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Inflationary perturbation spectra at next-to-leading slow-roll order in effective field theory of inflation

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Abstract The effective field theory (EFT) of inflation provides an essential picture to explore the effects of the unknown high energy physics in the single scalar field inflation models. For a generic EFT of inflation, possible high energy corrections to simple slow-roll inflation can modify both the propagating speed and dispersion relations of the cosmological scalar and tensor perturbations. With the arrival of the era of precision cosmology, it is expected that these high energy corrections become more important and have to be taken into account in the analysis with future precise observational data. In this paper we study the observational predictions of the EFT of inflation by using the third-order uniform asymptotic approximation method. We calculate explicitly the primordial power spectra, spectral indices, running of the spectral indices for both scalar and tensor perturbations, and the ratio between tensor and scalar spectra. These expressions are all written in terms of the Hubble flow parameters and the flow of four new slow-roll parameters and expanded up to the next-to-leading order in the slow-roll expansions so they represent the most accurate results obtained so far in the literature. The flow of the four new slow-roll parameters, which arise from the four new operators introduced in the action of the EFT of inflation, can affect the primordial perturbation spectra at the leading-order and the corresponding spectral indices at the next-to-leading order.

1 Introduction

Inflation is the leading paradigm for the hot Big Bang origin of the Universe [1–3] (see Ref. [4] for an updated review). The most remarkable features of the inflationary scenario is that it provides an elegant mechanism for generating structures in the Universe and the spectrum of cosmic microwave background (CMB) anisotropies, which are fully consistent with the cosmological observations with a spectacular precision [5–8]. In general, there are a lot of approaches to realize inflation that originate from very different background physics. The recent precise observational data set supports the key predictions of the standard single-field inflationary models. In this model, a single scalar degrees of freedom with a self-interacting potential produces a slow-roll phase during which the energy density of the matter field remains nearly constant and the space-time behaves like a quasi-de Sitter spacetime.

Instead of constructing inflation models with scalar degrees of freedom from different fundamental theories, the EFT provides a general framework for describing the most generic single scalar field theory and the associated fluctuations on a quasi de-Sitter background [9,10]. This framework is based on the decomposition of space and time with a preferred time foliation defined by the scalar field, which provides a clock that breaks time but preserves spatial diffeomorphism invariance. Similar idea has been used in the construction of the Horava-Lifshitz (HL) theory of quantum gravity [11–15], in which the symmetry of the theory is broken from the general covariance down to the foliation-preserving diffeomorphisms. With this symmetry, the action of the theory has to be constructed only in terms of three dimensional spatial diffeomorphism invariants. This allows us to characterize all the possible high energy corrections to simple slow-roll inflation and their impacts on the primordial perturbation spectra. Such considerations have attracted a lot of attention and the observational effects of high-order operators on inflationary perturbation spectra have been extensively studied both in the framework of the EFT of inflation and the inflation in HL theory [16–19] (see [20] for an updated review). With such thoughts, an extension of the EFT of inflation by

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adding high dimension operators has also been constructed and the corresponding primordial perturbation spectra have also been explored in refs. [21–23].

In the EFT of inflation, possible high energy corrections to simple slow-roll inflation can affect the propagations of both the cosmological scalar and tensor perturbations in two aspects. One is that it may lead to a time-dependent propagating sound speeds associated with the scalar and tensor perturbation modes, and another is that it could modify the conventional linear dispersion relation of the perturbation modes to nonlinear ones. Both the time-dependent propagating sound speeds and the modified dispersion relation can make important corrections in the primordial scalar and tensor perturbation spectra. In most of previous works, however, in order to estimate the spectra, the time-dependent speed has been assumed to be constant and the operators which produces nonlinear dispersion relations are ignored [9,24–26]. One of the main reasons for such treatment is that the considerations of the time variation of the sound speed and the nonlinear dispersion relations make it very difficult to calculate the corresponding power spectra and spectral indices.

However, to match with the accuracy of the current and forthcoming observations, as pointed out in [27–30], consideration of the contributions from the time variation of the sound speed and the modified dispersion relation are highly demanded. To achieve this goal, one has to calculate the primordial perturbation spectra up to the second-order in the slow-roll expansions. Recently, we have developed a powerful method, the uniform asymptotic approximation method [31–33], to calculate precisely the primordial perturbation spectra for a lot of inflation models. The robustness of this method has been verified for calculating primordial spectra in k-inflation [28,34,35], and inflation with nonlinear dispersion relations [32,33,36], inflation in loop quantum cosmology [37–39], and inflation with Gauss-Bonnet corrections [27] (For an alternative approach by using Green's function method, see [40,41] and its recent application to the scalar-tensor theories in different frames [42].). We note here that this method was first applied to inflationary cosmology in the framework of general relativity (GR) in [43–45], and then we have developed it, so it can be applied to more general cases and high accuracy [32,36]. The accurate results of some inflationary models derived from this method have also been used to the study of the adiabatic regularisation of the primordial power spectra [46,47]. It is worth noting that this approximation has also been applied to study the parametric resonance during reheating [48] and the quantization conditions in quantum mechanics [49]. The main purpose of the present paper is to use this powerful method to derive the inflationary observables at the second-order in slow-roll inflation in the framework of the EFT of inflation. With the general expressions of power spectra and spectral indices obtained in [33,34] in the uniform asymptotic approximation, we calculate explicitly these quantities for both scalar and tensor perturbations of EFT of inflation. Then, ratio between the tensor and scalar spectra is also calculated up to the second-order in the slow-roll expansions. These observables represent a significant improvement over the previous results obtained so far in the literature.

We organize the rest of the paper as follows. In Sect. 2, we present a brief review of the EFT of inflation and the corresponding equations of motion for both cosmological scalar and tensor perturbations. In Sect. 3, we give the most general formulas of the primordial perturbation spectra in the uniform asymptotic approximation. Then, in Sect. 3, with these general expressions we calculate explicitly the power spectra, spectral indices, and running of the spectral indices of both scalar and tensor perturbations of the slow-roll inflation in the EFT of inflation. Our main conclusions and discussions are presented in Sect. 4.

2 The effective field theory of inflation

In this section, we present a brief introduction of the EFT of inflation [9,10]. In general, the EFT provides a framework for describing the most generic single scalar field theory on a quasi de-Sitter background. With this framework, it is shown that the action of the EFT of inflation around a flat Friedmann-Lemaître-Robertson-Walker (FLRW) background reads

$$\begin{aligned} S_{\rm eft} = M_{\rm Pl}^2 \int d^4x\sqrt{-g}\Big\{ &\frac{R}{2} + \dot{H} g^{00} - (3H^2+\dot{H}) \\ &+\frac{M_2^4}{2M_{\rm Pl}^2}(g^{00}+1)^2 - \frac{\bar{M}_1^3}{2M_{\rm Pl}^2}(g^{00}+1)\delta K_\mu^\mu \\ &-\frac{\bar{M}_2^2}{2M_{\rm Pl}^2}(\delta K_\mu^\mu)^2 - \frac{\bar{M}_3^2}{2M_{\rm Pl}^2}\delta K_\nu^\mu \delta K_\mu^\nu \Big\}, \end{aligned} \tag{2.1}$$

where

$$K_{\mu\nu} = (g_{\mu\sigma} + n_\mu n_\sigma)\nabla^\sigma n_\nu$$

denotes the extrinsic curvature at constant time hypersurfaces with n_ν being the unit normal vector. $\delta K_{\mu\nu}$ denotes the perturbation of $K_{\mu\nu}$ about the flat FLRW background. For the calculation of the power spectra for both the scalar and tensor perturbation later, we only consider operators up to second order fluctuation in metric, which is sufficient to extract the information of two-point function from scalar and tensor modes. Therefore, we truncate the EFT action to (2.1) in the above by cutting terms, for example, with $n > 2$ in $\hat{\mathcal{O}}_{(i)} = \sum_{n=2}^{\infty} \frac{M_n^4(t)}{n!}(1+g^{00})^n$.

We would like to mention that, in writing the above action, one requires the perturbation in inflaton field ϕ to be zero, i.e., $\delta\phi = 0$. This is achieved by considering the following linear transformations under time diffeomorphism,

$$\tilde{t} = t + \xi^0(t, x^i), \quad \delta\tilde{\phi} = \delta\phi + \xi^0(t, x^i)\dot{\phi}_0(t), \tag{2.2}$$

which leads to a particular gauge (unitary gauge) with $\xi^0(t, x^i) = -\delta\phi/\dot{\phi}_0$ where there is no inflaton perturbation. Obviously the action (2.1) that respects the unitary gauge has no time diffeomorphism invariance. In order to restore the time diffeomorphism invariance, one can introduce a Goldstone mode $\pi(t, x^i)$ and require it transforms as $\pi(t, x^i) \to \pi(t, x^i) - \xi^0(t, x^i)$. By introducing the Goldstone mode, the perturbation of inflaton field is not required to be zero and it relates to $\pi(t, x^i)$ via $\delta\phi = \dot{\phi}_0\pi$.

It is worth noting that the standard single scalar slow-roll inflation can be simply recovered by setting $M_2,\ \bar{M}_1,\ \bar{M}_2,\ \bar{M}_3$ to zero. As mentioned in [9], the terms with $M_2,\ \bar{M}_1,\ \bar{M}_2,\ \bar{M}_3$ encodes the possible effects of high energy physics on the simple slow-roll model of inflation under the current accurate observation. As we will see later, these new terms can affect both the propagating speed and dispersion relations of the cosmological scalar and tensor perturbations. Therefore, studying their effects on the primordial perturbation spectra can provide a way to probe the possible high energy effects in the observational data.

