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The Ω_c -puzzle solved by means of quark model predictions

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Abstract The observation of five Ω_c = ssc states by LHCb [Aaij et al. Phys. Rev. Lett. **118**, 182001 (2017)] and the confirmation of four of them by Belle [Yelton et al. Phys. Rev. D **97**, 051102 (2018)], may represent an important milestone in our understanding of the quark organization inside hadrons. By providing results for the spectrum of Ω_c baryons and predictions for their $\Xi_c^+ K^-$ and $\Xi_c'^+ K^-$ decay amplitudes within an harmonic oscillator based model, we suggest a possible solution to the Ω_c quantum number puzzle and we extend our mass and decay width predictions to the Ω_b states. Finally, we discuss why the set of $\Omega_{c(b)}$ baryons is the most suitable environment to test the validity of three-quark and quark-diquark effective degrees of freedom.

1 Introduction

The discovery of new resonances always enriches the present experimental knowledge of the hadron zoo, and it also provides essential information to explain the fundamental forces that govern nature. As the hadron mass patterns carry information on the way the quarks interact one another, they provide a means of gaining insight into the fundamental binding mechanism of matter at an elementary level.

In 2017, the LHCb Collaboration announced the observation of five narrow Ω_c states in the $\Xi_c^+ K^-$ decay channel [1]: $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$ and $\Omega_c(3119)$. They also reported the observation of another structure around 3188 MeV, the so-called $\Omega_c(3188)$, though they did not have enough statistical significance to interpret it as a genuine resonance [1]. Later, Belle observed five resonant states in the $\Xi_c^+ K^-$ invariant mass distribution and unambiguously confirmed four of the states announced by LHCb, $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, and $\Omega_c(3090)$, but no signal was found for the $\Omega_c(3119)$ [2]. Belle also measured a signal excess at 3188 MeV, corresponding to the $\Omega_c(3188)$ state reported by LHCb [2]. A comparison between the results reported by the two collaborations is displayed in Table 1. Here, it is shown that the $\Omega_c(3188)$, even if not yet confirmed, was seen both by LHCb and Belle, while, on the contrary, the $\Omega_c(3119)$ was not observed by Belle. It is also worth to mention that the LHCb collaboration has just announced the observation of a new bottom baryon, $\Xi_b(6227)^-$, in both $\Lambda_b^0 K^-$ and $\Xi_b^0 \pi$ decay modes [3], and of two bottom resonances, $\Sigma_b(6097)^{\pm}$, in the $\Lambda_b^0 \pi^{\pm}$ channels [4].

However, neither LHCb nor Belle were able to measure the Ω_c angular momenta and parities. For this reason, several authors tried to provide different quantum number assignments for these states. The current Ω_c puzzle consists in the discrepancy between the experimental results, reported by LHCb [1] and Belle [2], and the existing theoretical predictions [5–9]. Indeed, for a given Ω_c experimental state, more than one quantum number assignment was suggested [5]. In particular, the $\Omega_c(3119)$ was allocated to possibly be a $J^P = \frac{1}{2}^+$ or a $J^P = \frac{3}{2}^+$ state [7], while the authors in Ref. [8] proposed a $J^P = \frac{5}{2}^-$ assignment.

From the previous discussion it comes out that, in the case of the $\Omega_c(3119)$, not only the quantum number assignments are not univocal, but also the quark structure of this baryon is still unclear. The issues we have to deal with are not restricted to the contrasts between the different interpretations provided in the previous studies, they are also related to the discrepancies on the quantum number assignments within a given study. For example, in Ref. [9] the authors provided different J^P assignments for the $\Omega_c(3066)$ and $\Omega_c(3090)$ based on mass and decay width estimates. Moreover, the nature of the

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Table 1	Measured masses (in MeV) of the six resonances of	oserved in the $\Xi_c^+ K^-$	decay channel	(see text) according	to the LHCb [1]	and the Belle
[2] colla	borations in pp and e^+e^-	collisions, respectively					

Ω_c excited state	3000	3050	3066	3090	3119	3188
Mass (LHCb [1])	$3000.4 \pm 0.2 \pm 0.1$	$3050.2 \pm 0.1 \pm 0.1$	$3065.6 \pm 0.1 \pm 0.3$	$3090.2 \pm 0.3 \pm 0.5$	$3119\pm0.3\pm0.9$	$3188 \pm 5 \pm 13$
Mass (Belle [2])	$3000.7 \pm 1.0 \pm 0.2$	$3050.2 \pm 0.4 \pm 0.2$	$3064.9 \pm 0.6 \pm 0.2$	$3089.3 \pm 1.2 \pm 0.2$	-	$3199\pm9\pm4$

 $\Omega_c(3188)$ state is not addressed in these studies [5–9]. These divergences between the theoretical interpretations created a puzzle which needs to be addressed urgently.

