Gauge dependence and multiplicative renormalization of Yang–Mills theory with matter fields

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Abstract In the paper, within the background field method, the renormalization and the gauge dependence is studied as for an SU(2) Yang–Mills theory with multiplets of spinor and scalar fields. By extending the quantum action of the BV-formalism with an extra fermion vector field and a constant fermion parameter, the multiplicative character of the renormalizability is proven. The renormalization of all the physical parameters of the theory under consideration is shown to be gauge-independent.

1 Introduction

When constructing modern models of fundamental interactions [1], non-Abelian gauge field theories [2] play a central role. Any gauge theory can be quantized in a covariant way within the BV-formalism [3,4] involving the gauge-fixing procedure as an important tool. Although at the quantum level the gauge symmetry is broken, nevertheless, it causes the existence of fundamental global supersymmetry known as the BRST symmetry [5,6]; application of the BRST symmetry to renormalization of gauge theories has been proposed in paper [7]. The respective conserved fermion nilpotent generator is known as the BRST charge [3,4] responsible for correct construction of physical state space [8,9]. Due to the equivalence theorem [10] and the BRST symmetry, it is succeeded to prove that the physical S -matrix is independent of the choice of gauge fixing.

The gauge dependence problem did appear by itself from the study of the effective potential, which appeared to be gauge-dependent in Yang–Mills theories with the spon-

taneous symmetry breaking, when calculating physically-sensible results (the energy of the ground state, the masses of the physical particles, and so on) [11,12]. In Refs. [13,14] it was established that the energy of the ground state was gauge-independent. Later, it was proved [15,16] that in Yang–Mills theories the dependence of gauge parameters in the effective action could be described in terms of gauge-invariant functional whose arguments (fields) were gauge-dependent (see also recent Refs. [19,20] devoted to that problem as resolved via the procedure of redefinition of the field variables, found in [15,16]). When studying the gauge dependence problem it was a nice idea as to enlarge the usual BRST transformations by transforming also the gauge parameter ξ into fermion variable χ , which results in the extension of the master equation [17]. We have used a similar extension in the present paper, together with introducing new fermion fields θ_μ^a [18].

Our investigation of renormalization and gauge dependence in an SU(2) Yang–Mills theory with spinor and scalar fields is based on using the background field formalism [21,22,24]. This formalism is the popular method for quantum studies and calculations in gauge theories because it allows one to work with the effective action invariant under the gauge transformations of the background fields, and to reproduce all usual physical results which can be obtained within the standard quantization approach. Various aspects of quantum properties of Yang–Mills theories and quantum gravity theories have been successfully studied in this technique [18,25–31] (among recent applications see, for example, [32–35]).

Although there are many papers devoted to various aspects of renormalizability of Yang–Mills theories, the gauge dependence of the renormalization constants has been studied explicitly only as for the gauge field sector [18]. In recent paper [34], we have studied a structure of the renormalization procedure and a gauge dependence as for an Yang–

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Mills theory with a multiplet of spinor fields based on an arbitrary simple compact gauge Lie group. We have proved the multiplicativity of the renormalization and the gauge independence of the renormalization constants of the physical parameters of the theory (gauge interaction constant and fermion mass parameter). The main goal of the present paper is to include scalar fields into a gauge model as to have the possibility of generating masses to physical particles through the spontaneous symmetry breaking [36]. Thereby, we have generalized the previous results to the maximal spectrum of spins, $\{0, 1/2, 1\}$, capable to meet the requirement of multiplicative renormalizability. In contrast to the case of spinor fields, the multiplicative renormalizability in the sector of scalar fields is not provided with the gauge symmetry only, and this requires for an additional proof. Also, specific problems may appear when studying the gauge independence of renormalization constants of new couplings of scalar fields which do not connect directly with Yang–Mills interactions. Thus, the full description of the renormalization structure requires for a detailed explicit analysis of the respective coupled equations, which is presented in the paper. To simplify the presentation and calculations, we restrict ourselves by the simplest Yang–Mills theory with $SU(2)$ gauge group. Generalization of our results to another gauge group can be made straightly.

The paper is organized as follows. In Sect. 2, the action of $SU(2)$ Yang–Mills theory with multiplets of spinor and scalar fields in the background field method is extended with the help of additional fermion vector field and constant fermion parameter which allows one later to arrive at a multiplicative renormalizable theory. The symmetries of this extended action are studied and presented in a set of equations for that action. In Sect. 3, it is established the structure and the arbitrariness is described for any local functional with the quantum numbers of the extended action that satisfies the same set of equations as the extended action does. In Sect. 4, the equations are derived for the generating functional of vertexes (effective action), as a consequence at the quantum level, of the symmetry property of the extended action; and it is shown that the generating functional of vertexes satisfies the same equations as the extended quantum action does. In Sect. 5, it is studied the renormalization procedure of the theory considered when using the loop expansion technique and the minimal subtraction scheme; and thus the multiplicative renormalizability of the theory is proved. In Sect. 6, the gauge independence of all physical parameters of the theory under consideration to any order of loop expansions is found. Concluding remarks are given in Sect. 7.

