The magnetic moment of $Z_c(3900)$ as an axial-vector molecular state

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Abstract In this paper, we tentatively assign $Z_c(3900)$ to be an axialvector molecular state, and calculate its magnetic moment using the QCD sum rule method in an external weak electromagnetic field. Starting with the two-point correlation function in the external electromagnetic field and expanding it in powers of the electromagnetic interaction Hamiltonian, we extract the mass and pole residue of the $Z_c(3900)$ state from the leading term in the expansion and the magnetic moment from the linear response to the external electromagnetic field. The numerical values are $m_{Z_c} = 3.97 \pm 0.12 \text{ GeV}$ in agreement with the experimental value $m_{Z_c}^{exp} = 3899.0 \pm 3.6 \pm 4.9 \text{ MeV}$, $\lambda_{Z_c} = 2.1 \pm 0.4 \times 10^{-2} \text{ GeV}^5$ and $\mu_{Z_c} = 0.19^{+0.04}_{-0.01} \mu_N$.

1 Introduction

$Z_c(3900)$, as a good candidate of exotic hadrons, was observed by the BESIII collaboration in 2013 in the $\pi^\pm J/\psi$ invariant mass distribution of the process $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ at a center-of-mass energy of 4.260 GeV [1]. Then the Belle and CLEO collaborations confirmed the existence of $Z_c(3900)$ [2,3]. In 2017, the BESIII collaboration determined the J^P quantum number of $Z_c(3900)$ to be $J^P = 1^+$ with a statistical significance larger than 7σ over other quantum numbers in a partial wave analysis of the process $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ [4]. Inspired by this experimental progress, there have been plentiful theoretical studies on $Z_c(3900)$'s properties through different approaches (see Ref. [5–7] and the references therein for details). However, the underlying structure of $Z_c(3900)$ is not understood completely and more endeavors are necessary in order to arrive at a better understanding for the properties of $Z_c(3900)$.

The electromagnetic multipole moments of hadron encode the spatial distributions of charge and magnetization in the hadron and provide important information about the quark configurations of the hadron and the underlying dynamics. So it is interesting to study the electromagnetic multipole moments of hadron.

The studies on the properties of hadrons inevitably involve the nonperturbative effects of quantum chromodynamics (QCD). The QCD sum rule method [8,9] is a nonperturbative analytic formalism firmly entrenched in QCD with minimal modeling and has been successfully applied in almost every aspect of strong interaction physics. In Refs. [10–12], the QCD sum rule method was extended to calculating the magnetic moments of the nucleon and hyperon in the external field method. In this method, a static electromagnetic field is introduced, which couples to the quarks and polarizes the QCD vacuum and magnetic moments can be extracted from the linear response to this field. Later, a more systematic study was made for the magnetic moments of the octet baryons [13–16], the decuplet baryons [17–20] and the ρ meson [21]. In the case of the exotic X, Y, Z states, only the magnetic moment of $Z_c(3900)$ as an axialvector tetraquark state was calculated through this method [22].

In this article, we study the magnetic moment of $Z_c(3900)$ as an axialvector molecular state with quantum number $J^P = 1^+$ by the QCD sum rule method. The mass and pole residue, two of the input parameters needed to determine the magnetic moment, are calculated firstly including contributions of operators up to dimension 10. Then the magnetic moment is extracted from the linear term in $F_{\mu\nu}$ (external electromagnetic field) of the correlation function.

The rest of the paper is arranged as follows. In Sect. 2, we derive the sum rules for the mass, pole residue and magnetic moment of $Z_c(3900)$ state. Section 3 is devoted to the numerical analysis and a short summary is given in Sect. 4. In Appendix B, the spectral densities are shown.

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2 The derivation of the sum rules

The starting point of our calculation is the time-ordered correlation function in the QCD vacuum in the presence of a constant background electromagnetic field $F_{\mu\nu}$,

$$\begin{aligned}\Pi_{\mu\nu}(p) &= i \int d^4x e^{ipx} \langle 0 | T[J_\mu(x) J_\nu^\dagger(0)] | 0 \rangle_F \\ &= \Pi_{\mu\nu}^{(0)}(p) + \Pi_{\mu\nu\alpha\beta}^{(1)}(p) F^{\alpha\beta} + \dots, \end{aligned} \quad (1)$$

where

$$\begin{aligned}J_\mu(x) &= \frac{1}{\sqrt{2}} \{ [\bar{u}(x) i \gamma^5 c(x)] [\bar{c}(x) \gamma_\mu d(x)] \\ &\quad + [\bar{u}(x) \gamma_\mu c(x)] [\bar{c}(x) i \gamma^5 d(x)] \} \end{aligned} \quad (2)$$

is the interpolating current of $Z_c(3900)$ as a molecular state with $J^P = 1^+$ [23]. The $\Pi_{\mu\nu}^{(0)}(p)$ term is the correlation function without external electromagnetic field, and it gives rise to the mass and pole residue of $Z_c(3900)$. The magnetic moment will be extracted from the linear response term, $\Pi_{\mu\nu\alpha\beta}^{(1)}(p) F^{\alpha\beta}$.

