}) - 2m_Q(q_\mu + \bar{u}\omega v_\mu),
\end{aligned} \quad (25)$$

m_Q is the mass of the translating heavy quark, and

$$\begin{aligned}
[(q + \bar{u}\omega v)^2 - m_Q^2]^2 &= \frac{\bar{u}\omega}{m_\Sigma} s + H(u, \omega, q^2) - m_Q^2, \\
s &= (p_\Sigma + q)^2, \quad H(u, \omega, q^2) = \bar{u}\omega(\bar{u}\omega - m_\Sigma) \\
&\quad + \left(1 - \frac{\bar{u}\omega}{m_\Sigma}\right) q^2.
\end{aligned} \quad (26)$$

Here we have also used two kinds of newly defined LCDAs

$$\begin{aligned}
\tilde{\psi}_i(\omega, u) &= \int_0^\omega d\tau \tau \psi_i(\tau, u), \\
\hat{\psi}_i(\omega, u) &= \int_0^\omega d\tau \tilde{\psi}_i(\tau, u) \quad (i = 2, 3\sigma, 3s, 4).
\end{aligned}$$

As a result, the correlation function at QCD level can be expressed by six Dirac structures $\{\gamma_\mu, v_\mu, q_\mu, \gamma_\mu \not{q}, v_\mu \not{q}, q_\mu \not{q}\}$. Denoting the corresponding coefficients as $C_{\text{structure}}$, one have

$$\begin{aligned}
\Pi_{\mu, V}^{QCD}((p_\Sigma + q)^2, q^2) &= \bar{u}_\Sigma(p_\Sigma) \{C_{\gamma_\mu} \gamma_\mu + C_{v_\mu} v_\mu + C_{q_\mu} q_\mu + C_{\gamma_\mu \not{q}} \gamma_\mu \not{q} \\
&\quad + C_{v_\mu \not{q}} v_\mu \not{q} + C_{q_\mu \not{q}} q_\mu \not{q}\}.
\end{aligned} \quad (27)$$

With the denotation $(q + \bar{u}\omega v)^2 - m_Q^2 = \Delta$, the expression of the coefficients are

$$\begin{aligned}
C_{\gamma_\mu} &= \int_0^{2s_0} d\omega \int_0^1 du \left\{ [f^{(1)} \bar{u}(\tilde{\psi}_\parallel^{\bar{n}*} - \psi_\parallel^{n*} \omega^2 - \tilde{\psi}_\parallel^{n*}) \right. \\
&\quad \left. - f^{(2)}(m_Q \psi_\parallel^{1*} \omega + 2\tilde{\psi}_\parallel^{n\bar{n}*} \bar{u} - 2\tilde{\psi}_\parallel^{1*} \bar{u})] \frac{1}{\Delta} \right. \\
&\quad \left. + \bar{u} \left[-2f^{(1)} \bar{u}(\hat{\psi}_\parallel^{\bar{n}*} + \hat{\psi}_\parallel^{n*})(m_Q + 3\bar{u}\omega) \right. \right.
\end{aligned}$$

$$\begin{aligned}
&\quad \left. + f^{(1)}(\tilde{\psi}_\parallel^{n*} - \tilde{\psi}_\parallel^{\bar{n}*}) (q^2 + \bar{u}\omega(2q \cdot v + \bar{u}\omega)) \right. \\
&\quad \left. + 2f^{(2)} m_Q (3\hat{\psi}_\parallel^{n\bar{n}*} \bar{u} + \tilde{\psi}_\parallel^{n\bar{n}*} q \cdot v + \tilde{\psi}_\parallel^{1*} \bar{u}\omega) \right. \\
&\quad \left. + 4f^{(2)} \hat{\psi}_\parallel^{n\bar{n}*} \bar{u}(q \cdot v + \bar{u}\omega) \right] \frac{1}{\Delta^2} \\
&\quad \left. + 4\bar{u}^2 \left[f^{(1)} \bar{u}\omega(\hat{\psi}_\parallel^{\bar{n}*} + \hat{\psi}_\parallel^{n*}) (q^2 + \bar{u}\omega(2q \cdot v + \bar{u}\omega)) \right. \right. \\
&\quad \left. \left. - 2f^{(2)} m_Q \hat{\psi}_\parallel^{n\bar{n}*} (q^2 + q \cdot v \bar{u}\omega) \right] \frac{1}{\Delta^3} \right\},
\end{aligned} \quad (28)$$

$$\begin{aligned}
C_{q_\mu} &= \int_0^{2s_0} d\omega \int_0^1 du \frac{2f^{(2)} \psi_\parallel^{1*} \omega}{\Delta} \\
&\quad + [-2\bar{u}(f^{(1)}(m_Q(\tilde{\psi}_\parallel^{\bar{n}*} \tilde{\psi}_\parallel^{n*}) + \bar{u}\omega(\tilde{\psi}_\parallel^{n*} - \tilde{\psi}_\parallel^{\bar{n}*})) \\
&\quad + 2f^{(2)}(m_Q(\tilde{\psi}_\parallel^{n\bar{n}*} - \tilde{\psi}_\parallel^{1*}) + 5\hat{\psi}_\parallel^{n\bar{n}*} \bar{u} + \tilde{\psi}_\parallel^{n\bar{n}*} q \cdot v \\
&\quad + \tilde{\psi}_\parallel^{1*} \bar{u}\omega))] \frac{1}{\Delta^2} \\
&\quad + 8\bar{u}^2 [f^{(1)} m_Q \bar{u}\omega(\hat{\psi}_\parallel^{\bar{n}*} + \hat{\psi}_\parallel^{n*}) + 2f^{(2)} \hat{\psi}_\parallel^{n\bar{n}*} \\
&\quad \times (\bar{u}\omega(m_Q + q \cdot v) + m_Q q \cdot v + q^2)] \frac{1}{\Delta^3},
\end{aligned} \quad (29)$$

$$\begin{aligned}
C_{v_\mu} &= \int_0^{2s_0} d\omega \int_0^1 du \left\{ 2[f^{(1)} \psi_\parallel^{n*} \omega(m_Q + \bar{u}\omega) \right. \\
&\quad \left. + f^{(2)} \bar{u}(\tilde{\psi}_\parallel^{n\bar{n}*} + \psi_\parallel^{1*} \omega^2)] \frac{1}{\Delta} \right. \\
&\quad \left. - 2\bar{u}^2 \omega \{ f^{(1)} [m_Q(\tilde{\psi}_\parallel^{\bar{n}*} + 3\tilde{\psi}_\parallel^{n*}) - 6\bar{u}(\hat{\psi}_\parallel^{\bar{n}*} + \hat{\psi}_\parallel^{n*}) \right. \right. \\
&\quad \left. \left. + \bar{u}\omega(\tilde{\psi}_\parallel^{n*} - \tilde{\psi}_\parallel^{\bar{n}*})] \right. \right. \\
&\quad \left. \left. + 2f^{(2)}(4\hat{\psi}_\parallel^{n\bar{n}*} \bar{u} + \tilde{\psi}_\parallel^{n\bar{n}*} q \cdot v + \tilde{\psi}_\parallel^{1*} \bar{u}\omega) \right\} \frac{1}{\Delta^2} \right. \\
&\quad \left. - 8\bar{u}^3 \omega [f^{(1)}(\hat{\psi}_\parallel^{\bar{n}*} + \hat{\psi}_\parallel^{n*}) \right. \\
&\quad \left. (\bar{u}\omega(-m_Q + 2q \cdot v + \bar{u}\omega) + q^2) \right. \\
&\quad \left. - 2f^{(2)} \hat{\psi}_\parallel^{n\bar{n}*} (q^2 + q \cdot v \bar{u}\omega) \right] \frac{1}{\Delta^3} \Big\},
\end{aligned} \quad (30)$$