Condensed DeWitt's notations [37] are used through the paper. Functional derivatives with respect to field variables are understood as the left. Right derivatives of a quantity f with respect to the variable φ are denoted as $f \overleftarrow{\frac{\delta}{\delta \varphi}}$.

2 Extended quantum action

Let us consider an $SU(2)$ -gauge theory of non-Abelian vector fields $A_\mu^\alpha = A_\mu^\alpha(x)$, a doublet of spinor fields $\psi_j = \psi_j(x)$, $\bar{\psi}_j = \bar{\psi}_j(x)$ and a triplet of real scalar fields $\varphi^\alpha = \varphi^\alpha(x)$ in the $d = 4$ Minkowski space-time, with the action

$$\mathcal{S}_{YM}(A, \Psi, \varphi) = \mathcal{S}_1(A) + \mathcal{S}_2(A, \Psi) + \mathcal{S}_3(A, \varphi) + \mathcal{S}_4(\Psi) + \mathcal{S}_5(\varphi) + \mathcal{S}_6(\Psi, \varphi), \quad (2.1)$$

$$\mathcal{S}_1(A) = -\frac{1}{4} \int dx G_{\mu\nu}^\alpha(A) G_{\mu\nu}^\alpha(A), \quad (2.2)$$

$$\mathcal{S}_2(A, \Psi) = \int dx i \bar{\psi}_j \gamma^\mu D_{\psi\mu jk}(A) \psi_k, \quad (2.2)$$

$$\mathcal{S}_3(A, \varphi) = \frac{1}{2} \int dx D_{\varphi\mu}^{\alpha\beta}(A) \varphi^\beta D_{\varphi\mu}^{\alpha\gamma}(A) \varphi^\gamma, \quad (2.3)$$

$$\mathcal{S}_4(\Psi) = -m \int dx \bar{\psi}_j \psi_j, \quad (2.3)$$

$$\mathcal{S}_5(\varphi) = \int dx \left(-\frac{M^2}{2} \varphi^2 - \frac{\lambda}{4} \varphi^4 \right), \quad (2.4)$$

$$\mathcal{S}_6(\Psi, \varphi) = i \vartheta \int dx \varphi^\alpha \bar{\psi}_j t_{jk}^\alpha \psi_k, \quad (2.4)$$

where the notations

$$G_{\mu\nu}^\alpha(A) = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + g \varepsilon^{\alpha\beta\gamma} A_\mu^\beta A_\nu^\gamma, \quad (2.5)$$

$$D_{\psi\mu jk}(A) = \delta_{jk} \partial_\mu + g t_{jk}^\alpha A_\mu^\alpha, \quad t_{jk}^\alpha = \frac{i}{2} \sigma_{jk}^\alpha, \quad (2.5)$$

$$\Psi = \{\psi, \bar{\psi}\}, \quad (2.5)$$

$$D_{\varphi\mu}^{\alpha\beta}(A) = \delta_{\alpha\beta} \partial_\mu + g E_{\alpha\beta}^\gamma A_\mu^\gamma, \quad (2.5)$$

$$\varphi^2 = \varphi^\alpha \varphi^\alpha, \quad \varphi^4 = (\varphi^2)^2, \quad (2.5)$$

$$E_{\alpha\beta}^\gamma = \varepsilon^{\alpha\gamma\beta}, \quad \alpha, \gamma, \beta = 1, 2, 3, \quad j, k = 1, 2, \quad (2.5)$$

were used. In the relations (2.1)–(2.4) and (2.5) $\varepsilon^{\alpha\beta\gamma}$ represent the structure coefficients of the $SU(2)$ gauge group, $t^\alpha = \{t_{jk}^\alpha\}$ and $E^\alpha = \{E_{\beta\gamma}^\alpha\}$ are the generators of gauge transformations of spinor and scalar fields with the properties

$$[E^\alpha, E^\beta] = \varepsilon^{\alpha\beta\gamma} E^\gamma, \quad [t^\alpha, t^\beta] = \varepsilon^{\alpha\beta\gamma} t^\gamma, \quad (2.6)$$

$$(t^\gamma)^+ = -t^\gamma, \quad [\gamma^\mu, t^\alpha] = 0, \quad (2.6)$$

γ^μ are the Dirac matrices, σ_{jk}^α are the Pauli matrices and g is a gauge coupling parameter, λ is a coupling constant of the scalar field, ϑ is a coupling constant of the scalar and spinor fields, m and M are mass parameters of the spinor and the scalar fields, respectively.

