Canonical interpretation of $Y(10750)$ and $\Upsilon(10860)$ in the Υ family

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Abstract Inspired by the new resonance $Y(10750)$, we calculate the masses and two-body OZI-allowed strong decays of the higher vector bottomonium states within both screened and linear potential models. We discuss the possibilities of $\Upsilon(10860)$ and $Y(10750)$ as mixed states via the $S-D$ mixing. Our results suggest that $Y(10750)$ and $\Upsilon(10860)$ might be explained as mixed states between $5S$ - and $4D$ -wave vector $b\bar{b}$ states. The $Y(10750)$ and $\Upsilon(10860)$ resonances may correspond to the mixed states dominated by the $4D$ - and $5S$ -wave components, respectively. The mass and the strong decay behaviors of the $\Upsilon(11020)$ resonance are consistent with the assignment of the $\Upsilon(6S)$ state in the potential models.

1 Introduction

Very recently, the Belle Collaboration reported a new measurement of the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) cross sections at energies from 10.52 to 11.02 GeV using data collected with the Belle detector at the KEKB asymmetric-energy e^+e^- collider [1]. Besides two old vector states $\Upsilon(10860)$ and $\Upsilon(11020)$, a new resonance near 10.75 GeV, i.e. $Y(10750)$ as named in Ref. [2], was obviously found in the cross sections. The Breit-Wigner mass and width of this new structure are found to be $M = (10752.7 \pm 5.9_{-1.1}^{+0.7})$ MeV and $\Gamma = (35.5_{-11.3-3.3}^{+17.6+3.9})$ MeV. The production processes indicate that the spin-parity numbers of these three states appearing in the cross sections should be $J^{PC} = 1^{--}$. It is a great challenge for our understanding these states with the conventional S -, and D -wave bottomonium ($b\bar{b}$) states in the potential models.

The general consensus is that $\Upsilon(10860)$ and $\Upsilon(11020)$ correspond to the S -wave vector $b\bar{b}$ states $\Upsilon(5S)$ and $\Upsilon(6S)$, respectively [3–13]. However, if one assigns $\Upsilon(10860)$ to $\Upsilon(5S)$, we should face several problems, such as (i) the mass of $\Upsilon(5S)$ from the recent potential model calculations is about 70–90 MeV lower than the observed value of $\Upsilon(10860)$ [9–13]; (ii) the mass splittings $m[\Upsilon(5S) - \Upsilon(4S)]_{\text{th}} \simeq 210$ MeV and $m[\Upsilon(6S) - \Upsilon(5S)]_{\text{th}} \simeq 180$ MeV predicted within various potential models [9–13] are inconsistent with the observations $m[\Upsilon(5S) - \Upsilon(4S)]_{\text{exp}} \simeq 306$ MeV and $m[\Upsilon(6S) - \Upsilon(5S)]_{\text{exp}} \simeq 115$ MeV [14]; (iii) the anomalously large hadronic transition rates of $\Upsilon(10860) \rightarrow \Upsilon(1S, 2S, 3S)\pi^+\pi^-$ and $\Upsilon(10860) \rightarrow h_b(1P, 2P)\pi^+\pi^-$ observed in experiments [14–16] are difficult to be understood in theory [9, 10, 17–19].

For the newly observed $Y(10750)$, we also meet several problems if we explain it with a pure S -, or D -wave vector bottomonium state. According to the predictions in potential models, the $Y(10750)$ resonance lies between the vector states $\Upsilon(5S)$ and $\Upsilon_1(3D)$ [8–12]. Thus, if the $Y(10750)$ resonance correspond to a pure vector $b\bar{b}$ state, it should be assigned to either $\Upsilon(5S)$ or $\Upsilon_1(3D)$. However, if one assigns the $Y(10750)$ resonance to $\Upsilon(5S)$, we should meet a problem at once: how do we assign the $\Upsilon(10860)$ in the bottomonium family? On the other hand, if one assigns the $Y(10750)$ resonance to the $\Upsilon_1(3D)$ state, we cannot understand the productions of $Y(10750)$ in the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) processes, where the production cross sections of the D -wave states should be strongly suppressed for their very tiny dielectron widths predicted in theory [9, 12, 20, 21].

The above analysis indicates that both $\Upsilon(10860)$ and $Y(10750)$ cannot be simply explained with a pure S - or D -wave $b\bar{b}$ state with $J^{PC} = 1^{--}$. Thus, these resonances may have an exotic nature, such as, compact tetraquarks, hadrobottomonium, or a mixing state with some hybrid components as suggested in the literature [2, 22–24]. It should be

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emphasized that although $\Upsilon(10860)$ and $Y(10750)$ are not good candidates of a pure S - or D -wave $b\bar{b}$ states, we cannot exclude them as mixed states between the S - and D -wave vector $b\bar{b}$ states. In Refs. [20,21], Badalian *et al.* studied the dielectron widths of the vector bottomonium states, their results indicate that there might be sizeable $S - D$ mixing between the nS - and $(n - 1)D$ -wave ($n \geq 4$) vector states. If there is $S - D$ mixing indeed, the masses of the pure S - and D -wave states should be shifted to the physical states by some interactions. Then we may overcome the mass puzzles of the $\Upsilon(10860)$ as a pure $\Upsilon(5S)$ state. On the other hand, if there is $S - D$ mixing indeed, the physical states might have sizeable components of both S - and D -wave states. Considering $Y(10750)$ as a mixed state dominated by the D -wave component, we may understand the production cross section as comparable as that of $\Upsilon(10860)$ in the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) processes for its sizeable S -wave component. In fact, the $S - D$ mixing might also exist in the other meson spectra, such as the $\psi(3770)$ is suggested to be a 1^3D_1 state with a small admixture of 2^3S_1 state in the $c\bar{c}$ family [6,25–27], while the $D_J^*(2600)$ and $D_{s1}^*(2700)$ might be mixed states via the $2^3S_1 - 1^3D_1$ mixing in the D and D_s meson families, respectively [28–34].

