Two-body charmed baryon decays involving decuplet baryon in the quark-diagram scheme

Y. K. Hsiao^a, Qian Yi, Shu-Ting Cai, H. J. Zhao

School of Physics and Information Engineering, Shanxi Normal University, Linfen 041004, China

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Abstract In the quark-diagram scheme, we study the charmed baryon decays of $\mathbf{B}_c \rightarrow \mathbf{B}^* M$, where $\mathbf{B}_c$ is Λ_c^+ or $\Xi_c^{+(0)}$, together with $\mathbf{B}^*$ (M) the decuplet baryon (pseudoscalar meson). It is found that only two W -exchange processes are allowed to contribute to $\mathbf{B}_c \rightarrow \mathbf{B}^* M$. Particularly, we predict $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^{*0(+)} \pi^{+(0)}) = (2.8 \pm 0.4) \times 10^{-3}$, which respects the isospin symmetry. Besides, we take into account the $SU(3)$ flavor symmetry breaking, in order to explain the observation of $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^{*+} \eta)$. For the decays involving $\Delta^{++}(uuu)$, we predict $\mathcal{B}(\Lambda_c^+ \rightarrow \Delta^{++} \pi^-, \Xi_c^+ \rightarrow \Delta^{++} K^-) = (7.0 \pm 1.4, 13.5 \pm 2.7) \times 10^{-4}$ as the largest branching fractions in the singly Cabibbo-suppressed $\Lambda_c^+, \Xi_c^+ \rightarrow \mathbf{B}^* M$ decay channels, respectively, which are accessible to the LHCb, BELLEII and BESIII experiments.

1 Introduction

To determine the mass and lifetime of the Λ_b baryon, Λ_c^+ is often taken as the final state in the Λ_b decays [1], which involves the favored Cabibbo–Kobayashi–Maskawa (CKM) matrix elements for bigger branching fractions. With the higher precision in the recent years [2, 3], the subsequent $\Lambda_c^+ \rightarrow p K^- \pi^+$ decay has helped to make more accurate observations for the Λ_b decays, which receives the significant contribution from $\Lambda_c^+ \rightarrow \Delta^{++} K^-, \Delta^{++} \rightarrow p \pi^+$. Similarly, one uses $\Xi_b^0 \rightarrow \Xi_c^+ \pi^-$ to determine the Ξ_b^0 lifetime, whereas we find that the subsequent process $\Xi_c^+ \rightarrow p K^- \pi^+$ and its resonant contribution from $\Xi_c^+ \rightarrow \Delta^{++} K^-, \Delta^{++} \rightarrow p \pi^+$ have not been well studied yet [1]. Therefore, $\mathbf{B}_c \rightarrow \mathbf{B}^* M$ plays the key role in the precision measurements for the multi-body decays of beauty and charm baryons, where $\mathbf{B}_c = (\Lambda_c^+, \Xi_c^{+(0)})$, $\mathbf{B}^*$ the decouplet baryon and M the meson state, such as $\Lambda_c^+(\Xi_c^+) \rightarrow \Delta^{++} K^-$.

The $\mathbf{B}_c \rightarrow \mathbf{B}^* M$ decays are not richly observed. Therefore, it is still unclear how the $\mathbf{B}_c \rightarrow \mathbf{B}^* M, \mathbf{B}^* \rightarrow \mathbf{B} M'$ decays mix with the non-resonant contributions to $\mathbf{B}_c \rightarrow \mathbf{B} M M'$. In addition, $\mathbf{B}_c \rightarrow \mathbf{B} V, V \rightarrow M M'$ with V the vector meson causes more complicated mixtures [1]. The $SU(3)$ flavor ($SU(3)_f$) symmetry has been widely applied to the charmed baryon decays [4–16]. By well explaining the data, the flavor symmetry does not appear to be severely broken in $\mathbf{B}_c \rightarrow \mathbf{B} M$ [17], where $\mathbf{B}$ denotes the octet baryon. By contrast, $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^{*+} \eta)$ not well interpreted by the $SU(3)_f$ symmetry might hint the broken effect in $\mathbf{B}_c \rightarrow \mathbf{B}^* M$ [7], which could be as large as that in the D meson decays [18–21].

For a better understanding of the hadronization in $\mathbf{B}_c \rightarrow \mathbf{B}^* M$, there have been some theoretical attempts, which are in terms of the pole model, quark model and irreducible $SU(3)$ flavor ($SU(3)_f$) symmetry [4–7, 22–24]. Particularly, the quark-diagram scheme with the topological $SU(3)_f$ symmetry provides a clear picture for the decay processes [17, 25–27]. Due to the fact that $\mathbf{B}^*$ is a spin-3/2 baryon with totally symmetric quark contents, it can be shown that the topological diagrams involving the flavor anti-symmetric quark pair in $\mathbf{B}^*$ are forbidden or suppressed. Therefore, we purpose to use the quark-diagram scheme to relate all possible $\mathbf{B}_c \rightarrow \mathbf{B}^* M$ decay channels. With the existing data, we will perform the numerical analysis, and determine different topological contributions. We can hence test the validity of the topological scheme, which involves the $SU(3)_f$ symmetry and its broken effect. Furthermore, we will give predictions for $\mathcal{B}(\mathbf{B}_c \rightarrow \mathbf{B}^* M)$ to be compared to the future measurements, which can help to clarify how $\mathbf{B}_c \rightarrow \mathbf{B}^* M, \mathbf{B}^* \rightarrow \mathbf{B} M'$ mixes with $\mathbf{B}_c \rightarrow \mathbf{B} V, V \rightarrow M M'$ and the non-resonant configuration in $\mathbf{B}_c \rightarrow \mathbf{B} M M'$.

