Vacuum stability of asymptotically safe two Higgs doublet models

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Abstract We study different types of Two Higgs Doublet Models (2HDMs) under the assumption that all quartic couplings' beta functions vanish simultaneously at the Planck scale. The Standard Model seems to display this property almost accidentally, because the Higgs boson mass is close to 125 GeV. This also ties closely into the question of whether the theory is stable or metastable. We investigate if such “fixed points” can exist in various $\mathbb{Z}_2$ -symmetric 2HDM subclasses, and if the theories that meet these conditions are phenomenologically viable, as well as vacuum stable. We find that the fixed point condition drastically reduces the parameter space of 2HDM theories, but can be met. Fixed points can only exist in type II and type Y models, in regions of large $\tan\beta$, and they are only compatible with all existing experimental bounds if the $\mathbb{Z}_2$ -symmetry is at least softly broken, with a soft breaking parameter of at least $M_{12} > 70$ GeV (380 GeV) for type Y (type II) models. The allowed region falls into the alignment limit, with the mixing angle combination $|\alpha - \beta| \approx \frac{\pi}{2}$. While there are both vacuum-stable and vacuum-unstable solutions, only the vacuum-unstable ones really agree with Standard-Model-like CP-even Higgs boson mass values of 125 GeV. The vacuum-stable solutions favour slightly higher values. While scenarios of asymptotically safe 2HDM exist, they cannot improve over the Standard Model regarding the question of vacuum stability.

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1 Introduction

The discovery and the mass measurement of a Standard Model (SM)-like Higgs boson by ATLAS and CMS in 2012 [1,2] so far rank among the most impactful events in this century's particle physics. It is an interesting situation that the Higgs mass of $m_H = (125.5 \pm 0.5)$ GeV lies right at the edge of the so-called stability bound [3–5]. Extrapolating from the Higgs mass value to very short distances shows that the LHC result seems to hint at a quartic coupling of $\lambda = 0$ at Planck scale-like energies, and similarly the renormalisation group (RG) beta function tends to $\beta_\lambda(m_{Pl}) \sim 0$.

The argument also works in reverse: Before the LHC experiments had discovered a Higgs boson, calculations were performed to show that initial conditions of $\lambda = 0$ and $\beta_\lambda = 0$ at high scales naturally point to Higgs mass values around 125 GeV [6], as do the combination of a vanishing beta function at high scales and the experimental measurement of the top quark mass, or of a vanishing quartic coupling at high scales and the top quark mass [7]. The idea of vanishing beta functions suggests a link to the field of *Asymptotic Safety* [8–10], in which RG flow fixed points play a critical role. Originally an approach to quantum gravity, Asymptotic Safety immediately ensures that theories remain valid up to highest scales. It has in recent years become a point of interest in SM extensions, as a mechanism for UV completion or generalised renormalisability

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[11–16]. Most notably, recent works have emphasized that the interplay of Yukawa interactions with Higgs-like scalar fields is one of the key aspects in understanding Asymptotic Safety [17].

The relation between vanishing quartic couplings and vanishing beta functions at high scales and the measured Higgs mass at the LHC may be coincidental. On the other hand, the question of vacuum stability remains. Experimental results suggest that the SM vacuum is metastable, although agreement on how strong a statement can be made has not yet been reached [18]. Here, we study how models with an enlarged scalar sector behave in this regard. Two Higgs Doublet Models (2HDMs) are arguably among the most minimal and natural extensions of the SM, but nevertheless possess immense phenomenological flexibility and offer avenues to explore common open questions, such as baryon asymmetry [19–21] or dark matter candidates [22–24], among others. At the same time, 2HDMs fit well with the experimental Higgs measurements collected so far [25–27]. On the other hand however, the parameter space introduced by the 2HDM is quite vast, and somewhat complex. To this end, we look at 2HDMs and investigate if they can support the Asymptotic Safety scenario in a way similar to the SM. Specifically, we examine 2HDMs that exhibit simultaneously vanishing quartic coupling beta functions $\beta_{\lambda_i}(\mu)$ at the Planck scale $m_{Pl} = 1.2 \cdot 10^{19}$ GeV. As such, the concept of Asymptotic Safety will function first as a goal in this work, as it is a fundamentally interesting question which theories exhibit UV fixed points under which circumstances. The second step however, is to utilise Asymptotic Safety as a tool: As these fixed point conditions are strong demands in a model with an extended scalar sector, that means that if fixed points can be found, demanding the presence of Asymptotic Safety then becomes a powerful lever to significantly reduce the possible open parameter space of the general 2HDM. If phenomenologically viable asymptotically safe 2HDMs can be found, we then ask if they can improve on the SM in the question of vacuum stability.

This paper is organized as follows: In Sect. 2, general properties of 2HDMs are reviewed. A detailed outline of how the analyses are performed are then given in Sect. 3. Sections 4 and 5 subsequently treat different types of 2HDMs, including the complete softly-broken $\mathbb{Z}_2$ -symmetric model. An Appendix contains the complete two-loop beta functions of all 2HDM couplings used in this work.

2 2HDM

In this section, we briefly review the features of the general 2HDM, before reviewing the current state of bounds on the model from different sources.

2.1 General properties of the 2HDM

A 2HDM contains two SU(2) doublets Φ_1, Φ_2 [28]. The most general scalar potential takes the form:

$$\begin{aligned} V = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \left(M_{12}^2 \Phi_1^\dagger \Phi_2 + h.c. \right) \\ & + \frac{1}{2} \lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2} \lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 \\ & + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) \\ & + \left[\frac{1}{2} \lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + \lambda_6 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_1^\dagger \Phi_2 \right) \right. \\ & \left. + \lambda_7 \left(\Phi_2^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + h.c. \right]. \end{aligned} \quad (2.1)$$

In this notation, following [29], m_{11}, m_{22} , and λ_1 to λ_4 are real-valued, whereas $M_{12}, \lambda_5, \lambda_6$, and λ_7 are complex parameters. Of these 14 degrees of freedom, only eleven are physical. The rest can be absorbed by making use of the freedom of choice of bases for the SU(2) doublets Φ_i .

For spontaneous symmetry breaking (SSB), both fields Φ_1 and Φ_2 are assigned a vacuum expectation value (VEV): $\langle \Phi_i \rangle_0 = \left(0, \frac{v_i}{\sqrt{2}} \right)^\top$, with v_i related to the SM VEV $v \simeq 246$ GeV via

$$v_1^2 + v_2^2 = v^2.$$

The SU(2) doublets contain eight physical fields $\Phi_i = \left(\phi_i^+, \frac{(v_i + \rho_i + i\eta_i)}{\sqrt{2}} \right)^\top$, three of which are absorbed during SSB. The remaining physical Higgs bosons after rotating into mass eigenstates are the charged Higgs $H^\pm$, a pseudoscalar Higgs A and two CP-even scalar Higgs h, H . The rotation angle diagonalising the CP-even scalar mass matrix is conventionally called α , the angle diagonalising the charged and CP-odd bosons is called β . The latter angle β also appears in the ratio of $\frac{v_2}{v_1} \equiv \tan\beta$.

In general, 2HDMs permit tree-level FCNCs. According to the Paschos–Glashow–Weinberg theorem [30,31], a necessary and sufficient condition for their absence is to have all fermions of the same charge and helicity couple to the same Higgs doublet. There are effectively only four different ways of distributing fermions to doublets, as Φ_1 and Φ_2 are inherently interchangeable: A model in which all fermions couple to the same Higgs doublet (usually Φ_2) is called *type I*, a model where up-type quarks couple to Φ_2 and down-type quarks couple to Φ_1 is called *type II*. Aligning the leptons with up-type instead of down-type quarks results in the so-called *lepton-specific* and *flipped* models, or *type X* and *type Y*, respectively. In practice, the different types are usually enforced through discrete $\mathbb{Z}_2$ -symmetries. The exact charge assignments of Higgs and fermion fields are listed in Table 1 [32]. For our purpose the leptons only contribute minor corrections when compared to the quarks,

Table 1 Z_2 charge assignments of Higgs doublets and fermion fields for different 2HDM types

	Φ_1	Φ_2	u^R	d^R	l^R	Q^l, L^l
Type I	-1	1	1	1	1	1
Type II	-1	1	1	-1	-1	1
Type X	-1	1	1	1	-1	1
Type Y	-1	1	1	-1	1	1

so the primary computational focus will be on type I and type II models.

