Electromagnetic transition form factors, $E2/M1$ and $C2/M1$ ratios of the baryon decuplet

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Abstract We investigate the electromagnetic transition form factors of the baryon decuplet to the baryon octet, based on the self-consistent SU(3) chiral quark-soliton model, taking into account the effects of explicit breaking of flavor SU(3) symmetry. We emphasize the Q^2 dependence of the electromagnetic $N \rightarrow \Delta$ transition form factors and the ratios of $E2/M1$ and $C2/M1$ in comparison with the experimental and empirical data. In order to compare the present results of the electromagnetic transition form factors of the $N \rightarrow \Delta$ with those from lattice QCD, we evaluate the form factors with the pion mass deviated from its physical value. The results of the $E2/M1$ and $C2/M1$ ratios are in good agreement with the lattice data. We also present the results of the electromagnetic transition form factors for the decuplet to the octet transitions.

1 Introduction

Understanding how a baryon is shaped electromagnetically has been one of the most important issues in hadronic physics. The EM structure of the baryon decuplet, which contains the first excitations of the nucleon and hyperons, is far from complete understanding. The reason is that it is very difficult to get access to their structures experimentally on account of their ephemeral nature. On the other hand, the EM transition $N\gamma^* \rightarrow \Delta$ can be examined experimentally by using the electroproduction of the pion [1–10]. The EM transitions from the baryon octet to the decuplet are also experimentally accessible [11–14]. Thus, the structure of the baryon decuplet can be investigated by using the EM transition from the baryon octet to the decuplet. Theoretically, the EM transitions of the $N\gamma^* \rightarrow \Delta$ have been extensively studied within

various frameworks over decades: for example, the linear sigma model [15], the Skyrme models [16–19], the chiral bag model [20], the Dyson–Schwinger approaches [21–23], constituent quark models [24], relativistic quark models [25], QCD sum rules [26–29], a πN dynamical quark model [30], lattice QCD [31–34], AdS/QCD [35], chiral perturbation theory [36], chiral effective field theory (χ EFT) [37] and so on.

Since the Δ isobar decays mostly into the nucleon and pion, it has been known that the pion clouds play an essential role in describing the structure of the Δ . For example, the naive nonrelativistic quark model (NRQM) underestimates the $N \rightarrow \Delta$ transition magnetic moment by approximately 30% and predicts wrongly the electric quadrupole transition moment $Q_{p \rightarrow \Delta^+} = 0$, which indicates that the Δ isobar from the NRQM has a completely spherically symmetric shape. However, experimental results indicate that the $N \rightarrow \Delta$ transition electric quadrupole moment is small but still finite. Other members of the baryon decuplet show similar decay modes except for the Ω^- . The first excited Σ^* and Ξ^* hyperons decay into the corresponding octet baryons and the pions. This implies at least two important points about the structure of the baryon decuplet. Firstly, any approach in describing the baryon decuplet should consider the pions seriously. Secondly, chiral symmetry and its spontaneous breakdown should come into play, since the pions are regarded as the pseudo-Nambu–Goldstone (pNG) bosons. Thus, any theoretical works on the baryon decuplet should include the pions as the pNG bosons.

The chiral quark-soliton model (χ QSM) has been developed as a pion mean-field approach for describing the structure of the nucleon [38]. In the large N_c limit [39–41], the nucleon can be viewed as a state of the N_c valence quarks bound by the pion mean fields that are produced by the presence of the N_c valence quarks self-consistently. The model was extended to the description of the lowest-lying SU(3) baryons [42] and was successfully applied to explaining var-

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ious properties of the SU(3) baryons. For details, we refer to the reviews [43, 44] and references therein. The χ QSM was also extended to the description of singly heavy baryons [45–49]. Thus, the model provides a unified pion mean-field approach for both the light and singly heavy baryons. In the present work, we want to investigate the EM transition form factors and related observables from the baryon octet to the decuplet within the framework of the self-consistent SU(3) chiral quark-soliton model (χ QSM). The EM transition $N\gamma \rightarrow \Delta$ was already studied in the χ QSM [50–54]. In Ref. [54] the EM transition form factors for the $N\gamma^* \rightarrow \Delta$ were computed without the effects of the flavor SU(3) symmetry. Moreover, the large N_c argument was used to improve quantitatively the magnetic dipole and electric quadrupole transition moments. However, it is known that the absolute magnitudes of the helicity amplitudes for the $N\gamma \rightarrow \Delta$ excitation based on the χ QSM are underestimated, compared with the experimental and empirical data. The lattice data are also known to be quite smaller than the experimental data. This indicates that we need to study the EM transitions of the baryon decuplet to the octet more in detail. While the χ QSM model underestimates the absolute values for the helicity amplitudes, it provides very interesting results, compared with the lattice data.

In the present work, we will compute all possible EM transitions from the baryon octet to the decuplet with the effects of flavor SU(3) symmetry breaking taken into account. Since radiative decays of the negative decuplet baryons vanish in the exact flavor SU(3) symmetric case due to the U -spin symmetry, it is of great importance to consider the explicit breaking of flavor SU(3) symmetry. For example, the transition form factors for the $\Sigma^- \gamma \rightarrow \Sigma^{*-}$ and $\Xi^- \gamma \rightarrow \Xi^{*-}$ radiative decays have null results without flavor SU(3) symmetry breaking. This means that the linear m_s contributions play the leading role in describing these EM transition form factors. We first examine the $N\gamma^* \rightarrow \Delta$ transition, because the lattice data as well as the experimental data exist. The existing lattice calculation [33] still uses the values of the unphysical pion mass, so that we employ the corresponding values of the pion mass to compare the numerical results with the lattice data. We then present the numerical results of all possible EM transition form factors from the baryon octet to the decuplet, focusing on the explicit breaking of flavor SU(3) symmetry. The results of the $E2/M1$ and $C2/M1$ ratios are compared with the experimental and lattice data.

The present work is organized as follows: In Sect. 2, we briefly explain the definition of the EM transition form factors and helicity amplitudes. In Sect. 3, we describe shortly how the expressions of the EM transition form factors are obtained. In Sect. 4, the numerical results are presented and are compared with the experimental data. We also discuss them in comparison with the lattice data. In the final section, summary and conclusions are given.

Fig. 1 Schematic diagram for the $\gamma^* B_8 \rightarrow B_{10}$ transition

2 EM transition form factors for radiative excitations $B_8 \gamma^* \rightarrow B_{10}$

The radiative excitation from an octet baryon to a decuplet baryon, $B_8 \gamma^* \rightarrow B_{10}$, is schematically shown in Fig. 1 in the rest frame of a decuplet baryon.

In this rest frame, p_{10} , p_8 , and q denote respectively the momenta of a decuplet baryon and an octet one, and the momentum transfer carried by the photon. They are expressed respectively as

$$p_{10} = (M_{10}, \mathbf{0}), \quad p_8 = (E_8, -\mathbf{q}), \quad q = (\omega_q, \mathbf{q}), \quad (1)$$

where $\mathbf{q}$ and ω_q stand for the three-momentum and energy of the virtual photon, respectively. In this rest frame, the energy-momentum relations are given as $E_8^2 = M_8^2 + |\mathbf{q}|^2$ and $E_{10}^2 = M_{10}^2$. Thus, the momentum and energy of the virtual photon are written by

$$|\mathbf{q}|^2 = \left(\frac{M_{10}^2 + M_8^2 + Q^2}{2M_{10}} \right)^2 - M_8^2, \\ \omega_q = \left(\frac{M_{10}^2 - M_8^2 - Q^2}{2M_{10}} \right), \quad (2)$$

where $Q^2 = -q^2 > 0$.

To explain the radiative excitation $B_8 \gamma^* \rightarrow B_{10}$, we need to evaluate the matrix element of the EM current V^μ between B_{10} and B_8 as follows as [56]

$$\langle B_{10}(p_{10}, \lambda') | V^\mu(0) | B_8(p_8, \lambda) \rangle \\ = i \sqrt{\frac{2}{3}} \bar{u}_\beta(p_{10}, 1/2) \Gamma^{\beta\mu} u(p_8, 1/2), \quad (3)$$

where the EM current V^μ is defined as

$$V^\mu(0) = \bar{\psi}(0) \gamma^\mu \hat{Q} \psi(0), \quad (4)$$

with the charge operator given by $\hat{Q} = \text{diag}(2/3, -1/3, -1/3)$. $\lambda(\lambda')$ denotes the helicity of the initial (final) baryon state. $u^\beta(p, \lambda)$ and $u(p, \lambda)$ stand for the Rarita-Schwinger and Dirac spinors, respectively. The baryon states and Dirac spinors are normalized by $\langle B(p', \lambda') | B(p, \lambda) \rangle = (p^0/M) \delta_{\lambda'\lambda} \delta^{(3)}(\mathbf{p}' - \mathbf{p})$ and $\bar{u}(p, \lambda') u(p, \lambda) = \delta_{\lambda'\lambda}$, respectively. $\Gamma^{\beta\mu}$ in Eq. (3) can be expressed in terms of the multipole form factors [56, 57]

$$\Gamma^{\beta\mu} = G_{M1}^*(Q^2)\mathcal{K}_{M1}^{\beta\mu} + G_{E2}^*(Q^2)\mathcal{K}_{E2}^{\beta\mu} + G_{C2}^*(Q^2)\mathcal{K}_{C2}^{\beta\mu}, \quad (5)$$

where G_{M1}^* , G_{E2}^* , and G_{C2}^* are called respectively as the magnetic dipole ($M1$) transition form factor, the electric quadrupole ($E2$) one, and the Coulomb quadrupole ($C2$) one. The corresponding Lorentz tensors $\mathcal{K}_{M1,E2,C2}^{\beta\mu}$ are written as

$$\begin{aligned} \mathcal{K}_{M1}^{\beta\mu} &= \frac{-3(M_{10} + M_8)}{2M_8[(M_{10} + M_8)^2 + Q^2]} \varepsilon^{\beta\mu\sigma\tau} P_\sigma q_\tau, \\ \mathcal{K}_{E2}^{\beta\mu} &= -\mathcal{K}_M^{\beta\mu} \\ &\quad - \frac{6}{4M_{10}^2|q|^2} \frac{M_{10} + M_8}{M_8} \varepsilon^{\beta\sigma\nu\gamma} P_\nu q_\gamma \varepsilon^\mu{}_\sigma{}^{\alpha\delta} p_{10\alpha} q_\delta i\gamma^5, \\ \mathcal{K}_{C2}^{\beta\mu} &= -\frac{3}{4M_{10}^2|q|^2} \frac{M_{10} + M_8}{M_8} q^\beta [q^2 P^\mu - q \cdot P q^\mu] i\gamma^5. \end{aligned} \quad (6)$$

The Lorentz tensors are required to satisfy the identities $q_\mu \mathcal{K}_{M1,E2,C2}^{\beta\mu} = 0$ by conservation of the EM current.