In the present article, we first study the Ω_c -mass spectra by estimating the contributions due to spin-orbit interactions, spin-, isospin- and flavour-dependent interaction from the well-established charmed baryon mass spectrum. We reproduce quantitatively the spectrum of the Ω_c states within a harmonic oscillator hamiltonian plus a perturbation term given by spin-orbit, isospin and flavour dependent contributions (Sects. 2.1 and 2.2). Based on our results, we describe these five states as *P*-wave λ -excitations of the *ssc* system; we also calculate their $\Xi_c^+ K^-$ and $\Xi_c'^+ K^-$ decay widths (Sect. 2.3). Similarly to Refs. [10-12], we suggest a molecular interpretation of the $\Omega_c(3119)$ state, which was not observed by Belle. Later, we extend our mass and decay width predictions to the Ω_b sector, which will be useful for future experimental searches. Finally, we calculate the mass splitting between the ρ - and λ -mode excitations of $\Omega_{c(b)}$ resonances (see Fig. 1 upper-pannel). This calculation is fundamental to access to inner heavy-light baryon structure, as the presence or absence of ρ -mode excitations in the experimental spectrum will be the key to discriminate between the three-quark (see Fig. 1 upper-pannel) and the quark-diquark structures (see Fig. 1 lower-pannel), as it will be discussed in Sect. 3.

2 Results

2.1 S- and P-wave ssQ states.

The three-quark system (*ssQ*, where Q = c or *b*) Hamiltonian can be written in terms of two coordinates [13], ρ and λ , which encode the system spatial degrees of freedom (see Fig. 1). Let $m_{\rho} = m_s$ and $m_{\lambda} = \frac{3m_s m_Q}{2m_s + m_Q}$ be the *ssQ* system reduced masses; then, the ρ - and λ -mode frequencies are $\omega_{\rho,\lambda} = \sqrt{\frac{3K_Q}{m_{\rho,\lambda}}}$, where K_Q is the spring constant, which implies that in three equal-mass-quark baryons, in which $m_{\rho} = m_{\lambda}$, the λ - and ρ -orbital excitation modes are completely mixed together. By contrast, in heavy-light baryons, in which $m_{\rho} \ll m_{\lambda}$, the two excitation modes can be decoupled from each other as long as the light-heavy quark mass difference increases.



Fig. 1 Comparison between three-quark and quark-diquark baryon effective degrees of freedom. Upper panel: three-quark picture with two excitation modes. Lower panel: quark-diquark picture with one excitation mode

First of all, we construct the *ssc* and *ssb* ground and excited states to establish the quantum numbers of the five confirmed Ω_c states. A single quark is described by its spin, flavor and color quantum numbers. As a fermion, its spin is $S = \frac{1}{2}$, its flavor, spin-flavor and color representations are $\mathbf{3}_{f}$, $\mathbf{6}_{sf}$, and $\mathbf{3}_{c}$, respectively. An *ssQ* state, $|ssQ, S_{\rho}, S_{tot}, l_{\rho}, l_{\lambda}, J\rangle$, is characterized by total angular momentum $\mathbf{J} = \mathbf{l}_{\rho} + \mathbf{l}_{\lambda} + \mathbf{S}_{\text{tot}}$, where $\mathbf{S}_{\text{tot}} = \mathbf{S}_{\rho} + \frac{1}{2}$. In order to construct an ss Q color singlet state, the light quarks must transform under $SU_c(3)$ as the anti-symmetric $\mathbf{3}_c$ representation. The Pauli principle postulates that the wave function of identical fermions must be anti-symmetric for particle exchange. Thus, the ss spin-flavor and orbital wave functions have the same permutation symmetry: symmetric spin-flavor in S-wave, or antisymmetric spin-flavor in antisymmetric Pwave. Two equal flavour quarks are necessarily in the $6_{\rm f}$ flavor-symmetric state. Thus, they are in an S-wave symmetric spin-triplet state, $S_{\rho} = 1$, or in a *P*-wave antisymmetric spin-singlet state, $S_{\rho} = 0$.

If $l_{\rho} = l_{\lambda} = 0$, then $S_{\rho} = 1$, and we find the two ground states, Ω_Q and Ω_Q^* : $|ssQ, 1, S_{tot}, 0_{\rho}, 0_{\lambda}, J\rangle$ with $J = S_{tot} = \frac{1}{2}$ and $\frac{3}{2}$, respectively. If $l_{\rho} = 0$ and $l_{\lambda} = 1$, then $S_{\rho} = 1$ and, by coupling the spin and orbital angular momentum, we find five excited states: $|ssQ, 1, S_{tot}, 0_{\rho}, 1_{\lambda}, J\rangle$ with $J = \frac{1}{2}, \frac{3}{2}$ for $S_{tot} = \frac{1}{2}$, and $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ for $S_{tot} = \frac{3}{2}$, which we interpret as λ -mode excitations of the ssQ system. On the other hand, if $l_{\rho} = 1$ and $l_{\lambda} = 0$, then $S_{\rho} = 0$, and we find two excited states $|ssQ, 0, \frac{1}{2}, 1_{\rho}, 0_{\lambda}, J\rangle$ with $J = \frac{1}{2}, \frac{3}{2}$ which we interpret as ρ - mode excitations of the ssQ system.¹