The action (2.1) is invariant under the $SU(2)$ -gauge transformations with gauge parameters $\omega_\alpha = \omega_\alpha(x)$,

$$\begin{aligned}\delta_\omega \mathcal{S}_{YM}(A, \Psi, \varphi) &= 0 \\ \delta_\omega A_\mu^\alpha &= (\delta_{\alpha\beta} \partial_\mu + g \varepsilon^{\alpha\beta\gamma} A_\mu^\gamma) \omega_\beta = D_\mu^{\alpha\beta}(A) \omega_\beta, \\ \delta_\omega \psi_j &= -g t_{jk}^\beta \psi_k \omega_\beta, \quad \delta_\omega \bar{\psi}_j = g \bar{\psi}_k t_{kj}^\beta \omega_\beta, \\ \delta_\omega \varphi^\alpha &= -g E_{\alpha\gamma}^\beta \varphi^\gamma \omega_\beta.\end{aligned}\quad (2.7)$$

Notice that the form (2.4) of the polynomial $\mathcal{S}_5(\varphi)$ is uniquely determined by the invariance requirement under global $SU(2)$ -transformations.

In the background-field formalism [21, 22, 24] the gauge field A_μ^α appearing in the classical action (2.1) is replaced by $A_\mu^\alpha + \mathcal{B}_\mu^\alpha$,

$$\mathcal{S}_{YM}(A, \Psi, \varphi) \rightarrow \mathcal{S}_{YM}(A + \mathcal{B}, \Psi, \varphi), \quad (2.8)$$

where $\mathcal{B}_\mu^\alpha$ is considered as an external vector field. Effective action for functional integral in quantum theory is constructed with using the BV-formalism [3, 4].

To study a renormalization structure and a gauge dependence of renormalization constants we use an extended action. This action $S_{ext} = S_{ext}(Q, Q^*, \bar{C}, B, \mathcal{B}, \xi, \theta, \chi)$ is constructed by introducing additional fermion fields $\theta_\mu^\alpha = \theta_\mu^\alpha(x)$ and a constant fermion parameter¹ χ , and has the form

$$\begin{aligned}S_{ext} &= \mathcal{S}_{YM}(A + \mathcal{B}, \Psi, \varphi) + \int dx Q^* \mathcal{R}_Q \\ &+ \int dx \left(\bar{C}^\alpha D_\mu^{\alpha\gamma}(B) D_\mu^{\gamma\beta}(A + \mathcal{B}) C^\beta \right. \\ &+ B^\alpha D_\mu^{\alpha\beta}(B) A_\mu^\beta + (\xi/2) B^\alpha B^\alpha \Big) \\ &+ \int dx \left(\theta_\mu^\alpha [D_\mu^{\alpha\beta}(A + \mathcal{B}) \bar{C}^\beta - A_\mu^{*\alpha}] \right. \\ &+ \chi [(A_\mu^{*\alpha} - D_\mu^{\alpha\beta}(B) \bar{C}^\beta) A_\mu^\alpha \\ &+ C^{*\alpha} C^\alpha + \psi_j^* \psi_j + \bar{\psi}_j^* \bar{\psi}_j + \varphi^{*\alpha} \varphi^\alpha] \Big),\end{aligned}\quad (2.9)$$

where Q means the set of fields $\{A_\mu^\alpha, \psi_j, \bar{\psi}_j, \varphi^\alpha, C^\alpha\}$, the symbol Q^* is used for the set of corresponding antifields and $\mathcal{R}_Q$ are generators of the BRST transformations [5, 6],

$$\begin{aligned}\mathcal{R}_{A_\mu^\alpha} &= D_\mu^{\alpha\beta}(A + \mathcal{B}) C^\beta, \quad \mathcal{R}_{\psi_j} = -g t_{jk}^\alpha \psi_k C^\alpha, \\ \mathcal{R}_{\bar{\psi}_j} &= g \bar{\psi}_k t_{kj}^\alpha C^\alpha, \\ \mathcal{R}_{\varphi^\alpha} &= -g E_{\alpha\gamma}^\beta \varphi^\gamma C^\beta, \quad \mathcal{R}_{C^\alpha} = \frac{g}{2} \varepsilon^{\alpha\beta\gamma} C^\beta C^\gamma.\end{aligned}\quad (2.10)$$

The action (2.9) is invariant,

$$S_{ext} \frac{\overleftarrow{\delta}}{\delta \Pi_I} \delta \Pi_I \epsilon = 0,$$

¹ For the first time such additional variables were used in [18].