$$\begin{aligned}
C_{\gamma_\mu \not{q}} &= \int_0^{2s_0} d\omega \int_0^1 du \left\{ \frac{f^{(1)} \psi_\parallel^{n*} \omega}{\Delta} + 2\bar{u}[3f^{(1)} \bar{u}(\hat{\psi}_\parallel^{\bar{n}*} + \hat{\psi}_\parallel^{n*}) \right. \\
&\quad \left. + f^{(2)} m_Q(\tilde{\psi}_\parallel^{n\bar{n}*} - \tilde{\psi}_\parallel^{1*})] \frac{1}{\Delta^2} \right. \\
&\quad \left. - 4\bar{u}^2 [f^{(1)}(\hat{\psi}_\parallel^{\bar{n}*} + \hat{\psi}_\parallel^{n*}) (q^2 + \bar{u}\omega(2q \cdot v + \bar{u}\omega)) \right. \\
&\quad \left. + 2f^{(2)} m_Q \hat{\psi}_\parallel^{n\bar{n}*} (q \cdot v + \bar{u}\omega)] \frac{1}{\Delta^3} \right\},
\end{aligned} \quad (31)$$

$$\begin{aligned}
C_{q_\mu \not{q}} &= \int_0^{2s_0} d\omega \int_0^1 du \left\{ 2\bar{u}[f^{(1)}(\tilde{\psi}_\parallel^{\bar{n}*} - \tilde{\psi}_\parallel^{n*}) \right. \\
&\quad \left. + 2f^{(2)}(\tilde{\psi}_\parallel^{n\bar{n}*} - \tilde{\psi}_\parallel^{1*})] \frac{1}{\Delta^2} \right. \\
&\quad \left. + 8\bar{u}^2 [f^{(1)} m_Q(\hat{\psi}_\parallel^{\bar{n}*} + \hat{\psi}_\parallel^{n*}) \right.
\end{aligned}$$

Fig. 1 Feynman diagram of the QCD level correlation function. The green ellipse denotes the final $\Sigma_{Q'}$ baryon with four-velocity v . The left black dot denotes the $V - A$ current while the right dot denotes the doubly-heavy baryon current. The left straight line denotes one of the light quarks inside the $\Sigma_{Q'}$. It has momentum $\bar{u}\omega v$, where $\bar{u}$ is its momentum fraction of the diquark momentum

$$-2f^{(2)}\hat{\psi}_{\parallel}^{n\bar{n}*}(q \cdot v + \bar{u}\omega)\left[\frac{1}{\Delta^3}\right\}, \quad (32)$$

$$\begin{aligned} & C_{v\mu q} \\ &= \int_0^{2s_0} d\omega \int_0^1 du \left\{ 2\bar{u} \left[f^{(1)}[\bar{u}\omega(\tilde{\psi}_{\parallel}^{n\bar{n}*} - \tilde{\psi}_{\parallel}^{n*}) - 2m_Q\tilde{\psi}_{\parallel}^{n*}] \right. \right. \\ & \quad + 2f^{(2)}\bar{u}[\hat{\psi}_{\parallel}^{n\bar{n}*} + \omega(\tilde{\psi}_{\parallel}^{n\bar{n}*} - \tilde{\psi}_{\parallel}^{1*})] \left. \right] \frac{1}{\Delta^2} \\ & \quad + 8\bar{u}^3 \omega [f^{(1)}m_Q(\hat{\psi}_{\parallel}^{n\bar{n}*} + \hat{\psi}_{\parallel}^{n*}) \\ & \quad \left. - 2f^{(2)}\hat{\psi}_{\parallel}^{n\bar{n}*}(q \cdot v + \bar{u}\omega)] \frac{1}{\Delta^3} \right\}, \quad (33) \end{aligned}$$

where we have omitted the arguments (ω, u) of the LCDAs $\psi_{\parallel}$, $\tilde{\psi}_{\parallel}$ and $\hat{\psi}_{\parallel}$ for compactness. Figure 1 shows the Feynman diagram describing the QCD level correlation function. Since the correlation function is a function of Lorentz invariants $(p_{\Sigma} + q)^2$ and q^2 . By extracting the discontinuity of the correlation function Eq. (24) acrossing the branch cut on the $(p_{\Sigma} + q)^2$ complex plane, the correlation function can be expressed as a dispersion integration form

$$\Pi_{\mu,V}^{QCD}(p_{\Sigma}, q) = \frac{1}{\pi} \int_{(m_Q+m_{Q'}+m_q)^2}^{\infty} ds \frac{\text{Im}\Pi_{\mu,V}^{QCD}(s, q^2)}{s - (p_{\Sigma} + q)^2}. \quad (34)$$

According to the global Quark-Hadron duality, the continuum contribution Eq. (19) should be equal to the QCD level contribution Eq. (34) in the same spectral region $s_{th} < s < \infty$.

The equivalence between Eqs. (34) and (20) enables one to extract the form factors F_i^+

$$\begin{aligned} & - \frac{f_{\Xi}}{(q + p_{\Sigma})^2 - m_{\Xi}^2} \bar{u}_{\Sigma}(p_{\Sigma}) F_1^+(q^2) \\ &= \frac{1}{\pi} \int_{(m_Q+m_{Q'}+m_q)^2}^{s_{th}} \frac{ds}{s - (p_{\Sigma} + q)^2} \\ & \quad \text{Im} \left\{ \frac{C_{\gamma\mu} + C_{\gamma\mu} q (m_{\Sigma} + m_{\Xi})}{2m_{\Xi}} \right\}, \quad (35) \end{aligned}$$

$$\begin{aligned} & - \frac{f_{\Xi}}{(q + p_{\Sigma})^2 - m_{\Xi}^2} \bar{u}_{\Sigma}(p_{\Sigma}) F_2^+(q^2) \\ &= \frac{1}{\pi} \int_{(m_Q+m_{Q'}+m_q)^2}^{s_{th}} \frac{ds}{s - (p_{\Sigma} + q)^2} \\ & \quad \times \frac{1}{2m_{\Sigma}m_{\Xi}} \text{Im} \left[-2m_{\Sigma}C_{\gamma\mu}q + (m_{\Sigma}^2 - m_{\Sigma}m_{\Xi})C_{q\mu}q \right. \\ & \quad \left. + (m_{\Xi} - m_{\Sigma})C_{v\mu}q - m_{\Sigma}C_{q\mu} + C_{v\mu} \right], \quad (36) \end{aligned}$$

$$\begin{aligned} & - \frac{f_{\Xi}}{(q + p_{\Sigma})^2 - m_{\Xi}^2} \bar{u}_{\Sigma}(p_{\Sigma}) F_3^+(q^2) \\ &= \frac{1}{\pi} \int_{(m_Q+m_{Q'}+m_q)^2}^{s_{th}} \frac{ds}{s - (p_{\Sigma} + q)^2} \\ & \quad \text{Im} \left\{ \frac{(m_{\Xi} - m_{\Sigma})C_{\gamma\mu}q + C_{q\mu}}{2m_{\Xi}} \right\}, \quad (37) \end{aligned}$$

which describing the positive parity baryon $\Xi_{QQ'}^{P+}$ decays. By constructing Borel transformation on the both sides of the three Eqs. (39)–(37), one can get the explicit expression of each form factors F_i^+ . Before Borel transformation, we make the following transform on the denominators in Eqs. (28)–(33) to enable the extraction of the discontinuity,

$$\begin{aligned} & \frac{1}{[(q + \bar{u}\omega v)^2 - m_Q^2]^n} \\ & \rightarrow \frac{1}{(n-1)!} \left(\frac{\partial}{\partial \Omega} \right)^{(n-1)} \frac{1}{(q + \bar{u}\omega v)^2 - \Omega} \Big|_{\Omega=m_Q^2}. \quad (38) \end{aligned}$$

