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Collapsing stellar filament and exotic matter in Palatini $f(R)$ gravity

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Abstract We explore the dynamics of collapsing stellar filament in the presence of exotic material like dark matter. We use Palatini $f(R)$ theory to include exotic substance in the collapsing process. We derive a collapse equation by applying Darmois junction conditions on collapsing surface boundary Σ. It is found that the radial pressure related to baryonic matter remains non-zero at Σ. We then discuss the stability criteria of the collapsing process in the framework of three parameteric model, $f(R)=R+\lambda R_c[1-(1+\frac{R^2}{R_c^2})^{-n}]$. It is concluded that the stability of collapsing filament depends upon a directly proportional relation of gravitational effects of exotic terms with the radial pressure of seen matter. Stability criteria of family of polytropic filamentary structures are also discussed. For all stable polytropic filaments, it is found that the density of seen material is exponentially related to the exotic forces. Finally, we explore theoretical relation between gravitational waves and dark terms. It is theoretically predicted that the presence of exotic material can affect the propagation of gravitational waves.

1 Introduction

The concepts of dark energy (DE) and dark matter (DM) are striking discoveries of modern physics. Cosmic observations from Supernova Ia, CMBR (cosmic microwave background radiation) and Wilkinson Microwave Anisotropy Probe showed that our universe is expanding with an accelerating rate. There is a mysterious form of energy (called DE) having gravitationally repulsive effect is responsible for this cosmic acceleration [1,2]. Also, DM is a form of non-baryonic matter which neither emits nor absorbs electromagnetic radiations and is detectable by its gravitational effects on the baryonic matter. The observational studies related to mass discrepancy in galactic clusters and galactic rotational curves problems indicated the existences and importance of DM in the stellar evolution [3–5]. According to Planck data, the cosmic energy is distributed as follows: 68% is DE, 27% is DM and rest of 5% is baryonic matter [6].

In order to understand the nature of DE and DM many models are introduced by various researchers. In this context, the ΛCDM model, with cosmological constant (Λ) representing a energy density of vacuum energy, has been widely used in the theory of general relativity (GR). But this proposal suffers two problems like the "fine tuning problem" [7] and the "cosmic coincidence problem" [8,9]. According to fine tuning problem the predicted value of Λ is 120 order of magnitude smaller than the value predicted by the particle physics. The cosmic coincidence problem is the coincidence between densities of dark matter and dark energy. The quantum gravity fails to explain DE issue because it requires vacuum energy to be greater than hundreds of order magnitude more than observed value. There are different solutions for these problems like the modification of matter-part of the Einstein Hilbert action such as quintessence [10], Chaplygin gases [11–13], K-essence [14], etc. The another approach is the modified gravity which deals with the modified geometric part of the Einstein field equations [15]. These alternative of GR are considered viable if they provide admirable cosmological results as compare to GR.

The $f(R)$ gravity theory is one of the viable and most explored example of modified gravity which involves higher order curvature invariant term as compare to GR. This theory generalizes the Einstein-Hilbert action by replacing linear scalar curvature term (R) with a generic function of curvature "$f(R)$" [16–22]. This gravity attains a lot of attention because of its higher order curvature terms (generic function

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$f(R)$) could interpret not only DE issue admirably (explain cosmic acceleration at low cosmic densities) [23] but also DM problem [24,25].

There are usually two different approaches of $f(R)$ gravity: the metric approach [23] and the Palatini approach [26,27]. Between these approaches, the Palatini approach got a reasonable attention. Its formulation uses metric as well as christoffel connection (affine connection, $\Gamma^{\rho}_{\mu\nu}$) as two independent geometric variables. The metric $f(R)$ gravity deals with fourth-order differential equations which are not easy to handle. In contrast, the Palatini $f(R)$ gravity provides second order singularity-free equations to represent exotic quantities [28]. This version of $f(R)$ gravity introduces modified form of Friedmann equations [29–31] which is compatible with observational data [32,33]. It has been shown that initial-value Cauchy problem is well-formulated as well as well-posed in this approach [34,35]. It is also indicated that Palatini $f(R)$ gravity can be an intersecting candidate for problems related DM [36]. Furthermore, this theory can reproduce the effective dynamics of Loop Quantum which gives a link between Palatini formalism and the Quantum gravity [37]. The Palatini $f(R)$ gravity is also applied in the study of astrophysical phenomena like neutron stars [38,39] black holes [40] etc.

On large-scale, galaxies are present in the form of bunches called clusters or super-clusters. These bunches of galaxies are connected by a bridge-like arrangement called galaxy filaments. A rich web of galaxy filaments containing galaxies and their clusters along with DM haloes are predicted by the N-body simulation phenomenon [41–44]. The cold DM model indicated a network of low-density filaments that connects DM haloes (DM filaments) [45]. Numerous researchers have studied DM filaments connecting massive clusters. Higuchi et al. [46] predicted filament connecting massive galaxy clusters $RXJ0018.3 + 1618$ and $CL0015.9 + 1609$. Some people predicted [43] galaxy filaments that have a typical cylindrical radius $\sim 2h^{-1}Mpc$.

