



On the LHC signatures of $SU(5) \times U(1)'$ F-theory motivated models

A. Karozas^{1,a}, G. K. Leontaris^{1,b}, I. Tavellaris^{1,c}, N. D. Vlachos^{2,d}

¹ Physics Department, University of Ioannina, 45110 Ioannina, Greece

² Department of Nuclear and Elementary Particle Physics, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece

Received: 14 July 2020 / Accepted: 23 December 2020 / Published online: 16 January 2021

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Abstract We study low energy implications of F-theory GUT models based on $SU(5)$ extended by a $U(1)'$ symmetry which couples non-universally to the three families of quarks and leptons. This gauge group arises naturally from the maximal exceptional gauge symmetry of an elliptically fibred internal space, at a single point of enhancement, $E_8 \supset SU(5) \times SU(5)' \supset SU(5) \times U(1)'^4$. Rank-one fermion mass textures and a tree-level top quark coupling are guaranteed by imposing a Z_2 monodromy group which identifies two abelian factors of the above breaking sequence. The $U(1)'$ factor of the gauge symmetry is an anomaly free linear combination of the three remaining abelian symmetries left over by Z_2 . Several classes of models are obtained, distinguished with respect to the $U(1)'$ charges of the representations, and possible extra zero modes coming in vector-like representations. The predictions of these models are investigated and are compared with the LHC results and other related experiments. Particular cases interpreting the B-meson anomalies observed in LHCb and BaBar experiments are also discussed.

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^ae-mail: akarozas@uoi.gr (corresponding author)

^be-mail: leonta@uoi.gr

^ce-mail: i.tavellaris@uoi.gr

^de-mail: vlachos@physics.auth.gr

1 Introduction

Despite its tremendous success, the Standard Model (SM) of the strong and electroweak interactions leaves many theo-

retical questions unanswered. Accumulating evidence of the last few decades indicates that new ingredients are required in order to describe various New Physics (NP) phenomena in particle physics and cosmology. Amongst other shortcomings, the minimal SM spectrum does not accommodate a dark matter candidate particle and the tiny neutrino masses cannot be naturally incorporated. Regarding this latter issue, in particular, an elegant way to interpret the tiny masses of the three neutrinos and their associated oscillations, is the see-saw mechanism [1] which brings into the scene right-handed neutrinos and a new (high) scale. Interestingly, this scenario fits nicely inside the framework of (supersymmetric) grand unified theories (GUTs) which unify the three fundamental forces at a high (GUT) scale. Besides, several ongoing neutrino experiments suggest the existence of a ‘sterile’ neutrino which could also be a suitable dark matter candidate [2, 3]. Many other lingering questions regarding the existence of possible remnants of a covering theory, such as leptoquarks, vectorlike families, supersymmetry signatures and neutral gauge bosons, are expected to find an answer in the experiments carried out at the Large Hadron Collider (LHC). Remarkably, many field theory GUTs incorporate most of the above novel fields into larger representations, while, after spontaneous symmetry breaking of the initial gauge symmetry takes place, cases where additional $U(1)$ factors survive down to low energies implying masses for the associated neutral gauge bosons accessible to ongoing experiments. However, while GUTs with the aforementioned new features are quite appealing, they come at a price. Various extra fields, including heavy gauge bosons and other colored states, contribute to fast proton decay and other rare processes.

In contrast to plain field theory GUTs, string theory alternatives are subject to selection rules and other restrictions, while new mechanisms are operative which, under certain conditions, could eliminate many of the problematic states and undesired features. F-theory models [4–6], in particular, appear to naturally include such attractive features which are attributed to the intrinsic geometry of the compactification manifold and the fluxes piercing matter curves where the various supermultiplets reside. In other words, the geometric properties and the fluxes can be chosen so that, among other things, determine the desired symmetry breaking, reproduce the known multiplicity of the chiral fermion families, and eliminate the colored triplets in Higgs representations. Moreover, in F-theory constructions, the gauge symmetry of the resulting effective field theory model is determined in terms of the geometric structure of the elliptically fibred internal compactification space. In particular, the non-abelian part of the gauge symmetry is associated with the codimension-one singular fibers, while possible abelian and discrete symmetries are determined in terms of the Mordell–Weil (MW) and

Tate–Shafarevich (TS) groups.¹ For elliptically fibred manifolds, the non-abelian gauge symmetry is a simply laced algebra (i.e. of type A , D or E in Lie classification), the highest one corresponding to the exceptional group of E_8 . At fibral singularities, certain divisors wrapped with 7-branes are associated with subgroups of E_8 , and are interpreted as the GUT group of the effective theory. In addition, $U(1)$ symmetries may accompany the non-abelian group. The origin of the latter could emerge either from the E_8 -part commutant to the GUT group or from MW and TS groups mentioned above. Among the various possibilities, there is a particularly interesting case where a neutral gauge boson Z' associated with some abelian factor with non-universal couplings to the quarks and leptons, obtains mass at the TeV region. Since the SM gauge bosons couple universally to quarks (and leptons) of the three families, the existence of non-universal couplings would lead to deviations from SM predictions that could be interpreted as an indication for NP beyond the SM.

Within the above context, in [17] a first systematic study of a generic class of F-theory semi-local models based on the E_8 subgroup $SU(5) \times U(1)'$ has been presented.² The anomaly-free $U(1)'$ symmetry has non-universal couplings to the three chiral families and the corresponding gauge boson receives a low energy (a few TeV) mass. In that work, some particular properties of representative examples were examined in connection with new flavour phenomena and in particular, the B-meson physics explored in LHCb [20–22]. In the present work we extend the previous analysis by performing a systematic investigation into the various predictions and the constraints imposed on all possible classes of viable models emerging from this framework. Firstly we distinguish classes of models with respect to their low energy spectrum and properties under the $U(1)'$ symmetry. We find a class of models with a minimal MSSM spectrum at low energies. The members of this class are differentiated by the charges under the additional $U(1)'$. A second class of anomaly free viable effective low energy models, contains additional MSSM multiplets coming in vector-like pairs. In the present work, we analyse the constraints imposed by various processes on the list of models of the first class. The phenomenological analysis of a characteristic example containing extra vector-like states is also presented, while the complete analysis of these models is postponed for a future publication. In the first category (i.e. the minimal models), anomaly cancellation conditions impose non-universal Z' couplings to the three fermion field families. As a result, in most cases, the stringent bounds coming from kaon decays imply a relatively large Z' gauge boson mass that lies beyond the accessibility of the present

¹ For a recent survey see for example [7]. For earlier F-theory reviews see [8–10]. For models with Mordell–Weil $U(1)$'s and other issues see [11–16].

² For similar works on anomaly free $U(1)$'s see also [18, 19].

day experiments. On the contrary, models with extra vector-like pairs offer a variety of possibilities. There are viable cases where the fermions of the first two generations are characterised by the same Z' couplings. In such cases, the stringent bounds of the Kaon oscillation ($K^0 - \bar{K}^0$) system can be evaded and a Z' mass can be as low as a few TeV.

The work is organised and presented in five sections. In Sect. 2 we start by developing the general formalism of a Z' boson coupled non-universally to MSSM. Then, we discuss flavour violating processes in the quark and lepton sectors, putting emphasis on contributions to B-meson anomalies and other deviations from the SM explored in LHC and other related experiments. (To make the paper self contained, all relevant recent experimental bounds are also given). In Sect. 3 we start with a brief review of local F-theory GUTs. Then, using generic properties of the compactification manifold and the flux mechanism, we apply well defined rules and spectral cover techniques to construct viable effective models. We concentrate on a $SU(5) \times U(1)'$ model embedded in E_8 and impose anomaly cancellation conditions to obtain a variety of consistent F-theory effective models. We distinguish between two categories; a class of models with a MSSM (charged) spectrum (possibly with some extra neutral singlet fields) and a second one where the MSSM spectrum is extended with vector-like quark and charged lepton representations. In Sect. 4 we analyse the phenomenological implications of the first class, paying particular attention to B-meson physics and lepton flavour violating decays. Some consequences of the models with extra vector-like fields are discussed in Sect. 5, while a detailed investigation into the whole class of models will be presented in a future publication. In Sect. 6 we present our conclusions. Computational details are given in the Appendix.

2 Non-universal Z' interactions

In the Standard Model, the neutral gauge boson couplings to fermions with the same electric charge are equal, therefore, the corresponding tree-level interactions are flavour diagonal. However, this is not always true in models with additional Z' bosons associated with extra $U(1)'$ factors emanating from higher symmetries. If the $U(1)'$ charges of all or some of the three fermion families are different, significant flavour mixing effects might occur even at tree-level. In this section we review some basic facts about non-universal $U(1)'$ s and develop the necessary formalism to be used subsequently.

2.1 Generalities and formalism

To set the stage, we first consider the neutral part of the Lagrangian including the Z' interactions with fermions in the gauge eigenstates basis [23,24]:

$$-\mathcal{L}_{NC} \supset e J_{EM}^\mu A_\mu + \frac{g}{c_W} J^{(0)\mu} Z_\mu^0 + g' J'^\mu Z'_\mu, \quad (2.1)$$

where A_μ is the massless photon field, Z^0 is the neutral gauge boson of the SM and Z' is the new boson associated with the extra $U(1)'$ gauge symmetry. Also g and g' are the gauge couplings of the weak $SU(2)$ gauge symmetry and the new $U(1)'$ symmetry respectively. For shorthand, we have denoted $\cos \theta_W$ ($\sin \theta_W$) as c_W (s_W) where θ_W is the weak mixing angle with $g = e/\tan \theta_W$. The neutral current associated with the Z' boson can be written as:

$$J'^\mu = \bar{f}_L^0 \gamma^\mu q'_{f_L} f_L^0 + \bar{f}_R^0 \gamma^\mu q'_{f_R} f_R^0, \quad (2.2)$$

where f_L^0 (f_R^0) is a column vector of left (right) chiral fermions of a given type (u, d, e or ν) in the gauge basis and $q'_{f_{L,R}}$ are diagonal 3×3 matrices of $U(1)'$ charges. f_L denotes chiral fermions in the mass eigenstates basis, related to gauge eigenstates via unitary transformations of the form

$$f_L^0 = V_{f_L}^\dagger f_L, \quad f_R^0 = V_{f_R}^\dagger f_R. \quad (2.3)$$

$V_{f_{L,R}}$ are unitary matrices responsible for the diagonalization of the Yukawa matrices Y_f ,

$$Y_f^{diag} = V_{f_R} Y_f V_{f_L}^\dagger, \quad (2.4)$$

with the CKM matrix defined by the combination:

$$V_{CKM} = V_{u_L} V_{d_L}^\dagger.$$

In the mass eigenbasis, the neutral current (2.2) takes the form:

$$J^\mu = \bar{f}_L \gamma^\mu Q'_{f_L} f_L + \bar{f}_R \gamma^\mu Q'_{f_R} f_R \quad (2.5)$$

where

$$Q'_{f_L} \equiv V_{f_L} q'_{f_L} V_{f_L}^\dagger, \quad Q'_{f_R} \equiv V_{f_R} q'_{f_R} V_{f_R}^\dagger. \quad (2.6)$$

If the $U(1)'$ charges in the q'_{f_L} matrix are equal, then q'_{f_L} is the unit matrix up to a common charge factor and due to the unitarity of V_f 's the current in (2.5) becomes flavour diagonal. For models with family non-universal $U(1)'$ charges, the mixing matrix Q'_{f_L} is non-diagonal and flavour violating terms appear in the effective theory.

2.2 Quark sector flavour violation

2.2.1 $b \rightarrow sl^+l^-$ and R_K anomalies

The possible existence of non-universal Z' couplings to fermion families, may lead to departures from the SM predictions and leave clear signatures in present day or near future experiments. Such contributions strongly depend on the mass $M_{Z'}$ of the Z' gauge boson, the $U(1)'$ gauge coupling, g' , the $U(1)'$ fermion charges and the mixing matrices V_f . A particularly interesting case reported by LHCb [22] and BaBar [25] collaborations, indicate that there may be

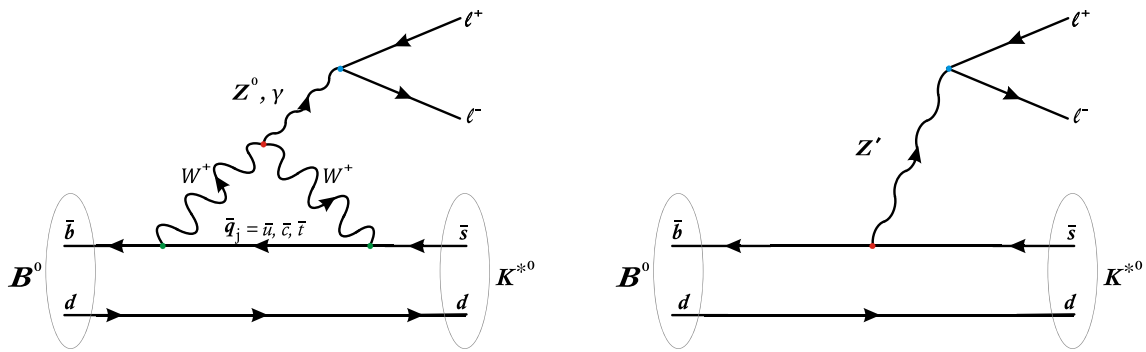


Fig. 1 Left panel: Example of a Feynman diagram contributing to $B^0 \rightarrow K^{*0} l^+ l^-$ in the SM context. Right panel: Tree level contribution in models with non-universal Z' 's

anomalies observed in B-meson decays, associated with the transition $b \rightarrow s l^+ l^-$, where $l = e, \mu, \tau$. Current LHCb measurements of b decays to different lepton pairs hint to deviations from lepton universality. In particular, the analysis performed in the q^2 invariant mass of the lepton pairs in the range $1.1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$ for the ratio of the branching ratios $Br(B \rightarrow K^{(*)} l^+ l^-)$, $l = \mu, e$ gives [22]

$$R_K \equiv \frac{Br(B \rightarrow K \mu^+ \mu^-)}{Br(B \rightarrow K e^+ e^-)} \simeq 0.846^{+0.016}_{-0.014} \text{ (stat/syst)}, \tag{2.7}$$

where statistical and systematic uncertainties are indicated. Similarly, the results for $B \rightarrow K^*(892) l^+ l^-$ (where $K^* \rightarrow K \pi$), for the same ratio (2.7) are found to be $R_{K^*} \simeq 0.69$. Since the SM strictly predicts $R_{K^{(*)}}^{SM} = 1$, these results strongly suggest that NP scenarios where lepton universality is violated should be explored. In the case $l = \mu$ in particular, additional experimental and theoretical arguments suggest that NP may be related with the muon channel [26–28].

In the SM, $B \rightarrow K^{(*)} l^+ l^-$ can only be realised at the one-loop level involving W^\pm flavour changing interactions (see left panel of Fig. 1). However, the existence of a Z' (neutral) gauge boson bearing non-universal couplings to fermions, can lead to tree-level contributions (right panel of Fig. 1) which might explain (depending on the model) the observed anomalies.

The effective Hamiltonian describing the interaction is given by [28]

$$H_{eff}^{b \rightarrow s ll} = -\frac{4G_F}{\sqrt{2}} \frac{e^2}{16\pi^2} (V_{tb} V_{ts}^*) \sum_{k=9,10} (C_k^{ll} O_k^{ll} + C_k'^{ll} \mathcal{O}_k'^{ll}) \tag{2.8}$$

where the symbols \mathcal{O}_n^{xx} stand for the following dimension-6 operators,

$$\begin{aligned} \mathcal{O}_9^{ll} &= (\bar{s} \gamma^\mu P_L b) (\bar{l} \gamma_\mu l), & \mathcal{O}_{10}^{ll} &= (\bar{s} \gamma^\mu P_L b) (\bar{l} \gamma_\mu \gamma_5 l) \\ \mathcal{O}_9'^{ll} &= (\bar{s} \gamma^\mu P_R b) (\bar{l} \gamma_\mu \gamma_5 l), & \mathcal{O}_{10}'^{ll} &= (\bar{s} \gamma^\mu P_R b) (\bar{l} \gamma_\mu \gamma_5 l), \end{aligned}$$

and C_k are Wilson coefficients displaying the strength of the interaction. Also, in (2.8), G_F is the Fermi coupling constant and V_{tb}, V_{ts}^* are elements of the CKM matrix.

The latest data for $R_{K^{(*)}}$ ratios can be interpreted by assuming a negative contribution to the Wilson coefficient $C_9^{\mu\mu}$, while all the other Wilson coefficients³ should be negligible, or vanishing [29–33]. The current best fit value is $C_9^{\mu\mu} \approx -0.95 \pm 0.15$.

In the presence of a non-universal Z' gauge boson, the $C_9^{\mu\mu}$ Wilson coefficient is given by :

$$C_9^{\mu\mu} = -\frac{\sqrt{2}}{4G_F} \frac{16\pi^2}{e^2} \left(\frac{g'}{M_{Z'}} \right)^2 \frac{(Q'_{d_L})_{23} (Q'_{e_L})_{22}}{V_{tb} V_{ts}^*}. \tag{2.9}$$

The desired value for the C_9 coefficient could be achieved by appropriately tuning the ratio $g'/M_{Z'}$. However, large suppressions may occur from the matrices Q'_f . In any case, the predictions must not create conflict with well known bounds coming from rare processes such as the mixing effects in neutral meson systems.

2.2.2 Meson mixing

Flavor changing Z' interactions in the quark sector can also induce significant contributions to the mass splitting in a neutral meson system. A representative example is given in Fig. 2. The diagrams show contributions to $B_s^0 [s\bar{b}]$ mixing in the SM (left) and tree-level contributions in non-universal Z' models (right).

For a meson P^0 with quark structure $[q_i \bar{q}_j]$, the contribution from Z' interactions to the mass splitting is given by [24]:

$$\Delta M_P \simeq 4\sqrt{2} G_F M_P F_P^2 \left(\frac{M_W}{g \cdot c_w} \right)^2 \left(\frac{g'}{M_{Z'}} \right)^2 \frac{1}{3} \text{Re}[(Q'_{q_L})_{ij}^2] \tag{2.10}$$

³ Alternative scenarios suggest: $C_{10}^{\mu\mu} \approx 0.73 \pm 0.14$ or $C_9^{\mu\mu} = -C_{10}^{\mu\mu} \approx -0.53 \pm 0.09$.

