Role of scalar $a_0(980)$ in the single Cabibbo suppressed process $D^+ \rightarrow \pi^+\pi^0\eta$

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Abstract Taking into account that the scalar $a_0(980)$ can be dynamically generated from the pseudoscalar-pseudoscalar interaction within the chiral unitary approach, we have studied the single Cabibbo suppressed process $D^+ \rightarrow \pi^+\pi^0\eta$. We find clear peaks of $a_0(980)^+$ and $a_0(980)^0$ in the $\pi^+\eta$ and $\pi^0\eta$ invariant mass distributions, respectively. The predicted Dalitz plots of $D^+ \rightarrow \pi^+\pi^0\eta$ also manifest the significant signals for $a_0(980)^+$ and $a_0(980)^0$ states. The uncertainties of the results due to the free parameters are also discussed. Our study shows that the process $D^+ \rightarrow \pi^+\pi^0\eta$ can be used to explore the nature of the scalar $a_0(980)$, thus we encourage the experimental physicists to measure this reaction with more precision.

1 Introduction

The studies of the charmed hadron decays are crucial to explore the strong and weak interaction effects, and to search for the CP violation [1–7]. Recently, the BESIII Collaboration has measured the absolute branching fraction of the $D^+ \rightarrow \pi^+\pi^0\eta$ decay of $(2.23 \pm 0.15 \pm 0.10) \times 10^{-3}$ [8], with much more precision than the prior measurement $(1.38 \pm 0.31 \pm 0.16) \times 10^{-3}$ of the CLEO Collaboration [9], and no evidence of CP violation is found. Although there is no significant ρ^+ and scalar $a_0(980)^{0,+}$ in the Dalitz plot of the $D^+ \rightarrow \pi^+\pi^0\eta$ decay, the BESIII Collaboration has pointed out that the phase-space Monte Carlo distributions do not agree well with the data distributions due to some possible resonances, and mentioned that the amplitude analyses of this decay in the near future with large data sample at BESIII and Belle II will offer the opportunity to explore the decay of $D^+ \rightarrow a_0(980)\pi$ [8].

It should be stressed that the signal of the $a_0(980)$ was found in many processes. For instance, Ref. [10] has studied the decay $D^0 \rightarrow K_s^0 a_0(980)$, and found a clear signal of the $a_0(980)$ in the $\pi^0\eta$ invariant mass distribution, and the Belle Collaboration has also observed the signal of the $a_0(980)$ in the decay $D^0 \rightarrow K^-\pi^+\eta$ [11]. In addition, there are significant peaks of the $a_0(980)^0$ and $a_0(980)^+$ in the $\pi^0\eta$ and $\pi^+\eta$ invariant mass distributions of the process $D_s^+ \rightarrow \pi^+\pi^0\eta$, as discussed in Refs. [12, 13]. Because the process $D^+ \rightarrow a_0(980)\pi$ can proceed in S -wave, and the scalar $a_0(980)$ has a large coupling to the $\pi\eta$ channel, we expect that there should be a sizeable signal of the $a_0(980)$ resonance if the large data sample are taken in near future. Another example is that the analysis of the reaction $\Lambda_b \rightarrow J/\psi p\pi$ shows the existence of the hidden-charm pentaquark [14], which is confirmed by the full amplitude analysis of the LHCb Collaboration [15].

On the other hand, the nature of the low-lying light scalar resonances are still problematic, and crucial for us to understand the spectrum of the scalar mesons [16], and there are many explanations about their nature, such as tetraquark, molecular states, and so on [see the review ‘Scalar mesons below 2 GeV’ of Particle Data Group (PDG) [17]]. The chiral unitary approach, which provides the amplitudes of the pseudoscalar-pseudoscalar interactions, has been tested successfully in many reactions where the scalar mesons are generated dynamically. For the chiral unitary approach with the coupled channels, the potentials of the Bethe–Salpeter (BS) equation are taken from the chiral Lagrangians [18, 19], and the scattering amplitudes are obtained by solving the Bethe–Salpeter equation in all possible coupled channels that couple within $SU(3)$ to certain given quantum numbers. Within the chiral unitary approach, the productions of the scalar $f_0(500)$, $f_0(980)$, and $a_0(980)$ have been studied in the decays of the D^0 [10], D_s^+ [12], $\bar{B}$ and $\bar{B}_s$ [20–23], χ_{c1} [24], τ^- [25], J/ψ [26], η_c [27], and Λ_c [28].

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Up to our knowledge, there is no theoretical analyses on the process $D^+ \rightarrow \pi^+\pi^0\eta$, so it is interesting to investigate the role of the $a_0(980)$ in this process. In this work, we will perform the study of the single Cabibbo suppressed process $D^+ \rightarrow \pi^+\pi^0\eta$ taking into account the final state interactions of the meson-meson interaction in coupled channels with the chiral unitary approach.

This paper is organized as follows. In Sect. 2, we will present the mechanism for the reaction of $D^+ \rightarrow \pi^+\pi^0\eta$, and in Sect. 3, we will show our results and discussions, followed by a short summary in the last section.