^a e-mail: yukuohsiao@gmail.com (corresponding author)

2 Formalism

2.1 Effective Hamiltonian in the flavor symmetry

To study the two-body charmed baryon decays, the corresponding quark-level effective Hamiltonian is given by [28]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{i=1,2} c_i (\lambda_a O_i^a + \lambda_p O_i^p + \lambda_c O_i^c), \quad (1)$$

with $\lambda_{(a,p,c)} \equiv (V_{cs}^* V_{ud}, V_{cp}^* V_{up}, V_{cd}^* V_{us})$ and $p = (d, s)$, where G_F is the Fermi constant, and c_i the Wilson coefficients. The current–current operators $O_i^{(a,p,c)}$ are written as

$$\begin{aligned} O_1^a &= (\bar{u}d)(\bar{s}c), & O_2^a &= (\bar{u}_\beta d_\alpha)(\bar{s}_\alpha c_\beta), \\ O_1^p &= (\bar{u}p)(\bar{p}c), & O_2^p &= (\bar{u}_\beta p_\alpha)(\bar{p}_\alpha c_\beta), \\ O_1^c &= (\bar{u}s)(\bar{d}c), & O_2^c &= (\bar{u}_\beta s_\alpha)(\bar{d}_\alpha c_\beta), \end{aligned} \quad (2)$$

where $(\bar{q}_1 q_2) = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$, and the subscripts (α, β) denote the color indices. With $s_c \equiv \sin \theta_c \simeq 0.22$, where θ_c denotes the Cabibbo angle for the quark-mixing in the weak interaction, the decays with $|\lambda_{(a,p,c)}| \simeq (1, s_c, s_c^2)$ are regarded as the Cabibbo-allowed (CA), singly Cabibbo-suppressed (SCS) and doubly Cabibbo-suppressed (DCS) processes, respectively.

For the lowest-lying anti-triplet charmed baryon states Ξ_c^0 , Ξ_c^+ and Λ_c^+ that consist of $(ds - sd)c$, $(su - us)c$ and $(ud - du)c$, respectively, we present them as

$$\mathbf{B}_c = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}. \quad (3)$$

The pseudoscalar meson states are given by

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}(\pi^0 + c\phi\eta + s\phi\eta') & \pi^- & K^- \\ \pi^+ & \frac{-1}{\sqrt{2}}(\pi^0 - c\phi\eta - s\phi\eta') & \bar{K}^0 \\ K^+ & K^0 & -s\phi\eta + c\phi\eta' \end{pmatrix}, \quad (4)$$

where (η, η') mix with $\eta_q = \sqrt{1/2}(u\bar{u} + d\bar{d})$ and $\eta_s = s\bar{s}$. The mixing angle $\phi = (39.3 \pm 1.0)^\circ$ in $(s\phi, c\phi) \equiv (\sin \phi, \cos \phi)$ comes from the mixing matrix, given by [29, 30]

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}. \quad (5)$$

The decuplet baryons are written as

$$\mathbf{B}^* = \frac{1}{\sqrt{3}} \left(\begin{pmatrix} \sqrt{3}\Delta^{++} & \Delta^+ & \Sigma^{*+} \\ \Delta^+ & \Delta^0 & \frac{\Sigma^{*0}}{\sqrt{2}} \\ \Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0} \end{pmatrix}, \begin{pmatrix} \Delta^+ & \Delta^0 & \frac{\Sigma^{*0}}{\sqrt{2}} \\ \Delta^0 & \sqrt{3}\Delta^- & \Sigma^{*-} \\ \frac{\Sigma^{*0}}{\sqrt{2}} & \Sigma^{*-} & \Xi^{*-} \end{pmatrix}, \begin{pmatrix} \Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0} \\ \frac{\Sigma^{*0}}{\sqrt{2}} & \Sigma^{*-} & \Xi^{*-} \\ \Xi^{*0} & \Xi^{*-} & \sqrt{3}\Omega^- \end{pmatrix} \right). \quad (6)$$

By neglecting the Lorentz indices, $\mathcal{H}_{eff}$ for the $c \rightarrow q_i \bar{q}_j q_k$ transition can be presented with the tensor notation, H_j^{ki} , and the nonzero entries are given by [27]

$$H_2^{31} = \lambda_a, \quad H_2^{21} = \lambda_d, \quad H_3^{31} = \lambda_s, \quad H_3^{21} = \lambda_c. \quad (7)$$

2.2 The quark-diagram scheme

In the quark-diagram scheme, there exist six different topological diagrams for the $\mathbf{B}_c \rightarrow \mathbf{B}^{(*)}M$ decays, as drawn in Fig. 1a–f, parameterized as the topological amplitudes (T, C, C') and (E', E_B, E_M) , respectively [17]. More explicitly, T and $C^{(\prime)}$ proceed with the W -boson emission (W_{EM}). By exchanging the W boson (W_{EX}), it gives rise to E' and $E_{B(M)}$. Since only (T, C) can be decomposed of two separate matrix elements based on the factorization, that is, $(T, C) \propto \langle M | \bar{q}_1 q_2 | 0 \rangle \langle \mathbf{B}^{(*)} | \bar{q}_3 c | \mathbf{B}_c \rangle$ [31], one classifies (T, C) and $(C', E', E_{B,M})$ as the factorizable and non-factorizable amplitudes, respectively.