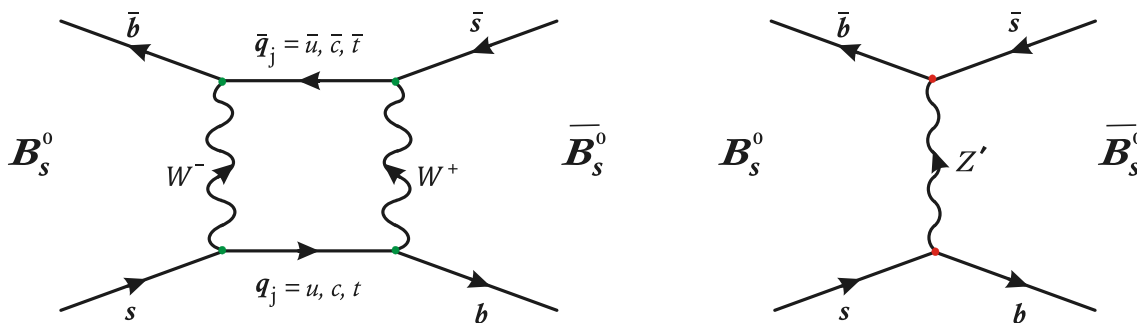


Fig. 2 Left figure: Representative *box* diagram contribute to $(B_s^0 - \bar{B}_s^0)$ mixing in the SM. Right figure: Tree level contribution in models with non-universal Z' gauge bosons

where M_W is the mass of the W^\pm gauge bosons and M_P, F_P is the mass and the decay constant of the meson P^0 respectively.

There are large uncertainties in the SM computations of ΔM_P , descending especially from QCD factors and the CKM matrix elements. Nevertheless, the experimental results suggest that there is still some room for NP contributions.

Next, we review theoretical and experimental constraints for $P^0 - \bar{P}^0$ meson systems to be taken into account in what follows.

• $B_s^0 - \bar{B}_s^0$ mixing:

B_s mixing can be described by the effective Lagrangian

$$\mathcal{L}^{NP} = -\frac{4G_F}{\sqrt{2}}(V_{tb}V_{ts}^*)^2[C_{bs}^{LL}(\bar{s}_L\gamma_\mu b_L)^2 + h.c.], \quad (2.11)$$

where C_{bs}^{LL} is a Wilson coefficient which modifies the SM prediction as follows [34]:

$$\Delta M_s^{pred} = |1 + C_{bs}^{LL}/R_{SM}^{loop}|\Delta M_s^{SM}, \quad (2.12)$$

with $R_{SM}^{loop} = 1.3397 \times 10^{-3}$.

A model with non-universal Z' couplings to fermions induces the following Wilson coefficient:

$$C_{bs}^{LL} = \frac{\eta^{LL}}{4\sqrt{2}G_F} \left(\frac{g'}{M_{Z'}}\right)^2 \frac{(Q'_{dL})_{23}^2}{(V_{tb}V_{ts}^*)^2} \quad (2.13)$$

where $\eta^{LL} \equiv \eta^{LL}(M_{Z'})$ is a constant which encodes renormalisation group effects. This constant has a weak dependence⁴ on the $M_{Z'}$ scale. In our analysis we consider that $\eta^{LL} = 0.79$ which corresponds to $M_{Z'} = 1$ TeV.

For the SM contribution ΔM_s^{SM} we consider the result obtained in Ref. [36],

$$\Delta M_s^{SM} = (18.5_{-1.5}^{+1.2}) ps^{-1},$$

which when compared with the experimental bound [37], $\Delta M_s^{exp} = (17.757_{-0.021}^{+0.021}) ps^{-1}$, shows through Eq. (2.12), that a small positive C_{bs}^{LL} is allowed.

⁴ For $M_{Z'} \in [1, 10]$ TeV it turns out that $\eta^{LL} \in [0.79, 0.75]$, see [34,35].

• $K^0 - \bar{K}^0$ mixing:

SM computations for the mass split in the neutral Kaon system are a combination of short-distance and long-distance effects, given as [38]

$$\Delta M_K^{SM} = (0.8 \pm 0.1)\Delta M_K^{Exp}, \quad (2.14)$$

where the experimental data are given by [37]:

$$\Delta M_K^{exp} \simeq 3.482 \times 10^{-15} \text{ GeV}.$$

This small discrepancy between SM computations and experiment can be explained by including NP effects into the analysis. Thus, according to (2.14), the contribution of a non-universal Z' boson to ΔM_K must satisfy the following constraint [39];

$$\Delta M_K^{NP} \lesssim 0.2 \times \Delta M_K^{exp}, \quad (2.15)$$

where ΔM_K^{NP} can be computed directly from the formula (2.10).

• $D^0 - \bar{D}^0$ mixing:

Neutral D mesons consist of up-type quarks, $D^0 \rightarrow [c\bar{u}]$. The experimental measurements for $D^0 - \bar{D}^0$ oscillations are sensitive to the ratio:

$$x_D \equiv \frac{\Delta M_D}{\Gamma_D}, \quad (2.16)$$

where Γ_D is the total decay width of D^0 and the observed value for the ratio is $x_D \simeq 0.32$ [40]. Since the process is subject to large theoretical and experimental uncertainties, we will simply consider NP contributions to x_D less or equal to the experimental value.

2.2.3 Leptonic meson decays: $P^0 \rightarrow l_i \bar{l}_i$

In the SM the decay of a neutral meson P^0 into a lepton (l_i) and its anti-lepton (\bar{l}_i) is realised at the one-loop level. While in the SM these processes are suppressed due to GIM [41] cancellation mechanism, in non-universal Z' models substantially larger tree-level contributions may be allowed. The decay width induced by Z' interactions can be written

in terms of the SM decay $P^- \rightarrow l_i \bar{\nu}_i$ as [24]:

$$\Gamma(P^0 \rightarrow l_i \bar{l}_i) \simeq 8 \frac{\Gamma(P^- \rightarrow l_i \bar{\nu}_i)}{|V_{kj}^{CKM}|^2} \frac{M_P^3 \sqrt{M_P^2 - 4m_{l_i}^2}}{(M_P^2 - m_{l_i}^2)^2} \times \left(\frac{g'}{M_{Z'}}\right)^4 \left(\frac{M_{Z_0}}{g}\right)^4 |(Q'_{qL})_{mn}(Q'_{eL})_{ii}|^2, \tag{2.17}$$

where the indices j, k refer to the quark structure $[q_j \bar{q}_k]$ of the meson P^- appearing in the SM interaction. Similarly, the indices m, n are used here to denote the quark structure of the neutral meson P^0 . All the relevant experimental bounds for this type of interactions can be found in [37].

2.3 Lepton flavour violation

2.3.1 $P^0 \rightarrow l_i \bar{l}_j$

The lepton flavour violation process $P^0 \rightarrow l_i \bar{l}_j$ is similar to the previous one where $i = j$. The decay width due to tree-level Z' contributions is given by [24]:

$$\Gamma(P^0 \rightarrow l_i \bar{l}_j) \simeq 4 \frac{\Gamma(P^- \rightarrow l_i \bar{\nu}_i)}{|V_{kr}^{CKM}|^2} \left(\frac{g'}{M_{Z'}}\right)^4 \times \left(\frac{M_{Z_0}}{g}\right)^4 |(Q'_{uL})_{mn}(Q'_{eL})_{ij}|^2. \tag{2.18}$$

As previously, the indices k, r are used to denote the quark structure $[q_r \bar{q}_k]$ of the meson participating in the SM interaction, while generation indices m, n refer to the quark structure of P^0 . Bounds and predictions for these rare interactions will be given in the subsequent analysis.

2.3.2 $(g - 2)_\mu$

The anomalous magnetic moment of the muon $a_\mu \equiv (g - 2)/2$, is measured with high accuracy. However there exists a discrepancy between experimental measurements and precise SM computations [37]:

$$\Delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = 261(63)(48) \times 10^{-11} \tag{2.19}$$

where $a_\mu^{SM} = 116591830(1)(40)(26) \times 10^{-11}$.

This difference may be explained by NP contributions. In the case of a Z' neutral boson, loop diagrams like the one shown on the left side of Fig. 3 contribute to Δa_μ . Collectively, the 1-loop contribution from a non-universal Z' bosons is [42]:

$$\Delta a_\mu^{Z'} = -\frac{m_\mu^2}{8\pi^2} \left(\frac{g'}{M_{Z'}}\right)^2 \sum_{j=1}^3 |(Q'_{eL})_{2j}|^2 F(x_{l_j}^{Z'}) \tag{2.20}$$

where $x_{l_j}^{Z'} := (m_{l_j}/M_{Z'})^2$ with the loop function defined as:

$$F(x) = \frac{5x^4 - 14x^3 + 39x^2 - 38x - 18x^2 \ln(x) + 8}{12(1-x)^4}. \tag{2.21}$$

In our analysis we will consider that $\Delta a_\mu^{Z'}$ must be less or equal to Δa_μ .

2.3.3 $l_i \rightarrow l_j \gamma$

A flavour violating Z' boson contributes also to radiative decays of the form $l_i \rightarrow l_j \gamma$. The 1-loop diagram of the strongly constrained decay $\mu^- \rightarrow e^- \gamma$ is displayed in Fig. 3 (right). Considering only Z' contributions, the branching ratio for this type of interactions is given by [43]:

$$\text{Br}(l_i \rightarrow l_j \gamma) = \frac{e^2}{16\pi \Gamma_{l_i}} \left(m_{l_i} - \frac{m_{l_j}^2}{m_{l_i}}\right)^3 (g')^2 \times \sum_f [y_2(Q'_{eL})_{fj}(Q'_{eL})_{fi}], \tag{2.22}$$

where the index $f = 1, 2, 3$ refers to the lepton running inside the loop, Γ_{l_i} is the total decay width of the lepton l_i and y_2 is a loop function that can be found in [43]. The most recent experimental bounds are:

$$\text{Br}(\mu \rightarrow e \gamma) < 4.2 \times 10^{-13}, \text{ Br}(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8} \text{ and } \text{Br}(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}.$$

Dominant constraints are expected to come from the muon decay.

2.3.4 $l_i \rightarrow l_j l_k \bar{l}_j$

A lepton flavour violating Z' boson mediates (at tree-level) three-body leptonic decays of the form $l_i \rightarrow l_j l_k \bar{l}_k$. The branching ratio is given by [44]:

$$\text{Br}(l_i \rightarrow l_j l_k \bar{l}_k) = \frac{m_{l_i}^5}{768\pi^3 \Gamma_{l_i}} \left(\frac{g'}{M_{Z'}}\right)^4 |(Q'_{eL})_{ij}(Q'_{eL})_{kj}|^2, \tag{2.23}$$

where the masses of the produced leptons have been neglected.

For decays of the form $l_i \rightarrow l_j l_k \bar{l}_j$ with $k \neq j$ the branching ratio is

$$\text{Br}(l_i \rightarrow l_j l_k \bar{l}_j) = \frac{m_{l_i}^5}{1536\pi^3 \Gamma_{l_i}} \times \left(\frac{g'}{M_{Z'}}\right)^4 |(Q'_{eL})_{ik}(Q'_{eL})_{jj} + (Q'_{eL})_{ij}(Q'_{eL})_{jk}|^2. \tag{2.24}$$

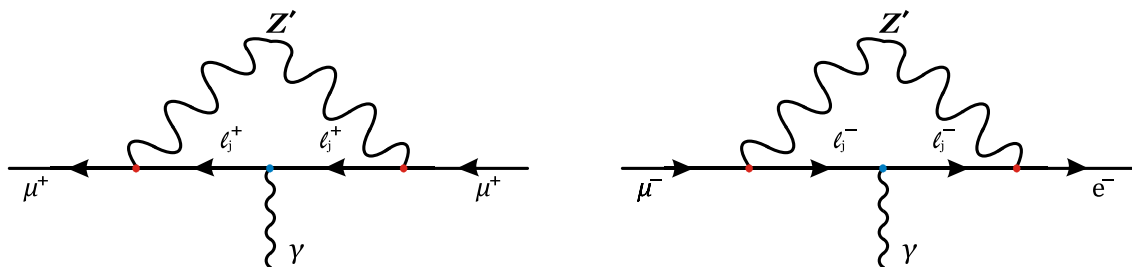


Fig. 3 Left side: Contribution of a non-universal Z' boson into the magnetic moment of (anti)muon. Right side: Contribution to the decay, $\mu^- \rightarrow e^- \gamma$. Any of the three (anti)leptons ($j = e, \mu, \tau$) could run in to the loop due to the non-universal charges under the extra $U(1)$ symmetry

The dominant constraint comes from the muon decay $\mu^- \rightarrow e^- e^- e^+$, with branching ratio bounded as $Br(\mu \rightarrow eee) < 10^{-12}$ at 90% confidence level [45].

3 Non-universal $U(1)'$ models from F-theory

We now turn on to the class of F-theory constructions accommodating abelian factors bearing non-universal couplings with the three families of the Standard Model. As already mentioned, we focus on constructions based on an elliptically fibred compact space with E_8 being the maximal singularity, and assume a divisor in the internal manifold where the associated non-abelian gauge symmetry is $SU(5)$. With this choice, E_8 decomposes as

$$E_8 \supset SU(5) \times SU(5)_\perp. \tag{3.1}$$

We will restrict our analysis in local constructions and describe the resulting effective theory in terms of the Higgs bundle picture which makes use of the adjoint scalars where only the Cartan generators acquire a non-vanishing vacuum expectation value (VEV).⁵ In the local picture we may work with the spectral data (eigenvalues and eigenvectors) which, for the case of $SU(5)$, are associated with the 5th degree polynomial

$$\begin{aligned} \mathcal{C}_5 &= \sum_{k=0}^5 b_k t^{5-k} = b_0 t^5 + b_1 t^4 \\ &+ b_2 t^3 + b_3 t^2 + b_4 t + b_5 = 0. \end{aligned} \tag{3.2}$$

This defines the spectral cover for the fundamental representation of $SU(5)$. Furthermore, as is the case for any $SU(n)$, the five roots

$$Q = \{t_1, t_2, t_3, t_4, t_5\}, \tag{3.3}$$

must add up to zero,

$$-b_1 \equiv \sum_{i=1}^5 t_i = 0. \tag{3.4}$$

⁵ For non-diagonal generalisations (T -branes) see [46].

The remaining coefficients are generically non-zero, $b_k \neq 0, k = 0, 2, 3, 4, 5$ and carry the geometric properties of the internal manifold.

The zero-mode spectrum of the effective low energy theory descends from the decomposition of the E_8 adjoint. With respect to the breaking pattern (3.1), it decomposes as follows:

$$248 \rightarrow (24, 1) + \boxed{(1, 24) + (10, 5) + (\bar{5}, 10) + (5, \bar{10})} + (\bar{10}, \bar{5}). \tag{3.5}$$

Ordinary matter and Higgs fields, including the appearance of possible singlets in the spectrum, appear in the box of the right-hand side in (3.5) and transform in bi-fundamental representations, with respect to the two $SU(5)$ s. From the above, we observe that the GUT decuplets transform in the fundamental of $SU(5)_\perp$, whilst the $\bar{5}$, 5-plets are in the anti-symmetric representation of the ‘perpendicular’ symmetry. For our present purposes however, it is adequate to work in the limit where the perpendicular symmetry reduces down to the Cartan subalgebra according to the breaking pattern $SU(5)_\perp \rightarrow U(1)_\perp^4$. In this picture, the GUT representations are characterised by the appropriate combinations of the five weights given in (3.3). The five 10-plets in particular, are counted by $t_{1,2,\dots,5}$ and the fiveplets which originally transform as decuplets under the second $SU(5)_\perp$ are characterised by the ten combinations $t_i + t_j$. In the geometric description, it is said that the $SU(5)$ GUT representations reside in Riemann surfaces (dubbed matter curves Σ_a) formed by the intersections of the $SU(5)$ GUT divisor with ‘perpendicular’ 7-branes. These properties are summarised in the following notation

$$\begin{aligned} \Sigma_{10_{t_i}} &: 10_{t_i}, \bar{10}_{-t_i}, \quad \Sigma_{5_{t_i+t_j}} : \bar{5}_{t_i+t_j}, 5_{-t_i-t_j}, \\ \Sigma_{1_{t_i-t_j}} &: 1_{t_i-t_j}. \end{aligned} \tag{3.6}$$

As we have seen above, since the weights $t_{i=1,2,3,4,5}$ associated with the $SU(5)_\perp$ group, are the roots of the polynomial (3.2), they can be expressed as functions of the coefficients b_k ’s which carry the information regarding the geometric properties of the compactification manifold. Based on this fact, in the subsequent analysis, we will make use of the

topological invariant quantities and flux data to determine the spectrum and the parameter space of the effective low energy models under consideration.