Due to the complex form of the analytic expression, here we just show the analytic expression of F_1^+ as an example

$$\begin{aligned} & F_1^+(q^2) \\ &= -\frac{1}{f_{\Xi}} \exp \left(\frac{m_{\Xi}^2}{M^2} \right) \int_0^{2s_0} d\omega \int_0^1 du \frac{m_{\Sigma}}{\bar{u}\omega} \\ & \quad \exp \left(-\frac{s_r}{M^2} \right) \theta(sth - s_r) \theta(s_r - (m_Q + m_{Q'} + m_q)^2) \\ & \quad \times \left\{ \left[f^{(1)}\bar{u}(\tilde{\psi}_{\parallel}^{n\bar{n}*} - \psi_{\parallel}^{n*}\omega^2 - \tilde{\psi}_{\parallel}^{n*}) - f^{(2)} \right. \right. \\ & \quad \left. \left. (m_Q\psi_{\parallel}^{1*}\omega + 2\tilde{\psi}_{\parallel}^{n\bar{n}*}\bar{u} - 2\tilde{\psi}_{\parallel}^{1*}\bar{u}) \right] \frac{1}{2m_{\Xi}} \right. \\ & \quad \left. + f^{(1)}\psi_{\parallel}^{n*} \frac{m_{\Sigma} + m_{\Xi}}{2m_{\Xi}} \right\} \\ & \quad - \frac{1}{f_{\Xi}} \exp \left(\frac{m_{\Xi}^2}{M^2} \right) \frac{\partial}{\partial \Omega} \int_0^{2s_0} d\omega \int_0^1 du \frac{m_{\Sigma}}{\bar{u}\omega} \end{aligned}$$

$$\begin{aligned}
& \exp\left(-\frac{s_r^\Omega}{M^2}\right) \theta(sth - s_r^\Omega) \theta(s_r^\Omega - (\sqrt{\Omega} + m_{Q'} + m_q)^2) \\
& \times \left\{ \bar{u} \left[-2f^{(1)} \bar{u} (\hat{\psi}_{\parallel}^{n*} + \hat{\psi}_{\parallel}^{n*}) (m_Q + 3\bar{u}\omega) \right. \right. \\
& + f^{(1)} (\tilde{\psi}_{\parallel}^{n*} - \tilde{\psi}_{\parallel}^{1*}) (q^2 + \bar{u}\omega(2q \cdot v + \bar{u}\omega)) \\
& + 2f^{(2)} m_Q (3\hat{\psi}_{\parallel}^{n*} \bar{u} + \tilde{\psi}_{\parallel}^{n*} q \cdot v + \tilde{\psi}_{\parallel}^{1*} \bar{u}\omega) \\
& + 4f^{(2)} \hat{\psi}_{\parallel}^{n*} \bar{u} (q \cdot v + \bar{u}\omega) \left. \right] \frac{1}{2m_\Xi} \\
& + 2\bar{u} [3f^{(1)} \bar{u} (\hat{\psi}_{\parallel}^{n*} + \hat{\psi}_{\parallel}^{n*}) \\
& + f^{(2)} m_Q (\tilde{\psi}_{\parallel}^{n*} - \tilde{\psi}_{\parallel}^{1*})] \frac{m_\Sigma + m_\Xi}{2m_\Xi} \left. \right\} \Big|_{\Omega=m_Q^2} \\
& - \frac{1}{f_\Xi} \exp\left(\frac{m_\Xi^2}{M^2}\right) \frac{1}{2} \left(\frac{\partial}{\partial \Omega} \right)^2 \int_0^{2s_0} d\omega \int_0^1 du \frac{m_\Sigma}{\bar{u}\omega} \\
& \exp\left(-\frac{s_r^\Omega}{M^2}\right) \theta(sth - s_r^\Omega) \theta(s_r^\Omega - (m_Q + m_{Q'} + m_q)^2) \\
& \times \left\{ 4\bar{u}^2 \left[f^{(1)} \bar{u}\omega (\hat{\psi}_{\parallel}^{n*} + \hat{\psi}_{\parallel}^{n*}) (q^2 + \bar{u}\omega(2q \cdot v + \bar{u}\omega)) \right. \right. \\
& - 2f^{(2)} m_Q \hat{\psi}_{\parallel}^{n*} (q^2 + q \cdot v \bar{u}\omega) \left. \right] \frac{1}{2m_\Xi} \\
& - 4\bar{u}^2 \left[f^{(1)} (\hat{\psi}_{\parallel}^{n*} + \hat{\psi}_{\parallel}^{n*}) (q^2 + \bar{u}\omega(2q \cdot v + \bar{u}\omega)) \right. \\
& + 2f^{(2)} m_Q \hat{\psi}_{\parallel}^{n*} (q \cdot v + \bar{u}\omega) \left. \right] \frac{m_\Sigma + m_\Xi}{2m_\Xi} \left. \right\} \Big|_{\Omega=m_Q^2}, \quad (39)
\end{aligned}$$

where s_r and s_r^Ω are defined as the singularity position of the heavy quark propagator in the correlation function. In other words, they are the roots of the following two equations respectively

$$\begin{aligned}
\frac{\bar{u}\omega}{m_\Sigma} s_r + H(u, \omega, q^2) - m_Q^2 &= 0, \\
\frac{\bar{u}\omega}{m_\Sigma} s_r^\Omega + H(u, \omega, q^2) - \Omega &= 0. \quad (40)
\end{aligned}$$

Further, since the G_i^+ can be obtained in a similar way, we will not show them explicitly here.

3 Numerical results

3.1 Transition form factors

In this work, the heavy quark masses are taken as $m_c = (1.35 \pm 0.10)$ GeV and $m_b = (4.7 \pm 0.1)$ GeV while the masses of light quarks are neglected. The masses, lifetimes and decay constants f_Ξ of doubly heavy baryons are shown in Table 3 [41–44]. While the masses and decay constants of Σ_Q are taken as $m_{\Sigma_c} = 2.454$ GeV, $m_{\Sigma_b} = 5.814$ GeV, and

Table 3 Masses, lifetimes and decay constants of doubly heavy baryons

Baryons	Mass (GeV)	Lifetime (fs)	f_Ξ (GeV ³) [17]
Ξ_{cc}^{++}	3.621 [1]	256 [3]	0.109 ± 0.021
Ξ_{cc}^+	3.621 [1]	45 [46]	0.109 ± 0.021
Ξ_{bc}^+	6.943 [47]	244 [41]	0.176 ± 0.040
Ξ_{bc}^0	6.943 [47]	93 [41]	0.176 ± 0.040
Ξ_{bb}^0	10.143 [47]	370 [41]	0.281 ± 0.071
Ξ_{bb}^-	10.143 [47]	370 [41]	0.281 ± 0.071

Table 4 Threshold s_{th} of $\Xi_{QQ'}$, Borel parameters M^2 , and q^2 range for fitting form factors

Channel	s_{th} (GeV ²)	M^2 (GeV ²)	Fit range (GeV ²)
$\Xi_{cc} \rightarrow \Sigma_c$	16 ± 1	15 ± 1	$0 < q^2 < 0.8$
$\Xi_{bb} \rightarrow \Sigma_b$	112 ± 2	20 ± 1	$0 < q^2 < 6$
$\Xi_{bc} \rightarrow \Sigma_c$	54 ± 1.5	20 ± 1	$0 < q^2 < 6$
$\Xi_{bc} \rightarrow \Sigma_b$	54 ± 1.5	20 ± 1	$0 < q^2 < 0.8$

$f^{(1)} = f^{(2)} = 0.038$ [45]. The upper limit of light quarks momentum is $s_0 = 1.2$ GeV [38].

The threshold s_{th} of $\Xi_{QQ'}$ and Borel parameters M^2 are shown in Table 4, which are chosen as to make the form factors stable. Since the light-cone OPE for heavy baryon transition is reliable in the region where q^2 is positive but not too large, to parameterize the form factors one needs to limit the q^2 region which are listed in the last column of Table 4. The parameterization formula used in this work is

$$F(q^2) = \frac{F(0)[1 + a(q^2) + b(q^2)^2]}{1 - \frac{q^2}{m_{\text{fit}}^2} + \delta \left(\frac{q^2}{m_{\text{fit}}^2} \right)^2}, \quad (41)$$

where the denominator reflects the pole structure of the form factors, while the nominator is due to the assumption that the form factors have a polynomial form at small q^2 . The numerical and fitting results for the form factors are given in Table 5, where the “Null” means that the corresponding parameter is set to be zero before the fitting.

Since we are interested in the error coming from our theoretical approach, the errors of the form factors are estimated from the thresholds s_{th} and Borel parameters M^2 , which are the only tunable parameters in LCSR. The q^2 dependence of the form factors corresponding to the four channels are shown in Fig. 2, with the parameters s_{th} , M^2 fixed at their center values as shown in Table 4.