2.1 Scalar perturbations

As mentioned above, the Goldstone mode $\pi(t, x^i)$ can be introduced to restore the time diffeomorphism invariance of action and it also describes the scalar perturbations around the flat FLRW background. Thus, in order to study the scalar perturbations, one can transform the action in (2.1) in unitary gauge to π-gauge by evaluating the action explicitly for π. Proceeding this procedure [9], one obtains

$$S^{\pi}_{\rm eft} = \int d^4x a^3\Big[A_0(\dot{\pi}^2 + 3H^2\epsilon_1\pi^2) + A_1\frac{(\partial_i\pi)^2}{a^2} + A_2\frac{(\partial^2\pi)^2}{a^4}\Big], \tag{2.3}$$

where

$$A_0 = -M_{\rm Pl}^2\dot{H} + 2M_2^4 - 3\bar{M}_1^3 H - \frac{9}{2}H^2\bar{M}_2^2 - \frac{3}{2}H^2\bar{M}_3^2, \tag{2.4}$$

$$A_1 = M_{\rm Pl}^2\dot{H} + \frac{1}{2}\bar{M}_1^3 H + \frac{3}{2}H^2\bar{M}_2^2 + \frac{1}{2}\bar{M}_3^2 H^2, \tag{2.5}$$

$$A_2 = -\frac{1}{2}\left(\bar{M}_2^2 + \bar{M}_3^2\right). \tag{2.6}$$

Here two remarks about the above action are in order. First, the above action for scalar perturbations is valid in the weak decoupling limit. In this way, we ignore most of the terms arising from mixing between gravity and Goldstone Boson, except the leading-order mixing term $M_{\rm Pl}^2\dot{H}\dot{\pi}\delta g^{00}$. This term gives rise to a mass term $3a^3H^2\epsilon_1\pi^2$ in the action for the scalar perturbations. Second, in order to match the precision of the current and future observations, both the scalar spectral index n_s and tensor-to-scalar ratio r are required be accurate up to the second-order in the slow-roll expansion. In this way, in order to simplify our calculations, we only interest in terms in the action which contribute to n_s and r at the first two orders in the slow-roll expansion. Other terms which lead to contributions beyond this order are all neglected in the above action.

Changing the variable π to $u = z\pi$ we cast the action into the form

$$S^{\pi}_{\rm eft} = \int d\eta d^3x\left[u'^2 + \left(\frac{z''}{z} + 3a^2H^2\epsilon_1\right)u^2 + \frac{A_1}{A_0}(\partial_i u)^2 + \frac{A_2}{A_0}\frac{(\partial^2 u)^2}{a^2}\right], \tag{2.7}$$

where a prime denotes the derivative with respect to the conformal time η and

$$z = a\sqrt{A_0}. \tag{2.8}$$

Then variation of the action with respect to the Fourier modes $u_k(\eta)$ of u leads to the equation of motion,

$$u_k'' + \left(\omega_k^2(\eta) - \frac{z''}{z} - 3a^2H^2\epsilon_1\right)u_k = 0, \tag{2.9}$$

where

$$c_s^2 \equiv -\frac{A_1}{A_0}, \tag{2.10}$$

and

$$\omega_k^2(\eta) = c_s^2k^2\left(1 - \frac{A_2}{A_0c_s^2}\frac{k^2}{a^2}\right). \tag{2.11}$$

It is easy to see that in the EFT of inflation, both the effective sound speed and the nonlinear terms in the dispersion relation receive contributions from the operators with M_2, $\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$. If one ignores $\bar{M}_2$ and $\bar{M}_3$ terms, the nonlinear dispersion relation reduces to the usual linear one. In Refs. [9,24–26], the nonlinear term (A_2 term in (2.11)) has been dropped and the time-dependent sound speed c_s has been assumed to be constant during inflation. In this paper, we are going to consider effects of both the nonlinear term and the time variation of the sound speed in the primordial power spectra.

2.2 Tensor perturbations

For tensor perturbations, the perturbed spacetime is set as

$$g_{ij} = a^2(\delta_{ij} + h_{ij}), \tag{2.12}$$

where h_{ij} represents the transverse and traceless tensor perturbations, which satisfies

$$h^i_i = 0 = \partial^i h_{ij}. \tag{2.13}$$

Then expanding the total action $S_{\rm eft}$ up to the second-order gives

$$S^2_h = \frac{M^2_{\rm Pl}}{8}\int dt d^3x a^3 \left(c_t^{-2}\partial_t h_{ij}\partial_t h^{ij} - a^{-2}\partial_k h_{ij}\partial^k h^{ij}\right), \tag{2.14}$$

where the effective sound speed c_t for tensor perturbations is given by

$$c_t^2 = \left(1 - \frac{\bar{M}_3^2}{M^2_{\rm Pl}}\right)^{-1}. \tag{2.15}$$

In order to avoid superluminal propagation for tensor modes, one has to require

$$\bar{M}_3^2 < 0. \tag{2.16}$$

Variation of the action with respect to h_{ij} leads to the equation of motion

$$\frac{d^2\mu_k^{(t)}(\eta)}{d\eta^2} + \left(c_t^2 k^2 - \frac{a''}{a}\right)\mu_k^{(t)}(\eta) = 0, \tag{2.17}$$

where $d\eta = dt/a$ is the conformal time and $\mu_k^{(t)}(\eta) = aM_{\rm Pl}h_k/\sqrt{2}$ with h_k denoting the Fourier modes of the two helicities of the tensor perturbations.

It is obvious to observe that the new operators introduced in the action (2.1) of EFT of inflation can affect sound speed of the tensor modes. As pointed out in [50,51], this effective sound speed can be redefined to unity by a disformal transformation. Under this disformal transformation, the new effects in the sound speed shift to the effective time-dependent mass term $a''/a \to \tilde{a}''/\tilde{a}$ in (2.17) such that the primordial tensor spectrum keeps invariant. With this property, one can either do the calculation with $c_t \neq 1$ or with $c_t = 1$ by using the disformal transformation. In this paper, we adopt the former case.

2.3 Slow-roll parameters

In order to consider the slow-roll inflation, we first need to impose the following slow-roll conditions,

$$\left|\frac{\dot{H}}{H^2}\right|, \quad \left|\frac{\ddot{H}}{H\dot{H}}\right| \ll 1. \tag{2.18}$$

With these conditions, it is convenient to introduce a set of the slow-roll parameters, the Hubble flow parameters, which are defined by

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2}, \quad \epsilon_{n+1} \equiv \frac{d\ln\epsilon_n}{d\ln a}. \tag{2.19}$$

In the EFT of inflation, in general one also requires all the coefficients $(M_2^4, \bar{M}_1^3, \bar{M}_2^2, \bar{M}_3^2)$ of the new operators in the action (2.1) satisfy the approximation conditions:

$$\left|\frac{M_2^4}{M^2_{\rm Pl}H^2\epsilon_1}\right|, \left|\frac{\bar{M}_1^3}{M^2_{\rm Pl}H\epsilon_1}\right|, \left|\frac{\bar{M}_2^2}{M^2_{\rm Pl}\epsilon_1}\right|, \left|\frac{\bar{M}_3^2}{M^2_{\rm Pl}\epsilon_1}\right| \ll 1. \tag{2.20}$$

In the slow-roll approximation, we also need all the above quantities are slow-varying. With this assumption, we can introduce four new sets of slow-roll parameters, which are defined by

$$\delta_1 \equiv \frac{M_2^4}{M^2_{\rm Pl}H^2\epsilon_1}, \quad \delta_{n+1} = \frac{d\ln\delta_n}{d\ln a}, \tag{2.21}$$

$$\sigma_1 \equiv \frac{\bar{M}_1^3}{M^2_{\rm Pl}H\epsilon_1}, \quad \sigma_{n+1} = \frac{d\ln\sigma_n}{d\ln a}, \tag{2.22}$$

$$\zeta_1 \equiv \frac{\bar{M}_2^2}{M^2_{\rm Pl}\epsilon_1}, \quad \zeta_{n+1} = \frac{d\ln\zeta_n}{d\ln a}, \tag{2.23}$$

$$\kappa_1 \equiv \frac{\bar{M}_3^2}{M^2_{\rm Pl}\epsilon_1}, \quad \kappa_{n+1} = \frac{d\ln\kappa_n}{d\ln a}. \tag{2.24}$$

Then using these slow-roll parameters, up to the second-order, the effective sound speed for scalar perturbations and the modified dispersion relation $\omega_k^2(\eta)$ can be expressed as

$$\begin{aligned} c_s^2 &= -\frac{3\zeta_1 + \kappa_1 + \sigma_1 - 2}{4\delta_1 - 9\zeta_1 - 3\kappa_1 - 6\sigma_1 + 2} \\ &\simeq 1 + \frac{1}{4}(-4\delta_1 + 6\zeta_1 + 2\kappa_1 + 5\sigma_1) \\ &\quad -6\delta_1\zeta_1 - 2\delta_1\kappa_1 - \frac{17\delta_1\sigma_1}{4} + \frac{3\delta_1^2}{2} \\ &\quad + \frac{15\zeta_1\kappa_1}{4} + \frac{33\zeta_1\sigma_1}{4} + \frac{45\zeta_1^2}{8} \\ &\quad + \frac{11\kappa_1\sigma_1}{4} + \frac{5\kappa_1^2}{8} + \frac{95\sigma_1^2}{32}, \end{aligned} \tag{2.25}$$

$$\omega_k^2(\eta) \simeq c_s^2 k^2 + b(\eta)k^4\eta^2, \tag{2.26}$$

where $b(\eta)$ is a small and slow-varying quantity which reads

$$\begin{aligned} b(\eta) \simeq{}& \frac{\zeta_1 + \kappa_1}{2} - \zeta_1\epsilon_1 - \kappa_1\epsilon_1 - \delta_1\zeta_1 - \delta_1\kappa_1 \\ &+ \frac{3\zeta_1\kappa_1}{2} + \frac{3\zeta_1\sigma_1}{2} + \frac{9\zeta_1^2}{4} + \frac{3\kappa_1\sigma_1}{2} - \frac{3\kappa_1^2}{4}. \end{aligned} \tag{2.27}$$

3 Scalar and tensor perturbation spectra in the uniform asymptotic approximation

3.1 General formulas of primordial spectra in the uniform asymptotic approximation

In this subsection, we present a very brief introduction of the general formulas of primordial perturbations with a slow-varying sound speed and parameter $b(\eta)$.