The Yukawa Lagrangian for type I and type II 2HDMs are hence given by:

$$-\mathcal{L}_Y^I = \left(\bar{Q}_i^l \Phi^2 Y_{ij}^d d_j^R + \bar{Q}_i^l \tilde{\Phi}^2 Y_{ij}^u u_j^R + \bar{L}_i^l \Phi^2 Y_{ij}^l l_j^R \right) + h.c., \quad (2.2)$$

$$-\mathcal{L}_Y^{II} = \left(\bar{Q}_i^l \Phi^2 Y_{ij}^d d_j^R + \bar{Q}_i^l \tilde{\Phi}^1 Y_{ij}^u u_j^R + \bar{L}_i^l \Phi^1 Y_{ij}^l l_j^R \right) + h.c., \quad (2.3)$$

where $Y^{u,d,l}$ are the Yukawa matrices for up-, down-, and lepton type particles, Q^l, L^l, u^R, d^R , and l^R are left- and right-handed quark and lepton fields respectively, and i, j denote the generations in flavour space. In our calculations, only the dominant y_{33} entries generated by the top quark, the bottom quark, and the tau lepton respectively, will be considered. Thus, the Yukawa matrices are assumed to have the simplified structures $Y^u = \text{diag}(0, 0, \lambda_t)$, $Y^d = \text{diag}(0, 0, \lambda_b)$ and $Y^l = \text{diag}(0, 0, \lambda_\tau)$.

Under the $\Phi_1 \rightarrow -\Phi_1$ $\mathbb{Z}_2$ symmetry mentioned above, it follows that $\lambda_6 = \lambda_7 = 0$, which leads to a mass matrix for the CP-even neutral scalars of the form:

$$M_{h/H}^2 = \begin{pmatrix} m_{11}^2 + \frac{3}{2}\lambda_1 v_1^2 + \frac{3}{2}\lambda_{345} v_2^2 & -\text{Re}(M_{12}^2) + \lambda_{345} v_1 v_2 \\ -\text{Re}(M_{12}^2) + \lambda_{345} v_1 v_2 & m_{22}^2 + \frac{3}{2}\lambda_2 v_2^2 + \frac{3}{2}\lambda_{345} v_1^2 \end{pmatrix}, \quad (2.4)$$

with $\lambda_{345} = \lambda_3 + \lambda_4 + \text{Re}(\lambda_5)$. The terms m_{11} and m_{22} can be eliminated using the minimum conditions from SSB, that is $\frac{\partial V}{\partial v_i} = 0$:

$$m_{11}^2 v_1 - \text{Re}(M_{12}^2) v_2 + \frac{\lambda_1}{2} v_1^3 + \frac{\lambda_{345}}{2} v_1 v_2^2 = 0, \quad (2.5)$$

$$m_{22}^2 v_2 - \text{Re}(M_{12}^2) v_1 + \frac{\lambda_2}{2} v_2^3 + \frac{\lambda_{345}}{2} v_1^2 v_2 = 0. \quad (2.6)$$

The charged and the pseudoscalar Higgs mass matrices are given by:

$$M_{H^\pm}^2 = \frac{v^2}{v_1 v_2} \left(\text{Re}(M_{12}^2) - \frac{\lambda_4 + \text{Re}(\lambda_5)}{2} v_1 v_2 \right) \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix}, \quad (2.7)$$

$$M_A^2 = \left(\text{Re}(M_{12}^2) - \text{Re}(\lambda_5) v_1 v_2 \right) \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix}. \quad (2.8)$$

Both have one zero eigenvalue, corresponding to the charged and the pseudoscalar Goldstone boson, respectively. The pseudoscalar mass vanishes for $M_{12} = \lambda_5 = 0$, because of an additional accidental spontaneously broken $U(1)$ -symmetry.

2.2 Vacuum stability

In 2HDMs, to be vacuum-stable the potential needs to be bounded from below in all directions. This is the case if and only if the following set of inequalities is met [22]:

$$\begin{aligned} \lambda_1 &> 0, & \lambda_2 &> 0, \\ \lambda_3 + \sqrt{\lambda_1 \lambda_2} &> 0, & \lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2} &> |\lambda_5|. \end{aligned} \quad (2.9)$$

For absolute stability, we require the conditions (2.9) to be fulfilled at all scales up to m_{Pl} . For ease of language, we will usually refer to a potential as *stable*, if it exhibits absolute stability, and as *unstable* otherwise. As will be discussed in more detail in Sect. 4.2, a number of models discussed in this paper exhibit very similar behaviour to the SM, in that a quartic coupling becomes negative, but not very negative, at high scales. To decide if these models are in fact truly unstable or only metastable, we calculate the tunnelling rate to the lower vacuum, and make an estimate on the vacuum lifetime. Unlike the SM, theories with more than one Higgs doublet can display a range of different vacuum configurations [33]: Not only can there be more than one minimum at the same time, but the minima can also be of CP breaking type, when the VEVs have a relative complex phase, or of charge breaking type, with one VEV carrying an electric charge. It has however been shown that minima of different types (i.e. CP-breaking, charge-breaking, or normal) cannot exist simultaneously within the same model [34–36]. By requiring the model to fulfil the minimum conditions for normal-type minima given by Eqs. (2.5) and (2.6), it is therefore assured that the absolute minimum of the theory is also normal. Nevertheless, the possible existence of potentially deeper minima means that even a potential bounded from below could just be a false vacuum state, and lie at risk to decay into the global vacuum. In this scenario, usually referred to as *panic vacuum*, it once again becomes a relevant question if the tunnelling time between the minimal is large compared to the age of the universe. A recent detailed treatment of the relations between different neutral 2HDM minima and the lifetimes of false vacua can be found in [33]. To check if the minimum at $v = 246$ GeV is global, true minimum of the

potential, the following necessary and sufficient condition has been established in [37]:

$$M_{12}^2 \left(m_{11}^2 - \sqrt{\frac{\lambda_1}{\lambda_2}} m_{22}^2 \right) \left(\tan\beta - \sqrt[4]{\frac{\lambda_1}{\lambda_2}} \right) > 0. \quad (2.10)$$

The authors in [33] also consider whether transitions are possible between degenerate vacuum states, i.e. solutions of the form $\{-v_1, -v_2\}$. We leave that possibility aside in this work.

2.3 Limits on 2HDM parameter space

While the 2HDM is a relatively simple SM extension, it still contains up to eleven new free parameters (six in the type II models studied below). On the other hand, the model's high popularity means that its parameter space has been comprehensively explored and constrained from both the theoretical and the experimental side, and in particular by recent LHC data [38–40]. At this point, we briefly review current bounds, more thorough discussions of different aspects can be found for example in [41–47].

In essence, constraints on the 2HDM parameter space can be sorted into three categories: Theory bounds are generated by requiring the model to possess certain features, commonly referred to as *positivity* (the Higgs potential must be bounded from below, cf. Eq. (2.9)), *perturbativity* (quartic couplings must not be large), and *unitarity* (of the S-matrix of $2 \rightarrow 2$ scattering amplitudes) [48, 49]. Secondly, there are mass bounds on the physical Higgs bosons from signal strength data by the ATLAS and CMS collaborations. These searches have confirmed the existence of a 125 GeV CP-even scalar eigenstate, and they also show that this boson couples to vector bosons and fermions in a very SM-Higgs-like fashion [50–53]. Furthermore, the absence of heavier resonances so far translates to mass bounds for the other Higgs eigenstates. Lastly, there are implications for the 2HDM from flavour physics [54]. Most notably, $\mathcal{B}(b \rightarrow s\gamma)$ measurements exclude charged Higgs masses smaller than $m_{H^\pm} = 580$ GeV [55] in type II/type Y models, lower bounds on $\tan\beta$ can be extracted from B_s mass differences and leptonic decays [56].

Together, these bounds can be combined to make a number of statements: The masses of the three additional Higgs bosons all must be large, the mass differences between them, however, small. The rotation angles must fulfil $|\beta - \alpha| \approx \frac{\pi}{2}$, ensuring that the mass basis of the CP-even scalar states aligns with the SM gauge eigenbasis. These features are thus usually referred to as *alignment limit* [57–60]. It should be noted that some studies have used fine-tuning arguments to impose stronger bounds on $\tan\beta$, and successively to the heavy Higgs boson masses [42]. Since the RG methods employed in this work contain a certain degree of tuning by design, they offer an alternative as to why these large $\tan\beta$

regions may yet be phenomenologically viable. As a consequence, our mass bounds are slightly more conservative than some.

3 Solving the fixed point equations

We pursue the question whether 2HDMs support “fixed points” at the Planck scale in the same way the SM does, and if the resulting models are vacuum-stable. The fixed point condition reads:

$$\beta_{\lambda_i}(m_{Pl}) = 0 \quad \forall i, \quad (3.1)$$

i.e., the beta functions of all quartic couplings λ_i present in the scalar potential are to have a root at the Planck mass m_{Pl} . Because of contributions from gauge and Yukawa couplings, the condition of $\beta_{\lambda_i} = 0$ is not technically sufficient to define a fixed point, nor does it necessarily lead to an asymptotically safe theory by itself; for recent progress in BSM model building see e.g. [11]. Still, because of similarities to the SM case and for convenience, the terms *fixed point* and *fixed point condition* will be used in this context, effectively interpreting effects disturbing the equilibrium into the realm of beyond the Planck scale physics.