In this work, we discuss the possibilities of $\Upsilon(10860)$ and $Y(10750)$ as mixed states via the $S - D$ mixing. By analyzing the mass spectrum of higher vector bottomonium states above the $B\bar{B}$ threshold within both screened and linear potential models, and calculating their strong decays with the 3P_0 model, we suggest that $Y(10750)$ and $\Upsilon(10860)$ might correspond to the two mixed states between $5S$ - and $4D$ -wave vector $b\bar{b}$ states with a sizeable mixing angle. The $Y(10750)$ and $\Upsilon(10860)$ resonances could be the $S - D$ mixed states dominated by the $4D$ - and $5S$ -wave components, respectively.

This paper is organized as follows. The mass spectrum of higher vector bottomonium is calculated in Sect. 2. The 3P_0 model is briefly introduced and strong decays the vector $b\bar{b}$ states are calculated in Sect. 3. Then, combining the mass and widths, we carry out a discussion about the properties of the $J^{PC} = 1^{--}$ states $Y(10750)$, $\Upsilon(10860)$ and $\Upsilon(11020)$ in Sect. 4. Finally, a short summary is presented in Sect. 5.

2 Mass spectrum

The mass spectrum of bottomonium has been calculated in our previous works [11,35] within the widely used potential models [3,6,8,36–43]. In these potential models, the effective potential of spin-independent term $V(r)$ is regarded as the sum of Lorentz vector $V_V(r)$ and Lorentz scalar $V_S(r)$ contributions [6], i.e.,

$$V(r) = V_V(r) + V_S(r). \quad (1)$$

The Lorentz vector potential $V_V(r)$ can be written as the standard color Coulomb form

$$V_V(r) = -\frac{4}{3} \frac{\alpha_s}{r}. \quad (2)$$

The Lorentz scalar potential $V_S(r)$ might be taken as a simple form with a linear potential [6,8,36–39] or the screened potential as suggested in Refs. [3,40–43], i.e.,

$$V_S(r) = \begin{cases} br & \text{Linear potential,} \\ \frac{b(1-e^{-\mu r})}{\mu} & \text{Screened potential.} \end{cases} \quad (3)$$

In the screened potential, the parameter μ is the screening factor which makes the long-range scalar potential of $V_S(r)$ behave like the linear potential br when $r \ll 1/\mu$, and become a constant b/μ when $r \gg 1/\mu$. The main effect of the screened potential on the spectrum is that the masses of the higher excited states are lowered. Such a screening effect might be important for us to reasonably describe the higher radial and orbital excitations.

Furthermore, we include three spin-dependent terms as follows. For the spin–spin contact hyperfine potential, we take [39]

$$H_{SS} = \frac{32\pi\alpha_s}{9m_b^2} \tilde{\delta}_\sigma(r) \mathbf{S}_b \cdot \mathbf{S}_{\bar{b}}, \quad (4)$$

where $\mathbf{S}_b$ and $\mathbf{S}_{\bar{b}}$ are spin matrices acting on the spins of the quark and antiquark. We take $\tilde{\delta}_\sigma(r) = (\sigma/\sqrt{\pi})^3 e^{-\sigma^2 r^2}$ as in Ref. [39]. For the spin–orbit and the tensor terms, we adopt [6]:

$$H_{SL} = \frac{1}{2m_b^2 r} \left(3 \frac{dV_V}{dr} - \frac{dV_S}{dr} \right) \mathbf{L} \cdot \mathbf{S}, \quad (5)$$

and

$$H_T = \frac{1}{12m_b^2} \left(\frac{1}{r} \frac{dV_V}{dr} - \frac{d^2 V_V}{dr^2} \right) S_T, \quad (6)$$

where $\mathbf{L}$ is the relative orbital angular momentum of b and $\bar{b}$ quarks, $\mathbf{S} = \mathbf{S}_b + \mathbf{S}_{\bar{b}}$ is the total quark spin, and the spin tensor S_T is defined by [6]

$$S_T = 6 \frac{\mathbf{S} \cdot \mathbf{r} \mathbf{S} \cdot \mathbf{r}}{r^2} - 2\mathbf{S}^2. \quad (7)$$

In the linear potential model, four model parameters (α_s , b , m_b , σ) in the above equations should be determined, while in the screened potential model, five model parameters (α_s , b , m_b , σ , μ) should be determined. These parameters of both linear and screened potential models have been reasonably determined by fitting the $b\bar{b}$ mass spectrum in our previous works [11,35]. In this work, we adopt the same parameter sets for both linear and screened potential models as the previous works [11,35]. For clarity, we collect these two parameter sets in Table 1.

Table 1 Quark model parameters for the linear potential (LP) model and screened potential (SP) model adopted from Refs. [11,35]

Parameter	LP model [35]	SP model [11]
m_b (GeV)	4.830	4.7572
α_s	0.4311	0.3683
b (GeV ²)	0.1425	0.2062
σ (GeV)	2.32	3.1025
r_c (fm)	0.0335	0.06012
μ (GeV)	...	0.05611

The radial Schrödinger equation is solved by using the three-point difference central method [44]. With this method, one can reasonably include the corrections from these spin-dependent potentials to both the mass and wave function of

a meson state. In the calculations, to overcome the singular behavior of $1/r^3$ in the spin-dependent potentials, we introduce a cutoff distance r_c . Within a small range $r \in (0, r_c)$, we let $1/r^3 = 1/r_c^3$. The details of the numerical method can be found in our previous works [11, 45].