We start by determining the zero-mode spectrum of the possible classes of models within the context discussed above. According to the spectral cover description, see Eqs. (3.2–3.6), the various matter curves of the theory accommodating the $SU(5)$ GUT multiplets are determined by the following equations:

$$\Sigma_{10_i} : P_{10} := b_5 \sim \prod_{i=1}^5 t_i = 0, \tag{3.7}$$

and

$$\begin{aligned} \Sigma_{5_{t_i+t_j}} : P_5 &:= b_3^2 b_4 - b_2 b_3 b_5 + b_0 b_5^2 \\ &\sim \prod_{i \neq j} (t_i + t_j) = 0. \end{aligned} \tag{3.8}$$

If all five roots t_i of the polynomial (3.2) are distinct and expressed as holomorphic functions of the coefficients b_k , then, simple counting shows that there can be five matter curves accommodating the tenplets(decuplets) and ten matter curves where the fiveplets(quintuplets) can reside. This would imply that the polynomial (3.2) could be expressed as a product $\prod_{i=1}^5 (\alpha_i t_i + \beta_i)$, with the coefficients α_i, β_i carrying the topological properties of the manifolds, while being in the same field as the original b_k . However, in the generic case not all five solutions $t_i(b_k)$ belong to the same field with b_k . In effect, there are monodromy relations among subsets of the roots t_i , reducing the number of independent matter curves. Depending on the specific geometric properties of the compactification manifold, we can have a variety of factorisations of the spectral cover polynomial \mathcal{C}_5 . (The latter are parametrised by the Cartan subalgebra modulo the Weyl group $W(SU(5)_\perp)$). In other words, generic solutions imply branch cuts and some roots are indistinguishable. The simplest case is when two of them are subject to a Z_2 monodromy,

$$Z_2 : t_1 = t_2. \tag{3.9}$$

Remarkably, there is an immediate implication of the Z_2 monodromy in the effective field theory model. It allows the tree-level coupling in the superpotential

$$10_{t_1} 10_{t_2} 5_{-t_1-t_2} \xrightarrow{Z_2} 10_{t_1} 10_{t_1} 5_{-2t_1}, \tag{3.10}$$

which can induce a heavy top-quark mass as required by low energy phenomenology.

Returning to the spectral cover description, under the Z_2 monodromy, the polynomial (3.2) is factorised accordingly to

$$\mathcal{C}_5 = (a_1 + a_2 t + a_3 t^2)(a_4 + a_7 t)(a_5 + a_8 t)(a_6 + a_9 t), \tag{3.11}$$

where the existence of the second degree polynomial is not factorisable in the sense presented above, indicating thus, that the corresponding roots t_1, t_2 are connected by Z_2 .

Comparing this with the spectral polynomial in (3.2), we can extract the relations between the coefficients b_k and a_j . Thus, one gets

$$\begin{aligned} b_0 &= a_3 a_7 a_8 a_9, \\ b_1 &= a_3 a_6 a_7 a_8 + a_3 a_4 a_9 a_8 + a_2 a_7 a_9 a_8 + a_3 a_5 a_7 a_9, \\ b_2 &= a_3 a_5 a_6 a_7 + a_2 a_6 a_8 a_7 + a_2 a_5 a_9 a_7 \\ &\quad + a_1 a_8 a_9 a_7 + a_3 a_4 a_6 a_8 + a_3 a_4 a_5 a_9 + a_2 a_4 a_8 a_9, \\ b_3 &= a_3 a_4 a_5 a_6 + a_2 a_5 a_7 a_6 + a_2 a_4 a_8 a_6 + a_1 a_7 a_8 a_6 \\ &\quad + a_2 a_4 a_5 a_9 + a_1 a_5 a_7 a_9 + a_1 a_4 a_8 a_9, \\ b_4 &= a_2 a_4 a_5 a_6 + a_1 a_5 a_7 a_6 + a_1 a_4 a_8 a_6 + a_1 a_4 a_5 a_9, \\ b_5 &= a_1 a_4 a_5 a_6. \end{aligned} \tag{3.12}$$

We impose the $SU(5)$ constraint $b_1 = 0$ assuming the Ansatz [47]

$$a_2 = -c(a_6 a_7 a_8 + a_5 a_7 a_9 + a_4 a_8 a_9), \quad a_3 = c a_7 a_8 a_9,$$

where a new holomorphic section c has been introduced. Substituting into (3.12) one gets

$$\begin{aligned} b_0 &= c a_7^2 a_8^2 a_9^2, \\ b_2 &= a_9(a_1 a_7 a_8 - (a_5^2 a_7^2 + a_4 a_5 a_8 a_7 + a_4^2 a_8^2) a_9 c) \\ &\quad - c a_6^2 a_7^2 a_8^2 - c a_6 a_7 (a_5 a_7 + a_4 a_8) a_9 a_8, \\ b_3 &= a_1(a_6 a_7 a_8 + (a_5 a_7 + a_4 a_8) a_9) \\ &\quad - (a_5 a_7 + a_4 a_8)(a_6 a_7 + a_4 a_9)(a_6 a_8 + a_5 a_9) c, \\ b_4 &= a_1(a_4 a_6 a_8 + a_5(a_6 a_7 + a_4 a_9)) - a_4 a_5 a_6 \\ &\quad \times (a_6 a_7 a_8 + (a_5 a_7 + a_4 a_8) a_9) c, \\ b_5 &= a_1 a_4 a_5 a_6. \end{aligned} \tag{3.13}$$

The equations of tenplets and fiveplets can now be expressed in terms of the holomorphic sections a_j 's and c . In the case of the tenplets we end up with four factors

$$P_{10} = a_1 \times a_4 \times a_5 \times a_6, \tag{3.14}$$

which correspond to four matter curves accommodating the tenplets of $SU(5)$. Substitution of (3.13) in to P_5 factorises the equation into seven factors corresponding to seven distinct fiveplets

$$\begin{aligned} P_5 &= (a_5 a_7 + a_4 a_8) \times (a_6 a_7 + a_4 a_9) \\ &\quad \times (a_6 a_8 + a_5 a_9) \\ &\quad \times (a_6 a_7 a_8 + a_4 a_9 a_8 + a_5 a_7 a_9) \\ &\quad \times (a_1 - a_5 a_6 a_7 c - a_4 a_6 a_8 c) \\ &\quad \times (a_1 - a_5 a_6 a_7 c - a_4 a_5 a_9 c) \\ &\quad \times (a_1 - a_4 a_6 a_8 c - a_4 a_5 a_9 c). \end{aligned} \tag{3.15}$$

Finally, we compute the homologies of the section a_j 's and c , and consequently of each matter curve. This can be

Table 1 Homology classes of the coefficients a_j and c . Note that $\chi = \chi_5 + \chi_7 + \chi_9$ where χ_7, χ_8, χ_9 are the unspecified homologies of the coefficients a_5, a_7 and a_9 respectively

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	c
$\eta - 2c_1 - \chi$	$\eta - c_1 - \chi$	$\eta - \chi$	$-c_1 + \chi_7$	$-c_1 + \chi_8$	$-c_1 + \chi_9$	χ_7	χ_8	χ_9	$\eta - 2\chi$

done by using the known homologies of the b_k coefficients:

$$[b_k] = (6 - k)c_1 - t = \eta - k c_1 \tag{3.16}$$

where c_1 is the 1st Chern class of the tangent bundle to S_{GUT} , $-t$ the 1st Chern class of the normal bundle to S_{GUT} and $\eta = 6c_1 - t$. The homologies of a_j 's and c are presented in Table 1, while the homologies of the various matter curves are given in Table 2. Because there are more a 's than b 's, three homologies which are taken to be $[a_7] = \chi_7, [a_8] = \chi_8$ and $[a_9] = \chi_9$, remain unspecified.

3.1 $SU(5) \times U(1)'$ in the spectral cover description

Our aim is to examine $SU(5) \times U(1)'$ models and particularly the rôle of the non-universal $U(1)'$ which should be consistently embedded in the covering group E_8 . Clearly, the $U(1)'$ symmetry should be a linear combination of the abelian factors residing in $SU(5)_\perp$. A convenient abelian basis to express the desired $U(1)'$ emerges in the following sequence of symmetry breaking

$$E_8 \supset E_6 \times SU(3)_\perp \supset E_6 \times U(1)_\perp \times U(1)'_\perp \tag{3.17}$$

$$\supset SO(10) \times U(1)_\psi \times U(1)_\perp \times U(1)'_\perp \tag{3.18}$$

$$\supset SU(5)_{GUT} \times U(1)_\chi \times U(1)_\psi \times U(1)_\perp \times U(1)'_\perp. \tag{3.19}$$

Then, the Cartan generators corresponding to the four $U(1)$'s are expressed as:

$$\begin{aligned} Q'_\perp &= \frac{1}{2} \text{diag}(1, -1, 0, 0, 0), \\ Q_\perp &= \frac{1}{2\sqrt{3}} \text{diag}(1, 1, -2, 0, 0), \\ Q_\psi &= \frac{1}{2\sqrt{6}} \text{diag}(1, 1, 1, -3, 0), \\ Q_\chi &= \frac{1}{2\sqrt{10}} \text{diag}(1, 1, 1, 1, -4). \end{aligned} \tag{3.20}$$

The monodromy $t_1 \leftrightarrow t_2$ imposed in the previous section, eliminates the abelian factor corresponding to Q'_\perp with $t_1 \neq t_2$. Then we are left with the remaining three $SU(5)_\perp$ generators

$$Q_\perp, Q_\psi, Q_\chi, \tag{3.21}$$

given in (3.20). Next, we assume that a low energy $U(1)'$ is generated by a linear combination of the unbroken $U(1)$'s:

$$Q' = c_1 Q_\perp + c_2 Q_\psi + c_3 Q_\chi. \tag{3.22}$$

Regarding the coefficients c_1, c_2, c_3 the following normalisation condition will be assumed

$$c_1^2 + c_2^2 + c_3^2 = 1, \tag{3.23}$$

while, further constraints will be imposed by applying anomaly cancellation conditions.

3.2 The Flux mechanism

We now turn into the symmetry breaking procedure. In F-theory, fluxes are used to generate the observed chirality of the massless spectrum. Most precisely, we may consider two distinct classes of fluxes. Initially, a flux is introduced along a $U(1)_\perp$ and its geometric restriction along a specific matter curve Σ_{n_j} is parametrised with an integer number. Then, the chiralities of the $SU(5)$ representations are given by

$$\#10_i - \#\overline{10}_i = m_i, \tag{3.24}$$

$$\#5_j - \#\overline{5}_j = M_j. \tag{3.25}$$

The integers M_i, m_j are subject to the chirality condition

$$\sum_i m_i = - \sum_j M_j = 3 \tag{3.26}$$

which coincides with the SM anomaly conditions [48,49] Next, a flux in the direction of hypercharge, denoted as \mathcal{F}_Y , is turned on in order to break the $SU(5)_{GUT}$ down to the SM gauge group. This ‘‘hyperflux’’ is also responsible for the splitting of $SU(5)$ representations. If some integers $N_{i,j}$ represent hyperfluxes piercing certain matter curves, then the combined effect of the two type of fluxes into the 10-plets and 5-plets is described according to:

$$10_{t_j} = \begin{cases} n_{(3,2)_{\frac{1}{6}}} - n_{(\overline{3},2)_{-\frac{1}{6}}} = m_j \\ n_{(\overline{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} = m_j - N_j \\ n_{(1,1)_{+1}} - n_{(1,1)_{-1}} = m_j + N_j \end{cases}, \tag{3.27}$$

$$5_{t_i} = \begin{cases} n_{(3,1)_{-\frac{1}{3}}} - n_{(\overline{3},1)_{+\frac{1}{3}}} = M_i \\ n_{(1,2)_{+\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_i + N_i. \end{cases} \tag{3.28}$$

We note in passing that since the Higgs field is accommodated on a matter curve of type (3.28), an elegant solution to the doublet-triplet splitting problem is realised. Indeed, imposing $M_i = 0$ the colour triplet is eliminated, while choosing $N_i \neq 0$ we ensure the existence of massless doublets in the low energy spectrum.

Table 2 Matter curves along with their $U(1)_\perp$ weights (\pm refer to $10/\overline{10}$ and $\overline{5}/5$ respectively), their defining equation and the corresponding homology class

Matter curve	Σ_{10_1}	Σ_{10_2}	Σ_{10_3}	Σ_{10_4}	Σ_{5_1}	Σ_{5_2}	Σ_{5_3}	Σ_{5_4}	Σ_{5_5}	Σ_{5_6}	Σ_{5_7}
Weights	$\pm t_1$	$\pm t_2$	$\pm t_3$	$\pm t_4$	$\pm 2t_1$	$\pm(t_1 + t_3)$	$\pm(t_1 + t_4)$	$\pm(t_1 + t_5)$	$\pm(t_3 + t_4)$	$\pm(t_3 + t_5)$	$\pm(t_4 + t_5)$
Def. equation	a_1	a_4	a_5	a_6	$a_6 a_7 a_8 + \dots$	$a_1 - \dots$	$a_1 - \dots$	$a_1 - \dots$	$a_5 a_7 + \dots$	$a_6 a_7 + \dots$	$a_6 a_8 + \dots$
Homology	$\eta - 2c_1 - \chi$	$\chi_7 - c_1$	$\chi_8 - c_1$	$\chi_9 - c_1$	$\chi - c_1$	$\eta - 2c_1 - \chi$	$\eta - 2c_1 - \chi$	$\eta - 2c_1 - \chi$	$\chi_7 + \chi_8 - c_1$	$\chi_7 + \chi_9 - c_1$	$\chi_8 + \chi_9 - c_1$

The $U(1)_Y$ flux is subject to the conditions

$$\mathcal{F}_Y \cdot \eta = \mathcal{F}_Y \cdot c_1 = 0,$$

in order to avoid a heavy Green–Schwarz mass for the corresponding gauge boson. Furthermore, assuming $\mathcal{F}_Y \cdot \chi_i = N_i$ (with $i = 7, 8, 9$) and correspondingly $\mathcal{F}_Y \cdot \chi = N$, with $N = N_7 + N_8 + N_9$, we can find the effect of hyperflux on each matter curve. While m_i and M_j are subject to the constraint (3.26), hyperflux integers $N_{7,8,9}$ are related to the undetermined homologies $\chi_{7,8,9}$ and as such, they are free parameters of the theory. The flux data and the SM content of each matter curve are presented in Table 3. The particle content of the matter curves arises from the decomposition of $10 + \overline{10}$ and $5 + \overline{5}$ pairs which reside on the appropriate matter curves. The MSSM chiral fields arise from the decomposition of 10 and 5, and are denoted by Q, L, u^c, d^c, e^c . Depending on the choice of the flux parameters, it is also possible that some of their conjugate fields appear in the light spectrum (provided of course that there are only three chiral families in the effective theory). These conjugate fields arise from $\overline{10}$ and $\overline{5}$ and in Table 3 and are denoted by $\overline{Q}, \overline{L}, \overline{u^c}, \overline{d^c}, \overline{e^c}$.

In the same table we have also included the charges of the remaining $U(1)'$ symmetry. We observe that the charges are functions of the $c_{1,2,3}$ coefficients which can be computed by applying anomaly cancellation conditions.

There are also singlet fields defined in (3.6) which play an important rôle in the construction of realistic F-theory models. In the present framework, these singlet states are parameterised by the vanishing combination $\pm(t_i - t_j) = 0, i \neq j$, therefore, due to Z_2 monodromy we end up with twelve singlets, denoted by θ_{ij} . Their $U(1)'$ charges and multiplicities are collectively presented in Table 4. Details on their rôle in the effective theory will be given in the subsequent sectors.

3.3 Anomaly cancellation conditions

In the previous sections we elaborated on the details of the F- $SU(5)$ GUT supplemented by a flavour-dependent $U(1)'$ extension where this abelian factor is embedded in the $SU(5)_\perp \supset E_8$. Since the effective theory has to be renormalisable and ultra-violet complete, the $U(1)'$ extension must be anomaly free. This requirement imposes significant restrictions on the $U(1)'$ charges of the spectrum and consequently, on the coefficients c_i defining the linear combination in (3.22). In this section we will work out the anomaly cancellation conditions to determine the appropriate linear combinations (3.22). This procedure will also specify all the possibly allowed $U(1)'$ charge assignments of the zero-mode spectrum. Consequently, each such set of charges will correspond to a distinct low energy model which can give definite predictions to be confronted with experimental data.

Table 3 Matter curves along with their $U(1)'$ charges, flux data and the corresponding SM content. Note that $N = N_7 + N_8 + N_9$

Matter curve	Q'	N_Y	M	SM content
$\Sigma_{10_1, \pm t_1}$	$\frac{10\sqrt{3}c_1 + 5\sqrt{6}c_2 + 3\sqrt{10}c_3}{60}$	$-N$	m_1	$m_1 Q + (m_1 + N)u^c + (m_1 - N)e^c$
$\Sigma_{10_2, \pm t_3}$	$\frac{-20\sqrt{3}c_1 + 5\sqrt{6}c_2 + 3\sqrt{10}c_3}{60}$	N_7	m_2	$m_2 Q + (m_2 - N_7)u^c + (m_2 + N_7)e^c$
$\Sigma_{10_3, \pm t_4}$	$\frac{\sqrt{10}c_3 - 5\sqrt{6}c_2}{20}$	N_8	m_3	$m_3 Q + (m_3 - N_8)u^c + (m_3 + N_8)e^c$
$\Sigma_{10_4, \pm t_5}$	$-\sqrt{\frac{2}{5}}c_3$	N_9	m_4	$m_4 Q + (m_4 - N_9)u^c + (m_4 + N_9)e^c$
$\Sigma_{5_1, (\pm 2t_1)}$	$-\frac{c_1}{\sqrt{3}} - \frac{c_2}{\sqrt{6}} - \frac{c_3}{\sqrt{10}}$	N	M_1	$M_1 \bar{d}^c + (M_1 + N)\bar{L}$
$\Sigma_{5_2, \pm(t_1+t_3)}$	$\frac{5\sqrt{3}c_1 - 5\sqrt{6}c_2 - 3\sqrt{10}c_3}{30}$	$-N$	M_2	$M_2 \bar{d}^c + (M_2 - N)\bar{L}$
$\Sigma_{5_3, \pm(t_1+t_4)}$	$-\frac{c_1}{2\sqrt{3}} + \frac{c_2}{\sqrt{6}} - \frac{c_3}{\sqrt{10}}$	$-N$	M_3	$M_3 \bar{d}^c + (M_3 - N)\bar{L}$
$\Sigma_{5_4, \pm(t_1+t_5)}$	$\frac{-10\sqrt{3}c_1 - 5\sqrt{6}c_2 + 9\sqrt{10}c_3}{60}$	$-N$	M_4	$M_4 \bar{d}^c + (M_4 - N)\bar{L}$
$\Sigma_{5_5, \pm(t_3+t_4)}$	$\frac{c_1}{\sqrt{3}} + \frac{c_2}{\sqrt{6}} - \frac{c_3}{\sqrt{10}}$	$N_7 + N_8$	M_5	$M_5 \bar{d}^c + (M_5 + N_7 + N_8)\bar{L}$
$\Sigma_{5_6, \pm(t_3+t_5)}$	$\frac{20\sqrt{3}c_1 - 5\sqrt{6}c_2 + 9\sqrt{10}c_3}{60}$	$N_7 + N_9$	M_6	$M_6 \bar{d}^c + (M_6 + N_7 + N_9)\bar{L}$
$\Sigma_{5_7, \pm(t_4+t_5)}$	$\frac{5\sqrt{6}c_2 + 3\sqrt{10}c_3}{20}$	$N_8 + N_9$	M_7	$M_7 \bar{d}^c + (M_7 + N_8 + N_9)\bar{L}$