While in the SM there is only one quartic Higgs coupling λ , the 2HDM potential with a $\mathbb{Z}_2$ symmetry protecting flavour conservation can contain up to five quartic terms (one of which may be complex). This means that compared to the SM, the fixed point condition has a much higher impact in terms of limiting the parameter space of the theory.

The search for fixed points comes down to solving the system of differential equations given by the beta functions of the running couplings of the theory. It involves the gauge couplings g_1, g_2, g_3 , the quartic scalar couplings λ_i and the Yukawa couplings $\lambda_t, \lambda_b, \lambda_\tau$. The complete two-loop expressions for the most general beta functions used are calculated with the Mathematica package SARAH [61, 62], and listed in Appendix A. Since the coefficients m_{11} and m_{22} of the dimension-two-operators do not appear directly in the beta functions of any other couplings, m_{ii} can be ignored at this point and determined with help of the minimum conditions at the electroweak scale, see Eqs. (2.5) and (2.6). The soft breaking parameter M_{12} also does not appear in the beta functions of quartic, gauge, or Yukawa couplings, and will be treated as a free parameter.

While the quartic couplings are already fixed implicitly by (3.1), the remaining initial conditions are given explicitly at low scales: Both gauge couplings and Yukawa couplings can be determined from experimental measurements. The $\overline{\text{MS}}$ gauge coupling initial conditions for g_1 and g_2 are calculated using the fine structure constant $\alpha^{-1}(M_Z) = 127.95 \pm 0.017$ and the weak mixing angle $\sin^2\theta_W = 0.23129 \pm 5 \cdot 10^{-5}$ [63, 64] to:

$$g_1(M_Z) = 0.35, \quad g_2(M_Z) = 0.65, \quad g_3(M_Z) = 1.2.$$

Uncertainties on gauge coupling initial values, including the strong coupling, are small enough to be inconsequential. The relations between Yukawa couplings and quark masses are model-dependent. In a type II model, the Yukawa couplings are related to the quark masses by the tree-level relations:

$$\lambda_t(m_t) = \frac{\sqrt{2}m_t}{v_2}, \quad \lambda_b(m_b) = \frac{\sqrt{2}m_b}{v_1}, \quad \lambda_\tau(m_\tau) = \frac{\sqrt{2}m_\tau}{v_1},$$

while in the type I model the bottom quark Yukawa coupling is instead determined by the other VEV: $\lambda_b(m_b) = \frac{\sqrt{2}m_b}{v_2}$. The $\overline{MS}$ quark masses used are $m_b(m_b) = 4.18 \pm 0.03$ GeV and $m_t(m_t) = 160^{+4.8}_{-4.3}$ GeV, the τ -lepton mass is given by $m_\tau(m_\tau) = 1.78$ GeV [64]. For the purpose of this paper, it is assumed that all non-SM effects only affect the running from the electroweak scale onwards. In other words, the Yukawa couplings are run up to M_Z under SM-like conditions, at which point the $\tan\beta$ -enhancement is switched on. The effective initial values used are thus:

$$\lambda_t(M_Z) = \frac{0.95}{\sin\beta}, \quad \lambda_b(M_Z) = \frac{0.176}{\cos\beta}, \quad \lambda_\tau(M_Z) = \frac{0.98}{\cos\beta}.$$

The so defined initial value problem is solved numerically.

With the full RG flow of all couplings known, their low scale values are used to determine the mass spectrum of the theory using the matrices given in Sect. 2. The results depend on the parameters treated as free (in these cases $\tan\beta$ and later M_{12}) and on the experimentally determined coupling initial conditions, but beyond this are a direct consequence of the theory itself and the fixed point assumption.

With all couplings known at all scales, the question of vacuum stability can also be answered: For a solution to be absolutely vacuum-stable, the quartic couplings must fulfil the inequalities given by Eq. ((2.9)) at all energy scales up to $\mu = m_{Pl}$.

4 The CP-conserving 2HDM with $\mathbb{Z}_2$ symmetry

We study a 2HDM with a discrete $\mathbb{Z}_2$ symmetry ($\Phi_1 \rightarrow -\Phi_1$, $\Phi_2 \rightarrow \Phi_2$) introduced in order to ensure CP-conservation in the scalar sector. The scalar potential reads:

$$\begin{aligned} V = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 \\ & + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\ & + \frac{1}{2} \lambda_5 \left[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right], \end{aligned} \quad (4.1)$$

with real-valued mass parameters m_{ii} and quartic couplings λ_i . The restriction $\text{Im}(\lambda_5) = 0$ does not follow immediately from the $\mathbb{Z}_2$ symmetry, but can be assumed in this case without loss of generality due to the structure of the beta functions: λ_5 always appears in the other quartic couplings' beta functions in the form of the norm squared, $|\lambda_5|^2$, and the function β_{λ_5} is proportional to λ_5 , with the remainder being comprised by real-valued terms only. It follows that the function β_{λ_5} must have a constant phase. It is also easy to see that in these circumstances for every function $\lambda_5(\mu)$ that solves the system of differential equations, a phase-shifted $e^{i\theta} \cdot \lambda_5(\mu)$ is also a solution for every constant phase θ . Therefore, it suffices to look at the real values for λ_5 when searching for fixed points.

The different Yukawa Lagrangians in type I and type II models lead to differences in the beta functions, as the Yukawa terms will appear in the running of different couplings. As an example, in the type I model one-loop beta function of quartic coupling λ_1 can be written as:

$$\begin{aligned} \beta_{\lambda_1}^{1l} = & \frac{1}{64\pi^2} \left(6g_1^2 g_2^2 + (3g_2^2 - 6\lambda_1)^2 + 3(g_1^2 - 2\lambda_1)^2 \right. \\ & \left. + 8(6\lambda_1^2 + \lambda_3^2 + (\lambda_3 + \lambda_4)^2 + \lambda_5^2) \right). \end{aligned} \quad (4.2)$$

The right hand side is expressed as a sum of perfect squares. Furthermore, it does not contain any Yukawa terms, as in type I models, Φ_1 does not couple to any fermions. As was also discussed in [17], Yukawa couplings are often times indispensable for enabling fixed points, as they can be the only terms with a negative sign in the beta functions. The consequence of this is that the $\beta_{\lambda_1}^{1l}$ above will never vanish, and type I models are excluded as viable candidates, when looking for 2HDMs with fixed points. As a corollary, models like the one discussed in [65] extending the SM by a singlet can also not support fixed points in their new quartic coupling.

The full two-loop beta functions for the type II $\mathbb{Z}_2$ -symmetric 2HDM are given in Appendix A. In this model, $\tan\beta$ is a free parameter that fixes the exact starting conditions. For fixed points to exist, Yukawa contributions to both Higgs field have to be big enough. This translates into a lower bound on $\tan\beta$, as the bottom-type Yukawa couplings in a type II model are $(\sin\beta)^{-1}$ -enhanced compared to the SM [cf. Eq. (4.2)]. On the other hand, $\tan\beta$ cannot be too large either, or the running Yukawa couplings will become divergent below the Planck scale. Accordingly, there is a $\tan\beta$ -interval in which fixed points can exist. In this two-loop framework, fixed points can be found for:

$$\tan\beta \in [60, 70]. \quad (4.3)$$

The beta functions being of polynomial form, there is usually a finite set of fixed point solutions. We find that there are

Fig. 1 Mass eigenstates of the heavier CP-even neutral scalar Higgs boson H in the type II $\mathbb{Z}_2$ -symmetric model. Vacuum-stable solutions are shown as blue circles with dashed borders, vacuum-unstable solutions as violet triangles, with the black vertical line marking the transition point. The uncertainty estimate is given by the difference between results for calculating the running of all couplings at one-loop and two-loop level, respectively

two branches: One branch is vacuum-stable for all values of $\tan\beta$ over all scales, the other one breaks the condition $\lambda_2 > 0$ at higher scales for certain values of $\tan\beta$, rendering the corresponding potential unbounded from below. For simplicity, we will in this work refer to the two branches as the *vacuum-stable* and the *vacuum-unstable* one, respectively. Before we take a closer look at the exact nature of the vacua in Sect. 4.2, the values of λ_i can be translated into a spectrum of masses of the physical Higgs bosons using Eqs. (2.4)–(2.7).