The transition magnetic moment¹ and the transition electric quadrupole moment are defined respectively by [58,59]:

$$\begin{aligned} \mu_{B_8 B_{10}} &= \frac{M_N}{M_8} \sqrt{\frac{M_{10}}{M_8}} G_{M1}^*(0) \mu_N, \\ \mathcal{Q}_{B_8 B_{10}} &= -\frac{6}{M_8} \frac{2M_{10}}{M_{10}^2 - M_8^2} \sqrt{\frac{M_{10}}{M_8}} G_{E2}^*(0), \end{aligned} \quad (7)$$

where μ_N denotes the nuclear magneton defined by $\mu_N = e/2M_N$.

It is also of great interest to examine the helicity amplitudes, since they can be extracted from experimental data. The transverse and Coulomb helicity amplitudes are defined respectively in terms of the spatial and temporal components of the EM current

$$\begin{aligned} A_\lambda &= -\frac{e}{\sqrt{2}\omega_q} \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} \boldsymbol{\epsilon}_{+1} \\ &\quad \cdot \langle B_{10}(3/2, \lambda) | \bar{\psi}(\mathbf{r}) \hat{\mathbf{Q}} \boldsymbol{\gamma} \psi(\mathbf{r}) | B_8(1/2, \lambda - 1) \rangle, \\ S_{1/2} &= -\frac{e}{\sqrt{2}\omega_q} \frac{1}{\sqrt{2}} \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} \\ &\quad \langle B_{10}(3/2, 1/2) | \bar{\psi}(\mathbf{r}) \hat{\mathbf{Q}} \gamma^0 \psi(\mathbf{r}) | B_8(1/2, 1/2) \rangle, \end{aligned} \quad (8)$$

where λ is the corresponding value of the helicity of the decuplet baryon B_{10} , i.e. $\lambda = 3/2$ or $1/2$. The transverse photon polarization vector is defined as $\hat{\epsilon} = -1/\sqrt{2}(1, i, 0)$.

¹ Note that the definition of the magnetic transition moment in the present work is different from that in Ref. [54], where the following approximation was used $\sqrt{M_{10}/M_8} \approx 1 + \mathcal{O}(N_c^{-2})$. In the present work, we strictly follow the definition used in experiments.

The helicity amplitudes can be expressed in terms of the multipole form factors [56,58]

$$\begin{aligned} A_{1/2} &= -\frac{e}{\sqrt{2}\omega_q} \frac{1}{4c_\Delta} (G_{M1}^* - 3G_{E2}^*), \\ A_{3/2} &= -\frac{e}{\sqrt{2}\omega_q} \frac{\sqrt{3}}{4c_\Delta} (G_{M1}^* + G_{E2}^*), \\ S_{1/2} &= \frac{e}{\sqrt{2}\omega_q} \frac{|q|}{4c_\Delta M_{10}} G_{C2}^*. \end{aligned} \quad (9)$$

where $c_\Delta = \sqrt{\frac{M_8^3}{2M_{10}|q|^2}} \sqrt{1 + \frac{Q^2}{(M_{10} + M_8)^2}}$. The multipole form factors can be explicitly expressed as follows:

$$\begin{aligned} G_{M1}^*(Q^2) &= -2c_\Delta \int d^3r 3j_1(|\mathbf{q}||\mathbf{r}|) \\ &\quad \langle B_{10}(3/2, 1/2) | [\hat{\mathbf{r}} \times \mathbf{V}]_{11} | B_8(1/2, -1/2) \rangle, \\ G_{E2}^*(Q^2) &\simeq -2c_\Delta \int d^3r \sqrt{\frac{20\pi}{27}} \frac{\omega_q}{|q|} \left(\frac{\partial}{\partial r} r j_2(|\mathbf{q}||\mathbf{r}|) \right) \\ &\quad \langle B_{10}(3/2, 1/2) | Y_{21}(\hat{\mathbf{r}}) V_0 | B_8(1/2, -1/2) \rangle, \\ G_{C2}^*(Q^2) &= 4c_\Delta \frac{M_{10}}{|q|} \int d^3r \sqrt{10\pi} j_2(|\mathbf{q}||\mathbf{r}|) \\ &\quad \langle B_{10}(3/2, 1/2) | Y_{20}(\hat{\mathbf{r}}) V_0 | B_8(1/2, 1/2) \rangle. \end{aligned} \quad (10)$$

Note that we neglect a term that gives a tiny correction to $E2$ at low-energy region and implements the current conservation on that multipole form factor [15,60]. Once we have evaluated the form factors, the well-known ratios R_{EM} and R_{SM} , which are defined respectively as

$$\begin{aligned} R_{EM}(Q^2) &= -\frac{G_{E2}^*(Q^2)}{G_{M1}^*(Q^2)}, \\ R_{SM}(Q^2) &= -\frac{|q|}{2M_{10}} \frac{G_{C2}^*(Q^2)}{G_{M1}^*(Q^2)}, \end{aligned} \quad (11)$$

can be obtained. The decay width in terms of the helicity amplitudes is expressed as [61]:

$$\Gamma(B_{10} \rightarrow B_8 \gamma) = \frac{\omega_q^2}{\pi} \frac{M_8}{2M_{10}} (|A_{1/2}|^2 + |A_{3/2}|^2). \quad (12)$$

3 Expressions of the EM transition form factors

We start from the effective chiral action $S_{\text{eff}}[U]$ given by

$$S_{\text{eff}}[U] = -N_c \text{Tr} \ln D(U), \quad (13)$$

where N_c is the number of colors and $D(U)$ represents the Dirac operator defined by

$$D(U) = i\not{D} + iMU^{\gamma_5} + i\hat{m}. \quad (14)$$

The U^{γ_5} denotes the chiral field, which is written as

$$U^{\gamma_5} = U \frac{1 + \gamma_5}{2} + U^\dagger \frac{1 - \gamma_5}{2}. \quad (15)$$

The U field is represented by

$$U(x) = \exp[i\pi^a(x)\lambda^a], \quad (16)$$

where $\pi^a(x)$ stand for the pNG fields and λ^a designate the Gell–Mann matrices in flavor SU(3). Since the classical pion fields have a hedgehog symmetry, we keep only the pNG fields for $a = 1, 2, 3$ whereas all other fields are set equal to zero. Then the static SU(3) soliton field can be constructed by Witten's trivial embedding

$$U(\mathbf{r}) = \begin{pmatrix} U_{\text{SU}(2)}(\mathbf{r}) & 0 \\ 0 & 1 \end{pmatrix}, \quad (17)$$

where the $U_{\text{SU}(2)}$ is expressed by incorporating the hedgehog symmetry as

$$U_{\text{SU}(2)}(\mathbf{r}) = \exp[i\boldsymbol{\pi} \cdot \boldsymbol{\tau}] = \exp[iP(\mathbf{r})\hat{\mathbf{r}} \cdot \boldsymbol{\tau}]. \quad (18)$$

The soliton profile function $P(r)$ is derived by minimizing the energy of the soliton. $\hat{m}$ in Eq. (14) stands for the current-quark mass matrix. Assuming isospin symmetry, i.e., $m_u = m_d = \bar{m}$, we can express it as $\hat{m} = \text{diag}(\bar{m}, \bar{m}, m_s) = \bar{m} + \delta m$. The δm is defined by

$$\delta m = \frac{-\bar{m} + m_s}{3} \mathbf{1}_3 + \frac{\bar{m} - m_s}{\sqrt{3}} \lambda^8 = m_1 \mathbf{1}_3 + m_8 \lambda^8, \quad (19)$$

and will be treated as a small perturbation. m_1 and m_8 are respectively written as $m_1 = (-\bar{m} + m_s)/3$, $m_8 = (\bar{m} - m_s)/\sqrt{3}$. To find the single-quark energies and states, we can define the Dirac Hamiltonian $h(U)$ as

$$h(U) = i\gamma_4 \gamma_i \partial_i - \gamma_4 M U^{\gamma_5} - \gamma_4 \bar{m}. \quad (20)$$

In the large N_c limit, the $1/N_c$ pion-loop fluctuations can be neglected. Thus, the classical equation of motion gives rise to the pion mean fields, which is derived by $\delta S_{\text{eff}}/\delta P(\mathbf{r}) = 0$. However, we have to consider the zero modes of the soliton completely, since they are not at all small. This is known as the zero-mode collective quantization, which can be achieved by rotating and translating the soliton as follows:

$$A(t)U_c(\mathbf{r} - \mathbf{z}(t))A^\dagger(t). \quad (21)$$

Here, $A(t)$ is a time-dependent SU(3) matrix in the flavor space and $\mathbf{z}(t)$ represents a time-dependent translation of the soliton. The rotational velocity is defined as

$$\Omega = \frac{1}{2i} \text{Tr}(A^\dagger \dot{A} \lambda^\alpha) \lambda^\alpha = \frac{1}{2} \Omega_\alpha \lambda^\alpha, \quad (22)$$

which is of order $1/N_c$. Assuming that the soliton rotates slowly, we can deal with Ω_α as a perturbative parameter. The translational corrections are taken to the zeroth order. Note that the translational zero modes naturally provides the Fourier transform for the EM transition form factors. Having performed the zero-mode quantization, we arrive at the collective Hamiltonian, which consists of the flavor SU(3) symmetric and symmetry-breaking terms, given as

$$H_{\text{coll}} = H_{\text{sym}} + H_{\text{sb}}, \quad (23)$$

with

$$H_{\text{sym}} = M_{\text{sol}} + \frac{1}{2I_1} \sum_{i=1}^3 \hat{J}_i \hat{J}_i + \frac{1}{2I_2} \sum_{a=4}^7 \hat{J}_a \hat{J}_a$$

$$H_{\text{sb}} = \alpha D_{88}^{(8)}(A) + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} D_{8i}^{(8)}(A) \hat{J}_i. \quad (24)$$

M_{sol} stands for the classical soliton mass that is obtained by the minimization procedure. $D_{ab}^{(\mathcal{R})}(A)$ and $\hat{J}$ denote the SU(3) Wigner D functions and the angular momentum operators. The explicit expressions for the moments of inertia (I_1, I_2) and the inertial parameters (α, β, γ) are given in Appendix B.