Furthermore, it is found in Fig. 1a, b that $\mathbf{B}_c$ with $(qaqb - qbqa)c$ cannot be turned into $\mathbf{B}^*(qaqbqk(i))$, where $qaqbqk(i)$ are totally symmetric, such that (T, C) give no contributions to $\mathbf{B}_c \rightarrow \mathbf{B}^*M$. Thus, the $\mathbf{B}_c \rightarrow \mathbf{B}^*M$ decays are purely non-factorizable processes. In addition, C' and E' are suppressed in $\mathbf{B}_c \rightarrow \mathbf{B}^*M$ [25], which is in accordance with the Körner–Pati–Woo theorem [32–34]. With the current–current structure of $(\bar{q}_i q_j)_{V-A}(\bar{q}_k c)_{V-A}$ in Eq. (2), q_i and q_k are color anti-symmetric. When combined as the constituents of the baryon, $q_{i,k}$ are flavor anti-symmetric, such that the topological diagrams (C', E') in Fig. 1c, d contribute to $\mathbf{B}_c \rightarrow \mathbf{B}M$, instead of $\mathbf{B}_c \rightarrow \mathbf{B}^*M$. Consequently, we are left with the W_{EX} topological diagrams (E_B, E_M) in Fig. 1e, f for $\mathbf{B}_c \rightarrow \mathbf{B}^*M$.

To proceed, we derive the amplitudes as $\mathcal{A}(\mathbf{B}_c \rightarrow \mathbf{B}^*M) = (G_F/\sqrt{2})T(\mathbf{B}_c \rightarrow \mathbf{B}^*M)$. Explicitly, the T amplitudes (T -amps) read [17, 25, 27]

$$T(\mathbf{B}_c \rightarrow \mathbf{B}^*M) = E_B^{(s)}(\mathbf{B}_c)^{ja} H_j^{ki}(\mathbf{B}^*)_{kab}(M)_i^b + E_M^{(s)}(\mathbf{B}_c)^{ja} H_j^{ki}(\mathbf{B}^*)_{iab}(M)_k^b, \quad (8)$$

where the parameters $E_{\mathbf{B},M}^{(s)}$ correspond to the topological diagrams in Fig. 1e, f, respectively. The W_{EX} decay process needs an additional quark pair from $g \rightarrow q\bar{q}$, where $q\bar{q}$ could be $u\bar{u}$, $d\bar{d}$ or $s\bar{s}$. To take into account the broken $SU(3)_f$ symmetry, $E_{\mathbf{B}(M)}$ with $g \rightarrow s\bar{s}$ can be more specifically denoted by $E_{\mathbf{B}(M)}^s$. Under the exact $SU(3)_f$ symmetry, it leads to $E_{\mathbf{B}(M)}^s = E_{\mathbf{B}(M)}$ [26]. In Tables 1, 2 and 3, we

Fig. 1 Topological diagrams for the $B_c \rightarrow B^{(*)}M$ decays

present the full expansions of $T(B_c \rightarrow B^{(*)}M)$ for the CA, SCS and DCS decay modes, respectively. For the branching fractions, we use the equation for the two-body decays, given by [1]

$$\mathcal{B}(B_c \rightarrow B^{(*)}M) = \frac{G_F^2 |\vec{p}_{B^{(*)}}| \tau_{B_c}}{16\pi m_{B_c}^2} |T(B_c \rightarrow B^{(*)}M)|^2, \quad (9)$$

$$|\vec{p}_{B^{(*)}}| = \frac{\sqrt{(m_{B_c}^2 - m_{+}^2)(m_{B_c}^2 - m_{-}^2)}}{2m_{B_c}},$$

with $m_{\pm} = m_{B^{(*)}} \pm m_M$, where τ_{B_c} stands for the B_c baryon lifetime.

3 Numerical analysis and discussions

In the numerical analysis, we adopt the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements as [1]

$$(V_{cs}, V_{ud}, V_{us}, V_{cd}) = (1 - \lambda^2/2, 1 - \lambda^2/2, \lambda, -\lambda), \quad (10)$$

with $\lambda = s_c = 0.22453 \pm 0.00044$ in the Wolfenstein parameterization. Besides, the B_c and B^{*} masses, together with the lifetime for B_c , are adopted from the PDG [1]. We perform a minimum χ^2 -fit with $\chi^2 = \sum (\mathcal{B}_{th} - \mathcal{B}_{ex})^2 / \sigma_{ex}^2$, where $\mathcal{B}_{th(ex)}$ represents the theoretical (experimental) input of the branching ratio, and σ_{ex} the experimental error. We calculate $\mathcal{B}_{th}$ with the equation in Eq. (9), together with $(\mathcal{B}_{ex}, \sigma_{ex})$ from Table 1. Note that $\mathcal{B}(\Xi_c^+ \rightarrow \Sigma^{*+} \bar{K}^0, \Xi^{*0} \pi^+)$ are not involved in the fit.

We use two scenarios for the global fit. In the first scenario (S1), we take $E_{B(M)}^s = E_{B(M)}$ under the exact $SU(3)_f$ symmetry. Since E_B and E_M are complex numbers, it leads to three independent parameters, given by

$$|E_B|, |E_M| e^{i\delta_{EM}}, \quad (11)$$

where E_B is set to be real, and δ_{EM} is a relative strong phase. Using the χ^2 -fit, we extract that

$$(|E_B|, |E_M|) = (0.41 \pm 0.03, 0.34 \pm 0.03) \text{ GeV}^3,$$

$$\delta_{EM} = (180.0 \pm 35.8)^\circ, \quad (12)$$

with $\chi^2/n.d.f = 4.5$, where $n.d.f = 1$ is the number of the degrees of freedom. For δ_{EM} , its information is from $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^{*+} \eta)$. Although $\delta_{EM} = 180^\circ$ has induced the largest positive interference between E_B and E_M , our result of $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^{*+} \eta) = (5.3 \pm 0.7) \times 10^{-3}$ is still shown to be in tension with the observation of $(9.1 \pm 2.0) \times 10^{-3}$. Sizeably, it adds 3.6 to the total χ^2 value.

