Eur. Phys. J. C (2019) 79:800
https://doi.org/10.1140/epjc/s10052-019-7318-y

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

dS_4 vacua from matter-coupled 4D $N = 4$ gauged supergravity

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Received: 13 July 2019 / Accepted: 17 September 2019 / Published online: 26 September 2019

Abstract We study dS_4 vacua within matter-coupled $N = 4$ gauged supergravity in the embedding tensor formalism. We derive a set of conditions for the existence of dS_4 solutions by using a simple ansatz for solving the extremization and positivity of the scalar potential. We find two classes of gauge groups that lead to dS_4 vacua. One of them consists of gauge groups of the form $G_e \times G_m \times H$ with H being a compact group and $G_e \times G_m$ a non-compact group with $SO(3)\times SO(3)$ subgroup and dyonically gauged. These gauge groups are the same as those giving rise to maximally supersymmetric AdS_4 vacua. The dS_4 and AdS_4 vacua arise from different coupling ratios between G_e and G_m factors. Another class of gauge groups is given by $SO(2,1)_e \times SO(2,1)_m \times G_{nc} \times G'_{nc} \times H$ with $SO(2,1)$, G_{nc} and G'_{nc} dyonically gauged. We explicitly check that all known dS_4 vacua in $N = 4$ gauged supergravity satisfy the aforementioned conditions, hence the two classes of gauge groups can accommodate all the previous results on dS_4 vacua in a simple framework. Accordingly, the results provide a new approach for finding dS_4 vacua. In addition, relations between the embedding tensors for gauge groups admitting dS_4 and dS_5 vacua are studied, and a new gauge group, $SO(2,1) \times SO(4,1)$, with a dS_4 vacuum is found by applying these relations to $SO(1,1) \times SO(4,1)$ gauge group in five dimensions.

Contents

1 Introduction

De Sitter (dS) vacua are solutions of general relativity and gauged supergravity with positively constant curvature. Although these solutions are originally of mathematical interest, cosmological observations, see for example [1–3], suggest that the universe has a very small positive value

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of cosmological constant. Furthermore, these solutions have attracted much attention during the past twenty years due to the proposed dS/CFT correspondence [4], a holographic duality between a theory of gravity on dS background and a Euclidean conformal field theory along the line of the AdS/CFT correspondence [5].

Unlike the AdS counterpart found naturally in many gauged supergravities, dS vacua are very rare and (if they exist) the embedding in string/M-theory is highly non-trivial. Various approaches have been devoted to search for these vacua with only a small number of solutions found, see [6–23] for an incomplete list. All these results even suggest that string/M-theory does not admit de Sitter solutions, for a recent review see [24] and references therein. In addition, there are a number of previous works considering de Sitter solutions of gauged supergravities. In four dimensions, de Sitter vacua are extensively studied, see for example [25–29]. On the other hand, de Sitter vacua in higher dimensions are less known [30–36].

In this paper, we study dS_4 vacua in four-dimensional $N = 4$ gauged supergravity coupled to vector multiplets constructed in [37] using the embedding tensor formalism, see [38,39] for an earlier construction. We do not attempt to find new dS_4 solutions but to introduce a new approach for finding dS_4 vacua. We will extend a recent result initiated in the study of dS_5 vacua in five-dimensional $N = 4$ gauged supergravity given in [36]. Unlike the previous results on dS_4 solutions mostly obtained from using the old construction of [39], working in the embedding tensor formalism has the advantage that different deformations, gaugings and non-trivial $SL(2)$ phases, are encoded in a single framework. Furthermore, an explicit form of the gauge group under consideration needs not be specified at the beginning. This allows to formulate a general setup and subsequently apply the results to a particular gauge group.

We now describe the procedure used in our analysis. In general, the scalar potential of a gauged supergravity can be written as a quadratic function of fermion-shift matrices. In [36], the extremization and positivity of the scalar potential are solved by using a particular form of an ansatz such that the gravitino-shift matrix (usually denoted by the A_1 tensor) vanishes. This guarantees the positivity of the potential and, with a suitable condition, the potential can be extremized. With the help of the embedding tensor formalism, a general form of gauge groups that lead to dS_4 vacua can be determined from the conditions imposed on the fermion-shift matrices.

It should be noted that the procedure and the resulting conditions are very similar to those arising from the existence of supersymmetric AdS_4 vacua given in [40]. However, there is a crucial difference in the sense that the conditions for dS_4 are derived from a particular ansatz, but those for AdS_4 are obtained by requiring unbroken supersymmetry. While the latter guarantee that the results are vacuum solutions of the $N = 4$ gauged supergravity and, in particular, extremize the scalar potential, we need to explicitly impose the extremization of the potential in the former case. We will see that some of these extra conditions are already implied by the quadratic constraint. The remaining ones imply that the gauge groups must be dyonically embedded in the global symmetry group similar to the AdS_4 case.

The paper is organized as follows. In Sect. 2, we review relevant formulae for computing the scalar potential of $N = 4$ gauged supergravity coupled to vector multiplets in the embedding tensor formalism. In Sect. 3, we derive the conditions for the scalar potential to admit dS_4 vacua by solving the extremization and positivity of the potential. A general form of gauge groups is also determined by solving these conditions subject to the quadratic constraint. In Sect. 4, we explicitly verify that all the previously known dS_4 vacua in $N = 4$ gauged supergravity are encoded in our results. Some relations between four- and five-dimensional embedding tensors for gauge groups that admit de Sitter vacua are given in Sect. 5. Conclusions and comments on the results are given in Sect. 6. We also include an appendix collecting useful identities for $SO(6)$ gamma matrices.

2 Four-dimensional $N = 4$ gauged supergravity coupled to vector multiplets

In this section, we breifly review $N = 4$ gauged supergravity coupled to an arbitrary number n of vector multiplets. We will mainly give relevant formulae for finding the scalar potential which is the most important part in our analysis. For more detail, interested readers are referred to [37].

For $N = 4$ supersymmetry, there are two types of supermultiplets, gravity and vector multiplets. The former contains the graviton $e^{\hat{\mu}}_\mu$, four gravitini $\psi_{\mu i}$, $i = 1, \ldots, 4$, six vectors A^m_μ, $m = 1, \ldots, 6$, four spin-$\frac{1}{2}$ fermions λ_i, and a complex scalar τ. The field content of the latter is given by a vector field A_μ, four gaugini λ^i and six scalars ϕ^m. We use the following conventions on various types of indices. Spacetime and tangent space indices will be denoted by $\mu, \nu, \ldots = 0, 1, 2, 3$ and $\hat{\mu}, \hat{\nu}, \ldots = 0, 1, 2, 3$, respectively. Indices $m, n, \ldots = 1, 2, \ldots, 6$ label vector representation of $SO(6) \sim SU(4)$ R-symmetry while $i, j, \ldots$ denote chiral spinor of $SO(6)$ or fundamental representation of $SU(4)$. The n vector multiplets are labeled by indices $a, b, \ldots = 1, \ldots, n$. Accordingly, the field content of the vector multiplets can be collectively written as $(A^a_\mu, \lambda^{ai}, \phi^{am})$.

The $(6n + 2)$ scalars from both the gravity and vector multiplets are described by the coset manifold

$$\mathcal{M} = \frac{SL(2)}{SO(2)} \times \frac{SO(6,n)}{SO(6) \times SO(n)}. \quad (1)$$

The first factor is parametrized by a complex scalar τ consisting of the dilaton ϕ and the axion χ from the gravity multiplet. The second factor incorporates the $6n$ scalars from the vector multiplets. A useful parametrization for these two coset manifolds is given in term of the coset representatives. For $SL(2)/SO(2)$, we will use the following form of the coset representative

$$\mathcal{V}_\alpha = \frac{1}{\sqrt{\text{Im}\tau}}\begin{pmatrix}\tau\\ 1\end{pmatrix} \tag{2}$$

with an index $\alpha = (+,-)$ denoting the $SL(2)$ fundamental representation. $\mathcal{V}_\alpha$ satisfies the relation

$$M_{\alpha\beta} = \text{Re}(\mathcal{V}_\alpha \mathcal{V}^*_\beta) \quad \text{and} \quad \epsilon_{\alpha\beta} = \text{Im}(\mathcal{V}_\alpha \mathcal{V}^*_\beta) \tag{3}$$

in which $M_{\alpha\beta}$ is a symmetric matrix with unit determinant. $\epsilon_{\alpha\beta}$ is anti-symmetric with $\epsilon_{+-} = \epsilon^{+-} = 1$.

For $SO(6,n)/SO(6)\times SO(n)$, we use the coset representative $\mathcal{V}_M{}^A$ transforming under the global $SO(6,n)$ and local $SO(6)\times SO(n)$ symmetry by left and right multiplications, respectively. The local index A can be split as $A = (m, a)$ with $m = 1, 2, \ldots, 6$ and $a = 1, 2, \ldots, n$ denoting vector representations of $SO(6)\times SO(n)$. The coset representative can then be written as

$$\mathcal{V}_M{}^A = (\mathcal{V}_M{}^m, \mathcal{V}_M{}^a). \tag{4}$$

Since $\mathcal{V}_M{}^A$ is an $SO(6,n)$ matrix, it satisfies the following relation

$$\eta_{MN} = -\mathcal{V}_M{}^m\mathcal{V}_N{}^m + \mathcal{V}_M{}^a\mathcal{V}_N{}^a \tag{5}$$

where $\eta_{MN} = \text{diag}\,(-1,-1,-1,-1,-1,-1,1,\ldots,1)$ is the $SO(6,n)$ invariant tensor. η_{MN} and its inverse η^{MN} can be used to lower and raise $SO(6,n)$ indices, $M, N, \ldots$. The $SO(6,n)/SO(6)\times SO(n)$ coset can also be described by a symmetric matrix

$$M_{MN} = \mathcal{V}_M{}^m\mathcal{V}_N{}^m + \mathcal{V}_M{}^a\mathcal{V}_N{}^a \tag{6}$$

which is manifestly $SO(6)\times SO(n)$ invariant.

Gaugings of the matter-coupled $N = 4$ supergravity can be efficiently implemented by using the embedding tensor formalism [37]. This constant tensor describes the embedding of a gauge group G_0 in the global symmetry group $SL(2)\times SO(6,n)$. It turns out that $N = 4$ supersymmetry allows only the following components of the embedding tensor $\xi_{\alpha M}$ and $f_{\alpha MNP} = f_{\alpha[MNP]}$. To describe a closed subalgebra of $SL(2)\times SO(6,n)$, the embedding tensor needs to satisfy the quadratic constraints

$$\begin{aligned}
&\xi_\alpha{}^M\xi_{\beta M} = 0, \qquad \epsilon^{\alpha\beta}\left(\xi_\alpha{}^P f_{\beta MNP} + \xi_{\alpha M}\xi_{\beta N}\right) = 0,\\
&\xi_{(\alpha}{}^P f_{\beta)MNP} = 0, \qquad 3f_{\alpha R[MN}f_{\beta PQ]}{}^R + 2\xi_{(\alpha[M}f_{\beta)NPQ]} = 0,\\
&\epsilon^{\alpha\beta}\left(f_{\alpha MNR}f_{\beta PQ}{}^R - \xi_\alpha{}^R f_{\beta R[M[P}\eta_{Q]N]}\right.\\
&\left.-\xi_{\alpha[M}f_{\beta N]PQ} + \xi_{\alpha[P}f_{\beta Q]MN}\right) = 0.
\end{aligned} \tag{7}$$

In this paper, we are only interested in maximally symmetric solutions of $N = 4$ gauged supergravity with only the metric and scalars non-vanishing. Therefore, we will set all the other fields to zero from now on. In this case, the bosonic Lagrangian takes the form of

$$\begin{aligned}
e^{-1}\mathcal{L} = &\frac{1}{2}R + \frac{1}{16}\partial_\mu M_{MN}\partial^\mu M^{MN}\\
&-\frac{1}{4(\text{Im}\tau)^2}\partial_\mu\tau\partial^\mu\tau^* - V.
\end{aligned} \tag{8}$$

The scalar potential V reads

$$\begin{aligned}
V = \frac{g^2}{16}\Bigg[&f_{\alpha MNP}f_{\beta QRS}M^{\alpha\beta}\left[\frac{1}{3}M^{MQ}M^{NR}M^{PS}\right.\\
&\left.+\left(\frac{2}{3}\eta^{MQ} - M^{MQ}\right)\eta^{NR}\eta^{PS}\right]\\
&-\frac{4}{9}f_{\alpha MNP}f_{\beta QRS}\epsilon^{\alpha\beta}M^{MNPQRS}\\
&+3\xi_\alpha{}^M\xi_\beta{}^N M^{\alpha\beta}M_{MN}\Bigg]
\end{aligned} \tag{9}$$

where M^{MN} and $M^{\alpha\beta}$ are the inverse matrices of M_{MN} and $M_{\alpha\beta}$, respectively. M^{MNPQRS} is obtained by raising indices of M_{MNPQRS} defined by

$$M_{MNPQRS} = \epsilon_{mnpqrs}\mathcal{V}_M{}^m\mathcal{V}_N{}^n\mathcal{V}_P{}^p\mathcal{V}_Q{}^q\mathcal{V}_R{}^r\mathcal{V}_S{}^s. \tag{10}$$

In subsequent analysis, it is useful to rewrite the potential in terms of the fermion-shift matrices A_1^{ij}, A_2^{ij} and $A_{2ai}{}^j$ that appear in the fermion mass-like terms and supersymmetry transformations of fermions. In general, the scalar potential can be determined by the supersymmetric Ward identity

$$\frac{1}{4}\delta^i_j V = \frac{1}{2}A_{2aj}{}^k A^*_{2ak}{}^i + \frac{1}{9}A_2^{ik}A^*_{2jk} - \frac{1}{3}A_1^{ik}A^*_{1jk} \tag{11}$$

which, after a contraction of indices, gives

$$V = \frac{1}{2}A_{2ai}{}^j A^*_{2aj}{}^i + \frac{1}{9}A_2^{ij}A^*_{2ij} - \frac{1}{3}A_1^{ij}A^*_{1ij}. \tag{12}$$

In terms of the scalar coset representatives, the fermion shift-matrices are given by

$$A_1^{ij} = \epsilon^{\alpha\beta}(\mathcal{V}_\alpha)^* \mathcal{V}_{kl}{}^M \mathcal{V}_N{}^{ik} \mathcal{V}_P{}^{jl} f_{\beta M}{}^{NP},$$
$$A_2^{ij} = \epsilon^{\alpha\beta}\mathcal{V}_\alpha \mathcal{V}_{kl}{}^M \mathcal{V}_N{}^{ik} \mathcal{V}_P{}^{jl} f_{\beta M}{}^{NP} + \frac{3}{2}\epsilon^{\alpha\beta}\mathcal{V}_\alpha \mathcal{V}_M{}^{ij}\xi_\beta{}^M,$$
$$A_{2ai}{}^j = \epsilon^{\alpha\beta}\mathcal{V}_\alpha \mathcal{V}_a{}^M \mathcal{V}_{ik}{}^N \mathcal{V}_P{}^{jk} f_{\beta MN}{}^P - \frac{1}{4}\delta_i^j \epsilon^{\alpha\beta}\mathcal{V}_\alpha \mathcal{V}_a{}^M \xi_{\beta M}. \quad (13)$$

$\mathcal{V}_M{}^{ij}$ is obtained from $\mathcal{V}_M{}^m$ by converting the $SO(6)$ vector index to an anti-symmetric pair of $SU(4)$ fundamental indices using the chiral $SO(6)$ gamma matrices.