In the present work, we will present here only the final expressions of the EM transition form factors, since the general formalism can be found in previous works. For a detailed calculation, we refer to a recent work on the EM form factors of the baryon decuplet [62] and a review [43]. Having taking into account the rotational $1/N_c$ and linear m_s corrections, we obtain the magnetic dipole form factor G_{M1}^* as

$$G_{M1}^{B_8 \rightarrow B_{10}^*}(Q^2) = -c_\Delta \int d^3r \frac{6}{\sqrt{2}} j_1(|\mathbf{q}||\mathbf{r}|) \mathcal{G}_{M1}^{B_8 \rightarrow B_{10}}(\mathbf{r}), \quad (25)$$

where the corresponding magnetic dipole density $\mathcal{G}_{M1}^{B_8 \rightarrow B_{10}}(\mathbf{r})$ is defined as

$$\mathcal{G}_{M1}^{B_8 \rightarrow B_{10}}(\mathbf{r}) = \left(\mathcal{Q}_0(\mathbf{r}) + \frac{1}{I_1} \mathcal{Q}_1(\mathbf{r}) \right) \langle B_{10} | D_{Q3}^{(8)} | B_8 \rangle$$

$$- \frac{1}{\sqrt{3}} \frac{1}{I_1} \mathcal{X}_1(\mathbf{r}) \langle B_{10} | D_{Q8}^{(8)} J_3 | B_8 \rangle$$

$$- \frac{1}{I_2} \mathcal{X}_2(\mathbf{r}) \langle B_{10} | d_{pq3} D_{Qp}^{(8)} J_q | B_8 \rangle$$

$$\begin{aligned}
& + \frac{2}{\sqrt{3}} m_8 \left(\frac{K_1}{I_1} \mathcal{X}_1(\mathbf{r}) - \mathcal{M}_1(\mathbf{r}) \right) \\
& \times \langle B_{10} | D_{83}^{(8)} D_{Q8}^{(8)} | B_8 \rangle \\
& + 2m_8 \left(\frac{K_2}{I_2} \mathcal{X}_2(\mathbf{r}) - \mathcal{M}_2(\mathbf{r}) \right) \\
& \times \langle B_{10} | d_{pq3} D_{8p}^{(8)} D_{Qq}^{(8)} | B_8 \rangle \\
& - 2m_1 \mathcal{M}_0(\mathbf{r}) \langle B_{10} | D_{Q3}^{(8)} | B_8 \rangle \\
& - \frac{2}{\sqrt{3}} m_8 \mathcal{M}_0(\mathbf{r}) \langle B_{10} | D_{88}^{(8)} D_{Q3}^{(8)} | B_8 \rangle. \quad (26)
\end{aligned}$$

The explicit expressions for the densities $\mathcal{Q}_i$, $\mathcal{X}$, and $\mathcal{M}_i$ can be found in Appendix A. The results on the matrix elements of collective operators $\langle B_{10} | \dots | B_8 \rangle$ are given in Appendix B. The expression of the electric quadrupole form factor is given as

$$\begin{aligned}
G_{E2}^{B_8 \rightarrow B_{10}^*}(Q^2) \\
= c_\Delta \int d^3r \sqrt{\frac{10}{9}} \frac{\omega}{|q|} \left(\frac{\partial}{\partial r} r j_2(|q||r|) \right) \mathcal{G}_{E2}^{B_8 \rightarrow B_{10}}(\mathbf{r}), \quad (27)
\end{aligned}$$

with the corresponding density $\mathcal{G}_{E2}^{B_8 \rightarrow B_{10}}(\mathbf{r})$

$$\begin{aligned}
\mathcal{G}_{E2}^{B_8 \rightarrow B_{10}}(\mathbf{r}) = & -\frac{2}{I_1} \mathcal{I}_{1E2}(\mathbf{r}) \\
& \times \left(3 \langle B_{10} | D_{Q3}^{(8)} J_3 | B_8 \rangle - \langle B_{10} | D_{Qi}^{(8)} J_i | B_8 \rangle \right) \\
& + 4m_8 \left(\frac{K_1}{I_1} \mathcal{I}_{1E2}(\mathbf{r}) - \mathcal{K}_{1E2}(\mathbf{r}) \right) \\
& \times \left(3 \langle B_{10} | D_{83}^{(8)} D_{Q3}^{(8)} | B_8 \rangle - \langle B_{10} | D_{8i}^{(8)} D_{Qi}^{(8)} | B_8 \rangle \right). \quad (28)
\end{aligned}$$

The explicit expressions for $\mathcal{I}_{1E2}(\mathbf{r})$ and $\mathcal{K}_{1E2}(\mathbf{r})$ can be found in Appendix A. The Coulomb quadrupole form factor $G_{C2}^{B_8 \rightarrow B_{10}^*}$ is written as

$$G_{C2}^{B_8 \rightarrow B_{10}^*}(Q^2) = c_\Delta \sqrt{40} \int d^3r \frac{M_{10}}{|q|} j_2(|q||r|) \mathcal{G}_{C2}^{B_8 \rightarrow B_{10}}(\mathbf{r}), \quad (29)$$

where $\mathcal{G}_{C2}^{B_8 \rightarrow B_{10}}(\mathbf{r})$ is simply the same as $\mathcal{G}_{E2}^{B_8 \rightarrow B_{10}}(\mathbf{r})$.

It is more convenient to decompose the densities into three different terms

$$\begin{aligned}
\mathcal{G}_{(M1, E2, C2)}^{B_8 \rightarrow B_{10}}(\mathbf{r}) = & \mathcal{G}_{(M1, E2, C2)}^{B_8 \rightarrow B_{10}(0)}(\mathbf{r}) + \mathcal{G}_{(M1, E2, C2)}^{B_8 \rightarrow B_{10}(\text{op})}(\mathbf{r}) \\
& + \mathcal{G}_{(M1, E2, C2)}^{B_8 \rightarrow B_{10}(\text{wf})}(\mathbf{r}). \quad (30)
\end{aligned}$$

The first term represents the SU(3)-symmetric ones including both the leading and rotational $1/N_c$ terms, the second one

denotes the linear m_s corrections arising from the current-quark mass term of the effective chiral action. The last term is originated from the collective wave functions. If the effects of the flavor SU(3) symmetry breaking are considered, a collective baryon wave function is not any longer in a pure state but a state mixed with higher representations, as shown in Eq. (B2). Thus, there are two different terms that provide the effects of flavor SU(3) symmetry breaking. The explicit expressions of these three terms are given in Appendix C.

Finally, we want to mention that while the leading-order contributions in the rotational $1/N_c$ expansion vanish for the $E2$ transition form factor because of the hedgehog ansatz in the present approach. This means that the rotational $1/N_c$ corrections take a role of the leading-order contributions. Moreover, we have only the single rotational $1/N_c$ term that contains the density $\mathcal{I}_{1E2}(\mathbf{r})$. The corresponding expression can be found in Eq. (A2) in Appendix A. Similarly, the $C2$ form factors does not get any contribution from the leading-order term.

4 Results and discussion

The parameters in the χ QSM except for the dynamical quark mass were already fixed by reproducing properties of the pion. Since the contributions of the sea quarks need to be regularized, the cutoff mass Λ should be introduced. This is fixed by reproducing the pion decay constant, $f_\pi = 93$ MeV. the average mass of the up and down current quarks $\bar{m}$ is determined by the physical pion mass $m_\pi = 139$ MeV. While the dynamical quark mass M can be considered as a free parameter, it is also fixed by describing the electric form factor of the proton. Once we fix all these parameters, we compute various observables including both the lowest-lying light and singly heavy baryons. Therefore, we do not have any room to fit the parameters in the present calculation.

It is already known from previous investigations [51, 52, 54] that the magnitudes of the EM transition form factors of the Δ are rather underestimated, compared with the experimental data while the $E2/M1$ and $C2/M1$ ratios are well described. There are several reasons why it is so. In fact, the pion-loop effects come into essential play in explaining the nature of the Δ isobar, since it decays strongly into the πN . Moreover, the Δ isobar has a rather broad width, so that the corresponding wavefunction should contain such information arising from this broad width. In Ref. [63] the strong decay widths of the baryon decuplet were scrutinized based on the χ QSM in a model-independent approach, where all the dynamical parameters were fixed by experimental data without calculating them self-consistently. While the strong decay width of the Δ is still underestimated, the widths of all the other members of the baryon decuplet are in good agreement with the experimental data. It implies that one should

go beyond the pion mean-field approximation to describe the strong properties of the Δ baryon quantitatively. Since it is rather difficult to take into account the pion-loop corrections beyond the pion mean-field approximation in the present framework, we will consider the Q^2 dependence of the form factors and the ratios of $E2/M1$ and $C2/M1$. These effects beyond the mean fields may be cancelled in the calculation of these ratios. Note that the lattice calculations also have similar problems in reproducing the experimental data.

We first discuss the results of the $N \rightarrow \Delta$ EM transition form factors, focusing on the Q^2 dependence of the form factors. In order to compare the Q^2 dependence of the present results with the experimental and empirical data, we normalize the values of the form factors at $Q^2 = 0.06 \text{ GeV}^2$, using the experimental data on the helicity amplitudes by the A1 Collaboration [3]. We explicitly multiply the present values of the $M1$, $E2$, and $C2$ form factors by 1.82, 3.13, and 3.18, respectively. The upper left panel of Fig. 2 draws the result of the magnetic dipole transition form factor as a function of Q^2 with the strange current quark mass taken to be $m_s = 180 \text{ MeV}$. We take the experimental and empirical data from Refs. [1, 3–6]. The present result seems to fall off more slowly than those of the empirical and experimental data, as Q^2 increases. However, the result is in agreement with the data. In the upper right panel of Fig. 2 we show the result of the electric quadrupole form factor for the $N \rightarrow \Delta$ EM transition as a function of Q^2 . The result exhibits different Q^2 dependence from that of the $M1$ form factor. It falls off faster than the empirical and experimental data. As we will discuss explicitly later, the present results of the $E2/M1$ ratio deviate from the experimental data because of this Q^2 dependence of the $E2$ form factor. Actually, one can understand this Q^2 behavior of the present results. Since the $E2$ transition form factor is proportional to ω_q within this model expression, it is strongly suppressed when $\omega_q(Q^2) = 0$, which corresponds approximately to $Q^2 \simeq 0.6 \text{ GeV}^2$ for the $N\gamma^* \rightarrow \Delta$ excitation. That explains why the $E2$ form factor decreases drastically as Q^2 increases. On the other hand, The result of the Coulomb form factor describes relatively well the experimental data, as shown in the lower panel of Fig. 2.