Since $\Lambda_c^+ \rightarrow \Sigma^{*+} \eta$ is in association with $|E_M^s|$, the tension hints the broken $SU(3)_f$ symmetry, where $|E_M^s|$ is not equal to $|E_M|$. On the other hand, $\mathcal{B}(\Xi_c^0 \rightarrow \Omega^- K^+)$ is fitted to agree with the data, indicating that $|E_B^s|$ is not deviating from $|E_B|$. Currently, the data points are not sufficient for an independent extraction of $|E_M^s|$. We hence adopt the numerical results from the two-body D meson decays, where the similar W_{EX} contributions have been found to induce the severe $SU(3)_f$ symmetry breaking [18–21]. In the second scenario (S2), we take $|E_M^s| = n_q \times |E_M|$ and $|E_B^s| \simeq |E_B|$, with $n_q = 1.4$ adopted from [21]. Consequently, we obtain

$$(|E_B|, |E_M|) = (0.40 \pm 0.03, 0.34 \pm 0.03) \text{ GeV}^3, \quad (13)$$

$$\delta_{EM} = (180.0 \pm 46.8)^\circ,$$

where $\chi^2/n.d.f$ is reduced as 1.3. As the demonstration, we obtain $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^{*+} \eta) = (7.3 \pm 1.5) \times 10^{-3}$, which alleviates the deviation from the observation. With the fit values of $(|E_B|, |E_M|, \delta_{EM})$ in S1 and S2, we present the branching ratios of the $B_c \rightarrow B^{(*)}M$ decays in Tables 1, 2 and 3, along with the recent theoretical results for comparison.

We get some useful relations in the quark-diagram scheme. For example, we find out three triangle sum rules for $B_c \rightarrow \Delta \pi$, given by

$$\begin{aligned} T(\Lambda_c^+ \rightarrow \Delta^0 \pi^+) - T(\Lambda_c^+ \rightarrow \Delta^{++} \pi^-) \\ - \sqrt{6} T(\Lambda_c^+ \rightarrow \Delta^+ \pi^0) &= 0, \\ T(\Xi_c^+ \rightarrow \Delta^0 \pi^+) - T(\Xi_c^+ \rightarrow \Delta^{++} \pi^-) \\ - \sqrt{6} T(\Xi_c^+ \rightarrow \Delta^+ \pi^0) &= 0, \end{aligned}$$

Table 1 Cabibbo-allowed $\mathbf{B}_c \rightarrow \mathbf{B}^* M$ decays

Decay modes	T -amp	$10^3 \mathcal{B} (S1, S2)$ [our work]	$10^3 \mathcal{B} (S_{pm}, S_{em})$ [7]	$10^3 \mathcal{B}_{ex}$ [1,35]
$\Lambda_c^+ \rightarrow \Delta^{++} K^-$	$-\lambda_a E_M$	$(12.0 \pm 2.2, 11.7 \pm 2.3)$	$(15.3 \pm 2.4, 12.4 \pm 1.0)$	10.8 ± 2.5
$\Lambda_c^+ \rightarrow \Delta^+ \bar{K}^0$	$-\lambda_a \frac{1}{\sqrt{3}} E_M$	$(4.0 \pm 0.7, 3.9 \pm 0.8)$	$(5.1 \pm 0.8, 4.1 \pm 0.3)$	–
$\Lambda_c^+ \rightarrow \Sigma^{*0} \pi^+$	$-\lambda_a \frac{1}{\sqrt{6}} E_{\mathbf{B}}$	$(2.9 \pm 0.4, 2.8 \pm 0.4)$	$(2.2 \pm 0.4, 2.1 \pm 0.2)$	–
$\Lambda_c^+ \rightarrow \Sigma^{*+} \pi^0$	$-\lambda_a \frac{1}{\sqrt{6}} E_{\mathbf{B}}$	$(2.9 \pm 0.4, 2.8 \pm 0.4)$	$(2.2 \pm 0.4, 2.1 \pm 0.2)$	–
$\Lambda_c^+ \rightarrow \Sigma^{*+} \eta$	$-\lambda_a \frac{1}{\sqrt{6}} (E_{\mathbf{B}} c \phi - \sqrt{2} E_M^{(s)} s \phi)$	$(5.3 \pm 0.8, 7.3 \pm 1.5)$	$(3.1 \pm 0.6, 6.2 \pm 0.5)$	9.1 ± 2.0
$\Lambda_c^+ \rightarrow \Sigma^{*+} \eta'$	$-\lambda_a \frac{1}{\sqrt{6}} (E_{\mathbf{B}} s \phi + \sqrt{2} E_M^{(s)} c \phi)$	$(0, 0)$	–	–
$\Lambda_c^+ \rightarrow \Xi^{*0} K^+$	$-\lambda_a \frac{1}{\sqrt{3}} E_{\mathbf{B}}$	$(3.9 \pm 0.6, 3.9 \pm 0.6)$	$(1.0 \pm 0.2, 4.1 \pm 0.3)$	4.3 ± 0.9
$\Xi_c^0 \rightarrow \Sigma^{*+} K^-$	$\lambda_a \frac{1}{\sqrt{3}} E_M$	$(1.8 \pm 0.3, 1.7 \pm 0.3)$	$(3.1 \pm 0.5, 2.3 \pm 0.2)$	–
$\Xi_c^0 \rightarrow \Sigma^{*0} \bar{K}^0$	$\lambda_a \frac{1}{\sqrt{6}} E_M$	$(0.9 \pm 0.2, 0.9 \pm 0.2)$	$(1.6 \pm 0.2, 1.2 \pm 0.1)$	–
$\Xi_c^0 \rightarrow \Xi^{*-} \pi^+$	$\lambda_a \frac{1}{\sqrt{3}} E_{\mathbf{B}}$	$(2.6 \pm 0.4, 2.5 \pm 0.4)$	$(2.8 \pm 0.5, 2.3 \pm 0.2)$	–
$\Xi_c^0 \rightarrow \Xi^{*0} \pi^0$	$\lambda_a \frac{1}{\sqrt{6}} E_{\mathbf{B}}$	$(1.3 \pm 0.2, 1.3 \pm 0.2)$	$(1.4 \pm 0.2, 1.2 \pm 0.1)$	–
$\Xi_c^0 \rightarrow \Xi^{*0} \eta$	$\lambda_a \frac{1}{\sqrt{6}} (E_{\mathbf{B}} c \phi - \sqrt{2} E_M^{(s)} s \phi)$	$(2.4 \pm 0.4, 3.4 \pm 0.7)$	$(2.1 \pm 0.4, 3.5 \pm 0.3)$	–
$\Xi_c^0 \rightarrow \Xi^{*0} \eta'$	$\lambda_a \frac{1}{\sqrt{6}} (E_{\mathbf{B}} s \phi + \sqrt{2} E_M^{(s)} c \phi)$	$(0.01 \pm 0.04, 0.08 \pm 0.10)$	–	–
$\Xi_c^0 \rightarrow \Omega^- K^+$	$\lambda_a E_{\mathbf{B}}^{(s)}$	$(4.8 \pm 0.7, 4.8 \pm 0.7)$	$(2.3 \pm 0.5, 7.0 \pm 0.6)$	4.2 ± 1.0
$\Xi_c^+ \rightarrow \Sigma^{*+} \bar{K}^0$	0	0	0	28.6 ± 16.8
$\Xi_c^+ \rightarrow \Xi^{*0} \pi^+$	0	0	0	< 4.0