2 Formalism

In analogy to Refs. [10, 12], the mechanism of the Cabibbo suppressed process $D^+ \rightarrow \pi^+\pi^0\eta$ includes three steps, weak decay, hadronization, and the final state interactions. The weak decay of the D^+ can happen by means of W^+ internal emission, where the c quark decays into a W^+ boson and a d quark, and then W^+ goes to $\bar{d}$ and u quarks. In order to produce the final hadrons, the $d\bar{d}$ or $u\bar{d}$ pair need to hadronize to a pair of pseudoscalar mesons with the $\bar{q}q$ ($= \bar{u}u + \bar{d}d + \bar{s}s$) produced from the vacuum as depicted in Fig. 1a or b, and we have,

$$\sum_i d(\bar{q}_i q_i) \bar{d} = \sum_i M_{2i} M_{i2} = (M^2)_{22}, \quad (1)$$

$$\sum_i u(\bar{q}_i q_i) \bar{d} = \sum_i M_{1i} M_{i2} = (M^2)_{12}, \quad (2)$$

for Fig. 1a, b, respectively, where M is the matrix in terms of the pseudoscalar mesons.

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \frac{2\eta'}{\sqrt{6}} \end{pmatrix}. \quad (3)$$

Since the η' has a large mass and does not play a role in the generation of the $a_0(980)$ [29], we ignore the η' component in this work, and Eqs. (1) and (2) can be re-written as,

$$(M^2)_{22} = \pi^+\pi^- + \frac{1}{2}\pi^0\pi^0 - \sqrt{\frac{2}{3}}\pi^0\eta + \frac{1}{3}\eta\eta + K^0\bar{K}^0, \quad (4)$$

$$(M^2)_{12} = \frac{2}{\sqrt{3}}\pi^+\eta + K^+\bar{K}^0. \quad (5)$$

Because the channels $\pi^+\pi^-$, $\pi^0\pi^0$, and $\eta\eta$ of Eq. (4) do not couple to the system of isospin $I = 1$, they have no

Fig. 1 Diagrammatic representation of the D^+ decay. **a** The internal emission of $D^+ \rightarrow \pi^+ d \bar{d}$ and hadronization of the $d \bar{d}$ through $\bar{q}q$ with vacuum quantum numbers. **b** The internal emission of $D^+ \rightarrow \pi^0 u \bar{d}$ and hadronization of the $u \bar{d}$ through $\bar{q}q$ with vacuum quantum numbers. **c** The external emission of $D^+ \rightarrow \pi^+ d \bar{d}$ and hadronization of the $d \bar{d}$ through $\bar{q}q$ with vacuum quantum numbers. **d** The external emission of $D^+ \rightarrow K^+ s \bar{d}$ and hadronization of the $s \bar{d}$ through $\bar{q}q$ with vacuum quantum numbers

contribution in the $a_0(980)$ production, thus we have the final states after the hadronization as follows,

$$H^{(a)} = V^{(a)} V_{cd} V_{ud} \left(-\sqrt{\frac{2}{3}} \pi^0 \eta + K^0 \bar{K}^0 \right) \pi^+, \quad (6)$$

$$\begin{aligned} H^{(b)} &= V^{(b)} V_{cd} V_{ud} \left(\frac{2}{\sqrt{3}} \pi^+ \eta + K^+ \bar{K}^0 \right) \left(-\frac{1}{\sqrt{2}} \pi^0 \right) \\ &= V^{(b)} V_{cd} V_{ud} \left(-\sqrt{\frac{2}{3}} \pi^+ \eta - \frac{1}{\sqrt{2}} K^+ \bar{K}^0 \right) \pi^0, \end{aligned} \quad (7)$$

where the factor $-\frac{1}{\sqrt{2}}$ of π^0 in Eq. (7) is due to the flavor component of the $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$. The elements of the CKM matrix are $V_{cd} = V_{us} = -0.22534$ and $V_{ud} = V_{cs} = 0.97427$ [17]. The $V^{(a)}$ and $V^{(b)}$ are the factors of the production vertices of Fig. 1a, b containing all the dynamics, and these two factors should be similar because the weak processes of Fig. 1a, b are the same before hadronizations.

In addition, the mechanisms of the W^+ external emission shown in Fig. 1c, d also contribute to the process $D^+ \rightarrow \pi^+ \pi^0 \eta$. The hadronization step of Fig. 1c is the same as the one of Fig. 1a, but with an extra color factor C accounting for the relative weight of the external emission mechanism with respect to the internal emission mechanism. Because the $u\bar{d}$ or $u\bar{s}$ pair from the W^+ decay in the external emission (Fig. 1c, d) is constrained to form the color singlet π^+ and K^+ within three choices of colors, while the u and $\bar{d}$ quarks from the W^+ decay in the internal emission (Fig. 1a, b) have the fixed colors, the factor C should be around 3 [30–32]. Thus, we have the possible final states for Fig. 1c,

$$H^{(c)} = C \times V^{(a)} V_{cd} V_{ud} \left(-\sqrt{\frac{2}{3}} \pi^+ \pi^0 \eta + K^0 \bar{K}^0 \pi^+ \right). \quad (8)$$