The phenomenon of gravitational collapse is responsible to the birth of stellar structures (stars, planets and cluster of galaxies), radiations and gravitational waves. During collapse process a stable astronomical body (like star) or a stable system of bodies (clusters or super clusters of stars) turns into unstable one due to its own gravity. That's why a great numbers of researchers explored characteristics of gravitational collapse for the study of evolving stellar distributions. Herrera and Santos [47] described dynamics of non-dissipative collapsing cylindrically symmetric anisotropic filamentary structure based on baryonic matter only. They used Darmois junction conditions to framework the collapse process and found that the radial pressure remains non zero throughout the collapse. This collapse discussion is based on baryonic matter only and doesn't involve the role of DM or DE.

The study of stellar evolution in the presence of exotic terms (DE and DM) may provide modified dynamics of stellar evolution, as compare to GR, which in turns may reveal the modification hidden in the phenomenon of structure formation of the universe [48–56]. In this paper, we investigate the dynamics of collapsing cylindrically symmetric stellar filament in the presence of baryonic and non-baryonic matter. It is an attempt to find how the dark part of the universe (exotic terms) relates with the dynamics of baryonic matter, radiations and gravitational waves. For this purpose, we use higher order curvature invariant base modified gravity " the Palatini $f(R)$ gravity". The paper is formulated as follows: the next section describes the Palatini $f(R)$ gravity and the galaxy filament. Section 3 discusses collapse of cylindrically symmetric filamentary structures by using Darmois junction conditions. Section 4 applies the three parameteric Palatini $f(R)$ model to the collapsing process to determine the effects of exotic matter on the stability criteria and gravitational waves. Finally, Sect. 5 summarizes the results.

2 Basic scenario

In this section, we dicuss the basic scenario regarding galaxy filament in the Palatini $f(R)$ gravity.

2.1 Palatini $f(R)$ gravity

The notion of $f(R)$ theory for a given time moderation in the gravitational scheme of GR is one of the successful approaches. It has given very useful results in the field of cosmology as well as physics which explain the cosmic expansion. The main concern of this theory is to substitute an algebraic general function of Ricci scalar "$f(R)$" instead of the cosmological constant in the standard Einstein-Hilbert action [57]. It can be written as

$$S_f(R) = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + S_M, \tag{1}$$

where $f(R)$, κ and S_M are non-linear functions, coupling constant and the matter action, respectively. The variation of the action with respect to metric ($g_{\mu\nu}$) and connection ($\Gamma^{\rho}_{\mu\nu}$), provides the equations of motion for the Palatini formalism in following forms

$$f_R(R)R_{\mu\nu} - \frac{1}{2}[g_{\mu\nu}f(R)] = \kappa T_{\mu\nu}, \tag{2}$$

$$\nabla_\mu(g^{\mu\nu}\sqrt{-g}f_R(R)) = 0. \tag{3}$$

The trace of Eq. (2) gives a direct relation between $R \equiv R(\Gamma)$ and $T \equiv g^{\mu\nu}T_{\mu\nu}$ as follows,

$$Rf_R(R) - 2f(R) = \kappa T. \tag{4}$$

The Palatini $f(R)$ gravity is consistent with classical theory of gravity for those observational cases which provide roots of above equation. The vacuum case of Eq. (4) gives $R=\bar{R}=constant$ which in turns along with Eq. (3) provide conservation of metric tensor as well as fixing $(\Gamma^{\rho}_{\mu\nu})$ as Levi-Civita connection. Consequently, Eq. (2) reduces for the vacuum case to

$$\bar{R}_{\mu\nu}-\Lambda(\bar{R})g_{\mu\nu}=0, \tag{5}$$

where $\bar{R}_{\mu\nu}$ becomes metric Ricci tensor of $g_{\mu\nu}$ and $\Lambda(\bar{R})=\frac{\bar{R}}{4}$. We can notice that this theory can reduces to GR (with and without the cosmological constant) according to the chosen $f(R)$ model. Solving Eq. (3) for $\Gamma^{\rho}_{\mu\nu}$, inserting the obtain result in Eq. (2) and describing it in terms of $g_{\mu\nu}$, we may express the single formulation of the Palatini $f(R)$ field equation given by

$$\frac{1}{f_R}(\bar{\nabla}_\mu\bar{\nabla}_\nu-g_{\mu\nu}\bar{\Box})f_R+\frac{1}{2}g_{\mu\nu}\bar{R}+\frac{\kappa}{f_R}T_{\mu\nu}+\frac{1}{2}g_{\mu\nu}\left(\frac{f}{f_R}-R\right)+\frac{3}{2f_R^2}\left[\frac{1}{2}g_{\mu\nu}(\bar{\nabla}f_R)^2-\bar{\nabla}_\mu f_R\bar{\nabla}_\nu f_R\right]-\bar{R}_{\mu\nu}=0. \tag{6}$$