$$T(\Xi_c^0 \rightarrow \Delta^+ \pi^-) - T(\Xi_c^0 \rightarrow \Delta^- \pi^+) - \sqrt{6} T(\Xi_c^0 \rightarrow \Delta^0 \pi^0) = 0. \quad (14)$$

Besides, we obtain

$$T(\Lambda_c^+ \rightarrow \Delta^+ K^0, \Delta^0 K^+) = 0, \\ T(\Xi_c^+ \rightarrow \Sigma^{*+} \bar{K}^0, \Xi^{*0} \pi^+) = 0, \quad (15)$$

as the consequence of C' being set to give no contribution to $\mathbf{B}_c \rightarrow \mathbf{B}^* M$. Indeed, C' is the only topology that takes part in the decays in Eq. (15), but suppressed due to the Körner–Pati–Woo theorem [32–34]. According to the other theoretical calculations [6,7,22–24], $\mathcal{B}(\Xi_c^+ \rightarrow \Sigma^{*+} \bar{K}^0, \Xi^{*0} \pi^+) = 0$ is also predicted, which supports that $C' = 0$. Experimentally, $\mathcal{B}_{ex}(\Xi_c^+ \rightarrow \Sigma^{*+} \bar{K}^0, \Xi^{*0} \pi^+)$ in Table 2 can be used to test the suppression. With $\mathcal{B}(\Xi_c^+ \rightarrow \Xi^{*0} \pi^+)/\mathcal{B}(\Xi_c^+ \rightarrow \Xi^- \pi^+ \pi^+) < 0.1$ and $\mathcal{B}(\Xi_c^+ \rightarrow \Xi^- \pi^+ \pi^+) = (2.86 \pm 1.21 \pm 0.38) \times 10^{-2}$ [1,35], we determine $\mathcal{B}_{ex}(\Xi_c^+ \rightarrow \Xi^{*0} \pi^+) < 4.0 \times 10^{-3}$, which can be seen as the non-observation to agree with $T(\Xi_c^+ \rightarrow \Xi^{*0} \pi^+) = 0$. However, $\mathcal{B}_{ex}(\Xi_c^+ \rightarrow \Sigma^{*+} \bar{K}^0) = (2.9 \pm 1.7) \times 10^{-2}$ seems to disagree with the prediction of $\mathcal{B}(\Xi_c^+ \rightarrow \Sigma^{*+} \bar{K}^0) = 0$, despite of the large uncertainty. While the Körner–Pati–Woo theorem is regarded as a tree-level approximation, allowing possible corrections to C' and E' , we need more accurate observations to test if $C'(E') = 0$.

Uniquely, the decuplet baryon can contain three identical quarks, denoted by $\mathbf{B}^*(qqq)$, which leads to an additional weight factor of $\sqrt{3}$ among the decuplet baryons in Eq. (6).

The factor can be considered as the main reason why $\Lambda_c^+ \rightarrow \Delta^{++} K^-$ and $\Xi_c^0 \rightarrow \Omega^- K^+$ are measured with the largest branching fractions in the CA decay channels of Λ_c^+ , $\Xi_c^0 \rightarrow \mathbf{B}^* M$, respectively. Accordingly, the T -amps with $\mathbf{B}^*(qqq)$ are listed as

$$T(\Lambda_c^+ \rightarrow \Delta^{++} K^-, \Delta^{++} \pi^-) = -(\lambda_a, \lambda_d) E_M, \\ T(\Xi_c^+ \rightarrow \Delta^{++} K^-, \Delta^{++} \pi^-) = -(\lambda_s, \lambda_c) E_M, \\ T(\Xi_c^0 \rightarrow \Omega^- K^+, \Delta^- \pi^+) = (\lambda_a, -\lambda_c) E_{\mathbf{B}}. \quad (16)$$