2.2 Mass spectra of Ω_Q states

We make use of a three-dimensional harmonic oscillator hamiltonian (h.o.) plus a perturbation term given by spin– orbit, isospin and flavour dependent contributions:

$$H = H_{\text{h.o.}} + A \,\mathbf{S}^2 + B \,\mathbf{S} \cdot \mathbf{L} + E \,\mathbf{I}^2 + G \,\mathbf{C_2}(\text{SU}(3)_{\text{f}}); \ (1)$$

here S, *I* and $C_2(SU(3)_f)$ are the spin, the isospin and the $C_2(SU(3)_f)$ Casimir operators, and

$$H_{\text{h.o.}} = \sum_{i=1}^{3} m_i + \frac{\mathbf{p}_{\rho}^2}{2m_{\rho}} + \frac{\mathbf{p}_{\lambda}^2}{2m_{\lambda}} + \frac{1}{2}m_{\rho}\omega_{\rho}^2\boldsymbol{\rho}^2 + \frac{1}{2}m_{\lambda}\omega_{\lambda}^2\boldsymbol{\lambda}^2$$
(2)

is the three-dimensional harmonic oscillator Hamiltonian written in terms of Jacobi coordinates, ρ and λ , and conjugated momenta, \mathbf{p}_{ρ} and \mathbf{p}_{λ} , whose eigenvalues are $\sum_{i=1}^{3} m_i + \omega_{\rho} n_{\rho} + \omega_{\lambda} n_{\lambda}$, where $\omega_{\rho(\lambda)} = \sqrt{\frac{3K_Q}{m_{\rho(\lambda)}}}$, $n_{\rho(\lambda)} = 2k_{\rho(\lambda)} + l_{\rho(\lambda)}$, $k_{\rho(\lambda)} = 0, 1, ...,$ and $l_{\rho(\lambda)} = 0, 1, ...$

We set the quark masses to reproduce the $\Omega_c(2695)$, $\Omega_c^*(2765)$, $\Xi_{cc}(3621)$ and $\Sigma_b(5814)$ ground state masses [15]: $m_q = 295$ MeV, $m_s = 450$ MeV, $m_c = 1605$ MeV and $m_b = 4920$ MeV; the spring constant K_c is set to reproduce the mass difference between $\Xi_c(2790)$, with $J^P = \frac{1}{2}^{-1}$, and the $\Xi_c(2469)$ ground state: $K_c = 0.0328 \text{ GeV}^3$, while K_b is set to reproduce the mass difference between Λ_b (5919), with $J^P = \frac{1}{2}^{-1}$, and the $\Lambda_b(5619)$ ground state: $K_b = 0.0235$ GeV³. In order to calculate the mass difference between the ρ and λ orbital excitations of ssQ states, we scale the h.o. frequency by the ρ and λ oscillator masses. From the definition of m_{ρ} and m_{λ} , one finds $m_{\rho} = m_s = 450$ MeV and $m_{\lambda} = \frac{3m_s m_c}{2m_s + m_c} \simeq 865$ MeV for Ω_c states, and $m_{\lambda} = \frac{3m_s m_b}{2m_s + m_b} \simeq 1141$ MeV for Ω_b states; the ρ - and λ mode frequencies are $\omega_{\rho,\lambda} = \sqrt{\frac{3K_Q}{m_{\rho,\lambda}}}$. Finally, the mass splitting parameters, A, B, E and \dot{G} , calculated in the following, are reported in Table 2.

 Table 2
 Values of the parameter reported in Eq. (1) with the corresponding uncertainties expressed in MeV

State	Α	В
Charm	21.54 ± 0.37	23.91 ± 0.31
Bottom	6.73 ± 1.63	5.15 ± 0.33
State	E	G
Charm	30.34 ± 0.23	54.37 ± 0.58
Bottom	26.00 ± 1.80	70.91 ± 0.49

We estimate the mass splittings due to the spin-orbit, spin-, isospin- and flavor-dependent interactions from the well established charmed (bottom) baryon mass spectrum. The spin-orbit interaction, which is mysteriously small in light baryons [16-18], turns out to be fundamental to describe the heavy-light baryon mass patterns, as it is clear from those of the recently observed Ω_c states. The spin-, isospin-, and flavour-dependent interactions are necessary to reproduce the masses of charmed baryon ground states, as observed in Ref. [19]. By means of these estimates, we predict in a parameterfree procedure the spectrum of the ssQ excited states constructed in the previous section. The predicted masses of the λ - and ρ -orbital excitations of the Ω_c and Ω_b baryons are reported in Tables 3 and 4, respectively. In particular, Table 3 shows that we are able to reproduce quantitatively the mass spectra of the Ω_c states observed both by LHCb and Belle; the latter are reported in Table 1.

We estimate the energy splitting due to the spin–spin interaction from the (isospin-averaged) mass difference between $\Sigma_c^*(2520)$ and $\Sigma_c(2453)$. This value (65 ± 8 MeV) agrees with the mass difference between Ω_c (2695) and Ω_c^* (2770), a value close to 71 MeV. As a consequence, the spin-spin mass splitting between two orbitally excited states characterized by the same flavor configuration but different spins, specifically $S_{\text{tot}} = \frac{1}{2}$ and $S_{\text{tot}} = \frac{3}{2}$, is around 65 MeV plus corrections due the spin–orbit contribution which can be calculated, for example, from the $\Lambda_c(2595)$ - $\Lambda_c(2625)$ mass difference. According to the quark model, $\Lambda_c(2595)$ and $\Lambda_c(2625)$ are the charmed counterparts of $\Lambda(1405)$ and $\Lambda(1520)$, respectively; their spin-parities are $\frac{1}{2}^-$ and $\frac{3}{2}^-$, and their mass difference, about 36 MeV, is due to spin–orbit effects.