The above equation can be rewritten in Einstein-type field equations as

$$\bar{G}_{\mu\nu}=\frac{\kappa}{f_R}(T^{(M)}_{\mu\nu}+T^{(D)}_{\mu\nu}). \tag{7}$$

Here

$$T^{(D)}_{\mu\nu}=\frac{1}{\kappa}(\bar{\nabla}_\mu\bar{\nabla}_\nu-g_{\mu\nu}\bar{\Box})f_R+\frac{f_R}{2\kappa}g_{\mu\nu}\left(\frac{f}{f_R}-R\right)+\frac{3}{2\kappa f_R}\left[\frac{1}{2}g_{\mu\nu}(\bar{\nabla}f_R)^2-\bar{\nabla}_\mu f_R\bar{\nabla}_\nu f_R\right], \tag{8}$$

shows the stress-energy tensor associated to the Palatini $f(R)$ gravity, $\bar{\Box}=\bar{\nabla}_\mu\bar{\nabla}_\nu g^{\mu\nu}$, $\bar{G}_{\mu\nu}\equiv\bar{R}_{\mu\nu}-\frac{1}{2}g_{\mu\nu}\bar{R}$, while $\bar{\nabla}_\mu$ indicates covariant derivative related to the Levi-Civita connection of metric tensor. Moreover, f_R and f are functions of $R(\Gamma)\equiv g^{\mu\nu}R_{\mu\nu}(\Gamma)$.

2.2 Galaxy Filament

Here, we describe geometry of a collapsing filament containing clusters of galaxies and dark-terms. For this, we assume cylindrically symmetric configuration that is bounded by a cylindrical surface and represented by the line element

$$ds^2=-A^2dt^2+A^2dr^2+B^2dz^2+C^2d\phi^2. \tag{9}$$

Here, $x^0=t,\ x^1=r,\ x^3=\phi$ with variations

$$-\infty\le t\le\infty,\ -\infty<z<\infty,\ 0\le\phi\le 2\Pi. \tag{10}$$

Consider the collapsing filament is composed of anisotropic dissipative baryonic fluid whose energy-momentum tensor is

$$T_{\mu\nu}=(\rho+P_r)V_\mu V_\nu+P_rg_{\mu\nu}+(P_\phi-P_r)K_\mu K_\nu+(P_z-P_r)S_\mu S_\nu+q(l_\mu V_\nu+l_\nu V_\mu), \tag{11}$$

where

$$l_\nu V^\mu=0,\ V_\mu=-A\delta^0_\mu,\ l_\mu=A\delta^1_\mu,\ S_\mu=B\delta^2_\mu,\ K_\mu=C\delta^3_\mu. \tag{12}$$

Equations (7), (8), (9) and (11) provide five non-zero components of the Einstein tensor but we need the following ones to discuss the dynamics of the collapsing structures

$$G_{11}=-\frac{B_{,tt}}{B}-\frac{C_{,tt}}{C}+\frac{A_{,t}}{A}\left(\frac{B_{,t}}{B}+\frac{C_{,t}}{C}\right)+\frac{A_{,r}}{A}\left(\frac{B_{,r}}{B}+\frac{C_{,r}}{C}\right)-\frac{B_{,t}}{B}\frac{C_{,t}}{C}+\frac{B_{,r}}{B}\frac{C_{,r}}{C}=\kappa P_rA^2+T^{(D)}_{11}A^2. \tag{13}$$

$$G_{01}=-\frac{B_{,tr}}{B}+\frac{1}{A}\left(A_{,r}\frac{B_{,t}}{B}+\frac{B_{,r}}{B}A_{,t}+A_{,r}\frac{C_{,t}}{C}\right)-\frac{C_{,tr}}{C}+\frac{A_{,t}}{A}\frac{C_{,r}}{C}=qA^2+T^{(D)}_{01}A^2. \tag{14}$$

3 Collapse of Stellar Filament

It is well known that collapsing stellar is always bounded by exterior configuration. In this study, we assume Einstein–Rosen coordinates [58] to represent exterior vacuum distribution corresponding to the hypersurface Σ, thus the exterior geometry is given by [47],

$$ds^2=-e^{2(\xi-\varpi)}(dT^2-d\tilde{R}^2)+e^{2\varpi}dZ^2+e^{-2\varpi}\tilde{R}^2d\phi^2. \tag{15}$$

Here ξ and ϖ depend upon T and $\tilde{R}$. The equations $R_{\beta\gamma}=0$ provides the gravitational wave equation as

$$\varpi_{,TT}-\varpi_{,\tilde{R}\tilde{R}}-\frac{\varpi_{,\tilde{R}}}{\tilde{R}}=0, \tag{16}$$

$$2\tilde{R}\varpi_{,T}\varpi_{,\tilde{R}}=\tilde{R}(\varpi^2_{,T}+\varpi^2_{,\tilde{R}}). \tag{17}$$

We use Darmois junction conditions [47,59,60] for the matching of interior collapsing boundary with the exterior one. So, we assume continuity of the first fundamental form (continuity of interior and exterior spacetimes) and continuity of the second fundamental (continuity of extrinsic curvatures). To apply junction conditions, the derived equation of collapsing boundary surface Σ is given by

$$N_-=r-r_\Sigma=0, \tag{18}$$

$$N_+=\tilde{R}-\tilde{R}_\Sigma(T)=0. \tag{19}$$

Here N_- and N_+ represent interior as well as exterior spacetimes at boundaries Σ_- and Σ_+, respectively. The radial term r_Σ is a constant because Σ describes a comoving collapsing boundary of the fluid. For the application of junction conditions, we arrange that Σ has the same parametrization whether it is assumed as embedded in N_- or N_+.