Tables 6 and 7 show the comparison between this work and other works in the previous literatures. From the comparison one can find that the vector form factors f_i obtained in this work fit well with those of other works, especially that

Table 5 Form factors of the transition $\Xi_{QQ'q_2} \rightarrow \Sigma_{Q'q_1q_2}$. $F(0)$, m_{fit} , δ , a and b correspond to the five fitting parameters in Eq. (41). The form factors of $\Xi_{QQ'q} \rightarrow \Sigma_{Q'qq}$ are just $\sqrt{2}$ times those of $\Xi_{QQ'q_2} \rightarrow \Sigma_{Q'q_1q_2}$, which are not shown explicitly in this table

F	$F(0)$	m_{fit}	δ	a	b
$f_1^{\Xi_{cc}^+ \rightarrow \Sigma_c^+}$	-0.86 ± 0.08	1.22 ± 0.06	0.46 ± 0.02	Null	Null
$f_2^{\Xi_{cc}^+ \rightarrow \Sigma_c^+}$	-1.19 ± 0.06	1.20 ± 0.01	0.25 ± 0.01	-0.67 ± 0.02	-0.18 ± 0.0
$f_3^{\Xi_{cc}^+ \rightarrow \Sigma_c^+}$	-0.66 ± 0.17	0.93 ± 0.06	0.43 ± 0.03	Null	Null
$g_1^{\Xi_{cc}^+ \rightarrow \Sigma_c^+}$	0.07 ± 0.02	1.24 ± 0.01	0.21 ± 0.01	1.15 ± 0.28	1.88 ± 0.36
$g_2^{\Xi_{cc}^+ \rightarrow \Sigma_c^+}$	2.08 ± 0.1	0.85 ± 0.12	0.26 ± 0.04	-1.63 ± 0.47	0.66 ± 0.39
$g_3^{\Xi_{cc}^+ \rightarrow \Sigma_c^+}$	7.0 ± 0.02	1.13 ± 0.03	0.24 ± 0.0	Null	Null
$f_1^{\Xi_{bc}^+ \rightarrow \Sigma_b^0}$	-0.38 ± 0.03	1.14 ± 0.02	0.48 ± 0.01	Null	Null
$f_2^{\Xi_{bc}^+ \rightarrow \Sigma_b^0}$	-1.46 ± 0.06	2.32 ± 1.15	-6.37 ± 6.71	-0.18 ± 0.55	-0.86 ± 0.60
$f_3^{\Xi_{bc}^+ \rightarrow \Sigma_b^0}$	-0.84 ± 0.08	0.89 ± 0.01	0.46 ± 0.01	Null	Null
$g_1^{\Xi_{bc}^+ \rightarrow \Sigma_b^0}$	0.05 ± 0.01	1.6 ± 0.54	0.89 ± 0.36	1.92 ± 0.45	-2.54 ± 0.77
$g_2^{\Xi_{bc}^+ \rightarrow \Sigma_b^0}$	4.46 ± 0.22	1.98 ± 0.01	1.17 ± 0.22	-0.59 ± 0.00	0.07 ± 0.02
$g_3^{\Xi_{bc}^+ \rightarrow \Sigma_b^0}$	11.64 ± 0.35	0.94 ± 0.02	0.28 ± 0.02	Null	Null
$f_1^{\Xi_{bc}^0 \rightarrow \Sigma_c^+}$	-0.21 ± 0.01	6.48 ± 0.0	-0.41 ± 0.01	Null	Null
$f_2^{\Xi_{bc}^0 \rightarrow \Sigma_c^+}$	-0.09 ± 0.0	3.43 ± 6.86	0.45 ± 0.0	Null	Null
$f_3^{\Xi_{bc}^0 \rightarrow \Sigma_c^+}$	0.03 ± 0.0	3.3 ± 0.04	0.43 ± 0.02	-0.33 ± 0.02	0.01 ± 0.0
$g_1^{\Xi_{bc}^0 \rightarrow \Sigma_c^+}$	0.01 ± 0.0	3.96 ± 0.93	0.42 ± 0.22	1.58 ± 0.53	-0.08 ± 0.02
$g_2^{\Xi_{bc}^0 \rightarrow \Sigma_c^+}$	0.12 ± 0.01	4.43 ± 0.21	1.01 ± 0.32	-0.06 ± 0.0	0.0 ± 0.0
$g_3^{\Xi_{bc}^0 \rightarrow \Sigma_c^+}$	0.47 ± 0.02	2.78 ± 0.01	0.3 ± 0.0	-0.12 ± 0.0	0.01 ± 0.0
$f_1^{\Xi_{bb}^- \rightarrow \Sigma_b^0}$	-0.3 ± 0.01	3.5 ± 0.15	1.5 ± 0.19	-0.07 ± 0.01	0.01 ± 0.0
$f_2^{\Xi_{bb}^- \rightarrow \Sigma_b^0}$	0.1 ± 0.01	5.6 ± 1.72	2.85 ± 2.02	-0.28 ± 0.03	0.0 ± 0.01
$f_3^{\Xi_{bb}^- \rightarrow \Sigma_b^0}$	-0.26 ± 0.01	3.56 ± 0.01	0.52 ± 0.05	0.07 ± 0.01	-0.01 ± 0.0
$g_1^{\Xi_{bb}^- \rightarrow \Sigma_b^0}$	-0.01 ± 0.01	4.55 ± 3.87	1.4 ± 1.43	-2.0 ± 6.6	0.12 ± 12.27
$g_2^{\Xi_{bb}^- \rightarrow \Sigma_b^0}$	0.05 ± 0.02	4.35 ± 0.11	1.21 ± 0.07	-0.32 ± 0.03	0.07 ± 0.01
$g_3^{\Xi_{bb}^- \rightarrow \Sigma_b^0}$	1.02 ± 0.13	3.26 ± 0.12	0.56 ± 0.07	-0.32 ± 0.01	0.02 ± 0.0

from QCDSR. However, there are certain difference for other form factors g_i . Some comments on such difference between various works are given as follows.

In terms of this work and the work based on QCDSR, the difference between the form factors derived from the two approaches can be attributed to following two points:

- In the QCDSR work, the authors performed a leading order calculation for a three-point correlation function by OPE, where all the non-perturbative effects are produced by the condensates of the dimension 3 to 5 operators. In the LCSR work, the non-perturbative effects are produced by the LCDAs of Σ_Q baryons, which were also derived by QCDSR [38], but with only dimen-

sion 3 condensates included. As a result, the amount of non-perturbative effects introduced in this work and the QCDSR work are different.

- Ideally, when all order QCD corrections and the complete series of OPE are included, the results from QCDSR and LCSR calculation should be equivalent. However, both in the QCDSR and LCSR works, only leading order calculations are performed. In addition, the QCDSR work only contains contribution from several low dimensional operator condensates, while in this work, only several leading twist LCDAs are introduced. Thus we have in fact extracted two parts of the same form factor respectively in the two works. Generally, these two parts will overlap but will not be the same.

Fig. 2 q^2 dependence of the $\Xi_{QQ'q_2} \rightarrow \Sigma_{Q'q_1q_2}$ form factors. The first two graphs correspond to $\Xi_{cc} \rightarrow \Sigma_c$, the second two graphs correspond to $\Xi_{bc} \rightarrow \Sigma_b$, the third two graphs correspond to $\Xi_{bc} \rightarrow \Sigma_c$ and the fourth two graphs correspond to $\Xi_{bb} \rightarrow \Sigma_b$. Here the parameters

s_{th}, M^2 are fixed at their center values as shown in Table 4. For the case of $\Xi_{QQ'q} \rightarrow \Sigma_{Q'qq}$, the vertical scale needs to be enlarged by a factor $\sqrt{2}$

Table 6 Comparison of Ξ_{cc} decay form factors derived in this work with the results from QCD sum rules (QCDSR) [39], light-front quark model (LFQM) [8], the non-relativistic quark model (NRQM) and the MIT bag model (MBM) [48]

Transitions	$F(0)$	This work	QCDSR [39]	LFQM [8]	NRQM [48]	MBM [48]
$\Xi_{cc}^{++} \rightarrow \Sigma_c^+$	$f_1(0)$	-0.86 ± 0.08	-0.35 ± 0.04	-0.46	-0.28	-0.30
	$f_2(0)$	-1.19 ± 0.06	1.15 ± 0.12	1.04	0.14	0.91
	$f_3(0)$	-0.66 ± 0.17	-1.40 ± 0.39	$-$	-0.10	0.07
	$g_1(0)$	0.07 ± 0.02	-0.23 ± 0.06	-0.62	-0.70	-0.56
	$g_2(0)$	2.08 ± 0.1	-0.26 ± 0.15	0.04	-0.02	0.05
	$g_3(0)$	7.0 ± 0.02	2.68 ± 0.39	$-$	0.10	2.59