The external electromagnetic field can interact directly with the quarks inside the hadron and also polarize the QCD vacuum. As a consequence, the vacuum condensates involved in the operator product expansion of the correlation function in the external electromagnetic field $F_{\mu\nu}$ are

- dimension-2 operator,

$$F_{\mu\nu}, \quad (3)$$

- dimension-3 operator,

$$\langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle_F, \quad (4)$$

- dimension-5 operators,

$$\langle 0 | \bar{q} q | 0 \rangle_F, \langle 0 | \bar{q} g_s G_{\mu\nu} q | 0 \rangle_F, \epsilon_{\mu\nu\alpha\beta} \langle 0 | \bar{q} g_s G^{\alpha\beta} q | 0 \rangle_F, \quad (5)$$

- dimension-6 operators,

$$\langle 0 | \bar{q} q | 0 \rangle \langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle_F, \langle 0 | g_s^2 G G | 0 \rangle_F, \dots, \quad (6)$$

- dimension-7 operators,

$$\langle 0 | g_s^2 G G | 0 \rangle \langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle_F, \langle 0 | g_s \bar{q} \sigma \cdot G q | 0 \rangle_F, \dots, \quad (7)$$

- dimension-8 operators,

$$\begin{aligned} &\langle 0 | \bar{q} q | 0 \rangle^2 F_{\mu\nu}, \langle 0 | g_s \bar{q} \sigma \cdot G q | 0 \rangle \\ &\times \langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle_F, \langle 0 | \bar{q} q | 0 \rangle \langle 0 | \bar{q} g_s G_{\mu\nu} q | 0 \rangle_F, \end{aligned}$$

$$\epsilon_{\mu\nu\alpha\beta} \langle 0 | \bar{q} q | 0 \rangle \langle 0 | \bar{q} g_s G^{\alpha\beta} q | 0 \rangle_F, \dots \quad (8)$$

The new vacuum condensates induced by the external electromagnetic field $F_{\mu\nu}$ can be described by introducing new parameters, χ , κ and ξ , called vacuum susceptibilities, as follows:

$$\begin{aligned} \langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle_F &= e e_q \chi \langle 0 | \bar{q} q | 0 \rangle_F F_{\mu\nu}, \\ \langle 0 | \bar{q} g_s G_{\mu\nu} q | 0 \rangle_F &= e e_q \kappa \langle 0 | \bar{q} q | 0 \rangle_F F_{\mu\nu}, \\ \epsilon_{\mu\nu\alpha\beta} \langle 0 | \bar{q} g_s G^{\alpha\beta} q | 0 \rangle_F &= i e e_q \xi \langle 0 | \bar{q} q | 0 \rangle_F F_{\mu\nu}. \end{aligned} \quad (9)$$

In order to express the two-point correlation function (1) physically, we expand it in powers of the electromagnetic interaction Hamiltonian $H_{int} = -ie \int d^4y j_\alpha^{em}(y) A^\alpha(y)$,

$$\begin{aligned} \Pi_{\mu\nu}(p) &= i \int d^4x e^{ipx} \langle 0 | T[J_\mu(x) J_\nu^\dagger(0)] | 0 \rangle \\ &\quad + i \int d^4x e^{ipx} \langle 0 | T \left\{ J_\mu(x) \right. \\ &\quad \times \left. \left[-ie \int d^4y j_\alpha^{em}(y) A^\alpha(y) \right] J_\nu^\dagger(0) \right\} | 0 \rangle + \dots, \end{aligned} \quad (10)$$

where $j_\alpha^{em}(y)$ is the electromagnetic current and $A^\alpha(y)$ is the electromagnetic four-vector.

Inserting complete sets of relevant states with the same quantum numbers as the current operator $J_\mu(x)$ and carrying out the involved integrations, we have

$$\begin{aligned} \Pi_{\mu\nu}^{had}(p) &= \frac{\lambda_{Z_c}^2}{m_{Z_c}^2 - p^2} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) \\ &\quad - i \frac{\lambda_{Z_c}^2 G_2(0)}{(p^2 - m_{Z_c}^2)^2} F_{\mu\nu} + i \frac{a}{m_{Z_c}^2 - p^2} F_{\mu\nu} + \dots, \end{aligned} \quad (11)$$

where we make use of the following matrix elements

$$\langle 0 | J_\mu(0) | Z_c(p) \rangle = \lambda_{Z_c} \epsilon_\mu(p) \quad (12)$$

with λ_{Z_c} and $\epsilon_\mu(p)$ being the pole residue and polarization vector of $Z_c(3900)$, respectively,

$$\begin{aligned} \langle Z_c(p) | j_\alpha^{em}(0) | Z_c(p') \rangle &= G_1(Q^2) \epsilon^*(p) \cdot \epsilon(p') (p + p')_\alpha \\ &\quad + G_2(Q^2) [\epsilon_\alpha(p') \epsilon^*(p) \cdot q - \epsilon_\alpha^*(p) \epsilon(p') \cdot q] \\ &\quad - \frac{G_3(Q^2)}{2m_{Z_c}^2} \epsilon^*(p) \cdot q \epsilon(p') \cdot q (p + p')_\alpha \end{aligned} \quad (13)$$

with $q = p' - p$ and $Q^2 = -q^2$. The Lorentz-invariant functions $G_1(Q^2)$, $G_2(Q^2)$ and $G_3(Q^2)$ are related to the charge, magnetic and quadrupole form-factors,