4.1 Higgs boson mass spectra

The masses for all three physical bosons in this model are shown in Figs. 1, 2 and 3, with the vacuum-stable cases shown as blue circles, and the vacuum-unstable solutions marked with purple triangles. The critical $\tan\beta$ value for stability in the lower branch is marked by a black vertical line. For $\tan\beta$ values greater than this, λ_2 in the unstable branch will not stay positive at all scales up to the fixed point. An estimate for the theoretical uncertainty is given by the differences between performing the RG evolution of all couplings at one-loop or at two-loop level, before determining the masses using Eqs. (2.4)–(2.7).

It becomes quickly evident that the mass spectrum produced is not in line with experimental bounds. The CP-even neutral scalar mass eigenstates are split apart wide due to the large parameter $\tan\beta$: without significant mixing, one of the eigenstates is of the order of v_1 , the other of the order v_2 . The heavier CP-even eigenstate takes on mass values of around 125–130 GeV for the (mostly) vacuum-unstable branch, and 135–140 GeV for the stable one. The lighter CP-even eigenstate lies in the O(1) GeV region, which is ruled out experimentally. This will be addressed in Sect. 5.

Fig. 2 Mass eigenstates of the light CP-even neutral scalar Higgs boson h in the type II $\mathbb{Z}_2$ -symmetric model, as labelled in Fig. 1

Fig. 3 Mass eigenstates of the charged Higgs boson H^+ in the type II $\mathbb{Z}_2$ -symmetric model, as labelled in Fig. 1

The heavier CP-even eigenstate in the vacuum-unstable case has roughly the correct mass to be considered as a candidate for a SM-like Higgs. However, while it is theoretically possible that the observed Higgs boson at 125 GeV is the *heavier* of the two CP-even eigenstates, this configuration is heavily disfavoured by experimental observations, due to strong bounds from the $H \rightarrow hh$ decay [66]. It is therefore usually assumed in 2HDMs that the 125 GeV Higgs is the lighter of the two CP-even neutral eigenstates.

The charged Higgs boson mass is below 200 GeV, which falls into the regions excluded by $\bar{B} \rightarrow X_s \gamma$ measurements, as mentioned in Sect. 2.3. The pseudoscalar mass is not shown, because it vanishes completely: As it turns out, all fixed point solutions contain $\lambda_5 \equiv 0$. In this case (with M_{12} forbidden by the $\mathbb{Z}_2$ -symmetry), this means that the model displays an accidental U(1)-symmetry which forces the pseudoscalar into the role of a pseudo-Goldstone boson, and hence to become massless. The mixing angle α is close to zero in all cases.

Together with the large $\tan\beta$ values, this ensures that the type II alignment limit condition of $|\beta - \alpha| \sim \frac{\pi}{2}$ (cf. Sect. 2.3) is always met, as is shown in Fig. 4.

Fig. 4 Mixing angle difference $|\beta - \alpha|$ with difference between one-loop and two-loop fixed point results as uncertainty. The vacuum-stable and -unstable branches are marked as labelled in Fig. 1

To summarise, the mass spectrum produced by the solutions to the fixed point equations in the $\mathbb{Z}_2$ -symmetric 2HDM exhibits a potential SM-Higgs candidate in the vacuum-unstable branch, but is excluded by experimental observations because of the remaining boson spectrum. Both the light CP-even scalar and the charged Higgs boson are clearly too light to present a phenomenologically viable model. We fix this problem in the next section. Performing the same analysis in a type-Y 2HDM leads to generally analogous results at slightly elevated $\tan\beta$ -values.

4.2 Vacuum stability of the fixed point solutions

As alluded to in Sect. 2.2, further steps need to be taken to answer the question of vacuum stability beyond the first broad classification given by (2.9). While the following discussions are presented using the example of the fixed point solutions in the $\mathbb{Z}_2$ -symmetric model, the results can easily be generalised and hold true for the softly-broken model as well.

In the case of the vacuum-stable branch of solutions, the 2HDM potential is bounded from below at all scales. However, it is not yet ensured that the minimum at $v = 246$ GeV is the global minimum, or if there is another, deeper one. Since all couplings are known, checking condition (2.10) is only a matter of plugging in the values at the correct scale. We find that our stable branch solutions all satisfy Eq. (2.10) easily. While there usually exists another normal minimum, the SM-like minimum is always the deeper, global one.

In the case of the vacuum-unstable solutions, the situation turns out to be slightly more complex. With growing values of $\tan\beta$, λ_2 at the fixed point can become negative. Examining the RG evolution of $\lambda_2(\mu)$ to low scales shows that it only becomes negative at high energy scales. The instability scales for different values of $\tan\beta$ are shown in Fig. 5 (left plot). As can be inferred from that plot, the instability issue becomes worse with growing $\tan\beta$.

This behaviour is reminiscent of the SM quartic coupling. To decide if the models are really vacuum-unstable, the lifetime of these vacua has to be compared to the age of the universe. The tunnelling probability is calculated by using a semiclassical approximation [67], given as

$$p \simeq \left(\frac{T_U}{R}\right)^4 e^{-S_0}, \quad (4.4)$$

where T_U is the age of the universe taken as $T_U \simeq 10^{10}$ years, S_0 is the Euclidean bounce action, and R is a dimensional factor. We combine our two-loop RG-improved running couplings with a one-loop Coleman–Weinberg effective potential approximation. A very detailed account of such calculations performed for the SM case to a much higher degree of precision is given in [67, 68], a similar approach has been applied to the 2HDM in [69]. The effective potential can be approximated as

$$V_{eff}(\varphi_1, \varphi_2) \simeq \frac{\lambda_2^{eff}}{8} \varphi_2^4, \quad (4.5)$$

with λ_2^{eff} absorbing all one-loop terms V_{eff}^{1l} , using the assumption that φ_2 contributes dominantly. All squared mass terms in V_{eff} can be expressed through the known running couplings, which are evaluated at the scale $\varphi_2 = \mu$.

The result is shown in Fig. 5 (right) for the value of $\tan\beta = 66$, which was the most unstable point in our solution space. The purple line marks the metastability bound, below which the tunnelling probability (4.4) becomes larger than one, and a vacuum catastrophe is expected to take place. The blue band shows the full evolution of λ_2^{eff} , evaluated at two-loop RG and one-loop effective potential level. Even in this “most unstable” case, it stays above the instability region.

We find that in our fixed point solutions, the bounded-from-below conditions (2.9) are not broken severely enough to really make the model absolutely unstable. Instead, the situation seems to mirror the SM case: in our models, the solutions labelled “unstable” really fall in the metastability region. They do however cut quite close to the instability boundary.

It should be noted that especially compared to the SM, the 2HDM calculations presented above do not yet operate with the same degree of precision. Comparing one-loop and two-loop results in the both plots of Fig. 5 suggests that the theoretical uncertainties are too large to make concrete statements about vacuum stability. A full analysis combining the different approximations consistently to a higher order could be necessary and very interesting.

For this reason as much as for clarity’s sake, we will continue to employ the terminology of “stable” and “unstable” in this work based on whether the conditions (2.9) are fulfilled when distinguishing between branches of fixed point

Fig. 5 Left: scale μ_{crit} at which the quartic coupling $\lambda_2(\mu)$ turns negative as a function of $\tan\beta$. The two-loop RG improved $\lambda_2^{2l}(\mu)$ is shown as a blue line, the one-loop result as comparison is shown dashed in purple. Right: RG flow of the full quartic coupling $\lambda_2^{eff}(\mu)$ (blue line)

solutions, regardless of their precise nature as discussed in this section.

5 The 2HDM with softly broken $\mathbb{Z}_2$

In Sect. 4 we have found fixed points, but the resulting models are all experimentally excluded due to small mass values. To allow for larger, phenomenologically viable masses for m_h , m_{H^+} , and m_A , it is necessary to go beyond the $\mathbb{Z}_2$ -symmetric model. The least invasive way to generate heavier masses is to include the so-called *softly-broken* $\mathbb{Z}_2$ -symmetric 2HDMs. The assumption of a $\mathbb{Z}_2$ -symmetry under the transformation $\Phi_1 \rightarrow -\Phi_1$ is not completely dropped, but a mass term $M_{12}^2(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1)$ mixing between the two Higgs field is allowed.

Like the other mass parameters, M_{12} does not appear in any quartic, gauge, or Yukawa beta function. It can instead be treated as a free parameter. For this reason, M_{12} does not influence the fixed point search itself. Phenomenologically, on the other hand, the mixing parameter can have a big impact, especially in the case of $\lambda_5 = 0$ observed in our models. The additional global U(1) symmetry is now broken by non-vanishing M_{12} -terms, which allows the pseudoscalar Higgs boson to acquire mass. Additionally, three of the other four bosons grow approximately linear with M_{12} , which allows them to evade experimental constraints. The influence of M_{12} on the different boson masses for the vacuum-unstable branch is illustrated in Fig. 6. The behaviour is analogous in the vacuum-stable branch.