Table 7 Comparison of Ξ_{bb} and Ξ_{bc} decay form factors derived in this work with the results from QCD sum rules (QCDSR) [39] and light-front quark model (LFQM) [8]

Transitions	$F(0)$	This work	QCDSR [39]	LFQM [8]
$\Xi_{bc}^+ \rightarrow \Sigma_b^0$	$f_1(0)$	-0.38 ± 0.03	-0.28 ± 0.03	-0.32
	$f_2(0)$	-1.46 ± 0.06	2.04 ± 0.21	1.54
	$f_3(0)$	-0.84 ± 0.08	-3.78 ± 1.38	$-$
	$g_1(0)$	0.05 ± 0.01	-0.13 ± 0.06	-0.41
	$g_2(0)$	4.46 ± 0.22	-0.18 ± 0.25	0.18
	$g_3(0)$	11.64 ± 0.35	10.1 ± 1.4	$-$
$\Xi_{bc}^0 \rightarrow \Sigma_c^+$	$f_1(0)$	-0.21 ± 0.01	-0.22 ± 0.03	-0.07
	$f_2(0)$	-0.09 ± 0.0	0.36 ± 0.06	0.10
	$f_3(0)$	0.03 ± 0.0	-0.45 ± 0.07	$-$
	$g_1(0)$	0.01 ± 0.0	-0.22 ± 0.03	-0.10
	$g_2(0)$	0.12 ± 0.01	-0.31 ± 0.05	-0.003
	$g_3(0)$	0.47 ± 0.02	0.47 ± 0.07	$-$
$\Xi_{bb}^- \rightarrow \Sigma_b^0$	$f_1(0)$	-0.3 ± 0.01	-0.12 ± 0.01	-0.06
	$f_2(0)$	0.1 ± 0.01	0.22 ± 0.03	0.15
	$f_3(0)$	-0.26 ± 0.01	-0.46 ± 0.06	$-$
	$g_1(0)$	-0.01 ± 0.01	-0.12 ± 0.01	-0.09
	$g_2(0)$	0.05 ± 0.02	-0.19 ± 0.03	-0.02
	$g_3(0)$	1.02 ± 0.13	0.49 ± 0.07	$-$

In terms of this work and the work based on LFQM, the difference between the form factors derived from the two approaches can be attributed to two points:

- In this work, the hadron transition matrix element is extracted from a correlation function by finding the mass pole residues, where uncertainties may occur when one separates the single particle state from the continuum spectrum. However, the LFQM directly expresses the initial and final hadronic states in terms of the quark level wave functions. This operation avoids the potential pollution from heavier spectrum but introduces more tunable parameters, which may be inconsistent with LCSR.
- The baryon pictures between LCSR and LFQM are very different. In LCSR, the Σ_Q baryons are described by their LCDAs, which are defined in full QCD theory and describe a three-body system. In LFQM, the baryons are described by a two-body Gaussian shape wave function,

where the two spectator quarks are combined to be a point-like diquark.

3.2 Semi-leptonic decays

The effective Hamiltonian inducing the semi-leptonic decays $\Xi_{QQ'} \rightarrow \Sigma_{Q'}$ is

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(V_{ub} [\bar{u} \gamma_\mu (1 - \gamma_5) b] [\bar{l} \gamma^\mu (1 - \gamma_5) \nu] + V_{cd}^* [\bar{d} \gamma_\mu (1 - \gamma_5) c] [\bar{\nu} \gamma^\mu (1 - \gamma_5) l] \right), \quad (42)$$

where the Fermi constant G_F and CKM matrix elements are taken from Refs. [49, 50]

$$G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}, \quad |V_{ub}| = 0.00357, \quad |V_{cd}| = 0.225. \quad (43)$$

The decay amplitudes induced by vector current and axial-vector current can be expressed in terms of the following helicity amplitudes

$$\begin{aligned}
 H_{\frac{1}{2},0}^V &= -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} \left((M_1 + M_2) f_1 - \frac{q^2}{M_1} f_2 \right), \\
 H_{\frac{1}{2},0}^A &= -i \frac{\sqrt{Q_+}}{\sqrt{q^2}} \left((M_1 - M_2) g_1 + \frac{q^2}{M_1} g_2 \right), \\
 H_{\frac{1}{2},1}^V &= i \sqrt{2Q_-} \left(-f_1 + \frac{M_1 + M_2}{M_1} f_2 \right), \\
 H_{\frac{1}{2},1}^A &= i \sqrt{2Q_+} \left(-g_1 - \frac{M_1 - M_2}{M_1} g_2 \right), \\
 H_{\frac{1}{2},t}^V &= -i \frac{\sqrt{Q_+}}{\sqrt{q^2}} \left((M_1 - M_2) f_1 + \frac{q^2}{M_1} f_3 \right), \\
 H_{\frac{1}{2},t}^A &= -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} \left((M_1 + M_2) g_1 - \frac{q^2}{M_1} g_3 \right), \quad (44)
 \end{aligned}$$

where $Q_{\pm} = (M_1 \pm M_2)^2 - q^2$. M_1 and M_2 are the masses of the initial and final baryon. The amplitudes with negative helicity external states have the following simple relations with those having positive helicities

$$H_{-\lambda_2, -\lambda_W}^V = H_{\lambda_2, \lambda_W}^V \quad \text{and} \quad H_{-\lambda_2, -\lambda_W}^A = -H_{\lambda_2, \lambda_W}^A, \quad (45)$$

λ_2 and λ_W denotes the polarizations of the final $\Sigma_{Q'}$ and the intermediate W boson, respectively. The total helicity amplitudes induced by the $V - A$ current are

$$H_{\lambda_2, \lambda_W} = H_{\lambda_2, \lambda_W}^V - H_{\lambda_2, \lambda_W}^A. \quad (46)$$

Decay widths of $\Xi_{QQ'} \rightarrow \Sigma_{Q'} l \nu$ can be classified by the polarization of the $l \nu$ pairs. The decay widths with longitudinally or transversely polarized $l \nu$ pairs are

$$\begin{aligned}
 \frac{d\Gamma_L}{dq^2} &= \frac{G_F^2 |V_{CKM}|^2 q^2 p (1 - \hat{m}_l^2)^2}{384\pi^3 M_1^2} \\
 &\times \left((2 + \hat{m}_l^2) (|H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},0}|^2) \right. \\
 &\left. + 3\hat{m}_l^2 (|H_{-\frac{1}{2},t}|^2 + |H_{\frac{1}{2},t}|^2) \right), \quad (47)
 \end{aligned}$$

$$\begin{aligned}
 \frac{d\Gamma_T}{dq^2} &= \frac{G_F^2 |V_{CKM}|^2 q^2 p (1 - \hat{m}_l^2)^2 (2 + \hat{m}_l^2)}{384\pi^3 M_1^2} \\
 &\times (|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2), \quad (48)
 \end{aligned}$$

where $\hat{m}_l \equiv m_l/\sqrt{q^2}$. Note that when calculating form factors with LCSR, we choose the rest frame of the final baryon $\Sigma_{Q'}$. While for simplicity, we calculate the decay width in the rest frame of the initial particle $\Xi_{QQ'}$. $p = \sqrt{Q_+ Q_-}/(2M_1)$ is the three-momentum magnitude of $\Sigma_{Q'}$ in the rest frame of $\Xi_{QQ'}$. By integrating out the squared transfer momentum q^2 , one can obtain the total decay width

$$\Gamma = \int_{m_l^2}^{(M_1 - M_2)^2} dq^2 \left(\frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2} \right). \quad (49)$$

Table 8 shows the integrated partial decay widths, branching ratios and the ratios of Γ_L/Γ_T for various semi-leptonic $\Xi_{QQ'} \rightarrow \Sigma_{Q'} l \nu_l$ processes, where $l = e, \mu, \tau$. The masses of e and μ have been neglected while the mass of τ is taken as 1.78 GeV [49]. Figure 3 shows the q^2 dependence of the differential decay widths corresponding to four channels. Table 9 gives a comparison of our decay width results with those given in the literatures. It can be found that most of the decay widths from this and other works are of the same order of magnitude.

For these phenomenology results, here are some remarks:

- The errors of the decay widths given in Table 8 and Fig. 3 totally results from the errors of form factors.
- From Table 8, one can find that the decay widths of Ξ_{cc} and $\Xi_{bc} \rightarrow \Sigma_b$ decays are several orders of magnitude larger than those of Ξ_{bb} decays.