In order to compare the present results with the lattice data [34], we have to compute the EM transition form factors, employing the values of the unphysical pion mass, which were used by Ref. [34]. To do that, we have to derive the solutions of the pion mean fields with specific values of the unphysical pion mass, i.e. $m_\pi = 297 \text{ MeV}$ and 353 MeV . Then we can compute the EM transition form factors, using these solutions with the values of m_π . In fact, Goeke et al. [64] showed that the stable mean-field soliton still exists in the wide range of the pion mass $0 \leq m_\pi \leq 1500 \text{ MeV}$. It indicates that the results from the χ QSM can be directly compared with those from lattice QCD, the unphysical pion mass being employed. They indeed described remarkably the

mass of the nucleon in comparison with the lattice data. The same method was extended to the description of the energy-momentum form factors of the nucleon [65] and the EM form factors of singly heavy baryons [66]. Thus, we compare the present results with the lattice data, using the values of the unphysical pion mass employed by the lattice calculation.

The upper left panel of Fig. 3 illustrates the results of the $M1$ transition form factors as functions of Q^2 . The solid curve depicts the original values of the $M1$ form factors with the physical pion mass, whereas the long-dashed and short-dashed ones draw those obtained by using $m_\pi = 297 \text{ MeV}$ and 353 MeV , respectively. The sizes of the form factors get smaller as the value of m_π increases. Moreover, the $M1$ form factors fall off more slowly as the pion mass increases, which are well-known behaviors in the lattice results of the EM form factors of the proton and Δ [67–70]. While the general Q^2 dependence of results of the $M1$ transition form factor is similar to those of the lattice calculation, the present results decrease still faster than the lattice ones as Q^2 increases. In the upper right panel of Fig. 3 we represent the results of the $E2$ transition form factors with the pion mass varied as in the case of the $M1$ form factor. The sizes of the $E2$ form factor are drastically diminished by increasing the value of the pion mass. This tendency was already seen in the $E2$ form factors of the Δ and Ω^- in Ref. [62]. As shown in the upper right panel of Fig. 3, the present results with larger pion masses are in better agreement with the lattice data in the lower Q^2 region ($Q^2 \lesssim 0.5 \text{ GeV}^2$). In the lower panel of Fig. 3, we depict the results of the $C2$ transition form factor. As the pion mass increases, the sizes of the $C2$ one decreases as in the case of the $E2$ form factor. However, the results get underestimated compared with the lattice data in the lower Q^2 region.

The left and right panels of Fig. 4 show respectively the results of the $E2/M1$ and $C2/M1$ ratios for the $N\gamma^* \rightarrow \Delta$ excitation as functions of Q^2 , being compared with the experimental and empirical data [1, 4, 6–10] as well as those of the lattice calculation [34]. The results for the $E2/M1$ ratios are in qualitatively good agreement with the experimental data near $Q^2 \approx 0$, the present ones fall off faster than the experimental and empirical data. This arises from the Q^2 dependence of the $E2$ transition form factors, for which the results decrease much faster than those of the $M1$ form factors. On the other hand, the results for the $C2/M1$ form factors are more or less in agreement with the data. Note that the lattice data on the $C2/M1$ are underestimated in comparison with the experimental data. This may be due to the unphysical values of the pion mass used in the lattice calculations.

Since larger unphysical values of the pion mass are taken in lattice calculations, it is of great importance to employ the same values of the pion mass used in lattice QCD to make a quantitative comparison with the lattice data. Moreover, it is also very interesting to examine the dependence of the

Fig. 2 The normalized results for the magnetic dipole transition form factor for the $N \rightarrow \Delta$ EM transition as functions of Q^2 with the strange current quark mass m_s taken to be 180 MeV. In the upper left and right panels, we draw the results of the magnetic dipole and electric quadrupole transition form factors respectively, whereas in the lower panel, we depict those of the Coulomb quadrupole form factor. They are compared with the experimental and empirical data. We normalize

the numerical results by the experimental data at $Q^2 = 0.06 \text{ GeV}^2$, i.e. the $M1$ form factors by 1.82, the $E2$ ones by 3.13, and $C2$ ones by 3.18. The solid curve illustrates the result of the EM transition form factors with $m_s = 180 \text{ MeV}$. The result is normalized by the data from Ref. [3], i.e. by the factor of 1.82. The red triangle denotes the data taken from Ref. [6], the black circles from Ref. [3], the brown square from Ref. [4], the blue squares from Ref. [1], and the violet triangle from Ref. [5]

numerical results on the pion mass. In Fig. 5, we show the present results for R_{EM} and R_{SM} at $Q^2 = 0.1 \text{ GeV}^2$ with the pion mass squared varied from 0 to 0.26 GeV^2 . As the pion mass increases, the numerical results for R_{EM} decrease mildly. Those for R_{SM} show a very interesting feature. As m_π^2 decreases, the present results get closer to the lattice data. The results from chiral effective field theory (χ EFT) behave very differently from both the present and lattice results, which show very strong dependence on the pion mass.

The ratios R_{EM} and R_{SM} ratios for all the members of the baryon decuplet have not been much investigated. The R_{EM} is an only known ratio experimentally [6, 61]. Moreover, there are no experimental data and are few theoretical results on the $C2/M1$ ratios for the whole baryon decuplet. In Table 1, we

list the numerical results of the $E2/M1$ (R_{EM}) and $C2/M1$ (R_{SM}) ratios at $Q^2 = 0$ in comparison with those from other models. The second and fourth columns list the results of the R_{EM} without and with the effects of flavor SU(3) symmetry breaking. Comparing the results in these two columns with each other, we find that the contributions of the m_s corrections seem to be not at all small. However, one has to keep in mind that the effects of flavor SU(3) symmetry breaking to the $M1$ form factors are smaller than to the $E2$ and $C2$ form factors, as we will show later explicitly. Thus, the effects of flavor SU(3) symmetry breaking apparently look amplified. We want to mention that while the $M1$, $E2$, and $C2$ form factors for the EM $\Sigma^- \rightarrow \Sigma^{*-}$ and $\Xi^- \rightarrow \Xi^{*-}$ transitions vanish in exact flavor SU(3) symmetry due to the U -spin

Fig. 3 The normalized results for the magnetic dipole transition form factor for the $N \rightarrow \Delta$ EM transition as a function of Q^2 with the strange current quark mass m_s taken to be 180 MeV. In the upper left and right panels, we draw the results of the magnetic dipole and electric quadrupole transition form factors respectively, whereas in the lower panel, we depict those of the Coulomb quadrupole form factor. They are compared with the lattice data taken from Ref. [34]. We normalize

the numerical results by the lattice data with $m_\pi = 297$ MeV, i.e. the $M1$ form factors by 1.51, the $E2$ ones by 6.93, and $C2$ ones by 1.57. The solid curve illustrates the result of the EM transition form factors with the value of the physical pion mass, the long-dashed one shows those with $m_\pi = 297$ MeV, and the short-dashed one draws those with $m_\pi = 353$ MeV. The “+” and “x” symbols denote respectively the lattice data with $m_\pi = 353$ MeV and $m_\pi = 297$ MeV

symmetry, the ratios R_{EM} and R_{SM} do not vanish. The reason can be easily understood by examining Eqs. (C1) and (C4), which are the SU(3) symmetric leading contributions to the $M1$ and $E2$ form factors, respectively. The matrix elements of the collective operators for both the $M1$ and $E2$ form factors have basically the same structures, so that the ratios of these form factors are proportional to the ratios of the densities given in terms of Q_0 and so on. Therefore, even though form factors for the $\Sigma^- \rightarrow \Sigma^{*-}$ and $\Xi^- \rightarrow \Xi^{*-}$ photo-transitions vanish in exact flavor SU(3) symmetry, the ratios R_{EM} and R_{SM} turn out finite.

The present value of R_{EM} for the $N\gamma \rightarrow \Delta$ is underestimated by about 20% in comparison with the experimental data. This discrepancy may be overcome by going beyond

the pion mean-field approximation, as was hinted by the results of χ PT. The results of R_{EM} for all decuplet hyperons are comparable with those of chiral perturbation theory (χ PT) [36] except for those of R_{EM} for the $\Sigma^-\gamma \rightarrow \Sigma^{*-}$ and $\Xi^-\gamma \rightarrow \Xi^{*-}$ excitations. Interestingly, both channels are forbidden by the U -spin symmetry. On the other hand, the results from the chiral constituent quark model [71] are overall larger than the present ones. The results for the $\Sigma^-\gamma \rightarrow \Sigma^{*-}$ and $\Xi^-\gamma \rightarrow \Xi^{*-}$ transitions from the QCD sum rules [29] are very large, compared with those of the present work.

The EM transition form factors should comply with the U -spin symmetry in the exact flavor-SU(3) symmetric case. The U -spin symmetry is inherited in Eqs. (C1) and (C4) as

Fig. 4 The numerical results of the $E2/M1$ and $C2/M1$ ratios, i.e., $R_{EM}(Q^2)$ and $R_{SM}(Q^2)$ for the $N\gamma^* \rightarrow \Delta$ excitation in the left and right panels, respectively, without any normalization. The solid curves draw the results with the effects of flavor SU(3) symmetry breaking

whereas the dashed ones depict those in exact SU(3) symmetry. The results are compared with the experimental and empirical data taken from Refs. [1, 4, 6–10] as well as the lattice data taken from [34]

Fig. 5 The results for R_{EM} and R_{SM} given in percentage with the pion mass squared varied from 0 to 0.26 GeV^2 . The stars (*) denote the present results whereas the inverted triangles (▼) stand for the lattice

data. The long-dashed curves illustrate the results from chiral effective theory (χ EFT). The lattice data are taken from Ref. [32], and the results from χ EFT are taken from Ref. [37]

it should be. The U -spin relations for the magnetic transition moments were given in Refs. [53, 72]. In particular, the magnetic transition form factors for the negatively charged decuplet baryons should vanish in exact flavor SU(3) symmetry, which one can easily see from Eqs. (C1). Some years ago, the SELEX Collaboration measured the upper limit of the partial width for the radiative decay of Σ^{*-} , which is given as $\Gamma(\Sigma^{*-} \rightarrow \Sigma^- \gamma) < 9.5 \text{ keV}$. It indicates that the corresponding magnetic transition moment should satisfy the upper limit $|\mu_{\Sigma^{*-} \Sigma^-}| < 0.82 \mu_N$ [53]. Thus, the experimen-

tal data can provide a clue as to how much the U -spin symmetry is broken in the case of the EM transitions for the baryon decuplet.