$$G_C(Q^2) = G_1(Q^2) + \frac{2}{3} \eta G_Q(Q^2),$$

$$G_M(Q^2) = -G_2(Q^2),$$

$$G_Q(Q^2) = G_1(Q^2) + G_2(Q^2) + (1 + \eta)G_3(Q^2), \quad (14)$$

respectively, where $\eta = \frac{Q^2}{4m_{Z_c}^2}$. At zero momentum transfer, these form-factors are proportional to the usual static quantities of the charge e , magnetic moment μ_{Z_c} and quadrupole moment Q_1 ,

$$eG_C(0) = e,$$

$$eG_M(0) = 2m_Z\mu_{Z_c},$$

$$eG_Q(0) = m_{Z_c}^2 Q_1. \quad (15)$$

The constant a parameterizes the contributions from the pole–continuum transitions.

On the other hand, $\Pi_{\mu\nu}(p)$ can be calculated theoretically via the OPE method at the quark–gluon level. To this end, one can insert the interpolating current $J_\mu(x)$ (2) into the correlation function (1), contract the relevant quark fields via Wick's theorem and obtain

$$\begin{aligned} \Pi_{\mu\nu}^{OPE}(p) &= \frac{i}{2} \int d^4x e^{ipx} \\ &\times (Tr[(i\gamma_5)S_{ca}^{(u)}(-x)(i\gamma_5)S_{ac}^{(c)}(x)]Tr[\gamma_\mu S_{bd}^{(d)}(x)\gamma_\nu S_{db}^{(c)}(-x)] \\ &+ Tr[(i\gamma_5)S_{db}^{(c)}(-x)\gamma_\mu S_{bd}^{(d)}(x)]Tr[(i\gamma_5)S_{ac}^{(c)}(x)\gamma_\nu S_{ca}^{(u)}(-x)] \\ &+ Tr[(i\gamma_5)S_{ca}^{(u)}(-x)\gamma_\mu S_{ac}^{(c)}(x)]Tr[(i\gamma_5)S_{bd}^{(d)}(x)\gamma_\nu S_{db}^{(c)}(-x)] \\ &+ Tr[(i\gamma_5)S_{bd}^{(d)}(x)(i\gamma_5)S_{db}^{(c)}(-x)]Tr[\gamma_\mu S_{ac}^{(c)}(x)\gamma_\nu S_{ca}^{(u)}(-x)]), \end{aligned} \quad (16)$$

where $S^{(c)}(x) = \langle 0|T[c(x)\bar{c}(0)]|0\rangle$ and $S^{(q)}(x) = \langle 0|T[q(x)\bar{q}(0)]|0\rangle$, $q = u, d$ are the full charm- and up (down)-quark propagators, whose expressions are given in Appendix A, Tr denotes the trace of the Dirac spinor indices, and a, b, c and d are color indices. Through the dispersion relation, $\Pi_{\mu\nu}^{OPE}(p)$ can be written as

$$\begin{aligned} \Pi_{\mu\nu}^{OPE}(p) &= \int_{4m_c^2}^{\infty} ds \frac{\rho^{(0)}(s)}{s-p^2} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) \\ &+ \int_{4m_c^2}^{\infty} ds \frac{\rho^{(1)}(s)}{s-p^2} (iF_{\mu\nu}) + \text{other Lorentz structures}, \end{aligned} \quad (17)$$

where $\rho^i(s) = \frac{1}{\pi} \text{Im}\Pi_i^{OPE}(s)$, $i = 0, 1$ are the spectral densities. The spectral densities $\rho^i(s)$ are given in Appendix B.

Finally, matching the phenomenological side (11) and the QCD representation (17), we obtain

$$\frac{\lambda_{Z_c}^2}{m_{Z_c}^2 - p^2} + \dots = \int_{4m_c^2}^{\infty} ds \frac{\rho^{(0)}(s)}{s - p^2}, \quad (18)$$

Table 1 Some input parameters needed in the calculations

Parameter	Value
$\langle \bar{q}q \rangle$	$-(0.24 \pm 0.01)^3 \text{ GeV}^3$
$\langle g_s \bar{q} \sigma G q \rangle$	$(0.8 \pm 0.1) \langle \bar{q}q \rangle \text{ GeV}^2$
$\langle g_s^2 G G \rangle$	$0.88 \pm 0.25 \text{ GeV}^4$
m_c	$1.275_{-0.035}^{+0.025} \text{ GeV}$ [24]

for the Lorentz-structure $(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2})$, and

$$\frac{\lambda_{Z_c}^2 G_M(0)}{(m_{Z_c}^2 - p^2)^2} + \frac{a}{m_{Z_c}^2 - p^2} + \dots = \int_{4m_c^2}^{\infty} ds \frac{\rho^{(1)}(s)}{s - p^2}, \quad (19)$$

for the Lorentz-structure $iF_{\mu\nu}$.