3 de Sitter vacua of $N = 4$ four-dimensional gauged supergravity

In this section, we will look for gauge groups that lead to de Sitter vacua. The analysis is similar to that given in [36] for dS_5 vacua. Furthermore, the procedure is closely parallel to the case of maximally supersymmetric AdS_4 vacua given in [40].

Denoting the chiral $SO(6)$ gamma matrices by Γ_m^{ij}, we can write $\mathcal{V}_M{}^{ij}$ in term of the coset representative $\mathcal{V}_M{}^m$ as

$$\mathcal{V}_M{}^{ij} = \mathcal{V}_M{}^m \Gamma_m^{ij}. \quad (14)$$

Similarly, the inverse element $\mathcal{V}_{ij}{}^M$ is given by

$$\mathcal{V}_{ij}{}^M = \mathcal{V}_m{}^M (\Gamma_m^{ij})^*. \quad (15)$$

In order to have dS_4 vacua, we require that

$$\langle \delta V \rangle = 0 \quad \text{and} \quad \langle V \rangle > 0 \quad (16)$$

where, as in [40], the bracket $\langle\ \rangle$ indicates that the quantity inside is evaluated at the vacuum. In terms of the fermion-shift matrices, these conditions read

$$\langle \delta V \rangle = -\frac{1}{3}\langle \delta A_1^{ij} A_{1ij}^* \rangle - \frac{1}{3}\langle \delta A_{1ij}^* A^{1ij} \rangle + \frac{1}{9}\langle \delta A_2^{ij} A_{2ij}^* \rangle + \frac{1}{9}\langle \delta A_{2ij}^* A_2^{ij} \rangle + \frac{1}{2}\langle \delta A_{2ai}{}^j A_{2aj}^*{}^i \rangle + \frac{1}{2}\langle A_{2ai}{}^j \delta A_{2aj}^*{}^i \rangle = 0, \quad (17)$$
$$\text{and} \quad \frac{1}{9}\langle A_2^{ij} A_{2ij}^* \rangle + \frac{1}{2}\langle A_{2ai}{}^j A_{2aj}^*{}^i \rangle > \frac{1}{3}\langle A_1^{ij} A_{1ij}^* \rangle. \quad (18)$$

In general, there are various solutions to these conditions. However, as in five dimensions, we will consider only the following two possibilities:

1. $\langle A_1^{ij} \rangle = \langle A_{2ai}{}^j \rangle = 0$ and $\langle A_2^{ik} A_{2jk}^* \rangle = \frac{9}{4}|\mu|^2 \delta_j^i$ with $A_{2ij}^* \delta A_2^{ij} + \delta A_{2ij}^* A_2^{ij} = 0$.
2. $\langle A_1^{ij} \rangle = \langle A_2^{ij} \rangle = 0$ and $\langle A_{2ai}{}^k A_{2ak}^*{}^j \rangle = \frac{1}{2}|\mu|^2 \delta_i^j$ with $\delta A_{2ai}{}^j A_{2aj}^*{}^i + A_{2ai}{}^j \delta A_{2aj}^*{}^i = 0$.

$|\mu|^2 = V_0$ denotes the cosmological constant. We now consider these two sets of conditions separately.

3.1 $\langle A_1^{ij} \rangle = \langle A_{2ai}{}^j \rangle = 0$ and $\langle A_2^{ik} A_{2jk}^* \rangle = \frac{9}{4}|\mu|^2 \delta_j^i$

We begin with the $\langle A_{2ai}{}^j \rangle = 0$ condition. From equation (13), we see that the first term in $\langle A_{2ai}{}^j \rangle = 0$ is traceless in i and j indices while the second term is the trace part. They must then vanish separately and lead to the following condition

$$\xi_{\alpha a} = 0 \quad (19)$$

where $\xi_\alpha{}^a = \langle \mathcal{V}_M{}^a \rangle \xi_\alpha{}^M$. Using the first condition in the quadratic constraint (7) and $\xi_\alpha{}^m = \langle \mathcal{V}_M{}^m \rangle \xi_\alpha{}^M$, we find

$$\xi_\alpha{}^M \xi_{\beta M} = -\xi_\alpha{}^m \xi_\beta{}^m + \xi_\alpha{}^a \xi_\beta{}^a = -\xi_\alpha{}^m \xi_\beta{}^m = 0 \quad (20)$$

for any values of α and β. This implies that $\xi_{\alpha m} = 0$. Together with (19), we then have

$$\xi_{\alpha M} = 0. \quad (21)$$

This condition implies that the gauge group is entirely embedded in $SO(6, n)$.

With $\xi_{\alpha M} = 0$, the $\langle A_{2ai}{}^j \rangle = 0$ condition gives

$$\epsilon^{\alpha\beta}\langle \mathcal{V}_\alpha \mathcal{V}_a{}^M \mathcal{V}_{ik}{}^N \mathcal{V}_P{}^{jk} \rangle f_{\beta MN}{}^P = 0\,. \quad (22)$$

In subsequent analysis, as in [40], it is useful to introduce "dressed" complex components of the embedding tensor

$$\mathfrak{f}_{ABC} = \mathfrak{f}_{1\,ABC} + i\mathfrak{f}_{2\,ABC} = \langle \mathcal{V}_A{}^M \mathcal{V}_B{}^N \mathcal{V}_C{}^P \mathcal{V}_\alpha \rangle \epsilon^{\alpha\beta} f_{\beta NMP}. \quad (23)$$

The real and imaginary parts are given explicitly by

$$\mathfrak{f}_{1\,ABC} = \frac{1}{\sqrt{\langle \text{Im}\tau \rangle}} \langle \mathcal{V}_A{}^M \mathcal{V}_B{}^N \mathcal{V}_C{}^P \rangle \times [\langle \text{Re}\tau \rangle f_{-NMP} - f_{+NMP}], \quad (24)$$
$$\mathfrak{f}_{2\,ABC} = \langle \sqrt{\text{Im}\tau}\, \mathcal{V}_A{}^M \mathcal{V}_B{}^N \mathcal{V}_C{}^P \rangle f_{-NMP}. \quad (25)$$

It should also be noted that by working at the origin of the scalar manifold $SL(2)/SO(2)$ with $\text{Im}\tau = 1$ and $\text{Re}\tau = 0$, we see that $\mathfrak{f}_{1\,ABC}$ and $\mathfrak{f}_{2\,ABC}$ correspond to electric and magnetic components of $f_{\alpha MNP}$, respectively. In particular, we have for $\xi_{\alpha M} = 0$,

$$A_1^{ij} = -\mathfrak{f}^*_{mnp}(\Gamma^m\Gamma^{*n}\Gamma^p)^{ij}, \tag{26}$$

$$A_2^{ij} = -\mathfrak{f}_{mnp}(\Gamma^m\Gamma^{*n}\Gamma^p)^{ij}, \tag{27}$$

$$A_{2ai}{}^j = \mathfrak{f}_{amn}(\Gamma^{mn})_i{}^j. \tag{28}$$

With all these ingredients, we can rewrite equation (22) as

$$\mathfrak{f}_{amn}(\Gamma^{mn})_i{}^j = 0 \tag{29}$$

which gives

$$\mathfrak{f}_{amn} = 0. \tag{30}$$

This also implies that both real and imaginary parts of $\mathfrak{f}_{amn}$ vanish or equivalently

$$f_{\alpha amn} = 0. \tag{31}$$

The conditions $\langle A_1^{ij}\rangle = 0$ and $\langle A_2^{ik}A^*_{2jk}\rangle = \frac{9}{4}|\mu|^2\delta^i_j$ give

$$\mathfrak{f}_{mnp}(\Gamma^n\Gamma^{*m}\Gamma^p)^{ij} = -\sqrt{3}\mu P^{ij} \quad \text{and}$$
$$\mathfrak{f}^*_{mnp}(\Gamma^n\Gamma^{*m}\Gamma^p)^{ij} = 0 \tag{32}$$

where we have introduced a constant matrix P^{ij} with the property $P^{ik}P^*_{jk} = \delta^i_j$.

It can be seen that all of these conditions are very similar to those for the existence of supersymmetric AdS_4 vacua with $\mathfrak{f}_{mnp}$ and $\mathfrak{f}^*_{mnp}$ interchanged and μ replaced by $\sqrt{3}\mu$. Using the identity given in (174), we obtain

$$\mathfrak{f}^*_{mnp}\mathfrak{f}^{mnp} = 6|\mu|^2 \quad \text{and} \quad \mathfrak{f}^*_{mnp} + i\epsilon_{mnpqrs}\mathfrak{f}^*_{qrs} = 0. \tag{33}$$

In terms of real and imaginary parts, these can be written as

$$\mathfrak{f}_{1\,mnp}\mathfrak{f}_1{}^{mnp} + \mathfrak{f}_{2\,mnp}\mathfrak{f}_2{}^{mnp} = 6|\mu|^2, \tag{34}$$

$$\mathfrak{f}_{1\,mnp} = \epsilon_{mnpqrs}\mathfrak{f}_{2\,qrs} \quad \text{and} \quad \mathfrak{f}_{2\,mnp} = -\epsilon_{mnpqrs}\mathfrak{f}_{1\,qrs} \tag{35}$$

which imply that both $\mathfrak{f}_1$ and $\mathfrak{f}_2$ cannot be zero, hence the gauge groups are essentially dyonic.

All these conditions must be solved subject to the quadratic constraint which for $\xi_{\alpha M} = 0$ simplifies considerably

$$f_{\alpha R[MN}f_{\beta PQ]}{}^R = 0 \quad \text{and} \quad \epsilon^{\alpha\beta}f_{\alpha MNR}f_{\beta PQ}{}^R = 0. \tag{36}$$

In terms of $\mathfrak{f}_{ABC}$, these constraints read

$$\mathfrak{f}_{[AB}{}^E\mathfrak{f}_{CD]E} = 0, \qquad \text{Re}(\mathfrak{f}_{[AB}{}^E\mathfrak{f}^*_{CD]E}) = 0,$$
$$\text{Im}(\mathfrak{f}_{AB}{}^E\mathfrak{f}^*_{CDE}) = 0. \tag{37}$$

It has been shown in [40] that the conditions (33) and the $(mnpq)$-components of the quadratic constraint (37) has a unique solution of the form

$$\mathfrak{f}_{123} = \frac{1}{\sqrt{2}}\mu \quad \text{and} \quad \mathfrak{f}_{456} = \frac{i}{\sqrt{2}}\mu \tag{38}$$

or, equivalently,

$$\mathfrak{f}_{1\,123} = \frac{1}{\sqrt{2}}\mu \quad \text{and} \quad \mathfrak{f}_{2\,456} = \frac{1}{\sqrt{2}}\mu. \tag{39}$$

Note some numerical changes especially the different relative sign between $\mathfrak{f}_{1\,123}$ and $\mathfrak{f}_{2\,456}$ which is opposite to that of the AdS_4 case.

At this point, all the remaining parts of the whole analysis are essentially the same as in [40]. In particular, the resulting gauge groups that can give rise to dS_4 vacua must take the same form as in the AdS_4 case. We will not repeat all the details here but simply summarize the structure of possible gauge groups. First of all, it should be noted that other components of the embedding tensors, $\mathfrak{f}_{mab}$ and $\mathfrak{f}_{abc}$, are not constrained by the existence of dS_4 vacua.

For $\mathfrak{f}_{mab} = \mathfrak{f}_{abc} = 0$, the gauge group is only generated by $\mathfrak{f}_{mnp}$. With the solution (38), we find the gauge group of the form

$$SO(3)_{\text{e}} \times SO(3)_{\text{m}} \tag{40}$$

which can be embedded entirely in the $SO(6)$ R-symmetry. The full gauge group is dyonically gauged by the six graviphotons with the two $SO(3)$ factors being electrically and magnetically gauged, respectively.

For $\mathfrak{f}_{mab} = 0$ but $\mathfrak{f}_{abc} \neq 0$, $\mathfrak{f}_{mmp}$ and $\mathfrak{f}_{abc}$ lead to gauge groups of the form

$$SO(3)_{\text{e}} \times SO(3)_{\text{m}} \times G_0^v \subset SO(6,n). \tag{41}$$

with G_0^v being a compact group gauged by vector fields from the vector multiplets.

