Eur. Phys. J. C (2019) 79:996
https://doi.org/10.1140/epjc/s10052-019-7524-7

THE EUROPEAN PHYSICAL JOURNAL C

Letter

Contributions from Φ_{B2} to the $B \to PP$ decays within the QCD factorization

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Received: 19 August 2019 / Accepted: 1 December 2019 / Published online: 9 December 2019

Abstract With the potential for the improvements of measurement precision, the refinement of theoretical calculation on hadronic B weak decays is necessary. In this paper, we study the contributions of B mesonic distribution amplitude Φ_{B2} within the QCD factorization approach, and find that Φ_{B2} contributes to only the nonfactorizable annihilation amplitudes for the $B \to PP$ decays (P denotes the ground $SU(3)$ pseudoscalar mesons). Although small, the Φ_{B2} contributions might be helpful for improving the performance of the QCD factorization approach, especially for the pure annihilation $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$ decays.

Because of successive impetus from both experiments and theoretical improvements, the study of nonleptonic B meson weak decays has been one of the hot topics of particle physics. Most of the two-body hadronic B decays with branching ratio larger than 10^{-6} have been investigated thoroughly and carefully at the BaBar and Belle experiments [1,2] in the past years. A huge amount of B meson experimental data will be accumulated at the high luminosity colliders in the near future, about 50 ab^{-1} by the Belle-II detector at the e^+e^- SuperKEKB collider [3] and about 300 fb^{-1} by the LHCb Upgrade II detector at the hadron HL-LHC collider [4,5]. With the advent of a new age of B physics at the intensity frontier, besides some new phenomena, the unprecedented precision will offer a much more rigorous test on the standard model of elementary particles. The prospective experimental sensitivities for B mesons require more and more accuracy of theoretical calculation.

As is well known, the participation of the strong interactions make it very complicated to calculate the B meson weak decays, especially for the nonleptonic cases. Based on power-counting rules in the heavy quark limits and perturbative QCD theory, some phenomenological models, such as QCD factorization (QCDF) [6–11], perturbative QCD (pQCD) approach [12–15] and so on, have been developed and employed to compute the hadronic matrix elements (HMEs) describing the transformations between the initial B meson and final hadrons through local quark interactions. However, the nonperturbative contributions to HMEs bring theoretical results on branching ratios with many and large uncertainties, particularly for the internal W-boson emission and the neutral current processes. To reduce theoretical uncertainties and satisfy the precision requirements of experimental analysis, a careful and comprehensive examination of all possible nonperturbative factors within a phenomenological model is necessary. In this paper, the contributions from the B meson wave functions will be reassessed in detail within the theoretical framework of QCDF.

Wave functions (WFs) or distribution amplitudes (DAs) of the B meson are the essential ingredients of the master formulas in QCDF [7] and pQCD [13] approaches to evaluate the nonfactorizable contributions to HMEs, such as the spectator scattering amplitudes. However, the knowledge of the B mesonic WFs and DAs is still limited so far. It is intuitive that the component quarks of a hadron should move with the same velocity to form a color singlet, and thus the valence quarks would share momentum fractions according to their masses. It is expected that the B mesonic DAs should be very asymmetric with ξ at the scales of order m_b or smaller, if the light spectator quark carries a longitudinal momentum fraction $\xi \sim \mathcal{O}(\Lambda_{\rm QCD}/m_b)$, where $\Lambda_{\rm QCD}$ and m_b are respectively the characteristic QCD scale and the mass of b quark. Generally, the B meson is described by two scalar functions up to the leading power in $1/m_b$ [16–19], which is written as [7]

$$\langle 0|\bar{q}_\alpha(z)[\ldots]b_\beta(0)|\bar{B}(p)\rangle = -\frac{i\,f_B}{4}\Big\{\big(\not{p}+m_b\big)\gamma_5\Big\}_{\beta\gamma}\int d\xi\, e^{-i\xi p_+ z_-}\Big[\Phi_{B1}(\xi) + \not{n}_-\Phi_{B2}(\xi)\Big]_{\gamma\alpha}, \quad (1)$$

where the dots denote the path-ordered exponential gauge factor; the light spectator quark moves along the light-like