under the following global supersymmetry transformations of the variables (here $\{\Pi_I\}$ is the set of all the variables entering the action, ϵ is a constant fermion parameter of the transformation, $\varepsilon(\epsilon) = 1, \epsilon^2 = 0$),

$$\delta A_\mu^\alpha = D_\mu^{\alpha\beta}(A + \mathcal{B}) C^\beta + \theta_\mu^\alpha = \frac{\delta}{\delta A_\mu^{*\alpha}} S_{ext} + \chi A_\mu^\alpha, \quad (2.11)$$

$$\begin{aligned}\delta B^\alpha &= -\frac{1}{\xi} \left[D_\mu^{\alpha\gamma}(B) D_\mu^{\gamma\beta}(A + \mathcal{B}) C^\beta \right. \\ &+ D_\mu^{\alpha\beta}(A + \mathcal{B}) \theta_\mu^\beta + \chi D_\mu^{\alpha\beta}(B) A_\mu^\beta \Big] - \chi B^\alpha \\ &= -\frac{1}{\xi} \frac{\delta}{\delta C^\alpha} S_{ext} - \chi B^\alpha,\end{aligned}\quad (2.12)$$

$$\delta C^\alpha = \frac{g}{2} \varepsilon^{\alpha\beta\gamma} C^\beta C^\gamma = \frac{\delta}{\delta C^{*\alpha}} S_{ext} - \chi C^\alpha, \quad (2.13)$$

$$\begin{aligned}\delta \bar{C}^\alpha &= -\frac{1}{\xi} D_\mu^{\alpha\beta}(B) A_\mu^\beta + \chi \bar{C}^\alpha \\ &= -\frac{1}{\xi} \frac{\delta}{\delta B^\alpha} S_{ext} + B^\alpha + \chi \bar{C}^\alpha,\end{aligned}\quad (2.14)$$

$$\delta \psi_j = -g t_{jk}^\alpha \psi_k C^\alpha = \frac{\delta}{\delta \psi_j^*} S_{ext} - \chi \psi_j, \quad (2.15)$$

$$\delta \bar{\psi}_j = g \bar{\psi}_k t_{kj}^\alpha C^\alpha = \frac{\delta}{\delta \bar{\psi}_j^*} S_{ext} - \chi \bar{\psi}_j, \quad (2.16)$$

$$\delta \varphi^\alpha = -g E_{\alpha\gamma}^\beta \varphi^\gamma C^\beta = \frac{\delta}{\delta \varphi^{*\alpha}} S_{ext} + \chi \varphi^\alpha, \quad (2.17)$$

$$\begin{aligned}\delta A_\mu^{*\alpha} &= \chi A_\mu^{*\alpha}, \quad \delta C^{*\alpha} = -\chi C^{*\alpha}, \quad \delta \psi_j^* = \\ &= -\chi \psi_j^*, \quad \delta \bar{\psi}_j^* = -\chi \bar{\psi}_j^*, \quad \delta \varphi^{*\alpha} = \chi \varphi^{*\alpha},\end{aligned}\quad (2.18)$$

$$\delta \xi = 2\xi \chi, \quad \delta \mathcal{B}_\mu^\alpha = -\theta_\mu^\alpha, \quad \delta \theta_\mu^\alpha = 0, \quad \delta \chi = 0. \quad (2.19)$$

Taking into account the right-hand sides of the relations (2.11)–(2.14) and omitting indices of all variables, the invariance condition of the action S_{ext} rewrites in the form of the following equation

$$\begin{aligned}\int dx \left(S_{ext} \frac{\overleftarrow{\delta}}{\delta Q} \frac{\delta}{\delta Q^*} S_{ext} - B \frac{\delta}{\delta \bar{C}} S_{ext} - \theta \frac{\delta}{\delta B} S_{ext} \right) \\ + \chi \int dx \left[Q \frac{\delta}{\delta Q} - Q^* \frac{\delta}{\delta Q^*} - \bar{C} \frac{\delta}{\delta \bar{C}} - B \frac{\delta}{\delta B} \right] S_{ext} \\ + 2\chi \xi \frac{\partial}{\partial \xi} S_{ext} = 0.\end{aligned}\quad (2.20)$$

Also, the action (2.9) satisfies the equation

$$S_{ext} \overleftarrow{H}^\alpha \omega_\alpha = 0, \quad (2.21)$$

where

$$\begin{aligned} \overleftarrow{H}^\alpha \omega_\alpha = \int dx \left\{ \left[\frac{\overleftarrow{\delta}}{\delta B_\mu^\beta} D_\mu^{\beta\alpha} (B) + g \varepsilon^{\beta\gamma\alpha} \right. \right. \\ \times \left(\frac{\overleftarrow{\delta}}{\delta A_\mu^\beta} A_\mu^\gamma + \frac{\overleftarrow{\delta}}{\delta B^\beta} B^\gamma + \frac{\overleftarrow{\delta}}{\delta C^\beta} C^\gamma + \frac{\overleftarrow{\delta}}{\delta \overline{C}^\beta} \overline{C}^\gamma \right) \\ - g E_{\beta\gamma}^\alpha \left(\frac{\overleftarrow{\delta}}{\delta \varphi^\beta} \varphi^\gamma + \frac{\overleftarrow{\delta}}{\delta \varphi^{*\beta}} \varphi^{*\gamma} \right) \\ - g t_{jk}^\alpha \left(\frac{\overleftarrow{\delta}}{\delta \psi_j} \psi_k + \frac{\overleftarrow{\delta}}{\delta \overline{\psi}_j^*} \overline{\psi}_k^* \right) \\ + g \left(\frac{\overleftarrow{\delta}}{\delta \overline{\psi}_j} \overline{\psi}_k + \frac{\overleftarrow{\delta}}{\delta \psi_j^*} \psi_k^* \right) t_{kj}^\alpha \\ \left. + g \varepsilon^{\beta\gamma\alpha} \left(\frac{\overleftarrow{\delta}}{\delta A_\mu^{*\beta}} A_\mu^{*\gamma} + \frac{\overleftarrow{\delta}}{\delta C^{*\beta}} C^{*\gamma} + \frac{\overleftarrow{\delta}}{\delta \theta_\mu^\beta} \theta_\mu^\gamma \right) \right] \omega_\alpha \right\}, \end{aligned} \quad (2.22)$$