For $\mathfrak{f}_{mab} \neq 0$ and $\mathfrak{f}_{abc} \neq 0$, there can be two subsets of $\mathfrak{f}_{abc}$ in which one has common indices with $\mathfrak{f}_{mab}$, but the other one does not. The latter again forms a separate compact group. The $(mnpq)$-components of the quadratic constraint implies that $\mathfrak{f}_{1\,mab}$ and $\mathfrak{f}_{2\,mab}$ are non-vanishing only for $m = 1, 2, 3$ and $m = 4, 5, 6$, respectively. Therefore, $\mathfrak{f}_{mab}$ extend $SO(3)_{\text{e}} \times SO(3)_{\text{m}}$ to a product of non-compact groups $\text{G}_{\text{e}} \times G_{\text{m}}$ containing $SO(3)_{\text{e}} \times SO(3)_{\text{m}}$ as a subgroup. The general form of gauge groups is then given by

$$G_{\text{e}} \times G_{\text{m}} \times G_0^v \subset SO(6,n). \tag{42}$$

Finally, we will explicitly check that solutions to all of the above conditions indeed extremize the scalar potential. This is crucial because our conditions do not arise from the requirement of supersymmetric vacua as in the AdS_4 case. To proceed, we first note a number of useful relations

$$\delta \mathcal{V}_M{}^m = \mathcal{V}_M{}^a \delta\phi^{ma}, \qquad \delta \mathcal{V}_M{}^a = \mathcal{V}_M{}^m \delta\phi^{ma},$$
$$\delta \mathcal{V}_\alpha = \frac{i}{2\text{Im}\tau}\left(\mathcal{V}_\alpha^* \delta\tau - \mathcal{V}_\alpha \delta\text{Re}\tau\right) \tag{43}$$

from which it follows that

$$\delta \mathfrak{f}_{npq} = -3\delta_{m[n}\mathfrak{f}_{pq]a}\delta\phi^{ma} + \frac{1}{\text{Im}\tau}\text{Im}\mathfrak{f}_{npq}\delta\text{Re}\tau$$
$$-\frac{1}{2}\text{Im}\tau \mathfrak{f}^*_{npq}\delta\text{Im}\tau, \tag{44}$$
$$\delta \mathfrak{f}_{npb} = (2\delta_{m[n}\mathfrak{f}_{p]ab} - \delta_{ab}\mathfrak{f}_{mnp})\delta\phi^{ma} + \frac{1}{\text{Im}\tau}\text{Im}\mathfrak{f}_{npb}\delta\text{Re}\tau$$
$$-\frac{1}{2}\text{Im}\tau \mathfrak{f}^*_{npb}\delta\text{Im}\tau. \tag{45}$$

Using (27), it is straightforward to compute

$$\delta V = \frac{1}{9}\left(A_2^{ij}\delta A^*_{2ij} + \delta A_2^{ij} A^*_{2ij}\right)$$
$$= \frac{1}{9}\mathfrak{f}_{mnp}\delta\mathfrak{f}_{qrs}(\Gamma^m\Gamma^{*n}\Gamma^p)^{ij}(\Gamma^q\Gamma^{*r}\Gamma^s)^*_{ij} + \text{c.c.} \tag{46}$$

Using the identity (178), we can reduce this to

$$\delta V = -\frac{8}{3}\mathfrak{f}_{mnp}\delta\mathfrak{f}^{*mnp} + \text{c.c.}. \tag{47}$$

Since $\mathfrak{f}_{amn} = 0$ (from $\langle A_{2ai}{}^j\rangle = 0$), we immediately see from (44) that $\langle \delta_\phi V\rangle = 0$.

Furthermore using (26) and (27), we can derive the following results

$$\delta_{\text{Im}\tau} A_1^{ij} = -\frac{1}{2\text{Im}\tau} A_2^{ij}\delta\text{Im}\tau \quad \text{and}$$
$$\delta_{\text{Im}\tau} A_2^{ij} = -\frac{1}{2\text{Im}\tau} A_1^{ij}\delta\text{Im}\tau. \tag{48}$$

Using $\langle A_1^{ij}\rangle = 0$, we obtain $\langle \delta_{\text{Im}\tau} A_2^{ij}\rangle = 0$, hence $\langle \delta_{\text{Im}\tau} V\rangle = 0$. Finally, we consider the variation with respect to $\text{Re}\tau$ which reads

$$\delta_{\text{Re}\tau} V = -\frac{2}{3\text{Im}\tau}\text{Im}\mathfrak{f}_{mnp}(\mathfrak{f}^{*mnp} + \mathfrak{f}^{mnp})\delta\text{Re}\tau$$
$$= -\frac{4}{3\text{Im}\tau}\mathfrak{f}_1{}^{mnp}\mathfrak{f}_{2\,mnp}\delta\text{Re}\tau. \tag{49}$$

This vanishes due to the condition (35) which implies that $\mathfrak{f}_{1\,mnp}$ and $\mathfrak{f}_{2\,mnp}$ have no common indices. We then conclude that the conditions $\langle A_1^{ij}\rangle = \langle A_{2ai}{}^j\rangle = 0$ and $\langle A_2^{ik} A^*_{2jk}\rangle = \frac{9}{4}|\mu|^2\delta^i_j$ extremize the scalar potential and give dS_4 vacua of the matter-coupled $N = 4$ gauged supergravity.

As a final comment, we note that although the gauge groups giving rise to dS_4 vacua are exactly the same as those leading to supersymmetric AdS_4 solutions, the two types of vacua occur at different values of the ratio between the coupling constants of $SO(3)_\text{e}$ (G_e) and $SO(3)_\text{m}$ (G_m). More precisely, the two cases have the coupling ratios with opposite sign, recall the sign change in (39) as compared to the results of [40]. We will see this in explicit examples in the next section.

3.2 $\langle A_1^{ij}\rangle = \langle A_2^{ij}\rangle = 0$ and $\langle A_{2ai}{}^k A^*_{2ak}{}^j\rangle = \frac{1}{2}|\mu|^2\delta^j_i$

We now look at another possibility for dS_4 vacua to exist. In this case, we require that

$$\langle A_1^{ij}\rangle = \langle A_2^{ij}\rangle = 0 \quad \text{and} \quad \langle A_{2ai}{}^k A^*_{2ak}{}^j\rangle = \frac{1}{2}|\mu|^2\delta^j_i. \tag{50}$$

Since A_2^{ij} consists of two parts, one symmetric and the other anti-symmetric in i and j indices, these two parts must vanish separately. Setting the anti-symmetric part to zero gives

$$\xi_{\alpha m} = 0. \tag{51}$$

We again use the quadratic constraint $\xi_\alpha{}^M\xi_{\beta M} = 0$ and find that

$$\xi_\alpha{}^a\xi_\beta{}^a = 0 \tag{52}$$

which gives $\xi_{\alpha a} = 0$. Therefore, we have $\xi_{\alpha M} = 0$ as in the previous case.

With this result, the first two conditions in (50) give

$$\mathfrak{f}_{mnp}(\Gamma^n\Gamma^{*m}\Gamma^p)^{ij} = 0 \quad \text{and} \quad \mathfrak{f}^*_{mnp}(\Gamma^n\Gamma^{*m}\Gamma^p)^{ij} = 0 \tag{53}$$

which imply that

$$\mathfrak{f} = \mathfrak{f}^* = 0 \quad \text{or} \quad f_{\alpha mnp} = 0, \tag{54}$$

so, in this case, compact non-abelian subgroups of the $SO(6)$ R-symmetry are not gauged.

For the last condition in (50), a straightforward computation gives

$$\frac{1}{2}|\mu|^2\delta^i_j = \langle A_{2aj}{}^k A^*_{2ak}{}^i\rangle = -\frac{1}{2}\mathfrak{f}_{amn}\mathfrak{f}^*_{apq}\{\Gamma^{mn},\Gamma^{pq}\}^i{}_j. \tag{55}$$

Using the identity (177), we arrive at

$$|\mu|^2\delta^i_j = -2\mathfrak{f}^{amn}\mathfrak{f}^{*apq}(\Gamma_{mnpq})^i{}_j + 4\mathfrak{f}_{amn}\mathfrak{f}^{*amn}\delta^i_j \tag{56}$$

which gives

$$\mathfrak{f}_{amn}\mathfrak{f}^{*amn} = \frac{1}{4}|\mu|^2 \quad \text{and} \quad \mathfrak{f}^{a[mn}\mathfrak{f}^{*pq]a} = 0. \tag{57}$$

From the first condition, we see that $\mathfrak{f}_{amn}$ must be non-vanishing for the dS_4 vacua to exist ($\mu \neq 0$). Therefore, the gauge groups must be necessarily non-compact.

We then consider the extremization of the scalar potential

$$\delta V = \frac{1}{2}\left(\delta A_{2ai}{}^{j} A^{*}_{2aj}{}^{i} + A_{2ai}{}^{j}\delta A^{*}_{2aj}{}^{i}\right) = -\frac{1}{2}\mathfrak{f}^{*}_{apq}\delta\mathfrak{f}_{amn}\mathrm{Tr}\{\Gamma^{mn},\Gamma^{pq}\} + \text{c.c.}. \quad (58)$$

Using $\delta\mathfrak{f}_{amn}$ from (45) and the result in (54), we obtain, upon setting $\delta V = 0$,

$$\delta_\phi V = 0: \quad \mathfrak{f}^{*amn}\mathfrak{f}_{abm} = 0, \quad (59)$$

$$\delta_{\mathrm{Re}\tau} V = 0: \quad \mathrm{Im}\mathfrak{f}_{amn}\mathrm{Re}\mathfrak{f}^{amn} = 0 \quad \text{or} \quad \mathfrak{f}_{1\,amn}\mathfrak{f}_{2}{}^{amn} = 0, \quad (60)$$

$$\delta_{\mathrm{Im}\tau} V = 0: \quad \mathfrak{f}_{amn}\mathfrak{f}^{amn} = 0 \quad \text{or} \quad \mathfrak{f}_{1\,amn}\mathfrak{f}_{1}{}^{amn} = \mathfrak{f}_{2\,amn}\mathfrak{f}_{2}{}^{amn} \quad \text{and} \quad \mathfrak{f}_{1\,amn}\mathfrak{f}_{2}{}^{amn} = 0. \quad (61)$$

Note that the second condition in (61) is the same as (60) and implies that $\mathfrak{f}_{1\,amn}$ and $\mathfrak{f}_{2\,amn}$ have no common indices.

To determine the form of possible gauge groups, we need to solve all the above conditions subject to the quadratic constraint (37). By substituting the results from (60) and (61) in the first condition of (57), we find

$$\mathfrak{f}_{1\,amn}\mathfrak{f}_{1}{}^{amn} = \mathfrak{f}_{2\,amn}\mathfrak{f}_{2}{}^{amn} = \frac{1}{8}|\mu|^2. \quad (62)$$

This result and the condition (60) imply that, apart from being non-compact, the gauge group must be a product of at least two non-compact groups and dyonically embedded in $SO(6,n)$ since both $\mathfrak{f}_{1\,amn}$ and $\mathfrak{f}_{2\,amn}$ or $f_{\pm amn}$ are non-vanishing.

Finally, using the result from (54), we find that the second condition in (57) is already implied by the $(mnpq)$-component of the quadratic constraint. Therefore, we have a set of consistent conditions to be imposed on the embedding tensor. In the following, we will look for explicit solutions and possible forms of the corresponding gauge groups. The analysis will be closely parallel to that in the previous case.

We first note that components $\mathfrak{f}_{mab}$ and $\mathfrak{f}_{abc}$ are not constrained by the existence of dS_4 vacua. These components can be anything without affecting the dS_4 vacua. However, the structure of gauge groups will be different for different values of $\mathfrak{f}_{mab}$ and $\mathfrak{f}_{abc}$. We now look at various possibilities.

For the simplest case of $\mathfrak{f}_{mab} = \mathfrak{f}_{abc} = 0$, the only non-vanishing components of the embedding tensor are given by $\mathfrak{f}_{amn}$. Since $\mathfrak{f}_{abc} = 0$, the compact part of the gauge group must be an abelian $SO(2)$ group. $\mathfrak{f}_{amn}$ then leads to $SO(2,1)$ gauge group. The full gauge group generated by both real and imaginary parts of $\mathfrak{f}_{amn}$, or $f_{\pm amn}$, is given by a product of two $SO(2,1)$ factors, electrically and magnetically gauged,

$$SO(2,1)_{\mathrm{e}} \times SO(2,1)_{\mathrm{m}}. \quad (63)$$

In this gauge group, the two compact generators are embedded in the matter directions, $a, b = 1, 2, \ldots, n$. In notation of [28], this gauge group can be written as

$$SO(2,1)_{+} \times SO(2,1)_{+}. \quad (64)$$

It should be noted that the plus sign indicates that the $SO(2) \times SO(2)$ compact subgroup is embedded in the positive part of the $SO(6,n)$ invariant tensor η_{MN}.

We then move to the case of $\mathfrak{f}_{mab} = 0$ but $\mathfrak{f}_{abc} \neq 0$. The $(mnpa)$- and $(mabc)$-components of the quadratic constraint are trivially satisfied while the $(abcd)$-component reduces to the standard Jacobi's identity for $\mathfrak{f}_{abc}$ corresponding to a compact group H_{c}. The $(mnab)$-component of the quadratic constraint implies that $\mathfrak{f}_{amn}$ together with $\mathfrak{f}_{abc}$ generate a non-compact group G_{nc}. The full gauge group with both electric and magnetic factors taken into account is then given by

$$G_{\mathrm{nc,e(m)}} \times G'_{\mathrm{nc,m(e)}}. \quad (65)$$

It is useful to note that since $\mathfrak{f}_{1\,amn}$ and $\mathfrak{f}_{2\,amn}$ cannot have common indices, the number of non-compact generators n_{nc} for G_{nc} and G'_{nc} must satisfy $2 \leq n_{\mathrm{nc}} \leq 4$. An example for the gauge groups with $n = 6$ vector multiplets is given by

$$SO(2,1)_{\mathrm{e(m)}} \times SU(2,1)_{\mathrm{m(e)}} \quad \text{or} \quad SO(2,1)_{+} \times SU(2,1)_{+}. \quad (66)$$

It should be noted that, for $n = 6$, there are in total 12 vector fields, so the full gauge group also contains an additional abelian $SO(2)$ factor corresponding to the gauge symmetry of the remaining gauge field. However, matter fields are not charged under this $SO(2)$ factor as in the ungauged $N = 4$ supergravity. We have accordingly omitted this factor in Eq. (66).