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z_- line; $n_- = (1, 0, 0, -1)$ is a null vector; and the normalization conditions of DAs are [7]

$$\int_0^1 d\xi \, \Phi_{B1}(\xi) = 1, \quad (2)$$

$$\int_0^1 d\xi \, \Phi_{B2}(\xi) = 0. \quad (3)$$

According to the conventions of Refs. [16,17], $\Phi_{B1} = \phi_B^+$ and $\Phi_{B2} = (\phi_B^+ - \phi_B^-)/2$. Generally, the two functions $\phi_B^\pm$ are not identical, $\phi_B^+ \neq \phi_B^-$, and satisfy the relation $\phi_B^+(\xi) + \xi\, \phi_B^{-\prime}(\xi) = 0$ [17]. So, $\Phi_{B2} \neq 0$. The contributions of Φ_{B2} part are suppressed by the power factor of $\Lambda_{\rm QCD}/m_b$, compared with those of Φ_{B1}. In the actual calculations for the $B \to PP$ decays with the QCDF approach (P denotes the light $SU(3)$ ground pseudoscalar meson), for example in Ref. [8], only the contributions from Φ_{B1} part are considered appropriately, while those from Φ_{B2} part are not included explicitly. It should be pointed out that the value of $\Lambda_{\rm QCD}/m_b$ is not a negligible number, because the mass of the b quark is finite rather than infinite. It has been shown in Refs. [18–22] that there is a large contribution of Φ_{B2} to the hadronic $B \to \pi$ transition formfactors within the pQCD approach, and its share could reach up to $\sim 30\%$ with some specific inputs [21,22]. This means that the contributions of Φ_{B2} to branching ratios for the W emission processes can reach up to $\sim 70\%$ for some cases. The Φ_{B2} contribution that were neglected in most cases should be given due attention with the QCDF approach, which is the focus of this paper.

Here, it should be pointed out that a possibly large contribution of Φ_{B2} to formfactors is present only with the pQCD approach rather than the QCDF approach, due to different understandings on the nature of the hadronic transition formfactors. With the pQCD approach [12–15], it is assumed that the light quark with a soft momentum of $\mathcal{O}(\Lambda_{\rm QCD})$ in the initial B meson should interact with a hard gluon, so it could receive a large boost in order to form a colorless final state with a light energetic quark originating from the b quark decaying interaction point. It is therefore arguable that the hadronic transition formfactors are computable perturbatively with the help of the Sudakov factor regulation on soft contributions. The hadronic transition formfactors are written as the convolution of wave functions of both the B meson and final hadron. Contrarily, it is argued [7,17] with the QCDF approach that the hard and soft contributions to the heavy-to-light formfactors have the same scaling behavior, and the hard contributions are suppressed by one power of α_s compared with the soft contributions. Because of the dominance of soft contributions, the formfactors for the transition between B meson and light hadron are not fully calculable with the perturbative QCD theory. So, the formfactors are regarded as nonperturbative inputs with the QCDF approach, and therefore have nothing to do with the B mesonic wave functions.

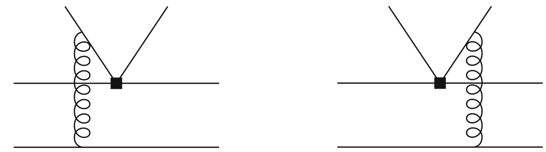

Fig. 1 The spectator scattering interactions

We will concentrate on the $B \to PP$ decays for the moment. Up to power corrections of $1/m_b$, the general QCDF formula of HMEs for an effective operator $\hat{O}_i$ is written as [7],

$$\begin{aligned}\langle P_1 P_2 | \hat{O}_i | \bar{B} \rangle = {} & F_0^{B\to P_1} \int_0^1 dx \, T_i^I(y)\, \phi_{P_2}(x) \\ & + F_0^{B\to P_2} \int_0^1 dy \, H_i^I(x)\, \phi_{P_1}(y) \\ & + \int_0^1 d\xi \, dx \, dy \, T_i^{II}(\xi, x, y)\, \phi_B(\xi)\, \phi_{P_1}(y)\, \phi_{P_2}(x),\end{aligned} \quad (4)$$

where $F_0^{B\to P_i}$ denotes the formfactor; T^I, H^I and T^{II} are hard scattering kernels; the mesonic DAs, $\phi_{P_2}(x)$ and $\phi_{P_1}(y)$, are the functions of longitudinal momentum fractions x and y of light quarks.