is the operator of the gauge transformations in the sector of the fields B_μ , φ , ψ , $\overline{\psi}$ and, at the same time, of the tensor transformations in the sector of the fields A_μ , C , $\overline{C}$, B , θ_μ , A_μ^* , φ^* , ψ^* , $\overline{\psi}^*$, C^* .

Finally, let us notice the two important relations linear in the fields A_μ , B and their derivatives, which the action (2.9) satisfies to,

$$\frac{\delta}{\delta B^\alpha} S_{\text{ext}} = D_\mu^{\alpha\beta} (B) A_\mu^\beta + \xi B^\alpha, \quad (2.23)$$

$$D_\mu^{\alpha\beta} (B) \frac{\delta}{\delta A_\mu^{*\beta}} S_{\text{ext}} - \frac{\delta}{\delta \overline{C}^\alpha} S_{\text{ext}} = -g \varepsilon^{\alpha\beta\gamma} A_\mu^\beta \theta_\mu^\gamma. \quad (2.24)$$

The equation (2.24) means that the action S_{ext} (2.9) depends on the variables $A_\mu^{*\alpha}$ and $\overline{C}^\alpha$ only in combination $A_\mu^{*\alpha} - D_\mu^{\alpha\beta} (B) \overline{C}^\beta$ when $\theta_\mu^\beta = 0$.

The action S_{ext} can be represented in another useful form,

$$S_{\text{ext}} = \Gamma_{0|0} + \sum_{k=6}^{22} \Gamma_{0|k} + \chi \sum_{k=1}^5 \Gamma_{0|k}, \quad (2.25)$$

where

$$\Gamma_{0|0} = \int dx \left(B^\alpha D_\mu^{\alpha\beta} (B) A_\mu^\beta + \frac{\xi}{2} B^\alpha B^\alpha + g \theta_\mu^\alpha \varepsilon^{\alpha\beta\gamma} A_\mu^\beta \overline{C}^\gamma \right), \quad (2.26)$$

$$\begin{aligned} \Gamma_{0|1} &= \int dx [\mathcal{A}_\mu^{*\alpha} \mathcal{A}_\mu^\alpha], \\ \Gamma_{0|2} &= \int dx [C^{*\alpha} C^\alpha], \quad \Gamma_{0|3} = \int dx [\psi_j^* \psi_j], \end{aligned} \quad (2.27)$$

$$\begin{aligned} \Gamma_{0|4} &= \int dx [\overline{\psi}_j^* \overline{\psi}_j], \quad \Gamma_{0|5} = \int dx [\varphi^{*\alpha} \varphi^\alpha], \\ \Gamma_{0|6} &= \int dx [\mathcal{A}_\mu^{*\alpha} \theta_\mu^\alpha], \end{aligned} \quad (2.28)$$

$$\Gamma_{0|7} = \int dx [\mathcal{A}_\mu^{*\gamma} D_\mu^{\gamma\sigma} (B) C^\sigma],$$

$$\Gamma_{0|8} = \int dx [g \mathcal{A}_\mu^{*\gamma} \varepsilon^{\gamma\alpha\sigma} \mathcal{A}_\mu^\alpha C^\sigma], \quad (2.29)$$

$$\Gamma_{0|9} = \int dx \left[\frac{g}{2} C^{*\alpha} \varepsilon^{\alpha\beta\gamma} C^\beta C^\gamma \right],$$

$$\Gamma_{0|10} = - \int dx [g \psi^* t^\alpha \psi C^\alpha], \quad (2.30)$$

$$\Gamma_{0|11} = \int dx [g \overline{\psi} t^\alpha \overline{\psi}^* C^\alpha],$$

$$\Gamma_{0|12} = - \int dx [g \varphi^{*\gamma} \varepsilon^{\gamma\alpha\sigma} \varphi^\sigma C^\alpha], \quad (2.31)$$

$$\Gamma_{0|13} = i \int dx [\overline{\psi} \gamma^\mu D_\mu \psi (B)],$$

$$\Gamma_{0|14} = i g \int dx [\overline{\psi} \gamma^\mu t^\alpha \mathcal{A}_\mu^\alpha \psi], \quad (2.32)$$