As mentioned previously, it has been known that the effects of the pion clouds play an essential role in describing the structure of the Δ isobar [37]. In particular, they contribute noticeably to the $E2$ form factor of Δ . Thus, it is of great importance to consider the pion clouds in explaining a deformed shape of the Δ . It was already shown in the χ QSM that the $E2$ form factor of the Δ is mainly governed by the

Table 1 The R_{EM} and R_{SM} on $B_8\gamma \rightarrow B_{10}$ within the chiral quark-soliton model with and without flavor SU(3) symmetry breaking in comparison those from Experimental data [6], Skyrmemodel [19], Linear sigma model (LSM) [15], non-relativistic quark model(NQM) [24], QCD sum rule (QCDSR) [29], chiral constituent quark model (χ CQM) [71] and chiral perturbation theory (χ PT) [36]

[%]	χ QSM($m_s = 0$ MeV)		χ QSM($m_s = 180$ MeV)		PDG [61]	Skyrme [19]	LSM [15]	NQR [24]	QCDSR [29]	χ CQM [71]	χ PT [36]
	R_{EM}	R_{SM}	R_{EM}	R_{SM}							
$p\gamma \rightarrow \Delta^+$	-1.7	-2.2	-1.8	-2.3	-2.5	-2.1	-1.8	-3.5	-1.7	-3.7	-2.5
$n\gamma \rightarrow \Delta^0$	-1.7	-2.2	-1.8	-2.3	-2.5	-2.1	-1.8	-3.5	-1.7	-3.7	-2.5
$\Sigma^+\gamma \rightarrow \Sigma^{*+}$	-1.5	-1.7	-1.0	-1.1	-	-1.2	-	-	-2.9	-2.9	-1.1
$\Sigma^0\gamma \rightarrow \Sigma^{*0}$	-1.5	-1.7	-0.9	-0.9	-	-1.0	-	-	-2.3	-2.3	-0.9
$\Sigma^-\gamma \rightarrow \Sigma^{*-}$	-1.5	-1.7	-2.5	-3.0	-	-1.9	-	-	-8.0	-5.5	3.7
$\Xi^0\gamma \rightarrow \Xi^{*0}$	-1.6	-1.8	-2.1	-2.5	-	-1.2	-	-	-1.6	-1.3	-0.9
$\Xi^-\gamma \rightarrow \Xi^{*-}$	-1.6	-1.8	0.9	1.5	-	-2.0	-	-	-12.4	-2.8	-3.9
$\Lambda^0\gamma \rightarrow \Sigma^{*0}$	-1.7	-2.1	-2.0	-2.5	-	-1.8	-	-	-4.6	-2.0	-1.8

contributions to from pion clouds or the polarized sea quarks in lower Q^2 regions. This implies that the effects of the polarized sea quarks will be also dominant on the $E2$ transition form factors of the baryon decuplet. In Figs. 6 and 7, we draw the results for the $M1$, $E2$, and $C2$ transition form factors of the baryon decuplet with the valence- and sea-quark contributions separated. As shown in Fig. 6, the sea-quark effects contribute to the $M1$ transition form factors by about (20–30)%. However, when it comes to the $E2$ and $C2$ transition form factors, the sea-quark effects are dominant over those of the valence quarks in lower Q^2 regions for all possible radiative transitions. The sea-quark contributions fall off in general drastically as Q^2 increases, compared with those of the valence quarks. So, the valence-quark contributions take over those of the sea quarks in higher Q^2 regions. Thus, we conclude that the sea-quark contributions or the effects of the pion clouds play indeed a major role in explaining how the decuplet baryons are deformed.

Figure 9 draws the results of the magnetic dipole transition form factors without and with the effects of flavor SU(3) symmetric breaking. The solid curves depict those with the linear m_s corrections whereas the dashed ones exhibit those in the exact SU(3) symmetric case. One can see that the effects of flavor SU(3) symmetry breaking contribute to the $M1$ form factors in general below 10%, which is in agreement with the quark-model prediction [73]. Note that they have almost a negligible contribution to the $\Xi\gamma^* \rightarrow \Xi^*$ transition. However, when it comes to the EM transitions for the negatively charged decuplet hyperons, the linear m_s terms take a leading role, since the flavor-SU(3) symmetric contributions vanish because of the U -spin symmetry. The magnitudes of these forbidden transition form factors lie below the upper limit imposed by the SELEX experiment. In Figs. 10 and 11, we draw respectively the $E2$ and $C2$ transition form factors of all the hyperons of the baryon decuplet. Since the densities for the $E2$ and $C2$ form factors are in fact the same each other, the general behaviors of these form factors are

very similar, as shown in Figs. 10 and 11. Except for the $N\gamma^* \rightarrow \Delta$ transition, the effects of flavor SU(3) symmetry breaking are noticeable. In particular, when it comes to the $\Sigma^0\gamma \rightarrow \Sigma^{*0}$ transition, the contributions of flavor SU(3) symmetry breaking are of almost the same order as the SU(3)-symmetric term. However, as we mentioned already in the previous section, the $E2$ and $C2$ transition form factors do not have any leading-order contributions. It means that the rotational $1/N_c$ correction plays a role of the leading-order contribution. While the linear m_s corrections should be usually smaller than the leading-order contributions as in the case of the $M1$ transition form factor, they become rather important when the leading-order contributions vanish. A typical example can be found in the calculation of the singlet axial-vector charge [74] for which the leading-order contribution disappears too. Thus, the linear m_s corrections come into significant play, when the $E2$ and $C2$ transition form factors are discussed.

As already mentioned previously, the magnitudes of the radiative decay rates for the baryon decuplet based on the χ QSM are quite underestimated. However, their ratios are still interesting. For example, the results of the ratios for some decay widths are given as

$$\begin{aligned}
 \frac{\Gamma^{\Delta \rightarrow N\gamma}}{\Gamma^{\Sigma^* \rightarrow \Sigma\gamma}} &= 3.27 \text{ (Exp: 2.64),} \\
 \frac{\Gamma^{\Delta \rightarrow N\gamma}}{\Gamma^{\Sigma^* \rightarrow \Lambda^0\gamma}} &= 1.50 \text{ (Exp: 1.40),} \\
 \frac{\Gamma^{\Sigma^* \rightarrow \Lambda^0\gamma}}{\Gamma^{\Sigma^* \rightarrow \Sigma\gamma}} &= 2.18 \text{ (Exp: 1.88).}
 \end{aligned} \tag{31}$$

Thus, the present results for these ratios are in qualitative agreement with the data.

Fig. 6 Results for the magnetic dipole transition form factors of all the other members of the baryon decuplet with the valence and sea-quark contributions separated. The long-dashed curves draw the valence-

quark (level-quark) contributions whereas the short-dashed ones depict the sea-quark (continuum) contributions. The solid curves show the total results

Fig. 7 Results for the electric quadrupole transition form factors of all the other members of the baryon decuplet with the valence and sea-quark contributions separated. The long-dashed curves draw the valence-

quark (level-quark) contributions whereas the short-dashed ones depict the sea-quark (continuum) contributions. The solid curves show the total results

Fig. 8 Results for the Coulomb quadrupole transition form factors of all the other members of the baryon decuplet with the valence and sea-quark contributions separated. The long-dashed curves draw

the valence-quark (level-quark) contributions whereas the short-dashed ones depict the sea-quark (continuum) contributions. The solid curves show the total results

Fig. 9 Results for the $M1$ transition form factors of all the other members of the baryon decuplet with and without the effects of flavor $SU(3)$ symmetry breaking. The solid curves depict the results with $m_s = 180$ MeV, whereas the dashed ones draw those in exact flavor $SU(3)$ symmetry

Fig. 10 Results for the $E2$ transition form factors of all the other members of the baryon decuplet with and without the effects of flavor $SU(3)$ symmetry breaking. The solid curves depict the results with $m_s = 180$ MeV, whereas the dashed ones draw those in exact flavor $SU(3)$ symmetry

Fig. 11 Results for the C_2 transition form factors of all the other members of the baryon decuplet with and without the effects of flavor $SU(3)$ symmetry breaking. The solid curves depict the results with $m_s = 180$ MeV, whereas the dashed ones draw those in exact flavor $SU(3)$ symmetry

5 Summary and conclusions

In the present work, we aimed at investigating the electromagnetic transition form factors of the baryon decuplet within the framework of the self-consistent SU(3) chiral quark-soliton model, taking into account the effects of flavor SU(3) symmetry breaking. We emphasized how the electromagnetic transition form factors depend on the momentum transfer squared, comparing the corresponding results with both the experimental and lattice data. The present results of the Q^2 dependence of the magnetic dipole and Coulomb quadrupole form factors are more or less well reproduced in comparison with the experimental and empirical data. On the other hand, the results of the electric quadrupole form factors fall off faster than the data. In order to compare the present results of the form factors with the lattice data, we employed the values of the unphysical pion mass, i.e. $m_\pi = 297$ MeV and 353 MeV, so that we are directly able to compare the results with the lattice data. In addition, we normalized the present results with the lattice data at $Q^2 = 0.06 \text{ GeV}^2$ such that we can see how the Q^2 dependences are different from those of the lattice calculation. The form factors fall off more slowly as the pion mass increases. Moreover, the magnitudes of the $E2$ and $C2$ form factors are much reduced by using the unphysical pion masses. We then computed the $E2/M1$ and $C2/M1$ ratios as functions of Q^2 . The $E2/M1$ ratios fall off faster than the experimental data. On the other hand, the results of the Q^2 dependence of the $C2/M1$ ratios are in good agreement with the experimental data.

We have examined the valence- and sea-quark contributions separately. While the sea-quark effects, which can be regarded as those of the pion clouds, contribute to the $M1$ transition form factors by about (20–30)%, they are dominant over those of the valence quarks in lower Q^2 regions for all possible radiative $E2$ and $C2$ transition form factors. The sea-quark contributions fall off fast as Q^2 increases, compared with those of the valence quarks. This means that in higher Q^2 regions the valence-quark contributions take over those of the sea quarks or the pion clouds. Thus, the sea-quark contributions play an essential role in describing how the decuplet baryons are deformed.

We then presented the results of the $E2/M1$ and $C2/M1$ ratios at $Q^2 = 0$. There exists an experimental data only on the $E2/M1$ ratio for the EM $N \rightarrow \Delta$ transition. The comparison of the present result with the data shows around 20% deviation from it. We then examined the effects of flavor SU(3) symmetry breaking on the electromagnetic transition form factors of the decuplet hyperons. While they are rather marginal on the magnetic dipole transition form factors, they play an important role in describing the $E2$ and $C2$ transition form factors. The reason is that the leading-order contributions vanish for the $E2$ and $C2$ transition form factors. Thus,

the rotational $1/N_c$ and linear m_s corrections are equally important.