For the first two terms of Eq. (4), soft contributions are assumed to be embodied in the formfactors $F_0^{B\to P_i}$ and DAs. Contributions of T^I and H^I are dominated by hard gluon exchange. So these contributions, which are irrelevant to B mesonic wave functions, are considered as perturbative corrections to the naive factorization formula, which involve only decay constants and formfactors, but no DAs.

The third term of Eq. (4) corresponds to nonfactorizable contributions. The spectator scattering interactions (see Fig. 1) entangle the initial B meson with the final hadrons, which make separating one hadron from others impossible. Therefore, the spectator scattering amplitudes are usually written as the convolution integral of the hard kernels T^{II} and all participating DAs. The hard spectator scattering amplitudes contain the contributions from both Φ_{B1} and Φ_{B2}, and can be written as

$$H_k(P_1, P_2) = H_k^{B1}(P_1, P_2) + H_k^{B2}(P_1, P_2), \quad (5)$$

where P_1 is the emitted meson; P_2 is the recoiled meson that incorporates the spectator quark from B meson into itself; H_k^{B1} (H_k^{B2}) is the contribution from Φ_{B1} (Φ_{B2}); the subscript k on H_k refers to the possible Dirac current structure $\Gamma\otimes\Gamma$ of an operator $\hat{O}$, namely, $k = 1$, 2 and 3 correspond to $\Gamma\otimes\Gamma = (V-A)\otimes(V-A)$, $(V-A)\otimes(V+A)$ and $-2(S-P)\otimes(S+P)$ respectively. After the straightforward calculation, we find that considering the $SU(3)$ flavor symmetry, the expressions of H_1^{B1} and H_2^{B1} are entirely consistent with Eqs. (47) and (48) of Ref. [23], and $H_3^{B1} = 0$. Our calculations also show that H_k^{B2} corresponding to Fig. 1a, b

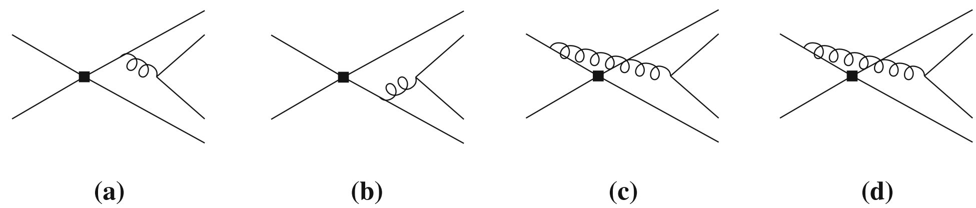

Fig. 2 The weak annihilation interactions, where **a** , **b** are factorizable diagrams, **c**, **d** are nonfactorizable diagrams

are nonzero. Moreover, the terms of both $\int_0^1 \frac{\Phi_{P1}(y)}{\bar{y}^2} dy$ and $\int_0^1 \frac{\Phi_{P1}^p(y)}{\bar{y}^2} dy$ appear in H_k^{B2}, where $\Phi_{P1}(y)$ and $\Phi_{P1}^p(y)$ are the leading twist (twist-2) and twist-3 DAs of the emitted meson P_1 and $\bar{y} = 1 - y$. It is clearly seen that with the asymptotic forms of $\Phi_{P1}(y) = 6\, y\, \bar{y}$ and $\Phi_{P1}^p(y) = 1$, the integrals of $\int_0^1 \frac{\Phi_{P1}(y)}{\bar{y}^2} dy$ and $\int_0^1 \frac{\Phi_{P1}^p(y)}{\bar{y}^2} dy$ exhibit logarithmic and linear infrared divergences. Fortunately, because of the opposite sign between the emitted quark and antiquark propagators plus the condition of Eq. (3), the contributions of H_k^{B2} exactly cancel each other out. The total contributions from Φ_{B2} to spectator scattering amplitudes are zero.