The weak process of Fig. 1d is the same as the one of Fig. 1c except for the elements of CKM matrix, thus we have,

$$\begin{aligned} H^{(d)} &= C \times V^{(a)} V_{cs} V_{us} \left(M_{32}^2 K^+ \right. \\ &\quad \left. + K^+ K^- \pi^+ - \frac{1}{\sqrt{2}} K^+ \bar{K}^0 \pi^0 \right). \end{aligned} \quad (9)$$

Now, we sum the contributions from Fig. 1a–d,

$$\begin{aligned} H &= H^{(a)} + H^{(b)} + H^{(c)} + H^{(d)} \\ &= V^{(a)} \left[-\sqrt{\frac{2}{3}} (1 + C + R) \pi^+ \pi^0 \eta + (1 + C) K^0 \bar{K}^0 \pi^+ \right. \\ &\quad \left. + C \times K^+ K^- \pi^+ - \frac{1}{\sqrt{2}} (C + R) K^+ \bar{K}^0 \pi^0 \right], \end{aligned} \quad (10)$$

where the elements of the CKM matrix have been absorbed in the normalization factor $V^{(a)}$, and the $R = V^{(b)}/V^{(a)}$ stands

Fig. 2 The mechanisms of the $D^+ \rightarrow \pi^+ \pi^0 \eta$. **a** Tree diagram, **b** the final state interaction of $\pi^+ \eta$, $K^+ \bar{K}^0$ and **c** the final state interaction of $\pi^0 \eta$, $K^0 \bar{K}^0$, and $K^+ K^-$

for the relative weight of Fig. 1b with respect to Fig. 1a. Since the mechanisms of weak process of Fig. 1a, b are the same, we expect the $R = V^{(b)}/V^{(a)}$ to be around 1, and will calculate the results with different values of R .

The full amplitude for the decay $D^+ \rightarrow \pi^+ \pi^0 \eta$ can be easily obtained as follows,

$$\begin{aligned} \mathcal{M} &= V^{(a)} [h_{\pi^0 \pi^+ \eta} + h_{\pi^0 \pi^+ \eta} G_{\pi^0 \eta}(M_{\pi^0 \eta}) t_{\pi^0 \eta \rightarrow \pi^0 \eta}(M_{\pi^0 \eta}) \\ &\quad + h_{K^0 \bar{K}^0 \pi^+} G_{K^0 \bar{K}^0}(M_{\pi^0 \eta}) t_{K^0 \bar{K}^0 \rightarrow \pi^0 \eta}(M_{\pi^0 \eta}) \\ &\quad + h_{K^+ K^- \pi^+} G_{K^+ K^-}(M_{\pi^0 \eta}) t_{K^+ K^- \rightarrow \pi^0 \eta}(M_{\pi^0 \eta})] \\ &\quad + V^{(a)} [h_{\pi^0 \pi^+ \eta} G_{\pi^+ \eta}(M_{\pi^+ \eta}) t_{\pi^+ \eta \rightarrow \pi^+ \eta}(M_{\pi^+ \eta}) \\ &\quad + h_{K^+ \bar{K}^0 \pi^0} G_{K^+ \bar{K}^0}(M_{\pi^+ \eta}) t_{K^+ \bar{K}^0 \rightarrow \pi^+ \eta}(M_{\pi^+ \eta})], \end{aligned} \quad (11)$$

with $h_{\pi^0 \pi^+ \eta} = -\sqrt{\frac{2}{3}}(1 + C + R)$, $h_{K^0 \bar{K}^0 \pi^+} = 1 + C$, $h_{K^+ K^- \pi^+} = C$, and $h_{K^+ \bar{K}^0 \pi^0} = -\frac{1}{\sqrt{2}}(C + R)$, as taken

from Eq. (10). G_i is the loop function, and the transition amplitudes $t_{i \rightarrow j}$ are obtained in the chiral unitary approach by solving the Bethe–Salepter equation,

$$T = [1 - VG]^{-1}V, \quad (12)$$

where V is a 2×2 matrix with the transition potential between the isospin channels $K\bar{K}$ and $\pi\eta$. With the isospin multiplets $K = (K^+, K^0)$, $\bar{K} = (\bar{K}^0, -K^-)$, and $\pi = (-\pi^+, \pi^0, \pi^-)$, the 2×2 matrix V can be easily obtained as follows [10],

$$V_{K\bar{K} \rightarrow K\bar{K}} = -\frac{1}{4f^2}s, \quad (13)$$

$$V_{K\bar{K} \rightarrow \pi\eta} = \frac{\sqrt{6}}{12f^2} \left(3s - \frac{8}{3}m_K^2 - \frac{1}{3}m_\pi^2 - m_\eta^2 \right), \quad (14)$$

$$V_{\pi\eta \rightarrow \pi\eta} = -\frac{1}{3f^2}m_\pi^2, \quad (15)$$

and the transition amplitudes $t_{i \rightarrow j}$ between charged coupled channels can be related to the ones between coupled channels with isospin base,

$$t_{K^+K^- \rightarrow \pi^0\eta} = -\frac{1}{\sqrt{2}}t_{K\bar{K} \rightarrow \pi\eta}^{I=1}, \quad (16)$$

$$t_{K^0\bar{K}^0 \rightarrow \pi^0\eta} = \frac{1}{\sqrt{2}}t_{K\bar{K} \rightarrow \pi\eta}^{I=1}, \quad (17)$$

$$t_{K^+\bar{K}^0 \rightarrow \pi^+\eta} = -t_{K\bar{K} \rightarrow \pi\eta}^{I=1}. \quad (18)$$

