Eur. Phys. J. C (2020) 80:1066
https://doi.org/10.1140/epjc/s10052-020-08619-y

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Charmed Ω_c weak decays into Ω in the light-front quark model

Yu-Kuo Hsiao[1,a], **Ling Yang**[1,b], **Chong-Chung Lih**[2,c], **Shang-Yuu Tsai**[1,d]

[1] School of Physics and Information Engineering, Shanxi Normal University, Linfen 041004, China
[2] Department of Optometry, Central Taiwan University of Science and Technology, Taichung 40601, Taiwan

Received: 3 October 2020 / Accepted: 29 October 2020 / Published online: 19 November 2020

Abstract More than ten Ω_c^0 weak decay modes have been measured with the branching fractions relative to that of $\Omega_c^0 \to \Omega^-\pi^+$. In order to extract the absolute branching fractions, the study of $\Omega_c^0 \to \Omega^-\pi^+$ is needed. In this work, we predict $\mathcal{B}_\pi \equiv \mathcal{B}(\Omega_c^0 \to \Omega^-\pi^+) = (5.1 \pm 0.7) \times 10^{-3}$ with the $\Omega_c^0 \to \Omega^-$ transition form factors calculated in the light-front quark model. We also predict $\mathcal{B}_\rho \equiv \mathcal{B}(\Omega_c^0 \to \Omega^-\rho^+) = (14.4 \pm 0.4) \times 10^{-3}$ and $\mathcal{B}_e \equiv \mathcal{B}(\Omega_c^0 \to \Omega^-e^+\nu_e) = (5.4 \pm 0.2) \times 10^{-3}$. The previous values for $\mathcal{B}_\rho/\mathcal{B}_\pi$ have been found to deviate from the most recent observation. Nonetheless, our $\mathcal{B}_\rho/\mathcal{B}_\pi = 2.8 \pm 0.4$ is able to alleviate the deviation. Moreover, we obtain $\mathcal{B}_e/\mathcal{B}_\pi = 1.1 \pm 0.2$, which is consistent with the current data.

1 Introduction

The lowest-lying singly charmed baryons include the anti-triplet and sextet states $\mathbf{B}_c = (\Lambda_c^+, \Xi_c^0, \Xi_c^+)$ and $\mathbf{B}'_c = (\Sigma_c^{(0,+,++)}, \Xi_c^{\prime(0,+)}, \Omega_c^0)$, respectively. The $\mathbf{B}_c$ and Ω_c^0 baryons predominantly decay weakly [1–5], whereas the Σ_c (Ξ'_c) decays are strong (electromagnetic) processes. There have been more accurate observations for the $\mathbf{B}_c$ weak decays in the recent years, which have helped to improve the theoretical understanding of the decay processes [6–14]. With the lower production cross section of $\sigma(e^+e^- \to \Omega_c^0 X)$ [4], it is an uneasy task to measure Ω_c^0 decays. Consequently, most of the Ω_c^0 decays have not been reanalyzed since 1990s [15–23], except for those in [24–29].

One still manages to measure more than ten Ω_c^0 decays, such as $\Omega_c^0 \to \Omega^-\rho^+$, $\Xi^0\bar{K}^{(*)0}$ and $\Omega^-\ell^+\nu_\ell$, but with the branching fractions relative to $\mathcal{B}(\Omega_c^0 \to \Omega^-\pi^+)$ [5]. To extract the absolute branching fractions, the study of $\Omega_c^0 \to \Omega^-\pi^+$ is crucial. Fortunately, the $\Omega_c^0 \to \Omega^-\pi^+$ decay involves a simple topology, which benefits its theoretical exploration. In Fig. 1a, $\Omega_c^0 \to \Omega^-\pi^+$ is depicted to proceed through the $\Omega_c^0 \to \Omega^-$ transition, while π^+ is produced from the external W-boson emission. Since it is a Cabibbo-allowed process with $V_{cs}^* V_{ud} \simeq 1$, a larger branching fraction is promising for measurements. Furthermore, it can be seen that $\Omega_c^0 \to \Omega^-\pi^+$ has a similar configuration to those of $\Omega_c^0 \to \Omega^-\rho^+$ and $\Omega_c^0 \to \Omega^-\ell^+\nu_\ell$, as drawn in Fig. 1, indicating that the three Ω_c^0 decays are all associated with the $\Omega_c^0 \to \Omega^-$ transition. While Ω is a decuplet baryon that consists of the totally symmetric identical quarks sss, behaving as a spin-3/2 particle, the form factors of the $\Omega_c^0 \to \Omega^-$ transition can be more complicated, which hinders the calculation for the decays. As a result, a careful investigation that relates $\Omega_c^0 \to \Omega^-\pi^+, \Omega^-\rho^+$ and $\Omega_c^0 \to \Omega^-\ell^+\nu_\ell$ has not been given yet, despite the fact that the topology associates them together.

Based on the quark models, it is possible to study the Ω_c^0 decays into Ω^- with the $\Omega_c^0 \to \Omega^-$ transition form factors. However, the validity of theoretical approach needs to be tested, which depends on if the observations, given by

$$\frac{\mathcal{B}(\Omega_c^0 \to \Omega^-\rho^+)}{\mathcal{B}(\Omega_c^0 \to \Omega^-\pi^+)} = 1.7 \pm 0.3\,[4]\,(> 1.3\,[5])\,,$$
$$\frac{\mathcal{B}(\Omega_c^0 \to \Omega^-e^+\nu_e)}{\mathcal{B}(\Omega_c^0 \to \Omega^-\pi^+)} = 2.4 \pm 1.2\,[5]\,, \quad (1)$$

can be interpreted. Since the light-front quark model has been successfully applied to the heavy hadron decays [27,30–46], in this report we will use it to study the $\Omega_c^0 \to \Omega^-$ transition form factors. Accordingly, we will be enabled to calculate the absolute branching fractions of $\Omega_c^0 \to \Omega^-\pi^+(\rho^+)$ and $\Omega_c^0 \to \Omega^-\ell^+\nu_\ell$, and check if the two ratios in Eq. (1) can be well explained.