In the uniform asymptotic approximation, we first write Eqs. (2.9) and (2.17) in the standard form [32,52]

$$\frac{d^2\mu(y)}{dy^2} = \{\lambda^2\hat{g}(y) + q(y)\}\mu(y), \tag{3.1}$$

where we introduce a new variable $y = -k\eta$, $\mu(y) = \mu_{\mathcal{R}}(y)$ and $\mu_h(y)$ corresponding to scalar and tensor perturbations respectively, and

$$\lambda^2\hat{g}(y) + q(y) = \frac{\nu^2(\eta) - 1/4}{y^2} - c^2(\eta) - b^2(\eta)y^2. \tag{3.2}$$

Here $c(\eta) = c_{s,t}(\eta)$ is the effective sound speed for scalar and tensor perturbation modes respectively. For scalar perturbation, we have [24,26]

$$\nu_s^2(\eta) = \eta^2\frac{z''(\eta)}{z(\eta)} + \frac{1}{4} + 3a^2H^2\epsilon_1, \tag{3.3}$$

and for tensor modes,

$$\nu_t^2(\eta) = \eta^2\frac{a''(\eta)}{a(\eta)} + \frac{1}{4} \quad \text{and} \quad b(\eta) = 0. \tag{3.4}$$

Note that in the above equation, λ is supposed to be a large parameter and used to label the different orders of the approximation. As we will see later in the general formulas of power spectrum in Eq. (3.8), λ with different power in the terms in square bracket denote different approximate orders in the uniform asymptotic approximation. In the finial calculation we can set $\lambda = 1$ for simplification. Now in order to construct the approximate solutions of the above equation by using the uniform asymptotic approximation, one needs to choose [32]

$$q(y) = -\frac{1}{4y^2}, \tag{3.5}$$

to ensure the convergence of the errors of the approximate solutions. Then, we have

$$\lambda^2\hat{g}(y) = \frac{\nu^2(\eta)}{y^2} - c^2(\eta) - b(\eta)y^2. \tag{3.6}$$

In order to get a healthy UV limit, for scalar perturbation we have to impose the condition $b(\eta) > 0$.

Considering that $b(\eta)$ is a small quantity, it is obvious to see that the function $\lambda^2\hat{g}(y)$ has a single turning point, which can be expressed as

$$y_0^2(\bar{\eta}_0) = \frac{-c_0^2(\bar{\eta}_0) + \sqrt{\bar{c}_0^4(\eta_0) + 4b(\bar{\eta}_0)\bar{\nu}_0^2(\bar{\eta}_0)}}{2b(\bar{\eta}_0)}. \tag{3.7}$$

Then following [34], the general formula of the power spectrum reads

$$\begin{aligned}\Delta^2(k) &\equiv \frac{k^3H^2}{4\pi^2}\left|\frac{u(y)}{z(\eta)}\right|^2_{y\to 0^+} \\ &= \frac{k^2}{8\pi^2}\frac{-k\eta}{z^2(\eta)\nu(\eta)}\exp\left(2\lambda\int_y^{\bar{y}_0}\sqrt{\hat{g}(y')}dy'\right) \\ &\quad\times\left[1 + \frac{\mathscr{H}(+\infty)}{\lambda} + \frac{\mathscr{H}^2(+\infty)}{2\lambda^2} + \mathcal{O}\left(\frac{1}{\lambda^3}\right)\right].\end{aligned} \tag{3.8}$$

We would like to mention that, in the square bracket of the above equation, the parameter λ with different powers denote different approximate orders in the uniform asymptotic approximation, which shows clearly the above formula is at the third-order approximation. In this formulas, the integral of $\sqrt{g}$ is given by Eqs. (A.8) with I_0 and I_1 being given by Eqs. (A.11) and (A.13) and the error control function is given by (A.15) with $\mathscr{H}_0$ and $\mathscr{H}_1$ being given by Eqs. (A.16) and (A.17).

Now we turn to consider the corresponding spectral indices. In order to do this, we first specify the k-dependence of $\bar{\nu}_0(\eta_0)$, $\bar{\nu}_1(\eta_0)$ through $\eta_0 = \eta_0(k)$. From the relation $-k\eta_0 = \bar{y}_0$, we have

$$\frac{d\ln(-\eta_0)}{d\ln k} = -1 + \frac{d\ln\bar{y}_0}{d\ln(-\eta_0)}\frac{d\ln(-\eta_0)}{d\ln k}, \tag{3.9}$$

which leads to

$$\frac{d\ln(-\eta_0)}{d\ln k} \simeq -1 - \frac{d\ln\bar{y}_0}{d\ln(-\eta_0)}. \tag{3.10}$$

Then using this relation, the spectral index is given by

$$\begin{aligned}n_s - 1,\ n_t &\equiv \frac{d\ln\Delta^2_{s,t}}{d\ln k} \\ &\simeq 3 - 2\bar{\nu}_0 - \frac{2\bar{b}_1\bar{\nu}_0}{3\bar{c}_0^4} + \frac{2\bar{c}_1\bar{\nu}_0}{\bar{c}_0} + \frac{2\bar{b}_1\bar{\nu}_0^3}{3\bar{c}_0^4} \\ &\quad -2\bar{\nu}_1\ln 2 + \frac{\bar{\nu}_1}{6\bar{\nu}_0^2}.\end{aligned} \tag{3.11}$$

Similarly, we find that the running of the spectral index $\alpha \equiv dn/d\ln k$ is given by

$$\begin{aligned}\alpha_{s,\,t}(k) &\simeq 2\bar{\nu}_1 + \frac{2\bar{b}_2\bar{\nu}_0}{3\bar{c}_0^4} - \frac{2\bar{c}_2\bar{\nu}_0}{\bar{c}_0} - \frac{2\bar{b}_2\bar{\nu}_0^3}{3\bar{c}_0^4} \\ &\quad +2\bar{\nu}_2\ln 2 - \frac{\bar{\nu}_2}{6\bar{\nu}_0^2}.\end{aligned} \tag{3.12}$$

In the above, we present all the formulas (Eqs. (3.8), (3.11), and (3.12)) that can be directly used to calculate the primordial perturbation spectra from different inflation models. Note that these formulas are easy to use because they only depend on the quantities $H(\eta)$, (c_0, c_1, c_2), (ν_0, ν_1, ν_2) evaluated at the turning point. These quantities can be easily calculated from Eqs. (2.25, 3.3) for scalar perturbations and Eqs. (2.15, 3.4) for tensor perturbations. In the following subsections, we apply these formulas to calculate the slow-roll power spectra for both scalar and tensor perturbations in the EFT of inflation.

3.2 Scalar spectrum

We first consider the scalar perturbations. As we already pointed out in the introduction, in order to match the accuracy of forthcoming observations, we need to calculate the spectral indices up to the next-to-leading order (second-order) in the expansions of the slow-roll approximation. For this purpose, we only need to consider the quantities $(\bar{c}_s,\ c_{\mathcal{R}1})$ and $(\nu_{\mathcal{R}0},\ \nu_{\mathcal{R}1},\ \nu_{\mathcal{R}2})$ up to the second-order in the slow-roll expansions.

For the slow-varying sound speed $c_{\mathcal{R}}$, from Eq. (2.25) we find

$$\begin{aligned}\bar{c}_{\mathcal{R}0} \simeq\ & 1-\bar{\delta}_1+\frac{3\bar{\zeta}_1}{2}+\frac{\bar{\kappa}_1}{2}+\frac{5\bar{\sigma}_1}{4}-6\bar{\delta}_1\bar{\zeta}_1-2\bar{\delta}_1\bar{\kappa}_1\\ &-\frac{17\bar{\delta}_1\bar{\sigma}_1}{4}+\frac{3\bar{\delta}_1^2}{2}+\frac{15\bar{\zeta}_1\bar{\kappa}_1}{4}+\frac{33\bar{\zeta}_1\bar{\sigma}_1}{4}\\ &+\frac{45\bar{\zeta}_1^2}{8}+\frac{11\bar{\kappa}_1\bar{\sigma}_1}{4}+\frac{5\bar{\kappa}_1^2}{8}+\frac{95\bar{\sigma}_1^2}{32}. \end{aligned} \tag{3.13}$$

For $\nu_{\mathcal{R}}$, from Eqs. (A.2) and (3.3) we find

$$\begin{aligned}\bar{\nu}_{\mathcal{R}0} \simeq\ & \frac{3}{2}+\bar{\epsilon}_1+\frac{\bar{\epsilon}_2}{2}+\bar{\epsilon}_1^2+\frac{11\bar{\epsilon}_1\bar{\epsilon}_2}{6}+\frac{\bar{\epsilon}_2\bar{\epsilon}_3}{6}\\ &+\bar{\delta}_1\bar{\delta}_2-\frac{9\bar{\zeta}_1\bar{\zeta}_2}{4}-\frac{3\bar{\kappa}_1\bar{\kappa}_2}{4}-\frac{3\bar{\sigma}_1\bar{\sigma}_2}{2}, \end{aligned} \tag{3.14}$$

$$\bar{\nu}_{\mathcal{R}1} \simeq -\bar{\epsilon}_1\bar{\epsilon}_2-\frac{\bar{\epsilon}_2\bar{\epsilon}_3}{2}, \tag{3.15}$$

and

$$\bar{\nu}_{\mathcal{R}2} \equiv \frac{d^2\nu_{\mathcal{R}}}{d\ln^2(-\eta)} = \mathcal{O}(\bar{\epsilon}_i^3). \tag{3.16}$$