The loop function G_i of Eqs. (11) and (12) is given by

$$G_i = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q-P)^2 - m_2^2 + i\epsilon} \frac{1}{q^2 - m_1^2 + i\epsilon}, \quad (19)$$

where m_1 and m_2 are the masses of the two mesons involved in the loop of the i th channel, and P is the four-momentum of the two mesons. The function G_i is logarithmically divergent, there are two methods to solve this singular integral, either using the three-momentum cut-off method [10, 12, 29], or the dimensional regularization method [33, 35–39]. The choice of a particular regularization scheme does not, of course, affect our argumentation [33, 34]. In this work, we use the dimensional regularization method, and the function G_i can be re-expressed as,

$$G_i = \frac{1}{16\pi^2} \left\{ a(\mu) + \ln \frac{m_2^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s} \ln \frac{m_2^2}{m_1^2} + \frac{P}{\sqrt{s}} \left[\ln(s - (m_2^2 - m_1^2) + 2p\sqrt{s}) + \ln(s + (m_2^2 - m_1^2) + 2p\sqrt{s}) - \ln(-s + (m_2^2 - m_1^2) + 2p\sqrt{s}) - \ln(-s - (m_2^2 - m_1^2) + 2p\sqrt{s}) \right] \right\} \quad (20)$$

with

$$p = \frac{\sqrt{[(s - (m_1 + m_2)^2)][s - (m_1 - m_2)^2]}}{2\sqrt{s}}, \quad (21)$$

where μ is the scale of dimensional regularization. Following Eq. (17) in Ref. [40], we take $\mu = 600$ MeV, $a(\mu)_{\pi\eta} = -1.71$, and $a(\mu)_{K\bar{K}} = -1.66$.

With the relationship of Eqs. (16–18), the full amplitude of Eq. (11) can be re-written as,

$$\begin{aligned} \mathcal{M} = V^{(a)} & \left[h_{\pi^0\pi^+\eta} + h_{\pi^0\pi^+\eta} G_{\pi^0\eta}(M_{\pi^0\eta}) t_{\pi\eta \rightarrow \pi\eta}^{I=1}(M_{\pi^0\eta}) \right. \\ & + \frac{h_{K^0\bar{K}^0\pi^+}}{\sqrt{2}} G_{K^0\bar{K}^0}(M_{\pi^0\eta}) t_{K\bar{K} \rightarrow \pi\eta}^{I=1}(M_{\pi^0\eta}) \\ & - \frac{h_{K^+K^-\pi^+}}{\sqrt{2}} G_{K^+K^-}(M_{\pi^0\eta}) t_{K\bar{K} \rightarrow \pi\eta}^{I=1}(M_{\pi^0\eta}) \left. \right] \\ & + V^{(a)} [h_{\pi^0\pi^+\eta} G_{\pi^+\eta}(M_{\pi^+\eta}) t_{\pi\eta \rightarrow \pi\eta}^{I=1}(M_{\pi^+\eta}) \\ & - h_{K^+\bar{K}^0\pi^0} G_{K^+\bar{K}^0}(M_{\pi^+\eta}) t_{K\bar{K} \rightarrow \pi\eta}^{I=1}(M_{\pi^+\eta})]. \quad (22) \end{aligned}$$

Since the amplitude of Eq. (22) depends on two independent invariant masses $M_{\pi^0\eta}$ and $M_{\pi^+\eta}$, the double differential width for the process $D^+ \rightarrow \pi^+\pi^0\eta$ is,

$$\frac{d^2\Gamma}{dM_{\pi^0\eta}dM_{\pi^+\eta}} = \frac{1}{(2\pi)^3} \frac{M_{\pi^0\eta}M_{\pi^+\eta}}{8m_{D^+}^3} |\mathcal{M}|^2. \quad (23)$$

We can obtain $d\Gamma/dM_{\pi^0\eta}$ and $d\Gamma/dM_{\pi^+\eta}$, by integrating Eq. (23) over each of the invariant mass variables with relations as follows,

$$\begin{aligned} (M_{\pi^0\eta}^2)_{\max} &= (E_{\pi^0}^* + E_\eta^*)^2 \\ &- \left(\sqrt{E_{\pi^0}^{*2} - m_{\pi^0}^2} - \sqrt{E_\eta^{*2} - m_\eta^2} \right)^2, \\ (M_{\pi^0\eta}^2)_{\min} &= (E_{\pi^0}^* - E_\eta^*)^2 \\ &- \left(\sqrt{E_{\pi^0}^{*2} - m_{\pi^0}^2} + \sqrt{E_\eta^{*2} - m_\eta^2} \right)^2, \quad (24) \end{aligned}$$

here $E_{\pi^0}^*$ and E_η^* are the energies of π^0 and η in the $\pi^+\eta$ rest frame, respectively,

$$\begin{aligned} E_{\pi^0}^* &= \frac{m_{D^+}^2 - M_{\pi^+\eta}^2 - m_{\pi^0}^2}{2M_{\pi^+\eta}}, \\ E_\eta^* &= \frac{M_{\pi^+\eta}^2 - m_{\pi^+}^2 + m_\eta^2}{2M_{\pi^+\eta}}. \quad (25) \end{aligned}$$

The $\pi^0\eta$ invariant mass distributions can be obtained by interchanging the π^0 and π^+ in Eqs. (24) and (25).