To study the higher vector $b\bar{b}$ states above the $B\bar{B}$ threshold, first we predict the masses for the nS ($n = 4, 5, 6$) and nD ($n = 3, 4, 5$) states within both the linear and screened potential models. The results are listed in Table 2 and also shown in Fig. 1. It is found that both the linear and screened potential models give a similar prediction of the masses. Considering the $\Upsilon(10580)$ and $\Upsilon(11020)$ resonances as the $\Upsilon(4S)$ and $\Upsilon(6S)$ states, respectively, their observed masses are in good agreement with the potential model predictions. However, considering the $\Upsilon(10860)$ as the $\Upsilon(5S)$ state, the observed mass is obviously (about 70 – 90 MeV) larger

Table 2 Predicted masses (MeV) of higher vector bottomonium states with both the linear potential (LP) and screened potential (SP) models

State	$n^{2S+1}L_J$	LP	SP	Ref. [12]	Ref. [9]	Ref. [10]	Exp.
$\Upsilon(4S)$	4^3S_1	10581	10597	10612	10635	10607	10579 [14]
$\Upsilon(5S)$	5^3S_1	10794	10811	10822	10878	10818	10885 [1]
$\Upsilon(6S)$	6^3S_1	10986	10997	11001	11102	10995	11000 [1]
$\Upsilon_1(3D)$	3^3D_1	10646	10658	10675	10698	10653	...
$\Upsilon_1(4D)$	4^3D_1	10846	10858	10871	10928	10853	...
$\Upsilon_1(5D)$	5^3D_1	11029	11036	11041	...	11023	...

For a comparison, the results from recent works and experimental observations are also listed

Fig. 1 The spectrum of the higher vector bottomonium above the $B\bar{B}$ threshold predicted within both screened potential (SP) and liner potential (LP) models. For a comparison, the experimental observations [1, 14] are plotted in the figure as well

than the model predictions. Figure 1 shows that the newly observed state $Y(10750)$ lies about 100 MeV above $\Upsilon_1(3D)$, while about 40–50 MeV below $\Upsilon(5S)$.

3 Strong decays

In this section, we use the 3P_0 model [46–48] to evaluate the Okubo-Zweig-Iizuka (OZI) allowed two-body strong decays of the vector bottomonium. In this model, it assumes that the vacuum produces a light quark-antiquark pair with the quantum number 0^{++} and the bottomonium decay takes place though the rearrangement of the four quarks. The transition operator $\hat{T}$ can be written as

$$\hat{T} = -3\gamma\sqrt{96\pi} \sum_m \langle 1m1 - m | 00 \rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \times \mathcal{Y}_1^m \left(\frac{\mathbf{p}_3 - \mathbf{p}_4}{2} \right) \chi_{1-m}^{34} \phi_0^{34} \omega_0^{34} b_{3i}^\dagger(\mathbf{p}_3) d_{4j}^\dagger(\mathbf{p}_4), \quad (8)$$

where γ is a dimensionless constant that denotes the strength of the quark-antiquark pair creation with momentum $\mathbf{p}_3$ and $\mathbf{p}_4$ from vacuum; $b_{3i}^\dagger(\mathbf{p}_3)$ and $d_{4j}^\dagger(\mathbf{p}_4)$ are the creation operators for the quark and antiquark, respectively; the subscripts, i and j , are the SU(3)-color indices of the created quark and anti-quark; $\phi_0^{34} = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $\omega_0^{34} = \frac{1}{\sqrt{3}}\delta_{ij}$ correspond to flavor and color singlets, respectively; $\chi_{1,-m}^{34}$ is a spin triplet state; and $\mathcal{Y}_\ell(\mathbf{k}) \equiv |\mathbf{k}|^\ell Y_{\ell m}(\theta_{\mathbf{k}}, \phi_{\mathbf{k}})$ is the ℓ -th solid harmonic polynomial.

For an OZI allowed two-body strong decay process $A \rightarrow B + C$, the helicity amplitude $\mathcal{M}^{M_{JA} M_{JB} M_{JC}}(\mathbf{P})$ can be calculated as follow

$$\langle BC | T | A \rangle = \delta(\mathbf{P}_A - \mathbf{P}_B - \mathbf{P}_C) \mathcal{M}^{M_{JA} M_{JB} M_{JC}}(\mathbf{P}). \quad (9)$$

With the Jacob-Wick formula [49,50], the helicity amplitudes $\mathcal{M}^{M_{JA} M_{JB} M_{JC}}(\mathbf{P})$ can be converted to the partial wave amplitudes $\mathcal{M}^{JL}$ via

$$\mathcal{M}^{JL}(A \rightarrow BC) = \frac{\sqrt{4\pi(2L+1)}}{2J_A+1} \sum_{M_{JB}, M_{JC}} \langle L0JM_{JA} | J_A M_{JA} \rangle \times \langle J_B M_{JB} J_C M_{JC} | J M_{JA} \rangle \mathcal{M}^{M_{JA} M_{JB} M_{JC}}(\mathbf{P}). \quad (10)$$

In the above equations, $(J_A, J_B$ and $J_C)$, $(L_A, L_B$ and $L_C)$ and $(S_A, S_B$ and $S_C)$ are the quantum numbers of the total angular momenta, orbital angular momenta and total spin for hadrons A, B, C , respectively; $M_{JA} = M_{JB} + M_{JC}$, $\mathbf{J} \equiv \mathbf{J}_B + \mathbf{J}_C$ and $\mathbf{J}_A \equiv \mathbf{J}_B + \mathbf{J}_C + \mathbf{L}$. In the c.m. frame of hadron A , the momenta $\mathbf{P}_B$ and $\mathbf{P}_C$ of mesons B and C satisfy $\mathbf{P}_B = -\mathbf{P}_C \equiv \mathbf{P}$.

To partly remedy the inadequacy of the nonrelativistic wave function as the momentum $\mathbf{P}$ increases, a Lorentz boost factor γ_f is introduced into the decay amplitudes [51],

$$\mathcal{M} \rightarrow \gamma_f \mathcal{M}(\gamma_f \mathbf{P}). \quad (11)$$

where $\gamma_f = M_B/E_B$. In the decays with small phase space, the three momenta $\mathbf{P}$ carried by the final state mesons and corrections from the Lorentz boost are relatively small, while the relativistic effects may be essential for the decay channels with larger phase space.

Finally, the partial width of the $A \rightarrow B + C$ process can be given by

$$\Gamma = 2\pi |\mathbf{P}| \frac{E_B E_C}{M_A} \sum_{JL} |\mathcal{M}^{JL}|^2, \quad (12)$$

where M_A is the mass of the initial hadron A , while E_B and E_C stand for the energies of final hadrons B and C , respectively. The details of the formula of the 3P_0 model can be found in Refs. [51,52].