Then, using the above expansions, the power spectrum for the curvature perturbation $\mathcal{R}$ can be calculated via Eq. (3.8). After tedious calculations we obtain,

$$\begin{aligned}\Delta^2_{\mathcal{R}}(k) = \bar{A}_s\Bigg[& 1-\left(2-2\bar{D}_p\right)\bar{\epsilon}_1-\bar{D}_p\bar{\epsilon}_2+\bar{\delta}_1+\left(\bar{D}_a-\frac{9}{8}\right)\bar{\zeta}_1\\ &-\frac{3\bar{\sigma}_1}{4}+\left(\bar{D}_a-\frac{9}{8}\right)\bar{\kappa}_1\\ &+\left(\bar{D}_p^2-\bar{D}_p+\bar{\Delta}_1+2\bar{\Delta}_2+\frac{7\pi^2}{12}-8\right)\bar{\epsilon}_1\bar{\epsilon}_2\\ &+\left(\frac{\bar{D}_p^2}{2}+\frac{\bar{\Delta}_1}{4}+\frac{\pi^2}{8}-\frac{3}{2}\right)\bar{\epsilon}_2^2\\ &+\left(-\frac{\bar{D}_p^2}{2}+\bar{\Delta}_2+\frac{\pi^2}{24}\right)\bar{\epsilon}_2\bar{\epsilon}_3\\ &+\left(2\bar{D}_p^2+2\bar{D}_p+\bar{\Delta}_1+\frac{\pi^2}{2}-5\right)\bar{\epsilon}_1^2+\frac{9\bar{\delta}_1\bar{\sigma}_1}{4}\\ &+\left(-\frac{3\bar{D}_p}{4}-\frac{5}{2}\right)\bar{\sigma}_1\bar{\sigma}_2\\ &+\left(-2\bar{D}_a\bar{D}_p-\frac{9031\bar{D}_a}{2715}+\frac{9\bar{D}_p}{4}+\frac{9}{4}\right)\bar{\zeta}_1\bar{\epsilon}_1\\ &-\frac{\bar{\delta}_1^2}{2}+\left(-2\bar{D}_p-2\right)\bar{\delta}_1\bar{\epsilon}_1+\left(3\bar{D}_a+\frac{9}{8}\right)\bar{\delta}_1\bar{\zeta}_1\\ &+\left(\bar{D}_a\bar{D}_p-\frac{449\bar{D}_a}{724}-\frac{9\bar{D}_p}{8}-\frac{4405}{2896}\right)\bar{\kappa}_1\bar{\kappa}_2\\ &+\frac{3}{4}\bar{D}_p\bar{\sigma}_1\bar{\epsilon}_2-\frac{57\bar{\sigma}_1^2}{32}+\left(\bar{D}_p+2\right)\bar{\delta}_1\bar{\delta}_2-\bar{D}_p\bar{\delta}_1\bar{\epsilon}_2\\ &+\left(3\bar{D}_a-\frac{15}{8}\right)\bar{\delta}_1\bar{\kappa}_1\\ &+\left(-\bar{D}_a\bar{D}_p+\frac{1829\bar{D}_a}{5430}+\frac{9\bar{D}_p}{8}-\frac{9}{8}\right)\bar{\zeta}_1\bar{\epsilon}_2\\ &+\left(-2\bar{D}_a\bar{D}_p-\frac{9031\bar{D}_a}{2715}+\frac{9\bar{D}_p}{4}+\frac{9}{4}\right)\bar{\kappa}_1\bar{\epsilon}_1\\ &+\left(\frac{39}{32}-\frac{11\bar{D}_a}{4}\right)\bar{\kappa}_1\bar{\sigma}_1\\ &+\left(-\bar{D}_a\bar{D}_p+\frac{1829\bar{D}_a}{5430}+\frac{9\bar{D}_p}{8}-\frac{9}{8}\right)\bar{\kappa}_1\bar{\epsilon}_2\\ &+\left(-\frac{11\bar{D}_a}{4}-\frac{81}{32}\right)\bar{\zeta}_1\bar{\sigma}_1+\left(\frac{297}{640}-\frac{69\bar{D}_a}{10}\right)\bar{\zeta}_1^2\\ &+\left(\frac{3\bar{D}_p}{2}+\frac{3}{2}\right)\bar{\sigma}_1\bar{\epsilon}_1\\ &-\left(\frac{2457}{320}-\frac{79\bar{D}_a}{5}\right)\bar{\zeta}_1\bar{\kappa}_1+\left(\frac{3657}{640}-\frac{89\bar{D}_a}{10}\right)\bar{\kappa}_1^2\\ &+\left(\bar{D}_a\bar{D}_p-\frac{449\bar{D}_a}{724}-\frac{9\bar{D}_p}{8}-\frac{10197}{2896}\right)\bar{\zeta}_1\bar{\zeta}_2\Bigg], \end{aligned} \tag{3.17}$$

where $\bar{A}_s \equiv \frac{181\bar{H}^2}{72e^3\pi^2\bar{\epsilon}_1 M_{\rm pl}}$, $\bar{D}_p \equiv \frac{67}{181}-\ln 2$, $\bar{D}_a \equiv \frac{90}{181}$, $\bar{\Delta}_1 \equiv \frac{183606}{32761}-\frac{\pi^2}{2}$, and $\bar{\Delta}_2 \equiv \frac{9269}{589698}$. Note that a letter with an over bar denotes quantity evaluated at the turning point $\bar{y}_0$. From the scalar spectrum (3.17), we observe that the effects of the four new operators affect the spectrum at the leading-order in the slow-roll expansion. We would like to mention that when we wrote down the action (2.3), we have neglected all the terms those do not contribute to the scalar power spectrum at the leading-order. But they may lead to contributions in the scalar spectrum at the second-order which are not included in the above expression. This is because their contributions only affect the spectral index n_s and tensor-to-scalar ratio r at the third-order and our intension here would be to obtain results which are accurate up to the second-order in the spectral index n_s and r for matching the accuracy of future observational data.

Then with the scalar power spectrum given above, the scalar spectral index is,

$$n_s - 1 \simeq -2\bar{\epsilon}_1 - \bar{\epsilon}_2 - 2\bar{\epsilon}_1^2 + \bar{\delta}_1\bar{\delta}_2 - \left(2\bar{D}_n + 3\right)\bar{\epsilon}_1\bar{\epsilon}_2 - \bar{D}_n\bar{\epsilon}_2\bar{\epsilon}_3 - \frac{5\bar{\zeta}_1\bar{\zeta}_2}{8} - \frac{5\bar{\kappa}_1\bar{\kappa}_2}{8} - \frac{3\bar{\sigma}_1\bar{\sigma}_2}{4}, \tag{3.18}$$

and the running of the scalar spectral index is expressed as

$$\begin{aligned}\alpha_s \simeq& -2\bar{\epsilon}_1\bar{\epsilon}_2 - \bar{\epsilon}_2\bar{\epsilon}_3 - 6\bar{\epsilon}_1^2\bar{\epsilon}_2 - \left(2\bar{D}_n + 3\right)\bar{\epsilon}_1\bar{\epsilon}_2^2 - \bar{D}_n\bar{\epsilon}_2\bar{\epsilon}_3^2 \\ &- \left(2\bar{D}_n + 4\right)\bar{\epsilon}_1\bar{\epsilon}_2\bar{\epsilon}_3 - \bar{D}_n\bar{\epsilon}_2\bar{\epsilon}_3\bar{\epsilon}_4 + \bar{\delta}_1\bar{\delta}_2^2 + \bar{\delta}_1\bar{\delta}_3\bar{\delta}_2 \\ &- \frac{5}{8}\left(\bar{\zeta}_1\bar{\zeta}_2^2 + \bar{\zeta}_1\bar{\zeta}_2\bar{\zeta}_3 + \bar{\kappa}_1\bar{\kappa}_2^2 + \bar{\kappa}_1\bar{\kappa}_2\bar{\kappa}_3\right) \\ &- \frac{3}{4}\left(\bar{\sigma}_1\bar{\sigma}_2^2 + \bar{\sigma}_1\bar{\sigma}_2\bar{\sigma}_3\right).\end{aligned} \tag{3.19}$$

For scalar spectral index, the new effects denoted by the four new sets of the slow-roll parameters appear at the next-to-leading order, while for the running of the index, they only contribute to the third-order of the slow-roll approximation.

3.3 Tensor spectrum

Now we consider the tensor spectrum. First we need to derive the expressions of $\nu_{t0}, \nu_{t1}, \nu_{t2}$, and c_{t0}, c_{t1}, c_{t2}. Repeating similar calculations for scalar perturbations, we obtain

$$\bar{\nu}_{t0} = \frac{3}{2} + \bar{\epsilon}_1 + \bar{\epsilon}_1^2 + \frac{4\bar{\epsilon}_1\bar{\epsilon}_2}{3} + \mathcal{O}(\bar{\epsilon}^3), \tag{3.20}$$

$$\bar{\nu}_{t1} \equiv \frac{d\nu_t}{d\ln(-\eta)} = -\bar{\epsilon}_1\bar{\epsilon}_2 + \mathcal{O}(\bar{\epsilon}^3), \tag{3.21}$$

$$\bar{\nu}_{t2} \equiv \frac{d^2\nu_t}{d\ln^2(-\eta)} = \mathcal{O}(\bar{\epsilon}^3), \tag{3.22}$$

and

$$\bar{c}_{t0} = 1 + \frac{\bar{\kappa}_1\bar{\epsilon}_1}{2} + \mathcal{O}(\bar{\epsilon}_i^3), \tag{3.23}$$

$$\bar{c}_{t1} \equiv \frac{dc_t}{d\ln(-\eta)} = \mathcal{O}(\bar{\epsilon}_i^3). \tag{3.24}$$

Then, the power spectrum for the tensor perturbation h_k reads

$$\begin{aligned}\Delta_t^2(k) = \bar{A}_t\Big[&1 + \left(-2\bar{D}_p - 2\right)\bar{\epsilon}_1 \\ &+ \left(2\bar{D}_p^2 + 2\bar{D}_p + \bar{\Delta}_1 + \frac{\pi^2}{2} - 5\right)\bar{\epsilon}_1^2 \\ &+ \left(-\bar{D}_p^2 - 2\bar{D}_p + 2\bar{\Delta}_2 + \frac{\pi^2}{12} - 2\right)\bar{\epsilon}_1\bar{\epsilon}_2 - \frac{\bar{\epsilon}_1\bar{\kappa}_1}{2}\Big],\end{aligned} \tag{3.25}$$

where $\bar{A}_t \equiv \frac{181\bar{H}^2}{36e^3\pi^2}$. Also the tensor spectral index and its running are given by

$$n_t \simeq -2\bar{\epsilon}_1 - 2\bar{\epsilon}_1^2 - \left(\bar{D}_n + 1\right)2\bar{\epsilon}_1\bar{\epsilon}_2, \tag{3.26}$$

and

$$\begin{aligned}\alpha_t \simeq& -2\bar{\epsilon}_1\bar{\epsilon}_2 - 6\bar{\epsilon}_1^2\bar{\epsilon}_2 - \left(\bar{D}_n + 1\right)2\bar{\epsilon}_1\bar{\epsilon}_2^2 \\ &- 2\left(\bar{D}_n + 1\right)\bar{\epsilon}_1\bar{\epsilon}_2\bar{\epsilon}_3.\end{aligned} \tag{3.27}$$

We observe that the new effects (represented by $\bar{\kappa}_1$) can only affect the tensor spectrum at the next-to-leading order. In this case, the tensor spectral index and its running are keeping the same as the standard slow-roll inflation.