Using (18) in (9), we get a interior metric on surface Σ_- as

$$ds_-^2 = -d\tau^2 + B^2 dz^2 + C^2 d\phi^2, \tag{20}$$

where τ is the time coordinates defined on Σ by

$$d\tau = A dt. \tag{21}$$

We shall use $\varepsilon^0 = \tau,\ \varepsilon^2 = z\ and\ \varepsilon^3 = \phi$ as parameters on the boundary surface Σ. Equations (15) and (19) give exterior metric on Σ given by

$$ds_+^2 = -e^{-2(\xi-\varpi)}\left[1-\left(\frac{d\tilde{R}}{dT}\right)^2\right]dT^2 + e^{2\varpi}dz^2 + e^{-2\varpi}\tilde{R}^2 d\phi^2. \tag{22}$$

According to continuity of first fundamental form, the interior metric (20) is the same as exterior metric (22) on the boundary Σ if

$$e^{-2(\xi-\varpi)}\left[1-\left(\frac{d\tilde{R}}{dT}\right)^2\right]^{\frac{1}{2}} dT = d\tau, \tag{23}$$

$$e^{\varpi} = B, \tag{24}$$

$$e^{-\varpi}\tilde{R} = C. \tag{25}$$

Here, we consider

$$1-\left(\frac{d\tilde{R}}{dT}\right)^2 > 0 \tag{26}$$

on Σ, so that T behave as a timelike coordinate.

The second fundamental form on boundary Σ describes continuity of extrinsic curvatures on interior as well as exterior geometry and is given by

$$K_{uv} d\varepsilon^u d\varepsilon^v, \quad u, v = 0, 2, 3, \tag{27}$$

where K_{uv} describes the extrinsic curvature given as follows

$$K^{\pm}_{uv} = -n^{\pm}_{\alpha}\left(\frac{\partial^2 z^{\alpha}}{\partial\chi^u\partial\chi^v} + \Gamma^{(\alpha)}_{\beta\gamma}\frac{\partial z^{\beta}}{\partial\chi^u}\frac{\partial z^{\gamma}}{\partial\chi^v}\right). \tag{28}$$

Here $n^{\pm}_{\alpha}$ represents outward unit normals to Σ and z^{α} refers to the equation of Σ in N_- or N_+, namely, (18) or (19). The term $\Gamma^{(\alpha)}_{\beta\gamma}$ represents christoffel symbols which can be calculated from the appropriate exterior or interior spacetimes.

From Eqs. (18) and (19), the calculated unit normals are

$$n^-_{\alpha} = (0, A, 0, 0), \tag{29}$$

$$n^+_{\alpha} = e^{(\xi-\varpi)}\left[1-\left(\frac{d\tilde{R}}{dT}\right)^2\right]^{-\frac{1}{2}}\left(-\frac{d\tilde{R}}{dT}, 1, 0, 0\right) = e^{-2(\xi-\varpi)}(-\dot{\tilde{R}}, \dot{T}, 0, 0). \tag{30}$$

Here dot indicates the differentiation with respect to τ given by means of (23). The derived normal vectors (29) and (30) are space like under the constraint (26). Thus, the obtained non-zero components of $K^{\pm}_{uv}$ are as follows

$$K^-_{00} = -\frac{A_{,r}}{A^2}, \tag{31}$$

$$K^-_{22} = -\frac{BB_{,r}}{A}, \tag{32}$$

$$K^-_{33} = -\frac{CC_{,r}}{A}, \tag{33}$$

$$K^+_{00} = e^{-2(\xi-\varpi)}T''\ddot{\tilde{R}} - \tilde{R}''\ddot{T} + \dot{\tilde{R}}(\dot{\tilde{R}}^2 + \dot{T}^2)[(\xi, T-\varpi, T) + (\xi, \tilde{R}-\varpi, \tilde{R})], \tag{34}$$

$$K^+_{22} = e^{2\varpi}(\dot{\tilde{R}}\varpi_{,T} + \dot{T}\varpi_{,\tilde{R}}), \tag{35}$$

$$K^+_{33} = -e^{-2\varpi}\tilde{R}^2(\dot{\tilde{R}}\varpi_{,T} + \dot{T}\varpi_{,\tilde{R}} - \frac{\dot{T}}{\tilde{R}}). \tag{36}$$

Equations (21) and (23)–(26) along with continuity of K_{uv} represent complete junction conditions at Σ.