In the calculations, the wave functions of the initial vector bottomonium states are taken from our quark model predictions. Furthermore, we need the wave functions of the final hadrons, i.e., the $B^{(*)}$ and $B_s^{(*)}$ mesons and some of their excitations, which are adopted from the quark model predictions of Refs. [31,53]. The masses of the final hadron states in the decay processes are adopted from the Particle Data Group (PDG) [14] if there are data, while if there are no observations we adopt the predicted values from Refs. [31,53]. The quark pair creation strength is determined to be $\gamma = 0.232$ by reproducing the measured width $\Upsilon(10580) \rightarrow B\bar{B} = 20.5$ MeV [14] with the wave function calculated from the screened potential model. The γ determined in this work is also close to the values 0.217/0.234 adopted in the study of the strong decays of excited charmonium states [51].

The strong decay properties for the vector bottomonium are presented in Table 3. For the 4^3S_1 state, both the linear and screened potential models give a compatible prediction. For the 5^3S_1 state, the main decay modes B^*B^* and $B_s^*B_s$ are predicted in both the LP and SP models, although the some branching fractions of some decay modes show some differences. For the 6^3S_1 state, the dominant decay mode BB^* is predicted in both the LP and SP models, however, obvious difference exists in the predictions for the B^*B^* decay mode, in the LP model the partial width for the B^*B^* mode is tiny, while in the SP model the branching fraction of the B^*B^* mode can reach up to $\sim 15\%$. For the 3^3D_1 state, both the LP and SP models give similar predictions, it should be mentioned that in the SP model the B^*B^* decay mode is open, which may be another main decay mode of 3^3D_1 . For the 4^3D_1 and 5^3D_1 states, the main decay modes predicted in the LP model are consistent with that of the SP model, although the the predicted partial width for a specific decay mode shows some differences. Finally, it should mentioned that the mass of the initial hadron not only affects its decay phase space but also affects the partial decay amplitude of a decay mode. Thus, the partial width ratios between differ-

Table 3 Strong decay properties for the higher vector bottomonium states predicted within both LP and SP models

State (LP/SP)	Decay mode	LP		SP		Ref. [12]		Ref. [9]		Ref. [10]	
		Γ_{th} (MeV)	B_r (%)	Γ_{th} (MeV)	B_r (%)	Γ_{th} (MeV)	B_r (%)	Γ_{th} (MeV)	B_r (%)	Γ_{th} (MeV)	B_r (%)
4^3S_1 (10581/10597)	BB	19.58	100	20.25	100	24.7	100	22.0	100	20.59	100
	Total	19.58	100	20.25	100	24.7	100	22.0	100	20.59	100
5^3S_1 (10794/10811)	BB	0.96	2.23	2.25	7.26	13.7	30.0	5.35	19.5	6.22	22.29
	BB^*	1.76	4.09	≈ 0	≈ 0	26.5	58.1	16.6	60.6	11.83	42.41
	B^*B^*	33.73	78.37	21.77	70.23	2.58	5.66	2.42	8.83	0.09	0.32
	B_sB_s	≈ 0	≈ 0	0.30	0.97	0.484	1.06	0.157	0.573	0.96	3.45
	$B_s^*B_s$	6.59	15.31	6.68	21.55	1.49	3.28	0.833	3.04	1.15	4.11
	$B_s^*B_s^*$					0.872	1.91	2.00	7.30	7.65	27.42
	Total	43.04	100	31.00	100	45.6	100	27.4	100	27.89	100
6^3S_1 (10986/10997)	BB	3.22	26.86	3.80	18.90	7.81	20.4	1.32	3.89	4.18	5.28
	BB^*	5.69	47.46	9.02	44.85	16.5	43.0	7.59	22.4	15.49	19.57
	B^*B^*	0.44	3.67	3.13	15.56	4.43	11.5	5.89	17.4	11.87	14.99
	B_sB_s	0.38	3.17	0.28	1.39	0.101	0.263	1.31	0.00386	0.07	0.09
	$B_s^*B_s$	1.94	16.18	2.38	11.83	0.780	2.04	0.136	0.401	1.50	1.89
	$B_s^*B_s^*$	0.32	2.67	1.50	7.46	0.448	1.17	0.310	0.914	2.02	2.56
	$BB(1P)$					8.27	21.6	7.81	23.0	40.08	50.64
	$BB(1P')$							10.8	31.8	3.95	4.98
	Total	12.4	100	20.11	100	38.3	100	33.9	100	79.16	100
3^3D_1 (10646/10658)	BB	≈ 0	≈ 0	0.95	3.41	5.47	10.1	23.8	23.0	...	...
	BB^*	26.41	100	18.76	67.39	15.2	28.1	0.245	0.236	...	...
	B^*B^*			8.13	29.20	33.4	61.8	79.5	76.7	...	...
	Total	26.41	100	27.84	100	54.1	100	103.6	100	...	...
4^3D_1 (10846/10858)	BB	11.82	30.17	12.84	23.87	27.4	31.4	3.85	5.36	...	...
	BB^*	5.55	14.17	7.45	13.85	15.1	17.3	14.0	19.5	...	...
	B^*B^*	18.25	46.58	27.28	50.71	42.1	48.3	50.6	70.5	...	...
	B_sB_s	1.67	4.3	1.86	3.48	0.560	0.642	0.101	0.141	...	...
	$B_s^*B_s$	0.05	0.13	0.62	1.15	0.360	0.412	0.332	0.462	...	...
	$B_s^*B_s^*$	1.84	4.70	3.75	6.97	1.66	1.91	2.94	4.09	...	...
	Total	39.18	100	53.8	100	87.2	100	71.8	100	...	...
5^3D_1 (11029/11036)	BB	8.28	14.27	8.13	10.39	20.0	16.4	...	...	...	...
	BB^*	7.89	13.60	8.49	10.85	19.3	15.8	...	...	...	...
	B^*B^*	24.08	41.51	27.41	35.02	47.1	38.7	...	...	...	...
	B_sB_s	≈ 0	≈ 0	0.02	0.03	0.0235	0.0193	...	...	...	...
	$B_s^*B_s$	0.44	0.76	0.36	0.46	0.103	0.0844	...	...	...	...
	$B_s^*B_s^*$	2.91	5.02	2.60	3.32	0.798	0.656	...	...	...	...
	$B^*B(1^3P_0)$	7.92	13.65	10.08	12.88	3.02	2.48	...	...	...	...
	$BB(1P)$	6.49	11.19	13.56	17.33	4.08	3.35	...	...	...	...
	$BB(1P')$			7.61	9.72	18.1	14.8	...	...	...	...
	$BB(1^3P_2)$					9.23	7.59	...	...	...	...
	Total	58.01	100	78.26	100	121.7	100	...	...	...	...