3.4 Expressions at horizon crossing

In the last two subsections, all the results are expressed in terms of quantities that are evaluated at the turning point. However, usually those expressions were expressed in terms of the slow-roll parameters which are evaluated at the time $\eta_\star$ when scalar or tensor perturbation modes cross the horizon, i.e., $a(\eta_\star)H(\eta_\star) = c_s(\eta_\star)k$ for scalar perturbations and $a(\eta_\star)H(\eta_\star) = c_t(\eta_\star)k$ for tensor perturbations. Consider modes with the same wave number k, it is easy to see that the scalar and tensor modes left the horizon at different times if $c_s(\eta) \neq c_t(\eta)$. When $c_s(\eta_\star) > c_t(\eta_\star)$, the scalar mode leaves the horizon later than the tensor mode, and for $c_s(\eta_\star) < c_t(\eta_\star)$, the scalar mode leaves the horizon before the tensor one.

As we have two different horizon crossing times, it is reasonable to rewrite all our results in terms of quantities evaluated at the later time, i.e., we should evaluate all expressions at scalar horizon crossing time $a(\eta_\star)H(\eta_\star) = c_s(\eta_\star)k$ for $c_s(\eta_\star) > c_t(\eta_\star)$ and at tensor-mode horizon crossing $a(\eta_\star)H(\eta_\star) = c_t(\eta_\star)k$ for $c_s(\eta_\star) < c_t(\eta_\star)$.

3.4.1 $c_s(\eta_\star) > c_t(\eta_\star)$

Then, we shall re-write all the expressions in terms of quantities evaluated at the time when the scalar-mode horizon crossing $c_s(\eta_\star) > c_t(\eta_\star)$. Skipping all the tedious calculations, we find that scalar spectrum can be written in the form

$$\begin{aligned}\Delta_s^2(k) = A_s^\star\Big[&1 - \left(2D_p^\star + 2\right)\epsilon_{\star1} \\ &- D_p^\star\epsilon_{\star2} - \left(D_a^\star - \frac{9}{8}\right)\zeta_{\star1} - \left(D_a^\star - \frac{9}{8}\right)\kappa_{\star1} \\ &+ \delta_{\star1} - \frac{3\sigma_{\star1}}{4} - \left(\frac{D_p^{\star2}}{2} - \Delta_2^\star - \frac{\pi^2}{24}\right)\epsilon_{\star2}\epsilon_{\star3} \\ &+ \left(2D_p^{\star2} + 2D_p^\star + \Delta_1^\star + \frac{\pi^2}{2} - 5\right)\epsilon_{\star1}^2\end{aligned}$$

$$
\begin{aligned}
&+\left(\frac{D_p^{\star 2}}{2}+\frac{\Delta_1^\star}{4}+\frac{\pi^2}{8}-1\right)\epsilon_{\star 2}^2\\
&+\left(D_p^{\star 2}-D_p^\star+\Delta_1^\star+2\Delta_2^\star+\frac{7\pi^2}{12}-7\right)\epsilon_{\star 1}\epsilon_{\star 2}\\
&-\left(3D_a^\star-\frac{15}{8}\right)\delta_{\star 1}\kappa_{\star 1}+\left(\frac{3D_p^\star}{4}-\frac{1}{2}\right)\sigma_{\star 1}\epsilon_{\star 2}\\
&-\left(D_a^\star D_p^\star+\frac{53117D_a^\star}{17376}-\frac{9D_p^\star}{8}\right)\zeta_{\star 1}\epsilon_{\star 2}\\
&-\frac{2509}{320}D_a^\star\zeta_{\star 1}\sigma_{\star 1}\\
&+\left(3D_a^\star+\frac{9}{8}\right)\delta_{\star 1}\zeta_{\star 1}\\
&+\left(\frac{3D_p^\star}{2}+\frac{1}{2}\right)\sigma_{\star 1}\epsilon_{\star 1}\\
&+\left(D_a^\star D_p^\star-\frac{223033D_a^\star}{28960}-\frac{9D_p^\star}{8}\right)\zeta_{\star 1}\zeta_{\star 2}\\
&+\frac{9\delta_{\star 1}\sigma_{\star 1}}{4}-\frac{38187D_a^\star\zeta_{\star 1}^2}{6400}\\
&+\left(D_a^\star D_p^\star-\frac{191789D_a^\star}{25920}-\frac{9D_p^\star}{8}\right)\kappa_{\star 1}\kappa_{\star 2}\\
&-\left(\frac{3D_p^\star}{4}+\frac{5}{2}\right)\sigma_{\star 1}\sigma_{\star 2}\\
&-\left(2D_a^\star D_p^\star+\frac{46213D_a^\star}{43440}-\frac{9D_p^\star}{4}\right)\zeta_{\star 1}\epsilon_{\star 1}-\frac{\delta_{\star 1}^2}{2}\\
&-\left(2D_a^\star D_p^\star+\frac{46213D_a^\star}{43440}-\frac{9D_p^\star}{4}\right)\kappa_{\star 1}\epsilon_{\star 1}\\
&-\left(D_a^\star D_p^\star+\frac{46213D_a^\star}{43440}-\frac{9D_p^\star}{8}\right)\kappa_{\star 1}\epsilon_{\star 2}-\frac{57\sigma_{\star 1}^2}{32}\\
&-\left(2D_p^\star+2\right)\delta_{\star 1}\epsilon_{\star 1}\\
&-D_p^\star\delta_{\star 1}\epsilon_{\star 2}-\frac{1147D_a^\star\zeta_{\star 1}\kappa_{\star 1}}{3200}-\frac{287}{960}D_a^\star\kappa_{\star 1}\sigma_{\star 1}\\
&\left.+\frac{49759D_a^\star\kappa_{\star 1}^2}{19200}+\left(D_p^\star+2\right)\delta_{\star 1}\delta_{\star 2}\right]. \qquad (3.28)
\end{aligned}
$$

where the subscript "$\star$" denotes evaluation at the horizon crossing, $A_s^\star \equiv \frac{181H_\star^2}{72e^3\pi^2\epsilon_{\star 1}}$, $D_p^\star \equiv \frac{67}{181}-\ln 3$, $D_a^\star \equiv \frac{90}{181}$, $\Delta_1^\star \equiv \frac{485296}{98283}-\frac{\pi^2}{2}$, and $\Delta_2^\star \equiv \frac{9269}{589698}$. For the scalar spectral index, one obtains

$$
\begin{aligned}
n_s-1 \simeq\; & -2\epsilon_{\star 1}-\epsilon_{\star 2}-2\epsilon_{\star 1}^2+\delta_{\star 1}\delta_{\star 2}\\
&-\left(2D_n^\star+3\right)\epsilon_{\star 1}\epsilon_{\star 2}-D_n^\star\epsilon_{\star 2}\epsilon_{\star 3}\\
&-\frac{5\zeta_{\star 1}\zeta_{\star 2}}{8}-\frac{5\kappa_{\star 1}\kappa_{\star 2}}{8}-\frac{3\sigma_{\star 1}\sigma_{\star 2}}{4}. \qquad (3.29)
\end{aligned}
$$

The running of the scalar spectral index reads

$$
\begin{aligned}
\alpha_s \simeq\; & -2\epsilon_{\star 1}\epsilon_{\star 2}-\epsilon_{\star 2}\epsilon_{\star 3}-3\epsilon_{\star 1}\epsilon_{\star 2}^2-6\epsilon_{\star 1}^2\epsilon_{\star 2}+\delta_{\star 1}\delta_{\star 2}\delta_{\star 3}\\
&-4\epsilon_{\star 1}\epsilon_{\star 2}\epsilon_{\star 3}+\delta_{\star 1}\delta_{\star 2}^2-2D_n^\star\epsilon_{\star 1}\epsilon_{\star 2}^2-\frac{3}{4}\sigma_{\star 1}\sigma_{\star 2}\sigma_{\star 3}\\
&-D_n^\star\epsilon_{\star 2}\epsilon_{\star 3}^2-D_n^\star\epsilon_{\star 2}\epsilon_{\star 3}\epsilon_{\star 4}-\frac{5}{8}\zeta_{\star 1}\zeta_{\star 2}^2-\frac{5}{8}\kappa_{\star 1}\kappa_{\star 2}\kappa_{\star 3}\\
&-\frac{5}{8}\zeta_{\star 1}\zeta_{\star 2}\zeta_{\star 3}-2D_n^\star\epsilon_{\star 1}\epsilon_{\star 2}\epsilon_{\star 3}-\frac{5}{8}\kappa_{\star 1}\kappa_{\star 2}^2-\frac{3}{4}\sigma_{\star 1}\sigma_{\star 2}^2. \qquad (3.30)
\end{aligned}
$$