While the present results of the EM transition form factors for the baryon decuplet describe well the Q^2 dependence, the magnitudes of the $N\gamma^* \rightarrow \Delta$ form factors are still underestimated, compared with the experimental data. This is already a well-known feature of the χ QSM. There is in fact a way of improving the present work. One can combine the Q^2 behavior of the form factors obtained by the present work with the magnetic transition moments evaluated in a model-independent approach [53]. In principle, the quadrupole transition moments can be determined in a similar way. Actually, it is of great importance to describe the electromagnetic transition form factors of the baryon decuplet, since all the determined dynamical parameters can be employed when we compute the strangeness-changing transitions. While there is no experimental information on semileptonic decays of the baryon decuplet except for the Ω^- , they are still very important in determining the strong vector and tensor coupling constants for the baryon decuplet and the octet to the vector meson vertices through the Goldberger–Treiman relations. The corresponding work is under way.

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Appendix A: Densities for the EM transition form factors

The densities of the magnetic dipole transition form factor are expressed explicitly as follows:

$$\begin{aligned}
 \frac{1}{N_c} \mathcal{Q}_0(\mathbf{r}) &= \langle \text{val} | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \cdot \boldsymbol{\tau} \langle \mathbf{r} | \text{val} \rangle \\
 &\quad + N_c \sum_n \mathcal{R}_1(E_n) \langle n | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \cdot \boldsymbol{\tau} \langle \mathbf{r} | n \rangle, \\
 \frac{1}{N_c} \mathcal{Q}_1(\mathbf{r}) &= i \frac{1}{2} \sum_{n \neq \text{val}} \frac{\text{sign}(E_n)}{E_n - E_{\text{val}}} \\
 &\quad \times \langle n | \mathbf{r} \rangle \gamma^5 [\{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \times \boldsymbol{\tau}] \langle \mathbf{r} | \text{val} \rangle \cdot \langle \text{val} | \boldsymbol{\tau} | n \rangle \\
 &\quad + i \frac{1}{4} \sum_{n,m} \mathcal{R}_4(E_n, E_m) \\
 &\quad \times \langle m | \mathbf{r} \rangle \gamma^5 [\{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \times \boldsymbol{\tau}] \langle \mathbf{r} | n \rangle \cdot \langle m | \boldsymbol{\tau} | n \rangle, \\
 \frac{1}{N_c} \mathcal{X}_1(\mathbf{r}) &= \sum_{n \neq \text{val}} \frac{1}{E_n - E_{\text{val}}} \\
 &\quad \times \langle \text{val} | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \langle \mathbf{r} | \text{val} \rangle \cdot \langle n | \boldsymbol{\tau} | \text{val} \rangle \\
 &\quad + \frac{1}{2} \sum_{n,m} \mathcal{R}_5(E_n, E_m) \\
 &\quad \times \langle n | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \langle \mathbf{r} | m \rangle \cdot \langle m | \boldsymbol{\tau} | n \rangle, \\
 \frac{1}{N_c} \mathcal{X}_2(\mathbf{r}) &= \sum_{n^0} \frac{1}{E_{n^0} - E_{\text{val}}} \\
 &\quad \times \langle \text{val} | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \cdot \boldsymbol{\tau} \langle \mathbf{r} | n^0 \rangle \langle n^0 | \text{val} \rangle \\
 &\quad + \sum_{n^0, m} \mathcal{R}_5(E_m, E_{n^0}) \\
 &\quad \times \langle m | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \cdot \boldsymbol{\tau} \langle \mathbf{r} | n^0 \rangle \langle n^0 | m \rangle, \\
 \frac{1}{N_c} \mathcal{M}_0(\mathbf{r}) &= \sum_{n \neq \text{val}} \frac{1}{E_n - E_{\text{val}}} \\
 &\quad \times \langle \text{val} | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \cdot \boldsymbol{\tau} \langle \mathbf{r} | n \rangle \langle n | \gamma^0 | \text{val} \rangle \\
 &\quad - \frac{1}{2} \sum_{n,m} \mathcal{R}_2(E_n, E_m) \\
 &\quad \times \langle m | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \cdot \boldsymbol{\tau} \langle \mathbf{r} | n \rangle \langle n | \gamma^0 | m \rangle, \\
 \frac{1}{N_c} \mathcal{M}_1(\mathbf{r}) &= \sum_{n \neq \text{val}} \frac{1}{E_n - E_{\text{val}}} \\
 &\quad \times \langle \text{val} | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \langle \mathbf{r} | n \rangle \cdot \langle n | \gamma^0 \boldsymbol{\tau} | \text{val} \rangle \\
 &\quad - \frac{1}{2} \sum_{n,m} \mathcal{R}_2(E_n, E_m) \\
 &\quad \times \langle m | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \langle \mathbf{r} | n \rangle \cdot \langle n | \gamma^0 \boldsymbol{\tau} | m \rangle, \\
 \frac{1}{N_c} \mathcal{M}_2(\mathbf{r}) &= \sum_{n^0} \frac{1}{E_{n^0} - E_{\text{val}}} \\
 &\quad \times \langle \text{val} | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \cdot \boldsymbol{\tau} \langle \mathbf{r} | n^0 \rangle \langle n^0 | \gamma^0 | \text{val} \rangle
 \end{aligned}$$

$$\begin{aligned}
 &- \sum_{n^0, m} \mathcal{R}_2(E_{n^0}, E_m) \\
 &\times \langle m | \mathbf{r} \rangle \gamma^5 \{ \hat{\mathbf{r}} \times \boldsymbol{\sigma} \} \cdot \boldsymbol{\tau} \langle \mathbf{r} | n^0 \rangle \langle n^0 | \gamma^0 | m \rangle. \quad (\text{A1})
 \end{aligned}$$

The densities of the electric quadrupole transition form factors are given as

$$\begin{aligned}
 (-\sqrt{10}) \frac{2}{N_c} \mathcal{I}_{1E2}(\mathbf{r}) &= \sum_{n \neq \text{val}} \frac{1}{E_n - E_{\text{val}}} \langle \text{val} | \boldsymbol{\tau} | n \rangle \\
 &\quad \cdot \langle n | \mathbf{r} \rangle \{ \sqrt{4\pi} Y_2 \otimes \tau_1 \}_1 \langle \mathbf{r} | \text{val} \rangle \\
 &\quad + \frac{1}{2} \sum_{n,m} \mathcal{R}_3(E_n, E_m) \langle n | \boldsymbol{\tau} | m \rangle \\
 &\quad \cdot \langle m | \mathbf{r} \rangle \{ \sqrt{4\pi} Y_2 \otimes \tau_1 \}_1 \langle \mathbf{r} | n \rangle, \\
 (-\sqrt{10}) \frac{2}{N_c} \mathcal{K}_{1E2}(\mathbf{r}) &= \sum_{n \neq \text{val}} \frac{1}{E_n - E_{\text{val}}} \langle \text{val} | \gamma^0 \boldsymbol{\tau} | n \rangle \\
 &\quad \cdot \langle n | \mathbf{r} \rangle \{ \sqrt{4\pi} Y_2 \otimes \tau_1 \}_1 \langle \mathbf{r} | \text{val} \rangle \\
 &\quad + \frac{1}{2} \sum_{n,m} \mathcal{R}_5(E_n, E_m) \langle n | \gamma^0 \boldsymbol{\tau} | m \rangle \\
 &\quad \cdot \langle m | \mathbf{r} \rangle \{ \sqrt{4\pi} Y_2 \otimes \tau_1 \}_1 \langle \mathbf{r} | n \rangle. \quad (\text{A2})
 \end{aligned}$$

The regularization functions in Eqs. (A1) and (A2) are defined by

$$\begin{aligned}
 \mathcal{R}_1(E_n) &= -\frac{1}{2\sqrt{\pi}} E_n \\
 &\quad \times \int_0^\infty \phi(u) \frac{du}{u} e^{-u E_n^2}, \\
 \mathcal{R}_2(E_n, E_m) &= \frac{1}{2\sqrt{\pi}} \\
 &\quad \times \int_0^\infty \phi(u) \frac{du}{\sqrt{u}} \frac{E_m e^{-u E_m^2} - E_n e^{-u E_n^2}}{E_n - E_m}, \\
 \mathcal{R}_3(E_n, E_m) &= \frac{1}{2\sqrt{\pi}} \int_0^\infty \phi(u) \frac{du}{\sqrt{u}} \\
 &\quad \times \left[\frac{e^{-u E_m^2} - e^{-u E_n^2}}{u(E_n^2 - E_m^2)} - \frac{E_m e^{-u E_m^2} + E_n e^{-u E_n^2}}{E_n + E_m} \right], \\
 \mathcal{R}_4(E_n, E_m) &= \frac{1}{2\pi} \int_0^\infty \phi(u) du \\
 &\quad \times \int_0^1 d\alpha e^{-u E_n^2 (1-\alpha) - u E_m^2 \alpha} \frac{E_n(1-\alpha) - \alpha E_m}{\sqrt{\alpha(1-\alpha)}}, \\
 \mathcal{R}_5(E_n, E_m) &= \frac{\text{sign}(E_n) - \text{sign}(E_m)}{2(E_n - E_m)}, \quad (\text{A3})
 \end{aligned}$$

where $|\text{val}\rangle$ and $|n\rangle$ denote the states of the valence and sea quarks with the corresponding eigenenergies E_{val} and E_n of the single-quark Hamiltonian $h(U_c)$, respectively [43].