We now consider the case $\mathfrak{f}_{abc} = 0$ but $\mathfrak{f}_{mab} \neq 0$. For $\mathfrak{f}_{abc} = 0$, we again find that $\mathfrak{f}_{amn}$ lead to $SO(2,1)$ gauge group. Furthermore, since the existence of dS_4 vacua requires $\mathfrak{f}_{mnp} = 0$, the gauge group generated by $\mathfrak{f}_{mab}$ must also be $SO(2,1)$. Note also that the compact parts of these two $SO(2,1)$ factors are embedded in the matter and R-symmetry direction, respectively. After taking into account both electric and magnetic components, we find that the gauge group takes the form of

$$SO(2,1)^2_{\mathrm{e(m)}} \times SO(2,1)^2_{\mathrm{m(e)}} \quad \text{or} \quad SO(2,2)_{+} \times SO(2,2)_{-}. \quad (67)$$

In this equation, we have used the isomorphism $SO(2,2)_\pm \sim SO(2,1)_\pm \times SO(2,1)_\pm$. We also note here that in this case, there can be three electric (magnetic) and one magnetic (electric) $SO(2,1)$ factors. The number of electric and magnetic factors needs not be equal, but both types of gaugings are required.

We finally consider the most general case of $\mathfrak{f}_{abc} \neq 0$ and $\mathfrak{f}_{mab} \neq 0$. In general, there can be a subspace in which a subset of $\mathfrak{f}_{abc}$ forms a separate compact group H_c. We will split $\mathfrak{f}_{abc}$ into two parts $\mathfrak{f}_{a'b'c'}$ and $\mathfrak{f}_{a''b''c''}$ with $\mathfrak{f}_{a''mn} = 0$. The components $\mathfrak{f}_{a'b'c'}$ together with $\mathfrak{f}_{a'mn}$ form a non-compact group G_{nc} as discussed above while $\mathfrak{f}_{a''b''c''}$ form a separate compact factor H_c. In addition, the quadratic constraint implies that $\mathfrak{f}_{amn}$ and $\mathfrak{f}_{mab}$ cannot have common indices, so $\mathfrak{f}_{mab}$ again generate an $SO(2,1)$ factor as in the previous case. The gauge group is then given by

$$SO(2,1)_{\text{e(m)}} \times SO(2,1)_{\text{m(e)}} \times G_{\text{nc, e(m)}} \times G'_{\text{nc, m(e)}} \times H_c. \qquad (68)$$

An example for this type of gauge groups with $n = 6$ vector multiplets and $\mathfrak{f}_{a''b''c''} = 0$ is given by

$$SO(3,1)_+ \times SO(2,1)_+ \times SO(2,1)_- \qquad (69)$$

with $SO(3,1)_+ \times SO(2,1)_+$ identified with $G_{\text{nc}} \times G'_{\text{nc}}$.

4 dS_4 vacua from different gauge groups

In this section, we consider gauge groups that lead to dS_4 vacua for the case of $n = 6$ vector multiplets. These gauge groups have been classified in [28]. There are nine semisimple gauge groups that can be embedded in $SO(6,6)$ given by

$$SO(2,1)_+^2 \times SO(2,1)_-^2, \qquad (70)$$
$$SO(3,1)_+ \times SO(2,1)_+ \times SO(2,1)_-, \qquad (71)$$
$$SO(3,1)_+ \times SO(3,1)_+, \qquad (72)$$
$$SO(3)_-^2 \times SO(3)_+^2, \qquad (73)$$
$$SO(3,1)_- \times SO(3)_- \times SO(3)_+, \qquad (74)$$
$$SO(3,1)_- \times SO(3,1)_-, \qquad (75)$$
$$SO(2,1)_+^3 \times SO(3)_+, \qquad (76)$$
$$SL(3,\mathbb{R})_- \times SO(3)_-, \qquad (77)$$
$$SU(2,1)_+ \times SO(2,1)_+ \qquad (78)$$

in which the extra $SO(2)$ factor in the last two gauge groups has been neglected. In the following analysis, we will explicitly compute the scalar potentials and fermion-shift matrices for these gauge groups and verify that the dS_4 vacua satisfy the two sets of conditions given in the previous section.

However, all these gauge groups have been originally constructed by using the old formulation of [39]. We need to recast them in the embedding tensor formalism. The first six gauge groups have already been done in [29] with the corresponding embedding tensors for various factors given by

$$SO(4)_{\text{e(m)}}: \begin{cases} f_{+123} = \sqrt{2}(g_1 - \tilde{g}_1), & f_{+789} = \sqrt{2}(g_1 + \tilde{g}_1), \\ f_{-456} = \sqrt{2}(g_2 - \tilde{g}_2), & f_{-10,11,12} = \sqrt{2}(g_2 + \tilde{g}_2) \end{cases} \qquad (79)$$

$$SO(3,1)_{\text{e(m)}}: \begin{cases} f_{+123} = -f_{+783} = f_{+729} = f_{+189} \\ \quad = \frac{1}{\sqrt{2}}(g_1 - \tilde{g}_1), \\ f_{+789} = -f_{+129} = f_{+183} = f_{+723} \\ \quad = \frac{1}{\sqrt{2}}(g_1 + \tilde{g}_1), \\ f_{-456} = -f_{10,11,12} = f_{-10,5,12} \\ \quad = f_{-4,11,12} = \frac{1}{\sqrt{2}}(g_2 - \tilde{g}_2), \\ f_{-10,11,12} = -f_{-4,5,12} = f_{-4,11,6} \\ \quad = f_{-10,5,6} = \frac{1}{\sqrt{2}}(g_2 + \tilde{g}_2) \end{cases} \qquad (80)$$

$$SO(2,2)_{\text{e(m)}}: \begin{cases} f_{+723} = \frac{1}{\sqrt{2}}(g_1 + \tilde{g}_1), \\ f_{+189} = \frac{1}{\sqrt{2}}(g_1 - \tilde{g}_1), \\ f_{-10,5,6} = \frac{1}{\sqrt{2}}(g_2 + \tilde{g}_2), \\ f_{-4,11,12} = \frac{1}{\sqrt{2}}(g_2 - \tilde{g}_2) \end{cases} \qquad (81)$$

The six gauge groups in (70) to (75) are obtained by combinations of these $SO(4)$, $SO(3,1)$ and $SO(2,2)$ groups with suitable choices of the coupling constants as shown in Table 1.

We also note here that there can be other possible assignments for which simple factor corresponding to electric or magnetic embedding. For example, in $SO(2,2) \times SO(2,2)$ gauge group, we can have only one electric factor of $SO(2,1)$ and three magnetic $SO(2,1)$ factors or vice versa. The embedding tensor in this case is given by

$$f_{+723} = \frac{1}{\sqrt{2}}(g_1 + \tilde{g}_1), \qquad f_{-189} = \frac{1}{\sqrt{2}}(g_1 - \tilde{g}_1),$$
$$f_{-10,5,6} = \frac{1}{\sqrt{2}}(g_2 + \tilde{g}_2), \qquad f_{-4,11,12} = \frac{1}{\sqrt{2}}(g_2 - \tilde{g}_2). \qquad (82)$$

The electric-magnetic dual with one magnetic and three electric $SO(2,1)$'s is simply obtained by interchanging $+$ and $-$.

The embedding tensors for the remaining three gauge groups $SO(3) \times SO(2,1)^3$, $SU(2,1) \times SO(2,1)$ and $SL(3,\mathbb{R}) \times SO(3)$ are obtained as follow.

- For $SO(3)_+ \times SO(2,1)_+^3$, we rewrite it as $SO(3) \times SO(2,1) \times SO(2,1)^2$ with the embedding tensor given by

Table 1 The six gauge groups giving rise to dS_4 vacua as given in [28]. The embedding tensors for these gauge groups are obtained by imposing some relations between the coupling constants as shown in the last column

Gauge groups in [28]	Gauge groups in [29]	Conditions
$SO(3)^2_- \times SO(3)^2_+$	$SO(4)_e \times SO(4)_m$	$g_1, \tilde{g}_1, g_2, \tilde{g}_2 \neq 0$
$SO(3,1)_+ \times SO(3,1)_+$	$SO(3,1)_e \times SO(3,1)_m$	$\tilde{g}_1 = g_1, \tilde{g}_2 = g_2,$ $g_1, g_2 \neq 0$
$SO(3,1)_- \times SO(3,1)_-$	$SO(3,1)_e \times SO(3,1)_m$	$\tilde{g}_1 = -g_1, \tilde{g}_2 = -g_2,$ $g_1, g_2 \neq 0$
$SO(3,1)_- \times SO(3)_- \times SO(3)_+$	$SO(3,1)_m \times SO(4)_e$	$\tilde{g}_2 = -g_2,$ $g_2, g_1, \tilde{g}_1 \neq 0$
	$SO(3,1)_e \times SO(4)_m$	$\tilde{g}_1 = -g_1,$ $g_1, g_2, \tilde{g}_2 \neq 0$
$SO(2,1)^2_+ \times SO(2,1)^2_-$	$SO(2,2)_e \times SO(2,2)_m$	$g_1, \tilde{g}_1, g_2, \tilde{g}_2 \neq 0$
$SO(3,1)_+ \times SO(2,1)_+ \times SO(2,1)_-$	$SO(3,1)_m \times SO(2,2)_e$	$\tilde{g}_2 = g_2,$ $g_2, g_1, \tilde{g}_1 \neq 0$
	$SO(3,1)_e \times SO(2,2)_m$	$\tilde{g}_1 = g_1,$ $g_1, g_2, \tilde{g}_2 \neq 0$

$$f_{\alpha\,569} = g_1, \quad f_{\alpha\,10,11,12} = g_2, \quad f_{\beta\,127} = \tilde{g}_1, \quad f_{\beta\,348} = \tilde{g}_2 \tag{83}$$

with $\alpha = \pm$ and $\beta = \mp$ corresponding to the following electric and magnetic factors $SO(3)_{e(m)} \times SO(2,1)_{e(m)} \times SO(2,1)^2_{m(e)}$

- For $SU(2,1)_+ \times SO(2,1)_+$, we choose the following gauge generators. The $SO(2,1)$ factor is generated by X_5, X_6 and X_{11} while the $SU(2,1)$ is generated by $X_1, \ldots, X_4, X_7, \ldots X_{10}$ with the compact generators being $X_7, \ldots X_{10}$. The associated embedding tensor is given by

$$\begin{aligned} f_{\alpha\,129} &= f_{\alpha\,138} = f_{\alpha\,147} = f_{\alpha\,248} = -g_1, \\ f_{\alpha\,237} &= f_{\alpha\,349} = g_1, \\ f_{\alpha\,789} &= 2g_1, \qquad f_{\alpha\,1,2,10} = f_{\alpha\,3,4,10} = -\sqrt{3}g_1, \\ f_{\beta\,5,6,11} &= g_2 \end{aligned} \tag{84}$$

with $\alpha = \pm$ and $\beta = \mp$ corresponding to $SU(2,1)_{e(m)} \times SO(2,1)_{m(e)}$.

- For $SL(3,\mathbb{R})_- \times SO(3)_-$, we choose the generators for $SO(3)$ to be X_4, X_5 and X_6 while $SL(3,\mathbb{R})$ is generated by the compact X_1, X_2, X_3 and non-compact $X_7, \ldots, X_{11}$ generators. Non-vanishing components of the embedding tensor are given by

$$\begin{aligned} f_{\alpha\,123} &= f_{\alpha\,1,9,10} = f_{\alpha\,279} = -f_{\alpha\,2,8,10} = f_{\alpha\,3,7,10} \\ &= f_{\alpha\,3,8,9} = -g_1, \\ f_{\alpha\,178} &= 2g_1, \qquad f_{\alpha\,2,10,11} = f_{\alpha\,3,9,11} = \sqrt{3}g_1, \\ f_{\beta\,456} &= g_2 \end{aligned} \tag{85}$$

with $\alpha = \pm$ and $\beta = \mp$ corresponding to $SL(3,\mathbb{R})_{e(m)} \times SO(3)_{m(e)}$.

We now compute the fermion-shift matrices and scalar potential. To do an explicit computation, we will work with the coset representative $\mathcal{V}_\alpha$ of the form

$$\mathcal{V}_\alpha = e^{\phi/2} \begin{pmatrix} \chi - ie^{-\phi} \\ 1 \end{pmatrix}. \tag{86}$$

Since all known dS_4 vacua are found only with vanishing scalars from vector multiplets, we will give the scalar potential only for non-vanishing dilaton ϕ and axion χ to simplify the results.