[a] e-mail: yukuohsiao@gmail.com
[b] e-mail: yangling@ihep.ac.cn
[c] e-mail: cclih@phys.nthu.edu.tw
[d] e-mail: shangyuu@gmail.com (corresponding author)

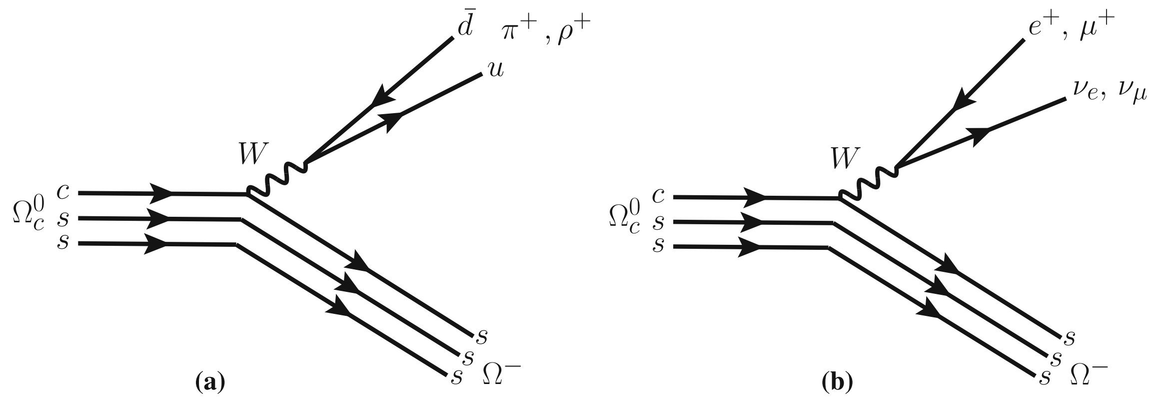

Fig. 1 Feynman diagrams for **a** $\Omega_c^0 \to \Omega^-\pi^+(\rho^+)$ and **b** $\Omega_c^0 \to \Omega^-\ell^+\nu_\ell$ with $\ell^+ = e^+$ or μ^+

2 Theoretical framework

2.1 General formalism

To start with, we present the effective weak Hamiltonians $\mathcal{H}_{H,L}$ for the hadronic and semileptonic charmed baryon decays, respectively [47]:

$$\mathcal{H}_H = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud}[c_1(\bar{u}d)(\bar{s}c) + c_2(\bar{s}d)(\bar{u}c)]\,,$$
$$\mathcal{H}_L = \frac{G_F}{\sqrt{2}} V_{cs}^* (\bar{s}c)(\bar{u}_\nu v_\ell)\,, \qquad (2)$$

where G_F is the Fermi constant, V_{ij} the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements, $c_{1,2}$ the effective Wilson coefficients, $(\bar{q}_1 q_2) \equiv \bar{q}_1\gamma_\mu(1-\gamma_5)q_2$ and $(\bar{u}_\nu v_\ell) \equiv \bar{u}_\nu\gamma^\mu(1-\gamma_5)v_\ell$. In terms of $\mathcal{H}_{H,L}$, we derive the amplitudes of $\Omega_c^0 \to \Omega^-\pi^+(\rho^+)$ and $\Omega_c^0 \to \Omega^-\ell^+\nu_\ell$ as [48,49]

$$\begin{aligned}\mathcal{M}_h &\equiv \mathcal{M}(\Omega_c^0 \to \Omega^- h^+)\\ &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud}\, a_1 \langle \Omega^-|(\bar{s}c)|\Omega_c^0\rangle\langle h^+|(\bar{u}d)|0\rangle\,,\\ \mathcal{M}_\ell &\equiv \mathcal{M}(\Omega_c^0 \to \Omega^-\ell^+\nu_\ell)\\ &= \frac{G_F}{\sqrt{2}} V_{cs}^* \langle \Omega^-|(\bar{s}c)|\Omega_c^0\rangle(\bar{u}_{\nu_\ell} v_\ell)\,, \qquad (3)\end{aligned}$$

where $h = (\pi, \rho)$, $\ell = (e, \mu)$, and $a_1 = c_1 + c_2/N_c$ results from the factorization [50], with N_c the color number.