For a comparison, other results predicted in the recent works [9, 10, 12] are also listed in the same table

Table 4 Strong decays for the $\Upsilon_1(4D)$ which is assigned to be $Y(10750)$ or $\Upsilon(10860)$ within the screened potential model

State	Mode	$Y(10750)$		$\Upsilon(10860)$	
		Γ_{th} (MeV)	B_r (%)	Γ_{th} (MeV)	B_r (%)
4^3D_1	BB	≈ 0	≈ 0	13.82	16.98
	BB^*	4.46	13.77	11.92	14.65
	B^*B^*	25.53	78.84	43.17	53.04
	$B_s B_s$	2.39	7.38	0.97	1.19
	$B_s^* B_s$	...	...	2.16	2.65
	$B_s^* B_s^*$	...	...	9.35	11.49
	Total	32.38	100	81.39	100

Table 5 Strong decays for the $\Upsilon(5S)$ which is assigned to be $Y(10750)$ or $\Upsilon(10860)$ within the screened potential model

State	Mode	$Y(10750)$		$\Upsilon(10860)$	
		Γ_{th} (MeV)	B_r (%)	Γ_{th} (MeV)	B_r (%)
5^3S_1	BB	0.20	0.38	7.46	16.95
	BB^*	15.63	29.50	20.84	47.34
	B^*B^*	35.52	67.03	8.26	18.76
	$B_s B_s$	1.64	3.09	1.02	2.31
	$B_s^* B_s$	...	...	5.63	12.79
	$B_s^* B_s^*$	...	...	0.81	1.84
	Total	52.99	100	44.02	100

ent decay modes of a hadron usually depend on its mass. This feature can be obviously seen from the results listed in Tables 4 and 5.

4 Discussions

4.1 $Y(10750)$ and $\Upsilon(10860)$

For the $\Upsilon(10860)$ resonance, the mass and spin-parity numbers indicate that it may be a candidate of the conventional $\Upsilon_1(4D)$. However, with this assignment it is found that the total decay width, ~ 81 MeV predicted from the quark models, is about a factor of 2 larger than the measured width ~ 37 MeV [1]. Furthermore, assigning $\Upsilon(10860)$ as $\Upsilon_1(4D)$ we will meet a problem in the explanation of its productions in the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$), because the production rates of D -wave states should be strongly suppressed for their tiny dielectron widths [9, 10, 12, 20, 21]. The $\Upsilon(10860)$ resonance is often explained as the $\Upsilon(5S)$ state in the literature [8–10, 12], with this assignment, the total width is predicted to be ~ 44 MeV (see Table 5), which is consistent with the data. However, if we assign $\Upsilon(10860)$ to $\Upsilon(5S)$ we should face some problems, for example, (i) the predicted mass of $\Upsilon(5S)$ state is about

70 MeV lower than that of $\Upsilon(10860)$; (ii) moreover, the mass splittings $m[\Upsilon(5S) - \Upsilon(4S)]_{th} \simeq 210$ MeV and $m[\Upsilon(6S) - \Upsilon(5S)]_{th} \simeq 180$ MeV predicted within various potential models (see Table 2) are inconsistent with the observations $m[\Upsilon(5S) - \Upsilon(4S)]_{exp} \simeq 306$ MeV and $m[\Upsilon(6S) - \Upsilon(5S)]_{exp} \simeq 115$ MeV [14].

It should be mentioned that when the $\Upsilon(10860)$ is considered as pure $\Upsilon(5S)$ state, the BB^* is the dominant decay channel of $\Upsilon(10860)$ in our calculation. Our prediction is consistent with other quark model estimations in Refs. [9, 10, 12]. However, our prediction is contradictory with the prediction with effective field theory based on the heavy quark spin symmetry at the hadronic level in Ref. [54], where the dominant decay channel of $\Upsilon(5S)$ is predicted to be B^*B^* . In the framework of effective field theory, the couplings between $\Upsilon(5S)$ and $B^{(*)}B^{(*)}$ are determined by the heavy quark symmetry, while the details of the wave functions of hadrons are neglected. While in the quark model framework, the decay amplitudes depend on the details of the wave functions, which include the effects of violation of the heavy quark symmetry. These effects may cause the discrepancy between the approach of effective field theory and quark potential models.

For the new structure $Y(10750)$ observed at Belle, from Fig. 1 one finds that it lies between the vector states $\Upsilon(5S)$ and $\Upsilon_1(3D)$. The predicted mass of $\Upsilon_1(3D)$ state is about 100 MeV lower than that of $Y(10750)$. If one assigns the $Y(10750)$ resonance to the $\Upsilon_1(3D)$ state, we cannot understand its production rates in the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) processes. The production cross sections of the D -wave states should be strongly suppressed for their very tiny dielectron widths predicted in theory [9, 10, 12, 20, 21]. Thus, the explanation of the $Y(10750)$ resonance as the $\Upsilon_1(3D)$ state should be excluded. On the other hand, if one assigns the $Y(10750)$ resonance to $\Upsilon(5S)$, it is found that the decays of $Y(10750)$ are dominated by the B^*B^* channel, and the decay width is predicted to be $\Gamma \sim 53$ MeV, which is close to the measured value $35.5^{+17.6+3.9}_{-11.3-3.3}$ MeV at Belle [1]. However, we will meet a problem that there are no S -wave vector states to be assigned to $\Upsilon(10860)$, then we cannot understand the largest production rates of $\Upsilon(10860)$ in the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) processes.