With all $SO(6,6)/SO(6) \times SO(6)$ scalars set to zero, we simply have

$$\mathcal{V} = \mathbf{I}_{12} \tag{87}$$

and $\mathcal{V}_M{}^{ij} = \frac{1}{2}\Gamma^{ij}_m \mathcal{V}_M{}^m$. Note that an extra factor of $\frac{1}{2}$ is added for consistency with the normalization used in [37] for the following $SO(6,n)$ identity

$$\eta_{MN} = -\frac{1}{2}\epsilon_{ijkl}\mathcal{V}_M{}^{ij}\mathcal{V}_N{}^{kl} + \mathcal{V}_M{}^a\mathcal{V}_N{}^a\,. \tag{88}$$

An explicit form of $SO(6)$ gamma matrices is given in the appendix. We are now in a position to consider each gauge group in detail. We emphasize that all of the critical points considered here are already known. Our main aim is to verify that they satisfy the conditions introduced in the previous section. For convenience, we collect the conditions for the existence of dS_4 vacua given in Sects. 3.1 and 3.2 here

$$\langle A_1^{ij}\rangle = \langle A_{2ai}{}^j\rangle = 0, \qquad \langle A_2^{ij} A^*_{2kj}\rangle = \frac{9}{4}V_0\delta^i_k, \tag{89}$$

$$\langle A_1^{ij}\rangle = \langle A_2^{ij}\rangle = 0, \qquad \langle A_{2ak}{}^j A^*_{2aj}{}^i\rangle = \frac{1}{2}V_0\delta^i_k. \tag{90}$$

We will also refer to dS_4 vacua as the first and second type dS_4 if they satisfy (89) and (90), respectively.

4.1 $SO(3)^2_+ \times SO(3)^2_-$

This case corresponds to the gauging of $SO(4)_{\rm e} \times SO(4)_{\rm m}$ group. The embedding tensor for this gauge group is given in (79). The scalar potential is found to be

$$V = -e^{\phi}\left[g_1^2 - 2g_1\tilde{g}_1 + \chi^2(g_2-\tilde{g}_2)^2 + \tilde{g}_1^2\right] + 4(g_1-\tilde{g}_1)(g_2-\tilde{g}_2) - e^{-\phi}(g_2-\tilde{g}_2)^2 \qquad (91)$$

with the following critical point

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{g_2-\tilde{g}_2}{g_1-\tilde{g}_1}\right]. \qquad (92)$$

To bring this critical point to the origin $\chi = \phi = 0$, we have two possibilities:

- Setting $\tilde{g}_2 - g_2 = g_1 - \tilde{g}_1$ leads to an AdS_4 critical point which is the trivial critical point of the same gauge group reported in [41] with $V_0 = -6(g_1-\tilde{g}_1)^2$. As expected, this critical point satisfies the AdS_4 conditions given in [40]

$$\langle A_2^{ij}\rangle = \langle A_{2ai}{}^j\rangle = 0, \qquad \langle A_1^{ij}A^*_{1kj}\rangle = -\frac{4}{3}V_0\delta^i_k. \qquad (93)$$

- Another possibility is to set $\tilde{g}_2 - g_2 = -(g_1-\tilde{g}_1)$ which leads to a dS_4 critical point with $V_0 = 2(g_1-\tilde{g}_1)^2$ and satisfying the conditions in (89)

$$\langle A_1^{ij}\rangle = \langle A_{2ai}{}^j\rangle = 0, \qquad \langle A_2^{ij}A^*_{2kj}\rangle = V_0\delta^i_k. \qquad (94)$$

4.2 $SO(3,1)_\pm \times SO(3,1)_\pm$

The embedding tensor for this gauge group is given in (80) with the scalar potential given by

$$V = \frac{1}{2}e^{-\phi}\left[e^{2\phi}\left(g_1^2+4g_1\tilde{g}_1+\chi^2\left(g_2^2+4g_2\tilde{g}_2+\tilde{g}_2^2\right)+\tilde{g}_1^2\right) + 2e^{\phi}(g_1-\tilde{g}_1)(g_2-\tilde{g}_2) + g_2^2+4g_2\tilde{g}_2+\tilde{g}_2^2\right]. \qquad (95)$$

There is a critical point at

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{g_2^2+4g_2\tilde{g}_2+\tilde{g}_2^2}{g_1^2+4g_1\tilde{g}_1+\tilde{g}_1^2}\right]. \qquad (96)$$

We now separately consider two types of gauge groups.

- For $SO(3,1)_+ \times SO(3,1)_+$, we choose $\tilde{g}_1 = g_1$ and $\tilde{g}_2 = g_2$ which eliminate the following components of the embedding tensor

$$f_{+123} = -f_{+783} = f_{+729} = f_{+189} = 0,$$
$$f_{-456} = -f_{10,11,12} = f_{-10,5,12} = f_{-4,11,12} = 0. \qquad (97)$$

Accordingly, the $SO(3)$ subgroups of both $SO(3,1)$ factors are embedded along the matter-multiplet directions $M = 7,8,9$ and $M = 10,11,12$. Setting $\tilde{g}_1 = g_1$ and $\tilde{g}_2 = g_2$, we can rewrite the critical point (96) as

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{g_2}{g_1}\right] \qquad (98)$$

To bring this critical point to the values $\chi = \phi = 0$, we set $g_2 = \pm g_1$, and both of these choices lead to the same dS_4 critical point with $V_0 = 6g_2^2$ and satisfying (90)

$$\langle A_1^{ij}\rangle = \langle A_2^{ij}\rangle = 0, \qquad \langle A_{2ai}{}^k A^*_{2ak}{}^j\rangle = 3g_2^2\delta^j_i. \qquad (99)$$

This dS_4 vacuum is then of the second type. There is no AdS_4 vacuum in this case since the existence of AdS_4 requires the embedding of $SO(3)\times SO(3)$ along the R-symmetry directions.

- For $SO(3,1)_- \times SO(3,1)_-$, we set $\tilde{g}_1 = -g_1$ and $\tilde{g}_2 = -g_2$ which give

$$f_{+789} = -f_{+129} = f_{+183} = f_{+723} = 0,$$
$$f_{-10,11,12} = -f_{-4,5,12} = f_{-4,11,6} = f_{-10,5,6} = 0.$$

The $SO(3)$ subgroups of both $SO(3,1)$ factors are now embedded along the R-symmetry directions $M = 1,2,3$ and $M = 4,5,6$. With this choice of the coupling constants, the critical point (96) becomes

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{g_2}{g_1}\right]. \qquad (100)$$

In this case, however, setting $g_2 = \pm g_1$ leads to two different critical points.

– Setting $g_2 = g_1$ leads to a dS_4 critical point satisfying (89)

$$V_0 = 2g_2^2, \qquad \langle A_1^{ij}\rangle = \langle A_{2ai}{}^j\rangle = 0,$$
$$\langle A_2^{ij}A^*_{2kj}\rangle = \frac{9}{4}V_0\delta^i_k. \qquad (101)$$

– The choice $g_2 = -g_1$ gives an AdS_4 critical point with $V_0 = -6g_2^2$ and satisfying

$$\langle A_2^{ij}\rangle = \langle A_{2ai}{}^j\rangle = 0, \qquad \langle A_1^{ij}A^*_{1kj}\rangle = -\frac{3}{4}V_0\delta^i_k. \qquad (102)$$

In this case, the dS_4 vacuum is of the first type. It should be noted that, as in the $SO(3)^2_+ \times SO(3)^2_-$ gauge group, the $SO(3,1)_- \times SO(3,1)_-$ gauge group gives two types of vacua with opposite ratios of the coupling constants.

It is useful to note that, gauge groups of the form $SO(3,1)_{\pm}\times SO(3,1)_{\mp}$, obtained by setting $\tilde{g}_1=\pm g_1$ and $\tilde{g}_2=\mp g_2$, lead to a Minkowski vacuum.

4.3 $SO(2,1)^2\times SO(2,1)^2$

In this case, the gauge group is given by $SO(2,2)\times SO(2,2)\sim SO(2,1)^4$, and there are two possible gaugings to consider depending on the assignment of electric or magnetic gaugings to each $SO(2,1)$ factor. One gauging is described by $SO(2,2)_{\mathrm{e}}\times SO(2,2)_{\mathrm{m}}\sim SO(2,1)_{\mathrm{e}}\times SO(2,1)_{\mathrm{e}}\times SO(2,1)_{\mathrm{m}}\times SO(2,1)_{\mathrm{m}}$ with the embedding tensor given in (81). The other one is $SO(2,1)_{\mathrm{e}}\times SO(2,1)_{\mathrm{m}}\times SO(2,1)_{\mathrm{m}}\times SO(2,1)_{\mathrm{m}}$ with the embedding tensor given in (82) and its electric-magnetic dual $SO(2,1)_{\mathrm{e}}\times SO(2,1)_{\mathrm{e}}\times SO(2,1)_{\mathrm{e}}\times SO(2,1)_{\mathrm{m}}$.

It turns out that all of these gaugings give rise to the same scalar potential of the form

$$V=\frac{1}{4}e^{-\phi}\left[e^{2\phi}\left[(g_1+\tilde{g}_1)^2+\chi^2(g_2+\tilde{g}_2)^2\right]+(g_2+\tilde{g}_2)^2\right] \tag{103}$$

with the following critical point

$$\chi=0,\qquad \phi=\ln\left[\pm\frac{g_2+\tilde{g}_2}{g_1+\tilde{g}_1}\right]. \tag{104}$$

The critical point can be shifted to the origin $\chi=0$ and $\phi=0$ by setting $\tilde{g}_2+g_2=\pm(g_1+\tilde{g}_1)$. Both choices lead to the same dS_4 critical point with $V_0=\frac{1}{2}(g_1+\tilde{g}_1)^2$ and satisfying (90)

$$\langle A_1^{ij}\rangle=\langle A_2^{ij}\rangle=0,\qquad \langle A_{2ai}{}^{k}A^{*}_{2ak}{}^{j}\rangle=\frac{1}{2}V_0\delta_i^j. \tag{105}$$

4.4 $SO(3)_+\times SO(3)_-\times SO(3,1)_-$

This case corresponds to $SO(4)\times SO(3,1)$ gauge group with two possible gaugings $SO(4)_{\mathrm{e(m)}}\times SO(3,1)_{\mathrm{m(e)}}$. The embedding tensors are given by

I: $SO(3)_{\mathrm{e}+}\times SO(3)_{\mathrm{e}-}\times SO(3,1)_{\mathrm{m}-}$:

$$f_{+123}=g_1,\quad f_{+789}=\tilde{g}_1$$
$$f_{-456}=-f_{-10,11,6}=f_{-10,5,12}=f_{-4,11,12}=g_2, \tag{106}$$

II: $SO(3,1)_{\mathrm{e}-}\times SO(3)_{\mathrm{m}-}\times SO(3)_{\mathrm{m}+}$:

$$f_{+123}=-f_{+783}=f_{+729}=f_{+189}=g_1$$
$$f_{-456}=g_2,\qquad f_{10,11,12}=\tilde{g}_2. \tag{107}$$

Note that the embedding tensors for $SO(3,1)$'s in both cases are obtained from (80) by setting $\tilde{g}_2=-g_2$ and $\tilde{g}_1=-g_1$, respectively. The two gaugings lead to the same scalar potential given by

$$V=2g_1g_2-\frac{1}{2}e^{-\phi}g_2^2-\frac{1}{2}e^{\phi}(g_1^2+g_2^2\chi^2). \tag{108}$$

This potential admits a critical point at

$$\chi=0,\qquad \phi=\ln\left[\pm\frac{g_2}{g_1}\right] \tag{109}$$

which can be shifted to the origin $\chi=\phi=0$ by setting $g_2=\pm g_1$. We now look at these two choices.

- The case of $g_2=g_1$ leads to a dS_4 solution with $V_0=g_1^2$ and satisfies (89)

$$\langle A_1^{ij}\rangle=\langle A_{2ai}{}^{j}\rangle=0,\qquad \langle A_2^{ij}A^{*}_{2kj}\rangle=\frac{9}{4}V_0\delta^i_k. \tag{110}$$

- For $g_2=-g_1$, the critical point is an AdS_4 vacuum with $V_0=-3g_1^2$.

4.5 $SO(3,1)_+\times SO(2,1)_+\times SO(2,1)_-$

This case corresponds to $SO(2,2)\times SO(3,1)$ gauge group with two possible gaugings, $SO(2,2)\sim SO(2,1)\times SO(2,1)$ and $SO(3,1)$ factors being electric and magnetic or vice versa.

I: $SO(2,2)_{\mathrm{e}}\times SO(3,1)_{\mathrm{m}}$:

$$f_{+723}=\frac{1}{\sqrt{2}}(g_1+\tilde{g}_1),\qquad f_{+189}=\frac{1}{\sqrt{2}}(g_1-\tilde{g}_1),$$
$$f_{-456}=-f_{10,11,12}=f_{-10,5,12}$$
$$=f_{-4,11,12}=\frac{1}{\sqrt{2}}(g_2-\tilde{g}_2),$$
$$f_{-10,11,12}=-f_{-4,5,12}=f_{-4,11,6}=f_{-10,5,6}$$
$$=\frac{1}{\sqrt{2}}(g_2+\tilde{g}_2). \tag{111}$$

II: $SO(2,2)_{\mathrm{m}}\times SO(3,1)_{\mathrm{e}}$:

$$f_{+123}=-f_{+783}=f_{+729}=f_{+189}=\frac{1}{\sqrt{2}}(g_1-\tilde{g}_1),$$
$$f_{+789}=-f_{+129}=f_{+183}=f_{+723}=\frac{1}{\sqrt{2}}(g_1+\tilde{g}_1),$$
$$f_{-10,5,6}=\frac{1}{\sqrt{2}}(g_2+\tilde{g}_2),\quad f_{-4,11,12}=\frac{1}{\sqrt{2}}(g_2-\tilde{g}_2). \tag{112}$$

In order to have $SO(3,1)_+$, we will set $\tilde{g}_2 = g_2$ and $\tilde{g}_1 = g_1$, respectively. These two gaugings lead to the scalar potentials

$$V_{\rm I} = \frac{1}{4}e^{\phi}\left[g_1^2 + 2g_1\tilde{g}_1 + 2\chi^2\left(g_2^2 + 4g_2\tilde{g}_2 + \tilde{g}_2^2\right) + \tilde{g}_1^2\right] + \frac{1}{2}e^{-\phi}\left(g_2^2 + 4g_2\tilde{g}_2 + \tilde{g}_2^2\right) \tag{113}$$

$$V_{\rm II} = \frac{1}{4}e^{-\phi}\left[e^{2\phi}\left[2g_1^2 + 8g_1\tilde{g}_1 + \chi^2(g_2+\tilde{g}_2)^2 + 2\tilde{g}_1^2\right] + (g_2+\tilde{g}_2)^2\right] \tag{114}$$

We now look at critical points of these potentials.