With $\mathbf{B}_c'$ ($\mathbf{B}'$) denoting the charmed sextet (decuplet) baryon, the matrix elements of the $\mathbf{B}_c' \to \mathbf{B}'$ transition can be parameterized as [28,45]

$$\begin{aligned}\langle T^\mu\rangle &\equiv \langle \mathbf{B}'(P', S', S_z')|\bar{q}\gamma^\mu(1-\gamma_5)c|\mathbf{B}_c'(P, S, S_z)\rangle\\ &= \bar{u}_\alpha(P', S_z')\left[\frac{P^\alpha}{M}\left(\gamma^\mu F_1^V + \frac{P^\mu}{M}F_2^V + \frac{P'^\mu}{M'}F_3^V\right)\right.\\ &\quad \left. + g^{\alpha\mu}F_4^V\right]\gamma_5 u(P, S_z)\\ &\quad -\bar{u}_\alpha(P', S_z')\left[\frac{P^\alpha}{M}\left(\gamma^\mu F_1^A + \frac{P^\mu}{M}F_2^A + \frac{P'^\mu}{M'}F_3^A\right)\right.\\ &\quad \left. + g^{\alpha\mu}F_4^A\right]u(P, S_z)\,, \qquad (4)\end{aligned}$$

where (M, M') and $(S, S') = (1/2, 3/2)$ represent the masses and spins of $(\mathbf{B}_c', \mathbf{B}')$, respectively, and $F_i^{V,A}$ $(i = 1, 2, \ldots, 4)$ the form factors to be extracted in the light-front quark model. The matrix elements of the meson productions are defined as [5]

$$\langle \pi(p)|(\bar{u}d)|0\rangle = i f_\pi q^\mu\,,$$
$$\langle \rho(\lambda)|(\bar{u}d)|0\rangle = m_\rho f_\rho \epsilon_\lambda^{\mu*}\,, \qquad (5)$$

where $f_{\pi(\rho)}$ is the decay constant, and ϵ_λ^μ is the polarization four-vector with λ denoting the helicity state.

2.2 The light-front quark model

The baryon bound state $\mathbf{B}_{(c)}'$ contains three quarks q_1, q_2 and q_3, with the subscript c for $q_1 = c$. Moreover, q_2 and q_3 are combined as a diquark state $q_{[2,3]}$, behaving as a scalar or axial-vector. Subsequently, the baryon bound state $|\mathbf{B}_{(c)}'(P, S, S_z)\rangle$ in the light-front quark model can be written as [31]

$$\begin{aligned}|\mathbf{B}_{(c)}'(P, S, S_z)\rangle &= \int \{d^3p_1\}\{d^3p_2\} 2(2\pi)^3\delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2)\\ &\quad \times \sum_{\lambda_1,\lambda_2} \Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2)|q_1(p_1, \lambda_1) q_{[2,3]}(p_2, \lambda_2)\rangle\,, \qquad (6)\end{aligned}$$

where Ψ^{SS_z} is the momentum-space wave function, and (p_i, λ_i) stand for momentum and helicity of the constituent (di)quark, with $i = 1, 2$ for q_1 and $q_{[2,3]}$, respectively. The tilde notations represent that the quantities are in the light-front frame, and one defines $P = (P^-, P^+, P_\perp)$ and $\tilde{P} = (P^+, P_\perp)$, with $P^\pm = P^0 \pm P^3$ and $P_\perp = (P^1, P^2)$.

Besides, $\tilde{p}_i$ are given by

$$\tilde{p}_i = (p_i^+, p_{i\perp}), \quad p_{i\perp} = (p_i^1, p_i^2), \quad p_i^- = \frac{m_i^2 + p_{i\perp}^2}{p_i^+}, \tag{7}$$

with

$$m_1 = m_{q_1}, \quad m_2 = m_{q_1} + m_{q_2},$$
$$p_1^+ = (1-x)P^+, \quad p_2^+ = xP^+,$$
$$p_{1\perp} = (1-x)P_\perp - k_\perp, \quad p_{2\perp} = xP_\perp + k_\perp, \tag{8}$$

where x and $k_\perp$ are the light-front relative momentum variables with $k_\perp$ from $\vec{k} = (k_\perp, k_z)$, ensuring that $P^+ = p_1^+ + p_2^+$ and $P_\perp = p_{1\perp} + p_{2\perp}$. According to $e_i \equiv \sqrt{m_i^2 + \vec{k}^2}$ and $M_0 \equiv e_1 + e_2$ in the Melosh transformation [30], we obtain

$$x = \frac{e_2 - k_z}{e_1 + e_2}, \quad 1 - x = \frac{e_1 + k_z}{e_1 + e_2}, \quad k_z = \frac{xM_0}{2} - \frac{m_2^2 + k_\perp^2}{2xM_0},$$
$$M_0^2 = \frac{m_1^2 + k_\perp^2}{1-x} + \frac{m_2^2 + k_\perp^2}{x}. \tag{9}$$

Consequently, Ψ^{SS_z} can be given in the following representation [41–45]:

$$\Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \frac{A^{(\prime)}}{\sqrt{2(p_1 \cdot \bar{P} + m_1 M_0)}} \bar{u}(p_1, \lambda_1)\Gamma_{S,A}^{(\alpha)} u(\bar{P}, S_z)\phi(x, k_\perp), \tag{10}$$

with

$$A = \sqrt{\frac{3(m_1 M_0 + p_1 \cdot \bar{P})}{3m_1 M_0 + p_1 \cdot \bar{P} + 2(p_1 \cdot p_2)(p_2 \cdot \bar{P})/m_2^2}},$$
$$\Gamma_S = 1, \quad \Gamma_A = -\frac{1}{\sqrt{3}}\gamma_5 \not{\epsilon}^*(p_2, \lambda_2),$$

and

$$A' = \sqrt{\frac{3m_2^2 M_0^2}{2m_2^2 M_0^2 + (p_2 \cdot \bar{P})^2}}, \quad \Gamma_A^\alpha = \epsilon^{*\alpha}(p_2, \lambda_2), \tag{11}$$

