




Strong coupling constants of doubly heavy baryons with vector mesons in QCD

T. M. Aliev^{1,a}, K. Şimşek^{2,b} 

¹ Physics Department, Middle East Technical University, Ankara 06800, Turkey

² Department of Physics and Astronomy, Northwestern University, Evanston, IL 60208, USA

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Abstract Using the most general form of the interpolating current for baryons, the strong electric and magnetic coupling constants of light vector mesons ρ and K^* with doubly heavy baryons are computed within the light-cone sum rules. We consider 2- and 3-particle distribution amplitudes of the aforementioned vector mesons. The obtained results can be useful in the analysis of experimental data on the properties of doubly heavy baryons conducted at LHC.

1 Introduction

The quark model has been very predictive in studying the properties of hadrons [1]. Many baryon states predicted by the quark model have already been observed experimentally.

The spectroscopy of doubly heavy baryons has been extensively investigated in many theoretical works. It has been studied in the framework of different approaches such as the Hamilton method [2], the hypercentral method [3], the lattice QCD [4,5], the QCD sum rules [6–12], the Bethe-Salpeter equation [13], and in an extended chromomagnetic model [14].

At the present time, only the Ξ_{cc} state is observed in experiments. This state was first observed by SELEX Collaboration in $\Xi_{cc} \rightarrow \Lambda_c K^- \pi^+$ and $p D^+ K^-$ reactions with the mass 3518.7 ± 1.7 MeV [15,16]. In 2017, the doubly heavy Ξ_{cc}^{++} was discovered by LHCb Collaboration with the measured mass of $3621.24 \pm 0.65 \pm 0.31$ MeV [17]. The LHCb Collaboration also measured the lifetime of this state: $\tau = 0.256^{+0.024}_{-0.022} \pm 0.014$ ps [18]. This discovery is stimulated by many theoretical works for a deeper understanding of the properties of these baryons via studying their electromagnetic, weak, and strong decays. The weak decays

of the doubly heavy baryons have been studied within various approaches such as the light-front approach [19,20], the QCD sum rules approach [21], the light-cone QCD sum rules [22,23], the quark model [24], and the covariant light-front quark model [25].

The radiative transitions of doubly heavy baryons in the framework of different approaches such as relativized quark model [26], in the chiral perturbation theory [27], in the light-cone QCD sum rules [28] are comprehensively studied. The strong coupling constant of light pseudoscalar mesons with doubly heavy baryons within the light-cone QCD sum rules are studied in [29,30].

In this work, we study the strong coupling constants of doubly heavy baryons with vector mesons ρ and K^* within the framework of the light-cone sum rules (LCSR). These coupling constants can play an important role in the description of doubly heavy baryons in terms of one boson exchange potential models.

This paper is organized as follows. In Sect. 2, we derive the LCSR for the electric- and magnetic-type strong coupling constants of doubly heavy baryons with vector mesons ρ and K^* , as well as present the details of the calculations. Section 3 is devoted to the numerical analysis of the sum rules for the said coupling constants. Section 4 contains our conclusion.

2 Light-cone sum rules for vector meson-baryon coupling constants

For determining the strong coupling constants of doubly heavy baryons with light vector mesons, we introduce the following correlation function:

$$\Pi = i \int d^4x e^{ipx} \langle V(q) | T \{ \eta(x) \bar{\eta}(0) \} | 0 \rangle \quad (1)$$

where $V(q)$ is a vector meson with momentum q and η denotes the interpolating current of the corresponding dou-

^a e-mail: taliev@metu.edu.tr

^b e-mail: ksimsek@u.northwestern.edu (corresponding author)

bly heavy baryon. By the virtue of the $SU(3)$ classification, there exist two types of interpolating currents, which are symmetric or antisymmetric under the exchange of two heavy quarks. Only when the two heavy quarks are different, we have the antisymmetric current. The most general forms of the interpolating currents, both symmetric and antisymmetric, for doubly heavy baryons with $J = 1/2$ can be written as

$$\eta^{(S)} = \frac{1}{\sqrt{2}} \epsilon^{abc} \sum_{i=1}^2 \left[(Q^{aT} A_1^i q^b) A_2^i Q^{c'} + (Q \leftrightarrow Q') \right] \quad (2)$$

and

$$\eta^{(A)} = \frac{1}{\sqrt{6}} \epsilon^{abc} \sum_{i=1}^2 \left[2 (Q^{aT} A_1^i Q'^b) A_2^i q^c + (Q^{aT} A_1^i q^b) A_2^i Q'^c - (Q'^{aT} A_1^i q^b) A_2^i Q^c \right] \quad (3)$$

where $a, b,$ and c are color indices, T is the transposition, and

$$A_1^1 = C, \quad A_1^2 = C\gamma_5, \quad A_2^1 = \gamma_5, \quad A_2^2 = \beta I \quad (4)$$

where β is an arbitrary parameter and C is the charge conjugation operator.

The main philosophy of the light-cone sum rules (LCSR) is the computation of the correlation function in two different domains. It can be calculated in terms of the hadrons, as well as in the deep Euclidean region $p^2 \rightarrow -\infty$ and $(p + q)^2 \rightarrow -\infty$ by using the operator product expansion (OPE) over twist. Afterwards, the corresponding double Borel transformation with respect to the variables $-p^2$ and $-(p + q)^2$ is performed to suppress the contributions from higher states and the continuum as well as to enhance the contributions by the ground state. Finally, matching the results, the desired sum rules is obtained.

We start the construction of the sum rules by considering the phenomenological part of the correlation function. To this end, we insert a complete set of intermediate states with the same quantum numbers as the interpolating currents. After isolating the ground-state baryons, we obtain

$$\Pi = \frac{\langle 0 | \eta | B_2(p_2) \rangle \langle B_2(p_2) | V(q) | B_1(p_1) \rangle \langle B_1(p_1) | \bar{\eta} | 0 \rangle}{(p_2^2 - m_{B_2}^2)(p_1^2 - m_{B_1}^2)} + \text{higher states} \quad (5)$$

where m_{B_1} and m_{B_2} are the masses of the initial and final doubly heavy baryons, respectively. The matrix elements in Eq. (5) are defined as follows:

$$\langle 0 | \eta | B_i(p_i) \rangle = \lambda_{B_i} u(p_i) \quad (6)$$

$$\langle B_2(p_2) | V(q) | B_1(p_1) \rangle = \bar{u}(p_2) \left(f_1 \gamma_\mu - f_2 \frac{i}{m_{B_1} + m_{B_2}} \sigma_{\mu\nu} q^\nu \right) u(p_1) \epsilon^\mu \quad (7)$$

where the λ_{B_i} are the residues, f_1 and f_2 are the relevant coupling constants of the doubly heavy baryons with the corresponding vector meson, ϵ_μ and q_μ are the 4-polarization and 4-momentum of the vector meson, and u is the Dirac spinor for the baryon which is normalized as $\bar{u}u = 2m$.

Using Eqs. (6) and (7) in (5), we obtain the following for the physical part of the correlation function:

$$\begin{aligned} \Pi^{\text{phys}} &= \frac{\lambda_{B_1} \lambda_{B_2}}{(p_1^2 - m_{B_1}^2)(p_2^2 - m_{B_2}^2)} \epsilon^\mu (\not{p}_2 + m_{B_2}) \\ &\times \left(f_1 \gamma_\mu - f_2 \frac{i}{m_{B_1} + m_{B_2}} \sigma_{\mu\nu} q^\nu \right) (\not{p}_1 + m_{B_1}) \\ &= \frac{\lambda_{B_1} \lambda_{B_2}}{(p^2 - m_{B_2}^2) \left[(p + q)^2 - m_{B_1}^2 \right]} \\ &\times \left[\not{p} \not{q} (f_1 + f_2) + 2(\epsilon \cdot p) \not{p} f_1 + \text{other structures} \right] \end{aligned} \quad (8)$$

where we have set $p_1 = p + q$ and $p_2 = p$.

On the other hand, the correlation function is calculated from the QCD side by using the OPE over twist. After applying the Wick theorem, from (1), we get the following results:

$$\begin{aligned} \Pi^{(SS)} &= \frac{1}{2} \epsilon^{abc} \epsilon^{a'b'c'} \int d^4x e^{ip \cdot x} \sum_{ij} (A_1^i)_{\alpha\beta} (A_2^j)_{\rho\gamma} \\ &\times (\tilde{A}_2^j)_{\gamma'\rho'} (\tilde{A}_1^i)_{\alpha'\beta'} \langle V(q) | \left[S_{Q\gamma\gamma'}^{cc'} S_{Q\alpha\beta'}^{aa'} \right. \\ &\left. + (Q \leftrightarrow Q') - S_{Q\alpha\gamma'}^{ca'} S_{Q'\gamma\beta'}^{ca'} - (Q \leftrightarrow Q') \right] q_\beta^b \bar{q}_{\alpha'}^{b'} | 0 \rangle \end{aligned} \quad (9)$$

$$\begin{aligned} \Pi^{(AA)} &= \frac{1}{6} \epsilon^{abc} \epsilon^{a'b'c'} \int d^4x e^{ip \cdot x} \sum_{ij} (A_1^i)_{\alpha\beta} (A_2^j)_{\rho\gamma} \\ &\times (\tilde{A}_2^j)_{\gamma'\rho'} (\tilde{A}_1^i)_{\alpha'\beta'} \langle V(q) | 4S_{Q\beta'\alpha'}^{bb'} S_{Q\alpha\beta'}^{aa'} q_\gamma^c \bar{q}_{\gamma'}^{c'} \\ &- 2S_{Q'\beta\gamma'}^{bc'} S_{Q\alpha\beta'}^{aa'} q_\gamma^c \bar{q}_{\alpha'}^{b'} - 2S_{Q\alpha\gamma'}^{ac'} S_{Q'\beta\beta'}^{ba'} q_\gamma^c \bar{q}_{\alpha'}^{b'} \\ &- 2S_{Q'\gamma\alpha'}^{cb'} S_{Q\alpha\beta'}^{aa'} q_\beta^b \bar{q}_{\gamma'}^{c'} + S_{Q'\gamma\gamma'}^{cc'} S_{Q\alpha\beta'}^{aa'} q_\beta^b \bar{q}_{\alpha'}^{b'} \\ &+ q_\beta^b \bar{q}_{\alpha'}^{b'} S_{Q\alpha\gamma'}^{ac'} S_{Q'\gamma\beta'}^{ca'} - 2q_\beta^b \bar{q}_{\alpha'}^{b'} S_{Q'\alpha\alpha'}^{ab'} S_{Q\gamma\beta'}^{ca'} \\ &+ q_\beta^b \bar{q}_{\alpha'}^{b'} S_{Q'\alpha\gamma'}^{ac'} S_{Q\gamma\beta'}^{ca'} + q_\beta^b \bar{q}_{\alpha'}^{b'} S_{Q\gamma\gamma'}^{cc'} S_{Q'\alpha\beta'}^{aa'} | 0 \rangle \end{aligned} \quad (10)$$

$$\begin{aligned} \Pi^{(SA)} &= \frac{1}{\sqrt{12}} \epsilon^{abc} \epsilon^{a'b'c'} \int d^4x e^{ip \cdot x} \sum_{ij} (A_1^i)_{\alpha\beta} (A_2^j)_{\rho\gamma} \\ &\times (\tilde{A}_2^j)_{\gamma'\rho'} (\tilde{A}_1^i)_{\alpha'\beta'} \langle V(q) | -2q_\gamma^c \bar{q}_{\alpha'}^{b'} S_{Q'\beta\gamma'}^{bc'} S_{Q\alpha\beta'}^{aa'} \\ &+ 2q_\gamma^c \bar{q}_{\alpha'}^{b'} S_{Q\alpha\gamma'}^{ac'} S_{Q'\beta\beta'}^{ba'} + q_\beta^b \bar{q}_{\alpha'}^{b'} S_{Q'\gamma\gamma'}^{cc'} S_{Q\alpha\beta'}^{aa'} \\ &- q_\beta^b \bar{q}_{\alpha'}^{b'} S_{Q\alpha\gamma'}^{ac'} S_{Q'\gamma\beta'}^{ca'} + q_\beta^b \bar{q}_{\alpha'}^{b'} S_{Q'\alpha\gamma'}^{ac'} S_{Q\gamma\beta'}^{ca'} \\ &- q_\beta^b \bar{q}_{\alpha'}^{b'} S_{Q'\alpha\beta'}^{aa'} S_{Q\gamma\gamma'}^{cc'} | 0 \rangle \end{aligned} \quad (11)$$

where S_Q is the heavy quark propagator. In these expressions, the superscripts (SS), (AA), and (SA) denote the symmetry property of the currents η and $\bar{\eta}$, and we have defined $\tilde{A}_i = \gamma^0 A_i^\dagger \gamma^0$. The heavy quark propagator in the presence of an

external background field in the coordinate space is given by

$$S_{Q\alpha\beta}^{aa'} = \frac{m_Q^2}{4\pi} \left[\frac{iK_2(m_Q\sqrt{-x^2})\not{x}}{(\sqrt{-x^2})^2} + \frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} \right]_{\alpha\beta} \times \delta^{aa'} - \frac{g_s}{16\pi^2} m_Q \int_0^1 du \left[\frac{iK_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} \times (\bar{u}\not{x}\sigma_{\lambda\tau} + u\sigma_{\lambda\tau}\not{x}) + K_0(m_Q\sqrt{-x^2})\sigma_{\lambda\tau} \right]_{\alpha\beta} \times G^{(n)\lambda\tau} \left(\frac{\lambda^{(n)}}{2}\right)^{aa'} \tag{12}$$

where $G_{\lambda\tau}^{(n)}$ is the gluon field strength tensor, the $\lambda^{(n)}$ are the Gell–Mann matrices, and the $K_i(m_Q\sqrt{-x^2})$ are the modified Bessel functions of the second kind. Using the Fierz identities, namely

$$q_\alpha^b \bar{q}_\beta^{b'} \rightarrow -\frac{1}{12} (\Gamma_i)_{\alpha\beta} \delta^{bb'} [\bar{q} \Gamma_i q] \tag{13}$$

and

$$q_\alpha^b \bar{q}_\beta^{b'} G_{\lambda\tau}^{(n)} \rightarrow -\frac{1}{16} \left(\frac{\lambda^{(n)}}{2}\right)^{bb'} (\Gamma_i)_{\alpha\beta} [\bar{q} \Gamma_i G_{\lambda\tau}^{(n)} q] \tag{14}$$

one can see that the following matrix elements appear in the calculation:

$$\langle V(q, \varepsilon) | \bar{q} \Gamma_i q | 0 \rangle \quad \text{and} \quad \langle V(q, \varepsilon) | \bar{q} \Gamma_i G_{\mu\nu}^{(n)} q | 0 \rangle \tag{15}$$