As can be seen in the plot, the CP-even neutral scalar eigenvalues (blue/violet) depend on M_{12} in different ways [cf. Eq. (2.4)]. The SM-like eigenstate (originally m_H) only shows a minor dependence, and hardly changes even for very large values of M_{12} . For the originally smaller eigenstate however, M_{12} can easily become the dominating contributor.

Fig. 6 Higgs boson masses for a softly broken $\mathbb{Z}_2$ -symmetry for $\tan\beta = 64$ against M_{12} in GeV for the vacuum-unstable branch of solutions. The masses shown correspond to the CP-even scalars m_h , m_H (violet squares, blue circles), the charged Higgs m_{H^+} (yellow diamonds) and the pseudoscalar m_A (green line)

While the original mass of this state was mainly generated by a small VEV v_2 , it soon starts to grow almost linearly with M_{12} , surpassing the mass of the former heavier eigenstate in a level crossing at values of roughly $M_{12} \approx 20$ GeV.

Both the charged Higgs (yellow) and the pseudoscalar Higgs (green) also grow together with M_{12} , and adopt an asymptotically linear behaviour as M_{12} becomes large. For the pseudoscalar, the linear dependence is actually exact as long as $\lambda_5 = 0$, as it is the case with our fixed point solutions. The mass eigenvalue in (2.7) then simplifies to:

$$m_A = \frac{M_{12}}{\sqrt{\sin\beta \cos\beta}} \sim M_{12} \sqrt{\tan\beta}. \quad (5.1)$$

For this reason the pseudoscalar mass m_A in Fig. 6 is shown as a straight line.

Compared to M_{12} , $\tan\beta$ has only a minor impact on the heavy boson masses. Figure 7 shows the SM-like M_{12} -independent eigenstate, henceforth referred to as m_h , with

the vacuum-stable case on the left and the vacuum-unstable case on the right. The distortion around $M_{12} \sim 18$ GeV arises from the point where the CP-even eigenvalues become degenerate. Figure 8 shows the corresponding plots for the M_{12} -dependent CP-even eigenstate m_H on the left for m_{H^+} on the right. Displayed in the figure is the vacuum stable case; the differences to the vacuum-unstable solution branch is absolutely minimal. The SM-like Higgs in Fig. 7 is the only eigenstate for which there remains a significant difference in stable and unstable branch. The pseudoscalar mass is not shown. The behaviour of m_A is nearly indistinguishable from m_H in these regions, as demonstrated already in Fig. 6. For the heavy bosons, contour lines are drawn in 25 GeV Intervals.

A finite M_{12} has a number of implications on the validity of the theory. Most importantly, it has the anticipated effect of allowing the model to produce phenomenologically viable mass spectra by opening up a way to drive m_h , m_A and m_{H^+} to higher values. The experimental bounds on the physical Higgs bosons can be translated to a lower bound on M_{12} . From the bound of $m_{H^+} > 580$ GeV [55] in type-Y models it follows that:

$$M_{12} \gtrsim 70 \text{ GeV}, \quad (5.2)$$

with the exact value depending on $\tan\beta$. Re-translated, this condition implies in terms of other boson masses:

$$m_A, m_H \geq 550 \text{ GeV}. \quad (5.3)$$

Because of the mixing angle α being close to zero, the SM-like Higgs mass stays almost unchanged. This means that in terms of vacuum stability, the situation also remains consistent with the $\mathbb{Z}_2$ -symmetric case: While there are vacuum-stable solutions to the fixed point equations, only the vacuum-unstable ones include masses around 125 GeV. The SM-like Higgs is thus in a unique position among the 2HDM bosons, in that its mass cannot be heavily adjusted in this model.

Whereas the SM-like CP-even scalar eigenstate is independent of M_{12} , the opposite is true for all other bosons: Even at its minimum, $M_{12} \sim 70$ GeV is already large enough to make it the controlling factor in generating the masses of the three bosons H , H^+ and A . For larger values of M_{12} , the degeneracy in masses becomes even stronger. Therefore, most of the parameter space of viable asymptotically safe 2HDMs falls into the decoupling limit [57], with one SM-like and three heavy bosons with $m_H \approx m_{H^+} \approx m_A \propto M_{12}$.

The high $\tan\beta$ -values necessary to find fixed points mean that in type II models specifically, bounds from $B_s \rightarrow \mu\mu$ decays are much more restrictive [56, 70]. They demand heavy boson masses upwards of

$$m_H \approx m_{H^+} \approx m_A > 3 \text{ TeV}. \quad (5.4)$$

5.1 Stability analysis

In order to understand the characteristics of a given fixed point, we study the linearised RG flow around the fixed point, described by the stability matrix given by:

$$M_{ij} = \left. \frac{\partial \beta_i}{\partial g_j} \right|_{FP}. \quad (5.5)$$

In this case, g_j includes gauge, Yukawa and quartic couplings. The number of negative eigenvalues of M_{ij} corresponds to the dimension of the critical surface from which trajectories run into the fixed point. However, it has less significance here: While it is important to confirm that the fixed points are indeed UV-attractive (which they are), both the exact fixed point scale and the low scale initial conditions give additional constraints that intersect non-trivially with the critical surface. The solution to Eq. (3.1) is always a single trajectory in parameter space. On the other hand, by construction our method of finding fixed points ensures that the solutions found connect to the critical surface.

It is therefore necessary to examine which of the initial conditions used is subject to uncertainties, and how these translate to changes in fixed point solutions and thus in Higgs boson mass spectra.

5.2 Uncertainty estimates

There are several factors that influence the fixed point analysis. Below, we look at changes in scale where the fixed point condition is applied, followed by a discussion about low scale top and bottom quark mass uncertainties. Unless stated differently, values for the SM-like CP-even scalar eigenstate (here h) will be evaluated using $M_{12, \text{Min}} = 380$ GeV. The pseudoscalar A and M_{12} -dependent CP-even scalar eigenstate H are entirely or almost entirely generated by the free parameter M_{12} and therefore have negligible uncertainties.

In general, the models are studied with the condition that the quartic coupling beta functions become zero at $m_{Pl} = 1.2 \cdot 10^{19}$ GeV. The mass spectrum shows a minor dependence on where exactly the fixed points are assumed to occur. The masses m_h are shown in Fig. 9 for different fixed point scales. We see that lower fixed point scales correspond to a larger difference between vacuum-stable (upper) and vacuum-unstable (lower) branches, and notably bring down the lower branch mass values. Also, the $\tan\beta$ -range in which fixed points can be found changes with the fixed point scale: when the fixed points scale is chosen at higher values than m_{Pl} , the divergence of large Yukawa couplings, especially y_b becomes an even more pronounced problem.

The mass spectra depend on the initial values chosen for the gauge and Yukawa couplings. The dependency on the top quark mass turns out to be especially strong. In Fig. 10 the

Fig. 7 Masses of M_{12} -independent CP-even neutral scalar m_h as a function of $\tan\beta$ and the soft breaking parameter M_{12} . The vacuum-stable branch is shown on the left, the vacuum-unstable branch on the right

Fig. 8 Masses of M_{12} -dependent CP-even neutral scalar m_H (left) and charged Higgs boson m_{H^+} (right) as a function of $\tan\beta$ and the soft breaking parameter M_{12}

Fig. 9 Masses of SM-like CP-even scalar Higgs boson, for $M_{12} = 380$ GeV and fixed point condition set at different scales of 10^{18} (circles), 10^{19} (triangles), and 10^{20} (diamonds) GeV. The upper (blue) branches correspond to the vacuum-stable solutions, the lower (purple) branches to the vacuum-unstable ones

SM-like Higgs boson masses are shown for the 1σ deviation bands of the $\overline{MS}$ top and bottom quark masses of $m_t(m_t) = (160^{+4.8}_{-4.3})$ GeV and $m_b(m_b) = (4.18 \pm 0.03)$ GeV [64]. For the charged Higgs mass, the uncertainty generally grows with $\tan\beta$, but also depends on the soft breaking parameter, as the quartic coupling contributions to the boson mass weaken with $\sqrt{M_{12}}^{-1}$. As such, the influence of quark initial values on the uncertainties becomes smaller with growing heavy boson masses. The absolute sizes of uncertainty bands Δm_{H^+} are shown against the central mass value m_{H^+} in Fig. 11. The vertical width of the bands in this plot indicates how Δm_{H^+} depends on $\tan\beta$. It has to be noted that the $\tan\beta$ -interval shown in the first (left) plots of Figs. 10 and 11 is smaller than the intervals shown in the corresponding right hand plot, or in Figs. 1, 2, 3, 4: the reason is that different Yukawa initial values not only change the mass spectrum, but also the region

in parameter space for which a fixed point exists: On one hand, smaller quark masses mean that the Yukawa couplings require larger $\tan\beta$ to fulfil the fixed point condition. On the other hand, larger quark masses move the Landau pole of the Yukawa couplings to lower scales. The first effect is very noticeable for lower values of m_t . This effect is also a reason why the variance in uncertainties at fixed values of M_{12} is much smaller in the left plot of Fig. 11 compared to the right one: As fixed points for all top quark initial values can only be found over a very small interval of $\tan(\beta)$, the influence on of $\tan(\beta)$ on m_H^+ at a fixed value of M_{12} stays limited.