According to quark–hadron duality, the excited and continuum states' spectral density can be approximated by the QCD spectral density above some effective threshold $s_{Z_c}^0$, whose value will be determined in Sect. 3,

$$\begin{aligned} \frac{\lambda_{Z_c}^2}{m_{Z_c}^2 - p^2} + \int_{s_{Z_c}^0}^{\infty} ds \frac{\rho^{(0)}(s)}{s - p^2} &= \int_{4m_c^2}^{\infty} ds \frac{\rho^{(0)}(s)}{s - p^2}, \\ \frac{\lambda_{Z_c}^2 G_M(0)}{(m_{Z_c}^2 - p^2)^2} + \frac{a}{m_{Z_c}^2 - p^2} &+ \int_{s_{Z_c}^0}^{\infty} ds \frac{\rho^{(1)}(s)}{s - p^2} = \int_{4m_c^2}^{\infty} ds \frac{\rho^{(1)}(s)}{s - p^2}. \end{aligned} \quad (20)$$

Subtracting the contributions of the excited and continuum states, one gets

$$\begin{aligned} \frac{\lambda_{Z_c}^2}{m_{Z_c}^2 - p^2} &= \int_{4m_c^2}^{s_{Z_c}^0} ds \frac{\rho^{(0)}(s)}{s - p^2}, \\ \frac{\lambda_{Z_c}^2 G_M(0)}{(m_{Z_c}^2 - p^2)^2} + \frac{a}{m_{Z_c}^2 - p^2} &= \int_{4m_c^2}^{s_{Z_c}^0} ds \frac{\rho^{(1)}(s)}{s - p^2}. \end{aligned} \quad (21)$$

In order to improve the convergence of the OPE series and suppress the contributions from the excited and continuum states, it is necessary to make a Borel transform. As a result, we have

$$\begin{aligned} \lambda_{Z_c}^2 e^{-m_{Z_c}^2/M_B^2} &= \int_{4m_c^2}^{s_{Z_c}^0} ds \rho^{(0)}(s) e^{-s/M_B^2}, \\ \lambda_{Z_c}^2 \left(\frac{G_M(0)}{M_B^2} + A \right) e^{-m_{Z_c}^2/M_B^2} &= \int_{4m_c^2}^{s_{Z_c}^0} ds \rho^{(1)}(s) e^{-s/M_B^2}, \end{aligned} \quad (22)$$

where M_B^2 is the Borel parameter and $A = \frac{a}{\lambda_{Z_c}^2}$. Taking the derivative of the first equation in (22) with respect to $-\frac{1}{M_B^2}$

Fig. 1 **a** The various condensates as functions of M_B^2 with $\sqrt{s_{Z_c}^0} = 4.6 \text{ GeV}$; **b** represents RP_0 and RH_0 varying with M_B^2 at $\sqrt{s_{Z_c}^0} = 4.6 \text{ GeV}$

Fig. 2 The dependence of the mass m_{Z_c} on the Borel parameter M_B^2 with $\sqrt{s_{Z_c}^0} = 4.5 \text{ GeV}$ (dot-dashed line), $\sqrt{s_{Z_c}^0} = 4.6 \text{ GeV}$ (real line) and $\sqrt{s_{Z_c}^0} = 4.6 \text{ GeV}$ (dashed line)

and dividing it by the original expression, one has

$$m_{Z_c}^2 = \frac{\frac{d}{d\left(-\frac{1}{M_B^2}\right)} \int_{4m_c^2}^{s_{Z_c}^0} ds \rho^{(0)}(s) e^{-\frac{s}{M_B^2}}}{\int_{4m_c^2}^{s_{Z_c}^0} ds \rho^{(0)}(s) e^{-\frac{s}{M_B^2}}}. \quad (23)$$

In the next section, (22) and (23) will be analyzed numerically to obtain the numerical values of the mass, the pole residue and the magnetic moment of the $Z_c(3900)$.

3 Numerical analysis

The input parameters needed in the numerical analysis are presented in Table 1. For the vacuum susceptibilities χ , κ and ξ , we take the values $\chi = -(3.15 \pm 0.30) \text{ GeV}^{-2}$, $\kappa = -0.2$ and $\xi = 0.4$ determined in the detailed QCD sum rules analysis of the photon light-cone distribution amplitudes [25]. Besides these parameters, we should determine the working intervals of the threshold parameter $s_{Z_c}^0$ and the Borel mass M_B^2 in which the mass, the pole residue and the magnetic moment vary weakly. The continuum threshold is related to

the square of the first excited states having the same quantum number as the interpolating field, while the Borel parameter is determined by demanding that the contributions of both the higher states and the continuum are sufficiently suppressed and the contributions coming from higher dimensional operators are small.

We define two quantities, the ratio of the pole contribution to the total contribution (RP) and the ratio of the highest dimensional term in the OPE series to the total OPE series (RH), as follows:

$$RP_i \equiv \frac{\int_{4m_c^2}^{s_{Z_c}^0} ds \rho^{(i)}(s) e^{-\frac{s}{M_B^2}}}{\int_{4m_c^2}^{\infty} ds \rho^{(i)}(s) e^{-\frac{s}{M_B^2}}},$$

$$RH_i \equiv \frac{\int_{4m_c^2}^{s_{Z_c}^0} ds \rho_i^{(d=n)}(s) e^{-\frac{s}{M_B^2}}}{\int_{4m_c^2}^{s_{Z_c}^0} ds \rho^{(i)}(s) e^{-\frac{s}{M_B^2}}}, \quad (24)$$

where $i = 0, 1$ and $n = 10(8)$ as $i = 0(1)$, respectively.