Table 4 Singlet fields θ_{ij} along with their corresponding $U(1)'$ charges and multiplicities M_{ij} . The “(−)” sign on the weights and charges refers to the singlets in the parentheses

Singlet fields	Weights	Q'_{ij} (Q'_{ji})	Multiplicity
$\theta_{13}, (\theta_{31})$	$\pm(t_1 - t_3)$	$\pm \frac{\sqrt{3}c_1}{2}$	$M_{13}, (M_{31})$
$\theta_{14}, (\theta_{41})$	$\pm(t_1 - t_4)$	$\pm \frac{c_1 + 2\sqrt{2}c_2}{2\sqrt{3}}$	$M_{14}, (M_{41})$
$\theta_{15}, (\theta_{51})$	$\pm(t_1 - t_5)$	$\pm \frac{1}{12} (2\sqrt{3}c_1 + \sqrt{6}c_2 + 3\sqrt{10}c_3)$	$M_{15}, (M_{51})$
$\theta_{34}, (\theta_{43})$	$\pm(t_3 - t_4)$	$\pm \frac{\sqrt{2}c_2 - c_1}{\sqrt{3}}$	$M_{34}, (M_{43})$
$\theta_{35}, (\theta_{53})$	$\pm(t_3 - t_5)$	$\pm \frac{1}{12} (-4\sqrt{3}c_1 + \sqrt{6}c_2 + 3\sqrt{10}c_3)$	$M_{35}, (M_{53})$
$\theta_{45}, (\theta_{54})$	$\pm(t_4 - t_5)$	$\pm \frac{1}{4} (\sqrt{10}c_3 - \sqrt{6}c_2)$	$M_{45}, (M_{54})$

Although the well known MSSM anomaly cancellation conditions coincide with the chirality condition (3.26) imposed by the fluxes, there are additional contributions to gauge anomalies due to the extra $U(1)'$ factor. In order to consistently incorporate the new abelian factor into the effective theory, the following six anomaly conditions should be considered:

$$\mathcal{A}_{331} : SU(3)_C SU(3)_C U(1)' \tag{3.29}$$

$$\mathcal{A}_{221} : SU(2)_L SU(2)_L U(1)' \tag{3.30}$$

$$\mathcal{A}_{YY1} : U(1)_Y U(1)_Y U(1)' \tag{3.31}$$

$$\mathcal{A}_{Y11} : U(1)_Y U(1)' U(1)' \tag{3.32}$$

$$\mathcal{A}_{111} : U(1)' U(1)' U(1)' \tag{3.33}$$

$$\mathcal{A}_G : \text{Gauge Gravity Anomaly} . \tag{3.34}$$

Using the data of Table 3, it is straightforward to compute the anomaly conditions (3.29–3.34). Analytical expressions are given in Appendix A. It turns out (up to overall factors) that $\mathcal{A}_{221} = \mathcal{A}_{331} = \mathcal{A}_{YY1} \equiv \mathcal{A}$, where \mathcal{A} depends on M_i, m_j, N_k and linearly on $c_{1,2,3}$. On the other hand, the mixed \mathcal{A}_{Y11} anomaly is not linear on $c_{1,2,3}$ and depends only on the hyperflux integers N_k .

The cubic (\mathcal{A}_{111}) and gravitational (\mathcal{A}_G) anomalies depend only on the $U(1)'$ charges (and flux integers), hence singlet fields come into play. The last terms of (A.4) and (A.3) display the contribution from the singlets. Since $Q'_{ij} = -Q'_{ji}$ as a first approximation, we can assume that the singlets always come in pairs ($M_{ij} = M_{ji}$), ensuring this way that their contribution to the anomalies always vanishes.

3.4 Solution strategy

The anomaly conditions displayed above are complicated functions of the c_i -coefficients and the flux integers m_i, M_j and N_k . In order to solve for the c_i 's we have to deal with the flux integers first. The precise determination of the spectrum in the present construction, depends on the choice of these flux parameters. While there is a relative freedom on the choice and the distribution of generations on the various matter curves, some phenomenological requirements may guide our choices. For example, the requirement for a tree-level top Yukawa coupling suggests that the top quark must be placed on the 10_1 matter curve (see Table 3) and the MSSM up-Higgs doublet at 5_1 since, due to Z_2 monodromy, the only renor-

Table 5 MSSM flux solutions along with the resulting c_i 's. For this class of models (Class A), singlets come in pairs ($M_{ij} = M_{ji}$)

Model	m_1	m_2	m_3	m_4	M_1	M_2	M_3	M_4	M_5	M_6	M_7	N_7	N_8	N_9	c_1	c_2	c_3
A1	1	2	0	0	0	-1	0	0	-1	-1	0	1	0	0	0	$-\frac{1}{2}\sqrt{\frac{3}{2}}$	$\frac{1}{2}\sqrt{\frac{5}{2}}$
A2	1	0	2	0	0	0	-1	0	-1	0	-1	0	1	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{2\sqrt{6}}$	$-\frac{1}{2}\sqrt{\frac{5}{2}}$
A3	1	0	0	2	0	0	0	-1	0	-1	-1	0	0	1	$\frac{1}{\sqrt{3}}$	$-\sqrt{\frac{2}{3}}$	0
A4	1	0	0	2	0	0	0	0	-1	-1	-1	0	0	1	$\frac{1}{\sqrt{3}}$	$-\sqrt{\frac{2}{3}}$	0
A5	1	0	2	0	0	0	0	0	-1	-1	-1	0	1	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{2\sqrt{6}}$	$-\frac{1}{2}\sqrt{\frac{5}{2}}$
A6	1	2	0	0	0	0	0	0	-1	-1	-1	1	0	0	0	$-\frac{1}{2}\sqrt{\frac{3}{2}}$	$\frac{1}{2}\sqrt{\frac{5}{2}}$

malisable top-like operator is: $10_{t_1} 10_{t_1} 5_{-2t_1} \equiv 10_1 10_1 5_1$. This suggests the following conditions on some of the flux integers:

$$m_1 = 1, \quad m_1 + N \geq 1, \quad M_1 + N \geq 1. \tag{3.35}$$

Furthermore, a solution to the doublet-triplet splitting problem implies that

$$|N_7| + |N_8| + |N_9| \neq 0. \tag{3.36}$$

Additional conditions can be imposed by demanding certain properties of the effective model and a specific zero-mode spectrum. In what follows, we will split our search into two major directions. Namely, minimal models which contain only the MSSM spectrum (no exotics), and models with vector-like pairs.

For each case we put conditions on the fluxes and then we scan for all possible combinations of flux integers satisfying all the constraints. Next, each set of flux solutions is applied to the anomaly conditions (A.1)–(A.4) and we check whether a solution for the c_i 's exists. Each solution for the c_i 's must also fulfill the normalisation condition (3.23).

4 Models with MSSM spectrum

We start with the minimal scenario where the models we are interested in have the MSSM spectrum accompanied only by pairs of conjugate singlet fields. In particular, three chiral families of quarks and leptons of the MSSM spectrum are ensured by the chirality condition (3.26).

On top of the conditions (3.35) and (3.36) we also assume that

$$M_1 = 0, \quad N = 1, \tag{4.1}$$

avoiding this way exotics since H_u will be the only MSSM state in 5_1 matter curve. In addition, absence of exotics necessarily implies that

$$m_i \geq 0, \quad -M_j \geq 0. \tag{4.2}$$

Then we search the flux parameter space for combinations of m_i, M_j and N_k which respect the conditions (3.26), (3.35), (3.36), (4.1) and (4.2). We allow the flux parameters to vary in the range $[-3, 3]$.

Our scan identifies 54 sets of flux integers that are consistent with all the MSSM spectrum criteria and a tree-level top term. From these 54 flux solutions, only 6 of them yield a solution for the c_i coefficients with equal pairs of singlets, $M_{ij} = M_{ji}$. This class of solutions are shown in Table 5 and the spectrum of the corresponding models are presented in Table 6. We refer to this class of models as *Class A*.

Note that the SM states of all the models above carry the same charges under the extra $U(1)'$ and differ only on how the SM states are distributed among the various matter curves. In all cases we expect similar low energy phenomenological implications.

Solutions for the remaining forty-eight set of fluxes arise if we relax the condition $M_{ij} = M_{ji}$ and allow for general multiplicities for the singlets. Scanning the parameter space, three new classes (named as *Class B*, *Class C* and *Class D*), of consistent solutions emerge. Some representative solutions from each class⁶ are shown in Table 7 while the corresponding models are presented in Table 8. A complete list of all the flux solutions, the corresponding charges and singlet spectrum is given in Appendix B.

It is being observed that for all the models presented so far, one of the tenplets $10_2, 10_3, 10_4$ acquires the same $U(1)'$ charge with the 10_1 matter curve accommodating the top-quark. Thus, at least one of the lightest left-handed quarks will have the same Q' charge with the top quark. In this case, the corresponding flavour processes associated with these two families are expected to be suppressed.

Next, we will investigate some phenomenological aspects of the models presented so far. We first write down all the possible $SU(5) \times U(1)'$ invariant tree-level Yukawa terms:

⁶ Each class consists of various flux and c_i solutions that results to the same Q' charges. The various models inside a class are differ on how the SM fields distributed on the matter curves.

Table 6 Models with MSSM spectrum plus pairs of singlet fields ($M_{ij} = M_{ji}$)

Model A1		Model A2		Model A3		Model A4		Model A5		Model A6	
Q'	SM	Q'	SM	Q'	SM	Q'	SM	Q'	SM	Q'	SM
0	$Q + 2u^c$	0	$Q + 2u^c$	0	$Q + 2u^c$	0	$Q + 2u^c$	0	$Q + 2u^c$	0	$Q + 2u^c$
0	$2Q + u^c + 3e^c$	-1/2	-	-1/2	-	-1/2	-	-1/2	-	0	$2Q + u^c + 3e^c$
1/2	-	0	$2Q + u^c + 3e^c$	1/2	-	1/2	-	0	$2Q + u^c + 3e^c$	1/2	-
-1/2	-	1/2	-	0	$2Q + u^c + 3e^c$	0	$2Q + u^c + 3e^c$	1/2	-	-1/2	-
0	H_u	0	H_u	0	H_u	0	H_u	0	H_u	0	H_u
0	$d^c + 2L$	-1/2	L	-1/2	L	-1/2	L	-1/2	L	0	L
1/2	L	0	$d^c + 2L$	1/2	L	1/2	L	0	L	1/2	L
-1/2	L	1/2	L	0	$d^c + 2L$	0	L	1/2	L	-1/2	L
1/2	d^c	-1/2	d^c	0	-	0	$d^c + L$	-1/2	d^c	1/2	d^c
-1/2	d^c	0	-	-1/2	d^c	-1/2	d^c	0	$d^c + L$	-1/2	d^c
0	-	1/2	d^c	1/2	d^c	1/2	d^c	1/2	d^c	0	$d^c + L$

- Renormalisable top-Yukawa type operator:

$$10_1 10_1 \bar{5}_1, \tag{4.3}$$

which is the only tree-level top quark operator allowed by the t_i weights (see Tables 7, 8) thanks to the Z_2 monodromy.

- Renormalisable bottom-type quarks operators:

$$10_1 \bar{5}_2 \bar{5}_7, 10_1 \bar{5}_3 \bar{5}_6, 10_1 \bar{5}_4 \bar{5}_5, \\ 10_2 \bar{5}_3 \bar{5}_4, 10_3 \bar{5}_2 \bar{5}_4, 10_4 \bar{5}_2 \bar{5}_3. \tag{4.4}$$

Depending on how the SM states are distributed among the various matters curves, tree level bottom and/or R-parity violation (RPV) terms may exist in the models.

4.1 Phenomenological analysis

Up till now we have sorted out a small number of phenomenologically viable models distinguished by their low energy predictions. In the remaining of this section, we will focus on Model D9. The implications of the remaining models will be explored in the Appendix.

Details for the fermion sectors of this model are given in Table 8, while the properties of the singlet sector can be found in Appendix B. In order to achieve realistic fermion hierarchies, we assume the following distribution of the MSSM spectrum in to the various matter curves:

$$10_1 \rightarrow Q_3 + u_{2,3}^c, \quad 10_2 \rightarrow Q_1 + u_1^c + e_1^c, \\ 10_4 \rightarrow Q_2 + e_{2,3}^c, \\ 5_1 \rightarrow H_u, \quad \bar{5}_2 \rightarrow H_d, \quad \bar{5}_3 \rightarrow L_3, \quad \bar{5}_4 \rightarrow L_2, \\ \bar{5}_5 \rightarrow d_1^c + L_1, \quad \bar{5}_6 \rightarrow d_2^c, \quad \bar{5}_7 \rightarrow d_3^c,$$

where the indices (1,2,3) on the SM states denote generation.

Top sector

The dominant contributions to the up-type quarks descend from the following superpotential terms

$$W \supset y_t 10_1 10_1 5_1 + \frac{y_1}{\Lambda} 10_1 10_2 5_1 \theta_{13} \\ + \frac{y_2}{\Lambda} 10_1 10_4 5_1 \theta_{15} + \frac{y_3}{\Lambda^2} 10_2 10_4 5_1 \theta_{13} \theta_{15} \\ + \frac{y_4}{\Lambda^2} 10_2 10_2 5_1 \theta_{13}^2 \\ + \frac{y_5}{\Lambda^2} 10_1 10_2 5_1 \theta_{15} \theta_{53} + \frac{y_6}{\Lambda^3} 10_2 10_2 5_1 \theta_{15} \theta_{53} \theta_{13}, \tag{4.5}$$

where y_i 's are coupling constant coefficients and Λ is a characteristic high energy scale of the theory. The operators yield the following mass texture:

Table 7 MSSM flux solutions along with the corresponding c_i 's for a general singlet spectrum

Model	m_1	m_2	m_3	m_4	M_1	M_2	M_3	M_4	M_5	M_6	M_7	N_7	N_8	N_9	c_1	c_2	c_3
B7	1	0	1	1	0	-1	0	0	-1	0	-1	0	1	0	$-\frac{\sqrt{5}}{3}$	$\frac{1}{6}\sqrt{\frac{5}{2}}$	$-\frac{1}{2}\sqrt{\frac{3}{2}}$
C8	1	0	0	2	0	0	-1	0	0	-1	-1	0	0	1	$-\frac{\sqrt{5}}{6}$	$\frac{7}{12}\sqrt{\frac{5}{2}}$	$-\frac{1}{4\sqrt{6}}$
D9	1	1	0	1	0	0	0	0	-1	-1	-1	0	0	1	$\frac{1}{2}\sqrt{\frac{5}{6}}$	$\frac{5}{8}\sqrt{\frac{5}{3}}$	$-\frac{3}{8}$

Table 8 MSSM like models accompanied by a general singlet spectrum

Curve	ModelB7		ModelC8		ModelD9	
	$\sqrt{15}Q'$	SM	$\sqrt{15}Q'$	SM	$\sqrt{10}Q'$	SM
10 ₁	-1	$Q + 2u^c$	1/4	$Q + 2u^c$	3/4	$Q + 2u^c$
10 ₂	3/2	-	3/2	-	-1/2	$Q + u^c + e^c$
10 ₃	-1	$Q + 2e^c$	-9/4	-	-7/4	-
10 ₄	3/2	$Q + u^c + e^c$	1/4	$2Q + u^c + 3e^c$	3/4	$Q + 2e^c$
5 ₁	2	H_u	-1/2	H_u	-3/2	H_u
$\bar{5}_2$	1/2	$d^c + 2L$	7/4	L	1/4	L
$\bar{5}_3$	-2	L	-2	$d^c + 2L$	-1	L
$\bar{5}_4$	1/2	L	1/2	L	3/2	L
$\bar{5}_5$	1/2	d^c	-3/4	-	-9/4	$d^c + L$
$\bar{5}_6$	3	d^c	7/4	d^c	1/4	d^c
$\bar{5}_7$	1/2	-	-2	d^c	-1	d^c

$$M_u = v_u \begin{pmatrix} y_4 \vartheta_{13}^2 + y_6 \vartheta_{15} \vartheta_{53} \vartheta_{13} & y_3 \vartheta_{13} \vartheta_{15} & y_1 \vartheta_{13} + y_5 \vartheta_{15} \vartheta_{53} \\ y_1 \vartheta_{13} + y_5 \vartheta_{15} \vartheta_{53} & y_2 \vartheta_{15} & \varepsilon y_t \\ y_1 \vartheta_{13} + y_5 \vartheta_{15} \vartheta_{53} & y_2 \vartheta_{15} & y_t \end{pmatrix}, \tag{4.6}$$

where $v_u = \langle H_u \rangle$, $\vartheta_{ij} = \langle \theta_{ij} \rangle / \Lambda$ and $\varepsilon \ll 1$ is a suppression factor introduced here to capture local effects of Yukawa couplings descending from a common tree-level operator [50–52]. The matrix has the appropriate structure to explain the hierarchy in the top sector.