- The critical point of $V_{\rm I}$ is given by

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{\sqrt{2(g_2^2 + 4g_2\tilde{g}_2 + \tilde{g}_2^2)}}{(g_1+\tilde{g}_1)}\right] \tag{115}$$

  which can be brought to the origin by choosing

$$\tilde{g}_1 = -g_1 \pm 2\sqrt{3}g_2 \tag{116}$$

  after setting $g_2 = \tilde{g}_2$. Both choices lead to the same dS_4 vacuum with $V_0 = 6g_2^2$ and satisfying (90).
- For $V_{\rm II}$, we find the following critical point

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{(g_2+\tilde{g}_2)}{\sqrt{2(g_1^2 + 4g_1\tilde{g}_t + \tilde{g}_1^2)}}\right]. \tag{117}$$

  After setting $\tilde{g}_1 = g_1$, this critical point can be brought to the origin by choosing

$$\tilde{g}_2 = -g_2 \pm 2\sqrt{3}g_1. \tag{118}$$

  Both sign choices again give the same dS_4 critical point which satisfies (90) and $V_0 = 6g_1^2$.

4.6 $SO(3)_+ \times SO(2,1)_+^3$

The embedding tensor for this gauge group is given in (83). The two possible gaugings $SO(3)_{\rm e(m)} \times SO(2,1)_{\rm e(m)} \times SO(2,1)^2_{\rm m(e)}$ respectively give the following scalar potentials and critical points

$$V_{\rm I} = \frac{1}{2}e^{-\phi}\left[g_1^2\left(\chi^2 e^{2\phi} + 1\right) + e^{2\phi}\left(\tilde{g}_1^2 + \tilde{g}_2^2\right)\right],$$

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{g_1}{\sqrt{\tilde{g}_1^2 + \tilde{g}_2^2}}\right] \tag{119}$$

and

$$V_{\rm II} = \frac{1}{2}e^{-\phi}\left[e^{2\phi}\left(g_1^2 + \chi^2\left(\tilde{g}_1^2 + \tilde{g}_2^2\right)\right) + \tilde{g}_1^2 + \tilde{g}_2^2\right],$$

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{\sqrt{\tilde{g}_1^2 + \tilde{g}_2^2}}{g_1}\right]. \tag{120}$$

These critical points can be shifted to the origin by setting $g_1 = \pm\sqrt{\tilde{g}_1^2 + \tilde{g}_2^2}$, leading to the same dS_4 solution with $V_0 = g_1^2$ and satisfying (90).

4.7 $SU(2,1)_+ \times SO(2,1)_+$

The embedding tensor for this gauge group is given in (84). This gauge group can be embedded either as $SU(2,1)_{\rm e} \times SO(2,1)_{\rm m}$ or $SU(2,1)_{\rm m} \times SO(2,1)_{\rm e}$. The scalar potentials and critical points for these two gaugings are given by

I: $$V_{\rm I} = \frac{1}{2}\left[e^{\phi}\left(12g_1^2 + g_2^2\chi^2\right) + e^{-\phi}g_2^2\right],$$

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{g_2}{2\sqrt{3}g_1}\right], \tag{121}$$

II: $$V_{\rm II} = \frac{1}{2}e^{\phi}\left(12g_1^2\chi^2 + g_2^2\right) + 6g_1^2e^{-\phi},$$

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{2\sqrt{3}g_1}{g_2}\right]. \tag{122}$$

Choosing $g_2 = \pm 2\sqrt{3}g_1$ leads to a dS_4 solution satisfying (90) with $V_0 = 12g_1^2$.

4.8 $SL(3,\mathbb{R})_- \times SO(3)_-$

The embedding tensor for this gauge group is given in (85). In this gauge group, both AdS_4 and dS_4 solutions are possible, and, unlike the previous cases, the choice of which gauge group factor is electric or magnetic affects the resulting solutions. We will consider each choice separately.

For $SL(3,\mathbb{R})_{\rm e} \times SO(3)_{\rm m}$ embedding, the scalar potential is given by

$$V = -\frac{1}{2}\left[e^{\phi}\left(g_1^2 + g_2^2\chi^2\right) + e^{-\phi}g_2^2 + 4g_1g_2\right] \tag{123}$$

with a critical point at

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{g_2}{g_1}\right]. \tag{124}$$

Choosing $g_2 = g_1$ and $g_2 = -g_1$ leads to AdS_4 and dS_4 vacua with $V_0 = -3g_1^2$ and $V_0 = g_1^2$, respectively. The dS_4 vacuum satisfies the relations given in (89).

For $SL(3,\mathbb{R})_{\rm m} \times SO(3)_{\rm e}$ embedding, we find a similar structure with the scalar potential and critical point given by

$$V = -\frac{1}{2}\left[e^{\phi}\left(g_1^2\chi^2 + g_2^2\right) + g_1^2 e^{-\phi} - 4g_1g_2\right] \quad (125)$$

and

$$\chi = 0, \qquad \phi = \ln\left[\pm\frac{g_1}{g_2}\right]. \quad (126)$$

Choosing $g_2 = -g_1$ and $g_2 = g_1$ leads to AdS_4 and dS_4 vacua with $V_0 = -3g_1^2$ and $V_0 = g_1^2$, respectively.

5 Relations between gaugings with dS_4 and dS_5 vacua

In this section, we give some relations between gaugings of $N = 4$ gauged supergravities in four and five dimensions with de Sitter vacua. In general, a circle reduction of $N = 4$ five-dimensional theory gives rise to four-dimensional theory with the same number of supersymmetries. As pointed out in [37], the relations between the embedding tensors in four and five dimensions can be obtained from an analysis of group structures. We will follow this procedure in relating four- and five-dimensional gaugings with de Sitter vacua.

A five-dimensional supergravity theory with $\hat{n}$ vector multiplets gives, via a reduction on S^1, a four-dimensional theory with $n = \hat{n} + 1$ vector multiplets. The global or duality symmetries in these two theories are given by $\hat{G} = SO(1,1) \times SO(5,\hat{n})$ and $G = SL(2) \times SO(6,\hat{n}+1)$, respectively. Accordingly, it is possible that gaugings in five dimensions can be encoded in those in four dimensions since $\hat{G} \subset G$.

Recall that components of the embedding tensor consistent with $N = 4$ supersymmetry in five dimensions are given by $\hat{\xi}_M$, $\hat{\xi}_{MN} = \hat{\xi}_{[MN]}$ and $\hat{f}_{MNP} = \hat{f}_{[MNP]}$, for more detail see [37]. To identify these components with those in four dimensions $\xi_{\alpha M}$ and $f_{\alpha MNP}$, we first consider the decomposition of a representation $(\mathbf{2}, \mathbf{n}+\mathbf{7})$ of $SL(2) \times SO(6, n+1)$ under its $SO(1,1)_B \times SO(1,1)_A \times SO(5,n)$ subgroup as follow

$$(\mathbf{2}, \mathbf{7}+\mathbf{n}) \to (\mathbf{2}, (\mathbf{n}+\mathbf{5})_0) + (\mathbf{2}, \mathbf{1}_{\frac{1}{2}}) + (\mathbf{2}, \mathbf{1}_{-\frac{1}{2}}) \quad (127)$$

for $SO(6, n+1) \to SO(1,1)_A \times SO(5,n)$ and

$$(\mathbf{2}, \mathbf{7}+\mathbf{n}) \to (\mathbf{7}+\mathbf{n})_{\frac{1}{2}} + (\mathbf{7}+\mathbf{n})_{-\frac{1}{2}} \quad (128)$$

for $SL(2) \to SO(1,1)_B$. The subscript denotes $SO(1,1)$ charges. These decompositions suggest the split of indices $M = (\hat{M}, \oplus, \ominus)$ and $\alpha = (+,-)$. Accordingly, the four-dimensional vector fields $A^{\alpha M}_\mu$ are split into

$$A^{\alpha M}_\mu = (A^{\hat{M}+}_\mu, A^{\hat{M}-}_\mu, A^{\oplus+}_\mu, A^{\oplus-}_\mu, A^{\ominus+}_\mu, A^{\ominus-}_\mu).$$

The $SO(1,1)$ factor in $\hat{G}$ is identified with the diagonal subgroup of $SO(1,1)_A \times SO(1,1)_B$. Generators of $SO(1,1) \times SO(5,n)$ are denoted by $\hat{t}_{\hat{0}}$ and $\hat{t}_{\hat{M}\hat{N}}$ and given in terms of $SL(2) \times SO(6, n+1)$ generators $(t_{\alpha\beta}, t_{MN})$ as follow

$$\hat{t}_{\hat{0}} = t_{+-} + t_{\ominus\oplus} \qquad \text{and} \qquad \hat{t}_{\hat{M}\hat{N}} = t_{\hat{M}\hat{N}}. \quad (129)$$

The five dimensional vector fields $(\hat{A}^{\hat{0}}_\mu, \hat{A}^{\hat{M}}_\mu)$ are given by

$$\hat{A}^{\hat{0}}_\mu = A^{\ominus-}_\mu \qquad \text{and} \qquad \hat{A}^{\hat{M}}_\mu = A^{\hat{M}+}_\mu. \quad (130)$$

The vector fields $A^{\hat{M}-}_\mu$ and $A^{\oplus+}_\mu$ are the magnetic dual of $A^{\hat{M}+}_\mu$ and $A^{\ominus-}_\mu$ which arise from the two-form fields in five dimensions. $A^{\oplus-}$ and $A^{\ominus+}$ are uncharged under the $SO(1,1)$ duality group and are the Kaluza–Klein vector coming from the five-dimensional metric and its dual.

By comparing the gauge covariant derivatives in four and five dimensions, we have the following identification of various components of the embedding tensors

$$\xi_{+\hat{M}} = \hat{\xi}_{\hat{M}}, \qquad f_{+\hat{M}\oplus\ominus} = \frac{1}{2}\hat{\xi}_{\hat{M}}, \qquad f_{-\ominus\hat{M}\hat{N}} = \hat{\xi}_{\hat{M}\hat{N}},$$
$$f_{+\hat{M}\hat{N}\hat{P}} = \hat{f}_{\hat{M}\hat{N}\hat{P}} \quad (131)$$

with all the remaining components set to zero in a simple circle reduction. In our analysis of de Sitter vacua, we have $\xi_{\hat{M}} = 0$ and $\xi_{\alpha M} = 0$, so the relevant relations are given by

$$f_{-\ominus\hat{M}\hat{N}} = \hat{\xi}_{\hat{M}\hat{N}} \qquad \text{and} \qquad f_{+\hat{M}\hat{N}\hat{P}} = \hat{f}_{\hat{M}\hat{N}\hat{P}}. \quad (132)$$

In the following analysis, we will give the connections between four- and five-dimensional gaugings that lead to de Sitter vacua. The five-dimensional gaugings have been classified in [36]. We will mainly work with $\hat{n} = 5$ except for the last example in which $\hat{n} = 7$. The latter leads to a new gauge group with a new dS_4 vacuum that has not been considered before since all the previous works have been done only for $n = \hat{n} + 1 = 6$. Another point to be noted is that, to apply the above decomposition and identification, we need to relabel some indices and coupling constants and interchange gauge generators while keep track of the R-symmetry and matter multiplet directions. The modifications do not qualitatively change the structure of the gauge groups, scalar potentials and the critical points.

Finally, we will divide the discussion into two parts since there are two classes of gauge groups in four dimensions that lead to de Sitter vacua. As we will see, the gauge groups with dS_4 vacua of the first type lead to gauge groups with only AdS_5 vacua in five dimensions in agreement with the absence of the five-dimensional analogue for dS_4 vacua of the first type. On the other hand, gauge groups giving rise to dS_4 vacua of the second type do lead to five-dimensional gauge groups with dS_5 vacua. This fact could possibly be inferred from the similar structure of four- and five-dimensional gauge groups namely a product of non-compact factors. Before discussing relations between these gauge groups in detail, we first give a summary of four- and five-dimensional gauge groups that are related to each other in Table 2.

5.1 Four-dimensional gaugings with AdS_4 and dS_4 vacua

In this case, the four-dimensional gaugings can give rise to both AdS_4 and dS_4 vacua with the dS_4 solutions satisfying the conditions given in (89). The related five-dimensional gauge groups only admit supersymmetric AdS_5 vacua. All the gauge groups considered here and the associated AdS_5 vacua have already been studied in [42,43].