Since it is difficult to assign the $Y(10750)$ and $\Upsilon(10860)$ as the pure S - and D -wave $b\bar{b}$ states simultaneously, we consider the possibilities of $Y(10750)$ and $\Upsilon(10860)$ as the $\Upsilon(5S)$ - $\Upsilon_1(4D)$ mixed states with the following mixing scheme

$$\begin{pmatrix} Y(10750) \\ \Upsilon(10860) \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \Upsilon_1(4D) \\ \Upsilon(5S) \end{pmatrix}. \quad (13)$$

The decay widths of $Y(10750)$ and $\Upsilon(10860)$ versus the mixing angle θ are presented in Fig. 2. In Refs. [20, 21],

Fig. 2 Strong decays of $Y(10750)$ and $\Upsilon(10860)$ versus the mixing angle θ within screened potential model. The horizontal line is the center value of the experiment, and the shaded region represents the error range of the experiment

Badalian et al. studied the dielectron widths of the vector bottomonium states, their results indicate that there might be sizeable $S - D$ mixing between the nS - and $(n - 1)D$ -wave ($n \geq 4$) vector states with a mixing angle $\sim 27^\circ$. With this mixing angle, the decay widths of $Y(10750)$ and $\Upsilon(10860)$ can be reasonably understood (see Fig. 2).

The $S - D$ mixing can shift the masses of the pure states $\Upsilon(5S)$ and $\Upsilon_1(4D)$ to the physical states $Y(10750)$ and $\Upsilon(10860)$. The Hamiltonian of the physical states is assumed to be

$$H = H_0 + H_I, \quad (14)$$

where H contributes diagonal elements to the mass matrix, while H_I could contribute non-diagonal elements to the mass matrix causing the $S - D$ mixing. Then, the masses of $Y(10750)$ and $\Upsilon(10860)$ can be determined by

$$M[\Upsilon(10860)] = \langle \Upsilon(10860) | H_0 + H_I | \Upsilon(10860) \rangle, \quad (15)$$

$$M[Y(10750)] = \langle Y(10750) | H_0 + H_I | Y(10750) \rangle. \quad (16)$$

Combining the mixing scheme defined in Eq. (13), one finds that

$$M[\Upsilon(10860)] = M[\Upsilon_1(4D)] \sin^2 \theta + M[\Upsilon(5S)] \cos^2 \theta - \Delta M_{SD} \sin(2\theta), \quad (17)$$

$$M[Y(10750)] = M[\Upsilon_1(4D)] \cos^2 \theta + M[\Upsilon(5S)] \sin^2 \theta + \Delta M_{SD} \sin(2\theta), \quad (18)$$

where $M[\Upsilon(5S)] = \langle \Upsilon(5S) | H | \Upsilon(5S) \rangle$ and $M[\Upsilon_1(4D)] = \langle \Upsilon_1(4D) | H | \Upsilon_1(4D) \rangle$ correspond to the masses of the pure states $\Upsilon(5S)$ and $\Upsilon_1(4D)$, respectively, while $\Delta M_{SD} = \langle \Upsilon_1(4D) | H_I | \Upsilon(5S) \rangle$ corresponds to the non-diagonal element. Taking a sizeable mixing angle $\theta \sim 20 - 30^\circ$ and a negative value $\Delta M_{SD} \sim -100$ MeV in Eq. (17), one finds that the physical masses of both $Y(10750)$ and $\Upsilon(10860)$ can be consistent with the experimental observations. Thus, with the $S - D$ mixing one may overcome the puzzle that the observed mass of $\Upsilon(10860)$ is obviously higher than the predicted masses of pure $\Upsilon(5S)$ in the potential models.

The $S - D$ mixing may explain the observations that production cross sections of $Y(10750)$ are comparable with those of $\Upsilon(10860)$ in the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) processes. As a pure $\Upsilon(5S)$ state, the dielectron width of $\Upsilon(10860)$ is predicted to be $\Gamma_{ee} = 0.348$ keV in a recent work [12]. Combining it with the mixing angle $\theta \simeq 27^\circ$ suggested in Refs. [20, 21] we obtain dielectron width $\Gamma_{ee} = 0.28$ keV for $\Upsilon(10860)$, which is consistent with the measured value 0.31 ± 0.07 keV [14]. As a mixed state containing sizeable S -wave component the dielectron width of $Y(10750)$ is estimated to be ~ 0.07 keV. The dielectron width ratio between $Y(10750)$ and $\Upsilon(10860)$ is predicted to be

$$R \sim \frac{\Gamma_{ee}[Y(10750)]}{\Gamma_{ee}[\Upsilon(10860)]} \simeq \frac{1}{4}, \quad (19)$$

which indicates that both $Y(10750)$ and $\Upsilon(10860)$ have a comparable coupling to e^+e^- . On the other hand, both $Y(10750)$ and $\Upsilon(10860)$ may have a comparable dipoinic transition strength. According to the studies in Refs. [4, 5, 54–61], the effects of bottom meson loops and/or intermediate Z_b states may play essential role to the dipoinic transitions of the higher S -wave bottomonia. The dipoinic transition rates of $Y(10750)$ via the intermediate Z_b states may be strongly suppressed by the phase space. However, $Y(10750)$ lies above the thresholds of $B^{(*)}B^{(*)}$, and dominantly decays into B^*B^* and BB^* channels, so the intermediate B^*B^* and BB^* loops may play an important role in its dipoinic transitions. Furthermore, the competition between the form factor and the final-state phase space plays an important role in the determination of these dipoinic decay rates [4]. Thus, the dipoinic transition strength of $Y(10750)$ via the B^*B^* and BB^* loops may have the same order of magnitude as the dipoinic transition strength of $\Upsilon(10860)$, which may result

in the observations of $Y(10750)$ and $\Upsilon(10860)$ in the same $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) processes. To further understand the productions of $Y(10750)$ and $\Upsilon(10860)$, more studies are urgently needed in theory.