As we known, the three-body decays of charm mesons often proceed as quasi-two-body decays with intermediate states, and the D^+ may decay into an η meson and an intermediate resonance ρ^+ , following by $\rho^+ \rightarrow \pi^+\pi^0$. Because

Fig. 3 The $\pi^+\eta$ (a) and $\pi^0\eta$ (b) mass distributions for the process $D^+ \rightarrow \pi^+\pi^0\eta$. The curves labeled as the ‘tree’, ‘ $a_0(980)^+$ ’, and ‘ $a_0(980)^0$ ’ correspond to the contributions from the tree diagram (Fig. 2a), the final state interactions of $\pi^+\eta$ (Fig. 2b), and the final state interactions of $\pi^0\eta$ (Fig. 2c), respectively. The curves labeled as ‘Total’ show the results from the total contributions of Eq. (22) with an energy cut $M_{\pi^+\pi^0} > 1$ GeV, and the ‘No restriction’ curves stand for the results from the total contributions without the energy cut on invariant mass $M_{\pi^+\pi^0}$

the broad ρ^+ resonance provides the background contributions for the $\pi^+\eta$ and $\pi^0\eta$ invariant mass distributions, we do the calculations by taking an energy restriction of $M_{\pi^+\pi^0} > 1$ GeV in order to eliminate the possible contribution from the intermediate ρ^+ , as done for the process $D_s^+ \rightarrow \pi^+\pi^0\eta$ in Refs. [12,41].

3 Results and discussions

In this section, we will present our results with above formalisms. In our model, we have three free parameters, (1) the normalization factor $V^{(a)}$ of Eq. (22), (2) the color factor C , and (3) the $R = V^{(b)}/V^{(a)}$. Since the process $D^+ \rightarrow \pi^+\pi^0\eta$ has been measured by the BESIII and CLEO Collaborations [8,9], which implies that the invariant mass distributions of this process are able to be measured exper-

Fig. 4 The Dalitz plots of ‘ $M_{\pi^0\eta}$ ’ vs ‘ $M_{\pi^+\eta}$ ’ (a) and ‘ $M_{\pi^0\eta}$ ’ vs ‘ $M_{\pi^+\pi^0}$ ’ (b) for the process $D^+ \rightarrow \pi^+\pi^0\eta$

imentally, thus we will take $V^{(a)} = 1$, and give our calculations up to an arbitrary normalization. As we discussed above, the color factor C should be around 3, we will take $C = 3$, and later will show the results by varying the value of C . Because the parameter $R = V^{(b)}/V^{(a)}$ is expected to be around 1, we take $R = 1$ at the first step, and then discuss the influence from the different value of R . The masses of the mesons involved in this work are taken from PDG [17].

In Fig. 3, we show the $\pi^+\eta$ and $\pi^0\eta$ invariant mass distributions. The curves labeled as the ‘tree’, ‘ $a_0(980)^+$ ’, and ‘ $a_0(980)^0$ ’ correspond to the contributions from the tree diagram (Fig. 2a), the final state interaction of $\pi^+\eta$ (Fig. 2b), and the final state interaction of $\pi^0\eta$ (Fig. 2c), respectively. The curves labeled as ‘Total’ show the results from the total contributions of Eq. (22) with an energy restriction of

Fig. 5 The $\pi^+\eta$ (a) and $\pi^0\eta$ (b) mass distributions for the process $D^+ \rightarrow \pi^+\pi^0\eta$ for different values of color factor C

$M_{\pi^+\pi^0} > 1$ GeV [12,41]. One can see a significant peak around $M_{\pi^+\eta} = 980$ MeV in the $\pi^+\eta$ invariant mass distribution and a significant peak around $M_{\pi^0\eta} = 980$ MeV in the $\pi^0\eta$ invariant mass distribution, which can be associated to the $a_0(980)^+$ and $a_0(980)^0$ resonances, respectively. In addition, we also present the results from the total contributions without the energy cut on invariant mass $M_{\pi^+\pi^0}$, labeled as ‘No restriction’ curves in Fig. 3. By comparing the curves of ‘Total’ with the ones of ‘No restriction’, one can easily found that the ρ^+ mainly contributes to the regions of $M_{\pi^+\eta} > 1$ GeV and $M_{\pi^0\eta} > 1$ GeV, and does not affect the peak positions of the $a_0(980)^+$ and $a_0(980)^0$. We also present the Dalitz plots of ‘ $M_{\pi^0\eta}$ ’ vs ‘ $M_{\pi^+\eta}$ ’ and ‘ $M_{\pi^0\eta}$ ’ vs ‘ $M_{\pi^+\pi^0}$ ’ for the process $D^+ \rightarrow \pi^+\pi^0\eta$ in Fig. 4a, b, which can be used to check our model in future.