where the vertex function $\Gamma_{S(A)}$ is for the scalar (axial-vector) diquark in $\mathbf{B}_c'$, and Γ_A^α for the axial-vector diquark in $\mathbf{B}'$. We have used the variable $\bar{P} \equiv p_1 + p_2$ to describe the internal motions of the constituent quarks in the baryon [32], which leads to $(\bar{P}_\mu \gamma^\mu - M_0)u(\bar{P}, S_z) = 0$, different from $(P_\mu \gamma^\mu - M)u(P, S_z) = 0$. For the momentum distribution, $\phi(x, k_\perp)$ is presented as the Gaussian-type wave function, given by

$$\phi(x, k_\perp) = 4\left(\frac{\pi}{\beta^2}\right)^{3/4} \sqrt{\frac{e_1 e_2}{x(1-x)M_0}} \exp\left(\frac{-\vec{k}^2}{2\beta^2}\right), \tag{12}$$

where β shapes the distribution.

Using $|\mathbf{B}_c'(P, S, S_z)\rangle$ and $|\mathbf{B}'(P,' S', S_z')\rangle$ from Eq. (6) and their components in Eqs. (10), (11) and (12), we derive the matrix elements of the $\mathbf{B}_c' \to \mathbf{B}'$ transition in Eq. (4) as

$$\begin{aligned}\langle \bar{T}^\mu \rangle &\equiv \langle \mathbf{B}'(P', S', S_z')|\bar{q}\gamma^\mu(1-\gamma_5)c|\mathbf{B}_c'(P, S, S_z)\rangle \\ &= \int \{d^3 p_2\} \frac{\phi'(x', k_\perp')\phi(x, k_\perp)}{2\sqrt{p_1^+ p_1'^+(p_1 \cdot \bar{P} + m_1 M_0)(p_1' \cdot \bar{P}' + m_1' M_0')}} \\ &\times \sum_{\lambda_2} \bar{u}_\alpha(\bar{P}', S_z')\left[\bar{\Gamma}_A'^\alpha(\not{p}_1' + m_1') \right.\\ &\times \left.\gamma^\mu(1-\gamma_5)(\not{p}_1 + m_1)\Gamma_A\right] u(\bar{P}, S_z),\end{aligned} \tag{13}$$

with $m_1 = m_c, m_1' = m_q$ and $\bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$. We define $J_{5j}^\mu = \bar{u}(\Gamma_5^{\mu\beta})_j u_\beta$ and $\bar{J}_{5j}^\mu = \bar{u}(\bar{\Gamma}_5^{\mu\beta})_j u_\beta$ with $j = 1, 2, ..., 4$, where

$$(\Gamma_5^{\mu\beta})_j = \{\gamma^\mu P^\beta, P'^\mu P^\beta, P^\mu P^\beta, g^{\mu\beta}\}\gamma_5,$$
$$(\bar{\Gamma}_5^{\mu\beta})_j = \{\gamma^\mu \bar{P}^\beta, \bar{P}'^\mu \bar{P}^\beta, \bar{P}^\mu \bar{P}^\beta, g^{\mu\beta}\}\gamma_5. \tag{14}$$

Then, we multiply J_{5j} ($\bar{J}_{5j}$) by $\langle T\rangle$ ($\langle \bar{T}\rangle$) as $F_{5j} \equiv J_{5j} \cdot \langle T\rangle$ and $\bar{F}_{5j} \equiv \bar{J}_{5j} \cdot \langle \bar{T}\rangle$ with $\langle T\rangle$ and $\langle \bar{T}\rangle$ in Eqs. (4) and (13), respectively, resulting in [45]

$$\begin{aligned}F_{5j} &= Tr\left\{u_\beta \bar{u}_\alpha \left[\frac{P^\alpha}{M}\left(\gamma^\mu F_1^V + \frac{P^\mu}{M}F_2^V + \frac{P'^\mu}{M'}F_3^V\right)\right.\right. \\ &\left.\left. + g^{\alpha\mu}F_4^V\right]\gamma_5 \bar{u}(\Gamma_{5\mu}^\beta)_j\right\}, \\ \bar{F}_{5j} &= \int \{d^3 p_2\} \frac{\phi'(x', k_\perp')\phi(x, k_\perp)}{2\sqrt{p_1^+ p_1'^+(p_1 \cdot \bar{P} + m_1 M_0)(p_1' \cdot \bar{P}' + m_1' M_0')}} \\ &\times \sum_{\lambda_2} Tr\left\{u_\beta \bar{u}_\alpha \left[\bar{\Gamma}_A'^\alpha(\not{p}_1' + m_1')\gamma^\mu(\not{p}_1 + m_1)\Gamma_A\right] u(\bar{\Gamma}_{5\mu}^\beta)_j\right\}.\end{aligned} \tag{15}$$

In the connection of $F_{5j} = \bar{F}_{5j}$, we construct four equations. By solving the four equations, the four form factors F_1^V, F_2^V, F_3^V and F_4^V can be extracted. The form factors F_i^A can be obtained in the same way.