$$\Gamma_{0|15} = -m \int dx [\overline{\psi} \psi],$$

$$\Gamma_{0|16} = \int dx \left[\frac{1}{2} (D_{\varphi\mu}^{\alpha\beta} (B) \varphi^\beta) D_{\varphi\mu}^{\alpha\gamma} (B) \varphi^\gamma \right], \quad (2.33)$$

$$\Gamma_{0|17} = \int dx [g \varepsilon^{\alpha\gamma\sigma} (D_{\varphi\mu}^{\alpha\beta} (B) \varphi^\beta) \mathcal{A}_\mu^\gamma \varphi^\sigma],$$

$$\Gamma_{0|18} = \int dx \left[\frac{g^2}{2} E_{\alpha\beta}^\rho \mathcal{A}_\mu^\alpha \varphi^\beta E_{\gamma\sigma}^\rho \mathcal{A}_\mu^\gamma \varphi^\sigma \right], \quad (2.34)$$

$$\Gamma_{0|19} = -\frac{M^2}{2} \int dx \varphi^2, \quad \Gamma_{0|20} = -\frac{\lambda}{4} \int dx \varphi^4,$$

$$\Gamma_{0|21} = i \vartheta \int dx [\varphi^\alpha \overline{\psi} t^\alpha \psi], \quad (2.35)$$

$$\Gamma_{0|22} = -\frac{1}{4} \int dx [G_{\mu\nu}^\alpha (\mathcal{A} + B) G_{\mu\nu}^\alpha (\mathcal{A} + B)], \quad (2.36)$$

and the notations

$$\mathcal{A}_\mu^{*\alpha} = A_\mu^{*\alpha} - D_\mu^{\alpha\beta} (B) \overline{C}^\beta, \quad \mathcal{A}_\mu^\alpha = A_\mu^\alpha \quad (2.37)$$

are used. Note that the functionals $\Gamma_{0|0} - \Gamma_{0|21}$ are homogeneous with respect to fields Ω and antifields Ω^* while the functional $\Gamma_{0|22}$ does not obey this property and in symbolic notation has the form

$$\Gamma_{0|22} = \sum_{l=0}^4 \Gamma_{0|22,l}, \quad \Gamma_{0|22,l} = \int dx [T_l(B) \mathcal{A}^l], \quad (2.38)$$

where $\Gamma_{0|22,l}$ is homogeneous functional with respect to variable $\mathcal{A}$ of order l and $T_l(B)$ is a tensor (differential operator) with l gauge and l Lorentz indices. In what follow we do not need in explicit representation of quantities $T_l(B)$.

Later on, we will see that the generating functional of vertex functions (effective action), counterterms and renormalized action satisfy the same Eqs. (2.20), (2.21), (2.22), (2.23), (2.24), as the action S_{ext} does. Moreover, the counterterms and the renormalized action are linear combinations of the same vertices as the action S_{ext} is. Explicit form of these vertices will be given in the next section.

Now we give the table of "quantum" numbers of fields, antifields, auxiliary fields and constant parameters used in construction of S_{ext} :

Quantity	A, B	$\psi, \bar{\psi}$	$\varphi,$	$C, \bar{C}$	B	ξ	θ	A^*	$\psi^*, \bar{\psi}^*$	φ^*	C^*	$dx(\equiv d^4x)$	∂_x	χ
ε	0	1	0	1	0	0	1	1	0	1	0	0	0	1
gh	0	0	0	1, -1	0	0	1	-1	-1	-1	-2	0	0	1
dim	1	3/2	1	1	2	0	2	2	3/2	2	2	-4	1	1
ε_f	0	1, -1	0	0	0	0	0	0	-1, 1	0	0	0	0	0

where " ε " describes the Grassmann parity, the symbol "gh" is used to denote the ghost number, "dim" means the canonical dimension and " ε_f " is the fermionic number. Using the table of "quantum" numbers it is easy to establish the quantum numbers of any quantities met in the text.

3 General structure of renormalized action

It will be shown below that the renormalized action is a local functional of field variables with quantum numbers of the action S_{ext} (2.9), which satisfies the same Eqs. (2.20)–(2.24) as the action S_{ext} does. In this section, we find a general solution to the Eqs. (2.20)–(2.24) under the conditions pointed out above.