5.1.1 $U(1) \times SU(2) \times SU(2)$ 5D gauge group

For five-dimensional gauge group $U(1) \times SU(2) \times SU(2)$, non-vanishing components of the embedding tensor can be written as

$$\hat{\xi}_{12} = g_1, \qquad \hat{f}_{345} = g_2, \qquad \hat{f}_{789} = \tilde{g}_2. \tag{133}$$

This gauge group arises from the four-dimensional gauge group $SO(3)^2_+ \times SO(3)^2_-$ with the non-vanishing components of the embedding tensor given by

$$\begin{aligned} &f_{+345} = g_2, \qquad f_{+789} = \tilde{g}_2, \\ &f_{-126} = g_1, \qquad f_{-10,11,12} = \tilde{g}_1. \end{aligned} \tag{134}$$

Therefore, we have the following identification

$$\begin{aligned} &\hat{f}_{\hat{M}\hat{N}\hat{P}} = f_{+\hat{M}\hat{N}\hat{P}}, \qquad \hat{M} = 3,4,5,7,8,9 \\ &\hat{\xi}_{\hat{M}\hat{N}} = f_{-\ominus\hat{M}\hat{N}}, \qquad \hat{M} = 1,2, \quad \ominus = 6, \oplus = 12. \end{aligned} \tag{135}$$

5.1.2 $U(1) \times SO(3,1)$ 5D gauge group

In this case, the $U(1) \times SO(3,1)$ gauge group is obtained from $SO(3)_- \times SO(3)_+ \times SO(3,1)_-$ in four dimensions. The corresponding five- and four-dimensional embedding tensors are given by

$$\hat{\xi}_{12} = g_1, \qquad \hat{f}_{345} = \hat{f}_{378} = -\hat{f}_{489} = -\hat{f}_{579} = g_2 \tag{136}$$

and

$$\begin{aligned} f_{-123} = g_1, \qquad &f_{+456} = f_{+489} = -f_{+5,9,10} \\ &= -f_{+6,8,10} = g_2, \quad f_{-7,11,12} = g_3. \end{aligned} \tag{137}$$

Comparing these two equations, we immediately find the following identification

$$\begin{aligned} &\hat{f}_{\hat{M}\hat{N}\hat{P}} = f_{+\hat{M}+1,\hat{N}+1,\hat{P}+1}, \qquad \hat{M} = 3,4,5,7,8,9, \\ &\hat{\xi}_{\hat{M}\hat{N}} = f_{-\ominus,\hat{M}+1,\hat{N}+1}, \qquad \hat{M} = 1,2, \quad \ominus = 1, \oplus = 12. \end{aligned} \tag{138}$$

Note that the embedding tensor for $SO(3)_-$ has been obtained from (79) by setting $\tilde{g}_1 = -g_1$ together with a scaling by $\frac{1}{2\sqrt{2}}$.

5.1.3 $U(1) \times SL(3,\mathbb{R})$ 5D gauge group

The $U(1) \times SL(3,\mathbb{R})$ gauge group in five dimensions is gauged by the following embedding tensor

$$\begin{aligned} &\hat{\xi}_{12} = g_1, \qquad \hat{f}_{367} = 2g_2, \qquad \hat{f}_{4,9,10} = \hat{f}_{5,8,10} = \sqrt{3}g_2, \\ &\hat{f}_{345} = \hat{f}_{389} = \hat{f}_{468} = \hat{f}_{497} = \hat{f}_{569} = \hat{f}_{578} = -g_2. \end{aligned} \tag{139}$$

This gauge group can be embedded in $SO(3)_- \times SL(3,\mathbb{R})_-$ gauge group in four dimensions with the embedding tensor given by

$$\begin{aligned} &f_{-123} = g_1, \quad f_{+478} = 2g_2, \quad f_{+5,10,11} = f_{+6,9,11} = \sqrt{3}g_2, \\ &f_{+456} = f_{+4,9,10} = f_{+579} = f_{+5,10,8} = f_{+6,7,10} \\ &\quad = f_{+689} = -g_2. \end{aligned} \tag{140}$$

We can write the relation between the embedding tensors for four- and five-dimensional gauge groups as follow

$$\begin{aligned} &\hat{f}_{\hat{M}\hat{N}\hat{P}} = f_{+,\hat{M}+1,\hat{N}+1,\hat{P}+1}, \qquad \hat{M} = 3,4,\ldots,10, \\ &\hat{\xi}_{\hat{M}\hat{N}} = f_{-\ominus,\hat{M}+1,\hat{N}+1}, \qquad \hat{M} = 1,2, \quad \ominus = 1, \oplus = 12. \end{aligned} \tag{141}$$

5.2 Four-dimensional gaugings with only dS_4 vacua

From the results of the previous sections and in [36], we know that when a non-abelian compact subgroup of the R-symmetry is not gauged, $\hat{f}_{\hat{m}\hat{n}\hat{p}} = 0$ and $f_{\alpha mnp} = 0$, the gauged supergravities admit de Sitter vacua. Both in four and five dimensions, these gauge groups take the form of a product of non-compact groups. We will give some relations between this type of gaugings in four and five dimensions.

5.2.1 $SO(1,1) \times SU(2,1)$ 5D gauge group

The five-dimensional gauge group $SO(1,1) \times SU(2,1)$ corresponds to the following embedding tensor

$$\begin{aligned} &\hat{\xi}_{5,10} = g_1, \\ &\hat{f}_{129} = \hat{f}_{138} = \hat{f}_{147} = \hat{f}_{248} = -\hat{f}_{349} = -\hat{f}_{237} = g_2, \\ &\hat{f}_{789} = -2g_2, \qquad \hat{f}_{346} = \hat{f}_{126} = \sqrt{3}g_2. \end{aligned} \tag{142}$$

Table 2 The identification between five- and four-dimensional gaugings. Gauge groups with #1, 2, 3 in five dimensions admit only AdS_5 vacua and are identified with four-dimensional gaugings with dS_4 vacua satisfying the conditions (89). Gauge groups with #4, ..., 11 give dS_5 vacua and are identified with four-dimensional gaugings that lead to dS_4 vacua satisfying the conditions (90)

$\#_{5D}$	5D gauge groups	4D gauge groups
1	$U(1)\times SU(2)\times SU(2)$	$SO(3)^2_+\times SO(3)^2_-$
2	$U(1)\times SO(3,1)$	$SO(3)_-\times SO(3)_+\times SO(3,1)_-$
3	$U(1)\times SL(3,\mathbb{R})$	$SO(3)_-\times SL(3,\mathbb{R})_-$
4	$SO(1,1)\times SU(2,1)$	$SO(2,1)_+\times SU(2,1)_+$
5	$SO(1,1)\times SO(2,1)$	$SO(2,1)_+\times SO(2,1)_+$
6	$SO(1,1)^{(2)}_{\text{diag}}\times SO(2,1)$	$SO(3,1)_+\times SO(2,1)_+\times SO(2,1)_-$
7	$SO(1,1)^{(3)}_{\text{diag}}\times SO(2,1)$	$SU(2,1)_+\times SO(2,1)_+$
8	$SO(1,1)\times SO(2,1)^2$	$SO(2,1)^2_+\times SO(2,1)^2_-$
9	$SO(1,1)\times SO(3,1)$	$SO(3,1)_+\times SO(2,1)_+\times SO(2,1)_-$
10	$SO(1,1)^{(2)}_{\text{diag}}\times SO(3,1)$	$SO(3,1)_+\times SO(3,1)_+$
11	$SO(1,1)\times SO(4,1)$	$SO(2,1)_+\times SO(4,1)_+$

This is identified with $SO(2,1)_+\times SU(2,1)_+$ gauge group in four dimensions with the embedding tensor

$$\begin{aligned} f_{+129} &= f_{+138}=f_{+147}=f_{+248}=-f_{+237}\\ &=-f_{+349}=g_2,\\ f_{+789}&=-2g_2,\quad f_{+126}=f_{+346}=\sqrt{3}g_2,\\ f_{-5,6,11}&=g_1. \end{aligned} \tag{143}$$

In this case, it is straightforward to see the relation between the two gauge groups

$$\begin{aligned} &\hat{f}_{\hat{M}\hat{N}\hat{P}}=f_{+\hat{M}\hat{N}\hat{P}},\qquad \hat{M}=1,2,3,4,7,8,9,\\ &\hat{\xi}_{\hat{M}\hat{N}}=f_{-\ominus,\hat{M}+1,\hat{N}+1},\qquad \hat{M}=5,10,\quad \ominus=5,\oplus=12. \end{aligned} \tag{144}$$

5.2.2 $SO(1,1)\times SO(2,1)$ 5D gauge group

As shown in [36], $SO(1,1)\times SO(2,1)$ gauge group can be embedded in $SO(5,n)$ in different forms. These forms depend on how the $SO(1,1)$ factor is gauged. In general, the $SO(1,1)$ can be embedded as a diagonal subgroup of d $SO(1,1)$ factors, $SO(1,1)^{(d)}_{\text{diag}}\sim[SO(1,1)^1\times\cdots\times SO(1,1)^d]_{\text{diag}}$ in the notation of [36]. It has also been shown in [36] that, due to the presence of a nonabelian non-compact factor, there are only three possibilities with $d=1,2,3$. For a simple embedding as $SO(1,1)\times SO(2,1)$ gauge group, with $SO(1,1)^{(1)}$ simply denoted by $SO(1,1)$, the embedding tensor is given by

$$\hat{\xi}_{56}=g_1,\qquad \hat{f}_{237}=g_2. \tag{145}$$

We identify this gauge group as arising from the $SO(2,1)_+\times SO(2,1)_+$ in four dimensions with the embedding tensor

$$f_{+237}=g_2,\qquad f_{-5,6,10}=g_1. \tag{146}$$

This embedding tensor has been obtained from (81) by setting $\tilde{g}_1=g_1$ and $\tilde{g}_2=g_2$ and renaming $\sqrt{2}g_{1,2}\to g_{2,1}$. We then see the following relation between the two embedding tensors

$$\begin{aligned} &\hat{f}_{\hat{M}\hat{N}\hat{P}}=f_{+\hat{M}\hat{N}\hat{P}},\qquad \hat{M}=2,3,7,\\ &\hat{\xi}_{\hat{M}\hat{N}}=f_{-\ominus\hat{M}\hat{N}},\qquad \hat{M}=5,6,\quad \ominus=10,\oplus=4. \end{aligned} \tag{147}$$

It should be pointed out that, in both five and four dimensions, there are additional $SO(2)$ factors under which matter fields are not charged. These abelian factors correspond to the gauge fields not participating in the above gauge groups. Furthermore, the four-dimensional gauge group $SO(2,1)_+\times SO(2,1)_+$ has not been separately listed in [28]. However, this gauge group can be obtained as a particular subgroup of either $SO(2,2)_+\times SO(2,2)_-$ or $SO(2,1)^3_+\times SO(3)_+$ gauge groups.

5.2.3 $SO(1,1)^{(2)}_{diag}\times SO(2,1)$ 5D gauge group

We then move to $SO(1,1)^{(2)}_{\text{diag}}\times SO(2,1)$ gauge group with the embedding tensor

$$\hat{\xi}_{18}=\hat{\xi}_{27}=g_1,\qquad \hat{f}_{4,5,10}=g_2. \tag{148}$$

This gauge group is related to $SO(3,1)_+\times SO(2,1)_+\times SO(2,1)_-$ gauge group in four dimensions with the embedding tensor

$$\begin{aligned} &f_{-789}=-f_{-129}=-f_{-138}=f_{-723}=-g_1,\\ &f_{+5,6,11}=g_2,\qquad f_{+4,10,12}=g_3 \end{aligned} \tag{149}$$

by the following relation

$$\begin{aligned} &\hat{f}_{\hat{M}\hat{N}\hat{P}}=f_{+\hat{M}+1,\hat{N}+1,\hat{P}+1},\quad \hat{M}=4,5,10,\\ &\hat{\xi}_{\hat{M}\hat{N}}=f_{-\ominus,\hat{M}+1,\hat{N}+1},\quad \hat{M}=1,2,7,8,\quad \ominus=1,\oplus=12. \end{aligned} \tag{150}$$

5.2.4 $SO(1,1)^{(3)}_{diag} \times SO(2,1)$ 5D gauge group

We now consider the final form of $SO(1,1)\times SO(2,1)$ gauge group in five dimensions namely $SO(1,1)^{(3)}_{\text{diag}} \times SO(2,1)$ with the following embedding tensor

$$\hat{\xi}_{18} = \hat{\xi}_{27} = \hat{\xi}_{36} = g_1, \qquad \hat{f}_{4,5,10} = g_2. \tag{151}$$

This gauge group is related to $SO(2,1)_+ \times SU(2,1)_+$ gauge group in four dimensions with the embedding tensor

$$f_{-129} = f_{-138} = f_{-147} = f_{-248} = -f_{-237} = -f_{-349} = g_1, \\ f_{-789} = -2g_1, \quad f_{-1,2,10} = f_{-3,4,10} = \sqrt{3}g_1, \quad f_{+5,6,11} = g_2 \tag{152}$$

by the following relation

$$\hat{f}_{\hat{M}\hat{N}\hat{P}} = f_{+\hat{M}+1,\hat{N}+1,\hat{P}+1}, \qquad \hat{M} = 4, 5, 10, \\ \hat{\xi}_{\hat{M}\hat{N}} = f_{-\ominus,\hat{M}+1,\hat{N}+1}, \\ \hat{M} = 1, 2, 3, 6, 7, 8, \quad \ominus = 1, \oplus = 12. \tag{153}$$

It should be noted that the embedding tensor (152) is just the one in (143) with $+ \to -$ and $g_2 \to -g_1$.

5.2.5 $SO(1,1) \times SO(2,2)$ 5D gauge group

In this case, the five-dimensional gauge group $SO(1,1) \times SO(2,2) \sim SO(1,1) \times SO(2,1) \times SO(2,1)$ is gauged by the embedding tensor

$$\hat{\xi}_{5,10} = g_1, \qquad \hat{f}_{237} = g_2, \qquad \hat{f}_{+189} = g_3. \tag{154}$$

We identify this gauge group as related to $SO(2,1)^2_+ \times SO(2,1)^2_-$ gauge group in four dimensions with the embedding tensor

$$f_{+237} = g_2, \qquad f_{+189} = g_3, \qquad f_{-5,6,10} = -g_1, \\ f_{-4,11,12} = g_4 \tag{155}$$

by the following relation

$$\hat{f}_{\hat{M}\hat{N}\hat{P}} = f_{+\hat{M}\hat{N}\hat{P}}, \qquad \hat{M} = 1, 8, 9, 2, 3, 7, \\ \hat{\xi}_{\hat{M}\hat{N}} = f_{-\ominus\hat{M}\hat{N}}, \qquad \hat{M} = 5, 10, \quad \ominus = 6, \oplus = 12. \tag{156}$$

It should be noted that the second $SO(2,1)$ factor in this case has the compact $SO(2)$ subgroup along the R-symmetry direction. This $SO(2,1)$ is called $SO(2,1)'$ in [36] to distinguish it from the other $SO(2,1)$ factor with the compact part along the matter direction.