Figure 12 shows a typical set of running quartic couplings each for the vacuum-stable and unstable branch and how different top quark mass initial values influence λ_i . For most couplings, the m_t -induced relative uncertainty becomes smaller as the scale decreases, even if the coupling itself becomes larger. This is most notable for λ_1 (blue) and λ_2 (violet). The quartic coupling λ_5 is not shown in these graphs, as $\lambda_5 = 0$ is an exact solution of the fixed point equations regardless of initial conditions. A last important detail to take note of is the range of λ_2 in the right graph: depending on the exact initial conditions, λ_2 may become negative (and thereby break the vacuum stability conditions of Eq. (2.9)) as early as 10^8 GeV, or not at all. To further analyse the influence of the exact starting parameters on the Higgs mass, all SM-like Higgs mass values generated by a fixed point solution can be shown against the corresponding top quark Yukawa initial value. Figure 13 shows the Higgs mass m_h for the 1σ regions of m_b and m_t for two different values of $\tan\beta$. The vertical spread in points is generated by shifting the bottom quark initial value. Once again, all vacuum-stable points are coloured blue, the vacuum-unstable ones violet. The right hand graph in particular illustrates well the two branches of fixed point solutions: The branch corresponding to higher Higgs boson masses is fully vacuum-stable, whereas the lower mass branch can only be vacuum-stable for top quark mass values on the lower end of its 1σ band. Comparing both graphs also shows how $\tan\beta$ needs to be of certain size to facilitate the existence of fixed points. For the central mass values of m_b and m_t , there are no fixed points at $\tan\beta = 60$. However, as shown in the left graph, fixed points can be found there if either or both initial values are slightly larger.

6 Summary

Proposing simultaneously vanishing quartic coupling beta functions at the Planck scale severely constrains the 2HDM parameter space, but is possible in a way similar to the SM. As such, the 2HDM likewise supports the idea of being extended to high scales through means of asymptotic safety.

The parameter $\tan\beta$ needs to be large, as both the up-type and the down-type Yukawa couplings have to be large in order to keep the positive contributions in quartic couplings beta functions in check. For the same reason, only type II and type Y models are viable, while type I and type X models are not. In the type II/type Y models studied, there always exists a $\tan\beta$ -interval in which fixed points can be found, see Eq. (4.3). Fixed points can be found in $\mathbb{Z}_2$ -symmetric models, but the resulting mass spectra are not phenomenologically viable. The most minimal model that also agrees with all experimental bounds is the softly-broken $\mathbb{Z}_2$ -symmetric 2HDM, cf. Figs. 6, 7 and 8.

The allowed parameter region defined by the fixed point assumption meets the characteristics of the decoupled alignment limit, with three heavy Higgs bosons $m_H \approx m_A \approx m_{H^\pm} \propto M_{12}$, and $|\beta - \alpha| \approx \frac{\pi}{2}$. To be consistent with experimental constraints, a lower bound is given on $M_{12} > 70$ GeV (380 GeV) for type Y (type II) models, corresponding to the charged Higgs limits for type Y. This implies lower limits on m_A and m_H , see Eqs. (5.3) and (5.4).

Similar to the SM, both the existence of fixed points and the vacuum stability depend strongly on the low scale initial values, most notably the exact top quark mass. Even more similar to the SM is the fact that while the bounded-from-below conditions (2.9) are sometimes violated in fixed point solutions, the violation only happens at high scales, and is mostly weak enough for the potential to remain in a metastable region, cf. Fig. 5. As illustrated in Fig. 13, the central values of Higgs and top quark mass indicate an unstable/metastable vacuum, but are rather close to the criticality border. Fixed point solutions with absolute stability can be possible by having $\overline{\text{MS}}$ top quark mass values lower than 160 GeV [64] or the fixed point scale set below $m_{Pl} = 1.2 \cdot 10^{19}$ GeV. A more precise determination of m_t and m_h by future experiments will allow a more definite statement.

In the end, the fact that Asymptotic Safety in the 2HDM is supported exactly in the parameter space region in which the 2HDM behaves most SM-like is intriguing. However, the extremely high values of $\tan\beta$ necessary to make fixed points work only just allow the model to exist in accordance with current experimental bounds. It would be interesting to see how the 2HDM behaves when combined with the types of SM-extensions that can support more complete UV-interactive fixed points in the gauge-Yukawa sector. An extended (possibly dark) fermion sector in particular could allow for easier model building, if it can lift some of the burden put on the SM Yukawa couplings by the fixed point conditions, and by extension on $\tan\beta$.

As it stands, the 2HDM does not solve the stability problem of the SM. Instead, the situation there too is a mirror to the SM. The branches of fixed point solutions with promising SM-like Higgs mass candidates seem to correspond to metastable vacuum potentials. A thorough NNLO analysis

Fig. 10 Masses of SM-like CP-even Higgs boson evaluated at $M_{12} = 380$ GeV with 1σ uncertainty regions from top quark (left) and bottom quark (right) mass initial values of $(160^{+4.8}_{-4.3})$ GeV and (4.18 ± 0.03) GeV respectively. Stable solutions are marked in blue with dashed outline, unstable solutions in violet

Fig. 11 Dependence of the sizes of the uncertainty bands Δm_{H^+} on the charged Higgs mass when driven by M_{12} . The left (right) plot shows the size of Δm_{H^+} generated by the 1σ top (bottom) quark uncertainty.

Stable solutions are marked in blue with dashed lines, unstable solutions in violet. The width of the bands is the variance in uncertainty generated by working at different values of $\tan(\beta)$

Fig. 12 Running quartic couplings with 1σ uncertainty intervals from low scale top quark mass initial value for the vacuum-stable (left) and vacuum-unstable (right) fixed point branch. The colours correspond to: λ_1 (blue), λ_2 (violet, dashed), λ_3 (yellow, dotted) and λ_4 (green, dot-dashed)

Fig. 13 Mass region of the SM-like CP-even scalar Higgs boson from 1σ quark mass uncertainties against the top Yukawa initial value $y_t(M_Z)$ for $\tan\beta = 60$ (left) and $\tan\beta = 64$ (right). Stable solutions are marked in blue, unstable solutions in violet

of the metastability border may be helpful to achieve clarity in this matter.

Note added: During the final phase of this project, an analysis of 2HDM fixed points using slightly different methodology appeared [71]. While our approach differs in details, we agree with the general conclusion that fixed points in type II 2HDMs are possible.

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Appendix: β functions

In this section, the β functions $\beta_{g_i} = \frac{dg_i(\mu)}{d \log(\mu)} = \mu \frac{dg_i(\mu)}{d\mu}$ are listed on 2-loop level for the most general model used (i.e. the softly-broken $\mathbb{Z}_2$ -symmetric 2HDM type II, which means that λ_6 and λ_7 do not appear). They were calculated with the Mathematica package SARAH [61, 62]. The general procedure of how to derive 2-loop RGEs for general field theories has been outlined in [72–74].

In the case of Yukawa couplings, first and second generation contributions have been neglected. The Yukawa matrices have therefore been restricted to their respective (3, 3)-entries.