Bottom sector

There is one tree-level and several non-renormalisable operators contributing to the down-type quarks. The dominant terms are:

$$W \supset y_b 10_1 \bar{5}_7 \bar{5}_2 + \frac{\kappa_1}{\Lambda} 10_1 \bar{5}_5 \bar{5}_2 \vartheta_{53} + \frac{\kappa_2}{\Lambda} 10_1 \bar{5}_6 \bar{5}_2 \theta_{43} + \frac{\kappa_3}{\Lambda} 10_2 \bar{5}_7 \bar{5}_2 \theta_{13} + \frac{\kappa_4}{\Lambda^2} 10_2 \bar{5}_6 \bar{5}_2 \theta_{13} \theta_{43} + \frac{\kappa_5}{\Lambda^2} 10_2 \bar{5}_5 \bar{5}_2 \theta_{13} \theta_{53} + \frac{\kappa_6}{\Lambda^2} 10_2 \bar{5}_7 \bar{5}_2 \theta_{15} \theta_{53} + \frac{\kappa_7}{\Lambda^3} 10_2 \bar{5}_5 \bar{5}_2 \theta_{15} \theta_{53}^2 + \frac{\kappa_8}{\Lambda^3} 10_2 \bar{5}_6 \bar{5}_2 \theta_{14} \theta_{43}^2 + \frac{\kappa_9}{\Lambda} 10_4 \bar{5}_7 \bar{5}_2 \theta_{15} + \frac{\kappa_{10}}{\Lambda} 10_4 \bar{5}_5 \bar{5}_2 \theta_{13} + \frac{\kappa_{11}}{\Lambda^2} 10_4 \bar{5}_6 \bar{5}_2 \theta_{13} \theta_{45} + \frac{\kappa_{12}}{\Lambda^2} 10_4 \bar{5}_5 \bar{5}_2 \theta_{15} \theta_{53} + \frac{\kappa_{13}}{\Lambda^3} 10_4 \bar{5}_6 \bar{5}_2 \theta_{15} \theta_{45} \theta_{53}, \tag{4.7}$$

with κ_i , y_b being coupling constant coefficients. These operators generate the following down quark mass matrix:

$$M_d = v_d \begin{pmatrix} \kappa_5 \vartheta_{53} \vartheta_{13} + \kappa_7 \vartheta_{15} \vartheta_{53}^2 & \kappa_{10} \vartheta_{13} + \kappa_{12} \vartheta_{15} \vartheta_{53} & \kappa_1 \vartheta_{53} \\ \kappa_4 \vartheta_{13} \vartheta_{43} + \kappa_8 \vartheta_{14} \vartheta_{43}^2 & \kappa_{11} \vartheta_{13} \vartheta_{45} + \kappa_{13} \vartheta_{15} \vartheta_{45} \vartheta_{53} & \kappa_2 \vartheta_{43} \\ \kappa_3 \vartheta_{13} + \kappa_6 \vartheta_{15} \vartheta_{53} & \kappa_9 \vartheta_{15} & y_b \end{pmatrix}, \tag{4.8}$$

where $v_d = \langle H_d \rangle$ is the VEV of the down-type MSSM Higgs. This matrix is subject to corrections from higher order terms and due to the many contributing operators, we expect large mixing effects.

Charged lepton sector

In the present construction, when flux pierces the various matter curves, the SM generations are distributed on different matter curves. As a consequence, in general, down type quarks and charged lepton sectors emerge from different couplings.

In the present model the common operators between bottom and charged lepton sector are those given in (4.8) with couplings κ_5 , κ_7 , κ_{10} and κ_{12} . All the other contributions descend from the operators

$$W \supset y_\tau 10_4 \bar{5}_3 \bar{5}_2 + \frac{\lambda_1}{\Lambda} 10_2 \bar{5}_4 \bar{5}_2 \theta_{43} + \frac{\lambda_2}{\Lambda} 10_2 \bar{5}_3 \bar{5}_2 \theta_{53} + \frac{\lambda_3}{\Lambda} 10_4 \bar{5}_4 \bar{5}_2 \theta_{45}, \tag{4.9}$$

where y_τ is a tree level Yukawa coefficient, λ_i coupling constants and $\eta \ll 1$ encodes local tree-level Yukawa coupling effects. Collectively we have the following mass texture for the charged leptons of the model:

$$M_e = v_d \begin{pmatrix} \kappa_5 \vartheta_{53} \vartheta_{13} + \kappa_7 \vartheta_{15} \vartheta_{53}^2 & \lambda_1 \vartheta_{43} & \lambda_2 \vartheta_{53} \\ \kappa_{10} \vartheta_{13} + \kappa_{12} \vartheta_{15} \vartheta_{53} & \lambda_3 \vartheta_{45} & \eta y_\tau \\ \kappa_{10} \vartheta_{13} + \kappa_{12} \vartheta_{15} \vartheta_{53} & \lambda_3 \vartheta_{45} & y_\tau \end{pmatrix}. \quad (4.10)$$

The μ -term

The bilinear term $5_1 \bar{5}_2$ is not invariant under the extra $U(1)'$ symmetry. However, the μ -term appears dynamically through the renormalisable operator:

$$\kappa 5_1 \bar{5}_3 \theta_{13} \longrightarrow \kappa \langle \theta_{13} \rangle H_u H_d \equiv \mu H_u H_d. \quad (4.11)$$

There are no constraints imposed on the VEV of singlet field θ_{13} , thus, a proper tuning of the values of κ and $\langle \theta_{13} \rangle$ can lead to an acceptable μ -parameter, $\mu \sim \mathcal{O}(TeV)$. As a result, the θ_{13} singlet which also contributes to the quarks and charged lepton sectors, must receive VEV at some energy scale close to the TeV region.

We also note that some of the singlet fields couple to the left-handed neutrinos and, in principle, can play the rôle of their right-handed partners. In particular, as suggested in [6], the six-dimensional massive KK-modes which correspond to the neutral singlets identified by the Z_2 symmetry $\theta_{12} \equiv \theta_{21}$ are the most appropriate fields to be identified as $\theta_{12} \rightarrow \nu^c$ and $\theta_{21} \rightarrow \bar{\nu}^c$ so that a Majorana mass term $M_N \nu^c \bar{\nu}^c$ is possible. We will not elaborate on this issue any further; some related phenomenological analysis can be found in [53].

CKM matrix

The square of the fermion mass matrices obtained so far can be diagonalised via the unitary matrices V_{f_L} . The various coupling constants and VEVs can be fitted to make the diagonal mass matrices satisfy the appropriate mass relations at the GUT scale. In our analysis we use the RGE results for a large $\tan \beta = v_u/v_d$ scenario produced in Ref. [54]. In addition, the combination $V_{u_L} V_{d_L}^\dagger$ must resemble as close as possible the CKM matrix.

For the various parameters of the present model, we use a natural set of numerical values

$$\begin{aligned} \kappa_i &\simeq 1, \quad y_1 = y_4 = y_5 = y_6 = 25y_2 = 25y_3 \\ &\simeq 0.5, \quad \epsilon = 10^{-4}, \quad y_t = 0.5, \quad y_b = 0.36. \end{aligned}$$

Then, the singlet VEV's ϑ_{ij} are fitted to:

$$\begin{aligned} \vartheta_{13} &\simeq 3.16 \times 10^{-12}, \quad \vartheta_{14} \simeq 3.98 \times 10^{-3}, \quad \vartheta_{15} \simeq 10^{-1}, \\ \vartheta_{43} &\simeq 1.9 \times 10^{-2}, \quad \vartheta_{53} \simeq 6.94 \times 10^{-3}, \quad \vartheta_{45} \simeq 10^{-2}. \end{aligned}$$

For the up and down quark diagonalising matrices, they yield

$$V_{u_L} = \begin{pmatrix} -1 & -0.000694 & 0.000694 \\ 0.000694 & -1 & 0.000116 \\ 0.0006939 & 0.000116 & 1 \end{pmatrix},$$

$$V_{d_L} = \begin{pmatrix} -0.9738 & 0.2273 & 0.00674 \\ -0.2266 & -0.9726 & 0.0519 \\ 0.0183 & 0.04908 & 0.9986 \end{pmatrix}. \quad (4.12)$$

The resulting CKM matrix is in agreement with the experimentally measured values

$$|V_{CKM}| \simeq \begin{pmatrix} 0.973659 & 0.227932 & 0.00601329 \\ 0.227325 & 0.972437 & 0.0518632 \\ 0.0176688 & 0.04913 & 0.998636 \end{pmatrix}. \quad (4.13)$$

It is clear that the CKM matrix is mostly influenced by the bottom sector while V_{u_L} is almost diagonal and unimodular.

Next, we compute the unitary matrix V_{e_L} which diagonalises the charged lepton mass matrix. The correct Yukawa relations and the charged lepton mass spectrum are obtained for

$$V_{e_L} = \begin{pmatrix} -0.801463 & 0.597943 & 0.0110641 \\ -0.597877 & -0.801539 & 0.00888511 \\ 0.0141811 & 0.000506117 & 0.999899 \end{pmatrix}, \quad (4.14)$$

where the remaining parameters were fitted to: $\lambda_1 = 0.4$, $\lambda_2 = \lambda_3 = 1$, $\eta = 10^{-4}$ and $y_\tau \simeq 0.51$.

R-parity violating terms

In the model under discussion, several tree-level as well as bilinear operators leading to RPV effects remain invariant under all the symmetries of the theory. More precisely, the tree-level operators :

$$10_1 \bar{5}_3 \bar{5}_6 \longrightarrow \lambda' Q_3 L_3 d_2^c, \quad (4.15)$$

$$10_2 \bar{5}_3 \bar{5}_4 \longrightarrow \lambda L_3 L_2 e_1^c, \quad (4.16)$$

violate both lepton and baryon number. Notice however, the absence of $u^c u^c d^c$ type of RPV terms which in combination with QLd^c terms can spoil the stability of the proton. There also exist bilinear RPV terms descending from tree-level operators. In the present model, these are:

$$5_1 \bar{5}_3 \theta_{14}, \quad 5_1 \bar{5}_4 \theta_{15}. \quad (4.17)$$

The effect of these terms strongly depend on the dynamics of the singlets, however it would be desirable to completely eliminate such operators.

One can impose an R-symmetry by hand [47] or to investigate the geometric origin of discrete Z_N symmetries that can eliminate such operators [55–58]. In addition, the study of such Yukawa coefficients at a local-level, shows that they can be suppressed for wide regions of the flux parameter space [59]. Since in this work we focus mostly in Z' flavour changing effects,⁷ we will assume that one of the aforementioned mechanisms protects the models from unwanted RPV terms.

⁷ Notice however that some RPV terms of the type Qld^c and lle^c contribute to flavour violation processes, see [60,61]. For an explanation of the LHCb anomalies through RPV interactions see [62–64].

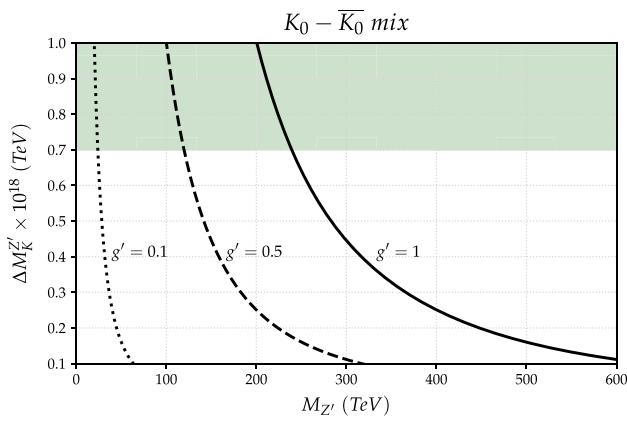


Fig. 4 Bounds to the neutral gauge boson mass $M_{Z'}$ of Model D9 due to $K_0 - \bar{K}_0$ mixing effects. The vertical axis displays Z' contributions ($\Delta M_K^{Z'}$) to the mass split of the neutral Kaon system. Dotted, dashed and solid black curves correspond to gauge coupling values: $g' = 0.1, 0.5$ and 1 respectively. The shaded region is excluded due the constrain $\Delta M_K^{NP} < 0.2 \Delta M_K^{exp}$

4.2 Z' bounds for model D9

Having obtained the V_f matrices for the top/bottom quark and charged lepton sectors, it is now straightforward to compute the flavour mixing matrices Q'_{fL} defined in (2.6). These matrices, along with the Z' mass ($M_{Z'}$) and gauge coupling (g'), enter the computation of the various flavour violating observables described in Sect. 2. Hence, we can use the constraints on these observables in order to derive bounds for the Z' mass and gauge coupling or, more precisely, for the ratio $g'/M_{Z'}$. In any case, the so derived bounds must be in accordance with LHC bounds coming from dilepton and diquark channels [65–67]. For heavy Z' searches, the LHC bounds on neutral gauge boson masses are strongly model dependent. For most of the GUT inspired Z' models, masses around $\sim 2-3$ TeV are excluded.

In the model at hand, we have seen that the lightest generations of the left-handed quarks have different $U(1)'$ charges. Consequently, strong constraints on the Z' mass are expected to come from the $K - \bar{K}$ mixing bounds. Hence, we first start from the neutral Kaon system.

$K^0 - \bar{K}^0$ mixing

Using Eq. (2.10) we find for the Kaon oscillation mass split that:

$$\Delta M_K^{Z'} \simeq 3.967 \times 10^{-14} \left(\frac{g'}{M_{Z'}} \right)^2.$$

The results are plotted in Fig. 4. As expected, the Kaon system puts strong bounds on $M_{Z'}$. To get an estimate, for $g' \simeq 0.5$ the constraint in (2.15) implies that $M_{Z'} \gtrsim 120$ TeV which lies far above the most recent collider searches.

$B_s^0 - \bar{B}_s^0$ mixing

From Eq. (2.13) we have that:

$$C_{bs}^{LL} \approx 1.9 \times 10^{-5} \left(\frac{g' \text{ TeV}}{M_{Z'}} \right)^2$$

which is too small in magnitude to significantly contribute to ΔM_s . This happens because the $U(1)'$ charges of b_L and s_L are equal.

$D^0 - \bar{D}^0$ mixing

For $M_D \simeq 1.86483$ GeV [37] and using for the decay constant the value $f_D \simeq 212$ MeV found in [68], the equation (2.10) gives:

$$\Delta M_D^{Z'} \simeq 2.71 \times 10^{-18} \left(\frac{g' \text{ TeV}}{M_{Z'}} \right)^2.$$

Then, for $\Gamma_D = 1/\tau_D \simeq 2.43843$ (ps) $^{-1}$ [37] we have that

$$x_D := \frac{\Delta M_D}{\Gamma_D} \simeq 0.0017 \left(\frac{g' \text{ TeV}}{M_{Z'}} \right)^2$$

which always obeys the bound $x_D \leq 0.32$.

$P^0 \rightarrow l_i \bar{l}_i$ decays

We have found that all the Z' contributions are well suppressed when compared to the experimental bounds. As an example, consider the decay $B_d^0 \rightarrow \mu^+ \mu^-$. Using Eq. (2.17) we obtain that

$$\text{Br}(B_d^0 \rightarrow \mu^+ \mu^-) \simeq 5.34 \times 10^{-9} \left(\frac{g' \text{ TeV}}{M_{Z'}} \right)^4$$

which always satisfies the experimental bound $\text{Br}(B_d^0 \rightarrow \mu^+ \mu^-) < 1.6_{-1.4}^{+1.6} \times 10^{-10}$, for $g' < 1$ and $M_{Z'} \sim \mathcal{O}(\text{TeV})$. Similar results were obtained for lepton flavour violating decays of the form $P^0 \rightarrow l_i \bar{l}_j$.

Muon anomalous magnetic moment and $\mu \rightarrow e \gamma$

Our results imply that Z' contributions to Δa_μ are always smaller than the observed discrepancy. Even for the limiting case where $g' = 1$ and $M_{Z'} = 1$ TeV our computations return: $\Delta a_\mu^{Z'} \simeq 3 \times 10^{-11}$. This suggests that for small Z' masses the model can explain the observed $(g-2)_\mu$ anomaly. However for larger $M_{Z'}$ values implied from the Kaon system the results are very suppressed.

For LFV radiative decays of the form $l_i \rightarrow l_j \gamma$, the strongest bounds are expected from the muon channel. For $g' = 1$, the present model predicts that $M_{Z'} \gtrsim 1.3$ TeV if

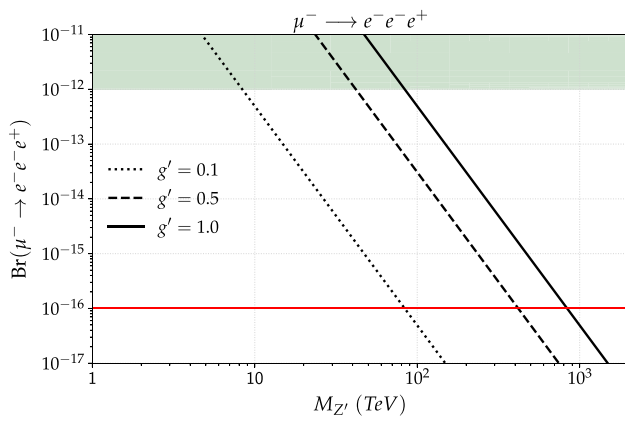


Fig. 5 Bounds to the neutral gauge boson mass $M_{Z'}$ as predicted in Model D9 from Z' contributions to the lepton flavour violation decay $\mu^- \rightarrow e^-e^-e^+$. The plot shows the branching ratio of the decay as function of the Z' mass for various values of the gauge coupling g' . Both axis are in logarithmic scale. Dotted, dashed and solid black curves correspond to $U(1)'$ gauge couplings: $g' = 0.1, 0.5$ and 1 respectively. The shaded region is excluded due to the current experimental bound: $\text{Br}(\mu^- \rightarrow e^-e^-e^+) < 10^{-12}$. The red horizontal line represents the estimated reach of future $\mu \rightarrow 3e$ experiments

the predicted $\mu \rightarrow e\gamma$ branching ratio is to satisfy the experimental bounds. Tau decays ($\tau \rightarrow e\gamma, \tau \rightarrow \mu\gamma$) are well suppressed, due to the short lifetime of the tau lepton.

$$\mu^- \rightarrow e^-e^-e^+$$

While all the three body lepton decays of the form $l_i \rightarrow l_j l_k \bar{l}_k$ are suppressed for the tau channel, strong constraints are obtained from the muon decay $\mu^- \rightarrow e^-e^-e^+$. In particular, the model predicts that

$$\text{Br}(\mu^- \rightarrow e^-e^-e^+) \simeq 4.92 \times 10^{-5} \left(\frac{g' \text{ TeV}}{M_{Z'}} \right)^4.$$

The results are compared with the experimental bounds in Fig. 5. We observe that, for $g' = 0.5$ (dashed line in the plot) we receive $M_{Z'} \gtrsim 42 \text{ TeV}$ in order the model to satisfy the current experimental bound, $\text{Br}(\mu^- \rightarrow e^-e^-e^+) < 10^{-12}$. While the constraints coming from this decay are stronger than the other lepton flavour violating processes discussed so far, they still are not compatible with the restrictions descending from the Kaon system.