$\{\Gamma_i\}$ is the full set of Dirac matrices, i.e.

$$\begin{aligned} \Gamma_1 &= I, & \Gamma_2 &= \gamma_5, & \Gamma_3 &= \gamma_\alpha, \\ \Gamma_4 &= i\gamma_\alpha\gamma_5, & \Gamma_5 &= \frac{1}{\sqrt{2}}\sigma_{\alpha\beta} \end{aligned} \tag{16}$$

Now if we insert Eqs. (12)–(16) into Eqs. (9)–(11), do calculations for the QCD part of the correlation function, and perform a double Borel transformation over variables $-p^2$ and $-(p+q)^2$ and equating the coefficients of the relevant structures from both parts, we obtain the desired sum rules for them. Our numerical analysis shows that the coefficients of the structures $\not{p}\not{q}$ and $(\varepsilon \cdot p)\not{p}$ exhibit the best convergence for the OPE series. For this reason, we choose the coefficients of these structures in our analysis. The analytical expressions of the coefficients of these structures are quite lengthy and therefore we present them in Appendix A.

As an example, we present the complete steps of calculations for one term, and the remaining ones are presented in Appendix B. Let us consider the term

$$\int du \int d^4x e^{i(p+\bar{u}q)\cdot x} \frac{K_i(m_Q\sqrt{-x^2})}{(\sqrt{-x^2})^i} \frac{K_j(m_{Q'}\sqrt{-x^2})}{(\sqrt{-x^2})^j} f(u) \tag{17}$$

where $f(u)$ is a generic 2-particle DA of the vector meson. Now we use the integral representation of the Bessel function,

namely

$$\frac{K_i(m_Q\sqrt{-x^2})}{(\sqrt{-x^2})^i} = \frac{1}{2} \int_0^\infty dt \frac{1}{t^{i+1}} e^{-\frac{m_Q}{2}(t-x^2/t)} \tag{18}$$

Then we introduce two new variables a and b as $a := \frac{2m_Q}{t}$ and $b := \frac{2m_{Q'}}{t}$. If we now do a Wick rotation, $x^0 \rightarrow ix_E^0$, and switch to the Euclid spacetime, $-x^2 \rightarrow x_E^2$, and perform the integration over d^4x_E , we obtain

$$\begin{aligned} &\frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} \int du \int_0^\infty da \int_0^\infty db \frac{a^{i-1} b^{j-1}}{(a+b)^2} \\ &\times f(u) e^{-k_E^2/(a+b)} e^{-m_Q^2/a - m_{Q'}^2/b} \end{aligned} \tag{19}$$

where $k_E := p_E + \bar{u}q_E$. Now, let us insert the identity $\int d\rho \delta(\rho - a - b) = 1$, make a scale transformation $a \rightarrow \rho\alpha$, $b \rightarrow \rho\beta$, and perform the integration over β to obtain

$$\begin{aligned} &\frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} \int du \int_0^\infty d\alpha \int d\rho e^{-k_E^2/\rho} \rho^{i+j-3} \\ &\times \alpha^{i-1} (1-\alpha)^{j-1} e^{-\left(\frac{m_Q^2}{\alpha} + \frac{m_{Q'}^2}{1-\alpha}\right)/\rho} f(u) \end{aligned} \tag{20}$$

Then, by writing out

$$k_E^2 = p_E^2(1-\bar{u}) - m_V^2(\bar{u}^2 - \bar{u}) + \bar{u}(p_E + q_E)^2 \tag{21}$$

and making use of the formula $\mathfrak{B}e^{-\alpha p_E^2} = \delta(\frac{1}{M^2} - \alpha)$ to perform the Borel transformations over the variables p_E^2 and $(p_E + q_E)^2$ and integrating over u and ρ , we get

$$\begin{aligned} &\frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} (M^2)^{i+j} e^{-m_V^2/2M^2} f\left(\frac{1}{2}\right) \\ &\times \int_0^\infty d\alpha \alpha^{i-1} (1-\alpha)^{j-1} e^{-\left(\frac{m_Q^2}{\alpha} + \frac{m_{Q'}^2}{1-\alpha}\right)/M^2} \end{aligned} \tag{22}$$

Now let

$$s := \frac{m_Q^2}{\alpha} + \frac{m_{Q'}^2}{1-\alpha} \tag{23}$$

Equating this to s_0 in order to perform the continuum subtraction, we find the bounds of α . As a result, we obtain

$$\begin{aligned} &\frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} (M^2)^{i+j} e^{-m_V^2/2M^2} f\left(\frac{1}{2}\right) \\ &\times \int_{\alpha_-}^{\alpha_+} d\alpha \alpha^{i-1} (1-\alpha)^{j-1} e^{-\left(\frac{m_Q^2}{\alpha} + \frac{m_{Q'}^2}{1-\alpha}\right)/M^2} \end{aligned} \tag{24}$$

Table 1 The values of the input parameters entering the sum rules. All the masses and decay constants are in units of GeV

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
m_s (1 GeV)	0.137	m_ρ	0.770	$m_{\mathcal{E}_{cc}}$	3.72 [8]	$\lambda_{\mathcal{E}_{cc}}$	0.16 [8]
m_c	1.4	f_ρ	0.216	$m_{\mathcal{E}_{bb}}$	9.96 [8]	$\lambda_{\mathcal{E}_{bb}}$	0.44 [8]
m_b	4.7	f_ρ^T	0.165	$m_{\mathcal{E}'_{bc}}$	6.72 [8]	$\lambda_{\mathcal{E}'_{bc}}$	0.28 [8]
		m_{K^*}	0.892	$m_{\Omega_{cc}}$	6.79 [8]	$\lambda_{\Omega_{cc}}$	0.30 [8]
		f_{K^*}	0.220	$m_{\Omega_{bb}}$	3.73 [8]	$\lambda_{\Omega_{bb}}$	0.18 [8]
		$f_{K^*}^T$	0.185	$m_{\Omega_{bc}}$	9.97 [8]	$\lambda_{\Omega_{bc}}$	0.45 [8]
				$m_{\Omega'_{bc}}$	6.75 [8]	$\lambda_{\Omega'_{bc}}$	0.29 [8]
				$m_{\Omega''_{bc}}$	6.80 [8]	$\lambda_{\Omega''_{bc}}$	0.31 [8]

which can be more conveniently rewritten as

$$\frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} (M^2)^{i+j} e^{-m_V^2/2M^2} f \left(\frac{1}{2}\right) \times \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \times \delta \left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha} \right) \tag{25}$$

where we have

$$\alpha_\pm := \frac{1}{2s} \left(s + m_Q^2 - m_{Q'}^2 \pm \sqrt{(s + m_Q^2 - m_{Q'}^2)^2 - 4sm_Q^2} \right) \tag{26}$$

and we have defined

$$\frac{1}{M^2} = \frac{1}{M_1^2} + \frac{1}{M_2^2} \tag{27}$$

Since in our case the mass of the initial and final baryons are practically the same, we take $M_1^2 = M_2^2$. The $1/2$ inside f is indeed given by u_0 which is defined as

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2} = \frac{1}{2} \tag{28}$$

Performing similar calculations for the remaining integrals and matching the two representations of the correlation function for the relevant coupling constants, we get the following sum rules:

$$f_1 = \frac{1}{2\lambda_{B_1}\lambda_{B_2}} e^{\frac{m_B^2}{M^2} + \frac{m_V^2}{4M^2}} \Pi_{f_1}^{\text{theo}} \tag{29}$$

$$f_1 + f_2 = \frac{1}{\lambda_{B_1}\lambda_{B_2}} e^{\frac{m_B^2}{M^2} + \frac{m_V^2}{4M^2}} \Pi_{f_1+f_2}^{\text{theo}} \tag{30}$$

3 Numerical analysis

In this section, we numerically analyze the LCSR for the electric-, f_1 , and magnetic-type, $f_1 + f_2$, strong coupling

constants of the vector meson ρ and K^* with the doubly heavy baryons \mathcal{E}_{cc} , \mathcal{E}_{bb} , \mathcal{E}_{bc} , \mathcal{E}'_{bc} , Ω_{cc} , Ω_{bb} , Ω_{bc} , and Ω'_{bc} by using Package X [31]. The LCSR for the coupling constants f_1 and $f_1 + f_2$ contain certain input parameters such as quark masses, the masses and decay constants of the vector mesons ρ and K^* , and the masses and residues of the aforementioned doubly heavy baryons. Some of these parameters are presented in Table 1. Other input parameters are present in the vector meson DAs of different twists. The complete list of these DAs together with the most recent values of the input parameters are given in Appendix C.

In addition to the above-mentioned input parameters, the LCSR also includes three auxiliary parameters, namely the Borel mass parameter, M^2 , the continuum threshold, s_0 , and the arbitrary parameter, β , which appear in the expression for the interpolating current. Physically measurable quantities should be independent of these parameters. Thus, we need to find the working regions of these auxiliary parameters such that the LCSR is reliable. The lower bound of the Borel mass parameter is obtained by requiring the contributions from the highest-twist terms should be considerably smaller than the contributions from the lowest-twist terms. The upper bound of M^2 can be determined by demanding that the continuum contribution should not be too large. Meantime, the continuum threshold, s_0 , is obtained by requiring that the two-point sum rules reproduce a 10% accuracy of the mass of doubly heavy baryons. These conditions lead to the values of M^2 and s_0 summarized in Table 2 for the channels considered.

Our analysis reveals that the twist-4 term contributions in the aforementioned domains of M^2 at the indicated values of s_0 are smaller than 15% and higher states contribute 30% at maximum for all the considered channels. As an illustration, we present the M^2 dependence of $f_1 + f_2$ and f_1 for $\mathcal{E}_{cc}\mathcal{E}_{cc}\rho$ at fixed values of s_0 and β in Figs. 1 and 2. Once we determine the working regions of M^2 and s_0 , we need to find the working region of the auxiliary parameter, β . To this end, we investigate the strong coupling constant $f_1 + f_2$ as a function of $\cos \theta$, where we have defined θ via $\beta = \tan \theta$. As an example, we give the dependence of the coupling constants $f_1 + f_2$ and f_1 for $\mathcal{E}_{cc}\mathcal{E}_{cc}\rho$ and $\mathcal{E}'_{bc}\mathcal{E}'_{bc}\rho$ at fixed values of M^2 and s_0 in Figs. 3, 4, 5 and 6, respectively.

Table 2 The working region of the parameters M^2 and s_0 for the channels considered in our work

Channel	M^2 (GeV ²)	$\sqrt{s_0}$ (GeV)
SS		
$\Xi_{cc} \rightarrow \Xi_{cc}\rho$	$3 \leq M^2 \leq 6$	4.6
$\Xi_{bb} \rightarrow \Xi_{bb}\rho$	$10 \leq M^2 \leq 15$	10.9
$\Omega_{bb} \rightarrow \Xi_{bb}K^*$	$10 \leq M^2 \leq 15$	10.9
$\Omega_{cc} \rightarrow \Xi_{cc}K^*$	$3 \leq M^2 \leq 6$	4.6
$\Omega_{bc} \rightarrow \Xi_{bc}K^*$	$6 \leq M^2 \leq 9$	7.5
AA		
$\Xi'_{bc} \rightarrow \Xi'_{bc}\rho$	$6 \leq M^2 \leq 9$	7.5
$\Omega'_{bc} \rightarrow \Xi'_{bc}K^*$	$6 \leq M^2 \leq 9$	7.5
SA		
$\Xi'_{bc} \rightarrow \Xi_{bc}\rho$	$6 \leq M^2 \leq 9$	7.5
$\Omega'_{bc} \rightarrow \Xi_{bc}K^*$	$6 \leq M^2 \leq 9$	7.5

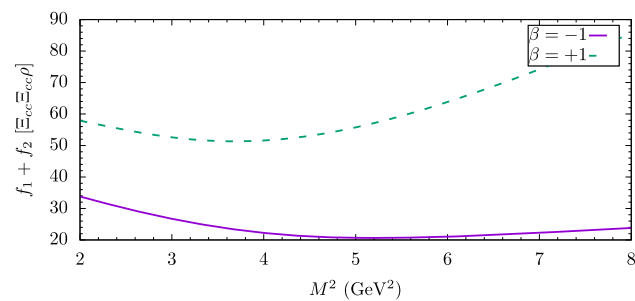


Fig. 1 The dependence of the coupling constant $f_1 + f_2$ for $\Xi_{cc}\Xi_{cc}\rho$ on M^2 for the shown values of β and at $\sqrt{s_0} = 4.6$ GeV

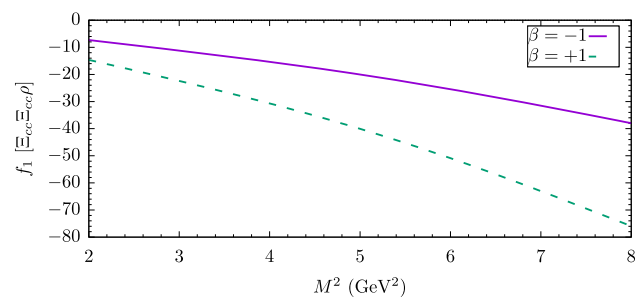


Fig. 2 The dependence of the coupling constant f_1 for $\Xi_{cc}\Xi_{cc}\rho$ on M^2 for the shown values of β and at $\sqrt{s_0} = 4.6$ GeV

In these figures, one can see that the coupling constants do not practically change when $|\cos \theta|$ varies between 0.6 and 1. Our numerical analysis for the strong coupling constants of doubly heavy baryons with vectors mesons leads to the results presented in Table 3. The uncertainties are due to the variation of the parameters M^2, s_0 , and the errors in the values of the input parameters.