In Fig. 1a, we compare the various terms in the OPE series as functions of M_B^2 with $\sqrt{s_{Z_c}^0} = 4.6$ GeV. From this figure one can see that except the quark condensate $\langle \bar{q}q \rangle$, other vacuum condensates are much smaller than the perturbative term. So the OPE series are under control. Figure 1(b) shows RP_0 and RH_0 varying with M_B^2 at $\sqrt{s_{Z_c}^0} = 4.6$ GeV. The figure shows that the requirement $RP_0 \geq 50\%$ ($RP_0 \geq 40\%$) gives $M_B^2 \leq 3.3$ GeV² ($M_B^2 \leq 3.7$ GeV²) and $RH_0 = 5\%$ at $M_B^2 = 1.25$ GeV².

From Fig. 2a, we know that the sum rule for the mass m_{Z_c} depends strongly on the Borel parameter M_B^2 as $M_B^2 \leq 3$ GeV². Along with the criterion of pole dominance, this fact confines M_B^2 from 3 to 3.7 GeV². In the analysis, we take $RP_0 \geq 40\%$ so that we can obtain a larger interval of the Borel parameter. Within the interval of M_B^2 determined above, the mass varies weakly with M_B^2 as depicted in Fig. 2b. Figure 2b also shows the weak dependence of the mass on the threshold parameter $s_{Z_c}^0$ as 4.5^2 GeV² $\leq s_{Z_c}^0 \leq 4.7^2$ GeV². As a result, we can reliably read off the value of the mass, $m_{Z_c} = 3.97 \pm 0.12$ GeV, in agreement with the experimental value $m_{Z_c}^{exp} = 3899.0 \pm 3.6 \pm 4.9$ MeV.

In Fig. 3, we show the variation of the pole residue with the Borel parameter M_B^2 in the determined interval at three different values of $s_{Z_c}^0$. It is obvious that the pole residue depends weakly on M_B^2 and $s_{Z_c}^0$ and $\lambda_{Z_c} = 2.1 \pm 0.4 \times 10^{-2}$ GeV⁵.

The same procedure can be done for the sum rule of the magnetic moment. The results are shown in Fig. 4, from which the value of $G_M(0)$ can be read off as $G_M(0) = 0.82_{-0.07}^{+0.17}$. Finally, we obtain

Fig. 3 The figure shows the dependence of the pole residue λ_{Z_c} on the Borel parameter M_B^2 in the determined interval at three different values of $s_{Z_c}^0$

$$\mu_{Z_c} = G_M(0) \frac{e}{2m_{Z_c}} = 0.82_{-0.07}^{+0.17} \frac{e}{2m_{Z_c}} = 0.19_{-0.01}^{+0.04} \mu_N, \quad (25)$$

where μ_N is the nucleon magneton.

In Ref. [22], the author gave $\mu_{Z_c} = 0.47_{-0.22}^{+0.27} \mu_N$ assuming $Z_c(3900)$ as an axialvector tetraquark state by the same method as used in this article. In Ref. [26], $\mu_{Z_c} = 0.67 \pm 0.32 \mu_N$ was predicted using the light-cone sum rule under the axialvector tetraquark assumption. In Table 2, we summarize the values of the magnetic moment of $Z_c(3900)$ under different assumptions about the quark configuration and with different methods. It is obvious that the magnetic moment of $Z_c(3900)$ has different values if $Z_c(3900)$ has different quark configurations. The theoretical predictions can be confronted to the experimental data in the future and give important information about the inner structure of the $Z_c(3900)$ state.

4 Conclusion

In this paper, we tentatively assign $Z_c(3900)$ to be an axialvector molecular state, calculate its magnetic moment using the QCD sum rule method in the external weak electromagnetic field. Starting with the two-point correlation function in the external electromagnetic field and expanding it in powers of the electromagnetic interaction Hamiltonian, we extract the mass and pole residue of the $Z_c(3900)$ state from the leading term in the expansion and the magnetic moment from the linear response to the external electromagnetic field. The numerical values are $m_{Z_c} = 3.97 \pm 0.12$ GeV, in agreement with the experimental value $m_{Z_c}^{exp} = 3899.0 \pm 3.6 \pm 4.9$ MeV, $\lambda_{Z_c} = 2.1 \pm 0.4 \times 10^{-2}$ GeV⁵ and $\mu_{Z_c} = 0.19_{-0.01}^{+0.04} \mu_N$ with μ_N the nucleon magneton. The prediction can be confronted

Fig. 4 **a** Shows the various condensates as functions of M_B^2 with $\sqrt{s_{Z_c}^0} = 4.6 \text{ GeV}$; **b** presents RP_1 and RH_1 varying with M_B^2 at $\sqrt{s_{Z_c}^0} = 4.6 \text{ GeV}$; **c** depicts the dependence of G_M on M_B^2 in the determined interval at three different values of $s_{Z_c}^0$

Table 2 The magnetic moment of $Z_c(3900)$ (μ_N is the nucleon magneton)

Quark configuration	Method	Value
Axialvector tetraquark	Light-cone sum rule	$0.67 \pm 0.32 \mu_N$ [26]
Axialvector tetraquark	QCD sum rule	$0.47^{+0.27}_{-0.22} \mu_N$ [22]
Axialvector molecule	QCD sum rule	$0.19^{+0.04}_{-0.01} \mu_N$ (this work)

to the experimental data in the future and give important information about the inner structure of the $Z_c(3900)$ state.