In conclusion, by taking into account the spin-spin and spin-orbit contributions, the mass difference between the lowest Ω_c excitation, $|ssc, 1, \frac{1}{2}, 0_{\rho}, 1_{\lambda}, \frac{1}{2}\rangle \equiv \Omega_c(3000)$ and $|ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{1}{2}\rangle$, is about $65 - 36 \simeq 30$ MeV. So, we identify the $|ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{1}{2}\rangle$ with the observed $\Omega_c(3050)$ (see Fig. 2 and Table 3). In the bottom sector, the energy splitting due to the spin-spin interaction through the (isospin-averaged) mass difference between Σ_b^* and Σ_b is 20 ± 7 MeV. In such a way, we expect a mass difference between the two S-wave ground states, Ω_b^* and Ω_b , close to

¹ A similar analysis is done in Ref. [14] where the authors considered $l_{\rho} = 0$ and $L = l_{\lambda}$ in a JJ coupling scheme.



Fig. 2 Ω_c mass spectra and tentative quantum number assignments. The theoretical predictions (red dots) are compared with the experimental results by LHCb [1] (blue line), Belle [2] (violet line) and Particle Data Group (black lines) [15]. Except the Ω_c (3188) case, the experimental error for the other states is too small to be appreciated in this energy scale. The spin- $\frac{1}{2}$ and $-\frac{3}{2}$ ground-state masses, Ω_c (2695) and Ω_c^* (2770) are indicated with \dagger because are inputs while all the others are predictions

 20 ± 7 MeV. Hence, we suggest the experimentalists to look for a Ω_b^* resonance with a mass of about 6082 MeV, as we can see in Fig. 3 and Table 4.

We estimate that the mass of $|ssc, 1, \frac{1}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\rangle$ is related to the previous spin-orbit splitting. We obtain a value of 3052 ± 15 MeV, which is compatible with the mass of the $\Omega_c(3066)$ within the experimental error. Thus, we identify the $|ssc, 1, \frac{1}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\rangle$ state with the $\Omega_c(3066)$ resonance. Through the estimation of orbital, spin-spin and spin-orbit interactions, we estimate the $|ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\rangle$ and $|ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{5}{2}\rangle$ mass values as 3080 ±13 MeV

Table 3 Our *ssc* state quantum number assignments (first column), predicted masses (second column) and open-flavor strong decay widths into $\Xi_c^+ K^-$ and $\Xi_c'^+ K^-$ channels (fourth column) are compared with the experimental masses (third column) and total decay widths (fifth column) [1,15]. An *ssc* state, $|ssc, S_{\rho}, S_{\text{tot}}, l_{\rho}, l_{\lambda}, J\rangle$, is characterized by total angular momentum $\mathbf{J} = \mathbf{I}_{\rho} + \mathbf{I}_{\lambda} + \mathbf{S}_{\text{tot}}$, where $\mathbf{S}_{\text{tot}} = \mathbf{S}_{\rho} + \frac{1}{2}$.

and 3140 ±14, respectively. Thence, we propose the following assignments: $|ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\rangle \rightarrow \Omega_{c}(3090)$ and $|ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{5}{2}\rangle \rightarrow \Omega_{c}(3188).$

In the bottom sector, the mass splitting due to the spinorbit interaction between $\Lambda_b(5912)$ and $\Lambda_b(5920)$ is 8 MeV and we estimated previously that the spin-spin splitting is 20 ± 7 MeV. Thus, we interpret the predicted $\Omega_b(6305)$, $\Omega_b(6313)$, $\Omega_b(6317)$, $\Omega_b(6325)$ and $\Omega_b(6338)$ states, reported in Table 4, as the bottom counterparts of the $\Omega_c(3000)$, $\Omega_c(3066)$, $\Omega_c(3050)$, $\Omega_c(3090)$ and $\Omega_c(3188)$, respectively. We observe that, unlike the charm sector, in the bottom sector the state $|ssb, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{1}{2}\rangle$ is heavier than the state $|ssb, 1, \frac{1}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\rangle$: this is due to the fact that in the charm sector the spin-orbit contribution is lesser than the spin-spin one, while in the bottom sector the situation is reversed (see Table 2).

In the charm sector, the mass splitting due to the flavordependent interaction can be estimated from the mass difference between Ξ_c and Ξ'_c , whose isospin-averaged masses are 2469.37 MeV and 2578.1 MeV, respectively; this leads to a value of 109 MeV, approximately. The bottom partner of Ξ_c and Ξ'_c are Ξ_b and Ξ'_b , with masses 5793.2 MeV and 5935.02 MeV, respectively. Therefore, in the bottom sector the flavordependent interaction gives a contribution of about 142 MeV, which is more than 30% larger than in the charm sector. The mass difference between the lightest charmed ground states, Σ_c and Λ_c , is related to the different isospin and flavor structures of the light quark multiplets: Λ_c is an isospin-singlet state belonging to an SU(3)_f flavor anti-triplet, while Σ_c is an isospin-triplet state belonging to an SU(3)_f flavor sectet. In the bottom sector, the isospin-flavor contribution to the

Our results are compatible with the experimental data, the predicted partial decay widths being lower than the total measured decay widths. Masses of states denoted with \dagger are used as inputs while all the others are predictions; partial decay widths denoted with \dagger^{\dagger} and with \dagger^{\dagger} \dagger^{\dagger} are zero for phase space and for selection rules, respectively