The β functions for the gauge couplings are given by:

$$16\pi^2 \beta_{g_1} = 7g_1^3 + \frac{1}{288\pi^2} \left(g_1^3 (208g_1^2 + 3(36g_2^2 + 88g_3^2 - 5\lambda_b^2 - 15\lambda_\tau^2 - 17\lambda_t^2)) \right), \quad (\text{A.1})$$

$$16\pi^2 \beta_{g_2} = -3g_2^3 + \frac{1}{32\pi^2} \left(-g_2^3 (-4g_1^2 - 16g_2^2 - 24g_3^2 + 3\lambda_b^2 + \lambda_\tau^2 + 3\lambda_t^2) \right), \quad (\text{A.2})$$

$$16\pi^2 \beta_{g_3} = -7g_3^3 + \frac{1}{96\pi^2} \left(-g_3^3 (3(4(13g_3^2 + \lambda_b^2 + \lambda_t^2) - 9g_2^2) - 11g_1^2) \right). \quad (\text{A.3})$$

The β functions for the quartic Higgs couplings λ_i for the softly-broken type II 2HDM are given by:

$$16\pi^2 \beta_{\lambda_1} = \frac{1}{4} \left(6g_1^2 (g_2^2 - 2\lambda_1) + 3g_1^4 - 36g_2^2 \lambda_1 + 9g_2^4 \right.$$

$$\begin{aligned} &+ 8 \left(2\lambda_1 (3\lambda_b^2 + \lambda_\tau^2) + 6\lambda_1^2 + 2\lambda_3 \lambda_4 \right. \\ &+ 2\lambda_3^2 + \lambda_4^2 + \lambda_5^2 - 6\lambda_b^4 - 2\lambda_\tau^4 \left. \right) \\ &+ \frac{1}{384\pi^2} \left[g_1^2 (6g_2^2 (39\lambda_1 + 20\lambda_4 \right. \\ &+ 36\lambda_b^2 + 44\lambda_\tau^2) - 303g_2^4 \\ &+ 4(25\lambda_1 (\lambda_b^2 + 3\lambda_\tau^2) + 108\lambda_1^2 \\ &+ 4(6(2\lambda_3 \lambda_4 + 2\lambda_3^2 + \lambda_4^2) \\ &- 3\lambda_5^2 + 4\lambda_b^4 - 12\lambda_\tau^4)) \left. \right) + g_1^4 (-573g_2^2 \\ &+ 651\lambda_1 + 60(2\lambda_3 + \lambda_4 + \lambda_b^2 - 5\lambda_\tau^2)) \\ &- 393g_1^6 + 3(-3g_2^4 (17\lambda_1 \\ &+ 4(-10\lambda_3 - 5\lambda_4 + 3\lambda_b^2 + \lambda_\tau^2)) \\ &+ 12g_2^2 (5\lambda_1 (3\lambda_b^2 + \lambda_\tau^2) + 36\lambda_1^2 \\ &+ 4(2\lambda_3 + \lambda_4)^2) + 291g_2^6 \\ &- 8(\lambda_1 (-80g_3^2 \lambda_b^2 + 20\lambda_3 \lambda_4 + 20\lambda_3^2 \\ &+ 12\lambda_4^2 + 14\lambda_5^2 + 3\lambda_b^4 + \lambda_\tau^4) \\ &+ 4(16g_3^2 \lambda_b^4 + \lambda_5^2 (10\lambda_3 + 11\lambda_4) \\ &+ 8\lambda_3 \lambda_4^2 + 6\lambda_3^2 \lambda_4 \\ &+ 4\lambda_3^3 + 3\lambda_4^3 - 5(3\lambda_b^6 + \lambda_\tau^6)) \\ &+ 24\lambda_1^2 (3\lambda_b^2 + \lambda_\tau^2) + 78\lambda_1^3) \\ &- 24\lambda_t^2 (3\lambda_1 \lambda_b^2 + 8\lambda_3 \lambda_4 + 8\lambda_3^2 \\ &+ 4(\lambda_4^2 + \lambda_5^2) - 4\lambda_b^4) \left. \right] \right]. \quad (\text{A.4}) \end{aligned}$$

$$\begin{aligned} 16\pi^2 \beta_{\lambda_2} = & \frac{1}{4} \left(6g_1^2 (g_2^2 - 2\lambda_2) + 3g_1^4 - 36g_2^2 \lambda_2 + 9g_2^4 \right. \\ &+ 8(6\lambda_2 \lambda_t^2 + 6\lambda_2^2 + 2\lambda_3 \lambda_4 + 2\lambda_3^2 \\ &+ \lambda_4^2 + \lambda_5^2 - 6\lambda_t^4) \left. \right) \\ &+ \frac{1}{384\pi^2} \left[g_1^2 (6g_2^2 (39\lambda_2 + 20\lambda_4 \right. \\ &+ 84\lambda_t^2) - 303g_2^4 \\ &+ 48(9\lambda_2^2 + 4\lambda_3 \lambda_4 + 4\lambda_3^2 + 2\lambda_4^2 \\ &- \lambda_5^2) + 340\lambda_2 \lambda_t^2 - 128\lambda_t^4) \\ &+ 3g_1^4 (-191g_2^2 + 217\lambda_2 + 40\lambda_3 \\ &+ 20\lambda_4 - 76\lambda_t^2) - 393g_1^6 \\ &+ 3(g_2^4 (-51\lambda_2 + 60(2\lambda_3 + \lambda_4) - 36\lambda_t^2) \end{aligned}$$

$$\begin{aligned}
& + 12g_2^2 \left(4(9\lambda_2^2 + (2\lambda_3 + \lambda_4)^2) \right. \\
& + 15\lambda_2\lambda_\tau^2 + 291g_2^6 \\
& + 8(\lambda_\tau^4(-64g_3^2 - 3\lambda_2 + 12\lambda_b^2) \\
& + \lambda_2\lambda_\tau^2(80g_3^2 - 9(8\lambda_2 + \lambda_b^2))) \\
& - 2\lambda_5^2(7\lambda_2 + 20\lambda_3 + 22\lambda_4) - 2(2\lambda_4^2(3\lambda_2 \\
& + 8\lambda_3) + 2\lambda_3\lambda_4(5\lambda_2 + 6\lambda_3) \\
& + 10\lambda_2\lambda_3^2 + 39\lambda_2^3 + 8\lambda_3^3 + 6\lambda_4^3) \\
& - 12\lambda_b^2(2\lambda_3\lambda_4 + 2\lambda_3^2 + \lambda_4^2 + \lambda_5^2) \\
& - 4\lambda_\tau^2(2\lambda_3\lambda_4 + 2\lambda_3^2 + \lambda_4^2 \\
& \left. + \lambda_5^2) + 60\lambda_\tau^6 \right) \Big]. \quad (A.5)
\end{aligned}$$

$$\begin{aligned}
16\pi^2\beta_{\lambda_3} = & \frac{1}{4} \left(-6g_1^2(g_2^2 + 2\lambda_3) + 3g_1^4 \right. \\
& - 36g_2^2\lambda_3 + 9g_2^4 \\
& + 8(\lambda_1(3\lambda_3 + \lambda_4) + \lambda_2(3\lambda_3 + \lambda_4) \\
& + \lambda_3(2\lambda_3 + 3\lambda_b^2 + \lambda_\tau^2) \\
& + 3\lambda_\tau^2(\lambda_3 - 2\lambda_b^2) + \lambda_4^2 + \lambda_5^2) \Big) \\
& + \frac{1}{384\pi^2} \left[g_1^2(6g_2^2(39\lambda_1 + 20\lambda_4 \right. \\
& + 36\lambda_b^2 + 44\lambda_\tau^2) - 303g_2^4 \\
& + 4(25\lambda_1(\lambda_b^2 + 3\lambda_\tau^2) + 108\lambda_1^2 \\
& + 4(6(2\lambda_3\lambda_4 + 2\lambda_3^2 + \lambda_4^2) \\
& - 3\lambda_5^2 + 4\lambda_b^4 - 12\lambda_\tau^4))) \\
& + g_1^4(-573g_2^2 + 651\lambda_1 + 60(2\lambda_3 \\
& + \lambda_4 + \lambda_b^2 - 5\lambda_\tau^2)) - 393g_1^6 \\
& + 3(-3g_2^4(17\lambda_1 + 4(-10\lambda_3 \\
& - 5\lambda_4 + 3\lambda_b^2 + \lambda_\tau^2)) \\
& + 12g_2^2(5\lambda_1(3\lambda_b^2 + \lambda_\tau^2) \\
& + 36\lambda_1^2 + 4(2\lambda_3 + \lambda_4)^2) + 291g_2^6 \\
& - 8(\lambda_1(-80g_3^2\lambda_b^2 + 20\lambda_3\lambda_4 + 20\lambda_3^2 \\
& + 12\lambda_4^2 + 14\lambda_5^2 + 3\lambda_b^4 + \lambda_\tau^4) \\
& + 4(16g_3^2\lambda_b^4 + \lambda_5^2(10\lambda_3 + 11\lambda_4) + 8\lambda_3\lambda_4^2 \\
& \left. + 6\lambda_3^2\lambda_4 + 4\lambda_3^3 + 3\lambda_4^3 - 5(3\lambda_b^6 + \lambda_\tau^6)) \right) \Big]
\end{aligned}$$