However, important progress is expected by future lepton flavour violation related experiments [69]. In particular, the *Mu3e* experiment at PSI [70] aim to improve the experimental sensitivity to $\sim 10^{-16}$. In the absence of a signal, three-body LFV muon decays can then be excluded for $\text{Br}(\mu^- \rightarrow e^-e^-e^+) < 10^{-16}$. In Fig. 5 the red horizontal line represents the estimated reach of future $\mu \rightarrow 3e$ experiments. For, $g' = 0.5$ we find that $M_{Z'} \gtrsim 420 \text{ TeV}$ in order the predicted branching ratio is to satisfy the foreseen *Mu3e* experimental bounds. Hence, for the present model,

the currently dominant bounds from the Kaon system will be exceeded in the near future by the limits of the upcoming $\mu^- \rightarrow e^-e^-e^+$ experiments.

R_K anomalies

The bounds derived from the Kaon oscillation system and the three-body decay $\mu \rightarrow e^-e^-e^+$ leaves no room for a possible explanation of the observed R_K anomalies. Indeed, for the relevant Wilson coefficient the model predicts that

$$C_9 \approx -0.079 \left(\frac{g' \text{ TeV}}{M_{Z'}} \right)^2$$

which has the desired sign ($C_9 < 0$), but for $M_{Z'} \sim 200 \text{ TeV}$ and $g' \simeq 1$ the resulting value is too small to explain the observed B meson anomalies.

Similar phenomenological analysis have been performed for all the other models presented so far. A discussion on their flavour violation bounds is given in Appendix C. Collectively, the results are very similar with those of Model D9. For all the $U(1)'$ models with MSSM spectrum the dominant bounds on $M_{Z'}$ comes from $K^0 - \bar{K}^0$ oscillation effects and the muon decay $\mu \rightarrow e^-e^-e^+$.

It is clear from the analysis so far that a successful explanation of the LHCb anomalies in the present F-theory framework, requires the use of some other type of mechanism. A common approach, is the explanation of the LHCb anomalies through the mixing of the conventional SM matter with extra vector-like fermions [71–79]. Next, we present such an F-theory model while a full classification of the various F-theory models with a complete family of vector-like fermions will be presented in a future work.

5 Models with vector-like exotics

We expand our analysis to models with the MSSM spectrum + vector-like (VL) states forming complete $(10 + \bar{10}), (5 + \bar{5})$ pairs under the $SU(5)$ GUT symmetry. Hence, as in the previous study, we choose appropriate fluxes, solve the anomaly cancellation conditions, and derive the $U(1)'$ charges of all the models with additional vector-like families.

Among the various models, particular attention is paid to models with different $U(1)'$ charges for the VL states, while keeping universal the $U(1)'$ charges for the SM fermion families. This way one can explain the observed B-meson anomalies due to the mixing of the SM fermions with the VL exotics while at the same time controlling other flavour violation observables. A model with these properties (first derived in [17]) is materialised with the following set of fluxes:

$$m_1 = 2, \quad m_2 = m_3 = -m_4 = 1, \\ M_1 = M_2 = M_3 = M_7 = 0, \quad M_4 = -M_6 = 1,$$

$$M_5 = -3,$$

which through anomaly cancellation gives the solution $(c_1, c_2, c_3) = (\frac{\sqrt{3}}{2}, -\frac{1}{4}\sqrt{\frac{3}{2}}, \frac{1}{4}\sqrt{\frac{5}{2}})$. This corresponds to the following $U(1)'$ charges for the various matter curves

$$\begin{aligned} 10_1 &: \frac{1}{4}, & 10_2 &: -\frac{1}{2}, & 10_3 &: \frac{1}{4}, & 10_4 &: -\frac{1}{4}, \\ 5_1 &: -\frac{1}{2}, & 5_2 &: \frac{1}{4}, & 5_3 &: -\frac{1}{2}, \\ 5_4 &: 0, & 5_5 &: \frac{1}{4}, & 5_6 &: \frac{3}{4}, & 5_7 &: 0. \end{aligned}$$

Assuming the following distribution of the fermion generations and Higgs fields into matter curves

$$\begin{aligned} 10_1 &\longrightarrow Q_{2,3} + u_{1,2,3}^c + e_3^c, & 10_2 &\longrightarrow Q_4 + u_4^c + e_4^c, \\ 10_3 &\longrightarrow Q_1 + e_{1,2}^c, & 10_4 &\longrightarrow \bar{Q}_4 + \bar{u}_4^c + \bar{e}_1^c, \\ 5_1 &\longrightarrow H_u, & \bar{5}_2 &\longrightarrow L_1, & \bar{5}_3 &\longrightarrow H_d, \\ 5_4 &\longrightarrow \bar{d}_4^c, & \bar{5}_5 &\longrightarrow d_{1,2,3}^c + L_{2,3}, & \bar{5}_6 &\longrightarrow d_4^c + L_4, & 5_7 &\longrightarrow \bar{L}_4, \end{aligned}$$

we obtain the desired $U(1)'$ charge assignment where all the SM families appear with a common charge ($Q'_{1,2,3} = 1/4$) while those of the VL states are non-universal.

Here $Q_i, u_i^c, e_i^c, L_i, \bar{d}_i^c$ with $i = 1, 2, 3$ refer to the three SM fermion generations while $u_4^c, \bar{u}_4^c, Q_4, \bar{Q}_4, e_4, \bar{e}_4^c, L_4, \bar{L}_4, d_4, \bar{d}_4^c$ represent the extra VL states. In a simplified notation, the components of the SM doublets are defined as $Q_i = (u_i, d_i)$ and similarly for the lepton doublets L_i . The components of the exotic doublets are $Q_4 \equiv (U', D')$ and $\bar{Q}_4 \equiv (\bar{D}', \bar{U}')$ and similar for the lepton exotic doublet. For the exotic singlets we use the notation $u_4^c = \bar{U}, \bar{u}_4^c = U$ and similar $e_4^c = \bar{E}, \bar{e}_4^c = E, d_4^c = \bar{D}, \bar{d}_4^c = D$.

The various mass terms can be written in a 5×5 notation as $F_R \mathbf{M}_F F_L$ where $F_R = (f_i^c, \bar{F}, \bar{F}')$ and $F_L = (f_i, F', F)^T$ with $f = u, d, e$ and $F = U, D, E$. We will focus on the down-type quark sector. The up quark sector can be treated similarly, while the parameters can be adjusted in such a way so that the CKM mixing is ensured. The various invariant operators yield a mass matrix of the form

$$M_d = \begin{pmatrix} k_0 \vartheta_{14} \vartheta_{54} v_d & k \varepsilon^3 \vartheta_{54} v_d & k \varepsilon^2 \vartheta_{54} v_d & k_4 \vartheta_{14} \vartheta_{53} v_d & k_3 \vartheta_{14} \vartheta_{53} \\ k_0 \vartheta_{14} \vartheta_{54} v_d & k \varepsilon^2 \vartheta_{54} v_d & k \varepsilon \vartheta_{54} v_d & k_4 \vartheta_{14} \vartheta_{53} v_d & k_3 \vartheta_{14} \vartheta_{53} \\ k_0 \vartheta_{14} \vartheta_{54} v_d & k \varepsilon \vartheta_{54} v_d & k \vartheta_{54} v_d & k_4 \vartheta_{14} \vartheta_{53} v_d & k_3 \vartheta_{14} \vartheta_{53} \\ k_2 \vartheta_{14} v_d & k_1 \xi v_d & k_1 v_d & k_9 \vartheta_{13} v_d & k_{10} \theta_{13} \\ k_6 \vartheta_{54} v_d & k_5 \xi \theta_{51} & k_5 \theta_{51} & k_8 \theta_{53} & k_7 \vartheta_{14} \vartheta_{53} v_u \end{pmatrix}, \tag{5.1}$$

where k 's are coupling constant coefficients and ε, ξ are small constant parameters encode local Yukawa effects. Here we represent the singlet VEVs simply as $\theta_{ij} = \langle \theta_{ij} \rangle$ while ϑ_{ij} represents the ratio $\langle \theta_{ij} \rangle / \Lambda$.

In order to simplify the matrix we consider that some terms are very small and that approximately vanish. In particular,

we assume that $k_2 = k_3 = k_5 \theta_{51} = k_6 = k_7 \vartheta_{14} \vartheta_{53} \approx 0$. Moreover, we introduce the following simplifications

$$\begin{aligned} k \vartheta_{54} v_d &= m, & k_0 \vartheta_{54} \vartheta_{14} v_d &= \alpha m, & k_4 \vartheta_{14} \vartheta_{53} &= \gamma \xi, \\ k_9 \vartheta_{13} v_d &= \beta \mu, & k_{10} \theta_{13} &\simeq k_8 \theta_{53} = M, & \varepsilon &\approx \xi, \end{aligned}$$

where the mass parameter M characterises the VL scale while $m = k \vartheta_{54} v_d$ is related to the low energy bottom quark mass scale. We have also assumed that the small Yukawa parameters are identical $\varepsilon \approx \xi$. With these modifications the matrix takes the following simplified form

$$M_d \approx \begin{pmatrix} \alpha m & m \xi^3 & m \xi^2 & \gamma \xi v_d & 0 \\ \alpha m & m \xi^2 & m \xi & \gamma \xi v_d & 0 \\ \alpha m & m \xi & m & \gamma \xi v_d & 0 \\ 0 & k_1 \xi v_d & k_1 v_d & \beta \mu & M \\ 0 & 0 & 0 & M & 0 \end{pmatrix}. \tag{5.2}$$

The local Yukawa parameter ξ connects the VL sector with the physics at the electroweak scale so we will use this small parameter to express the mixing between the two sectors. We proceed by perturbatively diagonalizing the down square mass matrix (M_d^2) using ξ as the expansion parameter.

Setting $k_1 \approx 0, \gamma v_d = c \mu$ and keeping up to $\mathcal{O}(\xi)$ terms we write the mass square matrix in the form $M_d^2 \approx \mathbf{A} + \xi \mathbf{B}$ where:

$$\begin{aligned} \mathbf{A} &= \begin{pmatrix} \alpha^2 m^2 & \alpha^2 m^2 & \alpha^2 m^2 & 0 & 0 \\ \alpha^2 m^2 & \alpha^2 m^2 & \alpha^2 m^2 & 0 & 0 \\ \alpha^2 m^2 & \alpha^2 m^2 & (\alpha^2 + 1)m^2 & 0 & 0 \\ 0 & 0 & 0 & M^2 & \beta \mu M \\ 0 & 0 & 0 & \beta \mu M & M^2 \end{pmatrix}, \\ \mathbf{B} &= \begin{pmatrix} 0 & 0 & 0 & c \beta \mu^2 & c \mu M \\ 0 & 0 & m^2 & c \beta \mu^2 & c \mu M \\ 0 & m^2 & 0 & c \beta \mu^2 & c \mu M \\ c \beta \mu^2 & c \beta \mu^2 & c \beta \mu^2 & 0 & 0 \\ c \mu M & c \mu M & c \mu M & 0 & 0 \end{pmatrix} \end{aligned} \tag{5.3}$$

The block-diagonal matrix \mathbf{A} , is the leading order part of the mass square matrix and can be diagonalised by a unitary matrix V_{bL}^0 as $V_{bL}^0 \mathbf{A} V_{bL}^{0T}$. Its mass square eigenvalues are

$$\begin{aligned} x_1 &= 0, & x_2 &= \frac{m^2}{2} \left(1 + 3\alpha^2 - \sqrt{1 - 2\alpha^2 + 9\alpha^4} \right), \\ x_3 &= \frac{m^2}{2} \left(1 + 3\alpha^2 + \sqrt{1 - 2\alpha^2 + 9\alpha^4} \right) \\ x_4 &= M(M - \beta \mu), & x_5 &= M(M + \beta \mu), \end{aligned} \tag{5.4}$$

where $x_{1,2,3}$ correspond to the mass squares of the three down type quark generations $d_{1,2,3}$ respectively. At this stage we ignore the small mass of the first generation down quark which can be generated by high order corrections. For the second and third generation we observe that the ratio $\sqrt{x_2/x_3}$ depends only on the parameter α . Hence, from the known

strange-bottom quark mass ratio (m_s/m_b) we estimate that $\alpha \simeq 10^{-2}$.

The corresponding normalised eigenvectors which form the columns of the diagonalising matrix are

$$\begin{aligned}
 v_{b1}^0 &= \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, & v_{b2}^0 &= \frac{1}{\sqrt{1+2q^2}} \begin{pmatrix} q \\ q \\ 1 \\ 0 \\ 0 \end{pmatrix}, \\
 v_{b3}^0 &= \frac{-1}{\sqrt{2(1+2q^2)}} \begin{pmatrix} 1 \\ 1 \\ -2q \\ 0 \\ 0 \end{pmatrix}, & v_{b4}^0 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \quad (5.5) \\
 v_{b5}^0 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix},
 \end{aligned}$$

where $q = 1 - \frac{m^2}{x_2}$ depends only on the parameter α , since $x_2 \sim m^2$.

The corrections to the above eigenvectors due to the perturbative part $\xi \mathbf{B}$ are given by the relation

$$v_{bi} \approx v_{bi}^0 + \xi \sum_{j \neq i} \frac{(V_{bL}^0 \mathbf{B} V_{bL}^{0\dagger})_{ji}}{x_i - x_j} v_{bj}^0 \quad (5.6)$$

where the second term displays the $\mathcal{O}(\xi)$ corrections to the basic eigenvectors of the leading order matrix \mathbf{A} . The corrected diagonalizing matrices schematically receive the form $V_{bL} = V_{bL}^0 + \xi V_{bL}^1$ and through them the mixing parameter ξ enters on the computation of the various flavour violation observables.

For the explanation of the LHCb anomalies we will consider that perturbative corrections are important for the corresponding bs coupling while almost vanish for the other flavour mixing coefficients. That way, due to the universal $U(1)'$ charges of the SM matter most of the flavour violation process are suppressed.

Assuming that the corresponding lepton contribution is $(Q'_{e_L})_{22} \approx 1$ and for $\alpha = 0.016$ we find for the $b \rightarrow s$ transition matrix element that :

$$(Q'_{d_L})_{23} \approx Q'_{1,2,3} \xi^2 - 0.7(c\beta)^2 \left(\frac{m}{M}\right)^2 \left(\frac{\mu}{M}\right)^4 Q'_4 \xi^2 \quad (5.7)$$

where $Q'_{1,2,3} = 1/4$ is the common charge of the MSSM fermions and $Q'_4 = -1/2$ is the charge of the extra matter descending from 10_2 matter curve. Note that the corresponding $U(1)'$ charge of the states descending from 5_4 matter curve is zero and consequently does not contribute to the above formula.

It is clear from Eq. (5.7) that the first term is dominant since the second one is suppressed due to the large VL mass scale characterized by the parameter M . Hence, keeping only the first term we have through Eq. (2.9) that

$$C_9 \approx -963 \left(\frac{g'}{M_{Z'}}\right)^2 Q'_{1,2,3} \xi^2 \quad (5.8)$$

and for $g' \lesssim 1$, $M_{Z'} \gtrsim 4$ TeV and $\xi^2 \sim \mathcal{O}(10^{-1})$ predicts $C_9 \approx -1$ which is the desired value for the explanation of the LHCb anomalies. It is emphasised here that this approach is valid in the small $\xi < 1$ regime. If ξ is large perturbation breaks down and a more general treatment is required.

6 Conclusions

In the present work we have examined the low energy implications of F-theory $SU(5) \times U(1)'$ GUT models embedded in $SU(5) \times SU(5)' \supset SU(5) \times U(1)^4$. This gauge symmetry emerges naturally from a single point of E_8 enhancement, associated with the maximal geometric singularity appearing in the elliptic fibration of the internal manifold. In order to ensure realistic fermion mass textures and a tree-level top quark Yukawa coupling, we have imposed a Z_2 monodromy group which acts on the geometric configuration of 7-branes and identifies two out of the four abelian factors descending from the $SU(5)'$ reduction. The $U(1)'$ symmetry of the so derived effective field theory models, is a linear combination of the three remaining abelian symmetries descending from $SU(5)'$. Imposing anomaly cancellation conditions we have constructed all possible $U(1)'$ combinations and found as a generic property the appearance of non-universal Z' -couplings to the three families of quarks and leptons. Introducing fluxes consistent with the anomaly cancellation conditions, and letting the various neutral singlet-fields acquire non-zero vevs, we obtained various effective models distinguished from each other by their different low energy spectra. We have focused on viable classes of models derived in this framework. We have investigated the predictions on flavour changing currents and other processes mediated by the Z' neutral gauge boson associated with the $U(1)'$ symmetry, which is supposed to break at some low energy scale. Using the bounds on such processes coming from current investigation at LHC and other related experiments we converted them to lower bounds on various parameters of the effective theory and in particular the Z' mass. The present work provides a comprehensive classification of semi-local effective F-theory constructions reproducing the MSSM spectrum either with or without vector-like fields. On the phenomenological side, the focus is mainly in explorations of models with the MSSM fields accompanied by several neutral singlets. Fifty four (54) MSSM models have been obtained and

are classified with respect to their predictions on the $U(1)'$ charges of the MSSM matter content. In most of these cases, $U(1)'$ couples non-universally to the first two fermion families, and consequently the $K_0 - \bar{K}_0$ oscillation system forces the strongest bound on the Z' mass. As such, assuming reasonable values of the $U(1)'$ gauge coupling g' we obtain $M_{Z'}$ bounds at few hundreds TeV, well above the most recent LHC searches. In other occasions various flavour violation processes are predicted that can be tested on the ongoing or future experiments. The dominant process mediated by Z' is the lepton flavour violating $\mu \rightarrow eee$ decay, whilst its associated $\mu \rightarrow e\gamma$ rare reaction remains highly suppressed. Future experiments designed to probe the lepton flavour violating process $\mu \rightarrow eee$ are expected to increase their sensitivity at about four orders of magnitude compared to the recent bounds. In this case the models analysed in the present work are a spearhead for the interpretations of a positive experimental outcome. Even in the absence of any signal, the foreseen bounds from $\mu \rightarrow eee$ searches will be compatible with, if not dominant compared to the current bounds obtained in our models from neutral Kaon oscillation effects. On the other hand, we have seen that, models with Z' coupled non-universally but only with MSSM spectrum, are not capable to interpret the recently observed LHCb B-meson anomalies. All the same, our classification includes a class of models with vector-like families with non-trivial Z' -couplings which are capable to account for such effects. These models display a universal nature of the Z' couplings to the first two families with negligible contributions to $K_0 - \bar{K}_0$ oscillations. Their main feature is that the $U(1)'$ charges of the vector-like fields differ from those of the first two generations inducing this way non-trivial mixing effects. As an example, we briefly described such a model which includes a complete family of vector-like of fields where the observed LHCb B-meson anomalies can be explained through the mixing of the extra fermions with the three generations of the SM. A detailed investigation of the whole class of these models will be presented in a future publication.