State	Predicted mass (MeV)	Experimental mass (MeV)	Predicted width Γ (open-flavor) (MeV)	Experimental width Γ_{tot} (MeV)
$\left ssc, 1, \frac{1}{2}, 0_{\rho}, 0_{\lambda}, \frac{1}{2}\right\rangle \equiv \Omega_{\rm c}(2695)^{\dagger}$	2702 ± 12	2695 ± 2	††	< 10 ⁻⁷
$\left ssc, 1, \frac{3}{2}, 0_{\rho}, 0_{\lambda}, \frac{3}{2}\right\rangle \equiv \Omega_{\rm c}^{*}(2770)^{\dagger}$	2767 ± 13	2766 ± 2	††	
$\left ssc, 1, \frac{1}{2}, 0_{\rho}, 1_{\lambda}, \frac{1}{2}\right\rangle \equiv \Omega_{\rm c}(3000)$	3016 ± 9	$3000.4 \pm 0.2 \pm 0.1 \pm 0.3$	0.48	$4.6\pm0.6\pm0.3$
$\left ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{1}{2}\right\rangle \equiv \Omega_{\rm c}(3050)$	3045 ± 13	$3050.2\pm0.1\pm0.1\pm0.3$	1.0	$0.8\pm0.2\pm0.1$
$\left ssc, 1, \frac{1}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\right\rangle \equiv \Omega_{c}(3066)$	3052 ± 15	$3065.6 \pm 0.1 \pm 0.3 \pm 0.3$	3.5	$3.5\pm0.4\pm0.2$
$\left ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\right\rangle \equiv \Omega_{\rm c}(3090)$	3080 ± 13	$3090.2\pm0.3\pm0.5\pm0.3$	1.09	$8.7\pm1.0\pm0.8$
$ ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{5}{2}\rangle \equiv \Omega_{\rm c}(3188)$	3140 ± 14	$3188 \pm 5 \pm 13$	9.87	60 ± 26
$\left ssc, 0, \frac{1}{2}, 1_{\rho}, 0_{\lambda}, \frac{1}{2}\right\rangle$	3146 ± 12		+ + +	
$\left ssc, 0, \frac{1}{2}, 1_{\rho}, 0_{\lambda}, \frac{3}{2}\right\rangle$	3182 ± 12		† † †	



Fig. 3 Ω_b mass spectrum predictions (red dots) and Ω_b ground-state experimental mass (black line) [15]. The experimental error on the $\Omega_b(6046)$ state, 2 MeV, is too small to be appreciated in this energy scale

baryon masses can be calculated from the mass difference between Σ_b and Λ_b .

We summarize all our proposed quantum number assignments for both Ω_c and Ω_b states in Figs. 2 and 3, respectively. In the charm sector, we find a good agreement between the mass pattern predicted for the spectrum and the experimental data: in particular, with the exception of the lightest and the heaviest resonant states, $\Omega_c(3000)$ and $\Omega_c(3188)$, respectively, also the absolute mass predictions are in agreement within the experimental error, which is very small (less than 1 MeV).

2.3 Decay widths of ssQ states

In the following, we compute the strong decays of ssQ baryons in sqQ - K (q = u, d) final states by means of the ³P₀ model [20–23] (see Appendix A).

Table 4 Our *ssb* state quantum number assignments (first column), predicted masses (second column) and open-flavor strong decay widths (fourth column) are compared with the experimental masses (third column) and total decay widths (fifth column) [15]. An *ssb* state,

In the ³*P*₀ model, the parameters depend on the harmonic oscillator frequency of the initial and final states. For charmed baryons, we expect the parameters α_{ρ} and α_{λ} to lie in the range 0.4–0.7 GeV. In principle, the values of the α_{ρ} and α_{λ} ho. parameters of lower- and higher-lying resonances should be different; see e.g. Ref. [24]. However, as widely discussed in the literature, it is customary to use constant values for α_{ρ} and α_{λ} . We also prefer not to take α_{ρ} and α_{λ} as free parameters, but to express them in terms of the baryon ρ - and λ -mode frequencies, $\omega_{\rho,\lambda} = \sqrt{3K_Q/m_{\rho,\lambda}}$, using the relation $\alpha_{\rho,\lambda}^2 = \omega_{\rho,\lambda}m_{\rho,\lambda}$ for both initial- and final-state baryon resonances; see Appendix B. In light of this, the only free parameter is the pair creation strength, $\gamma_0 = 9.2$, which is fitted to the reproduction of the Ω_c (3066) width. The frequency of *K* meson is set to be $\omega_c = 0.46$ GeV [25].

Tables 3 and 4 report our $\Omega_c \rightarrow \Xi_c^+ K^-, \Xi_c^{\prime+} K^-$ and $\Omega_b \to \Xi_b^0 K^-$ predicted decay widths. The $\Xi_c^+ K^-$ decay channel is where the Ω_c states were observed by LHCb and Belle; we also consider the $\Xi_c^{\prime+}K^-$ channel, which contributes to the $\Omega_{\rm c}(3090)$ and $\Omega_{\rm c}(3188)$ open-flavor decay widths. Both the $\Xi_c^+ K^-$ branching ratios and the quantum numbers of the Ω_c 's are unknown; we only have experimental informations on their total widths, Γ_{tot} . Thus, our predictions have to satisfy the constraint: $\Gamma(\Omega_c \to \Xi_c^+ K^-) \leq \Gamma_{\text{tot}}$. In light of this, we state that our strong decay width results, based both on our mass estimates and quantum number assignments, are compatible with the present experimental data. In particular, the λ -mode decay widths of the Ω_c states are in the order 1 MeV, while the $\Xi_c^+ K^-$ decay mode of the two ρ -excitations, $|ssc, 0, \frac{1}{2}, 1_{\rho}, 0_{\lambda}, \frac{1}{2}\rangle$ and $|ssc, 0, \frac{1}{2}, 1_{\rho}, 0_{\lambda}, \frac{3}{2}\rangle$, is forbidden by spin conservation. Similar considerations can be applied to the decay widths of ρ mode Ω_b states. The presence of inconsistencies between our predictions for the mass spectrum and the open-flavor strong decay widths of Table 3 can have two possible explanations:

 $|ssb, S_{\rho}, S_{tot}, l_{\rho}, l_{\lambda}, J\rangle$, is characterized by total angular momentum $\mathbf{J} = \mathbf{I}_{\rho} + \mathbf{I}_{\lambda} + \mathbf{S}_{tot}$, where $\mathbf{S}_{tot} = \mathbf{S}_{\rho} + \frac{1}{2}$. Partial decay widths denoted with $\dagger \dagger$ and with $\dagger \dagger \dagger$ are zero for phase space and for selection rules, respectively

State	Predicted mass (MeV)	Experimental mass (MeV)	Predicted width $\Gamma(\Omega_{\rm b} \to \Xi_b^0 K^-) ({\rm MeV})$	Experimental widths $\Gamma_{\rm tot}$ (MeV)
$ ssb, 1, \frac{1}{2}, 0_{\rho}, 0_{\lambda}, \frac{1}{2}\rangle \equiv \Omega_b$	6061 ± 15	6046 ± 2	††	< 10 ⁻⁹
$ ssb, 1, \frac{3}{2}, 0_{\rho}, 0_{\lambda}, \frac{3}{2}\rangle$	6082 ± 20		++	
$ ssb, 1, \frac{1}{2}, 0_{\rho}, 1_{\lambda}, \frac{1}{2}\rangle$	6305 ± 15		0.50	
$ ssb, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{1}{2}\rangle$	6317 ± 19		2.79	
$ ssb, 1, \frac{1}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\rangle$	6313 ± 15		1.14	
$\left ssb, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\right\rangle$	6325 ± 19		0.62	
$ ssb, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{5}{2}\rangle$	6338 ± 20		4.28	
$ ssb, 0, \frac{1}{2}, 1_{\rho}, 0_{\lambda}, \frac{1}{2}\rangle$	6452 ± 15		† † †	
$\left ssb, 0, \frac{1}{2}, 1_{\rho}, 0_{\lambda}, \frac{3}{2}\right\rangle$	6460 ± 15		† † †	



Fig. 4 Adapted from Fig. 2 of Ref. [1], APS copyright. Proposed spin- and parity-assignments for the $\Omega_c = css$ excited states reported by the LHCb Collaboration and later confirmed by Belle: $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$, and $\Omega_c(3188)$. We interpret $\Omega_c(3119)$ as a $\Xi_c^* K$ molecule

I) We used a single set of values for the α_{ρ} and α_{λ} h.o. parameters. Those values were extracted from a fit to the spectrum and not fitted to the reproduction of the Ω_c 's decay widths; II) There is not a single model which is capable of providing a completely satisfactory description of baryon open-flavor strong decay widths [26].

In conclusion, in addition to our mass estimates, also the ${}^{3}P_{0}$ model results suggest that the five Ω_{c} resonances, $\Omega_{c}(3000)$, $\Omega_{c}(3050)$, $\Omega_{c}(3066)$, $\Omega_{c}(3090)$, and $\Omega_{c}(3188)$, could be interpreted as *ssc* ground-state *P*-wave λ -excitations. In principle, both the $\Omega_{c}(3090)$ and $\Omega_{c}(3119)$ resonances observed by LHCb are compatible with the properties (mass and decay width) of the $|ssc, \frac{3}{2}, 1_{\lambda}, \frac{3}{2}\rangle$ theoretical state. As Belle could neither confirm nor deny the existence of the $\Omega_{c}(3119)$, given the low significance of its results for the previous state (0.4σ) , we prefer to: 1) Assign $|ssc, \frac{3}{2}, 1_{\lambda}, \frac{3}{2}\rangle$ to the $\Omega_{c}(3090)$; 2) Interpret the $\Omega_{c}(3119)$ as a $\mathcal{E}_{c}^{*}K$ bound state [10-12], the $\Omega_{c}(3119)$ lying 22 MeV below the $\mathcal{E}_{c}^{*}K$ threshold. See Fig. 4. Additionally, in Table 5, we present a comparison of different quantum number assignments for the Ω_{c} states.