This phenomenon is mainly due to the huge difference between the CKM matrix elements $|V_{cd}| = 0.225 \gg |V_{ub}| = 0.00357$. However, the decay width of $\Xi_{bc} \rightarrow \Sigma_c$ is also larger than those of Ξ_{bb} decays. This is due to the effect of broken heavy quark symmetry, which can be easily seen in the view of initial baryons masses

$$\frac{\Gamma(\Xi_{bc} \rightarrow \Sigma_c l \nu_l)}{\Gamma(\Xi_{bb} \rightarrow \Sigma_b l \nu_l)} \propto \left(\frac{M_{\Xi_{bb}}}{M_{\Xi_{bc}}} \right)^2 > 1. \quad (50)$$

For the same reason we can understand why $\Gamma(\Xi_{cc} \rightarrow \Sigma_c l \nu_l)$ is also larger than $\Gamma(\Xi_{bc} \rightarrow \Sigma_b l \nu_l)$. In fact, such feature also emerges in the case of B and D decays.

- According to the SU(3) symmetry, the decay widths of various semi-leptonic channels are related with each other. References [10] and [18] have offered a systematic SU(3) analysis of doubly heavy baryon decays as well as

Table 8 Decay widths and branching ratios of the semi-leptonic $\Xi_{QQ'} \rightarrow \Sigma_{Q'} l \nu_l$ decays, where $l = e, \mu$

Channels	Γ/GeV	$\mathcal{B}$	Γ_L/Γ_T
$\Xi_{cc}^{++} \rightarrow \Sigma_c^+ l^+ \nu_l$	$(7.12 \pm 1.33) \times 10^{-15}$	$(2.77 \pm 0.48) \times 10^{-3}$	6.92 ± 2.8
$\Xi_{cc}^+ \rightarrow \Sigma_c^0 l^+ \nu_l$	$(1.42 \pm 0.26) \times 10^{-14}$	$(9.72 \pm 1.68) \times 10^{-3}$	6.92 ± 2.8
$\Xi_{bc}^+ \rightarrow \Sigma_b^0 l^+ \nu_l$	$(2.17 \pm 0.19) \times 10^{-15}$	$(8.06 \pm 0.7) \times 10^{-4}$	2.91 ± 0.47
$\Xi_{bc}^0 \rightarrow \Sigma_b^- l^+ \nu_l$	$(4.34 \pm 0.38) \times 10^{-15}$	$(6.17 \pm 0.53) \times 10^{-4}$	2.91 ± 0.47
$\Xi_{bc}^0 \rightarrow \Sigma_c^+ l^- \bar{\nu}_l$	$(5.73 \pm 1.15) \times 10^{-17}$	$(8.1 \pm 1.62) \times 10^{-6}$	2.21 ± 1.6
$\Xi_{bc}^0 \rightarrow \Sigma_c^+ \tau^- \bar{\nu}_\tau$	$(3.5 \pm 0.62) \times 10^{-17}$	$(4.49 \pm 0.87) \times 10^{-6}$	2.45 ± 1.68
$\Xi_{bc}^+ \rightarrow \Sigma_c^{++} l^- \bar{\nu}_l$	$(1.14 \pm 0.23) \times 10^{-16}$	$(4.26 \pm 0.85) \times 10^{-5}$	2.21 ± 1.6
$\Xi_{bc}^+ \rightarrow \Sigma_c^{++} \tau^- \bar{\nu}_\tau$	$(7.0 \pm 1.24) \times 10^{-17}$	$(2.6 \pm 0.46) \times 10^{-5}$	2.45 ± 1.68
$\Xi_{bb}^- \rightarrow \Sigma_b^0 l^- \bar{\nu}_l$	$(1.26 \pm 0.32) \times 10^{-16}$	$(7.05 \pm 1.82) \times 10^{-5}$	1.57 ± 1.33
$\Xi_{bb}^- \rightarrow \Sigma_b^0 \tau^- \bar{\nu}_\tau$	$(6.34 \pm 1.62) \times 10^{-17}$	$(3.89 \pm 0.91) \times 10^{-5}$	1.36 ± 1.21
$\Xi_{bb}^0 \rightarrow \Sigma_b^+ l^- \bar{\nu}_l$	$(2.52 \pm 0.31) \times 10^{-16}$	$(1.41 \pm 0.36) \times 10^{-4}$	1.57 ± 1.33
$\Xi_{bb}^0 \rightarrow \Sigma_b^+ \tau^- \bar{\nu}_\tau$	$(1.26 \pm 0.32) \times 10^{-16}$	$(7.79 \pm 1.82) \times 10^{-5}$	1.36 ± 1.21

Fig. 3 q^2 dependence of the semi-leptonic $\Xi_{QQ'} \rightarrow \Sigma_{Q'} l \nu_l$ decay widths. $d\Gamma_L/dq^2$ are shown by the blue bands while $d\Gamma_T/dq^2$ are shown by the red bands. The dashed lines are the center value curves

and the band width reflects the corresponding error. For the case of $\Xi_{QQ'} \rightarrow \Sigma_{Q'} qq$, the vertical scale needs to be enlarged by a factor 2

a complete decay width relations. Several channels that we have not calculated in this work can still be estimated by SU(3) symmetry [10]

$$\begin{aligned}
 \Gamma(\Omega_{cc}^+ \rightarrow \Xi_c^0 l^+ \nu) &= \Gamma(\Xi_{cc}^{++} \rightarrow \Sigma_c^+ l^+ \nu) \\
 &= \frac{1}{2} \Gamma(\Xi_{cc}^+ \rightarrow \Sigma_c^0 l^+ \nu) \\
 &= (7.12 \pm 1.33) \times 10^{-15} \text{ GeV},
 \end{aligned}$$

Table 9 Comparison of the decay widths (in units of GeV) for the semi-leptonic decays in this work with the results derived from QCD sum rules (QCDSR) [39], the light-front quark model (LFQM) [8],

the heavy quark spin symmetry (HQSS) [51], the nonrelativistic quark model (NRQM) [48] and the MIT bag model (MBM) [48] in literatures

Channels	This work	QCDSR [39]	LFQM [8]	HQSS [51]	NRQM [48]	MBM [48]
$\Xi_{cc}^{++} \rightarrow \Sigma_c^+ l^+ \nu_l$	$(7.12 \pm 1.33) \times 10^{-15}$	$(2.3 \pm 0.4) \times 10^{-15}$	9.60×10^{-15}	5.22×10^{-15}	6.58×10^{-15}	2.63×10^{-15}
$\Xi_{cc}^+ \rightarrow \Sigma_c^0 l^+ \nu_l$	$(1.42 \pm 0.26) \times 10^{-14}$	$(4.6 \pm 0.9) \times 10^{-15}$	1.91×10^{-14}	1.04×10^{-14}	1.32×10^{-14}	5.92×10^{-15}
$\Xi_{bb}^- \rightarrow \Sigma_b^0 l^- \bar{\nu}_l$	$(1.26 \pm 0.32) \times 10^{-16}$	$(1.3 \pm 0.2) \times 10^{-16}$	3.33×10^{-17}	—	—	—
$\Xi_{bb}^0 \rightarrow \Sigma_b^+ l^- \bar{\nu}_l$	$(2.52 \pm 0.31) \times 10^{-16}$	$(2.5 \pm 0.4) \times 10^{-16}$	6.67×10^{-17}	—	—	—
$\Xi_{bc}^0 \rightarrow \Sigma_c^+ l^- \bar{\nu}_l$	$(5.73 \pm 1.15) \times 10^{-17}$	$(4.2 \pm 0.7) \times 10^{-16}$	4.74×10^{-17}	—	—	—
$\Xi_{bc}^+ \rightarrow \Sigma_c^{++} l^- \bar{\nu}_l$	$(1.14 \pm 0.23) \times 10^{-16}$	$(8.4 \pm 1.4) \times 10^{-16}$	9.48×10^{-17}	—	—	—
$\Xi_{bc}^+ \rightarrow \Sigma_b^0 l^+ \nu_l$	$(2.17 \pm 0.19) \times 10^{-15}$	$(1.5 \pm 0.3) \times 10^{-15}$	4.63×10^{-15}	—	—	—
$\Xi_{bc}^0 \rightarrow \Sigma_b^- l^+ \nu_l$	$(4.34 \pm 0.38) \times 10^{-15}$	$(3.0 \pm 0.5) \times 10^{-15}$	9.18×10^{-15}	—	—	—