From Table 3, we deduce the following conclusions:

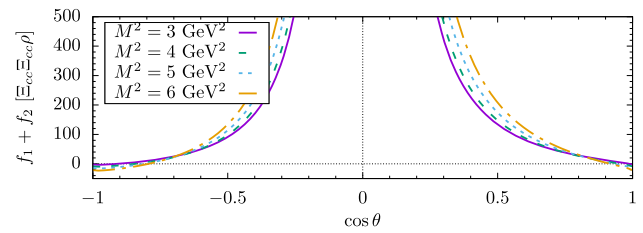


Fig. 3 The dependence of the coupling constant $f_1 + f_2$ for $\Xi_{cc}\Xi_{cc}\rho$ on $\cos \theta$ for the shown values of M^2 and at $\sqrt{s_0} = 4.6$ GeV

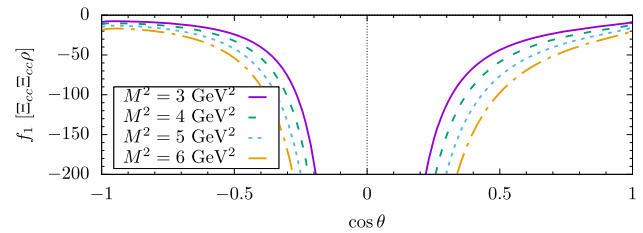


Fig. 4 The dependence of the coupling constant f_1 for $\Xi_{cc}\Xi_{cc}\rho$ on $\cos \theta$ for the shown values of M^2 and at $\sqrt{s_0} = 4.6$ GeV

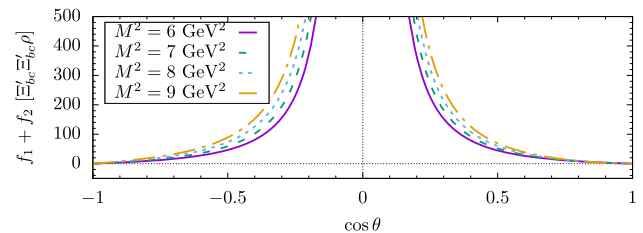


Fig. 5 The dependence of the coupling constant $f_1 + f_2$ for $\Xi'_{bc}\Xi'_{bc}\rho$ on $\cos \theta$ for the shown values of M^2 and at $\sqrt{s_0} = 7.5$ GeV

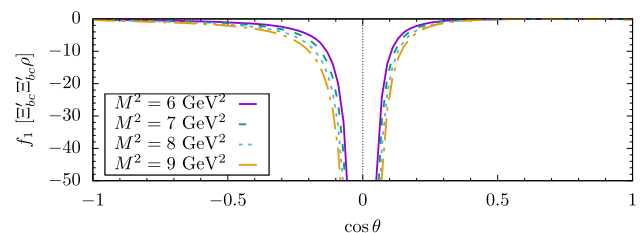


Fig. 6 The dependence of the coupling constant f_1 for $\Xi'_{bc}\Xi'_{bc}\rho$ on $\cos \theta$ for the shown values of M^2 and at $\sqrt{s_0} = 7.5$ GeV

- (a) The $SU(3)$ symmetry for the SS and AA cases works very well. The violation of the $SU(3)$ symmetry is about 10% at maximum.
- (b) In the SA case, $SU(3)$ symmetry works very well for electric strong coupling, f_1 , but considerably violated for the coupling $f_1 + f_2$ (about 50%).

4 Conclusion

The experimental discovery of the Ξ_{cc} baryon opened a new research area in theoretical studies for understanding the

Table 3 The numerical values for the strong coupling constants

Channel	$f_1 + f_2$	f_1
<i>SS</i>		
$\mathcal{E}_{cc} \rightarrow \mathcal{E}_{cc}\rho$	32.53 ± 2.30	-25.32 ± 8.49
$\mathcal{E}_{bb} \rightarrow \mathcal{E}_{bb}\rho$	23.66 ± 2.89	-7.69 ± 2.31
$\Omega_{bb} \rightarrow \mathcal{E}_{bb}K^*$	22.55 ± 2.63	-8.30 ± 2.74
$\Omega_{cc} \rightarrow \mathcal{E}_{cc}K^*$	28.36 ± 1.83	-23.64 ± 9.85
$\Omega_{bc} \rightarrow \mathcal{E}_{bc}K^*$	8.23 ± 0.77	-3.74 ± 1.37
<i>AA</i>		
$\mathcal{E}'_{bc} \rightarrow \mathcal{E}'_{bc}\rho$	-37.62 ± 6.48	-0.40 ± 0.12
$\Omega'_{bc} \rightarrow \mathcal{E}'_{bc}K^*$	-37.98 ± 6.75	-0.40 ± 0.14
<i>SA</i>		
$\mathcal{E}'_{bc} \rightarrow \mathcal{E}_{bc}\rho$	1.50 ± 0.31	-0.97 ± 0.30
$\Omega'_{bc} \rightarrow \mathcal{E}_{bc}K^*$	2.20 ± 0.40	-1.00 ± 0.36

properties of doubly heavy baryons by analyzing their weak, electromagnetic, and strong decays. In the present work, within the framework of the LCSR method, we estimate the electric and magnetic couplings of light vector mesons ρ and K^* with doubly heavy baryons of spin 1/2. In our analysis, we have used the general form of the interpolating currents in symmetric and antisymmetric forms with respect to the exchange of two heavy quarks. We obtained that in the case of symmetric-antisymmetric (*SA*) currents, couplings are affirmed to be smaller than the *SS* and *AA* cases. This circumstance can be explained by the fact that in the *SA* case, there are strong cancellations between the leading terms. Moreover, we obtained that our results for the considered couplings constants for the cases of *SS* and *AA* are in a good agreement with the *SU*(3) symmetry results and its violation is about 10% at maximum. For the *SA* case, for the electric coupling, the *SU*(3) symmetry violation is small (about 5%), but for magnetic coupling, its violation is about 50%.

Our final remark is that these results can be helpful in studies of the properties of doubly heavy baryons in experiments conducted at LHCb.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All the existing numerical data have been included in the manuscript and there is no other data regarding this work.]

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Appendix A: Analytical expressions of the correlation functions for the considered structures

In this section, we present the analytic expression of the correlation functions for the cases of *SS*, *AA*, and *SA* currents for the structures $\not{p}\not{q}$ and $(\varepsilon \cdot p)\not{p}$:

$$\begin{aligned}
 \Pi_{f_1+f_2}^{(SS)\text{theo}} = & 1/(96M^2\pi^4)im_Qm_{Q'}N^2(-(-1 + \beta^2)f_V^T \\
 & \times (3(m_Q + m_{Q'})m_V\mathcal{G}[\mathcal{S}(\alpha_1, \alpha_3), 0, 2] \\
 & + 9(m_Q + m_{Q'})m_V\mathcal{G}[\mathcal{S}(\alpha_1, \alpha_3), 1, 1] \\
 & - 3m_Qm_V(i\mathcal{G}[\tilde{\mathcal{S}}(\alpha_1, \alpha_3), 0, 2] \\
 & + f_V^T m_V(\mathcal{G}[\mathcal{T}_3^{(4)}(\alpha_1, \alpha_3), 1, 1] \\
 & - \mathcal{G}[\mathcal{T}_4^{(4)}(\alpha_1, \alpha_3), 1, 1] - 2\mathcal{G}[\mathcal{T}_3^{(4)}(\alpha_1, \alpha_3)u, 1, 1] \\
 & + 2\mathcal{G}[\mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)u, 1, 1] \\
 & + m_V^2(\mathcal{G}_2[\mathcal{H}_G[2, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 & - 2\mathcal{G}_2[u\mathcal{H}_G[2, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 & - \mathcal{G}_2[\mathcal{H}_G[2, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 & + 2\mathcal{G}_2[u\mathcal{H}_G[2, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1])) \\
 & - m_{Q'}(224m_Q\mathcal{I}[\phi_2^\perp(u), 1, 2] \\
 & + m_V(3i\mathcal{G}[\tilde{\mathcal{S}}(\alpha_1, \alpha_3), 0, 2] \\
 & + m_V(3f_V^T(\mathcal{G}[\mathcal{T}_3^{(4)}(\alpha_1, \alpha_3), 1, 1] - \mathcal{G}[\mathcal{T}_4^{(4)}(\alpha_1, \alpha_3), 1, 1] \\
 & - 2\mathcal{G}[\mathcal{T}_3^{(4)}(\alpha_1, \alpha_3)u, 1, 1] + 2\mathcal{G}[\mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)u, 1, 1] \\
 & + m_V^2(\mathcal{G}_2[\mathcal{H}_G[2, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 & - 2\mathcal{G}_2[u\mathcal{H}_G[2, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 & - \mathcal{G}_2[\mathcal{H}_G[2, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 & + 2\mathcal{G}_2[u\mathcal{H}_G[2, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1))) \\
 & + 14m_Q\mathcal{I}_2[\phi_4^\perp(u), 1, 2])) \\
 & + f_Vim_V(3(1 + \beta(6 + \beta))(m_Q + m_{Q'}) \\
 & \times \mathcal{G}[\mathcal{D}_G[1, \mathcal{A}(\alpha_1, \alpha_3)], 1, 2] \\
 & - 6(1 + \beta(6 + \beta))(m_Q + m_{Q'}) \\
 & \times \mathcal{G}[u\mathcal{D}_G[1, \mathcal{A}(\alpha_1, \alpha_3)], 1, 2] \\
 & - 3(3 + \beta(2 + 3\beta))m_Q\mathcal{G}[\mathcal{D}_G[1, \mathcal{V}(\alpha_1, \alpha_3)], 1, 2] \\
 & + m_{Q'}(-3(3 + \beta(2 + 3\beta)) \\
 & \mathcal{G}[\mathcal{D}_G[1, \mathcal{V}(\alpha_1, \alpha_3)], 1, 2] \\
 & - 28im_Q((-1 + \beta)^2\mathcal{I}[\psi_3^\perp(u), 1, 1] \\
 & - (1 + \beta)^2\mathcal{I}_2[\psi_3^\perp(u), 2, 2]))))
 \end{aligned} \tag{31}$$

$$\begin{aligned}
 \Pi_{f_1}^{(SS)\text{theo}} = & -1/(96M^4\pi^4)im_Qm_{Q'}m_VN^2 \\
 & \times (3i(-1 + \beta^2)(f_V^T)^2(m_Q + m_{Q'})m_V^3 \\
 & \times (6\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}(\alpha_1, \alpha_3)], 0, 2] \\
 & + 2(\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}(\alpha_1, \alpha_3)], 1, 1] \\
 & - \mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 & + \mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 & + \mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 0, 2]
 \end{aligned}$$