3 Comparison between the three-quark and quark-diquark structures

In the light sector, the quark model reproduces successfully the baryon spectrum by assuming that the constituent u, dand s quarks have roughly the same mass. This implies that the two oscillators, ρ and λ , have approximately the same frequency, $\omega_{\rho} \simeq \omega_{\lambda}$; therefore, the ρ - and λ -excitations are degenerate. By contrast, in the case of heavy-light baryons $m_{\rho} \ll m_{\lambda}$; thus, the two excitation modes are decoupled from one another; specifically $\omega_{\rho} - \omega_{\lambda} \simeq 130$ MeV for Ω_c states and $\omega_{\rho} - \omega_{\lambda} \simeq 150$ MeV for Ω_b states. Thus, the heavy-light baryon sector is the most suitable environment to test what are the correct effective spatial degrees of freedom for reproducing the mass spectra, as the presence or absence of ρ -mode excitations in the spectrum will be the key to discriminate between the three-quark and the quark-diquark structures (see Fig. 1). Specifically, if the predicted four ρ -excitations, $\Omega_c(3146)$, $\Omega_c(3182)$, $\Omega_b(6452)$, and $\Omega_b(6460)$, are not observed, then the other Ω_c states will be characterized by a quark-diquark structure.

Finally, we observe that in the case of a quark–diquarkpicture experimental confirmation, the model Hamiltonian employed, see Eq. (1), still holds because the quark–diquark h.o. Hamiltonian is the limit of the three-quark h.o. Hamiltonian, Eq. (2), when we freeze the ρ coordinate:²

$$H_{\text{h.o.}} = m_D + m_Q + \frac{\mathbf{p}_{\lambda}^2}{2m_{\lambda}} + \frac{1}{2}m_{\lambda}\omega_{\lambda}^2 \boldsymbol{\lambda}^2.$$
(3)

Here $m_D = 2m_s$ is the diquark mass. Indeed, the mass spectrum predicted with this definition of $H_{\text{h.o.}}$ is the same as that reported in Figs. 2 and 3, but without the frozen ρ excitations. We observe also that, if the quark–diquark scenario turns out to be the correct one, the suppression of the spin–spin interaction that we found going from the charmed to the bottom sector is consistent with the heavy quark symmetry, this suppression being an indication that heavy quark effective theory, HQET, still holds also in the heavy-light baryon sector.

4 Discussion

We calculated the $\Omega_{c(b)}$'s masses and $\Xi_{c(b)}^+ K^-$ strong decay amplitudes. By means of these mass and decay width predictions, we proposed an univocal assignment to the five Ω_c states observed both by LHCb [1] and Belle [2]:

$ssc, 1, \frac{1}{2}, 0_{\rho}, 1_{\lambda}, \frac{1}{2}$	$\rangle \rightarrow \Omega_c(3000)$,	(4)
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- $\left|ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{1}{2}\right\rangle \to \Omega_{c}(3050), \qquad (5)$
- $|ssc, 1, \frac{1}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\rangle \rightarrow \Omega_c(3066),$ (6)
- $\left|ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\right\rangle \to \Omega_{c}(3090), \qquad (7)$

$$|ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{5}{2}\rangle \rightarrow \Omega_c(3188)$$
 (8)

The latter was completely ignored in other studies [5–9]. In principle, both the $\Omega_c(3119)$ and $\Omega_c(3090)$ could

² In this limit, in fact, we recover the expression of the well known two-body h.o. Hamiltonian in the quark–diquark centre of mass frame, $H_{ho} = 2m_D + m_Q + \frac{p_r^2}{2\mu} + \frac{1}{2}\mu\omega_{\lambda}^2\mathbf{r}^2$, as $\frac{p_{\lambda}^2}{2m_{\lambda}} = \frac{p_r^2}{2\mu}$ and $\frac{1}{2}m_{\lambda}\omega_{\lambda}^2\boldsymbol{\lambda}^2 = \frac{1}{2}\mu\omega_{\lambda}^2\mathbf{r}^2$, where $\mu = \frac{m_Dm_Q}{m_D+m_Q}$ is the reduced mass, $\mathbf{p}_r = \frac{m_Q\mathbf{p}_D-m_D\mathbf{p}_Q}{m_D+m_Q}$ whose modulus is the quark and the diquark momentum in the centre of mass frame, in which $\mathbf{p}_{cm} = \mathbf{p}_D + \mathbf{p}_Q = 0$, and $\mathbf{r} = \mathbf{r}_D - \mathbf{r}_Q$ is the quark–diquark relative position vector.

State	Ours	Ref. [5]	Ref. [5]*	Ref [14]
$\Omega_{\rm c}(3000)$	1/2-	1/2-	3/2-	1/2-
$\Omega_{\rm c}(3050)$	3/2-	$1/2^{-}$	3/2-	3/2-
$\Omega_{\rm c}(3066)$	$1/2^{-}$	3/2-	$5/2^{-}$	$3/2^{-}$
$\Omega_{\rm c}(3090)$	3/2-	3/2-	$1/2^{+}$	$1/2^{+}$
$\Omega_{\rm c}(3119)$	Molecule	5/2-	3/2+	$5/2^{-}$
$\Omega_{\rm c}(3188)$	5/2-			$1/2^{-}(3/2^{-})$

Table 5 J^P quantum number assignments of Ω_c resonances from previous studies. * Ref. [5] provided two different sets of J^P assignments

be assigned to the $|ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\rangle$ state. However, as Belle could neither confirm nor deny the existence of the $\Omega_c(3119)$, we preferred the $\Omega_c(3119)$ interpretation as a $\Xi_c^* K$ meson-baryon molecule and assigned the $\Omega_c(3090)$ to the $|ssc, 1, \frac{3}{2}, 0_{\rho}, 1_{\lambda}, \frac{3}{2}\rangle \rightarrow \Omega_c(3090)$ state, providing a consistent solution to the Ω_c puzzle.