Compared with the leading contributions, the weak annihilation (WA) contributions are thought to be suppressed by one power of $\Lambda_{\rm QCD}/m_b$ [7]. However, the WA contributions are significant and can not be ignored in practical application of the QCDF approach to the hadronic B decays [8,23–26]. Therefore, the QCDF master formula of Eq. (4) is generalized to estimate the WA contributions. The WA interactions have two types of topologies within the QCDF approach. The nonfactorizable and factorizable topologies respectively correspond to gluon emission from the initial B meson and final quarks, see Fig. 2. The factorizable WA amplitudes can be written as the product of the time-like $0 \to P_1 P_2$ formfactors and the integral of B mesonic WFs, see Fig. 2a, b. With the normalization condition of Eq. (3), it is clearly seen that Φ_{B2} contributes nothing to the factorizable WA amplitudes A_k^f, where the superscript f means factorizable, *i.e.*, gluon emission from the final quarks; the subscript k has the same meaning as that of H_k in Eq. (5). The nonfactorizable WA amplitudes, corresponding to Fig. 2c, d, can be written as the convolution integral of all participating hadronic DAs, and contain the contributions from both Φ_{B1} and Φ_{B2}.

$$A_k^i = A_k^{i,B1} + A_k^{i,B2}, \tag{6}$$

where the superscript i means gluon emission from the initial B meson; $A_k^{i,B1}$ ($A_k^{i,B2}$) is the contribution from Φ_{B1} (Φ_{B2}). The expressions of $A_k^{i,B1}$ have been explicitly given by Eq. (62) of Ref. [8] and Eq. (54) of Ref. [23]. Here, we will give the new components $A_k^{i,B2}$.

$$A^{B2} = \pi\, \alpha_s \int_0^1 \xi\, \Phi_{B2}(\xi)\, d\xi = \pi\, \alpha_s\, \langle \xi \rangle_{B2}, \tag{7}$$

$$A_1^{i,B2} = -A^{B2} \int_0^1 \frac{dx}{\bar{x}} \int_0^1 \frac{dy}{y} \Bigg\{ 2\, \frac{\Phi_{P2}(x)\, \Phi_{P1}(y)}{1 - x\, \bar{y}}$$

$$- r_\chi^{P1}\, r_\chi^{P2}\, \frac{\bar{x}}{y}\, \frac{\Phi_{P2}^p(x)\, \Phi_{P1}^p(y)}{1 - x\, \bar{y}} \Bigg\}, \tag{8}$$

$$A_2^{i,B2} = A^{B2} \int_0^1 \frac{dx}{\bar{x}} \int_0^1 \frac{dy}{y} \Bigg\{ 2\, \frac{\Phi_{P2}(x)\, \Phi_{P1}(y)}{\bar{x}\, y} - r_\chi^{P1}\, r_\chi^{P2}\, \Phi_{P2}^p(x)\, \Phi_{P1}^p(y) \left[\frac{\bar{x}}{1 - x\, \bar{y}} - \frac{x}{\bar{x}\, y} \right] \Bigg\}, \tag{9}$$

$$A_3^{i,B2} = -A^{B2} \int_0^1 \frac{dx}{\bar{x}} \int_0^1 \frac{dy}{y} \Bigg\{ 2\, r_\chi^{P2}\, \frac{x\, \Phi_{P2}^p(x)\, \Phi_{P1}(y)}{1 - x\, \bar{y}} + r_\chi^{P1}\, \Phi_{P2}(x)\, \Phi_{P1}^p(y) \left[\frac{y - \bar{y}}{1 - x\, \bar{y}} + \frac{1}{\bar{x}\, y} \right] \Bigg\}, \tag{10}$$

where the factor $r_\chi^P = \frac{2\, m_P^2}{\bar{m}_b\, (\bar{m}_{q_1} + \bar{m}_{q_2})}$.

It is easy to find that contributions from Φ_{B2} to the WA amplitudes are nonzero, because the moment parameter $\langle \xi \rangle_{B2}$ is nonzero. Hence, Φ_{B2} may present nontrivial effects on the observables of hadronic B decays, especially for the WA dominant ones.