So, let P be a functional of the form

$$P = \int dx P(x), \quad (3.1)$$

where $P(x)$ is a local polynomial of all variables $Q, Q^*, \bar{C}, B, \mathcal{B}, \xi, \theta, \chi$ with $\dim(P(x)) = 4$. Let the functional P satisfies the Eqs. (2.20)–(2.24) (with the substitution $S_{ext} \rightarrow P$), and we represent it in the form

$$P = \Gamma_{0|0} + P^{(1)} + \chi P^{(2)}, \quad (3.2)$$

where the functional $\Gamma_{0|0}$ is defined in (2.26), the functionals $P^{(1)}, P^{(2)}$ do not depend on χ and obey the properties

$$\begin{aligned} \varepsilon(P^{(1)}) &= 0, \quad \text{gh}(P^{(1)}) = 0, \\ \dim(P^{(1)}) &= 0, \quad \varepsilon_f(P^{(1)}) = 0, \end{aligned} \quad (3.3)$$

$$\begin{aligned} \varepsilon(P^{(2)}) &= 1, \quad \text{gh}(P^{(2)}) = -1, \\ \dim(P^{(2)}) &= -1, \quad \varepsilon_f(P^{(2)}) = 0. \end{aligned} \quad (3.4)$$

From the Eq. (2.23) for P and the presentation (3.2) it follows that $P^{(1)}$ and $P^{(2)}$ do not depend on fields B^α ,

$$P^{(k)} = P^{(k)}(Q, Q^*, \bar{C}, \mathcal{B}, \xi, \theta), \quad k = 1, 2, \quad (3.5)$$

With the help of new variables $\mathcal{A}_\mu^{\alpha*}(x)$, $\mathcal{A}_\mu^\alpha$ (2.37), we define new functionals $\tilde{P}^{(k)}$ by the rule

$$\begin{aligned} \tilde{P}^{(k)} &= \tilde{P}^{(k)}(\Omega, \Omega^*, \mathcal{B}, \bar{C}, \xi, \theta) \\ &= P^{(k)}(Q, Q^*, \mathcal{B}, \bar{C}, \xi, \theta)|_{A^* \rightarrow \mathcal{A}^* + D(\mathcal{B})\bar{C}}, \quad k = 1, 2, \end{aligned} \quad (3.6)$$

and find that $\tilde{P}^{(k)}$ do not depend on fields $\bar{C}^\alpha$,

$$\tilde{P}^{(k)} = \tilde{P}^{(k)}(\Omega, \Omega^*, \mathcal{B}, \xi, \theta). \quad (3.7)$$

Omitting the indices of all variables in relations (3.6) and (3.7), the following notations

$$\Omega = \{A, \psi, \bar{\psi}, \varphi, C\}, \quad \Omega^* = \{A^*, \psi^*, \bar{\psi}^*, \varphi^*, C^*\} \quad (3.8)$$

are used. Independence of the functionals $\tilde{P}^{(k)}$ of the fields $\bar{C}^\alpha$ and the relations

$$\begin{aligned} P^{(k)} \int dx \left[\frac{\overleftarrow{\delta}}{\delta \mathcal{B}_\mu^\beta} D_\mu^{\beta\alpha}(\mathcal{B}) + g \varepsilon^{\beta\gamma\alpha} \left(\frac{\overleftarrow{\delta}}{\delta \bar{C}^\beta} \bar{C}^\gamma + \frac{\overleftarrow{\delta}}{\delta A_\mu^{\beta*}} A_\mu^{\gamma*} \right) \right] \\ = \tilde{P}^{(k)} \int dx \left[\frac{\overleftarrow{\delta}}{\delta \mathcal{B}_\mu^\beta} D_\mu^{\beta\alpha}(\mathcal{B}) + g \varepsilon^{\beta\gamma\alpha} \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^{\beta*}} \mathcal{A}_\mu^{\gamma*} \right], \end{aligned} \quad (3.9)$$

allow us to write down the following set of equations for $\tilde{P}^{(k)}$

$$\int dx \left[\tilde{P}^{(1)} \frac{\overleftarrow{\delta}}{\delta \Omega} \frac{\delta}{\delta \Omega^*} \tilde{P}^{(1)} - \theta_\mu^\alpha \frac{\delta}{\delta \mathcal{B}_\mu^\alpha} \tilde{P}^{(1)} \right] = 0, \quad (3.10)$$

$$\begin{aligned} 2\xi \frac{\partial}{\partial \xi} \tilde{P}^{(1)} &= \int dx \left[\tilde{P}^{(1)} \left(\frac{\overleftarrow{\delta}}{\delta \Omega} \frac{\delta}{\delta \Omega^*} \right. \right. \\ &\quad \left. \left. - \frac{\overleftarrow{\delta}}{\delta \Omega^*} \frac{\delta}{\delta \Omega} \right) \tilde{P}^{(2)} - \theta_\mu^\alpha \frac{\delta}{\delta \mathcal{B}_\mu^\alpha} \tilde{P}^{(2)} \right] \\ &\quad + \int dx \left[\left(\Omega^* \frac{\delta}{\delta \Omega^*} - \Omega \frac{\delta}{\delta \Omega} \right) \tilde{P}^{(1)} \right], \end{aligned} \quad (3.11)$$

$$\tilde{P}^{(k)} \overleftarrow{h}^\alpha \omega_\alpha = 0, \quad k = 1, 2, \quad (3.12)$$