Next, we obtain useful outcomes of junction conditions in the scenario of collapsing galactic filament [47]. In order to do that we use field equations to describe boundary conditions in a brief form and derive some useful results. From (23) we have

$$e^{-2(\xi-\varpi)}(\dot{T}^2 - \dot{\tilde{R}}^2) = 1, \tag{37}$$

and Eqs. (24) and (25) give

$$\tilde{R} = BC. \tag{38}$$

Differentiation of (38) along with (21) provide

$$\dot{\tilde{R}} = \frac{(BC)_{,t}}{A}. \tag{39}$$

The continuity of extrinsic curvatures K_{22} and K_{33} together with (24) and (25) imply

$$\dot{T} = (BC)_{,r}A^{-1}. \tag{40}$$

Now, differentiation of (39) and (40) along with (13) ,(14) as well as (21) give the relation

$$\ddot{T} - \ddot{\tilde{R}}\dot{T} = \frac{1}{A^4}[B_t(AC)_{,r} + C_t(AB)_{,r}](BC)_{,t} + [\kappa(P_r + T^{(D)}_{11})A^3BC - AB_tC_t - A_r(BC)_{,r} - AB_rC_r](BC)_{,r} + (q + T^D_{01})A^2. \tag{41}$$

From the continuity of K_{00} and K_{22} and Eqs. (17), (25), (37), (38), (40) and (41), we have

$$\frac{1}{A^4}[(B_tC_r - B_rC_t)^2 + \kappa(P_r + T_{11}^{(D)})A^2(BC)_r^2] + (q + T_{01}^{(D)})A^2 = (\dot{T}^2 - \dot{\tilde{R}}^2)^2\varpi_{,T}T^2. \quad (42)$$

Differentiation of Eqs.(24) and (25) with respect to (21) and (23), we obtain

$$e^{2\varpi-\xi}\left[1-\left(\frac{\dot{\tilde{R}}}{\dot{T}}\right)^2\right]^{\frac{-1}{2}}\varpi_{,T} = \frac{B_{,t}}{A}, \quad (43)$$

$$e^{-\xi}\left[1-\left(\frac{\dot{\tilde{R}}}{\dot{T}}\right)^2\right]^{\frac{-1}{2}}\left(\frac{\dot{\tilde{R}}}{\dot{T}} - \tilde{R}\varpi_{,T}\right) = \frac{C_{,t}}{A}. \quad (44)$$

The above derived Eqs. ((43) and (44)) along with the continuity of curvatures K_{22} and K_{33} give

$$\frac{1}{A^2}(B_{,t}C_{,r} - B_{,r}C_{,r}) = (\dot{T}^2 - \dot{\tilde{R}}^2)\varpi_{,T} - \dot{T}\dot{\tilde{R}}\psi_{,\tilde{R}}. \quad (45)$$

Finally, substituting Eq. (45) into (42) and using (16) as well as (40), we have

$$T_{11}^{(D)} + \kappa P_r - (q + T_{01}^{(D)})(BC)_r^{-2} = e^{2\varpi-\xi}\varpi_{,\tilde{R}}^2\left(2\frac{\varpi_{,T}}{\varpi_{,\tilde{R}}}\upsilon - \frac{\upsilon^2}{1-\upsilon^2}\right), \quad (46)$$

where $\upsilon = \frac{dR}{dT}$ represents the radial velocity of the collapsing boundary surface Σ. The result (46) denotes that the radial pressure P_r on surface Σ is non zero. The reason is that it may be due to the flux of momentum of the gravitational wave which is emitted from the cylinder. The above result can be used as a collapse equation of filamentary structure in presence of exotic terms.

4 Galaxy filament and the Palatini $f(R)$ gravity

Observations indicate that galaxies are attached together through cosmic webs which is actually a network of galaxy filaments connected via dark terms. It is believed that there exist filamentary structure made up of dark material among galaxies which behave like a bridge between galaxies. In this section, we will present model of galaxy filament in the Palatini $f(R)$ gravity and discuss the features of exotic material in the phenomenon of stability of filaments as well as gravitational waves.

4.1 Three parametric model of Palatini $f(R)$ formalism

In this section, we will convert our attention on physical perspective of Palatini $f(R)$ gravity. We consider a well-known three parametric form of Palatini $f(R)$ gravity for the explanation of physical expect of the galaxy filament. The model is described by [61,62]

$$f(R) = R + \lambda R_c\left[1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right], \quad (47)$$

where λ and n are positive real numbers, R_c represents a constant term having values of the order of the present effective Ricci scalar. It can be notice that this model represents absence of cosmological constant in flat spacetime at null contribution ($R = 0$). Moreover, $R = constant = \bar{R} = R_0x_1,\ x_1 > 0$, provides de-Sitter model for the following values

$$\lambda = \frac{(x_1^2+1)^{n+1}x_1}{2[(x_1^2+1)^{n+1} - (n+1)x_1^2 - 1]}. \quad (48)$$