In order to better investigate the Φ_{B2} contributions and eliminate other pollution, the pure WA decays $B_d \to K^+ K^-$ and $B_s \to \pi^+ \pi^-$ will be restudied in this paper. Although their branching ratios are tiny, they have been measured accurately by now [27].

$$\mathcal{B}(B_s \to \pi^+ \pi^-) = (6.7 \pm 0.8) \times 10^{-7}, \tag{11}$$

$$\mathcal{B}(B_d \to K^+ K^-) = (8.0 \pm 1.5) \times 10^{-8}. \tag{12}$$

With the asymptotic twist-2 and -3 DAs, $\Phi_P(u) = 6\, u\, \bar{u}$ and $\Phi_P^p(u) = 1$, the integrals in Eqs. (8)–(10) exhibit logarithmic and linear infrared divergences. For an estimation of the WA contributions from Φ_{B2}, these divergent endpoint integrals will be parameterized by the commonly used notations within the QCDF approach [8,23,26].

$$\int_0^1 \frac{du}{u} \to X_A, \tag{13}$$

$$\int_0^1 \frac{du}{u^2} \to X_L, \tag{14}$$

$$\int_0^1 du\, \frac{\ln u}{u} \to -\frac{1}{2} X_A^2, \tag{15}$$

$$\int_0^1 du\, \frac{\ln u}{u^2} \to X_L - X_A - X_L\, X_A. \tag{16}$$

The phenomenological parameters X_A and X_L, are usually treated as universal for hadronic B decays in previous literatures [8,23–26].

With the above parameterization scheme, the WA amplitudes can be rewritten as

$$A_1^{i,B2} = -A^{B2}\left\{12\,(\pi^2-6) - r_\chi^{P1} r_\chi^{P2}\left[\frac{\pi^2}{6} + \frac{1}{2}X_A^2 + X_L X_A + X_A - X_L\right]\right\}, \tag{17}$$

$$A_2^{i,B2} = A^{B2}\left\{72\,(X_A-1)^2 - r_\chi^{P1} r_\chi^{P2}\left[\frac{\pi^2}{6} + \frac{1}{2}X_A^2 + X_L X_A - X_L^2\right]\right\}, \tag{18}$$

$$A_3^{i,B2} = -A^{B2}\,6\left\{\left[\frac{\pi^2}{6} + \frac{1}{2}X_A^2 - X_A\right](2\,r_\chi^{P2} - r_\chi^{P1}) + r_\chi^{P1}\,(X_A X_L - X_L + 1)\right\}. \tag{19}$$

The parameters of X_A and X_L including part of strong phases are complex, and are usually parameterized as [8,23–26]

$$X_A = (1+\rho_A\, e^{i\phi_A})\ln\frac{m_B}{\Lambda_h}, \tag{20}$$

$$X_L = (1+\rho_A\, e^{i\phi_A})\,\frac{m_B}{\Lambda_h}, \tag{21}$$

where $\Lambda_h = 0.5$ GeV [8,23], and ϕ_A is an undetermined strong phase. In addition, according to the relations given by Refs. [16,17], the moment parameter in Eq. (7) is

$$\langle\xi\rangle_{B2} = \frac{1}{2}\Big(\langle\xi\rangle_+ - \langle\xi\rangle_-\Big) = \frac{\bar{\Lambda}}{3\,m_b}, \tag{22}$$

with $\langle\xi\rangle_+ = 2\,\langle\xi\rangle_- = \frac{4}{3}\,\frac{\bar{\Lambda}}{m_b}$ and $\bar{\Lambda} = m_B - m_b \approx 0.55$ GeV [16]. Using the exponential type model for B meson DAs

$$\phi_{B_q}^+(\xi) = N^+\,\xi\,\exp\left(-\frac{\xi\, m_{B_q}}{\omega_{B_q}}\right), \tag{23}$$

$$\phi_{B_q}^-(\xi) = N^-\,\exp\left(-\frac{\xi\, m_{B_q}}{\omega_{B_q}}\right), \tag{24}$$

where $N^\pm$ is the normalization constant determined via $\int_0^1 \phi_{B_q}^\pm(\xi)d\xi = 1$, one can obtain $\langle\xi\rangle_{B2} = 0.042 \pm 0.01$ with the shape parameter $\omega_{B_s} = 0.45 \pm 0.10$ GeV for B_s meson [28], and $\langle\xi\rangle_{B2} = 0.039 \pm 0.01$ with $\omega_{B_d} = 0.42 \pm 0.10$ GeV for B_d meson [22], which are basically in agreement with the estimation of Eq. (22).