$$\begin{aligned}
 &+4\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_3^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &+2\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &+\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &+3m_V^2\mathcal{G}_2[\mathcal{H}_G[3, \mathcal{T}(\alpha_1, \alpha_3)], 0, 2] \\
 &-2f_V(3(3+\beta(2+3\beta))(m_Q+m_{Q'})m_V^2 \\
 &\times\mathcal{G}[\mathcal{A}(\alpha_1, \alpha_3), 1, 2] \\
 &-3(3+\beta(2+3\beta))(m_Q+m_{Q'})m_V^2\mathcal{G}[\mathcal{V}(\alpha_1, \alpha_3), 1, 2] \\
 &+6(3+\beta(2+3\beta)) \\
 &(m_Q+m_{Q'})m_V^2\mathcal{G}[\mathcal{V}(\alpha_1, \alpha_3)u, 1, 2] \\
 &-14m_Qm_{Q'}(16(3+\beta(2+3\beta)) \\
 &\mathcal{I}[\phi_2^\parallel(u), 2, 2]+m_V^2(-8(-1+\beta)^2 \\
 &\times\mathcal{I}[\mathcal{H}[2, \phi_3^\perp(u)], 1, 1]+4(-1+\beta)^2 \\
 &\mathcal{I}[\mathcal{H}[2, \phi_2^\parallel(u)], 1, 1]+4(-1+\beta)^2 \\
 &\times\mathcal{I}[\mathcal{H}[2, \psi_4^\parallel(u)], 1, 1]+(3+\beta(2+3\beta)) \\
 &\mathcal{I}_2[\phi_4^\parallel(u), 2, 2]-4(5+\beta(2+5\beta)) \\
 &\times(-2\mathcal{I}_2[\mathcal{H}[2, \phi_3^\perp(u)], 2, 2] \\
 &+\mathcal{I}_2[\mathcal{H}[2, \phi_2^\parallel(u)], 2, 2]+\mathcal{I}_2[\mathcal{H}[2, \psi_4^\parallel(u)], 2, 2])))) \tag{32}
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{f_1+f_2}^{(AA)\text{theo}} &= (1/(576M^2\pi^4))m_Qm_{Q'}((-1+\beta) \\
 &\times f_V^T i(3(5+\beta)(m_Q+m_{Q'})m_V \\
 &\times\mathcal{G}[\mathcal{S}(\alpha_1, \alpha_3), 0, 2]-3(5+\beta)(m_Q+m_{Q'}) \\
 &m_V\mathcal{G}[\mathcal{S}(\alpha_1, \alpha_3), 1, 1]+(1+5\beta) \\
 &\times(-3i(m_Q+m_{Q'})m_V\mathcal{G}[\tilde{\mathcal{S}}(\alpha_1, \alpha_3), 0, 2] \\
 &-3f_V^T(m_Q+m_{Q'})m_V^2 \\
 &\times(\mathcal{G}[\mathcal{T}_3^{(4)}(\alpha_1, \alpha_3), 1, 1]-\mathcal{G}[\mathcal{T}_4^{(4)}(\alpha_1, \alpha_3), 1, 1] \\
 &-2\mathcal{G}[\mathcal{T}_3^{(4)}(\alpha_1, \alpha_3)u, 1, 1] \\
 &+2\mathcal{G}[\mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)u, 1, 1]+m_V^2(\mathcal{G}_2[\mathcal{H}_G[2, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &-2\mathcal{G}_2[u\mathcal{H}_G[2, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &-\mathcal{G}_2[\mathcal{H}_G[2, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &+2\mathcal{G}_2[u\mathcal{H}_G[2, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1])) \\
 &-14m_Qm_{Q'}(16\mathcal{I}[\phi_2^\perp(u), 1, 2] \\
 &+m_V^2\mathcal{I}_2[\phi_4^\perp(u), 1, 2])) \\
 &+2f_Vm_V(6((1+\beta+\beta^2)m_Q-\beta m_{Q'}) \\
 &\times\mathcal{G}[\mathcal{D}_G[1, \mathcal{A}(\alpha_1, \alpha_3)], 1, 2] \\
 &+3(m_Q+\beta(6+\beta)m_Q-(-1+\beta)^2m_{Q'}) \\
 &\times\mathcal{G}[u\mathcal{D}_G[1, \mathcal{A}(\alpha_1, \alpha_3)], 1, 2] \\
 &-3(1+\beta(4+\beta))m_Q\mathcal{G}[\mathcal{D}_G[1, \mathcal{V}(\alpha_1, \alpha_3)], 1, 2] \\
 &+m_{Q'}(3(1+\beta^2)\mathcal{G}[\mathcal{D}_G[1, \mathcal{V}(\alpha_1, \alpha_3)], 1, 2]-14i \\
 &m_Q(-1+\beta)(11+13\beta) \\
 &\times\mathcal{I}[\psi_3^\perp(u), 1, 1] \\
 &-(13+\beta(10+13\beta))\mathcal{I}_2[\psi_3^\perp(u), 2, 2])))) \tag{33}
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{f_1}^{(AA)\text{theo}} &= (1/(576M^4\pi^4))m_Qm_{Q'} \\
 &\times m_V(3(-1+\beta)(f_V^T)^2(m_Q+m_{Q'})m_V^3(2(5+\beta) \\
 &\times\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}(\alpha_1, \alpha_3)], 0, 2] \\
 &-2(5+\beta)(\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}(\alpha_1, \alpha_3)], 1, 1]
 \end{aligned}$$

$$\begin{aligned}
 &-\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &+\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 1, 1]) \\
 &-2(1+5\beta)\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &+12(1+\beta)\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &-2(5+\beta)\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &+(1+5\beta)m_V^2 \\
 &\times\mathcal{G}_2[\mathcal{H}_G[3, \mathcal{T}(\alpha_1, \alpha_3)], 0, 2] \\
 &+2if_V(3(-1+\beta)^2(m_Q+m_{Q'})m_V^2 \\
 &\times\mathcal{G}[\mathcal{A}(\alpha_1, \alpha_3), 1, 2]-3(-1+\beta)^2 \\
 &\times(m_Q+m_{Q'})m_V^2\mathcal{G}[\mathcal{V}(\alpha_1, \alpha_3), 1, 2] \\
 &+6(-1+\beta)^2(m_Q+m_{Q'})m_V^2\mathcal{G}[\mathcal{V}(\alpha_1, \alpha_3)u, 1, 2] \\
 &-14m_Qm_{Q'}(16(-1+\beta)^2 \\
 &\times\mathcal{I}[\phi_2^\parallel(u), 2, 2]+m_V^2(-8(-1+\beta)(13+11\beta) \\
 &\times\mathcal{I}[\mathcal{H}[2, \phi_3^\perp(u)], 1, 1]+4(-1+\beta) \\
 &\times(13+11\beta)\mathcal{I}[\mathcal{H}[2, \phi_2^\parallel(u)], 1, 1] \\
 &-52\mathcal{I}[\mathcal{H}[2, \psi_4^\parallel(u)], 1, 1]+4\beta(2+11\beta) \\
 &\times\mathcal{I}[\mathcal{H}[2, \psi_4^\parallel(u)], 1, 1]+(-1+\beta)^2\mathcal{I}_2[\phi_4^\parallel(u), 2, 2] \\
 &-60(-2\mathcal{I}_2[\mathcal{H}[2, \phi_3^\perp(u)], 2, 2] \\
 &+\mathcal{I}_2[\mathcal{H}[2, \phi_2^\parallel(u)], 2, 2] \\
 &+\mathcal{I}_2[\mathcal{H}[2, \psi_4^\parallel(u)], 2, 2]) - 12\beta(2+5\beta) \\
 &\times(-2\mathcal{I}_2[\mathcal{H}[2, \phi_3^\perp(u)], 2, 2]+\mathcal{I}_2[\mathcal{H}[2, \phi_2^\parallel(u)], 2, 2] \\
 &+\mathcal{I}_2[\mathcal{H}[2, \psi_4^\parallel(u)], 2, 2])))) \tag{34}
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{f_1+f_2}^{(SA)\text{theo}} &= (1/(32\sqrt{3}M^2\pi^4))m_Q \\
 &\times(m_Q-m_{Q'})m_{Q'}m_VN \\
 &\times(2f_V(-2\beta\mathcal{G}[\mathcal{D}_G[1, \mathcal{A}(\alpha_1, \alpha_3)], 1, 2] \\
 &+2(1+\beta(4+\beta))\mathcal{G}[u\mathcal{D}_G[1, \mathcal{A}(\alpha_1, \alpha_3)], 1, 2] \\
 &+(1+\beta^2)\mathcal{G}[\mathcal{D}_G[1, \mathcal{V}(\alpha_1, \alpha_3)], 1, 2] \\
 &+3(1+\beta)^2\mathcal{G}[u\mathcal{D}_G[1, \mathcal{V}(\alpha_1, \alpha_3)], 1, 2] \\
 &+(-1+\beta)f_V^T(-2i\beta\mathcal{G}[\mathcal{S}(\alpha_1, \alpha_3), 0, 2] \\
 &-2i(1+2\beta)\mathcal{G}[\mathcal{S}(\alpha_1, \alpha_3), 1, 1] \\
 &-2\mathcal{G}[\tilde{\mathcal{S}}(\alpha_1, \alpha_3), 0, 2]-(-1+\beta) \\
 &(\mathcal{G}[\tilde{\mathcal{S}}(\alpha_1, \alpha_3), 1, 1]-2\mathcal{G}[\tilde{\mathcal{S}}(\alpha_1, \alpha_3)u, 1, 1]) \\
 &+f_V^T im_V((1+3\beta)\mathcal{G}[\mathcal{T}_1^{(4)}(\alpha_1, \alpha_3), 0, 2] \\
 &+(1+3\beta)\mathcal{G}[\mathcal{T}_1^{(4)}(\alpha_1, \alpha_3), 1, 1] \\
 &-\mathcal{G}[\mathcal{T}_2^{(4)}(\alpha_1, \alpha_3), 0, 2]-\mathcal{G}[\mathcal{T}_2^{(4)}(\alpha_1, \alpha_3), 1, 1] \\
 &+3\mathcal{G}[\mathcal{T}_3^{(4)}(\alpha_1, \alpha_3), 0, 2] \\
 &-2\mathcal{G}[\mathcal{T}_3^{(4)}(\alpha_1, \alpha_3), 1, 1]-3\mathcal{G}[\mathcal{T}_4^{(4)}(\alpha_1, \alpha_3), 0, 2] \\
 &-\beta(3\mathcal{G}[\mathcal{T}_2^{(4)}(\alpha_1, \alpha_3), 0, 2] \\
 &+3\mathcal{G}[\mathcal{T}_2^{(4)}(\alpha_1, \alpha_3), 1, 1]-\mathcal{G}[\mathcal{T}_3^{(4)}(\alpha_1, \alpha_3), 0, 2] \\
 &+\mathcal{G}[\mathcal{T}_4^{(4)}(\alpha_1, \alpha_3), 0, 2]) \\
 &+2\mathcal{G}[\mathcal{T}_4^{(4)}(\alpha_1, \alpha_3), 1, 1]+4\mathcal{G}[\mathcal{T}_3^{(4)}(\alpha_1, \alpha_3)u, 1, 1] \\
 &-4\mathcal{G}[\mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)u, 1, 1]) \\
 &+f_V^T im_V^3((3+\beta)\mathcal{G}_2[\mathcal{H}_G[2, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &-2\mathcal{G}_2[\mathcal{H}_G[2, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 1, 1]
 \end{aligned}$$

$$\begin{aligned}
 &+4\mathcal{G}_2[u\mathcal{H}_G[2, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &-(3 + \beta)\mathcal{G}_2[\mathcal{H}_G[2, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &+2(\mathcal{G}_2[\mathcal{H}_G[2, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &-2\mathcal{G}_2[u\mathcal{H}_G[2, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1])) \\
 \Pi_{f_1}^{(SA)\text{theo}} = &-(1/(96\sqrt{3}M^4\pi^4))m_Q \\
 &\times m_Q m_V N(3(-1 + \beta)(f_V^T)^2(m_Q - m_{Q'})m_V^3 \\
 &\times ((4 + 8\beta)\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}(\alpha_1, \alpha_3)], 0, 2] \\
 &+ 2(2\beta\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}(\alpha_1, \alpha_3)], 1, 1] \\
 &+ 2\beta\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &+ 2\beta\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &- 2\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &+ 3\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &+ \beta\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &- 6\mathcal{G}[u\mathcal{H}_G[1, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &- 2\beta\mathcal{G}[u\mathcal{H}_G[1, \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &+ 2\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_3^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &+ 2\beta\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_3^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &+ 4\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &+ 4\beta\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)], 0, 2] \\
 &- 3\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &+ \beta\mathcal{G}[\mathcal{H}_G[1, \mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &+ 2(3 + \beta)\mathcal{G}[u\mathcal{H}_G[1, \mathcal{T}_4^{(4)}(\alpha_1, \alpha_3)], 1, 1] \\
 &+ m_V^2(2(2 + \beta)\mathcal{G}_2[\mathcal{H}_G[3, \mathcal{T}(\alpha_1, \alpha_3)], 0, 2] \\
 &- (3 + \beta)(\mathcal{G}_2[\mathcal{H}_G[3, \mathcal{T}(\alpha_1, \alpha_3)], 1, 1] \\
 &- 2\mathcal{G}_2[u\mathcal{H}_G[3, \mathcal{T}(\alpha_1, \alpha_3)], 1, 1])) \\
 &+ 2if_V(3(-1 + \beta)^2(m_Q - m_{Q'}) \\
 &\times m_V^2\mathcal{G}[\mathcal{A}(\alpha_1, \alpha_3), 1, 2] + 18(1 + \beta)^2 \\
 &\times (m_Q - m_{Q'})m_V^2\mathcal{G}[\mathcal{A}(\alpha_1, \alpha_3)u, 1, 2] \\
 &- 3m_Q m_V^2((-1 + \beta)^2 \\
 &\times \mathcal{G}[\mathcal{V}(\alpha_1, \alpha_3), 1, 2] - 8(1 + \beta + \beta^2) \\
 &\mathcal{G}[\mathcal{V}(\alpha_1, \alpha_3)u, 1, 2]) + m_{Q'}(3m_V^2 \\
 &\times ((-1 + \beta)^2\mathcal{G}[\mathcal{V}(\alpha_1, \alpha_3), 1, 2] \\
 &- 8(1 + \beta + \beta^2)\mathcal{G}[\mathcal{V}(\alpha_1, \alpha_3)u, 1, 2]) \\
 &+ 14(1 + \beta^2)m_Q(16\mathcal{I}[\phi_2^\parallel(u), 2, 2] \\
 &+ m_V^2(\mathcal{I}_2[\phi_4^\parallel(u), 2, 2] \\
 &+ 24(-2\mathcal{I}_2[\mathcal{H}[2, \phi_3^\perp(u)], 2, 2] \\
 &+ \mathcal{I}_2[\mathcal{H}[2, \phi_2^\parallel(u)], 2, 2] \\
 &+ \mathcal{I}_2[\mathcal{H}[2, \psi_4^\parallel(u)], 2, 2]))))
 \end{aligned} \tag{36}$$