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in this manuscript are available upon request by contacting with the corresponding author.].

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Appendix A: The quark propagators

The full quark propagators are

$$S_{ij}^q(x) = \frac{i \not{x}}{2\pi^2 x^4} \delta_{ij} - \frac{m_q}{4\pi^2 x^2} \delta_{ij} - \frac{\langle \bar{q}q \rangle}{12} \delta_{ij} + i \frac{\langle \bar{q}q \rangle}{48} m_q \not{x} \delta_{ij} - \frac{x^2}{192} \langle g_s \bar{q} \sigma G q \rangle \delta_{ij} + i \frac{x^2 \not{x}}{1152} m_q \langle g_s \bar{q} \sigma G q \rangle \delta_{ij} - i \frac{g_s t_{ij}^a G_{\mu\nu}^a}{32\pi^2 x^2} (\not{x} \sigma^{\mu\nu} + \sigma^{\mu\nu} \not{x}) + i \frac{\delta_{ij} e_q F_{\mu\nu}}{32\pi^2 x^2} (\not{x} \sigma^{\mu\nu} + \sigma^{\mu\nu} \not{x}) - \frac{\delta_{ij} e_q \chi \langle \bar{q}q \rangle \sigma^{\mu\nu} F_{\mu\nu}}{24} + \frac{\delta_{ij} e_q \langle \bar{q}q \rangle F_{\mu\nu}}{288} (\sigma^{\mu\nu} - 2\sigma^{\alpha\mu} x_\alpha x^\nu) + \frac{\delta_{ij} e_q \langle \bar{q}q \rangle F_{\mu\nu}}{576} [(\kappa + \xi) \sigma^{\mu\nu} x^2 - (2\kappa - \xi) \sigma^{\alpha\mu} x_\alpha x^\nu] + \dots \quad (\text{A.1})$$

for light quarks, and

$$S_{ij}^Q(x) = i \int \frac{d^4 k}{(2\pi)^4} e^{-ikx} \times \left[\frac{\not{k} + m_Q}{k^2 - m_Q^2} \delta_{ij} - \frac{g_s t_{ij}^a G_{\mu\nu}^a}{4} \frac{\sigma^{\mu\nu} (\not{k} + m_Q) + (\not{k} + m_Q) \sigma^{\mu\nu}}{(k^2 - m_Q^2)^2} + \frac{\langle g_s^2 GG \rangle}{12} \delta_{ij} m_Q \frac{k^2 + m_Q \not{k}}{(k^2 - m_Q^2)^4} + \frac{\delta_{ij} e_Q F_{\mu\nu}}{4} \frac{\sigma^{\mu\nu} (\not{k} + m_Q) + (\not{k} + m_Q) \sigma^{\mu\nu}}{(k^2 - m_Q^2)^2} + \dots \right] \quad (\text{A.2})$$

for heavy quarks. In these expressions $t^a = \frac{\lambda^a}{2}$ and λ^a are the Gell-Mann matrices, g_s is the strong interaction coupling constant, and i, j are color indices, $e_{Q(q)}$ is the charge of the heavy (light) quark and $F_{\mu\nu}$ is the external electromagnetic field.

Appendix B: The spectral densities

On the QCD side, we carry out the OPE up to dimension-10 and dimension-8 for the spectral densities $\rho^{(0)}(s)$ and $\rho^{(1)}(s)$, respectively. The explicit expressions of the spectral densities are given by

$$\rho^{(0)}(s) = \rho_0^{(d=0)} + \rho_0^{(d=3)}(s) + \rho_0^{(d=4)}(s) + \rho_0^{(d=5)}(s) + \rho_0^{(d=6)}(s) + \rho_0^{(d=7)}(s) + \rho_0^{(d=8)}(s) + \rho_0^{(d=10)}(s), \quad (\text{B.1})$$

with

$$\rho_0^{(d=0)}(s) = \frac{3}{4096\pi^6} \int_{a_{\min}}^{a_{\max}} da \int_{b_{\min}}^{1-a} db \frac{1}{a^3 b^3} \times (1-a-b)(1+a+b) (m_c^2(a+b) - \text{abs})^4, \quad (\text{B.2})$$

$$\rho_0^{(d=3)}(s) = -\frac{3m_c \langle 0|\bar{q}q|0 \rangle}{256\pi^4} \int_{a_{\min}}^{a_{\max}} da \int_{b_{\min}}^{1-a} db \frac{1}{a^2 b^2} \times (a+b)(1+a+b) (m_c^2(a+b) - \text{abs})^2, \quad (\text{B.3})$$