From Fig. 2, one can see that the partial widths of the strong decay modes and the ratios between them for the $Y(10750)$ and $\Upsilon(10860)$ resonances are sensitive to the mixing angle. If taking the mixing angle as $\theta \sim 30^\circ$, the decays of $\Upsilon(10860)$ are dominated by the B^*B^* , BB and $B_s^*B_s^*$ channels, while the decays of $Y(10750)$ might be governed by both the B^*B^* and BB^* channels. The large decay rates of $\Upsilon(10860)$ into the $B_s^*B_s^*$ channel can explain the observations at Belle [62] that the $B_s^*B_s^*$ cross section shows a prominent $\Upsilon(10860)$ signal, while the $B_s^*B_s$ and B_sB_s cross sections are relatively small and do not show any significant structures. From the PDG book [14], it is seen that the branching ratios of BB , BB^* , B^*B^* , B_sB_s , $B_sB_s^*$, and $B_s^*B_s^*$ decay modes are $5.5 \pm 1.0\%$, $13.7 \pm 1.6\%$, $38.1 \pm 3.4\%$, $0.5 \pm 0.5\%$, $1.35 \pm 0.32\%$, and $17.6 \pm 2.7\%$, respectively. Considering $\Upsilon(10860)$ as pure $\Upsilon(5S)$, we calculate the branching ratios of BB , BB^* , B^*B^* , B_sB_s , $B_sB_s^*$, and $B_s^*B_s^*$ decay modes, our results are listed in Table 5, it is found that these branching ratios of $\Upsilon(10860)$ can be hardly described with the pure $\Upsilon(5S)$ interpretation, which is consistent with the analysis in Refs. [9, 10, 12]. With the $S - D$ mixing, this problem can be partially overcome. Our results show that the B^*B^* dominates in the non-strange final channels and the $B_s^*B_s^*$ is prominent in the strange decay modes, which is consistent with the experimental data. However, the predicted large BB partial decay width is still in conflicting with the observations. More theoretical and experimental investigations are needed to clarify this puzzle.

The intermediate B^*B^* meson loop may play an important role in the $S - D$ mixing between $\Upsilon(5S)$ and $\Upsilon_1(4D)$. From Table 3, it is found that both $\Upsilon(5S)$ and $\Upsilon_1(4D)$ states strongly couple to the B^*B^* channel, the branching fractions for $\Upsilon(5S) \rightarrow B^*B^*$ and $\Upsilon_1(4D) \rightarrow B^*B^*$ can reach up to 70% and 50%, respectively. Thus, the intermediate B^*B^* meson loop may contribute a sizeable non-diagonal element to the mass matrix, which leads to the $S - D$ mixing. It is interesting to find that the mixing mechanism of axial-vectors $D_{s1}(2460)$ and $D_{s1}(2536)$ has been studied via intermediate hadron loops, e.g. DK , to which both states have strong couplings in Ref. [63]. Also, the $S - D$ mixing scheme of $\psi(4S)$ and $\psi(2D)$ states via the meson loops are investigated in Ref. [64]. Their results indicate that the intermediate hadron loops as the mixing mechanism can lead to strong configuration mixing effects and obvious mass shifts for the physical states.

Finally, it should be mentioned that the anomalously large hadronic dipion transition rates of $\Upsilon(10860) \rightarrow \Upsilon(1S, 2S, 3S)\pi^+\pi^-$, $h_b(1P, 2P)\pi^+\pi^-$ observed in experiments [14–16] may indicate an exotic nature of $\Upsilon(10860)$.

For example, in Refs. [22, 65], to interpret the anomalous production rates of $\Upsilon(10860)$ in the $e^+e^- \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$, the authors suggested that $\Upsilon(10860)$ should be a bound diquark-antidiquark tetraquark state. Recently, $\Upsilon(10860)$ was suggested to be the mixing of the $\Upsilon(5S)$ state with the lowest P -wave hybrid state, and with this picture the e^+e^- leptonic decay widths and the production rates of $\Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) and $h_b(1P, 2P)\pi^+\pi^-$ can be well interpreted [24]. However, assigning $\Upsilon(10860)$ as the $\Upsilon(5S)$ state, in Refs. [4, 5, 55–58] the authors pointed out that the unusually large dipion transition rates of $\Upsilon(10860)$ can be reasonably described via rescattering mechanism with B -meson loops. In this work, we mainly focus on the explanation of the mass, strong decay properties, and dielectron widths of $\Upsilon(10860)$, and with the $\Upsilon(5S)$ - $\Upsilon(4^3D_1)$ mixing one may obtain a reasonable understanding. Other sizeable exotic components, such as hybrid, tetraquark, may exist and can be added to the $5S - 4D$ mixing scheme as well. Further studies are needed to uncover the long-standing puzzles of $\Upsilon(10860)$.

4.2 $\Upsilon(10580)$ and $\Upsilon(11020)$

Taking $\Upsilon(10580)$ and $\Upsilon(11020)$ as the $\Upsilon(4S)$ and $\Upsilon(6S)$ states, respectively, from Fig. 1 one can find that their masses can be well described in both the LP and SP models. Furthermore, with these assignments from Table 3 it is found that the widths of $\Upsilon(10580)$ and $\Upsilon(11020)$ are predicted to be $\Gamma[\Upsilon(10580)]_{\text{th}} \simeq 20$ MeV and $\Gamma[\Upsilon(11020)]_{\text{th}} \simeq 12 - 20$ MeV in the potential models, respectively, which are reasonably compatible with the measured values $\Gamma[\Upsilon(10580)]_{\text{exp}} = 25.5 \pm 5.5$ MeV and $\Gamma[\Upsilon(11020)]_{\text{exp}} = 23.8_{-6.8}^{+8.0+0.7}$ for $\Upsilon(10580)$ and $\Upsilon(11020)$, respectively. However, their dielectron widths are overestimated as the pure S -wave states [12, 20]. To explain the dielectron widths, in Ref. [20], Badalian et al. suggested a $S - D$ mixing in the physical states $\Upsilon(10580)$ and $\Upsilon(11020)$.