Appendix B: Matrix elements of the SU(3) Wigner D function

The collective wavefunction of a baryon with flavor $F = (Y, T, T_3)$ and spin $S = (Y' = -N_c/3, J, J_3)$ in the representation ν is expressed in terms of a tensor with two indices, i.e. $\psi_{(\nu; F), (\bar{\nu}; \bar{S})}$, one running over the states F in the representation ν and the other one over the states $\bar{S}$ in the representation $\bar{\nu}$. Here, $\bar{\nu}$ denotes the complex conjugate of the ν , and the complex conjugate of S is written by $\bar{S} = (N_c/3, J, J_3)$. Thus, the collective wavefunction is expressed as

$$\psi_{(\nu; F), (\bar{\nu}; \bar{S})}(R) = \sqrt{\dim(\nu)}(-1)^{Q_S} [D_F^{(\nu)}(R)]^*, \quad (B1)$$

where $\dim(\nu)$ stands for the dimension of the representation ν and Q_S a charge corresponding to the baryon state S , i.e. $Q_S = J_3 + Y'/2$.

$$\begin{aligned} |B_{81/2}\rangle &= |\mathbf{8}_{1/2}, B\rangle + c_{10}^B |\mathbf{10}_{1/2}, B\rangle + c_{27}^B |\mathbf{27}_{1/2}, B\rangle, \\ |B_{103/2}\rangle &= |\mathbf{10}_{3/2}, B\rangle + a_{27}^B |\mathbf{27}_{3/2}, B\rangle + a_{35}^B |\mathbf{35}_{3/2}, B\rangle, \end{aligned} \quad (B2)$$

with the mixing coefficients

$$\begin{aligned} c_{10}^B &= c_{10} \begin{bmatrix} \sqrt{5} \\ 0 \\ \sqrt{5} \\ 0 \end{bmatrix}, \quad c_{27}^B = c_{27} \begin{bmatrix} \sqrt{6} \\ 3 \\ 2 \\ \sqrt{6} \end{bmatrix}, \\ a_{27}^B &= a_{27} \begin{bmatrix} \sqrt{15/2} \\ 2 \\ \sqrt{3/2} \\ 0 \end{bmatrix}, \quad a_{35}^B = a_{35} \begin{bmatrix} 5/\sqrt{14} \\ 2\sqrt{5/7} \\ 3\sqrt{5/14} \\ 2\sqrt{5/7} \end{bmatrix}, \end{aligned} \quad (B3)$$

respectively, in the basis $[N, \Lambda, \Sigma, \Xi]$ and $[\Delta, \Sigma^*, \Xi^*, \Omega]$. The parameters $c_{10}^B, c_{27}^B, a_{27}^B$ and a_{35}^B are given by

$$\begin{aligned} c_{10}^B &= -\frac{I_2}{15} \left(\alpha + \frac{1}{2} \gamma \right), \quad c_{27}^B = -\frac{I_2}{25} \left(\alpha - \frac{1}{6} \gamma \right), \\ a_{27}^B &= -\frac{I_2}{8} \left(\alpha + \frac{5}{6} \gamma \right), \quad a_{35}^B = -\frac{I_2}{24} \left(\alpha - \frac{1}{2} \gamma \right), \end{aligned} \quad (B4)$$

where α and γ are the parameters appearing in the collective Hamiltonian, which are written by

$$\begin{aligned} \alpha &= \left(-\frac{\Sigma_{\pi N}}{3m} + \frac{K_2}{I_2} Y' \right) m_s, \\ \beta &= -\frac{K_2}{I_2} m_s, \quad \gamma = 2 \left(\frac{K_1}{I_1} - \frac{K_2}{I_2} \right) m_s. \end{aligned} \quad (B5)$$

Here, $\Sigma_{\pi N}$ is the well-known πN sigma term. The moments of inertia (I_1, I_2) and anomalous moments of inertia (K_1, K_2) are given by

$$\begin{aligned} \frac{6}{N_c} I_1 &= \sum_{n \neq \text{val}} \frac{1}{E_n - E_{\text{val}}} \langle \text{val} | \boldsymbol{\tau} | n \rangle \cdot \langle n | \boldsymbol{\tau} | \text{val} \rangle \\ &+ \frac{1}{2} \sum_{n, m \neq n} \langle m | \boldsymbol{\tau} | n \rangle \cdot \langle n | \boldsymbol{\tau} | m \rangle \mathcal{R}_3(E_n, E_m), \\ \frac{4}{N_c} I_2 &= \sum_{n^0} \frac{1}{E_{n^0} - E_{\text{val}}} \langle \text{val} | n^0 \rangle \langle n^0 | \text{val} \rangle \\ &+ \sum_{n^0, m} \langle m | \boldsymbol{\tau} | n^0 \rangle \langle n^0 | m \rangle \mathcal{R}_3(E_{n^0}, E_m), \\ \frac{6}{N_c} K_1 &= \sum_{n \neq \text{val}} \frac{1}{E_n - E_{\text{val}}} \langle \text{val} | \boldsymbol{\tau} | n \rangle \cdot \langle n | \gamma^0 \boldsymbol{\tau} | \text{val} \rangle \\ &+ \frac{1}{2} \sum_{n, m \neq n} \langle m | \boldsymbol{\tau} | n \rangle \cdot \langle n | \gamma^0 \boldsymbol{\tau} | m \rangle \mathcal{R}_5(E_n, E_m), \\ \frac{4}{N_c} K_2 &= \sum_{n^0} \frac{1}{E_{n^0} - E_{\text{val}}} \langle \text{val} | n^0 \rangle \langle n^0 | \gamma^0 | \text{val} \rangle \\ &+ \sum_{n^0, m} \langle m | \boldsymbol{\tau} | n^0 \rangle \langle n^0 | \gamma^0 | m \rangle \mathcal{R}_5(E_{n^0}, E_m). \end{aligned} \quad (B6)$$

We list the results of the matrix elements of the relevant collective operators for the EM transition form factors in Tables 2, 3, 5, 4. Note that all the matrix elements of the mixed baryon wave functions arising from $\mathbf{10}$ and $\mathbf{35}$ components vanish.

Appendix C: M1, E2 and C2 densities

The explicit expressions of three terms in Eq. (30) are given for the magnetic dipole form factor

$$\begin{aligned} \mathcal{G}_{M1}^{B_8 \rightarrow B_{10}(0)}(\mathbf{r}) &= \frac{1}{6\sqrt{5}} \begin{pmatrix} 2 \\ -(\mathcal{Q}_{\Sigma \rightarrow \Sigma^*} + 1) \\ -2(\mathcal{Q}_{\Xi \rightarrow \Xi^*} + 1) \\ \sqrt{3}(\mathcal{Q}_{\Lambda \rightarrow \Sigma^*} + 1) \end{pmatrix} \\ &\times \left(\mathcal{Q}_0(\mathbf{r}) + \frac{1}{I_1} \mathcal{Q}_1(\mathbf{r}) + \frac{1}{2} \frac{1}{I_2} \mathcal{X}_2(\mathbf{r}) \right), \end{aligned} \quad (C1)$$

$$\begin{aligned} \mathcal{G}_{M1}^{B_8 \rightarrow B_{10}(\text{op})}(\mathbf{r}) &= \frac{m_8}{36\sqrt{15}} \begin{pmatrix} 1 \\ -3\mathcal{Q}_{\Sigma \rightarrow \Sigma^*} + 1 \\ -5\mathcal{Q}_{\Xi \rightarrow \Xi^*} - 1 \\ \sqrt{3}(\mathcal{Q}_{\Lambda \rightarrow \Sigma^*} + 1) \end{pmatrix} \\ &\times \left(\frac{K_1}{I_1} \mathcal{X}_1(\mathbf{r}) - \mathcal{M}_1(\mathbf{r}) \right) \\ &+ \frac{2m_8}{108\sqrt{5}} \begin{pmatrix} 7 \\ 3(-\mathcal{Q}_{\Sigma \rightarrow \Sigma^*} + 4) \\ 8\mathcal{Q}_{\Xi \rightarrow \Xi^*} + 5 \\ 4\sqrt{3}(\mathcal{Q}_{\Lambda \rightarrow \Sigma^*} + 1) \end{pmatrix} \end{aligned}$$

Table 2 The matrix elements of the collective operators for the leading-order contributions and the $1/N_c$ rotational corrections to the electromagnetic transition form factors

$B_8\gamma^* \rightarrow B_{10}$	$N\gamma^* \rightarrow \Delta$	$\Sigma\gamma^* \rightarrow \Sigma^*$	$\Xi\gamma^* \rightarrow \Xi^*$	$\Lambda\gamma^* \rightarrow \Sigma^*$
$\langle B_{10} D_{33}^{(8)} B_8\rangle$	$\frac{2}{3\sqrt{5}}$	$-\frac{1}{3\sqrt{5}}T_3$	$-\frac{2}{3\sqrt{5}}T_3$	$\frac{1}{15}$
$\langle B_{10} D_{83}^{(8)} B_8\rangle$	0	$-\frac{1}{15}$	$-\frac{1}{15}$	0
$\langle B_{10} D_{38}^{(8)}J_3 B_8\rangle$	0	0	0	0
$\langle B_{10} D_{88}^{(8)}J_3 B_8\rangle$	0	0	0	0
$\langle B_{10} d_{ab3}D_{3a}^{(8)}J_b B_8\rangle$	$-\frac{1}{3\sqrt{5}}$	$\frac{1}{6\sqrt{5}}T_3$	$\frac{1}{3\sqrt{5}}T_3$	$-\frac{1}{2\sqrt{15}}$
$\langle B_{10} d_{ab3}D_{8a}^{(8)}J_b B_8\rangle$	0	$\frac{1}{2\sqrt{15}}$	$\frac{1}{2\sqrt{15}}$	0
$\langle B_{10} D_{3i}^{(8)}J_i B_8\rangle$	0	0	0	0
$\langle B_{10} D_{8i}^{(8)}J_i B_8\rangle$	0	0	0	0

Table 3 The matrix elements of the collective operators for the m_s corrections to the electromagnetic transition form factors

$B_8\gamma^* \rightarrow B_{10}$	$N\gamma^* \rightarrow \Delta$	$\Sigma\gamma^* \rightarrow \Sigma^*$	$\Xi\gamma^* \rightarrow \Xi^*$	$\Lambda\gamma^* \rightarrow \Sigma^*$
$\langle B_{10} D_{88}^{(8)}D_{33}^{(8)} B_8\rangle$	$\frac{2}{9\sqrt{5}}$	0	$\frac{1}{18\sqrt{5}}T_3$	$\frac{1}{6\sqrt{15}}$
$\langle B_{10} D_{88}^{(8)}D_{83}^{(8)} B_8\rangle$	0	$\frac{1}{6\sqrt{15}}$	$\frac{1}{4\sqrt{15}}$	0
$\langle B_{10} D_{83}^{(8)}D_{38}^{(8)} B_8\rangle$	$\frac{1}{18\sqrt{5}}$	$-\frac{1}{6\sqrt{5}}T_3$	$-\frac{\sqrt{5}}{18}T_3$	$\frac{1}{6\sqrt{15}}$
$\langle B_{10} D_{83}^{(8)}D_{88}^{(8)} B_8\rangle$	0	$\frac{1}{6\sqrt{15}}$	$\frac{1}{4\sqrt{15}}$	0
$\langle B_{10} d_{ab3}D_{8a}^{(8)}D_{8b}^{(8)} B_8\rangle$	0	$\frac{2}{9\sqrt{5}}$	$\frac{1}{6\sqrt{5}}$	0
$\langle B_{10} d_{ab3}D_{3a}^{(8)}D_{8b}^{(8)} B_8\rangle$	$\frac{2}{18\sqrt{15}}$	$-\frac{1}{6\sqrt{15}}T_3$	$-\frac{4}{9\sqrt{15}}T_3$	$\frac{2}{9\sqrt{5}}$
$\langle B_{10} D_{83}^{(8)}D_{33}^{(8)} B_8\rangle$	$-\frac{1}{18\sqrt{15}}$	$\frac{1}{6\sqrt{15}}T_3$	$-\frac{2}{9\sqrt{15}}T_3$	$\frac{1}{9\sqrt{5}}$
$\langle B_{10} D_{83}^{(8)}D_{83}^{(8)} B_8\rangle$	0	$\frac{1}{9\sqrt{5}}$	$-\frac{1}{6\sqrt{5}}$	0
$\langle B_{10} D_{8i}^{(8)}D_{3i}^{(8)} B_8\rangle$	$\frac{1}{18\sqrt{15}}$	$-\frac{1}{6\sqrt{15}}T_3$	$\frac{2}{9\sqrt{15}}T_3$	$-\frac{1}{9\sqrt{5}}$
$\langle B_{10} D_{8i}^{(8)}D_{8i}^{(8)} B_8\rangle$	0	$-\frac{1}{9\sqrt{5}}$	$\frac{1}{6\sqrt{5}}$	0