Our results show that there will be the significant signal of the $a_0(980)$ in the Dalitz plots of Fig. 4. However, based on the present BESIII data sample, it is difficult to conclude whether the $a_0(980)$ does exist or not, which may be a reason why the $a_0(980)$ is not significant in BESIII Dalitz plot [8]. If we look at the BESIII measurement of Fig. 4(a) in Ref. [8], there are more events in the region of $M_{\eta\pi^0}^2 < 1$ GeV², which could be due to the background. The further measurements

Fig. 6 The $\pi^+\eta$ (a) and $\pi^0\eta$ (b) mass distributions for the process $D^+ \rightarrow \pi^+\pi^0\eta$ for different values of R

on the $\pi^0\eta$ and $\pi^+\eta$ invariant mass distributions are essential to test the predicted cusp structure of $a_0(980)$.

One issue should be pointed out that, although both the processes $D^+ \rightarrow \pi^+\pi^0\eta$ and $D_s^+ \rightarrow \pi^+\pi^0\eta$ have the same final states, they have the different $\pi^0\eta$ and $\pi^+\eta$ mass distributions by comparing the Fig. 3 with the ones of Refs. [12,13], which is because the $s\bar{s}$ component can be directly produced via the D_s^+ Cabibbo favored decay, but can not be produced via the D^+ Cabibbo suppressed decay.

As mentioned above, the color factor C should be around 3, since we taken $N_c = 3$ here. Indeed, the N_c scaling only indicates the relative strength of the absolute values, and the relative sign is not fixed [31]. We show the $\pi^+\eta$ and $\pi^0\eta$ mass distributions for the process $D^+ \rightarrow \pi^+\pi^0\eta$ with different values of $C = 3, 2, -2, -3$ in Fig. 5, and find that the peaks of $a_0(980)^+$ and $a_0(980)^0$ are very clear for the positive values of C , and become weaker for the negative values of C . It should be pointed out that the positive value of C is supported by the analyses of the process $\Lambda_c \rightarrow p\pi^+\pi^-$ measured by the BESIII Collaboration [28] and the process $B^+ \rightarrow J/\psi\omega K^+$ measured by the LHCb Collaboration [32].

In Fig. 6, we also show the $\pi^+\eta$ and $\pi^0\eta$ mass distributions for the process $D^+ \rightarrow \pi^+\pi^0\eta$ with different values of $R = 1.5, 1.0, 0.5$. Although the strength in both $\pi^+\eta$ and $\pi^0\eta$ mass distributions become a little weaker for a smaller R , while the peaks of $a_0(980)^+$ and $a_0(980)^0$ are still very clear, which implies that signals of $a_0(980)^+$ and $a_0(980)^0$ do not depend on the relative weight R too much.

It should be stressed that, due to some possible resonances, the phase-space Monte Carlo distributions do not agree well with the data distribution, as mentioned by the BESIII [8], and it is difficult to claim the existence of the resonances in the reaction $D^+ \rightarrow \pi^+\pi^0\eta$ basing on the present BESIII data sample. Thus we do not compare our results with the BESIII measurement in this work. One of our purposes is to encourage the experimentalists to measure the $\pi^+\eta$ and $\pi^0\eta$ invariant mass distributions with more data sample, and test our predicted cusp structure of the $a_0(980)$.

On the other hand, the interaction of the emitted η from the $\pi^+\eta$ (or $\pi^0\eta$) and the π^0 (or π^+) should be seriously considered. Indeed, from the Dalitz plot of Fig. 4a, one can find the $a_0(980)^+$ in the $\pi^+\eta$ invariant mass distribution has little effect on the $a_0(980)^0$ in the $\pi^0\eta$ invariant mass distribution, thus, we expect that the three-body interactions do not affect the signals of the $a_0(980)^+$ and $a_0(980)^0$ too much. Furthermore, there will be more uncertainties of our predictions due to the additional parameters if the three-body interactions are considered. Hence, we will leave these contributions to future studies when more experimental data sample become available.

4 Conclusions

In this work, we have investigated the single Cabibbo suppressed process $D^+ \rightarrow \pi^+\pi^0\eta$, by taking into account the pseudoscalar-pseudoscalar interaction in S -wave within the chiral unitary approach, where the scalar $a_0(980)$ can be dynamically generated. By including the mechanisms of the W^+ internal and external emissions, we have calculated the $\pi^+\eta$ and $\pi^0\eta$ mass distributions, and found the clear peaks of $a_0(980)^+$ and $a_0(980)^0$. The Dalitz plots of ' $M_{\pi^0\eta}$ ' vs ' $M_{\pi^+\pi^0}$ ' and ' $M_{\pi^0\eta}$ ' vs ' $M_{\pi^+\eta}$ ' are also predicted, and can be used to check our model in future.