While $\mathcal{B}(\Lambda_c^+ \rightarrow \Delta^{++} K^-)$ and $\mathcal{B}(\Xi_c^0 \rightarrow \Omega^- K^+)$ have been observed, the other branching fractions are given by

$$\mathcal{B}(\Lambda_c^+ \rightarrow \Delta^{++} \pi^-, \Xi_c^+ \rightarrow \Delta^{++} K^-) \\ = (7.0 \pm 1.4, 13.5 \pm 2.7) \times 10^{-4}, \\ \mathcal{B}(\Xi_c^+ \rightarrow \Delta^{++} \pi^-, \Xi_c^0 \rightarrow \Delta^- \pi^+) \\ = (7.8 \pm 1.6, 2.5 \pm 0.4) \times 10^{-5}, \quad (17)$$

which are predicted as the largest branching fractions in the SCS $\Lambda_c^+(\Xi_c^+)$ and DCS $\Xi_c^{+(0)}$ decay channels, respectively. Here, we present our predictions of $S2$, which is favored by the χ^2 -fit. The equality of $T(\Lambda_c^+ \rightarrow \Sigma^{*0} \pi^+) = T(\Lambda_c^+ \rightarrow \Sigma^{*+} \pi^0)$ corresponds to the isospin symmetry. The branching fraction, given by

$$\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^{*0(+)} \pi^{+(0)}) = (2.8 \pm 0.4) \times 10^{-3}, \quad (18)$$

can be used to test the broken effect. The decays $\mathbf{B}_c \rightarrow \mathbf{B}^* \eta^{(\prime)}, \Lambda_c^+ \rightarrow \Sigma^{*+} \eta, \Xi_c^0 \rightarrow \Xi^{*0} \eta$ and $\Xi_c^+ \rightarrow \Sigma^{*+} \eta$

Table 2 Singly Cabibbo-suppressed $\mathbf{B}_c \rightarrow \mathbf{B}^* M$ decays

Decay modes	T -amp	$10^4 \mathcal{B} (S1, S2)$ [our work]	$10^4 \mathcal{B} (S_{pm}, S_{em})$ [7]
$\Lambda_c^+ \rightarrow \Delta^{++} \pi^-$	$-\lambda_d E_M$	$(7.2 \pm 1.3, 7.0 \pm 1.4)$	$(12.5 \pm 2.0, 6.6 \pm 0.6)$
$\Lambda_c^+ \rightarrow \Delta^+ \pi^0$	$-\lambda_d \frac{1}{\sqrt{6}} (E_{\mathbf{B}} - E_M)$	$(5.8 \pm 0.9, 5.6 \pm 1.1)$	$(8.3 \pm 1.3, 4.4 \pm 0.4)$
$\Lambda_c^+ \rightarrow \Delta^0 \pi^+$	$-\lambda_d \frac{1}{\sqrt{3}} E_{\mathbf{B}}$	$(3.5 \pm 0.5, 3.3 \pm 0.5)$	$(4.2 \pm 0.7, 2.2 \pm 0.2)$
$\Lambda_c^+ \rightarrow \Delta^+ \eta$	$-\lambda_d \frac{1}{\sqrt{6}} (E_{\mathbf{B}} + E_M) c\phi$	$(0.03 \pm 0.28, 0.02 \pm 0.45)$	–
$\Lambda_c^+ \rightarrow \Delta^+ \eta'$	$-\lambda_d \frac{1}{\sqrt{6}} (E_{\mathbf{B}} + E_M) s\phi$	$(0.01 \pm 0.09, 0.01 \pm 0.15)$	–
$\Lambda_c^+ \rightarrow \Sigma^{*+} K^0$	$-\lambda_d \frac{1}{\sqrt{3}} E_M^{(s)}$	$(1.8 \pm 0.4, 3.5 \pm 0.7)$	$(1.3 \pm 0.2, 2.2 \pm 0.2)$
$\Lambda_c^+ \rightarrow \Sigma^{*0} K^+$	$-\lambda_d \frac{1}{\sqrt{6}} E_{\mathbf{B}}^{(s)}$	$(1.3 \pm 0.2, 1.3 \pm 0.2)$	$(0.7 \pm 0.1, 1.1 \pm 0.1)$
$\Xi_c^0 \rightarrow \Delta^+ K^-$	$-\lambda_s \frac{1}{\sqrt{3}} E_M$	$(1.1 \pm 0.2, 1.0 \pm 0.2)$	$(3.0 \pm 0.5, 1.2 \pm 0.1)$
$\Xi_c^0 \rightarrow \Delta^0 \bar{K}^0$	$-\lambda_s \frac{1}{\sqrt{3}} E_M$	$(1.1 \pm 0.2, 1.0 \pm 0.2)$	$(3.0 \pm 0.5, 1.2 \pm 0.1)$
$\Xi_c^0 \rightarrow \Sigma^{*-} \pi^+$	$\frac{1}{\sqrt{3}} (\lambda_d E_{\mathbf{B}} - \lambda_s E_{\mathbf{B}})$	$(6.1 \pm 0.9, 5.8 \pm 0.9)$	$(9.9 \pm 1.6, 4.9 \pm 0.4)$
$\Xi_c^0 \rightarrow \Sigma^{*+} \pi^-$	$\lambda_d \frac{1}{\sqrt{3}} E_M$	$(1.0 \pm 0.2, 1.0 \pm 0.2)$	$(2.5 \pm 0.4, 1.2 \pm 0.1)$
$\Xi_c^0 \rightarrow \Sigma^{*0} \pi^0$	$\frac{1}{\sqrt{12}} [\lambda_d (E_{\mathbf{B}} - E_M) - \lambda_s E_{\mathbf{B}}]$	$(3.1 \pm 0.4, 2.9 \pm 0.5)$	$(5.6 \pm 0.9, 2.8 \pm 0.2)$
$\Xi_c^0 \rightarrow \Sigma^{*0} \eta$	$\frac{1}{\sqrt{12}} [\lambda_d (E_{\mathbf{B}} + E_M) c\phi + \lambda_s (\sqrt{2} E_M^{(s)} s\phi - E_{\mathbf{B}} c\phi)]$	$(0.9 \pm 0.2, 1.2 \pm 0.3)$	$(1.1 \pm 0.2, 0.9 \pm 0.1)$
$\Xi_c^0 \rightarrow \Sigma^{*0} \eta'$	$\frac{1}{\sqrt{12}} [\lambda_d (E_{\mathbf{B}} + E_M) s\phi - \lambda_s (\sqrt{2} E_M^{(s)} c\phi + E_{\mathbf{B}} s\phi)]$	$(0.004 \pm 0.120, 0.050 \pm 0.250)$	–
$\Xi_c^0 \rightarrow \Xi^{*0} K^0$	$\frac{1}{\sqrt{3}} \lambda_d E_M^{(s)}$	$(0.8 \pm 0.2, 1.6 \pm 0.4)$	$(0.9 \pm 0.2, 1.2 \pm 0.1)$
$\Xi_c^0 \rightarrow \Xi^{*-} K^+$	$\frac{1}{\sqrt{3}} (\lambda_d E_{\mathbf{B}}^{(s)} - \lambda_s E_{\mathbf{B}}^{(s)})$	$(4.6 \pm 0.7, 4.6 \pm 0.7)$	$(3.6 \pm 0.6, 4.9 \pm 0.4)$
$\Xi_c^+ \rightarrow \Delta^{++} K^-$	$-\lambda_s E_M$	$(13.8 \pm 2.5, 13.5 \pm 2.7)$	$(35.0 \pm 5.7, 14.6 \pm 1.2)$
$\Xi_c^+ \rightarrow \Delta^+ \bar{K}^0$	$-\lambda_s \frac{1}{\sqrt{3}} E_M$	$(4.6 \pm 0.8, 4.5 \pm 0.9)$	$(11.7 \pm 1.9, 4.9 \pm 0.4)$
$\Xi_c^+ \rightarrow \Sigma^{*+} \pi^0$	$-\lambda_s \frac{1}{\sqrt{6}} E_{\mathbf{B}}$	$(3.4 \pm 0.5, 3.2 \pm 0.5)$	$(4.8 \pm 0.8, 2.4 \pm 0.2)$
$\Xi_c^+ \rightarrow \Sigma^{*0} \pi^+$	$-\lambda_s \frac{1}{\sqrt{6}} E_{\mathbf{B}}$	$(3.4 \pm 0.5, 3.2 \pm 0.5)$	$(4.8 \pm 0.8, 2.4 \pm 0.2)$
$\Xi_c^+ \rightarrow \Sigma^{*+} \eta$	$-\lambda_s \frac{1}{\sqrt{6}} (E_{\mathbf{B}} c\phi - \sqrt{2} E_M^{(s)} s\phi)$	$(6.4 \pm 1.0, 9.1 \pm 1.8)$	$(8.7 \pm 1.4, 7.3 \pm 0.6)$
$\Xi_c^+ \rightarrow \Sigma^{*+} \eta'$	$-\lambda_s \frac{1}{\sqrt{6}} (E_{\mathbf{B}} s\phi + \sqrt{2} E_M^{(s)} c\phi)$	$(0.1 \pm 0.3, 0.6 \pm 0.8)$	–
$\Xi_c^+ \rightarrow \Xi^{*0} K^+$	$-\lambda_s \frac{1}{\sqrt{3}} E_{\mathbf{B}}^{(s)}$	$(5.0 \pm 0.8, 5.0 \pm 0.8)$	$(3.5 \pm 0.6, 4.9 \pm 0.4)$