$$\begin{aligned}
\Gamma(\Omega_{bc}^0 \rightarrow \Xi_b'^- l^+ \nu) &= \Gamma(\Xi_{bc}^+ \rightarrow \Sigma_b^0 l^+ \nu) \\
&= \frac{1}{2} \Gamma(\Xi_{bc}^0 \rightarrow \Sigma_b^- l^+ \nu) \\
&= (2.17 \pm 0.19) \times 10^{-15} \text{ GeV}, \\
\Gamma(\Omega_{bc}^0 \rightarrow \Xi_c'^+ l^- \bar{\nu}) &= \Gamma(\Xi_{bc}^0 \rightarrow \Sigma_c^+ l^- \bar{\nu}) \\
&= \frac{1}{2} \Gamma(\Xi_{bc}^+ \rightarrow \Sigma_c^{++} l^- \bar{\nu}) \\
&= (5.73 \pm 1.15) \times 10^{-17} \text{ GeV}, \\
\Gamma(\Omega_{bb}^- \rightarrow \Xi_b'^0 l^- \bar{\nu}) &= \Gamma(\Xi_{bb}^- \rightarrow \Sigma_b^0 l^- \bar{\nu}) \\
&= \frac{1}{2} \Gamma(\Xi_{bb}^0 \rightarrow \Sigma_b^+ l^- \bar{\nu}) \\
&= (1.26 \pm 0.32) \times 10^{-16} \text{ GeV}.
\end{aligned} \tag{51}$$

- From the comparison shown in Table 9, it seems that our semi-leptonic decay widths fit well with those from other works. Especially, our prediction for $\Gamma(\Xi_{cc} \rightarrow \Sigma_c l \nu_l)$ is almost the same as those from LFQM, HQSS, and NRQM calculations. Our prediction for other decay channels fit with either QCDSR or LFQM calculations.

4 Conclusions

As a continuation of our previous work on $\Xi_{QQ'}$ decays into anti-triplets $\Lambda_{Q'}$, this work performs a phenomenological study on the semi-leptonic decay of $\Xi_{QQ'}$ into an sextet baryon $\Sigma_{Q'}$ with LCSR approach. The transition form factors have been derived with the parallel LCDAs of the final states $\Sigma_{Q'}$. With the numerical results of these form factors, the decay widths and branching ratios of the corresponding semi-leptonic process are predicted. The error estimation and theoretical analyses are also given in detail. Our prediction for semi-leptonic decay widths is almost the same as those from most of other works we have compared in this work. Phenomenologically, we hope our predictions for the

branching ratios could be tested by LHCb and other experiments in the future. Theoretically, we hope our calculation can help people to exam the singly heavy baryon LCDAs, and further improvement on these LCDAs as well as higher order corrections on this LCSR calculation are also expected in the future.

Acknowledgements The authors are very grateful to Prof. Wei Wang for useful discussions. This work is supported in part by National Natural Science Foundation of China under Grants Nos. 11575110, 11735010, and 11911530088, Natural Science Foundation of Shanghai under Grant No. 15DZ2272100, and by Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: There are no external data associated with the manuscript.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.
Funded by SCOAP³.

References

1. R. Aaij et al., [LHCb Collaboration], Phys. Rev. Lett. **119**(11), 112001 (2017). <https://doi.org/10.1103/PhysRevLett.119.112001>. [arXiv:1707.01621](https://arxiv.org/abs/1707.01621) [hep-ex]
2. F.S. Yu, H.Y. Jiang, R.H. Li, C.D. Lü, W. Wang, Z.X. Zhao, Chin. Phys. C **42**(5), 051001 (2018). <https://doi.org/10.1088/1674-1137/42/5/051001>. [arXiv:1703.09086](https://arxiv.org/abs/1703.09086) [hep-ph]
3. R. Aaij et al., [LHCb Collaboration], Phys. Rev. Lett. **121**(5), 052002 (2018). <https://doi.org/10.1103/PhysRevLett.121.052002>. [arXiv:1806.02744](https://arxiv.org/abs/1806.02744) [hep-ex]