In Eqs. (31)–(36), N is the normalization factor which is equal to $\frac{1}{\sqrt{2}}$ (1) for different (identical) heavy quark flavors; moreover, we have suppressed the second argument, $\alpha_2 = 1 - \alpha_1 - \alpha_3$, in the 3-particle distribution amplitudes for the sake of convenience, and defined the following integrals and operators:

$$\mathcal{I}[f(u), i, j] := \int du \int d^4x e^{i(p+\bar{u}q)\cdot x} K_i K_j f(u) \tag{37}$$

$$\mathcal{I}_2[f(u), i, j] := \int du \int d^4x e^{i(p+\bar{u}q)\cdot x} K_i K_j f(u)x^2 \tag{38}$$

$$\mathcal{I}_4[f(u), i, j] := \int du \int d^4x e^{i(p+\bar{u}q)\cdot x} K_i K_j f(u)x^4 \tag{39}$$

$$\begin{aligned}
 \mathcal{G}[f(u)\mathcal{F}(\alpha_i), i, j] &:= \int du \int \mathcal{D}\alpha_i \\
 &\times \int d^4x e^{i(p+(\alpha_1+u\alpha_3)q)\cdot x} K_i K_j f(u)\mathcal{F}(\alpha_i)
 \end{aligned} \tag{40}$$

$$\begin{aligned}
 \mathcal{G}_2[f(u)\mathcal{F}(\alpha_i), i, j] &:= \int du \int \mathcal{D}\alpha_i \\
 &\times \int d^4x e^{i(p+(\alpha_1+u\alpha_3)q)\cdot x} K_i K_j f(u)\mathcal{F}(\alpha_i)x^2
 \end{aligned} \tag{41}$$

$$\begin{aligned}
 \mathcal{G}_4[f(u)\mathcal{F}(\alpha_i), i, j] &:= \int du \int \mathcal{D}\alpha_i \\
 &\times \int d^4x e^{i(p+(\alpha_1+u\alpha_3)q)\cdot x} K_i K_j f(u)\mathcal{F}(\alpha_i)x^4
 \end{aligned} \tag{42}$$

$$\mathcal{H}[n, f(u)] := i^n \int_0^u dv_n \cdots \int_0^{v_3} dv_2 \int_0^{v_2} dv_1 f(v_1) \tag{43}$$

$$\begin{aligned}
 \mathcal{H}_G[n, \mathcal{F}(\alpha_i)] &:= (-iu)^n \int_0^{\alpha_3} d\alpha_3^{(n)} \\
 &\cdots \int_0^{\alpha_3^{(3)}} d\alpha_3^{(2)} \int_0^{\alpha_3^{(2)}} d\alpha_3^{(1)} \\
 &\mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3^{(1)}, \alpha_3^{(1)})
 \end{aligned} \tag{44}$$

$$\mathcal{D}[n, f(u)] := \left(-i \frac{\partial}{\partial u}\right)^n f(u) \tag{45}$$

$$\mathcal{D}_G[n, \mathcal{F}(\alpha_i)] := \left(\frac{i}{u} \frac{\partial}{\partial \alpha_3}\right)^n \mathcal{F}(\alpha_i) \tag{46}$$

where we have introduced the short-hand notation

$$K_i := \frac{K_i(m_Q\sqrt{-x^2})}{(\sqrt{-x^2})^i}, \quad K_j := \frac{K_j(m_{Q'}\sqrt{-x^2})}{(\sqrt{-x^2})^j} \tag{47}$$

Appendix B: Distribution amplitudes for vector mesons

In this section, we collect the matrix elements $\langle V(q, \varepsilon)|\bar{q}(x)\Gamma_i q(0)|0\rangle$ and $\langle V(q, \varepsilon)|\bar{q}(x)\Gamma_i G_{\mu\nu} q(0)|0\rangle$ and the relevant distribution amplitudes for vector mesons together with the most recent values for the DA parameters involved [32–35]. Up to twist-4 accuracy, the matrix elements $\langle V(q, \varepsilon)|\bar{q}(x)\Gamma q(0)|0\rangle$ and $\langle V(q, \varepsilon)|\bar{q}(x)\Gamma G_{\mu\nu} q(0)|0\rangle$ are given as follows:

$$\begin{aligned} & \langle V(q, \varepsilon) | \bar{q}_1(x) \gamma_\mu q_2(0) | 0 \rangle \\ &= f_V m_V \left\{ \frac{\varepsilon^\lambda \cdot x}{q \cdot x} q_\mu \int_0^1 du e^{i\bar{u}q \cdot x} \right. \\ & \times \left[\phi_2^\parallel(u) + \frac{m_V^2 x^2}{16} \phi_4^\parallel(u) \right] + \left(\varepsilon_\mu^\lambda - q_\mu \frac{\varepsilon^\lambda \cdot x}{q \cdot x} \right) \\ & \times \int_0^1 du e^{i\bar{u}q \cdot x} \phi_3^\perp(u) - \frac{1}{2} x_\mu \frac{\varepsilon^\lambda \cdot x}{(q \cdot x)^2} m_V^2 \\ & \times \int_0^1 du e^{i\bar{u}q \cdot x} \left[\psi_4^\parallel(u) + \phi_2^\parallel(u) - 2\phi_3^\perp(u) \right] \left. \right\} \end{aligned} \tag{48}$$

$$\begin{aligned} & \langle V(q, \varepsilon) | \bar{q}_1(x) \gamma_\mu \gamma_5 q_2(0) | 0 \rangle \\ &= -\frac{1}{4} \varepsilon_\mu^{\nu\alpha\beta} \varepsilon_\nu^\lambda q_\alpha x_\beta f_V m_V \int_0^1 du \psi_3^\perp(u) \end{aligned} \tag{49}$$

$$\begin{aligned} & \langle V(q, \varepsilon) | \bar{q}_1(x) \sigma_{\mu\nu} q_2(0) | 0 \rangle \\ &= -i f_V^T \left\{ (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu) \right. \\ & \times \int_0^1 du e^{i\bar{u}q \cdot x} \left[\phi_2^\perp(u) + \frac{m_V^2 x^2}{16} \phi_4^\perp(u) \right] \\ & + \frac{\varepsilon \cdot x}{(q \cdot x)^2} (q_\mu x_\nu - q_\nu x_\mu) \\ & \times \int_0^1 du e^{i\bar{u}q \cdot x} \left[\phi_3^\parallel(u) - \frac{1}{2} \phi_2^\perp(u) - \frac{1}{2} \psi_4^\perp(u) \right] \\ & + \frac{1}{2} (\varepsilon_\mu^\lambda x_\nu - \varepsilon_\nu^\lambda x_\mu) \frac{m_V^2}{q \cdot x} \\ & \times \int_0^1 du e^{i\bar{u}q \cdot x} \left[\psi_4^\perp(u) - \phi_2^\perp(u) \right] \left. \right\} \end{aligned} \tag{50}$$

$$\begin{aligned} & \langle V(q, \varepsilon) | \bar{q}_1(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(ux) q_2(0) | 0 \rangle \\ &= f_V^T m_V^2 \frac{\varepsilon^\lambda \cdot x}{2q \cdot x} \left(q_\alpha q_\mu g_{\beta\nu}^\perp - q_\beta q_\mu g_{\alpha\nu}^\perp - q_\alpha q_\nu g_{\beta\mu}^\perp + q_\beta q_\nu g_{\alpha\mu}^\perp \right) \\ & \times \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)q \cdot x} \mathcal{T}(\alpha_i) \\ & + f_V^T m_V^2 \left(q_\alpha \varepsilon_\mu^\lambda g_{\beta\nu}^\perp - q_\alpha \varepsilon_\nu^\lambda g_{\beta\mu}^\perp + q_\beta \varepsilon_\mu^\lambda g_{\alpha\nu}^\perp \right) \\ & \times \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)q \cdot x} \mathcal{T}_1^{(4)}(\alpha_i) \\ & + f_V^T m_V^2 \left(q_\mu \varepsilon_\alpha^\lambda g_{\beta\nu}^\perp - q_\mu \varepsilon_\beta^\lambda g_{\alpha\nu}^\perp - q_\nu \varepsilon_\alpha^\lambda g_{\beta\mu}^\perp + q_\nu \varepsilon_\beta^\lambda g_{\alpha\mu}^\perp \right) \\ & \times \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)q \cdot x} \mathcal{T}_2^{(4)}(\alpha_i) \\ & + \frac{f_V^T m_V^2}{q \cdot x} \left(q_\alpha q_\mu \varepsilon_\beta^\lambda x_\nu - q_\beta q_\mu \varepsilon_\alpha^\lambda x_\nu - q_\alpha q_\nu \varepsilon_\beta^\lambda x_\mu + q_\beta q_\nu \varepsilon_\alpha^\lambda x_\mu \right) \\ & \times \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)q \cdot x} \mathcal{T}_3^{(4)}(\alpha_i) \\ & + \frac{f_V^T m_V^2}{q \cdot x} \left(q_\alpha q_\mu \varepsilon_\nu^\lambda x_\beta - q_\beta q_\mu \varepsilon_\nu^\lambda x_\alpha - q_\alpha q_\nu \varepsilon_\mu^\lambda x_\beta + q_\beta q_\nu \varepsilon_\mu^\lambda x_\alpha \right) \\ & \times \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)q \cdot x} \mathcal{T}_4^{(4)}(\alpha_i) \end{aligned} \tag{51}$$

$$\begin{aligned} & \langle V(q, \varepsilon) | \bar{q}_1(x) g_s G_{\mu\nu}(ux) q_2(0) | 0 \rangle \\ &= -i f_V^T m_V (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)q \cdot x} \mathcal{S}(\alpha_i) \end{aligned} \tag{52}$$

$$\begin{aligned} & \langle V(q, \lambda) | \bar{q}_1(x) g_s \tilde{G}_{\mu\nu}(ux) \gamma_5 q_2(0) | 0 \rangle \\ &= -i f_V^T m_V (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)q \cdot x} \tilde{\mathcal{S}}(\alpha_i) \end{aligned} \tag{53}$$

$$\begin{aligned} & \langle V(q, \varepsilon) | \bar{q}_1(x) g_s \tilde{G}_{\mu\nu}(ux) \gamma_\alpha \gamma_5 q_2(0) | 0 \rangle \\ &= f_V m_V q_\alpha (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)q \cdot x} \mathcal{A}(\alpha_i) \end{aligned} \tag{54}$$

$$\begin{aligned} & \langle V(q, \varepsilon) | \bar{q}_1(x) g_s G_{\mu\nu}(ux) i \gamma_\alpha q_2(0) | 0 \rangle \\ &= f_V m_V q_\alpha (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)q \cdot x} \mathcal{V}(\alpha_i) \end{aligned} \tag{55}$$

where $\tilde{G}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}$ is the dual gluon field strength tensor and $\int \mathcal{D}\alpha_i = \int d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. Now we list the DAs.

2-particle twist-2 DAs:

$$\phi_2^\parallel(u) = 6\bar{u}(1 + a_1^\parallel C_1^{3/2}(\xi) + a_2^\parallel C_2^{3/2}(\xi))u \tag{56}$$

$$\phi_2^\perp(u) = 6\bar{u}(1 + a_1^\perp C_1^{3/2}(\xi) + a_2^\perp C_2^{3/2}(\xi))u \tag{57}$$