2.3 Branching fractions in the helicity basis

One can present the amplitude of $\Omega_c^0 \to \Omega^- h^+(\Omega^-\ell^+\nu_\ell)$ in the helicity basis of $H_{\lambda_\Omega \lambda_{h(\ell)}}$ [28,45], where $\lambda_\Omega = \pm 3/2, \pm 1/2$ represent the helicity states of the Ω^- baryon, and $\lambda_{h,\ell}$ those of h^+ and $\ell^+\nu_\ell$. Substituting the matrix elements in Eqs. (3) with those in Eqs. (4) and (5), the amplitudes in the helicity basis now read $\sqrt{2}\mathcal{M}_h = (i)\sum_{\lambda_\Omega, \lambda_h} G_F V_{cs}^* V_{ud} a_1 m_h f_h H_{\lambda_\Omega \lambda_h}$ and $\sqrt{2}\mathcal{M}_\ell = \sum_{\lambda_\Omega, \lambda_\ell} G_F V_{cs}^* H_{\lambda_\Omega \lambda_\ell}$, where $H_{\lambda_\Omega \lambda_f} = H_{\lambda_\Omega \lambda_f}^V - H_{\lambda_\Omega \lambda_f}^A$ with $f = (h, \ell)$. Explicitly, $H_{\lambda_\Omega \lambda_f}^{V(A)}$ is written as [28]

$$H_{\lambda_\Omega \lambda_f}^{V(A)} \equiv \langle \Omega^- |\bar{s}\gamma_\mu(\gamma_5)c|\Omega_c^0\rangle \varepsilon_f^\mu, \tag{16}$$

with $\varepsilon_h^\mu = (q^\mu/\sqrt{q^2}, \epsilon_\lambda^{\mu*})$ for $h = (\pi, \rho)$. For the semi-leptonic decay, since the $\ell^+\nu_\ell$ system behaves as a scalar or vector, $\varepsilon_\ell^\mu = q^\mu/\sqrt{q^2}$ or $\epsilon_\lambda^{\mu*}$. The π meson only has a zero helicity state, denoted by $\lambda_\pi = \bar{0}$. On the other hand, the three helicity states of ρ are denoted by $\lambda_\rho = (1, 0, -1)$. For the lepton pair, we assign $\lambda_\ell = \lambda_\pi$ or λ_ρ. Subsequently, we expand $H^{V(A)}_{\lambda_\Omega \lambda_f}$ as

$$H^{V(A)}_{\frac{1}{2}\bar{0}} = \sqrt{\frac{2}{3}\frac{Q_\pm^2}{q^2}}\left(\frac{Q_\mp^2}{2MM'}\right)(F_1^{V(A)} M_\pm \mp F_2^{V(A)}\bar{M}_+ \mp F_3^{V(A)}\bar{M}'_- \mp F_4^{V(A)} M)\,, \tag{17}$$

for $\varepsilon_f^\mu = q^\mu/\sqrt{q^2}$, where $M_\pm = M \pm M'$, $Q_\pm^2 = M_\pm^2 - q^2$, and $\bar{M}_\pm^{(\prime)} = (M_+ M_- \pm q^2)/(2M^{(\prime)})$. We also obtain

$$H^{V(A)}_{\frac{3}{2}1} = \mp\sqrt{Q_\mp^2}\, F_4^{V(A)}\,,$$
$$H^{V(A)}_{\frac{1}{2}1} = -\sqrt{\frac{Q_\mp^2}{3}}\left[F_1^{V(A)}\left(\frac{Q_\pm^2}{MM'}\right) - F_4^{V(A)}\right],$$
$$H^{V(A)}_{\frac{1}{2}0} = \sqrt{\frac{2}{3}\frac{Q_\mp^2}{q^2}}\left[F_1^{V(A)}\left(\frac{Q_\pm^2 M_\mp}{2MM'}\right) \mp \left(F_2^{V(A)} + F_3^{V(A)}\frac{M}{M'}\right)\left(\frac{|\vec{P}'|^2}{M'}\right) \mp F_4^{V(A)}\bar{M}'_-\right], \tag{18}$$

for $\varepsilon_f^\mu = \epsilon_\lambda^{\mu*}$, with $|\vec{P}'| = \sqrt{Q_+^2 Q_-^2}/(2M)$. Note that the expansions in Eqs. (17) and (18) have satisfied $\lambda_{\Omega_c} = \lambda_\Omega - \lambda_f$ for the helicity conservation, with $\lambda_{\Omega_c} = \pm 1/2$. The branching fractions then read

$$\mathcal{B}_h \equiv \mathcal{B}(\Omega_c^0 \to \Omega^- h^+) = \frac{\tau_{\Omega_c} G_F^2 |\vec{P}'|}{32\pi m_{\Omega_c}^2}|V_{cs}V_{ud}^*|^2 a_1^2 m_h^2 f_h^2 H_h^2\,,$$
$$\mathcal{B}_\ell \equiv \mathcal{B}(\Omega_c^0 \to \Omega^- \ell^+\nu_\ell) = \frac{\tau_{\Omega_c} G_F^2 |V_{cs}|^2}{192\pi^3 m_{\Omega_c}^2}\int_{m_\ell^2}^{(m_{\Omega_c}-m_\Omega)^2} dq^2 \left(\frac{|\vec{P}'|(q^2 - m_\ell^2)^2}{q^2}\right) H_\ell^2\,, \tag{19}$$

where

$$H_\pi^2 = \left|H_{\frac{1}{2}\bar{0}}\right|^2 + \left|H_{-\frac{1}{2}\bar{0}}\right|^2,$$
$$H_\rho^2 = \left|H_{\frac{3}{2}1}\right|^2 + \left|H_{\frac{1}{2}1}\right|^2 + \left|H_{\frac{1}{2}0}\right|^2 + \left|H_{-\frac{1}{2}0}\right|^2 + \left|H_{-\frac{1}{2}-1}\right|^2 + \left|H_{-\frac{3}{2}-1}\right|^2,$$
$$H_\ell^2 = \left(1 + \frac{m_\ell^2}{2q^2}\right) H_\rho^2 + \frac{3m_\ell^2}{2q^2} H_\pi^2\,, \tag{20}$$

with τ_{Ω_c} the Ω_c^0 lifetime.