Now, to obtain useful results, we consider a specific choice of x_1 and then calculate value of λ. It can be noticed from above equation that $2\lambda > x_1$ which gives $\Lambda(R_1) = \frac{R_1}{4} < \Lambda(\infty)$ at de-Sitter point. If we fixed the value of x_1 with $n >> 1$ and fixed n with $x_1 >> 1$, we get $x_1 \rightarrow 2\lambda$. In this situation, this model is consistent with ΛCDM model. The Einstein gravity can be obtained under the limit $f(R) \rightarrow R$. In this context, we have

$$T_{11}^{(D)} = \frac{B^2}{f_R}\left\{\frac{\ddot{f}_R}{A^2} - \frac{f_R''}{C^2} + \frac{\dot{f}_R}{A^2}\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) + \frac{\dot{f}_R^{\,2}}{4f_RA^2} - \left(R - \frac{f}{f_R}\right)\frac{f_R}{2} + \left(\frac{C'}{C} - \frac{B'}{B} - \frac{A'}{A}\right)\frac{f_R'}{C^2} + \frac{f_R'^2}{4f_RC^2}\right\} = P_r^D. \quad (49)$$

$$T_{01}^{(D)} = \frac{1}{f_R}\left\{\dot{f}'_R - \frac{5f_R'\dot{f}_R}{2} - \frac{f_R'\dot{C}}{C} - \frac{\dot{f}_R}{A}\right\} = q^D. \quad (50)$$

Here P_r^D represents radial pressure exerted by exotic terms and q^D shows dissipation of exotic material in the collapsing system. Their values are given in Appendix.

4.2 Stability of collapsing filament with exotic material

The stability of stellar body is of great importance. Any stable compact model is considered incompetent if it is unstable against fluctuations. The re-stability of collapsing process leads to the formation of stellar structures in the universe. Here, we obtain the stability criteria of collapsing galaxy filament in the presences of exotic material. We consider that the collapsing of the filament stops and the stellar body becomes stable. In this situation the collapsing velocity vanishes, $\upsilon = 0$ and the dissipation effects related to baryonic material becomes negligible ($q \approx 0$). In this case Eq. (46) yields

$$\kappa P_r = (q^D)(BC)_r^{-2} - P_r^D, \tag{51}$$

which shows a relation between exotic material and baryonic material in the stability of the collapsing filament in the Palatini $f(R)$ gravity. To discuss necessary and sufficient condition of the stability criteria, we assume inverse situation, let there is a non-dissipative case satisfying

$$\kappa P_r - (q^D)(BC)_r^{-2} + P_r^D = 0, \tag{52}$$

under these condition the collapse Eq. (46) reduces to

$$e^{2(\varpi-\xi)}\varpi_{,\tilde{R}}^2\left(2\frac{\varpi_{,T}}{\varpi_{,\tilde{R}}}\upsilon - \frac{\upsilon^2}{1-\upsilon^2}\right) = 0. \tag{53}$$

This gives

$$\upsilon = \frac{1}{y}(-1 + (1+y^2)^{\frac{1}{2}}) \tag{54}$$

with $y = \frac{\varpi_{,T}}{\varpi_{,\tilde{R}}}$.

Now, if the condition (52) is sufficient for stability then $\upsilon = 0$ and we get $y = \frac{\varpi_{,T}}{\varpi_{,\tilde{R}}} = 0$, which yield $\varpi = \varpi(\tilde{R})$. In this case, Eqs.(24), (40) and the continuity of K_{22} imply $B = B(r)$ and $C = C(r)$ at Σ. As r is constant at the boundary Σ, the metric terms $B = B(r)$ and $C = C(r)$ become constant at Σ. In this situation, Eq. (38) implies $\tilde{R} = constant$ at hypersurface which inturn shows that collapsing process is stop. Hence our supposition is correct, the condition (52) play the role of sufficient condition in the stability analysis of stellar filamentary structure.

In order to discuss stability criteria of family of filamentary structures, we assume polytropic type baryonic matter distribution that is $P_r = k\rho^\gamma$ with $\gamma = \frac{n+1}{n}$. Here γ represents a polytropic exponent, k is a constant and n indicates polytropic index. Under this consideration, Eq. (52) implies

$$\kappa(k\rho^\gamma) = (q^D)(BC)_R^{-2} + P_r^D, \tag{55}$$

or

$$\rho = \left[\frac{(q^D)(BC)_r^{-2} - P_r^D}{\kappa(k)}\right]^{\frac{n}{n+1}}. \tag{56}$$

The above result (56) indicates that in stable condition the density of baryonic matter is exponentially related to the effects due to exotic material. For polytropic models the filamentary structure becomes unstable, if

$$\rho \lessgtr \left[\frac{(q^D)(BC)_r^{-2} - P_r^D}{\kappa(k)}\right]^{\frac{n}{n+1}}. \tag{57}$$

It is well known that different values of polytropic index represent various stellar configurations [63], so polytropic equation along with stability condition give instability criteria for family of filamentary structure in the Palatini $f(R)$ gravity.