We calculated the mass splitting between the ρ - and λ -mode excitations of the $\Omega_{c(b)}$ resonances. This large mass splitting, that we predicted to be larger than 150 MeV, is fundamental to access to the inner heavy-light baryon structure. If the ρ -excitations in the predicted mass region are not observed in the future, then the three-quark model effective degrees of freedom for the heavy-light baryons will be ruled out, supporting the Heavy Quark Effective Theory (HQET) picture of the heavy-light baryons described as heavy quark–light diquark systems. If the HQET is valid for the heavy-light baryons, the heavy quark symmetry, predicted by the HQET in the heavy-light meson sector, can be extended to the heavy-quark–light-diquark baryon sector, opening the way to new future theoretical applications.

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Appendix A: ${}^{3}P_{0}$ Decay model

The ${}^{3}P_{0}$ is an effective model to compute the open-flavor strong decays of hadrons in the quark model formalism [20– 23]. In this model, a hadron decay takes place in its rest frame and proceeds via the creation of an additional $q\bar{q}$ pair with vacuum quantum numbers, i.e. $J^{PC} = 0^{++}$. We label the initial baryon- and final baryon- and meson-states as A, B and C, respectively. The final baryon-meson state BCis characterized by a relative orbital angular momentum ℓ between B and C and a total angular momentum $\mathbf{J} = \mathbf{J}_{B} + \mathbf{J}_{C} + \ell$. The decay widths can be calculated as [20,21,27]

$$\Gamma = \frac{2\pi\gamma_0^2}{2J_A + 1} \, \Phi_{A \to BC}(q_0) \sum_{M_{J_A}, M_{J_B}} \left| \mathcal{M}^{M_{J_A}, M_{J_B}} \right|^2.$$

Here, $\mathcal{M}^{M_{J_A}, M_{J_B}}$ is the $A \to BC$ amplitude which, for simplicity, is usually expressed in terms of hadron harmonicoscillator wave functions, γ_0 is the dimensionless paircreation strength. q_0 is the relative momentum between *B* and *C*, and the coefficient $\Phi_{A\to BC}(q_0)$ is the relativistic phase space factor [27],

$$\Phi_{A \to BC}(q_0) = 4\pi q_0 \frac{E_B(q_0)E_C(q_0)}{M_A}$$

with $E_{B,C} = \sqrt{M_{B,C}^2 + q_0^2}$.

Appendix B: Baryon wave functions

Differently from light baryon phenomenology, where the ρ and λ -modes of the mixed-symmetry spatial wave function are degenerate in energy, in the heavy-light sector the previous modes decouple; so, they can be distinguished through an analysis of the heavy-light baryon mass spectra. This happens because frequency of the ρ - and λ -modes are different,

$$\omega_{\rho} = \sqrt{\frac{3K_Q}{m_{\rho}}} \text{ and } \omega_{\lambda} = \sqrt{\frac{3K_Q}{m_{\lambda}}},$$
 (B.1)

where m_{ρ} and m_{λ} are defined in Sect. 2.1. We write the baryon wave functions in terms of ω_{ρ} and ω_{λ} by using the relation $\alpha_{\rho,\lambda}^2 = \omega_{\rho,\lambda} m_{\rho,\lambda}$.

For the S-wave charmed baryon,

$$\psi(0, 0, 0, 0) = 3^{3/4} \left(\frac{1}{\pi\omega_{\rho}m_{\rho}}\right)^{\frac{3}{4}} \left(\frac{1}{\pi\omega_{\lambda}m_{\lambda}}\right)^{\frac{3}{4}} \\ \times \exp\left[-\frac{\mathbf{P}_{\rho}^{2}}{2\omega_{\rho}m_{\rho}} - \frac{\mathbf{P}_{\lambda}^{2}}{2\omega_{\lambda}m_{\lambda}}\right].$$
(B.2)

For the P-wave charmed baryon,

$$\psi_{\rho}(1, m, 0, 0) = -i \left(\frac{8}{3\sqrt{\pi}}\right)^{1/2} \left(\frac{1}{\omega_{\rho}m_{\rho}}\right)^{5/4} \mathcal{Y}_{1}^{m}(\mathbf{P}_{\rho}) \\ \times \left(\frac{3}{\pi\omega_{\lambda}m_{\lambda}}\right)^{3/4} \exp\left[-\frac{\mathbf{P}_{\rho}^{2}}{2\omega_{\rho}m_{\rho}} - \frac{\mathbf{P}_{\lambda}^{2}}{2\omega_{\lambda}m_{\lambda}}\right],$$
(B.3)

$$\begin{split} \psi_{\lambda}(0,0,1,m) &= -i \left(\frac{8}{3\sqrt{\pi}}\right)^{1/2} \left(\frac{1}{\omega_{\lambda}m_{\lambda}}\right)^{5/4} \mathcal{Y}_{1}^{m}(\mathbf{P}_{\lambda}) \\ &\times \left(\frac{3}{\pi\omega_{\rho}m_{\rho}}\right)^{3/4} \exp\left[-\frac{\mathbf{P}_{\rho}^{2}}{2\omega_{\rho}m_{\rho}} - \frac{\mathbf{P}_{\lambda}^{2}}{2\omega_{\lambda}m_{\lambda}}\right]. \end{split} \tag{B.4}$$

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