Table 1 The $\Omega_c^0 \to \Omega^-$ transition form factors with $F(0)$ at $q^2 = 0$, where $\delta \equiv \delta m_c/m_c = \pm 0.04$ from Eq. (21)

	$F(0)$	a	b
F_1^V	$0.54 + 0.13\delta$	-0.27	1.65
F_2^V	$0.35 - 0.36\delta$	-30.00	96.82
F_3^V	$0.33 + 0.59\delta$	0.96	9.25
F_4^V	$0.97 + 0.22\delta$	-0.53	1.41
F_1^A	$2.05 + 1.38\delta$	-3.66	1.41
F_2^A	$-0.06 + 0.33\delta$	-1.15	71.66
F_3^A	$-1.32 - 0.32\delta$	-4.01	5.68
F_4^A	$-0.44 + 0.11\delta$	-1.29	-0.58

3 Numerical analysis

In the Wolfenstein parameterization, the CKM matrix elements are adopted as $V_{cs} = V_{ud} = 1 - \lambda^2/2$ with $\lambda = 0.22453 \pm 0.00044$ [5]. We take the lifetime and mass of the Ω_c^0 baryon and the decay constants $(f_\pi, f_\rho) = (132, 216)$ MeV from the PDG [5]. With $(c_1, c_2) = (1.26, -0.51)$ at the m_c scale [47], we determine a_1. In the generalized factorization, N_c is taken as an effective color number with $N_c = (2, 3, \infty)$ [28,29,46,50], in order to estimate the non-factorizable effects. For the $\Omega_c^+(css) \to \Omega^-(sss)$ transition form factors, the theoretical inputs of the quark masses and parameter β in Eq. (15) are given by [34,40]

$$m_1 = m_c = (1.35 \pm 0.05)\text{ GeV}\,,\quad m_1' = m_s = 0.38\text{ GeV}\,,$$
$$m_2 = 2m_s = 0.76\text{ GeV}\,,$$
$$\beta_c = 0.60\text{ GeV}\,,\quad \beta_s = 0.46\text{ GeV}\,, \tag{21}$$

where $\beta_{c(s)}$ is to determine $\phi^{(\prime)}(x^{(\prime)}, k_\perp^{(\prime)})$ for Ω_c^0 (Ω^-). We hence extract F_i^V and F_i^A in Table 1. For the momentum dependence, we have used the double-pole parameterization:

$$F(q^2) = \frac{F(0)}{1 - a\left(q^2/m_F^2\right) + b\left(q^4/m_F^4\right)}\,, \tag{22}$$

with $m_F = 1.86$ GeV.

Using the theoretical inputs, we calculate the branching fractions, whose results are given in Table 2.

4 Discussions and conclusions

In Table 2, we present $\mathcal{B}_\pi$ and $\mathcal{B}_\rho$ with $N_c = (2, 3, \infty)$. The errors come from the form factors in Table 1, of which the uncertainties are correlated with the charm quark mass. By comparison, $\mathcal{B}_\pi$ and $\mathcal{B}_\rho$ are compatible with the values in Ref. [28]; however, an order of magnitude smaller than those in Refs. [20,22], whose values are obtained with the total decay widths $\Gamma_{\pi(\rho)} = 2.09a_1^2(11.34a_1^2) \times 10^{11}$ s^{-1}

Table 2 Branching fractions of (non-)leptonic Ω_c^0 decays and their ratios, where $\mathcal{R}_{\rho(e)/\pi} \equiv \mathcal{B}_{\rho(e)}/\mathcal{B}_\pi$. The three numbers in the parenthesis correspond to $N_c = (2, 3, \infty)$, and the errors come from the uncertainties of the form factors in Table 1

$\mathcal{B}(\mathcal{R})$	Our work	Ref. [20]	Ref. [22]	Ref. [28]	Ref. [24]	Data [4,5]
$10^3\mathcal{B}_\pi$	$(5.1 \pm 0.7, 6.0 \pm 0.8, 8.0 \pm 1.0)$	(56.6, 66.5, 88.9)	(36.0, 42.3, 56.6)	(−, −, 2)		
$10^3\mathcal{B}_\rho$	$(14.4 \pm 0.4, 17.0 \pm 0.5, 22.1 \pm 0.6)$	(307.0, 361.1, 482.5)	(126.7, 149.0, 199.1)	(−, −, 19)		
$10^3\mathcal{B}_e$	5.4 ± 0.2				127	
$10^3\mathcal{B}_\mu$	5.0 ± 0.2					
$\mathcal{R}_{\rho/\pi}$	2.8 ± 0.4	5.4	3.5	9.5		1.7 ± 0.3 (> 1.3)
$\mathcal{R}_{e/\pi}$	$(1.1 \pm 0.2, 0.9 \pm 0.1, 0.7 \pm 0.1)$					2.4 ± 1.2

and $\Gamma_{\pi(\rho)} = 1.33a_1^2(4.68a_1^2) \times 10^{11}$ s^{-1}, respectively. We also predict $\mathcal{B}_e = (5.4 \pm 0.2) \times 10^{-3}$ as well as $\mathcal{B}_\mu \simeq \mathcal{B}_e$, which is much smaller than the value of 127×10^{-3} in [24]. Only the ratios $\mathcal{R}_{\rho/\pi}$ and $\mathcal{R}_{e/\pi}$ have been actually observed so far. In our work, $\mathcal{R}_{\rho/\pi} = 2.8 \pm 0.4$ is able to alleviate the inconsistency between the previous value and the most recent observation. We obtain $\mathcal{R}_{e/\pi} = 1.1 \pm 0.2$ with $N_c = 2$ to be consistent with the data, which indicates that $(\mathcal{B}_\pi, \mathcal{B}_\rho) = (5.1 \pm 0.7, 14.4 \pm 0.4) \times 10^{-3}$ with $N_c = 2$ are more favorable.