4.3 Gravitational waves and exotic material

The collapsing process of stellar bodies is one of the source of gravitational waves [64]. Recently, it is studied that the propagation of gravitational waves can be affected by the presence of dark matter just like light waves are disturbed by different medium of propagation. But this effects is so small that it would be far below the sensitivity of current detector [65]. Polarization modes of gravitational waves are also discussed in $f(R)$ gravity [66] which provide the existence of gravitational waves in this theory. In the present study of collapsing filament, we can describe the relation of gravitational waves with higher order curvature invariant terms (exotic terms) by using collapse equation (46).

Let the collapsing of stellar filament emits gravitational radiations directed outward from the axis. The waveform of the radiation is divided into three forms: (i) a precursor (quadrupole momenta formalism), (ii) sharp pluse, and (iii) the oscillatory tails. The quadrupole moment formalism take place at very early time. The sharp pulse of the radiation carries much of the energy of the wave and lasts longer than quadrupole moment phase. In the current study, we consider the sharp pulse of the gravitational radiation directed outward from the axis. In this condition the function ϖ can be described as [67]

$$\varpi = \frac{1}{2\pi}\int_{-\infty}^{T-\tilde{R}} \frac{h(T')dT'}{((T-T')-\tilde{R}^2)^{1/2}} + \varpi_{st}, \tag{58}$$

where ϖ_{st} shows a Levi-Civita static solution [68], $h(T)$ represents a time dependent function that shows the strength of the wave source and it is describes by $h(T) = h_0\delta(T)$. Here, h_0 indicates a constant and $\delta(T)$ is the dirac delta function. Equation (58) satisfies the wave equation (9) which provides

$$\varpi = \varpi_{st}, \quad \tilde{R} > T,$$
$$\varpi = \frac{h_0}{2\pi(T^2-\tilde{R}^2)^{1/2}} + \varpi_{st}, \quad \tilde{R} < T.$$

Equation (58) along with collapse Eq. (46) describe the relation of gravitational waves with exotic material in the presences of baryonic matter pressure and dissipation at Σ. If we consider the baryonic matter stress and dissipation is negligible at Σ ($p_r \approx 0,\ q \approx 0$), then from (46), we get a direct relation between dark terms and gravitational waves given by

$$T_{11}^{(D)} + (T_{01}^{(D)})(BC)_r^{-2} = e^{2\varpi-\xi}\varpi_{,\tilde{R}}^2\left(2\frac{\varpi_{,T}}{\varpi_{,\tilde{R}}}\upsilon - \frac{\upsilon^2}{1-\upsilon^2}\right), \tag{59}$$

The Eqs. (46), (58) and (59) show that the presences of higher order curvature invariant terms (representing DM) can disturb the propagation of gravitational waves in the collapsing of Stellar Filament.

5 Conclusion

Observational surveys indicate the DM as one of a fundamental constituent of filamentary structure on the galactic scales. We have studied a dissipative collapsing cylindrically symmetric filamentary structure in the presence of exotic matter. For this purpose, we have incorporate dark terms by using higher order curvature invariant principle (the Palatini approach of $f(R)$ gravity). The Darmois junction conditions is used to model the collapsing filament and to derive a collapse equation at the boundary surface Σ. From the collapse equation, it is found that the higher order curvature invariant terms (representing DM) along with dissipation related to baryonic matter preserve the radial pressure on the collapsing boundary. This result is consistent with model discussed in GR in the absence of DM [47]. Galactic observations predicted that the the existence and evolution of galaxies or their clusters rely on a big amount of unseen material like DM [3–5,45]. This indicates that the presence of exotic terms can affect the dynamics of evolving filamentary structures. We have used the Palatini $f(R)$ gravity formalism by considering three parameter model, $f(R)=R+\lambda R_c[1-(1+\frac{R^2}{R_c^2})^{-n}]$ as a candidate of exotic terms in the stability of collapsing filament. It is concluded that the stability criteria of collapsing filament rely upon the relation of higher order curvature invariant terms (exotic matter) and radial pressure (due to baryonic material). For stable configuration, radial pressure are directly proportional to the gravitational effects of dark terms. To discuss stability criteria for family of filamentary structures, we have applied polytropic equation of state for baryonic matter content. It is found that instability criteria of polytropic filaments provides a exponential relation between density of baryonic material and the gravitational effects of exotic terms which depends upon the polytropic index n.

In current phase, it is suspected that the phenomenon of gravitational waves can resolve the secret of exotic terms in the universe. The feature of dark terms as a medium of propagation of gravitational waves can reveal the secret of hidden terms [65,69]. In this study, we have investigated relation of gravitational waves emerging from collapsing filament with exotic materials. From the collapse equation a connection between gravitational waves and dark terms has been acquired which shows that the existence of dark terms can affect the propagation of gravitational waves.

It is mentioned here that further study on the stability criteria can help to find the ratio of baryonic matter and dark material in the filamentary structure.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This work is done on theoretical basis not on experimental basis. All the information has been given in the manuscript.]