Similar to the scalar perturbations, now let us turn to consider the tensor perturbations, which yield

$$
\begin{aligned}
\Delta_t^2(k) = & -2\left(1+D_p^\star\right)\epsilon_{\star 1}\\
&+\left(2D_p^{\star 2}+2D_p^\star-\Delta_1^\star+\frac{\pi^2}{2}-5\right)\epsilon_{\star 1}^2\\
&+\left(-D_p^{\star 2}-2D_p^\star+\frac{\pi^2}{12}-2+2\Delta_2^\star\right)\epsilon_{\star 1}\epsilon_{\star 2}\\
&-2\delta_{\star 1}\epsilon_{\star 1}+3\zeta_{\star 1}\epsilon_{\star 1}+\frac{\kappa_{\star 1}\epsilon_{\star 1}}{2}+\frac{3\sigma_{\star 1}\epsilon_{\star 1}}{2}. \qquad (3.31)
\end{aligned}
$$

For the tensor spectral index, we find

$$
n_t \simeq -2\epsilon_{\star 1}-\left(2D_n^\star+2\right)\epsilon_{\star 1}\epsilon_{\star 2}-2\epsilon_{\star 1}^2. \qquad (3.32)
$$

Then, the running of the tensor spectral index reads

$$
\begin{aligned}
\alpha_t \simeq & -2\epsilon_{\star 1}\epsilon_{\star 2}-6\epsilon_{\star 1}^2\epsilon_{\star 2}-2\left(D_n^\star+1\right)\epsilon_{\star 1}\epsilon_{\star 2}^2\\
&-2\left(2D_n^\star+1\right)\epsilon_{\star 1}\epsilon_{\star 2}\epsilon_{\star 3}. \qquad (3.33)
\end{aligned}
$$

Finally with both scalar and tensor spectra given above, we can evaluate the tensor-to-scalar ratio at the horizon crossing time($\eta_\star$), and find that

$$
\begin{aligned}
r \simeq 16\epsilon_{\star 1}\Bigg[& 1+D_p^\star\epsilon_{\star 2}-\delta_{\star 1}\\
&+\left(\frac{9}{8}-D_a^\star\right)\kappa_{\star 1}+\left(\frac{9}{8}-D_a^\star\right)\zeta_{\star 1}\\
&+\frac{3\sigma_{\star 1}}{4}+\left(D_p^\star-\Delta_1^\star-\frac{\pi^2}{2}+5\right)\epsilon_{\star 1}\epsilon_{\star 2}\\
&+\left(\frac{D_p^{\star 2}}{2}-\Delta_2^\star-\frac{\pi^2}{24}\right)\epsilon_{\star 2}\epsilon_{\star 3}\\
&+\left(\frac{D_p^{\star 2}}{2}-\frac{\Delta_1^\star}{4}-\frac{\pi^2}{8}+1\right)\epsilon_{\star 2}^2\\
&-\left(D_p^\star+2\right)\delta_{\star 1}\delta_{\star 2}\\
&+\left(\frac{3}{4}D_p^\star+\frac{1}{2}\right)\sigma_{\star 1}\epsilon_{\star 2}+\frac{421}{192}D_a^\star\kappa_{\star 1}\sigma_{\star 1}\\
&+\frac{3\delta_{\star 1}^2}{2}+\left(\frac{9}{8}D_p^\star-D_p^\star D_a^\star+\frac{223033D_a^{\star 2}}{14400}\right)\zeta_{\star 1}\zeta_{\star 2}\\
&+\left(\frac{9}{8}D_p^\star-D_p^\star D_a^\star+\frac{53117D_a^{\star 2}}{8640}\right)\zeta_{\star 1}\epsilon_{\star 2}\\
&-2\delta_{\star 1}\epsilon_{\star 1}-\left(D_a^\star+\frac{27}{8}\right)\delta_{\star 1}\zeta_{\star 1}
\end{aligned}
$$

$$+\frac{623}{64}D_a^\star\zeta_{\star1}\sigma_{\star1}+\frac{869993D_a^{\star2}\zeta_{\star1}^2}{64000}-\frac{6252109D_a^{\star2}\kappa_{\star1}^2}{1728000}$$
$$+\left(\frac{9}{8}D_p^\star-D_p^\star D_a^\star+\frac{191789D_a^{\star2}}{25920}\right)\kappa_{\star1}\kappa_{\star2}$$
$$+\frac{15481}{800}D_a^{\star2}\zeta_{\star1}\epsilon_{\star1}$$
$$+\frac{75\sigma_{\star1}^2}{32}+\frac{1125697D_a^{\star2}\zeta_{\star1}\kappa_{\star1}}{288000}$$
$$+\left(\frac{3}{4}D_p^\star+\frac{5}{2}\right)\sigma_{\star1}\sigma_{\star2}+\frac{5\sigma_{\star1}\epsilon_{\star1}}{2}$$
$$-\left(D_a^\star+\frac{3}{8}\right)\delta_{\star1}\kappa_{\star1}-\delta_{\star1}\epsilon_{\star2}-\left(D_p^\star-1\right)\delta_{\star1}\epsilon_{\star2}$$
$$-\frac{15\delta_{\star1}\sigma_{\star1}}{4}+\frac{598741D_a^{\star2}\kappa_{\star1}\epsilon_{\star1}}{64800}$$
$$+\left(\frac{9}{8}D_p^\star-D_p^\star D_a^\star+\frac{53117D_a^{\star2}}{8640}\right)\kappa_{\star1}\epsilon_{\star2}\Bigg]. \quad (3.34)$$

3.4.2 $c_s(\eta_\star) < c_t(\eta_\star)$

For $c_s(\eta_\star) < c_t(\eta_\star)$, as the scalar mode leaves the horizon before the tensor mode does,we shall rewrite all the expressions in terms of quantities evaluated at the time when the tensor mode leaves the Hubble horizon $a(\eta_\star)H(\eta_\star)=c_t(\eta_\star)k$. Skipping all the tedious calculations, we find that the scalar spectrum can be written in the form

$$\Delta_s^2(k)=A_s^\star\Bigg[1-\left(2D_p^\star+2\right)\epsilon_{\star1}$$
$$-D_p^\star\epsilon_{\star2}-\left(D_a^\star-\frac{9}{8}\right)\zeta_{\star1}-\left(D_a^\star-\frac{9}{8}\right)\kappa_{\star1}$$
$$+\delta_{\star1}-\frac{3\sigma_{\star1}}{4}-\left(\frac{D_p^{\star2}}{2}-\Delta_2^\star-\frac{\pi^2}{24}\right)\epsilon_{\star2}\epsilon_{\star3}$$
$$+\left(2D_p^{\star2}+2D_p^\star+\Delta_1^\star+\frac{\pi^2}{2}-5\right)\epsilon_{\star1}^2$$
$$+\left(\frac{D_p^{\star2}}{2}+\frac{\Delta_1^\star}{4}+\frac{\pi^2}{8}-1\right)\epsilon_{\star2}^2$$
$$+\left(D_p^{\star2}-D_p^\star+\Delta_1^\star+2\Delta_2^\star+\frac{7\pi^2}{12}-7\right)\epsilon_{\star1}\epsilon_{\star2}$$
$$-\left(3D_a^\star-\frac{15}{8}\right)\delta_{\star1}\kappa_{\star1}+\left(\frac{3D_p^\star}{4}-\frac{5}{4}\right)\sigma_{\star1}\epsilon_{\star2}$$
$$-\left(D_a^\star D_p^\star+\frac{175891D_a^{\star2}}{14400}-\frac{9D_p^\star}{8}\right)\zeta_{\star1}\epsilon_{\star2}$$
$$-\frac{2509}{320}D_a^\star\zeta_{\star1}\sigma_{\star1}$$
$$+\left(3D_a^\star+\frac{9}{8}\right)\delta_{\star1}\zeta_{\star1}+\left(\frac{3D_p^\star}{2}-1\right)\sigma_{\star1}\epsilon_{\star1}$$
$$+\left(D_a^\star D_p^\star-\frac{223033D_a^\star}{28960}-\frac{9D_p^\star}{8}\right)\zeta_{\star1}\zeta_{\star2}$$
$$+\frac{9\delta_{\star1}\sigma_{\star1}}{4}-\frac{38187D_a^\star\zeta_{\star1}^2}{6400}$$
$$+\left(D_a^\star D_p^\star-\frac{191789D_a^\star}{25920}-\frac{9D_p^\star}{8}\right)\kappa_{\star1}\kappa_{\star2}$$
$$-\left(\frac{3D_p^\star}{4}+\frac{5}{2}\right)\sigma_{\star1}\sigma_{\star2}$$
$$-\left(2D_a^\star D_p^\star+\frac{102767D_a^{\star2}}{7200}-\frac{9D_p^\star}{4}\right)\zeta_{\star1}\epsilon_{\star1}-\frac{\delta_{\star1}^2}{2}$$
$$-\left(2D_a^\star D_p^\star+\frac{400727D_a^{\star2}}{64800}-\frac{9D_p^\star}{4}\right)\kappa_{\star1}\epsilon_{\star1}$$
$$-\left(D_a^\star D_p^\star+\frac{269683D_a^{\star2}}{64800}-\frac{9D_p^\star}{8}\right)\kappa_{\star1}\epsilon_{\star2}$$
$$-\frac{57\sigma_{\star1}^2}{32}-\left(D_p^\star-1\right)\delta_{\star1}\epsilon_{\star2}$$
$$-2D_p^\star\delta_{\star1}\epsilon_{\star1}-\frac{1147D_a^\star\zeta_{\star1}\kappa_{\star1}}{3200}-\frac{287}{960}D_a^\star\kappa_{\star1}\sigma_{\star1}$$
$$+\frac{49759D_a^\star\kappa_{\star1}^2}{19200}+\left(D_p^\star+2\right)\delta_{\star1}\delta_{\star2}\Bigg]. \quad (3.35)$$