2-particle twist-3 DAs:

$$\begin{aligned} \phi_3^\parallel(u) &= 3\xi^2 + (3a_1^\perp \xi(-1 + 3\xi^2))/2 \\ & + ((15\kappa_3^\perp)/2 - (3\lambda_3^\perp)/4)\xi(-3 + 5\xi^2) \\ & + (3a_2^\perp \xi^2(-3 + 5\xi^2))/2 \\ & + (5\omega_3^\perp(3 - 30\xi^2 + 35\xi^4))/8 - (3f_V(m_{q_1} - m_{q_2})) \\ & \times \xi(2 + 9a_1^\parallel \xi 2a_2^\parallel(11 - 30\bar{u}u) \\ & + (1 + 6a_2^\parallel + 3a_1^\parallel) \ln(\bar{u}) + (1 + 6a_2^\parallel - 3a_1^\parallel) \ln(u)) / (2f_V^T m_V) \\ & + (3f_V(m_{q_1} + m_{q_2})(1 + 8a_1^\parallel \xi + 3a_2^\parallel(7 - 30\bar{u}u) \\ & + (1 + 6a_2^\parallel + 3a_1^\parallel)\xi \\ & \ln(\bar{u}) - (1 + 6a_2^\parallel - 3a_1^\parallel)\xi \ln(u))) / (2f_V^T m_V) \end{aligned} \tag{58}$$

$$\begin{aligned} \psi_3^\parallel(u) &= 6\bar{u}(1 + C_1^{3/2}(\xi)(a_1^\perp/3 + (5\kappa_3^\perp)/3) \\ & - (C_3^{3/2}(\xi)\lambda_3^\perp)/20 + C_2^{3/2}(\xi)(a_2^\perp/6 + (5\omega_3^\perp)/18)) \\ & \times u - (3f_V(m_{q_1} - m_{q_2})) \\ & \times (\bar{u}(9a_1^\parallel + 10a_2^\parallel \xi)u + (1 + 6a_2^\parallel + 3a_1^\parallel)\bar{u} \\ & \times \ln(\bar{u}) - (1 + 6a_2^\parallel - 3a_1^\parallel) \\ & \times u \ln(u))) / (f_V^T m_V) \\ & + (3f_V(m_{q_1} + m_{q_2})(\bar{u}u(1 + 2a_1^\parallel \xi + 3a_2^\parallel(7 - 5\bar{u}u)) \\ & + (1 + 6a_2^\parallel + 3a_1^\parallel)\bar{u} \ln(\bar{u}) \\ & + (1 + 6a_2^\parallel - 3a_1^\parallel)u \ln(u))) / (f_V^T m_V) \end{aligned} \tag{59}$$

$$\begin{aligned} \psi_3^\perp(u) &= 6\bar{u}(1 + C_1^{3/2}(\xi)(a_1^\perp/3 + (20\kappa_3^\perp)/9) \\ & + C_3^{3/2}(\xi)(-\lambda_3^\perp/8 + \tilde{\lambda}_3^\perp/4) \\ & + C_2^{3/2}(\xi)(a_2^\perp/6 + (5\omega_3^\perp)/12 - (5\tilde{\omega}_3^\perp)/24 \\ & + (10\zeta_3^\perp)/9))u - (6f_V^T(m_{q_1} - m_{q_2})(\bar{u}(9a_1^\perp + 10a_2^\perp \xi)u \\ & + (1 + 6a_2^\perp + 3a_1^\perp)\bar{u} \ln(\bar{u}) \\ & - (1 + 6a_2^\perp - 3a_1^\perp)u \ln(u))) / (f_V m_V) + (6f_V^T(m_{q_1} + m_{q_2})) \\ & \times (\bar{u}u(2 + 3a_1^\perp \xi + 2a_2^\perp(11 - 10\bar{u}u)) \\ & + (1 + 6a_2^\perp + 3a_1^\perp)\bar{u} \ln(\bar{u}) + (1 + 6a_2^\perp - 3a_1^\perp) \\ & \times u \ln(u))) / (f_V m_V) \end{aligned} \tag{60}$$

$$\begin{aligned} \phi_3^\perp(u) &= (3a_1^\parallel \xi^3)/2 + (3(1 + \xi^2))/4 \\ & + (5\kappa_3^\parallel - (15\lambda_3^\parallel))/16 \\ & + (15\tilde{\lambda}_3^\parallel)/8 \xi(-3 + 5\xi^2) \end{aligned}$$

$$\begin{aligned}
 & +((9a_2^{\parallel})/112 + (15\omega_3^{\parallel})/32 \\
 & - (15\tilde{\omega}_3^{\parallel})/64)(3 - 30\xi^2 + 35\xi^4) + (-1 + 3\xi^2)((3a_2^{\parallel})/7 \\
 & + 5\xi_3^{\parallel}) - (3f_V^T(m_{q_1} - m_{q_2}) \\
 & (2\xi + 2a_2^{\perp}\xi(11 - 20\bar{u}u) + 9a_1^{\perp}(1 - 2\bar{u}u) \\
 & + (1 + 6a_2^{\perp} + 3a_1^{\perp})\ln(\bar{u}) \\
 & - (1 + 6a_2^{\perp} - 3a_1^{\perp})\ln(u))/2f_V m_V \\
 & + (3f_V^T(m_{q_1} + m_{q_2})(2 + 9a_1^{\perp}\xi \\
 & + 2a_2^{\perp}(11 - 30\bar{u}u) + (1 + 6a_2^{\perp} + 3a_1^{\perp})\ln(\bar{u}) \\
 & + (1 + 6a_2^{\perp} - 3a_1^{\perp})\ln(u))/2f_V m_V) \tag{61}
 \end{aligned}$$

2-particle twist-4 DAs:

$$\begin{aligned}
 \psi_4^{\parallel}(u) = & 1 + C_3^{1/2}(\xi)((-9a_1^{\parallel})/5 \\
 & - (20\kappa_3^{\parallel})/3 - (16\kappa_4^{\parallel})/3) + C_1^{1/2}(\xi)((9a_1^{\parallel})/5 + 12\kappa_4^{\parallel}) \\
 & + C_3^{1/2}(\xi)(-5\theta_2^{\parallel} + 10\theta_1^{\parallel}) \\
 & + (6f_V^T(m_{q_1} - m_{q_2})(\xi + (a_1^{\perp}(-1 + 3\xi^2))/2 \\
 & + (5\kappa_3^{\perp}(-1 + 3\xi^2))/2 \\
 & + (a_2^{\perp}\xi(-3 + 5\xi^2))/2 + (5\omega_3^{\perp}\xi(-3 + 5\xi^2))/6 \\
 & - (\lambda_3^{\perp}(3 - 30\xi^2 + 35\xi^4))/16))/f_V m_V \\
 & + C_4^{1/2}(\xi)((-27a_2^{\parallel})/28 - (15\omega_3^{\parallel})/8 \\
 & - (15\tilde{\omega}_3^{\parallel})/16 + (5\xi_3^{\parallel})/4) \\
 & + C_2^{1/2}(\xi)(-1 - (2a_2^{\parallel})/7 + (40\xi_3^{\parallel})/3) \\
 & - (20C_2^{1/2}(\xi)\xi_4^{\parallel})/3 \tag{62}
 \end{aligned}$$

$$\begin{aligned}
 \phi_4^{\parallel}(u) = & (6\bar{u}f_V^T(m_{q_1} - m_{q_2})(-C_1^{3/2}(\xi)((82a_1^{\perp})/5 + 10\kappa_3^{\perp})) \\
 & + C_3^{3/2}(\xi)((2a_1^{\perp})/5 + (7\lambda_3^{\perp})/54) \\
 & + (2C_5^{3/2}(\xi)\lambda_3^{\perp})/135 + C_4^{3/2}(\xi) \\
 & \times (-2/315 + a_2^{\perp}/5 - \omega_3^{\perp}/21) + 20C_2^{3/2}(\xi)(10/189 \\
 & + a_2^{\perp}/3 - \omega_3^{\perp}/21)u)/f_V m_V \\
 & + (6\bar{u}f_V^T(m_{q_1} + m_{q_2})(2(3 + 16a_2^{\perp}) \\
 & + (10C_1^{3/2}(\xi)(-a_1^{\perp} \\
 & + \kappa_3^{\perp}))/3 - (C_3^{3/2}(\xi)\lambda_3^{\perp})/10 \\
 & + C_2^{3/2}(\xi)(-a_2^{\perp} + (5\omega_3^{\perp})/9))u)/f_V m_V \\
 & + 30\bar{u}^2(C_1^{5/2}(\xi)((17a_1^{\parallel})/50 \\
 & - \lambda_3^{\parallel}/5 + (2\tilde{\lambda}_3^{\parallel})/5) + (C_2^{5/2}(\xi)((9a_2^{\parallel})/7 + (7\omega_3^{\parallel})/6 \\
 & - (3\tilde{\omega}_3^{\parallel})/4 + \xi_3^{\parallel}/9))/10 + (4(1 + a_2^{\perp}/21 \\
 & + (10\xi_3^{\parallel})/9))/5)u^2 + 30\bar{u}^2(C_1^{5/2}(\xi)((2\theta_2^{\parallel})/3 \\
 & - (8\theta_1^{\parallel})/15) + (20\xi_4^{\parallel}/9)u^2 + (f_V^T(m_{q_1} - m_{q_2}) \\
 & \times ((-23 - 108a_2^{\perp} - 54a_1^{\perp} + 5u^2)\ln(\bar{u}) \\
 & - (-23 - 108a_2^{\perp} + 54a_1^{\perp} + 5\bar{u}^2)\ln(u))/f_V m_V \\
 & + (24f_V^T(m_{q_1} + m_{q_2})(1 + 6a_2^{\perp} \\
 & + 3a_1^{\perp})\bar{u}^2\ln(\bar{u}) + (1 + 6a_2^{\perp} - 3a_1^{\perp})u^2
 \end{aligned}$$

$$\begin{aligned}
 & \times \ln(u))/f_V m_V) + 4(a_1^{\parallel} - (40\kappa_3^{\parallel})/3)((11 \\
 & - 3\xi^2)/8 - (2 - \bar{u})\bar{u}^3\ln(\bar{u}) + (2 - u)u^3\ln(u) \\
 & + 80\psi_2^{\parallel}((11 - 3\xi^2)/8 \\
 & - (2 - \bar{u})\bar{u}^3\ln(\bar{u}) + (2 - u)u^3\ln(u) \\
 & - 80\tilde{\omega}_4^{\parallel}((\bar{u}(21 - 13\xi^2)u)/8 + \bar{u}^3(10 - 15\bar{u} \\
 & + 6\bar{u}^2)\ln(\bar{u}) + u^3(10 - 15u + 6u^2)\ln(u) \\
 & + 2(-2a_2^{\parallel} + 3\omega_3^{\parallel} - (14\xi_3^{\parallel})/3)((\bar{u}(21 \\
 & - 13\xi^2)u)/8 + \bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2) \\
 & \times \ln(\bar{u}) + u^3(10 - 15u + 6u^2)\ln(u)) \tag{63}
 \end{aligned}$$

$$\begin{aligned}
 \psi_4^{\perp}(u) = & 1 + C_1^{1/2}(\xi)((-3a_1^{\perp})/5 + 12\kappa_4^{\perp}) \\
 & + (C_5^{1/2}(\xi)\lambda_3^{\perp})/3 + C_4^{1/2}(\xi)((-3a_2^{\perp})/7 \\
 & - (5\omega_3^{\perp})/4) + C_3^{1/2}(\xi)((3a_1^{\perp})/5 \\
 & - 5\kappa_3^{\perp} - 12\kappa_4^{\perp} - \lambda_3^{\perp}/3 + 5((-\theta_2^{\perp} - \tilde{\theta}_2^{\perp})/2 + \theta_1^{\perp} \\
 & + \tilde{\theta}_1^{\perp})) + (f_V(m_{q_1} + m_{q_2})(3(1 + 6a_2^{\parallel}) \\
 & + 3a_1^{\parallel}C_1^{1/2}(\xi) + 5C_3^{1/2}(\xi)(4\kappa_3^{\parallel} - (3\lambda_3^{\parallel})/4 \\
 & + (3\tilde{\lambda}_3^{\parallel})/2) + (15C_4^{1/2}(\xi) \\
 & (2\omega_3^{\parallel} - \tilde{\omega}_3^{\parallel}))/4 + 5C_2^{1/2}(\xi)(-3a_2^{\parallel} + 4\xi_3^{\parallel}))/f_V^T m_V \\
 & + C_2^{1/2}(\xi)(-1 + (3a_2^{\perp})/7 - 10(\xi_4^{\perp} + \tilde{\xi}_4^{\perp})) \\
 & - (6\bar{u}f_V(m_{q_1} - m_{q_2})(9a_1^{\parallel} + 10a_2^{\parallel}\xi) \\
 & \times u)/f_V^T m_V) + (6f_V(m_{q_1} - m_{q_2}) \\
 & (-((1 + 6a_2^{\parallel} + 3a_1^{\parallel})\bar{u}\ln(\bar{u})) \\
 & + (1 + 6a_2^{\parallel} - 3a_1^{\parallel}) \\
 & \times u\ln(u))/f_V^T m_V) + (6f_V(m_{q_1} + m_{q_2}) \\
 & \times ((1 + 6a_2^{\parallel} + 3a_1^{\parallel})\bar{u}\ln(\bar{u}) \\
 & + (1 + 6a_2^{\parallel} - 3a_1^{\parallel}) \\
 & \times u\ln(u))/f_V^T m_V) \tag{64}
 \end{aligned}$$

$$\begin{aligned}
 \phi_4^{\perp}(u) = & 30\bar{u}^2(2/5 + (4a_2^{\perp})/35 - (4C_3^{5/2}(\xi)\lambda_3^{\perp})/1575 + C_2^{5/2}(\xi) \\
 & \times ((3a_2^{\perp})/35 + \omega_3^{\perp}/60) \\
 & + C_1^{5/2}(\xi)((3a_1^{\perp})/25 + \kappa_3^{\perp}/3 - \lambda_3^{\perp}/45 + (7\theta_2^{\perp})/30 \\
 & - (3\tilde{\theta}_2^{\perp})/20 - \theta_1^{\perp}/15 + \tilde{\theta}_1^{\perp}/5) \\
 & + (4\xi_4^{\perp})/3 - (8\tilde{\xi}_4^{\perp})/3)u^2 \\
 & + (-a_1^{\perp} + 5\kappa_3^{\perp} - 20\tilde{\phi}_2^{\perp})(\bar{u}\xi(-11 + 3\xi^2)u)/2 + 4(2 - \bar{u}) \\
 & \times \bar{u}^3\ln(\bar{u}) - 4(2 - u)u^3\ln(u) + ((-36a_2^{\perp})/11 \\
 & - (252\langle Q^{(1)} \rangle)/55 - (140\langle Q^{(3)} \rangle)/11 \\
 & + 2\omega_3^{\perp})(-\bar{u}(-21 + 13\xi^2)u)/8 + \bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2) \\
 & \ln(\bar{u}) + u^3(10 - 15u + 6u^2) \times \ln(u)) \tag{65}
 \end{aligned}$$