Acknowledgements This research is co-financed by Greece and the European Union (European Social Fund-ESF) through the Operational Programme “Human Resources Development, Education and Lifelong Learning 2014-2020” in the context of the project “Grand Unified Theories from Superstring Theory: Theoretical Predictions and Modern Particle Physics Experiments” (MIS 5047638).

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: There are no external data associated with the manuscript.]

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Appendices

A Anomaly conditions: analytic expressions

Up to overall factors, our computations give: $\mathcal{A}_{221} = \mathcal{A}_{331} = \mathcal{A}_{Y1} \equiv \mathcal{A}$, with

$$\begin{aligned} \mathcal{A} = & (30\sqrt{3}c_1 + 15\sqrt{6}c_2 + 9\sqrt{10}c_3) m_1 \\ & + (-60\sqrt{3}c_1 + 15\sqrt{6}c_2 + 9\sqrt{10}c_3) m_2 \\ & + (9\sqrt{10}c_3 - 45\sqrt{6}c_2) m_3 - 36\sqrt{10}c_3 m_4 \\ & + (-20\sqrt{3}c_1 - 10\sqrt{6}c_2 - 6\sqrt{10}c_3) M_1 \\ & + (10\sqrt{3}c_1 - 10\sqrt{6}c_2 - 6\sqrt{10}c_3) M_2 \\ & + (-10\sqrt{3}c_1 + 10\sqrt{6}c_2 - 6\sqrt{10}c_3) M_3 \\ & + (-10\sqrt{3}c_1 - 5\sqrt{6}c_2 + 9\sqrt{10}c_3) M_4 \\ & + (20\sqrt{3}c_1 + 10\sqrt{6}c_2 - 6\sqrt{10}c_3) M_5 \\ & + (20\sqrt{3}c_1 - 5\sqrt{6}c_2 + 9\sqrt{10}c_3) M_6 \\ & + (15\sqrt{6}c_2 + 9\sqrt{10}c_3) M_7 \\ & + 30\sqrt{3}c_1 N_7 + (10\sqrt{3}c_1 + 20\sqrt{6}c_2) N_8 \\ & + (10\sqrt{3}c_1 + 5\sqrt{6}c_2 + 15\sqrt{10}c_3) N_9. \end{aligned} \quad (\text{A.1})$$

For the mixed \mathcal{A}_{Y11} anomaly we have:

$$\begin{aligned} \mathcal{A}_{Y11} = & \frac{3}{2}\sqrt{\frac{3}{5}}c_1^2 N_7 + \frac{1}{30} \\ & \times (\sqrt{15}c_1^2 + 4\sqrt{30}c_2c_1 + 8\sqrt{15}c_2^2) N_8 \\ & + \frac{1}{60} (2\sqrt{15}c_1^2 + 2\sqrt{30}c_2c_1 + 30\sqrt{2}c_3c_1 \\ & + \sqrt{15}c_2^2 + 15\sqrt{15}c_3^2 + 30c_2c_3) N_9 \end{aligned} \quad (\text{A.2})$$

The $U(1)'$ -gravity anomaly yields the following expression:

$$\begin{aligned} \mathcal{A}_G = & (20\sqrt{3}c_1 + 10\sqrt{6}c_2 + 6\sqrt{10}c_3) m_1 \\ & + (-40\sqrt{3}c_1 + 10\sqrt{6}c_2 + 6\sqrt{10}c_3) m_2 \end{aligned}$$

$$\begin{aligned}
 &+ (6\sqrt{10}c_3 - 30\sqrt{6}c_2) m_3 \\
 &-24\sqrt{10}c_3m_4 + (-20\sqrt{3}c_1 - 10\sqrt{6}c_2 - 6\sqrt{10}c_3) M_1 \\
 &+ (10\sqrt{3}c_1 - 10\sqrt{6}c_2 - 6\sqrt{10}c_3) M_2 \\
 &+ (-10\sqrt{3}c_1 + 10\sqrt{6}c_2 - 6\sqrt{10}c_3) M_3 \\
 &+ (-10\sqrt{3}c_1 - 5\sqrt{6}c_2 + 9\sqrt{10}c_3) M_4 \\
 &+ (20\sqrt{3}c_1 + 10\sqrt{6}c_2 - 6\sqrt{10}c_3) M_5 \\
 &+ (20\sqrt{3}c_1 - 5\sqrt{6}c_2 + 9\sqrt{10}c_3) M_6 \\
 &+ (15\sqrt{6}c_2 + 9\sqrt{10}c_3) M_7 \\
 &+24\sqrt{3}c_1N_7 + (8\sqrt{3}c_1 + 16\sqrt{6}c_2) N_8 \\
 &+ (8\sqrt{3}c_1 + 4\sqrt{6}c_2 + 12\sqrt{10}c_3) N_9 + \sum_{i \neq j} M_{ij} Q'_{ij}. \tag{A.3}
 \end{aligned}$$

and the pure cubic $U(1)'$ anomaly is:

$$\begin{aligned}
 \mathcal{A}_{111} = & (20\sqrt{3}c_1^3 + 6(5\sqrt{6}c_2 + 3\sqrt{10}c_3) c_1^2 \\
 & + 6(5\sqrt{3}c_2^2 + 6\sqrt{5}c_3c_2 + 3\sqrt{3}c_3^2) c_1 \\
 & + 5\sqrt{6}c_2^3 + 9\sqrt{\frac{2}{5}}c_3^3 + 9\sqrt{6}c_2c_3^2 + 9\sqrt{10}c_2^2c_3) m_1 \\
 & + (-160\sqrt{3}c_1^3 + 24(5\sqrt{6}c_2 + 3\sqrt{10}c_3) c_1^2 \\
 & - 12(5\sqrt{3}c_2^2 + 6\sqrt{5}c_3c_2 + 3\sqrt{3}c_3^2) c_1 \\
 & + 5\sqrt{6}c_2^3 + 9\sqrt{\frac{2}{5}}c_3^3 + 9\sqrt{6}c_2c_3^2 + 9\sqrt{10}c_2^2c_3) m_2 \\
 & - 9 \left(15\sqrt{6}c_2^3 - 9\sqrt{10}c_3c_2^2 + 3\sqrt{6}c_3^2c_2 - \sqrt{\frac{2}{5}}c_3^3 \right) \\
 & \times m_3 - 576\sqrt{\frac{2}{5}}c_3^2m_4 \\
 & - (80\sqrt{3}c_1^3 + 24(5\sqrt{6}c_2 + 3\sqrt{10}c_3) c_1^2 \\
 & + 24(5\sqrt{3}c_2^2 + 6\sqrt{5}c_3c_2 + 3\sqrt{3}c_3^2) c_1 \\
 & + 20\sqrt{6}c_2^3 + 9\sqrt{10}c_3^2 \\
 & + 36\sqrt{6}c_2c_3^2 + 36\sqrt{10}c_2^2c_3) M_0 \\
 & + (10\sqrt{3}c_1^3 - 6(5\sqrt{6}c_2 + 3\sqrt{10}c_3) c_1^2 \\
 & + 12(5\sqrt{3}c_2^2 + 6\sqrt{5}c_3c_2 + 3\sqrt{3}c_3^2) c_1 - 20\sqrt{6}c_2^3 \\
 & - 36\sqrt{10}c_3c_2^2 - 36\sqrt{6}c_3^2c_2 - 36\sqrt{\frac{2}{5}}c_3^3) M_1 \\
 & - (10\sqrt{3}c_1^3 + 6(5\sqrt{6}c_2 - 3\sqrt{10}c_3) c_1^2 \\
 & - 12(5\sqrt{3}c_2^2 - 6\sqrt{5}c_3c_2 + 3\sqrt{3}c_3^2) c_1 \\
 & + 20\sqrt{6}c_2^3 - 36\sqrt{\frac{2}{5}}c_3^3 + 36\sqrt{6}c_2c_3^2 - 36\sqrt{10}c_2^2c_3) M_2 \\
 & - (10\sqrt{3}c_1^3 - 3(5\sqrt{6}c_2 - 9\sqrt{10}c_3) c_1^2
 \end{aligned}$$

Table 9 Singlets charges of Class A models

Class A Models	Charges					
	Q'_{13}	Q'_{14}	Q'_{15}	Q'_{34}	Q'_{35}	Q'_{45}
A1, A6	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1
A2, A5	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$
A3, A4	$-\frac{1}{2}$	$\frac{1}{2}$	0	1	$\frac{1}{2}$	$-\frac{1}{2}$

$$\begin{aligned}
 &-3(5\sqrt{3}c_2^2 - 18\sqrt{5}c_3c_2 + 27\sqrt{3}c_3^2) c_1 \\
 &- 5\sqrt{\frac{3}{2}}c_2^3 + \frac{243c_3^3}{\sqrt{10}} - 81\sqrt{\frac{3}{2}}c_2c_3^2 + 27\sqrt{\frac{5}{2}}c_2^2c_3) M_3 \\
 &+ \left(80\sqrt{3}c_1^3 + 20\sqrt{6}c_2^3 - 36\sqrt{\frac{2}{5}}c_3^3 \right. \\
 &+ 36\sqrt{6}c_2c_3^2 - 36\sqrt{10}c_2^2c_3 \\
 &+ 24(5\sqrt{6}c_2 - 3\sqrt{10}c_3) c_1^2 + 24(5\sqrt{3}c_2^2 \\
 &- 6\sqrt{5}c_3c_2 + 3\sqrt{3}c_3^2) c_1) M_4 \\
 &+ (80\sqrt{3}c_1^3 - 12(5\sqrt{6}c_2 - 9\sqrt{10}c_3) c_1^2 \\
 &+ 6(5\sqrt{3}c_2^2 - 18\sqrt{5}c_3c_2 + 27\sqrt{3}c_3^2) c_1 \\
 &- 5\sqrt{\frac{3}{2}}c_2^3 + \frac{243c_3^3}{\sqrt{10}} - 81\sqrt{\frac{3}{2}}c_2c_3^2 + 27\sqrt{\frac{5}{2}}c_2^2c_3) M_5 \\
 &+ \frac{27}{10}(25\sqrt{6}c_2^3 + 45\sqrt{10}c_3c_2^2 + 45\sqrt{6}c_3^2c_2 + 9\sqrt{10}c_3^3) \\
 &\times M_6 + \sum_{i \neq j} M_{ij} Q'^3_{ij} \tag{A.4}
 \end{aligned}$$

The sums in (A.3) and (A.4) represents the contribution from the singlets.

B List of models

In this Appendix all the flux solutions subject to MSSM spectrum criteria, the corresponding $U(1)'$ -charges and details about the singlet spectrum are presented. For each c_i -solution presented, a similar solution subject to $c_i \rightarrow -c_i$ is also predicted from the solution of the anomaly cancellation conditions. Hence, models with charges subject to $Q' \rightarrow -Q'$ are also exist.

As mentioned on the main text, there are fifty-four solutions that fall into four classes of models: *Class A*, *B*, *C* and *D*.

Class A

This class consists of six models. The flux data solutions along with the resulting c_i -coefficients have been presented in Table 5 of the main text. The corresponding models defined by these solutions along with their $U(1)'$ charges are given

Table 10 Class B models, flux data and the corresponding c_i -solutions

Class B Model	Flux data														c_i coefficients		
	m_1	m_2	m_3	m_4	M_1	M_2	M_3	M_4	M_5	M_6	M_7	N_7	N_8	N_9	c_1	c_2	c_3
B1	1	0	1	1	0	-1	0	0	0	-1	-1	0	0	1	$-\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{3}$	$\frac{1}{\sqrt{6}}$
B2	1	0	1	1	0	0	-1	0	0	-1	-1	0	0	1	$-\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{3}$	$\frac{1}{\sqrt{6}}$
B3	1	0	1	1	0	0	0	-1	-1	0	-1	0	1	0	$\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{6}$	$\frac{\sqrt{\frac{3}{2}}}{2}$
B4	1	0	1	1	0	0	0	0	-2	0	-1	0	1	0	$\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{6}$	$\frac{\sqrt{\frac{3}{2}}}{2}$
B5	1	0	1	1	0	0	0	0	-1	0	-2	0	1	0	$\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{6}$	$\frac{\sqrt{\frac{3}{2}}}{2}$
B6	1	0	1	1	0	0	0	0	0	-2	-1	0	0	1	$-\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{3}$	$\frac{1}{\sqrt{6}}$
B7	1	0	1	1	0	-1	0	0	-1	0	-1	0	1	0	$\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{6}$	$\frac{\sqrt{\frac{3}{2}}}{2}$
B8	1	0	1	1	0	0	0	0	0	-1	-2	0	0	1	$-\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{3}$	$\frac{1}{\sqrt{6}}$
B9	1	1	0	1	0	-1	0	0	0	-1	-1	0	0	1	$-\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{3}$	$\frac{1}{\sqrt{6}}$
B10	1	1	0	1	0	0	-1	0	-1	-1	0	1	0	0	0	$-\frac{\sqrt{\frac{3}{2}}}{2}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
B11	1	1	0	1	0	0	-1	0	0	-1	-1	0	0	1	$-\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{3}$	$\frac{1}{\sqrt{6}}$
B12	1	1	0	1	0	0	0	-1	-1	-1	0	1	0	0	0	$-\frac{\sqrt{\frac{3}{2}}}{2}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
B13	1	1	0	1	0	0	0	0	-2	-1	0	1	0	0	0	$-\frac{\sqrt{\frac{3}{2}}}{2}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
B14	1	1	0	1	0	0	0	0	-1	-2	0	1	0	0	0	$-\frac{\sqrt{\frac{3}{2}}}{2}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
B15	1	1	0	1	0	0	0	0	0	-2	-1	0	0	1	$-\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{3}$	$\frac{1}{\sqrt{6}}$
B16	1	1	0	1	0	0	0	0	0	-1	-2	0	0	1	$-\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{3}$	$\frac{1}{\sqrt{6}}$
B17	1	1	1	0	0	-1	0	0	-1	0	-1	0	1	0	$\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{6}$	$\frac{\sqrt{\frac{3}{2}}}{2}$
B18	1	1	1	0	0	0	-1	0	-1	-1	0	1	0	0	0	$-\frac{\sqrt{\frac{3}{2}}}{2}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
B19	1	1	1	0	0	0	0	-1	-1	-1	0	1	0	0	0	$-\frac{\sqrt{\frac{3}{2}}}{2}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
B20	1	1	1	0	0	0	0	-1	-1	0	-1	0	1	0	$\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{6}$	$\frac{\sqrt{\frac{3}{2}}}{2}$
B21	1	1	1	0	0	0	0	0	-2	-1	0	1	0	0	0	$-\frac{\sqrt{\frac{3}{2}}}{2}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
B22	1	1	1	0	0	0	0	0	-2	0	-1	0	1	0	$\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{6}$	$\frac{\sqrt{\frac{3}{2}}}{2}$
B23	1	1	1	0	0	0	0	0	-1	-2	0	1	0	0	0	$-\frac{\sqrt{\frac{3}{2}}}{2}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
B24	1	1	1	0	0	0	0	0	-1	0	-2	0	1	0	$\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{3}{2}}}{6}$	$\frac{\sqrt{\frac{3}{2}}}{2}$

in Table 6. Here we present only the singlet spectrum for this class of models.

As have been already discussed, in this particular class of models the singlets come in pairs, meaning that $M_{ij} =$

M_{ji} . Hence, a minimal singlet spectrum scenario implies that $M_{ij} = M_{ji} = 1$. The singlet charges Q'_{ij} for each model are given in Table 9, below.

Table 11 $U(1)'$ charges of Class B models

Class B Models	Charges $\times \sqrt{15}$										
	Q'_{10_1}	Q'_{10_2}	Q'_{10_3}	Q'_{10_4}	Q'_{5_1}	Q'_{5_2}	Q'_{5_3}	Q'_{5_4}	Q'_{5_5}	Q'_{5_6}	Q'_{5_7}
B1, B2, B6, B8, B9, B11, B15, B16	-1	3/2	3/2	-1	2	-1/2	-1/2	2	-3	-1/2	-1/2
B3, B4, B5, B7, B17, B20, B22, B24	1	-3/2	1	-3/2	-2	1/2	-2	1/2	1/2	3	1/2
B10, B12, B13, B14, B18, B19, B21, B23	-1	-1	3/2	3/2	2	2	-1/2	-1/2	-1/2	-1/2	-3

Table 12 Singlets spectrum of Class B models

Class B Models	Multiplicities											Charges $\times \sqrt{15}$						
	M_{13}	M_{14}	M_{15}	M_{34}	M_{35}	M_{45}	M_{31}	M_{41}	M_{51}	M_{43}	M_{53}	M_{54}	Q'_{13}	Q'_{14}	Q'_{15}	Q'_{34}	Q'_{35}	Q'_{45}
B1, B2, B6, B8, B9, B11, B15, B16	1	2	2	1	1	1	1	1	1	1	1	1	$-\frac{5}{2}$	$-\frac{5}{2}$	0	0	$\frac{5}{2}$	$\frac{5}{2}$
B3, B4, B5, B7, B17, B20, B22, B24	1	2	2	1	1	1	1	1	1	1	1	1	$\frac{5}{2}$	0	$\frac{5}{2}$	$-\frac{5}{2}$	0	$\frac{5}{2}$
B10, B12, B13, B14, B18, B19, B21, B23	1	2	2	1	1	1	1	1	1	1	1	1	0	$-\frac{5}{2}$	$-\frac{5}{2}$	$-\frac{5}{2}$	$-\frac{5}{2}$	0

Class B

This class of models consists of 24 solutions. All the relevant data characterized the models organized in three tables. In particular, Table 10 contains the flux data of the models along with the corresponding c_i -solutions, as those have been extracted from the solution of the anomaly cancellation conditions. In Table 11, the $U(1)'$ charges of the matter curves are given. Finally, details about the singlet spectrum presented in Table 12.