For the scalar spectral index, one obtains

$$n_s-1\simeq-2\epsilon_{\star1}-\epsilon_{\star2}-2\epsilon_{\star1}^2+\delta_{\star1}\delta_{\star2}$$
$$-\left(2D_n^\star+3\right)\epsilon_{\star1}\epsilon_{\star2}-D_n^\star\epsilon_{\star2}\epsilon_{\star3}$$
$$-\frac{5\zeta_{\star1}\zeta_{\star2}}{8}-\frac{5\kappa_{\star1}\kappa_{\star2}}{8}-\frac{3\sigma_{\star1}\sigma_{\star2}}{4}. \quad (3.36)$$

The running of the scalar spectral index reads

$$\alpha_s\simeq-2\epsilon_{\star1}\epsilon_{\star2}-\epsilon_{\star2}\epsilon_{\star3}-3\epsilon_{\star1}\epsilon_{\star2}^2-6\epsilon_{\star1}^2\epsilon_{\star2}+\delta_{\star1}\delta_{\star2}\delta_{\star3}$$
$$-4\epsilon_{\star1}\epsilon_{\star2}\epsilon_{\star3}+\delta_{\star1}\delta_{\star2}^2-2D_n^\star\epsilon_{\star1}\epsilon_{\star2}^2-\frac{3}{4}\sigma_{\star1}\sigma_{\star2}\sigma_{\star3}$$
$$-D_n^\star\epsilon_{\star2}\epsilon_{\star3}^2-D_n^\star\epsilon_{\star2}\epsilon_{\star3}\epsilon_{\star4}-\frac{5}{8}\zeta_{\star1}\zeta_{\star2}^2-\frac{5}{8}\kappa_{\star1}\kappa_{\star2}\kappa_{\star3}$$
$$-\frac{5}{8}\zeta_{\star1}\zeta_{\star2}\zeta_{\star3}-2D_n^\star\epsilon_{\star1}\epsilon_{\star2}\epsilon_{\star3}-\frac{5}{8}\kappa_{\star1}\kappa_{\star2}^2-\frac{3}{4}\sigma_{\star1}\sigma_{\star2}^2. \quad (3.37)$$

Similar to the scalar perturbations, now let us turn to consider the tensor perturbations, which yield

$$\Delta_t^2(k)=-2\left(1+D_p^\star\right)\epsilon_{\star1}$$
$$+\left(2D_p^{\star2}+2D_p^\star-\Delta_1^\star+\frac{\pi^2}{2}-5\right)\epsilon_{\star1}^2$$
$$+\left(-D_p^{\star2}-2D_p^\star+\frac{\pi^2}{12}-2+2\Delta_2^\star\right)\epsilon_{\star1}\epsilon_{\star2}$$
$$-2\delta_{\star1}\epsilon_{\star1}+3\zeta_{\star1}\epsilon_{\star1}+\frac{\kappa_{\star1}\epsilon_{\star1}}{2}+\frac{3\sigma_{\star1}\epsilon_{\star1}}{2}. \quad (3.38)$$

For the tensor spectral index, we find

$$n_t \simeq -2\epsilon_{\star 1} - \left(2D_n^\star + 2\right)\epsilon_{\star 1}\epsilon_{\star 2} - 2\epsilon_{\star 1}^2. \tag{3.39}$$

Then, the running of the tensor spectral index reads

$$\alpha_t \simeq -2\epsilon_{\star 1}\epsilon_{\star 2} - 6\epsilon_{\star 1}^2\epsilon_{\star 2} - 2\left(D_n^\star + 1\right)\epsilon_{\star 1}\epsilon_{\star 2}^2 \\ -2\left(2D_n^\star + 1\right)\epsilon_{\star 1}\epsilon_{\star 2}\epsilon_{\star 3}. \tag{3.40}$$

Finally with both scalar and tensor spectra given above, we can evaluate the tensor-to-scalar ratio at the horizon crossing time($\eta_\star$), and find that

$$\begin{aligned} r \simeq 16\epsilon_{\star 1}\Bigg[& 1 + D_p^\star\epsilon_{\star 2} - \delta_{\star 1} \\ &+ \left(\frac{9}{8} - D_a^\star\right)\kappa_{\star 1} + \left(\frac{9}{8} - D_a^\star\right)\zeta_{\star 1} + \frac{3\sigma_{\star 1}}{4} \\ &+ \left(D_p^\star - \Delta_1^\star - \frac{\pi^2}{2} + 5\right)\epsilon_{\star 1}\epsilon_{\star 2} - D_p^\star\delta_{\star 1}\epsilon_{\star 2} \\ &+ \left(\frac{D_p^{\star 2}}{2} - \Delta_2^\star - \frac{\pi^2}{24}\right)\epsilon_{\star 2}\epsilon_{\star 3} \\ &+ \left(\frac{D_p^{\star 2}}{2} - \frac{\Delta_1^\star}{4} - \frac{\pi^2}{8} + 1\right)\epsilon_{\star 2}^2 \\ &- \left(D_p^\star + 2\right)\delta_{\star 1}\delta_{\star 2} + \left(\frac{3}{4}D_p^\star + \frac{5}{4}\right)\sigma_{\star 1}\epsilon_{\star 2} - \frac{15\delta_{\star 1}\sigma_{\star 1}}{4} \\ &+ \left(\frac{9}{8}D_p^\star - D_p^\star D_a^\star + \frac{223033 D_a^{\star 2}}{14400}\right)\zeta_{\star 1}\zeta_{\star 2} \\ &- \left(D_a^\star + \frac{3}{8}\right)\delta_{\star 1}\kappa_{\star 1} \\ &+ \left(\frac{9}{8}D_p^\star - D_p^\star D_a^\star + \frac{175891 D_a^{\star 2}}{14400}\right)\zeta_{\star 1}\epsilon_{\star 2} - 2\delta_{\star 1}\epsilon_{\star 1} \\ &+ \frac{623}{64}D_a^\star\zeta_{\star 1}\sigma_{\star 1} + \frac{869993 D_a^{\star 2}\zeta_{\star 1}^2}{64000} - \frac{6252109 D_a^{\star 2}\kappa_{\star 1}^2}{1728000} \\ &+ \left(\frac{9}{8}D_p^\star - D_p^\star D_a^\star + \frac{191789 D_a^{\star 2}}{25920}\right)\kappa_{\star 1}\kappa_{\star 2} \\ &+ \frac{15481}{800}D_a^{\star 2}\zeta_{\star 1}\epsilon_{\star 1} \\ &+ \frac{75\sigma_{\star 1}^2}{32} + \frac{1125697 D_a^{\star 2}\zeta_{\star 1}\kappa_{\star 1}}{288000} \\ &+ \left(\frac{3}{4}D_p^\star + \frac{5}{2}\right)\sigma_{\star 1}\sigma_{\star 2} + \frac{3\delta_{\star 1}^2}{2} \\ &+ \frac{598741 D_a^{\star 2}\kappa_{\star 1}\epsilon_{\star 1}}{64800} + \frac{5\sigma_{\star 1}\epsilon_{\star 1}}{2} - \left(D_a^\star + \frac{27}{8}\right)\delta_{\star 1}\zeta_{\star 1} \\ &+ \left(\frac{9}{8}D_p^\star - D_p^\star D_a^\star + \frac{1058843 D_a^{\star 2}}{129600}\right)\kappa_{\star 1}\epsilon_{\star 2} \\ &+ \frac{421}{192}D_a^\star\kappa_{\star 1}\sigma_{\star 1} - \delta_{\star 1}\epsilon_{\star 2}\Bigg]. \end{aligned} \tag{3.41}$$

3.5 Comparison with results from generalized slow-roll approach

In this subsection, we would like to provide a roughly comparison between our results and those derived by using the generalized slow-roll approach in the unified EFT of inflation [55] at the leading slow-roll order. For this purpose, we can map the coefficients C_N, C_{KK}, C_{NK}, and $\tilde{C}_{KK}$ in Eq. (16) in [55] with the four new coefficients M_2, $\bar{M}_1$, $\bar{M}_2$, and $\bar{M}_3$ as

$$C_N + \frac{1}{2}C_{NN} \to -\dot{H}M_{\rm Pl}^2 + 2M_2^4, \tag{3.42}$$

$$C_{NK} \to -\bar{M}_1^3, \tag{3.43}$$

$$C_{KK} \to -\bar{M}_2^2 - M_{\rm Pl}^2, \tag{3.44}$$

$$\tilde{C}_{KK} \to -\bar{M}_3^2 + M_{\rm Pl}^2, \tag{3.45}$$

$$C_R \to \frac{1}{2}M_{\rm Pl}^2. \tag{3.46}$$

In [55], the authors only consider the second-order spatial derivative terms in the action and make the fourth spatial derivative terms vanishing by imposing the condition

$$C_{KK} = -\tilde{C}_{KK}, \tag{3.47}$$

which corresponds to $\bar{M}_2^2 = -\bar{M}_3^2$ in the notation of this paper. One of essential differences of our calculation in this paper is that we have included the contributions of the fourth spatial derivative terms in the calculation of the power spectra.