$$\rho_0^{(d=4)}(s) = \frac{m_c^2 \langle 0|g_s^2 GG|0 \rangle}{4096\pi^6} \int_{a_{\min}}^{a_{\max}} da \int_{b_{\min}}^{1-a} db \times \frac{1}{a^3 b^3} (a^3 + b^3)(1-a-b) (1+a+b) (m_c^2(a+b) - \text{abs}) + \frac{\langle 0|g_s^2 GG|0 \rangle}{4096\pi^6} \int_{a_{\min}}^{a_{\max}} da \int_{b_{\min}}^{1-a} db \times \frac{1}{a^2 b^2} (a+b)(2a+2b-1) (m_c^2(a+b) - \text{abs})^2, \quad (\text{B.4})$$

$$\rho_0^{(d=5)}(s) = \frac{3m_c \langle 0|g_s \bar{q} \sigma \cdot G q|0 \rangle}{256\pi^4} \int_{a_{\min}}^{a_{\max}} da \int_{b_{\min}}^{1-a} db \times \frac{1}{a^2 b^2} (a^2 + b^2)(a+b) (m_c^2(a+b) - \text{abs}) + \frac{3m_c \langle 0|g_s \bar{q} \sigma \cdot G q|0 \rangle}{512\pi^4} \int_{a_{\min}}^{a_{\max}} da \times \int_{b_{\min}}^{1-a} db \frac{1}{ab} (a+b) (m_c^2(a+b) - \text{abs}) - \frac{3m_c \langle 0|g_s \bar{q} \sigma \cdot G q|0 \rangle}{256\pi^4} \int_{a_{\min}}^{a_{\max}} da \frac{1}{a(1-a)} (m_c^2 - a(1-a)s), \quad (\text{B.5})$$

$$\rho_0^{(d=6)}(s) = \frac{m_c^2 \langle 0|\bar{q}q|0 \rangle^2 \sqrt{s(s-4m_c^2)}}{16\pi^2 s}, \quad (\text{B.6})$$

$$\rho_0^{(d=7)}(s) = -\frac{m_c \langle 0|\bar{q}q|0 \rangle \langle 0|g_s^2 GG|0 \rangle}{512\pi^4} \times \int_{a_{\min}}^{a_{\max}} da \int_{b_{\min}}^{1-a} db \frac{1}{a^2} b(1+a+b) - \frac{m_c \langle 0|\bar{q}q|0 \rangle \langle 0|g_s^2 GG|0 \rangle}{1536\pi^4} \int_{a_{\min}}^{a_{\max}} da (1+a+b_{\min}) \frac{as + m_c^2}{m_c^2}, \quad (\text{B.7})$$

$$\rho_0^{(d=8)}(s) = \frac{\langle 0|\bar{q}q|0 \rangle \langle 0|g_s \bar{q} \sigma \cdot G q|0 \rangle}{32\pi^2} \times \frac{m_c^2}{\sqrt{s(s-4m_c^2)}} \left(2 \frac{m_c^2}{M_B^2} + 2 \frac{m_c^2}{s} - 1 \right), \quad (\text{B.8})$$

$$\rho_0^{(d=10)}(s) = \frac{\langle 0|\bar{q}q|0 \rangle^2 \langle 0|g_s^2 GG|0 \rangle}{576\pi^2} \frac{1}{\sqrt{s(s-4m_c^2)}} \times \left(\frac{s(s-3m_c^2)}{M_B^4} - 3 \frac{s-2m_c^2}{M_B^2} \right) + \frac{\langle 0|g_s \bar{q} \sigma \cdot G q|0 \rangle^2}{128\pi^2} \frac{m_c^2 s}{M_B^4 \sqrt{s(s-4m_c^2)}}$$

$$\begin{aligned} & \times \left(1 - \frac{m_c^2}{M_B^2}\right). \\ \rho^{(1)}(s) &= \rho_1^{(d=2)} + \rho_1^{(d=3)}(s) \\ &+ \rho_1^{(d=5)}(s) + \rho_1^{(d=6)}(s) \\ &+ \rho_1^{(d=7)}(s) + \rho_1^{(d=8)}(s), \end{aligned} \quad (\text{B.9})$$

(B.10)

with

$$\begin{aligned} \rho_1^{(d=2)}(s) &= -\frac{3}{1024\pi^6} \int_{a_{\min}}^{a_{\max}} da \int_{b_{\min}}^{1-a} db \\ &\times \frac{1}{a^2 b^2} (a+b) (m_c^2(a+b) - \text{abs})^3, \end{aligned} \quad (\text{B.11})$$

$$\begin{aligned} \rho_1^{(d=3)}(s) &= \frac{3m_c \chi \langle 0|\bar{q}q|0\rangle}{256\pi^4} \int_{a_{\min}}^{a_{\max}} da \\ &\times \int_{b_{\min}}^{1-a} db \frac{1}{ab^2} (m_c^2(a+b) - \text{abs})^2, \end{aligned} \quad (\text{B.12})$$