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Appendix A

The quantities P_r^D and q^D are

$$
\begin{aligned}
P_r^D = \frac{B^2}{\kappa}\Bigg[& -\frac{\lambda}{2} \\
& \times\left(R_c+\left\{+\frac{2nR^2}{R_c}\left(1+\frac{R^2}{R_c^2}+R_c\right)^{-1}\left(\frac{R^2}{R_c^2}+1\right)^{-n}\right\}\right) \\
& -\frac{2n\lambda\ddot{R}}{A^2R_c}\left(\frac{R^2}{R_c^2}+1\right)^{-(n+1)} \\
& +\left(\frac{R^2}{R_c^2}+1\right)^{-(n+2)}\frac{4\lambda Rn(n+1)}{A^2R_c^3}\Bigg\{3\dot{R}^2+R\ddot{R} \\
& -\frac{2R^2\dot{R}^2(n+2)}{R_c^2}\left(\frac{R^2}{R_c^2}+1\right)^{-(n+1)}\Bigg\} \\
& -\left(\frac{R^2}{R_c^2}+1\right)^{-(n+1)}\frac{2\lambda nR''}{C^2R_c}-\frac{R^2\dot{R}}{R_c^2} \\
& \times(4n^2+n)\left(\frac{R^2}{R_c^2}+1\right)^{-1}\Bigg\{RR''+3R'^2 \\
& -\frac{(2n+4)R^2R'^2}{R_c^2}\left(\frac{R^2}{R_c^2}+1\right)^{-(n+1)}\Bigg\}+\left(\frac{R^2}{R_c^2}+1\right)^{-(n+1)} \\
& \times\frac{2n\lambda}{A^2R_c}\left\{\frac{2(n+1)R^2\dot{R}}{R_c^2}\left(\frac{R^2}{R_c^2}+1\right)^{-1}-\dot{R}\right\}\Bigg\{-\frac{\dot{A}}{A} \\
& +\frac{\dot{B}}{B}+\frac{\dot{C}}{C}+\left(\frac{R^2}{R_c^2}+1\right)^{(n+1)} \\
& \times\frac{n\lambda(R_c^2+R^2)^{(n+1)}}{\{2n\lambda RR_c^{2n+2}-2R_c(R_c^2+R^2)^{(n+1)}\}}\Bigg\{-\dot{R}\Bigg(1-\frac{2R^2}{R_c^2} \\
& +(n+1)\left(1+\frac{R^2}{R_c^2}\right)^{-1}\Bigg)\Bigg\}\Bigg\} \\
& +\left(\frac{R^2}{R_c^2}+1\right)^{(n+1)}\frac{2\lambda n}{C^2R_c}\Bigg\{\left(\frac{R^2}{R_c^2}+1\right)^{-1} \\
& \times\frac{2R^2(n+1)R'}{R_c^2}-R'\Bigg\}
\end{aligned}
$$

$$\times\left\{\frac{\lambda n(R_c^2+R^2)^{(n+1)}}{\{-2\lambda nRR_c^{2n+2}-R_c(R_c^2+R^2)^{(n+1)}\}}\left(\frac{R^2}{R_c^2}+1\right)^{(n+1)} \times\left\{-R'+\frac{2R^2(n+1)R'}{R_c^2}\left(\frac{R^2}{R_c^2}+1\right)^{-1}\right\} -\frac{A'}{A}+\frac{C'}{C}-\frac{B'}{B}\right\}\Bigg], \tag{A.1}$$

$$q^D=\frac{1}{\kappa}\Bigg[-\frac{2\lambda n\dot{R}'}{R}\left(\frac{R^2}{R_c^2}+1\right)^{-(n+1)} +\frac{\lambda(4n^2+4n)R}{R_c^3}\left(\frac{R^2}{R_c^2}+1\right)^{-(n+2)} \left\{-R'(n+1)\frac{2R^2}{R_c^2}+R'\left(\frac{R^2}{R_c^2}+1\right)^{-1}\right\} \times-\frac{2\lambda nA'}{AR_c}\left(\frac{R^2}{R_c^2}+1\right)^{(n+1)}\left\{\frac{2R^2(n+1)\dot{R}}{R_c^2} \times\left(\frac{R^2}{R_c^2}+1\right)^{-1}\right\}-\dot{R}-\frac{2\lambda n\dot{C}}{CR_c}\left(\frac{R^2}{R_c^2}+1\right)^{(n+1)} \times\left\{\frac{2R^2(n+1)R'}{R_c^2}\left(\frac{R^2}{R_c^2}+1\right)^{-1} \times-R'\right\}+\left(\frac{R^2}{R_c^2}+1\right)^{-(2n+2)} \times\frac{10\lambda^2n^2}{2n\lambda RR_c^{2n+2}-R_c\{R_c(R_c^2+R^2)^{(n+1)}\}}\left\{\dot{R}R' -\frac{(4n+4)^2R^4R'\dot{R}}{R_c^2}\left(\frac{R^2}{R_c^2}+1\right)^{-1} +\frac{(4n+4)^2R^4R'\dot{R}}{R_c^4}\left(\frac{R^2}{R_c^2}+1\right)^{-2}\right\}\Bigg]. \tag{A.2}$$

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