We have also presented the $\pi^+\eta$ and $\pi^0\eta$ mass distributions for different values of the free parameters, the color factor C and the relative weight R . One can find our results do not depend on the values of R in the range $0.5 < R < 1.5$ too much. Both the peaks of $a_0(980)^+$ and $a_0(980)^0$ are much clear for the positive values of C , and become weaker for the negative values of C .

In summary, our study indicates that the single Cabibbo suppressed process $D^+ \rightarrow \pi^+\pi^0\eta$ is suitable to explore the

nature of scalar $a_0(980)$, and we encourage the experimental physicists to measure this reaction with more precision.

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References

1. H.Y. Cheng, Charmed baryons circa 2015. *Front. Phys. (Beijing)* **10**, 101406 (2015)
2. D. Ebert, W. Kallies, Nonleptonic decays of charmed baryons in the MIT bag model. *Phys. Lett.* **131B**, 183 (1983) [Erratum: *Phys. Lett.* **148B**, 502 (1984)]
3. H.Y. Cheng, B. Tseng, Nonleptonic weak decays of charmed baryons. *Phys. Rev. D* **46**, 1042 (1992) [Erratum: *Phys. Rev. D* **55**, 1697 (1997)]
4. H.Y. Cheng, X.W. Kang, F. Xu, Singly Cabibbo-suppressed hadronic decays of Λ_c^+ . *Phys. Rev. D* **97**, 074028 (2018)
5. C.D. Lü, W. Wang, F.S. Yu, Test flavor $SU(3)$ symmetry in exclusive Λ_c decays. *Phys. Rev. D* **93**, 056008 (2016)
6. C.Q. Geng, Y.K. Hsiao, C.W. Liu, T.H. Tsai, Three-body charmed baryon decays with $SU(3)$ flavor symmetry. *Phys. Rev. D* **99**, 073003 (2019)
7. E. Oset et al., Weak decays of heavy hadrons into dynamically generated resonances. *Int. J. Mod. Phys. E* **25**, 1630001 (2016)
8. M. Ablikim et al., [BESIII], Observation of $D^+ \rightarrow \eta\eta\pi^+$ and improved measurement of $D^{0(+)} \rightarrow \eta\pi^+\pi^{-(0)}$. *Phys. Rev. D* **101**, 052009 (2020)
9. M. Artuso et al., [CLEO], Measurement of exclusive D meson decays to eta and eta-prime final states and $SU(3)$ amplitude analysis. *Phys. Rev. D* **77**, 092003 (2008)
10. J.J. Xie, L.R. Dai, E. Oset, The low lying scalar resonances in the D^0 decays into K_s^0 and $f_0(500)$, $f_0(980)$, $a_0(980)$. *Phys. Lett. B* **742**, 363 (2015)
11. Y.Q. Chen et al., [Belle], Dalitz analysis of $D^0 \rightarrow K^-\pi^+\eta$ decays at Belle. *Phys. Rev. D* **102**, 012002 (2020)
12. R. Molina, J. Xie, W. Liang, L. Geng, E. Oset, Theoretical interpretation of the $D_s^+ \rightarrow \pi^+\pi^0\eta$ decay and the nature of $a_0(980)$. *Phys. Lett. B* **803**, 135279 (2020)