have sizeable branching fractions, which is due to the constructive interferences between $E_{\mathbf{B}}$ and E_M . However, the other branching fractions of $\mathbf{B}_c \rightarrow \mathbf{B}^* \eta^{(\prime)}$ are typically small with the destructive interferences. Moreover, we find that $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^{*+} \eta') = 0$ with $m_{\Lambda_c^+} < m_{\Sigma^{*+}} + m_{\eta'}$.

The approach of the irreducible $SU(3)_f$ symmetry has been widely used in the hadron weak decays [4–16]. For $\mathbf{B}_c \rightarrow \mathbf{B}^* M$, there exist four parameters a_8 and $a_{9,10,11}$ [4, 6], which correspond to the decomposition of $\mathcal{H}_{eff} = H(6) + H(\bar{15})$ in the $SU(3)_f$ representation of 6 and $\bar{15}$, respectively. By comparison, we derive that

$$\begin{aligned}
 (E_{\mathbf{B}}, E_M) &= (-2a_8 + a_9, 2a_8 + a_9), \quad (E', C') \\
 &= (-2a_9 - 2a_{10}, -2a_{11}), \quad (19)
 \end{aligned}$$

such that a_i are found to correspond to the topologies. Since (E', C') have been the vanishing topological parameters, one has $a_9 = -a_{10}$ and $a_{11} = 0$. Moreover, our global fits for

$E_{\mathbf{B},M}$ indicate that $a_{9(10)}$ from $H(\bar{15})$ has a non-zero value. By contrast, the numerical analysis performed with the irreducible $SU(3)_f$ symmetry neglects the contributions from $H(\bar{15})$ [7], whose results are given in the tables. In the physical mass scenario (S_{pm}) for the global fit in Ref. [7], where $m_{\mathbf{B}_c}$, $m_{\mathbf{B}^*}$ and m_M are taken from the physical values in Ref. [1], $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^{*+} \eta, \Xi^{*0} K^+)$ and $\mathcal{B}(\Xi_c^0 \rightarrow \Omega^- K^+)$ are fitted to be a few times smaller than the observations. Instead of considering the $SU(3)_f$ symmetry breaking, one performs another fit in the equal mass scenario (S_{em}), where $m_{\Lambda_c} = m_{\Xi_c}$, $m_{\Delta} = m_{\Sigma^*} = m_{\Omega}$ and $m_{\pi} = m_{\eta} = m_K$, resulting in the raised values of the above branching fractions.