The scalar power spectrum in [55] (Eq. (85)) is calculated at the leading-order in the optimized slow-roll approximation. In order to compare with our results at least at the first-order in the slow-roll expansion, we have to recalculate their power spectra by including the $p = 1$ order in Eq. (65) in [55]. The scalar power spectrum can be calculated as (c.f. Eq. (65) in [55])

$$\ln \Delta_\zeta^2 \simeq G(\ln x_f) + p_1(\ln x_f)G'(\ln x_f), \tag{3.48}$$

where

$$G(\ln x_f) \simeq \ln\left(\frac{H^2}{8\pi^2 b_s \epsilon_H c_s}\right) - \frac{10}{3}\epsilon_H - \frac{2}{3}\delta_1 \\ -\frac{7}{3}\sigma_{s1} - \frac{1}{3}\xi_{s1} - \frac{8}{3}\sigma_{s2}, \tag{3.49}$$

$$G'(\ln x_f) \simeq 4\epsilon_H + 2\delta_1 + \sigma_{s1} + \xi_{s1} + \frac{2}{3}\delta_2 \\ +\frac{7}{3}\sigma_{s2} + \frac{1}{3}\xi_{s2}, \tag{3.50}$$

$$p_1(\ln x_f) = \frac{7}{3} - \ln 2 - \gamma_E - \ln x_f, \tag{3.51}$$

with γ_E being the Euler-Mascheroni constant. In the above expressions, $c_s, b_s, \delta_1, \epsilon_H, \sigma_{s1}, \sigma_{s2}, \xi_{s1}, \xi_{s2}, \delta_{s2}, x_1$, and x_f are all defined in [55]. In these quantities, σ_{s1}, σ_{s2}, δ_2, ξ_{s1}, and ξ_{s2} represent second-order contributions in the notation of our paper since they are defined as the derivatives of the slow-roll quantities c_s, δ_1, b_s. Therefore we do not consider these terms when we compare the power spectrum at the leading-order. Then we can transform the above power spectrum in our notation, which is

$$\Delta_\zeta^2 = \frac{H^2}{8\pi^2\epsilon_1}\left[1 + 2(\alpha_\star - 1)\epsilon_1 + \alpha_\star\epsilon_2 - \frac{3}{4}\sigma_1 + \delta_1\right]. \tag{3.52}$$

where $\alpha_\star \equiv 2 - \ln 2 - \gamma_E \simeq 0.729637$ and the power spectrum is evaluated at the Horizon crossing point $x_f = 1$. By comparing the coefficients of each term with those in (3.35) and considering the assumption $\zeta_1 = -\kappa_1$ if $\bar{M}_2^2 = -\bar{M}_3^2$ in [55], we observe the two expressions are almost the same. The overall amplitude has a relative difference $\lesssim 0.15\%$, and the coefficients of ϵ_1 and ϵ_2 have relative differences 0.44% and 0.16%. The coefficients of σ_1 and δ_1 are exactly the same in the two expressions.

4 Conclusion and outlook

The uniform asymptotic approximation method provides a powerful, systematically improvable, and error-controlled approach to construct accurate analytical solutions of linear perturbations. In this paper, by applying the third-order uniform asymptotic approximations, we have obtained explicitly the analytical expressions of power spectra, spectral indices, and running of spectral indices for both scalar and tensor perturbations in the EFT of inflation with the slow-roll approximation.

Comparing to the standard slow-roll inflation, the EFT of inflation introduces four new operators which can modify both the effective sound speed and the linear dispersion relation. In order to calculate the effects of these four new operators in the primordial spectra, we defined four new slow-roll parameters as in (2.25). All the final expressions are written in terms of both the Hubble flow parameters defined in (2.19) and the flow of four new slow-roll parameters and expanded up to the next-to-leading order in the slow-roll expansions so they represent the most accurate results obtained so far in the literature. We observe that the four new operators introduced in the action of the EFT of inflation can affect the primordial perturbation spectra at the leading-order and the corresponding spectral indices at the next-to-leading order. The running of the indices are keeping the same with that in the standard slow-roll inflation up to the next-to-leading order.

The next-to-leading order corrections to the scalar index n_s and the tensor-to-scalar ratio r in the slow- roll approximation are very important and useful in the future analysis with forthcoming observational data. As pointed out in [53,54], the future CMB measurements can achieve the errors of the measurements on both n_s and r are $\lesssim 10^{-3}$. This implies, if the contributions of the new effects (for examples, $\delta_{\star 1}\delta_{\star 2}$, $\zeta_{\star 1}\zeta_{\star 2}$, etc) at the magnitude of $O(10^{-3})$, then they can not be ignored in the analysis with future experiments. Therefore, it would be interesting to constrain these new effects by using the more precise forthcoming observational data in the future. We expect that such constraints could help us to understand the physics of the early Universe.

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Appendix A: Integrals of $\sqrt{g}$ and the error control function

Here we present the calculation of the integral of $\sqrt{g}$ and the error control function $\mathscr{H}(+\infty)$ in the general expression of the power spectra (3.8).

Let us first consider the integral of $\sqrt{g}$, which in general reads

$$\int_y^{\bar{y}_0} \sqrt{g(y')}dy' = \int_y^{\bar{y}_0} \sqrt{\frac{\nu^2(\eta)}{y'^2} - c^2(\eta) - b(\eta)y'^2}dy'. \tag{A.1}$$

Here we note that $\nu(\eta)$, $c(\eta)$, and $b(\eta)$ are all slow-varying quantities. In order to evaluate the integral with these time-dependent quantities, we consider expanding them about the turning point $y = \bar{y}_0$ as

$$\nu(\eta) = \bar{\nu}_0 + \bar{\nu}_1 \ln\frac{y}{\bar{y}_0} + \mathcal{O}\left(\ln^2\frac{y}{\bar{y}_0}\right), \tag{A.2}$$

$$c(\eta) = \bar{c}_0 + \bar{c}_1 \ln\frac{y}{\bar{y}_0} + \mathcal{O}\left(\ln^2\frac{y}{\bar{y}_0}\right), \tag{A.3}$$

$$b(\eta) = \bar{b}_0 + \bar{b}_1 \ln\frac{y}{\bar{y}_0} + \mathcal{O}\left(\ln^2\frac{y}{\bar{y}_0}\right), \tag{A.4}$$

where

$$\nu_1(\eta) \equiv \frac{d\nu(\eta)}{d\ln(-\eta)}, \tag{A.5}$$

$$c_1(\eta) \equiv \frac{dc(\eta)}{d\ln(-\eta)}, \tag{A.6}$$

$$b_1(\eta) \equiv \frac{db(\eta)}{d\ln(-\eta)}, \tag{A.7}$$

and $\bar{\nu}_0 = \nu(\eta_0), \bar{\nu}_1 = \nu_1(\eta_0), \bar{c}_0 = c(\eta_0), \bar{c}_1 = c_1(\eta_0)$, $\bar{b}_0 = b(\eta_0)$, and $\bar{b}_1 = b_1(\eta_0)$. With the above expansions, the integral (A.8) can be divided into two parts

$$\int_y^{\bar{y}_0} \sqrt{g(y')}dy' \simeq I_0 + I_1, \tag{A.8}$$

where

$$I_0 = \int_y^{\bar{y}_0} \sqrt{\frac{\bar{\nu}_0^2}{y'^2} - \bar{c}_0^2 - \bar{b}_0 y'^2} dy, \tag{A.9}$$

$$I_1 = -\int_y^{\bar{y}_0} \frac{\bar{b}_1 y'^4 + 2\bar{c}_0\bar{c}_1 y'^2 - 2\bar{\nu}_0\bar{\nu}_1}{2y'\sqrt{\bar{\nu}_0^2 - \bar{c}_0^2 y'^2 - \bar{b}_0 y'^4}} \ln\frac{y'}{\bar{y}_0} dy'. \tag{A.10}$$

Performing the integral I_0 gives

$$\lim_{y\to 0} I_0 = -\frac{\bar{\nu}_0}{2} - \bar{\nu}_0 \ln\frac{y}{2\bar{\nu}_0} - \frac{\bar{\nu}_0}{4}\ln(\bar{c}_0^4 + 4\bar{b}_0\bar{\nu}_0^2) - \frac{\bar{c}_0^2}{4\sqrt{\bar{b}_0}} \arctan\left(\frac{2\sqrt{\bar{b}_0}\bar{\nu}_0}{\bar{c}_0^2}\right). \tag{A.11}$$

For the second integral, since we only need to calculate the power spectrum up to the next-to-leading order in the slow-roll expansions, we can safely ignored the term with $\bar{b}_0$ in the squre root which only contributes to the third order corrections in the slow-roll expansions. With this simpfications, we have

$$I_1 \simeq -\int_y^{\bar{\nu}_0/\bar{c}_0} \frac{\bar{b}_1 y'^4 + 2\bar{c}_0\bar{c}_1 y'^2 - 2\bar{\nu}_0\bar{\nu}_1}{2y'\sqrt{\bar{\nu}_0^2 - \bar{c}_0^2 y'^2}} \ln\frac{y'}{\bar{\nu}_0/\bar{c}_0} dy', \tag{A.12}$$

which leads to

$$\lim_{y\to 0} I_1 \simeq \frac{(1-\ln 2)\bar{\nu}_0}{\bar{c}_0}\bar{c}_1 - \left(\frac{\pi^2}{24} - \frac{\ln^2 2}{2} + \frac{1}{2}\ln^2\frac{y}{\bar{\nu}_0/\bar{c}_0}\right)\bar{\nu}_1 + \frac{(5-6\ln 2)\bar{\nu}_0^3}{18\bar{c}_0^4}\bar{b}_1. \tag{A.13}$$

For error control function, the integral form is

$$\mathscr{H}(\xi) \simeq \frac{5}{36}\left\{\int_{\bar{y}_0}^{y} \sqrt{g(y')}dy'\right\}^{-1}\Bigg|_{\bar{y}_0}^{y} - \int_{\bar{y}_0}^{y} \left\{\frac{q}{g} - \frac{5g'^2}{16g^3} + \frac{g''}{4g^2}\right\}\sqrt{g(y')}dy'. \tag{A.14}$$

Using the expansions in Eqs. (A.2, A.3, A.4), $\mathscr{H}$ can also divided into two parts,

$$\mathscr{H}(\xi) \simeq \mathscr{H}_0(\xi) + \mathscr{H}_1(\xi), \tag{A.15}$$

where

$$\lim_{y\to 0} \mathscr{H}_0(\xi) \simeq \frac{\bar{c}_0^4 + 8\bar{b}_0\bar{\nu}_0^2}{6\bar{c}_0^4\bar{\nu}_0 + 24\bar{b}_0\bar{\nu}_0^3}, \tag{A.16}$$

$$\lim_{y\to 0} \mathscr{H}_1(\xi) \simeq \frac{\bar{c}_1}{6\bar{c}_0\bar{\nu}_0} - \frac{23 + 12\ln 2}{72\bar{\nu}_0^2}\bar{\nu}_1 + \frac{(1+4\ln 2)\bar{\nu}_0}{6\bar{c}_0}\bar{b}_1. \tag{A.17}$$

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