Table 4 The relevant transition matrix elements of the collective operators coming from the 27plet component of the baryon wave functions

$B_{27}\gamma^* \rightarrow B_{10}$	$N\gamma^* \rightarrow \Delta$	$\Sigma\gamma^* \rightarrow \Sigma^*$	$\Xi\gamma^* \rightarrow \Xi^*$	$\Lambda\gamma^* \rightarrow \Sigma^*$
$\langle B_{10} D_{33}^{(8)} B_{27}\rangle$	$\frac{1}{9}\sqrt{\frac{2}{15}}$	$-\frac{1}{6\sqrt{5}}T_3$	$-\frac{7}{9\sqrt{30}}T_3$	$\frac{2}{9\sqrt{15}}$
$\langle B_{10} D_{83}^{(8)} B_{27}\rangle$	0	$\frac{1}{3\sqrt{15}}$	$\frac{1}{6\sqrt{10}}$	0
$\langle B_{10} D_{38}^{(8)}J_3 B_{27}\rangle$	0	0	0	0
$\langle B_{10} D_{88}^{(8)}J_3 B_{27}\rangle$	0	0	0	0
$\langle B_{10} d_{ab3}D_{3a}^{(8)}J_b B_{27}\rangle$	$\frac{2}{9}\sqrt{\frac{2}{15}}$	$-\frac{1}{3\sqrt{5}}T_3$	$-\frac{14}{9\sqrt{30}}T_3$	$\frac{4}{9\sqrt{15}}$
$\langle B_{10} d_{ab3}D_{8a}^{(8)}J_b B_{27}\rangle$	0	$\frac{2}{3\sqrt{15}}$	$\frac{1}{3\sqrt{10}}$	0
$\langle B_{10} D_{3i}^{(8)}J_i B_{27}\rangle$	0	0	0	0
$\langle B_{10} D_{8i}^{(8)}J_i B_{27}\rangle$	0	0	0	0

Table 5 The relevant transition matrix elements of the collective operators coming from the 27plet component of the baryon wave functions

$B_8\gamma^* \rightarrow B_{27}$	$N\gamma^* \rightarrow \Delta$	$\Sigma\gamma^* \rightarrow \Sigma^*$	$\Xi\gamma^* \rightarrow \Xi^*$	$\Lambda\gamma^* \rightarrow \Sigma^*$
$\langle B_{27} D_{33}^{(8)} B_8\rangle$	$\frac{2}{9}\sqrt{\frac{2}{3}}$	0	$-\frac{2}{9}\sqrt{\frac{2}{15}}T_3$	$\frac{2}{3\sqrt{15}}$
$\langle B_{27} D_{83}^{(8)} B_8\rangle$	0	$\frac{2}{3\sqrt{15}}$	$\frac{1}{3}\sqrt{\frac{2}{5}}$	0
$\langle B_{27} D_{38}^{(8)}J_3 B_8\rangle$	0	0	0	0
$\langle B_{27} D_{88}^{(8)}J_3 B_8\rangle$	0	0	0	0
$\langle B_{27} d_{ab3}D_{3a}^{(8)}J_b B_8\rangle$	$\frac{1}{9}\sqrt{\frac{2}{3}}$	0	$-\frac{2}{9\sqrt{30}}T_3$	$\frac{1}{3\sqrt{15}}$
$\langle B_{27} d_{ab3}D_{8a}^{(8)}J_b B_8\rangle$	0	$\frac{1}{3\sqrt{15}}$	$\frac{1}{3\sqrt{10}}$	0
$\langle B_{27} D_{3i}^{(8)}J_i B_8\rangle$	0	0	0	0
$\langle B_{27} D_{8i}^{(8)}J_i B_8\rangle$	0	0	0	0

$$\begin{aligned}
& \times \left(\frac{K_2}{I_2} \mathcal{X}_2(\mathbf{r}) - \mathcal{M}_2(\mathbf{r}) \right) \\
& - \frac{m_1}{3\sqrt{5}} \begin{pmatrix} 2 \\ -(Q_{\Sigma \rightarrow \Sigma^*} + 1) \\ -2(Q_{\Xi \rightarrow \Xi^*} + 1) \\ \sqrt{3}(Q_{\Lambda \rightarrow \Sigma^*} + 1) \end{pmatrix} \mathcal{M}_0(\mathbf{r}) \\
& + \frac{m_8}{36\sqrt{15}} \begin{pmatrix} 4 \\ 1 \\ Q_{\Xi \rightarrow \Xi^*} + 2 \\ \sqrt{3}(Q_{\Lambda \rightarrow \Sigma^*} + 1) \end{pmatrix} \mathcal{M}_0(\mathbf{r}), \quad (C2)
\end{aligned}$$

$$\begin{aligned}
\mathcal{G}_{M1}^{B_8 \rightarrow B_{10}(\text{wf})}(\mathbf{r}) &= c_{27} \frac{1}{18\sqrt{5}} \begin{pmatrix} 2 \\ -3Q_{\Sigma \rightarrow \Sigma^*} + 2 \\ -(7Q_{\Xi \rightarrow \Xi^*} + 2) \\ 2\sqrt{3}(Q_{\Lambda \rightarrow \Sigma^*} + 1) \end{pmatrix} \\
& \times \left[\left(Q_0(\mathbf{r}) + \frac{1}{I_1} Q_1(\mathbf{r}) \right) - \frac{1}{2} \frac{1}{I_2} \mathcal{X}_2(\mathbf{r}) \right] \\
& + a_{27} \frac{\sqrt{5}}{45} \begin{pmatrix} 5 \\ 2 \\ -Q_{\Xi \rightarrow \Xi^*} + 1 \\ 2\sqrt{3}(Q_{\Lambda \rightarrow \Sigma^*} + 1) \end{pmatrix} \\
& \times \left[\left(Q_0(\mathbf{r}) + \frac{1}{I_1} Q_1(\mathbf{r}) \right) - \frac{1}{2} \frac{1}{I_2} \mathcal{X}_2(\mathbf{r}) \right], \quad (C3)
\end{aligned}$$

in the basis of $[N \rightarrow \Delta, \Sigma \rightarrow \Sigma^*, \Xi \rightarrow \Xi^*, \Lambda \rightarrow \Sigma^*]$. $Q_{B_8 \rightarrow B_{10}}$ stand for the charges of the corresponding baryons. c_{27} and a_{27} are the mixing coefficients in the collective baryon wave functions, of which the explicit expressions can be found in Appendix B.

Similarly, the densities for the electric quadrupole form factors are written by

$$\mathcal{G}_{E2}^{B_8 \rightarrow B_{10}f(0)}(\mathbf{r}) = -\frac{1}{2\sqrt{5}I_1}$$

$$\times \begin{pmatrix} 2 \\ -(Q_{\Sigma \rightarrow \Sigma^*} + 1) \\ -2(Q_{\Xi \rightarrow \Xi^*} + 1) \\ \sqrt{3}(Q_{\Lambda \rightarrow \Sigma^*} + 1) \end{pmatrix} \mathcal{I}_{1E2}(\mathbf{r}), \quad (C4)$$

$$\begin{aligned}
\mathcal{G}_{E2}^{B_8 \rightarrow B_{10}(\text{op})}(\mathbf{r}) &= \frac{4}{27\sqrt{15}} m_8 \left(\frac{K_1}{I_1} \mathcal{I}_{1E2}(\mathbf{r}) - \mathcal{K}_{1E2}(\mathbf{r}) \right) \\
& \times \begin{pmatrix} -1 \\ 3Q_{\Sigma \rightarrow \Sigma^*} + 2 \\ -(4Q_{\Xi \rightarrow \Xi^*} + 5) \\ 2\sqrt{3}(Q_{\Lambda \rightarrow \Sigma^*} + 1) \end{pmatrix}, \quad (C5)
\end{aligned}$$

$$\begin{aligned}
\mathcal{G}_{E2}^{B_8 \rightarrow B_{10}(\text{wf})}(\mathbf{r}) &= -\frac{1}{6\sqrt{5}I_1} \\
& \times \begin{pmatrix} 2 \\ -3Q_{\Sigma \rightarrow \Sigma^*} + 2 \\ 7Q_{\Xi \rightarrow \Xi^*} + 2 \\ 2\sqrt{3}(Q_{\Lambda \rightarrow \Sigma^*} + 1) \end{pmatrix} \\
& + a_{27} \begin{pmatrix} 2 \\ 4 \\ 2(-Q_{\Xi \rightarrow \Xi^*} + 1) \\ 4\sqrt{3}(Q_{\Lambda \rightarrow \Sigma^*} + 1) \end{pmatrix} \mathcal{I}_{1E2}(\mathbf{r}) \quad (C6)
\end{aligned}$$

in the basis of $[N \rightarrow \Delta, \Sigma \rightarrow \Sigma^*, \Xi \rightarrow \Xi^*, \Lambda \rightarrow \Sigma^*]$. The densities for the Coulomb quadrupole form factors are identical to those for the $E2$ form factors, i.e. $\mathcal{G}_{C2}^{B_8 \rightarrow B_{10}}(\mathbf{r}) = \mathcal{G}_{E2}^{B_8 \rightarrow B_{10}}(\mathbf{r})$.

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