4 Conclusions

In summary, we have studied the $\mathbf{B}_c \rightarrow \mathbf{B}^* M$ decays in the quark-diagram scheme. We have found that only two W -

Table 3 Doubly Cabibbo-suppressed $\mathbf{B}_c \rightarrow \mathbf{B}^* M$ decays

Decay modes	T -amp	$10^5 \mathcal{B} (S1, S2)$ [our work]	$10^5 \mathcal{B} (S_{pm}, S_{em})$ [7]
$\Lambda_c^+ \rightarrow \Delta^+ K^0$	0	0	0
$\Lambda_c^+ \rightarrow \Delta^0 K^+$	0	0	0
$\Xi_c^0 \rightarrow \Delta^+ \pi^-$	$-\lambda_c \frac{1}{\sqrt{3}} E_M$	$(0.6 \pm 0.1, 0.6 \pm 0.1)$	$(2.2 \pm 0.4, 0.7 \pm 0.1)$
$\Xi_c^0 \rightarrow \Delta^0 \pi^0$	$-\lambda_c \frac{1}{\sqrt{6}} (E_B - E_M)$	$(1.5 \pm 0.2, 1.4 \pm 0.3)$	$(4.3 \pm 0.7, 1.3 \pm 0.1)$
$\Xi_c^0 \rightarrow \Delta^- \pi^+$	$-\lambda_c E_B$	$(2.7 \pm 0.4, 2.5 \pm 0.4)$	$(6.5 \pm 1.1, 2.0 \pm 0.2)$
$\Xi_c^0 \rightarrow \Delta^0 \eta$	$-\lambda_c \frac{1}{\sqrt{6}} (E_B + E_M) c\phi$	$(0.01 \pm 0.07, 0.01 \pm 0.12)$	–
$\Xi_c^0 \rightarrow \Delta^0 \eta'$	$-\lambda_c \frac{1}{\sqrt{6}} (E_B + E_M) s\phi$	$(0.003 \pm 0.034, 0.003 \pm 0.055)$	–
$\Xi_c^0 \rightarrow \Sigma^{*0} K^0$	$-\lambda_c \frac{1}{\sqrt{6}} E_M^{(s)}$	$(0.2 \pm 0.1, 0.5 \pm 0.1)$	$(0.4 \pm 0.1, 0.3 \pm 0.0)$
$\Xi_c^0 \rightarrow \Sigma^{*-} K^+$	$-\lambda_c \frac{1}{\sqrt{3}} E_B^{(s)}$	$(0.7 \pm 0.1, 0.7 \pm 0.1)$	$(0.9 \pm 0.1, 0.7 \pm 0.1)$
$\Xi_c^+ \rightarrow \Delta^{++} \pi^-$	$-\lambda_c E_M$	$(8.0 \pm 1.5, 7.8 \pm 1.6)$	$(25.5 \pm 4.4, 7.8 \pm 0.7)$
$\Xi_c^+ \rightarrow \Delta^+ \pi^0$	$-\lambda_c \frac{1}{\sqrt{6}} (E_B - E_M)$	$(6.5 \pm 1.0, 6.3 \pm 1.2)$	$(17.0 \pm 2.9, 5.2 \pm 0.4)$
$\Xi_c^+ \rightarrow \Delta^0 \pi^+$	$-\lambda_c \frac{1}{\sqrt{3}} E_B$	$(3.9 \pm 0.6, 3.7 \pm 0.7)$	$(8.5 \pm 1.5, 2.6 \pm 0.2)$
$\Xi_c^+ \rightarrow \Delta^+ \eta$	$-\lambda_c \frac{1}{\sqrt{6}} (E_B + E_M) c\phi$	$(0.03 \pm 0.32, 0.03 \pm 0.52)$	–
$\Xi_c^+ \rightarrow \Delta^+ \eta'$	$-\lambda_c \frac{1}{\sqrt{6}} (E_B + E_M) s\phi$	$(0.01 \pm 0.15, 0.01 \pm 0.24)$	–
$\Xi_c^+ \rightarrow \Sigma^{*+} K^0$	$-\lambda_c \frac{1}{\sqrt{3}} E_M^{(s)}$	$(2.1 \pm 0.4, 4.2 \pm 0.8)$	$(3.5 \pm 0.6, 2.6 \pm 0.2)$
$\Xi_c^+ \rightarrow \Sigma^{*0} K^+$	$-\lambda_c \frac{1}{\sqrt{6}} E_B^{(s)}$	$(1.5 \pm 0.2, 1.5 \pm 0.2)$	$(1.7 \pm 0.3, 1.3 \pm 0.1)$

exchange diagrams, E_B and E_M , could give contributions to the observed branching fractions of $\Lambda_c^+ \rightarrow (\Delta^{++} K^-, \Sigma^{*+} \eta, \Xi^{*0} K^+)$ and $\Xi_c^0 \rightarrow \Omega^- K^+$. In addition, we have predicted $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^{*0(+)} \pi^{(0)}) = (2.8 \pm 0.4) \times 10^{-3}$, which respects the isospin symmetry. To interpret the observation of $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^{*+} \eta)$, we have taken into account the $SU(3)_f$ symmetry breaking. In particular, $\mathcal{B}(\Lambda_c^+ \rightarrow \Delta^{++} \pi^-, \Xi_c^+ \rightarrow \Delta^{++} K^-) = (7.0 \pm 1.4, 13.5 \pm 2.7) \times 10^{-4}$ and $\mathcal{B}(\Xi_c^+ \rightarrow \Delta^{++} \pi^-, \Xi_c^0 \rightarrow \Delta^- \pi^+) = (7.8 \pm 1.6, 2.5 \pm 0.4) \times 10^{-5}$ have been predicted as the largest branching fractions in the SCS Λ_c^+ (Ξ_c^+) and DSC $\Xi_c^{+(0)}$ decay channels, respectively.

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