Using the commonly used notations in the QCDF approach [8,23–26], the amplitudes for the pure WA decays $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$ are written as

$$\mathcal{A}(B_s\to\pi^+\pi^-) = i\,\frac{G_F}{\sqrt{2}}\,f_{B_s}\,f_\pi^2\left\{V_{ub}^* V_{us}\left(b_1 + 2\,b_4 + \frac{1}{2}\,b_{4,\mathrm{EW}}\right) + V_{cb}^* V_{cs}\left(2\,b_4 + \frac{1}{2}\,b_{4,\mathrm{EW}}\right)\right\}, \tag{25}$$

$$\mathcal{A}(B_d\to K^+K^-) = i\,\frac{G_F}{\sqrt{2}}\,f_{B_d}\,f_K^2\left\{V_{ub}^* V_{ud}\left(b_1 + 2\,b_4 + \frac{1}{2}\,b_{4,\mathrm{EW}}\right) + V_{cb}^* V_{cd}\left(2\,b_4 + \frac{1}{2}\,b_{4,\mathrm{EW}}\right)\right\}, \tag{26}$$

where the Fermi weak coupling constant $G_F \simeq 1.166\times10^{-5}$ GeV^{-2} [1]; f_{B_q}, f_π and f_K are decay constants; V_{ij} ($i = u, c$ and $j = d, s, b$) is the Cabibbo–Kobayashi–Maskawa (CKM) matrix element. The definition of parameter b_i is

$$b_1 = \frac{C_F}{N_c^2}\,C_1\,A_1^i, \tag{27}$$

$$b_4 = \frac{C_F}{N_c^2}\left[C_4\,A_1^i + C_6\,A_2^i\right], \tag{28}$$

$$b_{4,\mathrm{EW}} = \frac{C_F}{N_c^2}\left[C_{10}\,A_1^i + C_8\,A_2^i\right], \tag{29}$$

where $C_F = 4/3$ is the color factor; $N_c = 3$ is the number of colors; C_i is the Wilson coefficient; A_k^i is the amplitude building block of Eq. (6).

To provide a quantitative estimate of the Φ_{B2} contributions, the inputs listed in Table 1 are used in our numerical calculation. Their central values will be regarded as the default inputs unless otherwise specified.

The constraints on annihilation parameters from data are illustrated in Fig. 3. It is clearly seen from Fig. 3a that it is impossible to accommodate simultaneously $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$ decays within 2σ errors with the same values of ρ_A and ϕ_A when the Φ_{B2} contributions are overlooked. Other studies of B decays, such as Refs. [23,29], have uncovered similar results. It seems not easy to clarify discrepancies between data and the QCDF results with the same set of parameters ρ_A and ϕ_A. To clam down this situation, the factorizable and nonfactorizable annihilation parameters corresponding to different topologies are introduced in Refs. [30,31]. However, more annihilation parameters make the method uneconomical and unsatisfactory. Interestingly, by including the Φ_{B2} contributions, Fig. 3b shows overlapping areas of annihilation parameters, which implies that the Φ_{B2} contributions are nontrivial for accommodating the tension between data and QCDF predictions for $\mathcal{B}(B_d\to K^+K^-)$ and $\mathcal{B}(B_s\to\pi^+\pi^-)$. In addition, if theoretical uncertainties from inputs are taken into account, the overlapping bands will be inevitably enlarged. The same annihi-

Table 1 The input parameters [1]

$m_b = 4.78 \pm 0.06$ GeV,	$\bar{m}_b(\bar{m}_b) = 4.18^{+0.04}_{-0.03}$ GeV,	$\bar{m}_s(2\,\text{GeV}) = 95 \pm 5$ MeV,
$\frac{\bar{m}_s(2\,\text{GeV})}{\bar{m}_{u,d}(2\,\text{GeV})} = 27.3 \pm 0.7$,	$f_\pi = 130.2 \pm 1.7$ MeV,	$f_K = 155.6 \pm 0.4$ MeV,
$m_{B_d} = 5279.63 \pm 0.15$ MeV,	$f_{B_d} = 187.1 \pm 4.2$ MeV,	$\tau_{B_d} = 1.520 \pm 0.004$ ps,
$m_{B_s} = 5366.89 \pm 0.19$ MeV,	$f_{B_s} = 227.2 \pm 3.4$ MeV,	$\tau_{B_s} = 1.509 \pm 0.004$ ps.