Class C

Twelve models define this class. Gauge anomaly cancellation solutions are given in Table 13 while the corresponding matter curve $U(1)'$ charges are listed in Table 14. The properties of the singlet spectrum are described in Table 15.

Class D

This class contains twelve models. Flux data along with the corresponding solution for the c_i -coefficients are given in Table 16. The $U(1)'$ charges are listed in Table 17 while the properties (multiplicities and Q'_{ij} charges) of the singlet spectrum are described in Table 18.

Phenomenological analysis of Model D9 was presented in the main body of the present text.

Regarding the singlet sector of the models, their superpotential can be written as

$$W \supset \mu_{ij}^{\alpha\beta} \theta_{ij}^\alpha \theta_{ji}^\beta + \lambda_{ijk}^{\alpha\beta\gamma} \theta_{ij}^\alpha \theta_{kj}^\beta \theta_{ki}^\gamma \tag{B.1}$$

where $\mu_{ij}^{\alpha\beta}$ are mass parameters and $\lambda_{ijk}^{\alpha\beta\gamma}$ dimensionless coupling constants. The Greek indices run from 1 up to the mul-

tiplicity M_{ij} of the corresponding singlet. Minimalization of the superpotential ($\partial W / \partial \theta_{ij}^\alpha = 0$) leads to the F-flatness conditions.

C Flavour violation bounds for the various models

In the main text we have analysed in detail the low energy implications of model D9. A similar phenomenological analysis have been performed for all the MSSM spectrum models discussed so far. Due to the large number of models we do not present in detail the analysis for each model. Here we discuss the main flavor violation results for the four classes of MSSM models presented in the previous sections.

Models of the same class share common $U(1)'$ properties and consequently their phenomenological analysis is very similar. Next, we discuss the basic flavour violation bounds for each class of models. The main results collectively presented in Table 19.

Class A: The six models that compromised the Class A have very similar $U(1)'$ charges. More specifically, only two values allowed for the $|Q'|$ charges, 0 and 1/2. Matter fields descending from the $SU(5)$ tenplets have zero charge and as a result the corresponding flavor violation process appear very suppressed. The Q' charges appear (semi) non-universal in the lepton sector but again the corresponding LFV process are well suppressed in comparison with the experimental results. In summary, flavor violation process in Class A models appear to be suppressed and consequently $M_{Z'}$ bounds cannot be extracted for this class of models.

Class B: From the 24 models of this class, eight-teen of them have been analysed in detail. In particular, the models B4, B5, B8, B13, B15 and B16 predict inappropriate mass hierarchies

Table 13 Class C models, flux data along with the corresponding c_i -coefficients

Class C Model	Flux data														c_i coefficients		
	m_1	m_2	m_3	m_4	M_1	M_2	M_3	M_4	M_5	M_6	M_7	N_7	N_8	N_9	c_1	c_2	c_3
C1	1	0	0	2	0	-1	0	0	0	-1	-1	0	0	1	$-\frac{\sqrt{5}}{3}$	$\frac{5\sqrt{\frac{3}{2}}}{12}$	$\frac{1}{4\sqrt{6}}$
C2	1	0	0	2	0	0	0	0	0	-2	-1	0	0	1	$-\frac{\sqrt{5}}{3}$	$\frac{5\sqrt{\frac{3}{2}}}{12}$	$\frac{1}{4\sqrt{6}}$
C3	1	0	0	2	0	0	0	0	0	-1	-2	0	0	1	$-\frac{\sqrt{5}}{6}$	$\frac{7\sqrt{\frac{3}{2}}}{12}$	$-\frac{1}{4\sqrt{6}}$
C4	1	0	2	0	0	-1	0	0	-1	0	-1	0	1	0	$\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{5}{2}}}{6}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
C5	1	0	2	0	0	0	0	-1	-1	0	-1	0	1	0	$\frac{\sqrt{5}}{6}$	$-\frac{\sqrt{\frac{5}{2}}}{12}$	$-\frac{3\sqrt{\frac{3}{2}}}{4}$
C6	1	0	2	0	0	0	0	0	-2	0	-1	0	1	0	$\frac{\sqrt{5}}{3}$	$-\frac{\sqrt{\frac{5}{2}}}{6}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
C7	1	0	2	0	0	0	0	0	-1	0	-2	0	1	0	$\frac{\sqrt{5}}{6}$	$-\frac{\sqrt{\frac{5}{2}}}{12}$	$-\frac{3\sqrt{\frac{3}{2}}}{4}$
C8	1	0	0	2	0	0	-1	0	0	-1	-1	0	0	1	$-\frac{\sqrt{5}}{6}$	$\frac{7\sqrt{\frac{3}{2}}}{12}$	$-\frac{1}{4\sqrt{6}}$
C9	1	2	0	0	0	0	-1	0	-1	-1	0	1	0	0	0	$\frac{\sqrt{\frac{5}{2}}}{2}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
C10	1	2	0	0	0	0	0	-1	-1	-1	0	1	0	0	0	$\frac{\sqrt{\frac{5}{2}}}{4}$	$-\frac{3\sqrt{\frac{3}{2}}}{4}$
C11	1	2	0	0	0	0	0	0	-2	-1	0	1	0	0	0	$\frac{\sqrt{\frac{5}{2}}}{2}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
C12	1	2	0	0	0	0	0	0	-1	-2	0	1	0	0	0	$\frac{\sqrt{\frac{5}{2}}}{4}$	$-\frac{3\sqrt{\frac{3}{2}}}{4}$

Table 14 $U(1)'$ charges of Class C models. The charges are multiplied with $\sqrt{15}$

Class C Models	Charges $\times \sqrt{15}$										
	Q'_{10_1}	Q'_{10_2}	Q'_{10_3}	Q'_{10_4}	Q'_{5_1}	Q'_{5_2}	Q'_{5_3}	Q'_{5_4}	Q'_{5_5}	Q'_{5_6}	Q'_{5_7}
C1, C2	-1/4	9/4	-3/2	-1/4	1/2	-2	7/4	1/2	-3/4	-2	7/4
C3, C8	1/4	3/2	-9/4	1/4	-1/2	-7/4	2	-1/2	3/4	-7/4	2
C4, C6	1/4	-9/4	1/4	3/2	-1/2	2	-1/2	-7/4	2	3/4	-7/4
C5, C7	-1/4	-3/2	-1/4	9/4	1/2	7/4	1/2	-2	7/4	-3/4	-2
C9, C11	1/4	1/4	-9/4	3/2	-1/2	-1/2	2	-7/4	2	-7/4	3/4
C10, C12	-1/4	-1/4	-3/2	9/4	1/2	1/2	7/4	-2	7/4	-2	-3/4

Table 15 Singlets spectrum of Class C models

Class C Models	Multiplicities												Charges $\times \sqrt{15}$					
	M_{13}	M_{14}	M_{15}	M_{34}	M_{35}	M_{45}	M_{31}	M_{41}	M_{51}	M_{43}	M_{53}	M_{54}	Q'_{13}	Q'_{14}	Q'_{15}	Q'_{34}	Q'_{35}	Q'_{45}
C1, C2	1	1	1	1	1	1	1	1	1	2	1	1	$-\frac{5}{2}$	$\frac{5}{4}$	0	$\frac{15}{4}$	$\frac{5}{2}$	$-\frac{5}{4}$
C3, C8	1	1	1	2	1	1	1	1	1	1	1	1	$-\frac{5}{4}$	$\frac{5}{2}$	0	$\frac{15}{4}$	$\frac{5}{4}$	$-\frac{5}{2}$
C4, C6	1	1	1	1	1	1	1	1	1	1	2	1	$\frac{5}{2}$	0	$-\frac{5}{4}$	$-\frac{5}{2}$	$-\frac{15}{4}$	$-\frac{5}{4}$
C5, C7	1	1	1	1	2	1	1	1	1	1	1	1	$\frac{5}{4}$	0	$-\frac{5}{2}$	$-\frac{5}{4}$	$-\frac{15}{4}$	$-\frac{5}{2}$
C9, C11	1	1	1	1	1	1	1	1	1	1	1	2	0	$\frac{5}{2}$	$-\frac{5}{4}$	$\frac{5}{2}$	$-\frac{5}{4}$	$-\frac{15}{4}$
C10, C12	1	1	1	1	1	2	1	1	1	1	1	1	0	$\frac{5}{4}$	$-\frac{5}{2}$	$\frac{5}{4}$	$-\frac{5}{2}$	$-\frac{15}{4}$

Table 16 Class D models flux data

Class D	Flux data														c_i coefficients		
	Model	m_1	m_2	m_3	m_4	M_1	M_2	M_3	M_4	M_5	M_6	M_7	N_7	N_8	N_9	c_1	c_2
D1	1	0	1	1	0	0	-1	0	-1	0	-1	0	1	0	$\frac{\sqrt{5}}{2}$	$-\frac{\sqrt{3}}{8}$	$\frac{7}{8}$
D2	1	0	1	1	0	0	0	-1	0	-1	-1	0	0	1	$\frac{\sqrt{5}}{2}$	$\frac{5\sqrt{3}}{8}$	$-\frac{3}{8}$
D3	1	0	1	1	0	0	0	0	-1	-1	-1	0	0	1	$-\sqrt{\frac{5}{6}}$	$-\frac{\sqrt{3}}{8}$	$\frac{3}{8}$
D4	1	0	1	1	0	0	0	0	-1	-1	-1	0	1	0	$\sqrt{\frac{5}{6}}$	$-\frac{\sqrt{3}}{4}$	$\frac{1}{4}$
D5	1	1	0	1	0	-1	0	0	-1	-1	0	1	0	0	0	$-\frac{\sqrt{15}}{8}$	$-\frac{7}{8}$
D6	1	1	0	1	0	0	0	-1	0	-1	-1	0	0	1	$-\sqrt{\frac{5}{6}}$	$-\frac{\sqrt{3}}{8}$	$\frac{3}{8}$
D7	1	1	0	1	0	0	0	0	-1	-1	-1	1	0	0	0	$-\frac{\sqrt{15}}{4}$	$-\frac{1}{4}$
D8	1	1	1	0	0	-1	0	0	-1	-1	0	1	0	0	0	$-\frac{\sqrt{15}}{4}$	$-\frac{1}{4}$
D9	1	1	0	1	0	0	0	0	-1	-1	-1	0	0	1	$\frac{\sqrt{5}}{2}$	$\frac{5\sqrt{3}}{8}$	$-\frac{3}{8}$
D10	1	1	1	0	0	0	-1	0	-1	0	-1	0	1	0	$\sqrt{\frac{5}{6}}$	$-\frac{\sqrt{3}}{4}$	$\frac{1}{4}$
D11	1	1	1	0	0	0	0	0	-1	-1	-1	0	1	0	$\frac{\sqrt{5}}{2}$	$-\frac{\sqrt{3}}{8}$	$\frac{7}{8}$
D12	1	1	1	0	0	0	0	0	-1	-1	-1	1	0	0	0	$-\frac{\sqrt{15}}{8}$	$-\frac{7}{8}$

Table 17 $U(1)'$ charges of Class D models

Class D	Charges $\times \sqrt{10}$										
Models	Q'_{10_1}	Q'_{10_2}	Q'_{10_3}	Q'_{10_4}	Q'_{5_1}	Q'_{5_2}	Q'_{5_3}	Q'_{5_4}	Q'_{5_5}	Q'_{5_6}	Q'_{5_7}
D1, D11	3/4	-1/2	3/4	-7/4	-3/2	-1/4	-3/2	1	-1/4	9/4	1
D2, D9	3/4	-1/2	-7/4	3/4	-3/2	-1/4	1	-3/2	9/4	-1/4	1
D3, D6	-3/4	7/4	1/2	-3/4	3/2	-1	1/4	3/2	-9/4	-1	1/4
D4, D10	3/4	-7/4	3/4	-1/2	-3/2	1	-3/2	-1/4	1	9/4	-1/4
D5, D12	-3/4	-3/4	1/2	7/4	3/2	3/2	1/4	-1	1/4	-1	-9/4
D7, D8	-3/4	-3/4	7/4	1/2	3/2	3/2	-1	1/4	-1	1/4	-9/4

and as a result have been excluded from further analysis. For the remaining realistic models, the dominant constraints descents from the Kaon oscillation system. Approximately, the Z' contribution to the $K^0 - \bar{K}^0$ mass split is

$$\Delta M_K^{Z'} \simeq \frac{10^{-13} g'^2}{M_{Z'}^2} \tag{C.1}$$

which compared to the experimental bounds, for $g' = 0.5$ gives the constraint: $M_{Z'} \gtrsim 190$ TeV.

Class C: Due to the flux integers which characterize this class of models (see Table 13), all the matter fields descend-

ing from the $SU(5)$ tenplets have the same $U(1)'$ charges and as a result the corresponding flavour violation processes (like semi-leptonic meson decays and meson mixing effects) are suppressed. However, on the lepton sector the $U(1)'$ charges are non-universal leading to lepton flavor violation phenomena at low energies. The dominant constraint descent from the three body decay $\mu^- \rightarrow e^- e^- e^+$. Approximately for all the C-models, we find that the Z' contributions to the branching ratio of the decay is

$$Br(\mu^- \rightarrow e^- e^- e^+) \simeq 7.2 \times 10^{-6} \left(\frac{g' \text{ TeV}}{M_{Z'}} \right)^4$$

Table 18 Singlets spectrum of Class D models

Class D Models	Multiplicities											Charges $\times \sqrt{10}$						
	M_{13}	M_{14}	M_{15}	M_{34}	M_{35}	M_{45}	M_{31}	M_{41}	M_{51}	M_{43}	M_{53}	M_{54}	Q'_{13}	Q'_{14}	Q'_{15}	Q'_{34}	Q'_{35}	Q'_{45}
D1, D11	1	1	3	4	1	2	2	2	1	1	4	1	$\frac{5}{4}$	0	$\frac{5}{2}$	$-\frac{5}{4}$	$\frac{5}{4}$	$\frac{5}{2}$
D2, D9	1	1	1	1	4	1	3	1	2	3	1	4	$\frac{5}{4}$	$-\frac{5}{2}$	0	$\frac{5}{4}$	$-\frac{5}{4}$	$-\frac{5}{2}$
D3, D6	3	1	1	3	1	3	1	1	1	1	3	1	$-\frac{5}{2}$	$-\frac{5}{4}$	0	$\frac{5}{4}$	$\frac{5}{2}$	$\frac{5}{4}$
D4, D10	3	1	1	1	3	1	1	1	1	3	1	3	$\frac{5}{2}$	0	$\frac{5}{4}$	$-\frac{5}{2}$	$-\frac{5}{4}$	$\frac{5}{4}$
D5, D12	1	1	2	1	3	1	1	4	1	3	1	3	0	$-\frac{5}{4}$	$-\frac{5}{2}$	$-\frac{5}{4}$	$-\frac{5}{2}$	$-\frac{5}{4}$
D7, D8	1	2	1	2	1	4	3	1	3	1	3	1	0	$-\frac{5}{2}$	$-\frac{5}{4}$	$-\frac{5}{2}$	$-\frac{5}{4}$	$\frac{5}{4}$

Table 19 Dominant flavour violation process for each model along with the corresponding bounds on the mass of the flavour mixing Z' boson

Models	Dominant process	$(M_{Z'}/g')$ bound (TeV)
Class-B (excluded: B4, B5, B8, B13, B15, B16)	$K^0 - \bar{K}^0$ mixing	$M_{Z'}/g' \gtrsim 380$
	$\mu^- \rightarrow e^- e^- e^+$	$M_{Z'}/g' \gtrsim 51.8$
Class-C	Future $\mu^- \rightarrow e^- e^- e^+$ searches	$M_{Z'}/g' \gtrsim 518$
D1, D2, D5, D6, D8, D10	$K^0 - \bar{K}^0$ mixing	$M_{Z'}/g' \gtrsim 475$
	$K^0 - \bar{K}^0$ mixing	$M_{Z'}/g' \gtrsim 238$
D3, D4, D7, D9, D11, D12	Future $\mu^- \rightarrow e^- e^- e^+$ searches	$M_{Z'}/g' \gtrsim 420$

which compared to the current experimental bound implies that $M_{Z'} \gtrsim (51.8 \times g')$ TeV, where g' the $U(1)'$ gauge coupling. In the absence of any signal in future $\mu^- \rightarrow e^- e^- e^+$ searches, this bound is expected to increased by one order of magnitude: $M_{Z'} \gtrsim (518 \times g')$ TeV.

Class D: In this class of models the dominant constraints descend from the Kaon system. In some cases, strong bounds will be placed by future $\mu^- \rightarrow e^- e^- e^+$ searches. In particular, for the models D1, D2, D5, D6, D8 and D10 the constraints from Z' contributions to the $K^0 - \bar{K}^0$ mass split is: $M_{Z'} \gtrsim (475 \times g')$ TeV. For the rest of D-models (D3, D4, D7, D9, D11, D12), the results are similar with those of model D9 which have been detailed analysed in the main body of the present text.

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