In this work, we also consider the $\Upsilon(11020)$ resonance as a mixed state via the $\Upsilon(6S)$ - $\Upsilon_1(5D)$ mixing. The mixing scheme is adopted as follows:

$$\begin{pmatrix} \Upsilon(M_x) \\ \Upsilon(11020) \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \Upsilon_1(5D) \\ \Upsilon(6S) \end{pmatrix}. \quad (20)$$

The strong decay width of $\Upsilon(11020)$ versus the mixing angle θ is plotted in Fig. 3. It is seen that the total decay width is consistent with experimental data when the mixing angle varies in large range. The current total decay width alone cannot determine the mixing angle. However, the significant leptonic decay width $\Gamma_{ee} \simeq 0.130 \pm 0.030$ keV indicates the $\Upsilon(11020)$ resonance has a large S -wave $b\bar{b}$ component.

From the predicted strong decay properties of $\Upsilon(6S)$ and $\Upsilon_1(5D)$ states listed in Table 3, one finds that $\Upsilon(6S)$

Fig. 3 Strong decay of $\Upsilon(11020)$ versus the mixing angle θ within screened potential model. The horizontal line is the center value of the experiment, and the shaded region represents the error range of the experiment

and $\Upsilon_1(5D)$ states mainly couple to two different channels BB^* and B^*B^* , respectively. Furthermore, both $\Upsilon(6S)$ and $\Upsilon_1(5D)$ states are far from the thresholds of BB^* and B^*B^* . Thus, the $\Upsilon(6S)$ - $\Upsilon_1(5D)$ mixing effect via virtual meson loops may be smaller than that of $\Upsilon(5S)$ - $\Upsilon_1(4D)$. If the intermediate meson loops are the main mechanism causing the $S-D$ mixing, the mixing angle for $\Upsilon(6S)$ - $\Upsilon_1(5D)$ should be smaller than that for $\Upsilon(5S)$ - $\Upsilon_1(4D)$. To sum up, although the $\Upsilon(6S)$ - $\Upsilon_1(5D)$ mixing with a small $5D$ -wave component assignment cannot be excluded, we prefer to assign $\Upsilon(11020)$ as the pure $\Upsilon(6S)$ state. In Ref. [43] the authors also expected that there may less $S-D$ mixing for the $\Upsilon(5S)$ state with the consideration of coupled-channel effects. To better understand the nature of $\Upsilon(11020)$, the missing $\Upsilon_1(5D)$ is worth looking for in future experiments. Our predictions of the mass and strong decay widths of $\Upsilon_1(5D)$ may provide helpful information for future experimental observations.

Besides the possibility of $\Upsilon(5S)$ - $\Upsilon_1(4D)$ and $\Upsilon(6S)$ - $\Upsilon_1(5D)$ mixing, taking the same mixing scheme the decay width of $\Upsilon(10580)$ as $\Upsilon(4S)$ - $\Upsilon_1(3D)$ mixing is also shown in Fig. 4. The total decay width varies dramatically with the mixing angle, and the zero mixing angle is more favored (actually, we use this case to determine the quark pair creation strength γ). From Table 3, it is found that the $\Upsilon(4S)$ state mainly couples to the BB channel, while $\Upsilon_1(3D)$ dominantly decays into the BB^* channel. The dominant decay modes of $\Upsilon(4S)$ and $\Upsilon_1(3D)$ are very different, which indicates that the $\Upsilon(4S)$ - $\Upsilon_1(3D)$ mixing effects via virtual meson loops may be negligible. Thus, if the intermediate meson loops are the main mechanism causing the $S-D$ mixing, and the mix-

Fig. 4 Strong decay of $\Upsilon(10580)$ versus the mixing angle θ within screened potential model. The horizontal line is the center value of the experiment, and the shaded region represents the error range of the experiment

ing effects can be neglected. However, it should be mentioned that in Ref. [43] the authors expected that there may exist a sizeable $S-D$ mixing for the $\Upsilon(4S)$ state with the consideration of coupled-channel effects. It is interesting to note that the $\Upsilon_1(3D)$ state might be a narrow state with a width of $\sim 20-30$ MeV, this state might be first established in its dominant BB^* decay mode in forthcoming experiments. Looking for the missing $\Upsilon_1(3D)$ state is useful for better understanding the nature of $\Upsilon(10580)$.

5 Summary

In this paper, we calculate the spectrum of the higher vector bottomonium states above the $B\bar{B}$ threshold within both screened and linear potential models. Then, using the predicted masses and wave functions of these higher vector bottomonium states, their two-body OZI-allowed strong decays are investigated in the 3P_0 model.

Combining the productions, mass, and decay width of the higher vector bottomonium states with each other, we conclude that $Y(10750)$ and $\Upsilon(10860)$ might not be pure S -wave and D -wave vector bottomonium states. Then, we further discuss the possibility of $\Upsilon(10860)$ and $Y(10750)$ as mixed states via the $S-D$ mixing. Our results suggest that $Y(10750)$ and $\Upsilon(10860)$ might be mixed states via the $5^3S_1 - 4^3D_1$ mixing with a sizeable mixing angle $\theta \simeq 20^\circ - 30^\circ$. The components of $Y(10750)$ and $\Upsilon(10860)$ are dominated by the 4^3D_1 and 5^3S_1 states, respectively.

Moreover, the strong decay behaviors of the $\Upsilon(10580)$ and $\Upsilon(11020)$ resonances are also discussed. If the $\Upsilon(10580)$

and $\Upsilon(11020)$ resonances are assigned as the $\Upsilon(4S)$ and $\Upsilon(6S)$ states, respectively, their observed widths together with masses are consistent with the theoretical predictions.

Finally, it should be mentioned that the mechanism for the $S - D$ mixing is not clear. If $Y(10750)$ and $\Upsilon(10860)$ as mixed states, several questions should be clarified in future works: (i) what causes the mixing between the 5^3S_1 and 4^3D_1 states; (ii) and how the masses of the pure S - and D -wave states are shifted to the physical states by the configuration mixing. Furthermore, in this work we only provide one possible explanation of the $Y(10750)$ and $\Upsilon(10860)$ resonances with the $S - D$ mixing scheme from the point of view of the mass and OZI allowed two-body strong decays, our results do not exclude other explanations.

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