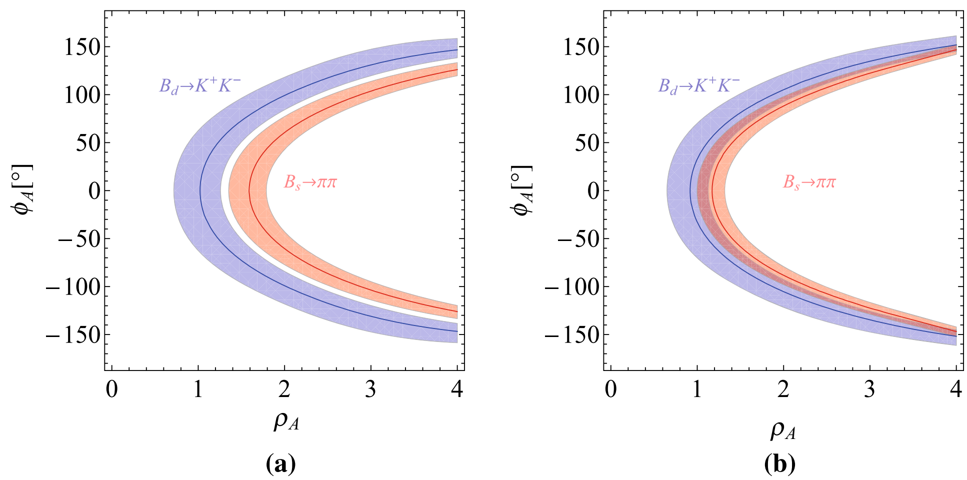

Fig. 3 The contour plots of branching ratios of $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$ decays as functions of the annihilation parameters ρ_A and ϕ_A without and with the Φ_{B2} contributions in **a** , **b**, respectively. The solid curves correspond to the central values of data and the bands correspond to the $2\,\sigma$ constraints

Table 2 The CP-averaged branching ratios in the unit of 10^{-7}, where the theoretical uncertainties are from the input parameters listed in Table 1. Different scenarios are explained in the text

Decay mode	Our results				Ref. [23]	Data
	S1		S2		S3	
	$A_k^{i,B2} = 0$	$A_k^{i,B2} \neq 0$	$A_k^{i,B2} = 0$	$A_k^{i,B2} \neq 0$	$A_k^{i,B2} = 0$	
$B_s \to \pi^+\pi^-$	$3.13^{+0.56}_{-0.43}$	$5.08^{+1.05}_{-0.86}$	$3.44^{+0.62}_{-0.47}$	$5.63^{+1.18}_{-0.97}$	1.49	6.7 ± 0.8
$B_d \to K^+K^-$	$0.85^{+0.17}_{-0.14}$	$1.01^{+0.20}_{-0.16}$	$0.78^{+0.15}_{-0.13}$	$0.91^{+0.18}_{-0.15}$	0.79	0.80 ± 0.15

lation parameters suitable for pure WA hadronic B decays might be obtained with the QCDF approach.

As is shown by Fig. 3b, strict limits on annihilation parameters ρ_A and ϕ_A can not be obtained only from experimental data on $\mathcal{B}(B_d \to K^+K^-)$ and $\mathcal{B}(B_s \to \pi^+\pi^-)$. In principle, considering more B decays, such as a global fit on nonleptonic B decays in Refs. [30,31], is helpful for extracting the informations of annihilation parameters. However, for many hadronic B decays, other contributions, such as spectator scattering interactions, will complicate the determination of annihilation parameters. How to get annihilation parameter spaces as compact as possible from data is beyond the scope of this paper.

It is seen from Fig. 3b that, in general, the value of ρ_A increase with the increasing value of $|\phi_A|$. A large value of parameter ρ_A will spoil the self-consistency and confidence level of the QCDF approach, and $\rho_A \leq 1$ is proposed in Refs. [8,23]. The strong phase ϕ_A describes the rescattering among hadrons and relates closely to CP violation of nonleptonic B decays. Focusing on the pure WA decays of $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$, to roughly estimate branching ratios, two scenarios based on Fig. 3b are considered in our numerical calculation. Scenario S1 is with parameters $\rho_A = 1$ and $\phi_A = 0°$, and scenario S2 is with $\rho_A = 1.2$ and $\phi_A = -40°$. Practically, for the scenario S1, it is intuitive that zero strong phase ϕ_A seems a little unnatural. Trying to combine the value of ρ_A as close to one as possible with a nonzero ϕ_A, the scenario S2 is considered. In addition, the scenario S2 is comparable with the scenario S3 of Ref. [23], where the "universal annihilation" parameters $\rho_A = 1$ and $\phi_A = -45°$ are used.