5.2.6 $SO(1,1) \times SO(3,1)$ 5D gauge group

In this case, the five-dimensional gauge group is gauged by the following embedding tensor

$$\hat{\xi}_{5,10} = g_1, \qquad \hat{f}_{789} = -\hat{f}_{129} = -\hat{f}_{138} = \hat{f}_{237} = g_2. \tag{157}$$

This gauge group is obtained from $SO(3,1)_+ \times SO(2,1)_+ \times SO(2,1)_-$ gauge group in four dimensions with the embedding tensor

$$f_{-4,6,11} = -g_1, \qquad f_{+789} = -f_{+129} = -f_{+138} \\ = f_{+237} = g_2, \qquad f_{-5,7,12} = g_3. \tag{158}$$

The relation between the two embedding tensors is then given by

$$\hat{f}_{\hat{M}\hat{N}\hat{P}} = f_{+\hat{M}\hat{N}\hat{P}}, \qquad \hat{M} = 1, 2, 3, 7, 8, 9, \\ \hat{\xi}_{\hat{M}\hat{N}} = f_{-\ominus,\hat{M}+1,\hat{N}+1}, \qquad \hat{M} = 5, 10, \quad \ominus = 6, \oplus = 12. \tag{159}$$

5.2.7 $SO(1,1)^{(2)}_{diag} \times SO(3,1)$ 5D gauge group

As shown in [36], there is another embedding of the five-dimensional $SO(1,1) \times SO(3,1)$ gauge group in the form of $SO(1,1)^{(2)}_{\text{diag}} \times SO(3,1)$ with the embedding tensor

$$\hat{\xi}_{29} = \hat{\xi}_{38} = g_1, \qquad \hat{f}_{789} = -\hat{f}_{129} = -\hat{f}_{138} = \hat{f}_{237} = g_2. \tag{160}$$

We identify that this gauge group is related to $SO(3,1)_+ \times SO(3,1)_+$ gauge group in four dimensions with the embedding tensor

$$f_{-789} = -f_{-129} = -f_{-138} = f_{-723} = -g_1, \\ f_{+10,11,12} = -f_{+4,5,12} = -f_{+4,6,11} = f_{+10,5,6} = g_2 \tag{161}$$

via the following relation

$$\hat{f}_{\hat{M}\hat{N}\hat{P}} = f_{+\hat{M}+3,\hat{N}+3\hat{P}+3}, \qquad \hat{M} = 1, 2, 3, 7, 8, 9, \\ \hat{\xi}_{\hat{M}\hat{N}} = f_{-\ominus\hat{M}\hat{N}}, \qquad \hat{M} = 2, 3, 8, 9, \quad \ominus = 1, \oplus = 7. \tag{162}$$

5.2.8 $SO(1,1) \times SO(4,1)$ 5D gauge group

To gauge this group, we need to couple the five-dimensional $N = 4$ supergravity to at least $\hat{n} = 7$ vector multiplets. The embedding tensor is given by

$$\hat{\xi}_{5,12} = g_1, \quad \hat{f}_{126} = \hat{f}_{137} = \hat{f}_{149} = \hat{f}_{238} = \hat{f}_{2,4,10} = \hat{f}_{3,4,11} = g_2,$$
$$\hat{f}_{678} = \hat{f}_{6,9,10} = \hat{f}_{7,9,11} = \hat{f}_{8,10,11} = -g_2. \qquad (163)$$

There is no known four-dimensional gauging that can be identified with this gauge group upon a circle reduction. We will use the relations given in (132) to construct the following four-dimensional embedding tensor

$$f_{+MNP} = \hat{f}_{M-1,N-1,P-1}, \quad M = 2, 3, 4, 5, 7, \ldots, 12,$$
$$f_{-\ominus MN} = \hat{\xi}_{M-1,N-1}, \quad M = 6, 13, \quad \ominus = 1. \qquad (164)$$

The result is given by

$$f_{+237} = f_{+248} = f_{+2,5,10} = f_{+349} = f_{+3,5,11} = f_{+4,5,12} = g_2,$$
$$f_{+789} = f_{+7,10,11} = f_{+8,10,12} = f_{+9,11,12} = -g_2,$$
$$f_{-1,6,13} = g_1. \qquad (165)$$

This corresponds to $SO(2,1)_+ \times SO(4,1)_+$ gauge group. In this case, the four-dimensional gauged supergravity also needs to couple to at least 7 vector multiplets.

It is now straightforward to compute the scalar potential and determine the critical points. With only the dilaton and axion non-vanishing, the result is given by

$$V = \frac{1}{2}e^{-\phi}\left[e^{2\phi}\left(g_1^2\chi^2 + 6g_2^2\right) + g_1^2\right] \qquad (166)$$

with a critical point at

$$\chi = 0, \quad \phi = \ln\left[\pm\frac{g_1}{\sqrt{6}g_2}\right]. \qquad (167)$$

After setting $g_2 = \pm\frac{1}{\sqrt{6}}g_1$, we obtain a dS_4 vacuum with $V_0 = g_1^2$ and satisfying (90). We also note that the electric-magnetic dual of (165), with $-$ and $+$ components interchanged, leads to the same potential and critical point.

Finally, scalar masses at this critical point are given by

$$m^2L^2 = 6_{\times 2}, \quad \frac{3}{2}(1+2\sqrt{2})_{\times 4}, \quad \frac{3}{2}(1-2\sqrt{2})_{\times 4},$$
$$0_{\times 6}, \quad 3_{\times 12}, \quad \left[\frac{3}{2}\right]_{\times 16} \qquad (168)$$

where we have used the dS_4 radius $L = \frac{3}{V_0}$. This dS_4 vacuum is unstable due to the negative mass value $\frac{3}{2}(1-2\sqrt{2})$. The mass value $m^2L^2 = 6$ corresponds to that of the dilaton and axion. The six massless scalars correspond to the Goldstone bosons of the symmetry breaking $SO(2,1) \times SO(4,1) \rightarrow SO(2) \times SO(4)$.

6 Conclusions

In this paper, we have studied dS_4 vacua of four-dimensional $N = 4$ gauged supergravity coupled to vector multiplets. By requiring that the scalar potential is extremized and positive, we have derived a set of conditions for determining a general form of gauge groups admitting dS_4 vacua by adopting a simple ansatz. This extends the previous result in five-dimensional $N = 4$ gauged supergravity and provides a useful approach for finding dS_4 vacua in $N = 4$ gauged supergravity. We have also given some relations between the embedding tensors of four- and five-dimensional gauge groups that could be related by a simple circle reduction. From this analysis, we have given a new example of four-dimensional gauge group, $SO(2,1) \times SO(4,1)$, that gives a dS_4 vacuum. This has not previously been studied since the gauging requires the coupling to at least seven vector multiplets.

Unlike in five dimensions, we find two large classes of gauge groups that give dS_4 vacua as maximally symmetric backgrounds of the matter-coupled $N = 4$ gauged supergravity. For the first class, the gauge groups take a general form of $G_e \times G_m \times G_0^v$ in which $G_{e(m)}$ is electrically (magnetically) gauged and contains an $SO(3)$ subgroup. G_0^v is a compact group gauged by vector fields in the vector multiplets. These gauge groups are precisely the ones that lead to supersymmetric AdS_4 vacua studied in [40]. Two different types of vacua, AdS_4 and dS_4, arise from different coupling ratios between G_e and G_m factors. This result is obtained from imposing the conditions that $\langle A_1^{ij}\rangle = \langle A_{2ai}{}^j\rangle = 0$. These conditions take a very similar form to those for the existence of supersymmetric AdS_4 vacua. We have explicitly verified that the potential is extremized by these conditions.

For the second class, we have imposed another set of conditions, $\langle A_1^{ij}\rangle = \langle A_2^{ij}\rangle = 0$, and found that the gauge groups generally take the form $SO(2,1) \times SO(2,1)' \times G_{nc} \times G'_{nc} \times H_c$. G_{nc} and G'_{nc} are non-compact groups with the compact parts embedded in the matter directions while the compact $SO(2) \times SO(2)' \subset SO(2,1) \times SO(2,1)'$ is embedded along the R-symmetry directions. As in the previous case, $SO(2,1) \times G_{nc}$ ($SO(2,1)' \times G'_{nc}$) is electrically (magnetically) gauged, and H_c is a compact group. It should be emphasized that only G_{nc} and G'_{nc} are necessary for the dS_4 vacua to exist.

Given that our ansatz is rather simple, it is remarkable that the above results encode all semi-simple gauge groups that are previously known to give dS_4 vacua of $N = 4$ gauged supergravity. Two different sets of these gauge groups have also been noted in [28], and these correspond to the two sets of conditions given in this paper. The results given here are hopefully useful for finding dS_4 vacua and could be interesting in the dS/CFT correspondence and cosmology.

In this paper, we have looked at only semisimple gauge groups. It is also interesting to consider non-semisimple gauge groups listed in [44] and those arising from flux compactification studied in [45] and [46]. In deriving all the conditions for the existence of dS_4 vacua, we have not restricted the gauge groups to be semisimple. Therefore, our conditions are also valid for non-semisimple gauge groups. In particular, it can be verified that, for the dS_4 vacuum from $ISO(3)\times ISO(3)$ gauge group considered in [46], we have $\langle A_1^{ij}\rangle = \langle A_{2ai}{}^j\rangle = 0$. This dS_4 solution is accordingly of the first type described by the criteria given in (89). A systematic classification of non-semisimple groups leading to dS_4 vacua in $N=4$ gauged supergravity is worth considering.

Given the success in $N = 4$ gauged supergravities in both four and five dimensions, it is natural to extend this approach to other gauged supergravities with different numbers of supersymmetries in various dimensions. The success of this approach also suggests that the conditions we have derived might have deeper meaning although they are originally obtained from a simple assumption. It would be of particular interest to have a definite conclusion whether there is some explanation for these conditions within gauged supergravity and string/M-theory or these conditions are just a tool for finding de Sitter solutions.

Acknowledgements P. K. is supported by The Thailand Research Fund (TRF) under grant RSA6280022.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This is a theoretical study and no experimental data has been listed.]

A Useful formulae

In this appendix, we collect some useful identities involving $SO(6)$ gamma matrices which are useful in the analysis of the constraints on the embedding tensor. This appendix closely follows the dicussion in [40] and [47]. The 8×8 gamma matrices of $SO(6)$, $\gamma_{mI}{}^I$, $I, J = 1, 2, \ldots, 8$, satisfy the Clifford algebra

$$\gamma_{mI}{}^K\gamma_{nK}{}^J + \gamma_{nI}{}^K\gamma_{mK}{}^J = 2\delta_I^J\delta_{mn}\,. \tag{169}$$

$\gamma_{mI}{}^J$ can be written in terms of the chirally projected 4×4 gamma matrices Γ_m^{ij}, $i, j = 1, 2, 3, 4$, and their complex conjugate $\Gamma_{mij} = (\Gamma_m^{ij})^* = \frac{1}{2}\epsilon_{ijkl}\Gamma_m^{kl}$ as follow

$$\gamma_{mI}{}^J = \begin{pmatrix} 0 & \Gamma_m^{ij} \\ \Gamma_{mij} & 0\end{pmatrix}. \tag{170}$$

Γ_m satisfy the Clifford algebra

$$\{\Gamma_m, \Gamma_n^*\} = 2\delta_{mn}\mathbf{I}_4. \tag{171}$$

An explicit form of these matrices can be chosen as

$$\Gamma_1 = i\mathbf{I}_2\otimes\sigma_2,\quad \Gamma_2 = i\sigma_2\otimes\sigma_3,\quad \Gamma_3 = i\sigma_2\otimes\sigma_1,$$
$$\Gamma_4 = -\sigma_3\otimes\sigma_2,\quad \Gamma_5 = -\sigma_2\otimes\mathbf{I}_2,\quad \Gamma_6 = -\sigma_1\otimes\sigma_2. \tag{172}$$

The $SO(6)$ generators in the chiral spinor representation or $SU(4)$ generators in the fundamental representation are given by

$$(\Gamma_{mn})^i{}_j = \frac{1}{2}\Gamma_m^{ik}(\Gamma_n^*)_{kj}. \tag{173}$$

The other antisymmetric products satisfy

$$(\Gamma_{mnp})^{ij} = \Gamma^{ik}_{[m}\Gamma_{nkl}\Gamma^{lj}_{p]} = i\epsilon_{mnpqrs}\Gamma_q^{ik}\Gamma_{rkl}\Gamma_s^{lj}, \tag{174}$$

$$(\Gamma_{mnpq})^i{}_j = \Gamma^{ik_1}_{[m}\Gamma_{nk_1k_2}\Gamma_p^{k_2k_3}\Gamma_{q]k_3j} = i\epsilon_{mnpqrs}\Gamma_r^{ik}\Gamma_{skj}, \tag{175}$$

$$\Gamma^{ik_1}_{[m}\Gamma_{nk_1k_2}\Gamma_p^{k_2k_3}\Gamma_{qk_3k_4}\Gamma_r^{k_4k_5}\Gamma_{s]k_5j} = i\delta^i_j\epsilon_{mnpqrs}. \tag{176}$$

Some useful identities are given by

$$\{\Gamma_{mn}, \Gamma_{pq}\} = 2\Gamma_{mnpq} + 2\delta_{np}\delta_{mq} - 2\delta_{mp}\delta_{nq}, \tag{177}$$

$$\mathrm{Tr}(\Gamma_{mnp}\Gamma_{qrs}) = -4\delta_{mq}\delta_{nr}\delta_{ps} + 4\delta_{mq}\delta_{ns}\delta_{pr} + 4\delta_{mr}\delta_{nq}\delta_{ps} - 4\delta_{mr}\delta_{ns}\delta_{pq} - 4\delta_{ms}\delta_{nq}\delta_{pr} + 4\delta_{ms}\delta_{nr}\delta_{pq}. \tag{178}$$

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