3-particle twist-3 DAs:

$$\begin{aligned}
 \mathcal{S}(\alpha_1, \alpha_3) = & 30\alpha_3^2(((-3\alpha_1^2 \\
 & + (1 - \alpha_1 - \alpha_3)^2))/2 + (1 - \alpha_3)\alpha_3)\psi_2^{\perp}
 \end{aligned}$$

$$\begin{aligned}
 &+(-6\alpha_1(1-\alpha_1-\alpha_3)+(1-\alpha_3)\alpha_3)\psi_1^\perp \\
 &+(1-\alpha_3)\psi_0^\perp - (-1+2\alpha_1+\alpha_3)((-3+5\alpha_3)\theta_2^\perp)/2 \\
 &+\alpha_3\theta_1^\perp + \theta_0^\perp) \tag{66}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{S}(\alpha_1, \alpha_3) &= 30\alpha_3^2(((-3(\alpha_1^2 + (1-\alpha_1-\alpha_3)^2))/2 \\
 &+(1-\alpha_3)\alpha_3)\tilde{\psi}_2^\perp + (-6\alpha_1(1-\alpha_1-\alpha_3) \\
 &+(1-\alpha_3)\alpha_3)\tilde{\psi}_1^\perp \\
 &+(1-\alpha_3)\tilde{\psi}_0^\perp - (\alpha_1-\alpha_3)((-3+5\alpha_3)\tilde{\theta}_2^\perp)/2 + \alpha_3\tilde{\theta}_1^\perp \\
 &+\tilde{\theta}_0^\perp) \tag{67}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{V}(\alpha_1, \alpha_3) &= 360\alpha_1(1-\alpha_1-\alpha_3)\alpha_3^2 \\
 &(\kappa_3^\parallel + ((-3+7\alpha_3)\lambda_3^\parallel)/2 + (-1+2\alpha_1+\alpha_3)\omega_3^\parallel) \tag{68}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}(\alpha_1, \alpha_3) &= 360\alpha_1(1-\alpha_1-\alpha_3)\alpha_3^2((-1+2\alpha_1+\alpha_3)\tilde{\lambda}_3^\parallel \\
 &+((-3+7\alpha_3)\tilde{\omega}_3^\parallel)/2 + \zeta_3^\parallel) \tag{69}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{T}(\alpha_1, \alpha_3) &= 360\alpha_1(1-\alpha_1-\alpha_3)\alpha_3^2(\kappa_3^\perp \\
 &+((-3+7\alpha_3)\lambda_3^\perp)/2 + (-1+2\alpha_1+\alpha_3)\omega_3^\perp) \tag{70}
 \end{aligned}$$

3-particle twist-4 DAs:

$$\begin{aligned}
 \mathcal{T}_1^{(4)}(\alpha_1, \alpha_3) &= 120\alpha_1(1-\alpha_1-\alpha_3) \\
 &\times \alpha_3((-1+2\alpha_1+\alpha_3)\phi_1^\perp + \phi_0^\perp \\
 &+(-1+3\alpha_3)\phi_2^\perp) \tag{71}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{T}_2^{(4)}(\alpha_1, \alpha_3) &= -30\alpha_3^2(-((-1+2\alpha_1+\alpha_3) \\
 &(((-3+5\alpha_3)\tilde{\psi}_2^\perp)/2 + \alpha_3\tilde{\psi}_1^\perp + \tilde{\psi}_0^\perp)) \\
 &+((-3(\alpha_1^2 + (1-\alpha_1-\alpha_3)^2))/2 \\
 &+(1-\alpha_3)\alpha_3)\tilde{\theta}_2^\perp \\
 &+(-6\alpha_1(1-\alpha_1-\alpha_3)+(1-\alpha_3)\alpha_3)\tilde{\theta}_1^\perp \\
 &+(1-\alpha_3)\tilde{\theta}_0^\perp) \tag{72}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{T}_3^{(4)}(\alpha_1, \alpha_3) &= -120\alpha_1(1-\alpha_1-\alpha_3)\alpha_3((-1+3\alpha_3) \\
 &\tilde{\phi}_2^\perp + (-1+2\alpha_1+\alpha_3)\tilde{\phi}_1^\perp + \tilde{\phi}_0^\perp) \tag{73}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{T}_4^{(4)}(\alpha_1, \alpha_3) &= 30\alpha_3^2(-((-1+2\alpha_1+\alpha_3) \\
 &\times (((-3+5\alpha_3)\psi_2^\perp)/2 + \alpha_3\psi_1^\perp + \psi_0^\perp)) + ((-3(\alpha_1^2 \\
 &+(1-\alpha_1-\alpha_3)^2))/2 + (1-\alpha_3)\alpha_3) \\
 &\theta_2^\perp + (-6\alpha_1(1-\alpha_1-\alpha_3)+(1-\alpha_3)\alpha_3)\theta_1^\perp \\
 &+(1-\alpha_3)\theta_0^\perp) \tag{74}
 \end{aligned}$$

where we have replaced $\alpha_2 = 1 - \alpha_1 - \alpha_3$ before the integration and we take $\xi = \bar{u}$ since the second quark is at the point $x = 0$. The $C_n^m(x)$ are the Gegenbauer polynomials. The q_1 and q_2 indicate the quark components of the vector meson. The ρ meson has both light quarks, hence $m_{q_1} = m_{q_2} = 0$. The K^* meson has one strange quark and one light quark, thus $m_{q_1} = m_s$ but $m_{q_2} = 0$. The derived DA parameters are given as follows:

$$\psi_0^\perp = \zeta_4^\perp \tag{75}$$

$$\tilde{\psi}_0^\perp = \tilde{\zeta}_4^\perp \tag{76}$$

$$\theta_0^\perp = -(1/6)\kappa_3^\perp - (1/3)\kappa_4^\perp \tag{77}$$

$$\tilde{\theta}_0^\perp = -(1/6)\kappa_3^\perp + (1/3)\kappa_4^\perp \tag{78}$$

$$\phi_0^\perp = (1/6)\kappa_3^\perp + (1/3)\kappa_4^\perp \tag{79}$$

$$\tilde{\phi}_0^\perp = (1/6)\kappa_3^\perp - (1/3)\kappa_4^\perp \tag{80}$$

$$\begin{aligned}
 \phi_1^\perp &= (9/44)a_2^\perp + (1/8)\omega_3^\perp + (63/220)\langle\langle Q^{(1)} \rangle\rangle \\
 &- (119/44)\langle\langle Q^{(3)} \rangle\rangle \tag{81}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{\phi}_1^\perp &= -(9/44)a_2^\perp + (1/8)\omega_3^\perp \\
 &- (63/220)\langle\langle Q^{(1)} \rangle\rangle - (35/44)\langle\langle Q^{(3)} \rangle\rangle \tag{82}
 \end{aligned}$$

$$\begin{aligned}
 \psi_1^\perp &= (3/44)a_2^\perp + (1/12)\omega_3^\perp + (49/110)\langle\langle Q^{(1)} \rangle\rangle \\
 &- (7/22)\langle\langle Q^{(3)} \rangle\rangle + (7/3)\langle\langle Q^{(5)} \rangle\rangle \tag{83}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{\psi}_1^\perp &= -(3/44)a_2^\perp + (1/12)\omega_3^\perp \\
 &- (49/110)\langle\langle Q^{(1)} \rangle\rangle + (7/22)\langle\langle Q^{(3)} \rangle\rangle \tag{84}
 \end{aligned}$$

$$\begin{aligned}
 \psi_2^\perp &= -(3/22)a_2^\perp - (1/12)\omega_3^\perp + (28/55)\langle\langle Q^{(1)} \rangle\rangle \\
 &+ (7/11)\langle\langle Q^{(3)} \rangle\rangle + (14/3)\langle\langle Q^{(5)} \rangle\rangle \tag{85}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{\psi}_2^\perp &= (3/22)a_2^\perp - (1/12)\omega_3^\perp - (28/55)\langle\langle Q^{(1)} \rangle\rangle \\
 &- (7/11)\langle\langle Q^{(3)} \rangle\rangle + (14/3)\langle\langle Q^{(5)} \rangle\rangle \tag{86}
 \end{aligned}$$

$$\theta_1^\parallel = -(7/10)a_1^\parallel \zeta_4^\parallel \tag{87}$$

$$\theta_2^\parallel = (7/5)a_1^\parallel \zeta_4^\parallel \tag{88}$$

$$\psi_2^\parallel = -(7/20)a_1^\parallel \zeta_4^\parallel \tag{89}$$

$$\theta_1^\perp = -(21/10)\zeta_4^\perp a_1^\perp \tag{90}$$

$$\tilde{\theta}_1^\perp = (21/10)\zeta_4^\perp a_1^\perp \tag{91}$$

$$\theta_2^\perp = (21/5)\zeta_4^\perp a_1^\perp \tag{92}$$

$$\tilde{\theta}_2^\perp = -(21/5)\zeta_4^\perp a_1^\perp \tag{93}$$

$$\tilde{\phi}_2^\perp = -(21/20)\zeta_4^\perp a_1^\perp \tag{94}$$

$$\langle\langle Q^{(1)} \rangle\rangle = -(10/3)\zeta_4^\perp \tag{95}$$

$$\langle\langle Q^{(3)} \rangle\rangle = -\zeta_4^\perp \tag{96}$$

$$\langle\langle Q^{(5)} \rangle\rangle = 0 \tag{97}$$

The numerical values for the DA parameters are given in Table 4.

The light quark masses are taken to be zero, namely $m_u = m_d = 0$, and the mass of the strange quark at $\mu = 1$ GeV is taken to be $m_s = 0.137$ GeV.

Appendix C: Details of calculations in the theoretical part

In this section, we present the integrals required in the theoretical analysis. In what follows, $f(u)$ and $\mathcal{F}(\alpha_i)$ denote generic 2- and 3-particle DAs, respectively, and we let $K_i := K_i(m_Q\sqrt{-x^2})/(\sqrt{-x^2})^i$ and $K_j := K_j(m_{Q'}\sqrt{-x^2})/(\sqrt{-x^2})^j$.

Table 4 The numerical values for the parameters in the DAs for vector mesons ρ and K^* . The renormalization scale is $\mu = 1$ GeV

Parameter	ρ	K^*	Parameter	ρ	K^*	Parameter	ρ	K^*	Parameter	ρ	K^*
f_V [GeV]	0.216	0.220	a_1^{\parallel}	0	0.03	ζ_3^{\parallel}	0.030	0.023	ζ_4^{\parallel}	0.07	0.02
f_V^T [GeV]	0.165	0.185	a_1^{\perp}	0	0.04	$\tilde{\lambda}_3^{\parallel}$	0	0.035	$\tilde{\omega}_4^{\parallel}$	-0.03	-0.02
m_V [GeV]	0.770	0.892	a_2^{\parallel}	0.15	0.11	$\tilde{\omega}_3^{\parallel}$	-0.09	-0.07	ζ_4^{\perp}	-0.08	-0.05
			a_2^{\perp}	0.14	0.10	κ_3^{\parallel}	0	0	$\tilde{\zeta}_4^{\perp}$	-0.08	-0.05
						ω_3^{\parallel}	0.15	0.10	κ_4^{\parallel}	0	-0.025
						λ_3^{\parallel}	0	-0.008	κ_4^{\perp}	0	0.013
						κ_3^{\perp}	0	0.003			
						ω_3^{\perp}	0.55	0.30			
						λ_3^{\perp}	0	-0.025			

Terms without $G_{\mu\nu}$: For the sake of simplicity, we suppress the integral measures, $\int du \int d^4x e^{i(p+\bar{u}q)x}$, on the left-hand side.