Using such inputs, we list the QCDF results for $\mathcal{B}(B_d \to K^+K^-)$ and $\mathcal{B}(B_s \to \pi^+\pi^-)$ with and without considering the Φ_{B2} contributions in Table 2, in which the theoretical predictions of scenario S3 of Ref. [23] and experimental data are also listed for convenience of comparison. In order to show the effects of Φ_{B2} much more clearly, we collect the numerical results of A_i^k in Table 3.

From Table 2, it can be found that: (i) The experimental data for both B_d and B_s decays can not be well explained simultaneously by QCDF approach without considering the Φ_{B2} contributions; (ii) The numerical difference between the

Table 3 The values of A_k^i in Eqs. (17)–(19). See text for explanations of different scenarios

Decay mode	Scenario	$A_k^{i,B2}=0$			$A_k^{i,B2}\neq 0$		
		A_1^i	A_2^i	A_3^i	A_1^i	A_2^i	A_3^i
$B_s \to \pi^+\pi^-$	S1	116	116	0	117	169	-20
	S2	$103-i\,64$	$103-i\,64$	0	$104-i\,67$	$139-i\,111$	$-15+i\,17$
$B_d \to K^+K^-$	S1	115	115	0	116	170	-21
	S2	$102-i\,64$	$102-i\,64$	0	$103-i\,67$	$140-i\,113$	$-15+i\,18$

case for $A_k^{i,B2}=0$ of scenario S2 and scenario S3 of Ref. [23] arises from different inputs, such as decay constants, the CKM parameters and so on, besides parameters ρ_A and ϕ_A. (iii) With the scenario S2, the Φ_{B2} contributions present about 60% and 20% corrections to $\mathcal{B}(B_d\to K^+K^-)$ and $\mathcal{B}(B_s\to\pi^+\pi^-)$, respectively, which significantly improve the QCDF predictions and can explain the data within uncertainty.

The results in Table 2 show that Φ_{B2} contributions to nonfactorizable WA amplitude building blocks A_k^i are small, due to the small moment $\langle\xi\rangle_{B2}$. In addition, according to the conventions of Refs. [8,23], building block A_3^i is always accompanied by the small value of Wilson coefficient C_5. Hence, on one hand, the dominant contributions to WA amplitudes come from Φ_{B1} part; on the other hand, to some certain extent, the Φ_{B2} contributions present un-negligible correction to the amplitude especially for the pure annihilation decay modes and can improve the performances of the QCDF approach.

In summary, the improvements of measurement precision with the running Belle-II and LHCb experiments call for the refinements of theoretical calculation on hadronic B weak decays. For the B mesons, there are two scalar DAs Φ_{B1} and Φ_{B2}. The Φ_{B2} contributions to formfactors and branching ratios can be significant for some cases with the pQCD approach. In this paper, we study the Φ_{B2} contributions with the QCDF approach, and find that for the $B \to PP$ decays, they can be safely neglected in the spectator scattering amplitudes, and contribute to only the nonfactorizable WA amplitudes. The Φ_{B2} contributions to WA amplitudes are small compared with the dominant Φ_{B1} contributions, due to the small moment $\langle\xi\rangle_{B2}$. However, the participation of Φ_{B2} plays a positive role in accommodating the pure WA decays $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$ to data with the universal annihilation parameters ρ_A and ϕ_A. The values of annihilation parameters ρ_A and ϕ_A with the QCDF approach have been under discussion for a long period. More information about WA parameters ρ_A and ϕ_A could be obtained by a comprehensive study on nonleptonic B decays.

Acknowledgements This work is supported by the National Natural Science Foundation of China (Grant Nos. 11875122, 11705047, U1632109 and 1191101296) and the Program for Innovative Research Team in University of Henan Province (Grant No. 19IRTSTHN018).

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This is a theoretical research work, no additional data are associated with this work.]

Funded by SCOAP[3].

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