4. R. Aaij et al., [LHCb Collaboration], Phys. Rev. Lett. **121**(16), 162002 (2018). <https://doi.org/10.1103/PhysRevLett.121.162002>. [arXiv:1807.01919](https://arxiv.org/abs/1807.01919) [hep-ex]
5. M.T. Traill, [LHCb Collaboration], PoS Hadron 2017, 067 (2018). <https://doi.org/10.22323/1.310.0067>
6. A. Cerri et al., [arXiv:1812.07638](https://arxiv.org/abs/1812.07638)
7. R. Aaij et al., [LHCb Collaboration], [arXiv:1909.12273](https://arxiv.org/abs/1909.12273) [hep-ex]
8. W. Wang, F.S. Yu, Z.X. Zhao, Eur. Phys. J. C **77**(11), 781 (2017). <https://doi.org/10.1140/epjc/s10052-017-5360-1>. [arXiv:1707.02834](https://arxiv.org/abs/1707.02834) [hep-ph]
9. L. Meng, N. Li, S. I. Zhu, Eur. Phys. J. A **54**(9), 143 (2018) <https://doi.org/10.1140/epja/i2018-12578-2>. [arXiv:1707.03598](https://arxiv.org/abs/1707.03598) [hep-ph]
10. W. Wang, Z.P. Xing, J. Xu, Eur. Phys. J. C **77**(11), 800 (2017). <https://doi.org/10.1140/epjc/s10052-017-5363-y>. [arXiv:1707.06570](https://arxiv.org/abs/1707.06570) [hep-ph]
11. T. Gutsche, M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij, Phys. Rev. D **96**(5), 054013 (2017). <https://doi.org/10.1103/PhysRevD.96.054013>. [arXiv:1708.00703](https://arxiv.org/abs/1708.00703) [hep-ph]
12. H.S. Li, L. Meng, Z.W. Liu, S.L. Zhu, Phys. Lett. B **777**, 169 (2018). <https://doi.org/10.1016/j.physletb.2017.12.031>. [arXiv:1708.03620](https://arxiv.org/abs/1708.03620) [hep-ph]
13. Z.H. Guo, Phys. Rev. D **96**(7), 074004 (2017). <https://doi.org/10.1103/PhysRevD.96.074004>. [arXiv:1708.04145](https://arxiv.org/abs/1708.04145) [hep-ph]
14. L.Y. Xiao, K.L. Wang, Q.F. Lu, X.H. Zhong, S.L. Zhu, Phys. Rev. D **96**(9), 094005 (2017). <https://doi.org/10.1103/PhysRevD.96.094005>. [arXiv:1708.04384](https://arxiv.org/abs/1708.04384) [hep-ph]
15. N. Sharma, R. Dhir, Phys. Rev. D **96**(11), 113006 (2017). <https://doi.org/10.1103/PhysRevD.96.113006>. [arXiv:1709.08217](https://arxiv.org/abs/1709.08217) [hep-ph]
16. Y.L. Ma, M. Harada, J. Phys. G **45**(7), 075006 (2018). <https://doi.org/10.1088/1361-6471/aac86e>. [arXiv:1709.09746](https://arxiv.org/abs/1709.09746) [hep-ph]
17. X.H. Hu, Y.L. Shen, W. Wang, Z.X. Zhao, Chin. Phys. C **42**(12), 123102 (2018). <https://doi.org/10.1088/1674-1137/42/12/123102>. [arXiv:1711.10289](https://arxiv.org/abs/1711.10289) [hep-ph]
18. Y.J. Shi, W. Wang, Y. Xing, J. Xu, Eur. Phys. J. C **78**(1), 56 (2018). <https://doi.org/10.1140/epjc/s10052-018-5532-7>. [arXiv:1712.03830](https://arxiv.org/abs/1712.03830) [hep-ph]
19. X. Yao, B. Müller, Phys. Rev. D **97**(7), 074003 (2018). <https://doi.org/10.1103/PhysRevD.97.074003>. [arXiv:1801.02652](https://arxiv.org/abs/1801.02652) [hep-ph]
20. D.L. Yao, Phys. Rev. D **97**(3), 034012 (2018). <https://doi.org/10.1103/PhysRevD.97.034012>. [arXiv:1801.09462](https://arxiv.org/abs/1801.09462) [hep-ph]
21. U. Özdem, J. Phys. G **46**(3), 035003 (2019). <https://doi.org/10.1088/1361-6471/aafffc>. [arXiv:1804.10921](https://arxiv.org/abs/1804.10921) [hep-ph]
22. A. Ali, A.Y. Parkhomenko, Q. Qin, W. Wang, Phys. Lett. B **782**, 412 (2018). <https://doi.org/10.1016/j.physletb.2018.05.055>. [arXiv:1805.02535](https://arxiv.org/abs/1805.02535) [hep-ph]
23. J.M. Dias, V.R. Debastiani, J.-J. Xie, E. Oset, Phys. Rev. D **98**(9), 094017 (2018). <https://doi.org/10.1103/PhysRevD.98.094017>. [arXiv:1805.03286](https://arxiv.org/abs/1805.03286) [hep-ph]
24. Z.X. Zhao, Eur. Phys. J. C **78**(9), 756 (2018). <https://doi.org/10.1140/epjc/s10052-018-6213-2>. [arXiv:1805.10878](https://arxiv.org/abs/1805.10878) [hep-ph]
25. Y. Xing, R. Zhu, Phys. Rev. D **98**(5), 053005 (2018). <https://doi.org/10.1103/PhysRevD.98.053005>. [arXiv:1806.01659](https://arxiv.org/abs/1806.01659) [hep-ph]
26. A. Ali, Q. Qin, W. Wang, Phys. Lett. B **785**, 605 (2018). <https://doi.org/10.1016/j.physletb.2018.09.018>. [arXiv:1806.09288](https://arxiv.org/abs/1806.09288) [hep-ph]
27. M.Z. Liu, Y. Xiao, L.S. Geng, Phys. Rev. D **98**(1), 014040 (2018). <https://doi.org/10.1103/PhysRevD.98.014040>. [arXiv:1807.00912](https://arxiv.org/abs/1807.00912) [hep-ph]
28. Z.P. Xing, Z.X. Zhao, Phys. Rev. D **98**(5), 056002 (2018). <https://doi.org/10.1103/PhysRevD.98.056002>. [arXiv:1807.03101](https://arxiv.org/abs/1807.03101) [hep-ph]
29. R. Aaij et al., [LHCb Collaboration], [arXiv:1808.08865](https://arxiv.org/abs/1808.08865)
30. W. Wang, R.L. Zhu, Phys. Rev. D **96**(1), 014024 (2017). <https://doi.org/10.1103/PhysRevD.96.014024>. [arXiv:1704.00179](https://arxiv.org/abs/1704.00179) [hep-ph]
31. R. Dhir, N. Sharma, Eur. Phys. J. C **78**(9), 743 (2018). <https://doi.org/10.1140/epjc/s10052-018-6220-3>
32. A.V. Berezhnuy, A.K. Likhoded, A.V. Luchinsky, Phys. Rev. D **98**(11), 113004 (2018). <https://doi.org/10.1103/PhysRevD.98.113004>. [arXiv:1809.10058](https://arxiv.org/abs/1809.10058) [hep-ph]
33. L.J. Jiang, B. He, R.H. Li, Eur. Phys. J. C **78**(11), 961 (2018). <https://doi.org/10.1140/epjc/s10052-018-6445-1>. [arXiv:1810.00541](https://arxiv.org/abs/1810.00541) [hep-ph]
34. Q.A. Zhang, Eur. Phys. J. C **78**(12), 1024 (2018). <https://doi.org/10.1140/epjc/s10052-018-6481-x>. [arXiv:1811.02199](https://arxiv.org/abs/1811.02199) [hep-ph]
35. G. Li, X.F. Wang, Y. Xing, Eur. Phys. J. C **79**(3), 210 (2019). <https://doi.org/10.1140/epjc/s10052-019-6729-0>. [arXiv:1811.03849](https://arxiv.org/abs/1811.03849) [hep-ph]
36. T. Gutsche, M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij, Z. Tyulemissov, Phys. Rev. D **99**(5), 056013 (2019). <https://doi.org/10.1103/PhysRevD.99.056013>. [arXiv:1812.09212](https://arxiv.org/abs/1812.09212) [hep-ph]
37. Y.J. Shi, Y. Xing, Z.X. Zhao, Eur. Phys. J. C **79**(6), 501 (2019). <https://doi.org/10.1140/epjc/s10052-019-7014-y>. [arXiv:1903.03921](https://arxiv.org/abs/1903.03921) [hep-ph]
38. A. Ali, C. Hambrock, A.Y. Parkhomenko, W. Wang, Eur. Phys. J. C **73**(2), 2302 (2013). <https://doi.org/10.1140/epjc/s10052-013-2302-4>. [arXiv:1212.3280](https://arxiv.org/abs/1212.3280) [hep-ph]
39. Y.J. Shi, W. Wang, Z.X. Zhao, [arXiv:1902.01092](https://arxiv.org/abs/1902.01092)
40. P. Colangelo, A. Khodjamirian, in *At the frontier of particle physics*, vol. 3, ed. by M. Shifman, pp. 1495–1576. <https://doi.org/10.1142/97898128104580033>. [arXiv:hep-ph/0010175](https://arxiv.org/abs/hep-ph/0010175)
41. M. Karliner, J.L. Rosner, Phys. Rev. D **90**(9), 094007 (2014). <https://doi.org/10.1103/PhysRevD.90.094007>. [arXiv:1408.5877](https://arxiv.org/abs/1408.5877) [hep-ph]
42. Z. Shah, K. Thakkar, A.K. Rai, Eur. Phys. J. C **76**(10), 530 (2016). <https://doi.org/10.1140/epjc/s10052-016-4379-z>. [arXiv:1609.03030](https://arxiv.org/abs/1609.03030) [hep-ph]
43. Z. Shah, A.K. Rai, Eur. Phys. J. C **77**(2), 129 (2017). <https://doi.org/10.1140/epjc/s10052-017-4688-x>. [arXiv:1702.02726](https://arxiv.org/abs/1702.02726) [hep-ph]
44. V.V. Kiselev, A.K. Likhoded, Phys. Usp. **45**, 455 (2002) [Usp. Fiz. Nauk **172**, 497 (2002)] <https://doi.org/10.1070/PU2002v045n05ABEH000958>. [arXiv:hep-ph/0103169](https://arxiv.org/abs/hep-ph/0103169)
45. S. Groote, J.G. Körner, O.I. Yakovlev, Phys. Rev. D **56**, 3943 (1997). <https://doi.org/10.1103/PhysRevD.56.3943>. [arXiv:hep-ph/9705447](https://arxiv.org/abs/hep-ph/9705447)
46. H.Y. Cheng, Y.L. Shi, Phys. Rev. D **98**(11), 113005 (2018). <https://doi.org/10.1103/PhysRevD.98.113005>. [arXiv:1809.08102](https://arxiv.org/abs/1809.08102) [hep-ph]
47. Z.S. Brown, W. Detmold, S. Meinel, K. Orginos, Phys. Rev. D **90**(9), 094507 (2014). <https://doi.org/10.1103/PhysRevD.90.094507>. [arXiv:1409.0497](https://arxiv.org/abs/1409.0497) [hep-lat]
48. R. Perez-Marcial, R. Huerta, A. Garcia, M. Avila-Aoki, Phys. Rev. D **40**, 2955 (1989) Erratum: [Phys. Rev. D **44**, 2203 (1991)]. <https://doi.org/10.1103/PhysRevD.44.2203>, <https://doi.org/10.1103/PhysRevD.40.2955>
49. C. Patrignani et al., [Particle Data Group], Chin. Phys. C **40**(10), 100001 (2016). <https://doi.org/10.1088/1674-1137/40/10/100001>
50. M. Tanabashi et al., [Particle Data Group], Phys. Rev. D **98**(3), 030001 (2018). <https://doi.org/10.1103/PhysRevD.98.030001>
51. C. Albertus, E. Hernandez, J. Nieves, PoS QNP **2012**, 073 (2012). <https://doi.org/10.22323/1.157.0073>. [arXiv:1206.5612](https://arxiv.org/abs/1206.5612) [hep-ph]