13. Y.K. Hsiao, Y. Yu, B.C. Ke, Resonant $a_0(980)$ state in triangle rescattering $D_s^+ \rightarrow \pi^+ \pi^0 \eta$ decays. *Eur. Phys. J. C* **80**, 895 (2020)
14. E. Wang, H.X. Chen, L.S. Geng, D.M. Li, E. Oset, Hidden-charm pentaquark state in $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decay. *Phys. Rev. D* **93**, 094001 (2016)
15. R. Aaij et al., [LHCb], Evidence for exotic hadron contributions to $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decays. *Phys. Rev. Lett.* **117**, 082003 (2016)
16. G.Y. Wang, S.C. Xue, G.N. Li, E. Wang, D.M. Li, Strong decays of the higher isovector scalar mesons. *Phys. Rev. D* **97**, 034030 (2018)
17. M. Tanabashi et al., [Particle Data Group], Review of particle physics. *Phys. Rev. D* **98**, 030001 (2018)
18. J. Gasser, H. Leutwyler, Chiral perturbation theory to one loop. *Ann. Phys.* **158**, 142 (1984)
19. V. Bernard, N. Kaiser, U.G. Meißner, Chiral dynamics in nucleons and nuclei. *Int. J. Mod. Phys. E* **4**, 193–346 (1995)
20. W.H. Liang, J.J. Xie, E. Oset, $\bar{B}^0$ decay into D^0 and $f_0(500)$, $f_0(980)$, $a_0(980)$, ρ and $\bar{B}_s^0$ decay into D^0 and $\kappa(800)$, K^{*0} . *Phys. Rev. D* **92**, 034008 (2015)
21. W.H. Liang, E. Oset, B^0 and B_s^0 decays into J/ψ $f_0(980)$ and J/ψ $f_0(500)$ and the nature of the scalar resonances. *Phys. Lett. B* **737**, 70 (2014)
22. W.H. Liang, J.J. Xie, E. Oset, $\bar{B}^0$, B^- and $\bar{B}_s^0$ decays into J/ψ and $K \bar{K}$ or $\pi \eta$. *Eur. Phys. J. C* **75**, 609 (2015)
23. J.J. Xie, G. Li, The decays of $\bar{B}^0$, $\bar{B}_s^0$ and B^- into η_c plus a scalar or vector meson. *Eur. Phys. J. C* **78**, 861 (2018)
24. W.H. Liang, J.J. Xie, E. Oset, $f_0(500)$, $f_0(980)$, and $a_0(980)$ production in the $\chi_{c1} \rightarrow \eta \pi^+ \pi^-$ reaction. *Eur. Phys. J. C* **76**, 700 (2016)
25. L.R. Dai, Q.X. Yu, E. Oset, Triangle singularity in $\tau^- \rightarrow \nu_\tau \pi^- f_0(980)$ ($a_0(980)$) decays. *Phys. Rev. D* **99**, 016021 (2019)
26. W.H. Liang, H.X. Chen, E. Oset, E. Wang, Triangle singularity in the $J/\psi \rightarrow K^+ K^- f_0(980)$ ($a_0(980)$) decays. *Eur. Phys. J. C* **79**, 411 (2019)
27. V. Debastiani, W.H. Liang, J.J. Xie, E. Oset, Predictions for $\eta_c \rightarrow \eta \pi^+ \pi^-$ producing $f_0(500)$, $f_0(980)$ and $a_0(980)$. *Phys. Lett. B* **766**, 59–64 (2017)
28. Z. Wang, Y.Y. Wang, E. Wang, D.M. Li, J.J. Xie, The scalar $f_0(500)$ and $f_0(980)$ resonances and vector mesons in the single Cabibbo-suppressed decays $\Lambda_c \rightarrow p K^+ K^-$ and $p \pi^+ \pi^-$. *Eur. Phys. J. C* **80**, 842 (2020)
29. J. Oller, E. Oset, Chiral symmetry amplitudes in the S wave isoscalar and isovector channels and the σ , $f_0(980)$, $a_0(980)$ scalar mesons. *Nucl. Phys. A* **620**, 438–456 (1997)
30. L.R. Dai, J.M. Dias, E. Oset, Disclosing $D^* \bar{D}^*$ molecular states in the $B_c^- \rightarrow \pi^- J/\psi \omega$ decay. *Eur. Phys. J. C* **78**, 210 (2018)
31. Y. Zhang, E. Wang, D.M. Li, Y.X. Li, Search for the $D^* \bar{D}^*$ molecular state $Z_c(4000)$ in the reaction $B^- \rightarrow J/\psi \rho^0 K^-$. *Chin. Phys. C* **44**, 093107 (2020)
32. L.R. Dai, G.Y. Wang, X. Chen, E. Wang, E. Oset, D.M. Li, The $B^+ \rightarrow J/\psi \omega K^+$ reaction and $D^* \bar{D}^*$ molecular states. *Eur. Phys. J. A* **55**, 36 (2019)
33. J.A. Oller, E. Oset, A. Ramos, Chiral unitary approach to meson–meson and meson–baryon interactions and nuclear applications. *Prog. Part. Nucl. Phys.* **45**, 157–242 (2000)
34. M. Doring, U.G. Meißner, E. Oset, A. Rusetsky, Unitarized chiral perturbation theory in a finite volume: scalar meson sector. *Eur. Phys. J. A* **47**, 139 (2011)
35. Z.H. Guo, L. Liu, U.G. Meißner, J.A. Oller, A. Rusetsky, Chiral study of the $a_0(980)$ resonance and $\pi \eta$ scattering phase shifts in light of a recent lattice simulation. *Phys. Rev. D* **95**, 054004 (2017)
36. L. Alvarez-Ruso, J.A. Oller, J.M. Alarcon, The $\phi(1020)a_0(980)$ S-wave scattering and hints for a new vector–isovector resonance. *Phys. Rev. D* **82**, 094028 (2010)
37. D. Gamermann, E. Oset, D. Strottman, M.J. Vicente Vacas, Dynamically generated open and hidden charm meson systems. *Phys. Rev. D* **76**, 074016 (2007)
38. J.A. Oller, E. Oset, N/D description of two meson amplitudes and chiral symmetry. *Phys. Rev. D* **60**, 074023 (1999)
39. H.A. Ahmed, C.W. Xiao, Study the molecular nature of σ , $f_0(980)$, and $a_0(980)$ states. *Phys. Rev. D* **101**, 094034 (2020)
40. J. Oller, U.G. Meißner, Chiral dynamics in the presence of bound states: kaon nucleon interactions revisited. *Phys. Lett. B* **500**, 263–272 (2001)
41. M. Ablikim et al., [BESIII], Amplitude analysis of $D_s^+ \rightarrow \pi^+ \pi^0 \eta$ and first observation of the pure W -annihilation decays $D_s^+ \rightarrow a_0(980)^+ \pi^0$ and $D_s^+ \rightarrow a_0(980)^0 \pi^+$. *Phys. Rev. Lett.* **123**, 112001 (2019)