$$\begin{aligned}
 K_i K_j f(u) &\rightarrow \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} (M^2)^{i+j} \\
 &\times f\left(\frac{1}{2}\right) e^{-m_V^2/2M^2} \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 &\times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right)
 \end{aligned} \tag{98}$$

$$\begin{aligned}
 x_\mu K_i K_j f(u) &\rightarrow \frac{-i(2p_\mu + q_\mu)}{M^2} \\
 &\times \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} (M^2)^{i+j} f\left(\frac{1}{2}\right) e^{-m_V^2/2M^2} \\
 &\times \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 &\times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right)
 \end{aligned} \tag{99}$$

$$\begin{aligned}
 x_\mu x_\nu K_i K_j f(u) &\rightarrow \frac{-(2p_\mu + q_\mu)(2p_\nu + q_\nu) - 2M^2 g_{\mu\nu}}{M^4} \\
 &\times \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} (M^2)^{i+j} f\left(\frac{1}{2}\right) e^{-m_V^2/2M^2} \\
 &\times \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 &\times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right)
 \end{aligned} \tag{100}$$

$$\begin{aligned}
 x^2 K_i K_j f(u) &\rightarrow \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} \\
 &\times M^2 e^{-m_V^2/2M^2} f\left(\frac{1}{2}\right) \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 &\times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) \\
 &(M^2)^{-3+i+j} \\
 &\times \frac{-4(-1+\alpha)(M^2(-1+i+j)\alpha + m_Q^2) + 4\alpha m_Q^2}{\alpha(\alpha-1)}
 \end{aligned} \tag{101}$$

$$\begin{aligned}
 x_\mu x^2 K_i K_j f(u) &\rightarrow \frac{-i(2p_\mu + q_\mu)}{M^2} \\
 &\times \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} M^2 e^{-m_V^2/2M^2} f\left(\frac{1}{2}\right)
 \end{aligned}$$

$$\begin{aligned}
 &\times \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 &\times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) \\
 &(M^2)^{-3+i+j} \\
 &\times \frac{-4(-1+\alpha)(M^2(-1+i+j)\alpha + m_Q^2) + 4\alpha m_Q^2}{\alpha(\alpha-1)}
 \end{aligned} \tag{102}$$

$$\begin{aligned}
 x_\mu x_\nu x^2 K_i K_j f(u) &\rightarrow \frac{-(2p_\mu + q_\mu)(2p_\nu + q_\nu) - 2M^2 g_{\mu\nu}}{M^4} \\
 &\times \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} M^2 e^{-m_V^2/2M^2} f\left(\frac{1}{2}\right) \\
 &\times \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 &\int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) \\
 &\times (M^2)^{-3+i+j} \frac{-4(-1+\alpha)(M^2(-1+i+j)\alpha + m_Q^2) + 4\alpha m_Q^2}{\alpha(\alpha-1)}
 \end{aligned} \tag{103}$$

$$\begin{aligned}
 x^4 K_i K_j f(u) &\rightarrow \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} M^2 e^{-m_V^2/2M^2} f\left(\frac{1}{2}\right) \\
 &\int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 &\times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) \\
 &\frac{16(M^2)^{-5+i+j}}{\alpha^2(-1+\alpha)^2} \\
 &\times (M^4(-2+i+j)(-1+i+j)(-1+\alpha)^2\alpha^2 \\
 &+ (-1+\alpha)^2 m_Q^2 (2M^2(-2+i+j)\alpha + m_Q^2) \\
 &- 2(-1+\alpha)\alpha(M^2(-2+i+j)\alpha + m_Q^2)m_{Q'}^2 + \alpha^2 m_{Q'}^4)
 \end{aligned} \tag{104}$$

$$\begin{aligned}
 x_\mu x^4 K_i K_j f(u) &\rightarrow \frac{-i(2p_\mu + q_\mu)}{M^2} \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} M^2 e^{-m_V^2/2M^2} f\left(\frac{1}{2}\right) \\
 &\int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 &\times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) \\
 &\frac{16(M^2)^{-5+i+j}}{\alpha^2(-1+\alpha)^2} \times (M^4(-2+i+j)(-1+i+j)(-1+\alpha)^2\alpha^2 \\
 &+ (-1+\alpha)^2 m_Q^2 (2M^2(-2+i+j)\alpha + m_Q^2)
 \end{aligned}$$

$$\begin{aligned}
 & -2(-1 + \alpha)\alpha(M^2(-2 + i + j)\alpha + m_Q^2)m_{Q'}^2 + \alpha^2m_{Q'}^4 \tag{105} \\
 x_\mu x_\nu x^4 K_i K_j f(u) & \rightarrow \frac{-(2p_\mu + q_\mu)(2p_\nu + q_\nu) - 2M^2 g_{\mu\nu}}{M^4} \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} M^2 e^{-m_V^2/2M^2} \\
 & f\left(\frac{1}{2}\right) \times \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 & \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) \times \frac{16(M^2)^{-5+i+j}}{\alpha^2(-1+\alpha)^2} \\
 & (M^4(-2+i+j)(-1+i+j)(-1+\alpha)^2\alpha^2 \\
 & + (-1+\alpha)^2 m_Q^2 (2M^2(-2+i+j)\alpha + m_Q^2) \\
 & - 2(-1+\alpha)\alpha(M^2(-2+i+j)\alpha + m_Q^2)m_{Q'}^2 + \alpha^2m_{Q'}^4) \tag{106}
 \end{aligned}$$

Terms with $G_{\mu\nu}$: For the sake of simplicity, we suppress the integral measures, $\int du \int d^4x \int \mathcal{D}\alpha_i e^{i(p+(\alpha_1+u\alpha_3)q)x}$, on the left-hand side.

$$\begin{aligned}
 K_i K_j f(u) \mathcal{F}(\alpha_i) & \rightarrow \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} (M^2)^{i+j} \\
 & e^{-m_V^2/2M^2} \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 & \times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \\
 & \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) \\
 & \times \int_0^{1/2} d\alpha_1 \\
 & \int_{1/2-\alpha_1}^{1-\alpha_1} d\alpha_3 f\left(\frac{1/2-\alpha_1}{\alpha_3}\right) \\
 & \frac{\mathcal{F}(\alpha_i)}{\alpha_3} \tag{107} \\
 x_\mu K_i K_j f(u) \mathcal{F}(\alpha_i) & \rightarrow \frac{-i(2p_\mu + q_\mu)}{M^2} \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} (M^2)^{i+j} \\
 & e^{-m_V^2/2M^2} \\
 & \times \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 & \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) \\
 & \times \int_0^{1/2} d\alpha_1 \int_{1/2-\alpha_1}^{1-\alpha_1} \\
 & d\alpha_3 f\left(\frac{1/2-\alpha_1}{\alpha_3}\right) \frac{\mathcal{F}(\alpha_i)}{\alpha_3} \tag{108} \\
 x_\mu x_\nu K_i K_j f(u) \mathcal{F}(\alpha_i) & \rightarrow \frac{-(2p_\mu + q_\mu)(2p_\nu + q_\nu) - 2M^2 g_{\mu\nu}}{M^4} \\
 & \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} (M^2)^{i+j} e^{-m_V^2/2M^2} \\
 & \times \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2}
 \end{aligned}$$

$$\begin{aligned}
 & \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) \\
 & \times \int_0^{1/2} d\alpha_1 \int_{1/2-\alpha_1}^{1-\alpha_1} d\alpha_3 \\
 & f\left(\frac{1/2-\alpha_1}{\alpha_3}\right) \frac{\mathcal{F}(\alpha_i)}{\alpha_3} \tag{109} \\
 x^2 K_i K_j f(u) \mathcal{F}(\alpha_i) & \rightarrow \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} e^{-m_V^2/2M^2} M^2 \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 & \times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) (M^2)^{-3+i+j} \\
 & \times \frac{-4(-1+\alpha)(M^2(-1+i+j)\alpha + m_Q^2) + 4\alpha m_{Q'}^2}{(-1+\alpha)\alpha} \\
 & \times \int_0^{1/2} d\alpha_1 \\
 & \int_{1/2-\alpha_1}^{1-\alpha_1} d\alpha_3 f\left(\frac{1/2-\alpha_1}{\alpha_3}\right) \frac{\mathcal{F}(\alpha_i)}{\alpha_3} \tag{110} \\
 x_\mu x^2 K_i K_j f(u) \mathcal{F}(\alpha_i) & \rightarrow \frac{-i(2p_\mu + q_\mu)}{M^2} \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} e^{-m_V^2/2M^2} M^2 \\
 & \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 & \times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) (M^2)^{-3+i+j} \\
 & \times \frac{-4(-1+\alpha)(M^2(-1+i+j)\alpha + m_Q^2) + 4\alpha m_{Q'}^2}{(-1+\alpha)\alpha} \\
 & \times \int_0^{1/2} d\alpha_1 \int_{1/2-\alpha_1}^{1-\alpha_1} d\alpha_3 f\left(\frac{1/2-\alpha_1}{\alpha_3}\right) \frac{\mathcal{F}(\alpha_i)}{\alpha_3} \tag{111}
 \end{aligned}$$

$$\begin{aligned}
 & x_\mu x_\nu x^2 K_i K_j f(u) \mathcal{F}(\alpha_i) \\
 & \rightarrow \frac{-(2p_\mu + q_\mu)(2p_\nu + q_\nu) - 2M^2 g_{\mu\nu}}{M^4} \\
 & \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} e^{-m_V^2/2M^2} M^2 \\
 & \times \int_{(m_Q+m_{Q'})^2}^{s_0} ds \\
 & e^{-s/M^2} \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) \\
 & \times (M^2)^{-3+i+j} \frac{-4(-1+\alpha)(M^2(-1+i+j)\alpha + m_Q^2) + 4\alpha m_{Q'}^2}{(-1+\alpha)\alpha} \\
 & \times \int_0^{1/2} d\alpha_1 \int_{1/2-\alpha_1}^{1-\alpha_1} d\alpha_3 f\left(\frac{1/2-\alpha_1}{\alpha_3}\right) \frac{\mathcal{F}(\alpha_i)}{\alpha_3} \tag{112} \\
 x^4 K_i K_j f(u) \mathcal{F}(\alpha_i) & \rightarrow \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} \\
 & M^2 e^{-m_V^2/2M^2} \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 & \times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s - \frac{m_Q^2}{\alpha} - \frac{m_{Q'}^2}{1-\alpha}\right) \frac{16(M^2)^{-5+i+j}}{(-1+\alpha)^2\alpha^2} \\
 & \times (M^4(-2+i+j)(-1+i+j)(-1+\alpha)^2\alpha^2
 \end{aligned}$$

$$\begin{aligned}
 &+(-1+\alpha)^2 m_Q^2 (2M^2(-2+i+j)\alpha+m_Q^2) \\
 &-2(-1+\alpha)\alpha(M^2(-2+i+j)\alpha+m_Q^2)m_{Q'}^2+\alpha m_{Q'}^4) \\
 &\times \int_0^{1/2} d\alpha_1 \\
 &\int_{1/2-\alpha_1}^{1-\alpha_1} d\alpha_3 f\left(\frac{1/2-\alpha_1}{\alpha_3}\right) \frac{\mathcal{F}(\alpha_i)}{\alpha_3}
 \end{aligned} \tag{113}$$

$$\begin{aligned}
 &x_\mu x^4 K_i K_j f(u) \mathcal{F}(\alpha_i) \\
 &\rightarrow \frac{-i(2p_\mu+q_\mu)}{M^2} \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} M^2 e^{-m_V^2/2M^2} \\
 &\int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 &\times \int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s-\frac{m_Q^2}{\alpha}-\frac{m_{Q'}^2}{1-\alpha}\right) \frac{16(M^2)^{-5+i+j}}{(-1+\alpha)^2 \alpha^2} \\
 &\times (M^4(-2+i+j)(-1+i+j)(-1+\alpha)^2 \alpha^2 \\
 &+(-1+\alpha)^2 m_Q^2 (2M^2(-2+i+j)\alpha+m_Q^2) \\
 &-2(-1+\alpha)\alpha(M^2(-2+i+j)\alpha+m_Q^2)m_{Q'}^2+\alpha m_{Q'}^4) \\
 &\times \int_0^{1/2} d\alpha_1 \int_{1/2-\alpha_1}^{1-\alpha_1} d\alpha_3 f\left(\frac{1/2-\alpha_1}{\alpha_3}\right) \frac{\mathcal{F}(\alpha_i)}{\alpha_3}
 \end{aligned} \tag{114}$$

$$\begin{aligned}
 &x_\mu x_\nu x^4 K_i K_j f(u) \mathcal{F}(\alpha_i) \\
 &\rightarrow \frac{-(2p_\mu+q_\mu)(2p_\nu+q_\nu)-2M^2 g_{\mu\nu}}{M^4} \frac{i}{4} \frac{16\pi^2}{(2m_Q)^i (2m_{Q'})^j} M^2 e^{-m_V^2/2M^2} \\
 &\times \int_{(m_Q+m_{Q'})^2}^{s_0} ds e^{-s/M^2} \\
 &\int d\alpha \alpha^{i-1} (1-\alpha)^{j-1} \delta\left(s-\frac{m_Q^2}{\alpha}-\frac{m_{Q'}^2}{1-\alpha}\right) \\
 &\times \frac{16(M^2)^{-5+i+j}}{(-1+\alpha)^2 \alpha^2} (M^4(-2+i+j)(-1+i+j)(-1+\alpha)^2 \alpha^2 \\
 &+(-1+\alpha)^2 m_Q^2 (2M^2(-2+i+j)\alpha+m_Q^2) \\
 &-2(-1+\alpha)\alpha(M^2(-2+i+j)\alpha+m_Q^2)m_{Q'}^2+\alpha m_{Q'}^4) \\
 &\times \int_0^{1/2} d\alpha_1 \int_{1/2-\alpha_1}^{1-\alpha_1} d\alpha_3 f\left(\frac{1/2-\alpha_1}{\alpha_3}\right) \frac{\mathcal{F}(\alpha_i)}{\alpha_3}
 \end{aligned} \tag{115}$$

For terms containing $q \cdot x$, we perform the following operations:

$$\begin{aligned}
 &(q \cdot x)^n f(u) \\
 &\rightarrow \begin{cases} (-i \frac{\partial}{\partial u})^n f(u), & n > 0 \\ i^{-n} \int_0^u dv_n \dots \int_0^{v_3} dv_2 \int_0^{v_2} dv_1 f(v_1), & n < 0 \end{cases}
 \end{aligned} \tag{116}$$

$$\begin{aligned}
 &(q \cdot x)^n \mathcal{F}(\alpha_i) \\
 &\rightarrow \begin{cases} \left(\frac{i}{u} \frac{\partial}{\partial \alpha_3}\right)^n \mathcal{F}(\alpha_i), & n > 0 \\ (-iu)^{-n} \int_0^{\alpha_3} d\alpha_3^{(n)} \dots \int_0^{\alpha_3^{(3)}} d\alpha_3^{(2)} \int_0^{\alpha_3^{(2)}} d\alpha_3^{(1)} \mathcal{F}(\alpha_1, 1-\alpha_1-\alpha_3^{(1)}, \alpha_3^{(1)}), & n < 0 \end{cases}
 \end{aligned} \tag{117}$$

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