The helicity amplitudes can be used to better understand how the form factors contribute to the branching fractions. With the identity $H^{V(A)}_{-\lambda_\Omega -\lambda_f} = \mp H^{V(A)}_{\lambda_\Omega \lambda_f}$ for the $\mathbf{B}'_c(J^P = 1/2^+)$ to $\mathbf{B}'(J^P = 3/2^+)$ transition [28], H_π^2 in Eq. (20) can be rewritten as $H_\pi^2 = 2(|H^V_{\frac{1}{2}\bar{0}}|^2 + |H^A_{\frac{1}{2}\bar{0}}|^2)$. From the prefactors in Eq. (17), we estimate the ratio of $|H^V_{\frac{1}{2}\bar{0}}|^2/|H^A_{\frac{1}{2}\bar{0}}|^2 \simeq 0.05$, which shows that $H^A_{\frac{1}{2}\bar{0}}$ dominates $\mathcal{B}_\pi$, instead of $H^V_{\frac{1}{2}\bar{0}}$. More specifically, it is the F_4^A term in $H^A_{\frac{1}{2}\bar{0}}$ that gives the main contribution to the branching fraction. By contrast, the $F^A_{1,3}$ terms in $H^A_{\frac{1}{2}\bar{0}}$ largely cancel each other, which is caused by $F_1^A M_- \simeq F_3^A \bar{M}'_-$ and a minus sign between F_1^A and F_3^A (see Table 1); besides, the F_2^A term with a small $F_2^A(0)$ is ignorable.

Likewise, we obtain $H_\rho^2 = 2(|H_\rho^V|^2 + |H_\rho^A|^2)$ for $\mathcal{B}_\rho$, where $|H_\rho^{V(A)}|^2 = |H^{V(A)}_{\frac{3}{2}1}|^2 + |H^{V(A)}_{\frac{1}{2}1}|^2 + |H^{V(A)}_{\frac{1}{2}0}|^2$. We find that $|H_\rho^A|^2$ is ten times larger than $|H_\rho^V|^2$. Moreover, $H^A_{\frac{1}{2}0}$ is similar to $H^A_{\frac{1}{2}\bar{0}}$, where the $F^A_{1,3}$ terms largely cancel each other, F_2^A is ignorable, and F_4^A gives the main contribution. While F_1^A and F_4^A in $H^A_{\frac{1}{2}1}$ have a positive interference, giving 20% of $\mathcal{B}_\rho$, F_4^A in $H^A_{\frac{3}{2}1}$ singly contributes 35%. In Eq. (20), the factor of m_ℓ^2/q^2 with $m_\ell \simeq 0$ should be much suppressed, such that $H_\ell^2 \simeq H_\rho^2$. Therefore, $\mathcal{B}_\ell$ receives the main contributions from the F_4^A terms in $H^A_{\frac{1}{2}0}$, $H^A_{\frac{1}{2}1}$ and $H^A_{\frac{3}{2}1}$, which is similar to the analysis for $\mathcal{B}_\rho$.

In summary, we have studied the $\Omega_c^0 \to \Omega^-\pi^+, \Omega^-\rho^+$ and $\Omega_c^0 \to \Omega^-\ell^+\nu_\ell$ decays, which proceed through the $\Omega_c^0 \to \Omega^-$ transition and the formation of the meson $\pi^+(\rho^+)$ or lepton pair from the external W-boson emission. With the form factors of the $\Omega_c^0 \to \Omega^-$ transition, calculated in the light-front quark model, we have predicted $\mathcal{B}(\Omega_c^0 \to \Omega^-\pi^+, \Omega^-\rho^+) = (5.1 \pm 0.7, 14.4 \pm 0.4) \times 10^{-3}$ and $\mathcal{B}(\Omega_c^0 \to \Omega^-e^+\nu_e) = (5.4 \pm 0.2) \times 10^{-3}$. While the previous studies have given the $\mathcal{R}_{\rho/\pi}$ values deviating from the most recent observation, we have presented $\mathcal{R}_{\rho/\pi} = 2.8 \pm 0.4$ to alleviate the deviation. Moreover, we have obtained $\mathcal{R}_{e/\pi} = 1.1 \pm 0.2$, consistent with the current data.

Acknowledgements YKH was supported in part by National Science Foundation of China (No. 11675030). CCL was supported in part by CTUST (No. CTU109-P-108).

Data Availability Statement This manuscript has associated data in a data repository. [Authors' comment: The data supporting the findings of this study are available at https://doi.org/10.1103/PhysRevD.98.030001 and https://doi.org/10.1103/PhysRevD.97.032001.]

Funded by SCOAP[3].