$$\begin{aligned}
& + 24\lambda_1^2(3\lambda_b^2 + \lambda_\tau^2) + 78\lambda_1^3) \\
& - 24\lambda_\tau^2(3\lambda_1\lambda_b^2 + 8\lambda_3\lambda_4 + 8\lambda_3^2 \\
& + 4(\lambda_4^2 + \lambda_5^2) - 4\lambda_b^4) \Big]. \quad (A.6)
\end{aligned}$$

$$\begin{aligned}
16\pi^2\beta_{\lambda_4} = & 3g_1^2(g_2^2 - \lambda_4) - 9g_2^2\lambda_4 + 2\lambda_4(\lambda_1 + \lambda_2 \\
& + 4\lambda_3 + 2\lambda_4 + 3\lambda_b^2 + \lambda_\tau^2) \\
& + 6\lambda_\tau^2(\lambda_4 + 2\lambda_b^2) + 8\lambda_5^2 \\
& + \frac{1}{384\pi^2} \left[2g_1^2(3g_2^2(20\lambda_1 + 20\lambda_2 + 8\lambda_3 \right. \\
& + 51\lambda_4 + 36\lambda_b^2 + 44\lambda_\tau^2 + 84\lambda_\tau^2) \\
& - 168g_2^4 + \lambda_4(48\lambda_1 + 48\lambda_2 + 48\lambda_3 \\
& + 96\lambda_4 + 25\lambda_b^2 + 75\lambda_\tau^2) \\
& + \lambda_\tau^2(85\lambda_4 + 16\lambda_b^2) + 192\lambda_5^2) \\
& + g_1^4(471\lambda_4 - 876g_2^2) \\
& + 3(6g_2^2(\lambda_4(48\lambda_3 + 24\lambda_4 + 5(3\lambda_b^2 + \lambda_\tau^2 \\
& + 3\lambda_\tau^2))) + 72\lambda_5^2) - 231g_2^4\lambda_4 \\
& + 4(-8\lambda_b^2(-10g_3^2\lambda_4 + 3\lambda_4(\lambda_1 \\
& + 2\lambda_3 + \lambda_4) + 6\lambda_5^2) \\
& - 2\lambda_\tau^2(3(4\lambda_4(\lambda_2 + 2\lambda_3 + \lambda_4) + \lambda_b^2(8\lambda_3 \\
& + 11\lambda_4) + 8\lambda_5^2 + 8\lambda_b^4) \\
& - 8g_3^2(5\lambda_4 + 8\lambda_b^2)) - 4\lambda_5^2(12(\lambda_1 + \lambda_2 \\
& + 2\lambda_3) + 13\lambda_4) \\
& - 2\lambda_4(20\lambda_1(2\lambda_3 + \lambda_4) + 7\lambda_1^2 + 20\lambda_2(2\lambda_3 + \lambda_4) \\
& + 7\lambda_2^2 + 28\lambda_3(\lambda_3 + \lambda_4) \\
& - 8\lambda_\tau^2(\lambda_4(\lambda_1 + 2\lambda_3 + \lambda_4) + 2\lambda_5^2) \\
& \left. - 3\lambda_\tau^4(9\lambda_4 + 16\lambda_b^2) - 27\lambda_4\lambda_b^4 - 9\lambda_4\lambda_\tau^4) \right) \Big]. \quad (A.7)
\end{aligned}$$

$$\begin{aligned}
16\pi^2\beta_{\lambda_5} = & \lambda_5(-3g_1^2 - 9g_2^2 + 2(\lambda_1 + \lambda_2 + 4\lambda_3 \\
& + 6\lambda_4 + 3\lambda_b^2 + \lambda_\tau^2 + 3\lambda_\tau^2)) \\
& + \frac{1}{384\pi^2} \left[\lambda_5(2g_1^2(57g_2^2 - 24\lambda_1 - 24\lambda_2 \right. \\
& + 192\lambda_3 + 288\lambda_4 + 25\lambda_b^2 + 75\lambda_\tau^2 + 85\lambda_\tau^2) \\
& + 471g_1^4 + 3(6g_2^2(48\lambda_3 + 96\lambda_4 + 5(3\lambda_b^2
\end{aligned}$$

$$\begin{aligned}
& + \lambda_\tau^2 + 3\lambda_t^2) - 231g_4^4 \\
& + 4(-8\lambda_b^2(-10g_3^2 + 3\lambda_1 + 6\lambda_3 + 9\lambda_4) \\
& - 2\lambda_t^2(-40g_3^2 + 12(\lambda_2 + 2\lambda_3 + 3\lambda_4) \\
& + 33\lambda_b^2) - 8\lambda_4(11(\lambda_1 + \lambda_2) + 19\lambda_3) \\
& - 8\lambda_3(10(\lambda_1 + \lambda_2) + 7\lambda_3) - 8\lambda_\tau^2(\lambda_1 + 2\lambda_3 \\
& + 3\lambda_4) - 64\lambda_4^2 - 3\lambda_b^4 - \lambda_\tau^4 - 3\lambda_t^4) \\
& - 56(\lambda_1^2 + \lambda_2^2) + 48\lambda_5^2) \Big]. \quad (A.8)
\end{aligned}$$

The β functions for the Yukawa couplings λ_t , λ_b and λ_τ are given by:

$$\begin{aligned}
16\pi^2\beta_{\lambda_t} = & \frac{1}{12}\lambda_t(-17g_1(t)^2 - 27g_2(t)^2 \\
& + 6(-16g_3(t)^2 + \lambda_b^2 + 9\lambda_t^2)) \\
& - \frac{1}{6912\pi^2}\Big[\lambda_t(-2534g_1^4 + 3g_1^2(108g_2^2 \\
& - 304g_3^2 + 41\lambda_b^2 - 1179\lambda_t^2) \\
& + 9(252g_2^4 - 9g_2^2(48g_3^2 + 11\lambda_b^2 + 75\lambda_t^2) \\
& + 4(1296g_3^4 - 16g_3^2(4\lambda_b^2 + 27\lambda_t^2) \\
& - 3(6\lambda_2^2 - 24\lambda_2\lambda_t^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 \\
& - 8\lambda_3\lambda_b^2 + 4\lambda_4^2 + 8\lambda_4\lambda_b^2 \\
& + 6\lambda_5^2 - 10\lambda_b^4 - 3\lambda_b^2\lambda_\tau^2 \\
& - 10\lambda_b^2\lambda_t^2 - 48\lambda_t^4))\Big] \quad (A.9)
\end{aligned}$$

$$\begin{aligned}
16\pi^2\beta_{\lambda_b} = & \frac{1}{12}\lambda_b(-5g_1^2 - 27g_2^2 + 6(-16g_3^2 \\
& + 9\lambda_b^2 + 2\lambda_\tau^2 + \lambda_t^2)) \\
& - \frac{1}{6912\pi^2}\Big[\lambda_b(226g_1^4 + 3g_1^2(324g_2^2 \\
& - 496g_3^2 - 711\lambda_b^2 - 450\lambda_\tau^2 + 53\lambda_t^2) \\
& + 9(252g_2^4 - 9g_2^2(48g_3^2 + 75\lambda_b^2 \\
& + 10\lambda_\tau^2 + 11\lambda_t^2) \\
& + 4(1296g_3^4 - 16g_3^2(27\lambda_b^2 + 4\lambda_t^2) \\
& - 3(6\lambda_1^2 - 24\lambda_1\lambda_b^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 \\
& - 8\lambda_3\lambda_t^2 + 4\lambda_4^2 + 8\lambda_4\lambda_t^2 + 6\lambda_5^2 - 48\lambda_b^4 \\
& - 9\lambda_b^2\lambda_\tau^2 - 10\lambda_b^2\lambda_t^2 - 9\lambda_\tau^4 - 10\lambda_t^4))\Big] \quad (A.10)
\end{aligned}$$

$$16\pi^2\beta_{\lambda_\tau} = \frac{1}{4}\lambda_\tau(-15g_1^2 - 9g_2^2 + 12\lambda_b^2 + 10\lambda_\tau^2)$$

$$\begin{aligned}
& - \frac{1}{768\pi^2}\Big[\lambda_\tau(-966g_1^4 - g_1^2(108g_2^2 \\
& + 50\lambda_b^2 + 537\lambda_\tau^2) \\
& + 3(84g_2^4 - 15g_2^2(6\lambda_b^2 + 11\lambda_\tau^2) \\
& - 4(80g_3^2\lambda_b^2 + 6\lambda_1^2 - 24\lambda_1\lambda_\tau^2 \\
& + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 4\lambda_4^2 + 6\lambda_5^2 - 27\lambda_b^4 \\
& - 27\lambda_b^2\lambda_\tau^2 - 9\lambda_b^2\lambda_t^2 - 12\lambda_\tau^4))\Big] \quad (A.11)
\end{aligned}$$

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