$$\begin{aligned} \rho_1^{(d=5)}(s) &= -\frac{m_c \langle 0|\bar{q}q|0\rangle}{32\pi^4} \int_{a_{\min}}^{a_{\max}} da \\ &\times \int_{b_{\min}}^{1-a} db \frac{1}{b} (m_c^2(a+b) - \text{abs}) \\ &- \frac{m_c(2\kappa - \xi) \langle 0|\bar{q}q|0\rangle}{512\pi^4} \int_{a_{\min}}^{a_{\max}} da \\ &\times \int_{b_{\min}}^{1-a} db \frac{1}{b} (m_c^2(a+b) - \text{abs}) \\ &+ \frac{3m_c(2\kappa + \xi) \langle 0|\bar{q}q|0\rangle}{256\pi^4} \int_{a_{\min}}^{a_{\max}} da \\ &\times \int_{b_{\min}}^{1-a} db \frac{1}{b^2} (m_c^2(a+b) - \text{abs}) \\ &+ \frac{m_c \langle 0|\bar{q}q|0\rangle}{64\pi^4} \int_{a_{\min}}^{a_{\max}} da \frac{1}{a} (m_c^2 - a(1-a)s) \\ &- \frac{m_c(\kappa + \xi) \langle 0|\bar{q}q|0\rangle}{256\pi^4} \int_{a_{\min}}^{a_{\max}} da \frac{1}{a} (m_c^2 - a(1-a)s) \end{aligned} \quad (\text{B.13})$$

$$\begin{aligned} \rho_1^{(d=6)}(s) &= -\frac{m_c^2 \langle 0|g_s^2 GG|0\rangle}{2048\pi^6} \int_{a_{\min}}^{a_{\max}} da \int_{b_{\min}}^{1-a} db \frac{1}{b^2} a(a+b) \\ &+ \frac{3 \langle 0|g_s^2 GG|0\rangle}{4096\pi^6} \int_{a_{\min}}^{a_{\max}} da \int_{b_{\min}}^{1-a} db \frac{1}{b} (m_c^2(a+b) - \text{abs}) \\ &- \frac{3 \langle 0|g_s^2 GG|0\rangle}{4096\pi^6} \int_{a_{\min}}^{a_{\max}} da \frac{1}{a} (m_c^2 - a(1-a)s) \\ &- \frac{m_c^2 \chi \langle 0|\bar{q}q|0\rangle^2}{32\pi^2} \frac{\sqrt{s(s-4m_c^2)}}{s}, \end{aligned} \quad (\text{B.14})$$

$$\begin{aligned} \rho_1^{(d=7)}(s) &= \frac{m_c \chi \langle 0|\bar{q}q|0\rangle \langle 0|g_s^2 GG|0\rangle}{1024\pi^4} \int_{a_{\min}}^{a_{\max}} da \int_{b_{\min}}^{1-a} db \frac{a}{b^2} \\ &+ \frac{\chi \langle 0|\bar{q}q|0\rangle \langle 0|g_s^2 GG|0\rangle}{3072\pi^4} \int_{a_{\min}}^{a_{\max}} da \frac{as + 3m_c^2}{m_c} \\ &+ \frac{3m_c \langle 0|g_s \bar{q}\sigma \cdot Gq|0\rangle}{256\pi^4} \int_{a_{\min}}^{a_{\max}} da \int_{b_{\min}}^{1-a} db \frac{a}{b} \\ &- \frac{3m_c \langle 0|g_s \bar{q}\sigma \cdot Gq|0\rangle}{256\pi^4} \int_{a_{\min}}^{a_{\max}} da \frac{1-a}{a} \\ &- \frac{3m_c^3 \langle 0|g_s \bar{q}\sigma \cdot Gq|0\rangle}{512\pi^4} \frac{1}{\sqrt{s(s-4m_c^2)}} \\ &+ \frac{3m_c \langle 0|g_s \bar{q}\sigma \cdot Gq|0\rangle}{1024\pi^4} \frac{\sqrt{s(s-4m_c^2)}}{s}, \end{aligned} \quad (\text{B.15})$$

$$\begin{aligned} \rho_1^{(d=8)}(s) &= -\frac{m_c^2 \chi \langle 0|\bar{q}q|0\rangle \langle 0|g_s \bar{q}\sigma \cdot Gq|0\rangle}{64\pi^2} \\ &\times \frac{1}{\sqrt{s(s-4m_c^2)}} \left(\frac{m_c^2}{M_B^2} + \frac{m_c^2}{s} - 1 \right) \\ &+ \frac{m_c^4 \langle 0|\bar{q}q|0\rangle^2}{48\pi^2} \frac{1}{\sqrt{s(s-4m_c^2)}} \left(\frac{1}{M_B^2} + \frac{2}{s} \right) \\ &+ \frac{m_c^4(2\kappa - \xi) \langle 0|\bar{q}q|0\rangle^2}{192\pi^2} \frac{1}{s\sqrt{s(s-4m_c^2)}} \\ &+ \frac{m_c^4(\kappa + \xi) \langle 0|\bar{q}q|0\rangle^2}{96\pi^2} \frac{1}{\sqrt{s(s-4m_c^2)}} \left(\frac{1}{M_B^2} + \frac{1}{s} \right) \\ &- \frac{m_c^2(2\kappa + \xi) \langle 0|\bar{q}q|0\rangle^2}{64\pi^2} \frac{1}{\sqrt{s(s-4m_c^2)}}. \end{aligned} \quad (\text{B.16})$$

In the above equations, $a_{\max} = \frac{1 + \sqrt{1 - \frac{4m_c^2}{s}}}{2}$, $a_{\min} = \frac{1 - \sqrt{1 - \frac{4m_c^2}{s}}}{2}$ and $b_{\min} = \frac{am_c^2}{as - m_c^2}$.

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