References

1. D. Cronin-Hennessy et al., CLEO Collaboration. Phys. Rev. Lett. **86**, 3730 (2001)
2. R. Ammar et al., CLEO Collaboration. Phys. Rev. Lett. **89**, 171803 (2002)
3. B. Aubert et al., BaBar Collaboration. Phys. Rev. Lett. **99**, 062001 (2007)
4. J. Yelton et al., Belle Collaboration. Phys. Rev. D **97**, 032001 (2018)
5. M. Tanabashi et al., Particle Data Group. Phys. Rev. D **98**, 030001 (2018)
6. C.D. Lu, W. Wang, F.S. Yu, Phys. Rev. D **93**, 056008 (2016)
7. C.Q. Geng, Y.K. Hsiao, Y.H. Lin, L.L. Liu, Phys. Lett. B **776**, 265 (2017)
8. C.Q. Geng, Y.K. Hsiao, C.W. Liu, T.H. Tsai, Phys. Rev. D **97**, 073006 (2018)
9. C.Q. Geng, Y.K. Hsiao, C.W. Liu, T.H. Tsai, Phys. Rev. D **99**, 073003 (2019)
10. Y.K. Hsiao, Y. Yu, H.J. Zhao, Phys. Lett. B **792**, 35 (2019)
11. H.J. Zhao, Y.L. Wang, Y.K. Hsiao, Y. Yu, JHEP **2002**, 165 (2020)
12. J. Zou, F. Xu, G. Meng, H.Y. Cheng, Phys. Rev. D **101**, 014011 (2020)
13. Y.K. Hsiao, Q. Yi, S.T. Cai, H.J. Zhao,. arXiv:2006.15291
14. P.Y. Niu, J.M. Richard, Q. Wang, Q. Zhao,. arXiv:2003.09323
15. M. Avila-Aoki, A. Garcia, R. Huerta, R. Perez-Marcial, Phys. Rev. D **40**, 2944 (1989)
16. R. Perez-Marcial, R. Huerta, A. Garcia, M. Avila-Aoki, Phys. Rev. D **40**, 2955 (1989)
17. R.L. Singleton, Phys. Rev. D **43**, 2939 (1991)
18. F. Hussain, J. Korner, Z. Phys, C **51**, 607 (1991)
19. J. Korner, M. Kramer, Z. Phys, C **55**, 659 (1992)
20. Q. Xu, A. Kamal, Phys. Rev. D **46**, 3836 (1992)
21. H.Y. Cheng, B. Tseng, Phys. Rev. D **48**, 4188 (1993)
22. H.Y. Cheng, Phys. Rev. D **56**, 2799 (1997)
23. M.A. Ivanov, J. Korner, V.E. Lyubovitskij, A. Rusetsky, Phys. Rev. D **57**, 5632 (1998)
24. M. Pervin, W. Roberts, S. Capstick, Phys. Rev. C **74**, 025205 (2006)
25. R. Dhir, C. Kim, Phys. Rev. D **91**, 114008 (2015)
26. C.Q. Geng, Y.K. Hsiao, C.W. Liu, T.H. Tsai, JHEP **1711**, 147 (2017)
27. Z.X. Zhao, Chin. Phys. C **42**, 093101 (2018)
28. T. Gutsche, M.A. Ivanov, J.G. Korner, V.E. Lyubovitskij, Phys. Rev. D **98**, 074011 (2018)
29. S. Hu, G. Meng, F. Xu, Phys. Rev. D **101**, 094033 (2020)
30. H.J. Melosh, Phys. Rev. D **9**, 1095 (1974)
31. H.G. Dosch, M. Jamin, B. Stech, Z. Phys, C **42**, 167 (1989)
32. W. Jaus, Phys. Rev. D **44**, 2851 (1991)
33. F. Schlumpf, Phys. Rev. D **47**, 4114 (1993); Erratum: [Phys. Rev. D **49**, 6246 (1994)]
34. C.Q. Geng, C.C. Lih, W.M. Zhang, Mod. Phys. Lett. A **15**, 2087 (2000)
35. C.R. Ji, C. Mitchell, Phys. Rev. D **62**, 085020 (2000)
36. B.L.G. Bakker, C.R. Ji, Phys. Rev. D **65**, 073002 (2002)
37. B.L.G. Bakker, H.M. Choi, C.R. Ji, Phys. Rev. D **67**, 113007 (2003)
38. H.Y. Cheng, C.K. Chua, C.W. Hwang, Phys. Rev. D **69**, 074025 (2004)
39. H.M. Choi, C.R. Ji, Few Body Syst. **55**, 435 (2014)
40. C.Q. Geng, C.C. Lih, Eur. Phys. J. C **73**, 2505 (2013)
41. H.W. Ke, X.H. Yuan, X.Q. Li, Z.T. Wei, Y.X. Zhang, Phys. Rev. D **86**, 114005 (2012)
42. H.W. Ke, N. Hao, X.Q. Li, J. Phys. G **46**, 115003 (2019)
43. X.H. Hu, R.H. Li, Z.P. Xing, Eur. Phys. J. C **80**, 320 (2020)
44. H.W. Ke, N. Hao, X.Q. Li, Eur. Phys. J. C **79**, 540 (2019)
45. Z.X. Zhao, Eur. Phys. J. C **78**, 756 (2018)
46. Y.K. Hsiao, S.Y. Tsai, C.C. Lih, E. Rodrigues, JHEP **2004**, 035 (2020)
47. G. Buchalla, A.J. Buras, M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996)
48. Y.K. Hsiao, C.Q. Geng, Eur. Phys. J. C **77**, 714 (2017)
49. Y.K. Hsiao, C.Q. Geng, Phys. Lett. B **782**, 728 (2018)
50. Y.K. Hsiao, S.Y. Tsai, E. Rodrigues, Eur. Phys. J. C **80**, 565 (2020)