where

$$\begin{aligned} \overleftarrow{h}^\alpha \omega_\alpha &= \int dx \left\{ \left[\frac{\overleftarrow{\delta}}{\delta \mathcal{B}_\mu^\beta} D_\mu^{\beta\alpha}(\mathcal{B}) + g \varepsilon^{\beta\gamma\alpha} \right. \right. \\ &\quad \times \left(\frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^{\beta*}} \mathcal{A}_\mu^{\gamma*} + \frac{\overleftarrow{\delta}}{\delta \varphi^\beta} \varphi^\gamma + \frac{\overleftarrow{\delta}}{\delta C^\beta} C^\gamma \right) \\ &\quad + g \varepsilon^{\beta\gamma\alpha} \left(\frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^{\beta*}} \mathcal{A}_\mu^{\gamma*} + \frac{\overleftarrow{\delta}}{\delta \varphi^{\beta*}} \varphi^{\gamma*} \right. \\ &\quad \left. \left. + \frac{\overleftarrow{\delta}}{\delta C^{\beta*}} C^{\gamma*} + \frac{\overleftarrow{\delta}}{\delta \theta_\mu^\beta} \theta_\mu^\gamma \right) \right. \\ &\quad \left. - g t_{jk}^\alpha \left(\frac{\overleftarrow{\delta}}{\delta \psi_j} \psi_k + \frac{\overleftarrow{\delta}}{\delta \bar{\psi}_j} \bar{\psi}_k^* \right) \right. \\ &\quad \left. + g \left(\frac{\overleftarrow{\delta}}{\delta \bar{\psi}_j} \bar{\psi}_k + \frac{\overleftarrow{\delta}}{\delta \psi_j^*} \psi_k^* \right) t_{kj}^\alpha \right\} \omega_\alpha. \end{aligned} \quad (3.13)$$

When studying the structure of functionals $\tilde{P}^{(k)}$ and in further research, it is helpful to have a consequence from equation (3.12) that corresponds to the case when $\omega_\alpha = \text{const}$,

$$\tilde{P}^{(k)} \overleftarrow{T}^\alpha = 0, \quad k = 1, 2, \quad (3.14)$$

where

$$\begin{aligned} \overleftarrow{T}^\alpha = \int dx \bigg\{ & \varepsilon^{\beta\gamma\alpha} \left(\frac{\overleftarrow{\delta}}{\delta \mathcal{B}_\mu^\beta} \mathcal{B}_\mu^\gamma \right. \\ & + \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^\beta} \mathcal{A}_\mu^\gamma + \frac{\overleftarrow{\delta}}{\delta \varphi^\beta} \varphi^\gamma + \frac{\overleftarrow{\delta}}{\delta C^\beta} C^\gamma \bigg) \\ & + \varepsilon^{\beta\gamma\alpha} \left(\frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^{*\beta}} \mathcal{A}_\mu^{*\gamma} + \frac{\overleftarrow{\delta}}{\delta \varphi^{*\beta}} \varphi^{*\gamma} \right. \\ & + \frac{\overleftarrow{\delta}}{\delta C^{*\beta}} C^{*\gamma} + \frac{\overleftarrow{\delta}}{\delta \theta_\mu^\beta} \theta_\mu^\gamma \bigg) \\ & - t_{jk}^\alpha \left(\frac{\overleftarrow{\delta}}{\delta \psi_j} \psi_k + \frac{\overleftarrow{\delta}}{\delta \bar{\psi}_j^*} \bar{\psi}_k^* \right) \\ & \left. + \left(\frac{\overleftarrow{\delta}}{\delta \bar{\psi}_j} \bar{\psi}_k + \frac{\overleftarrow{\delta}}{\delta \psi_j^*} \psi_k^* \right) t_{kj}^\alpha \right\}. \end{aligned} \quad (3.15)$$

We refer to the equation of the form (3.14) as the ones of the T -symmetry for the corresponding functionals.

Using the properties of the functional $\tilde{P}^{(2)}$: its locality and (3.4) as well as axial symmetry, Poincare- and T -symmetries, we find the general representation of $\tilde{P}^{(2)}$

$$\tilde{P}^{(2)} = \sum_{k=1}^5 Z_k \Gamma_{0|k} + \int dx [Z'_1 \mathcal{A}_\mu^{*\alpha} \mathcal{B}_\mu^\alpha], \quad (3.16)$$

where $Z_i, i = 1, 2, 3, 4, 5$ and Z'_1 are arbitrary constants and the functionals $\Gamma_{0|k}, k = 1, 2, 3, 4, 5$ were introduced in (2.27), (2.28). Later on any quantities "Z" with any set of indices do not depend on coordinates x and field variables.

Further, when using the Eq. (3.12) for $\tilde{P}^{(2)}$, we get that $Z'_1 = 0$. The final expression for the $\tilde{P}^{(2)}$ has the form

$$\tilde{P}^{(2)} = \sum_{k=1}^5 Z_k \Gamma_{0|k}, \quad (3.17)$$

Notice that the functional $\tilde{P}^{(2)}$ does not depend on the fields θ_μ^α and $\mathcal{B}_\mu^\alpha$. By taking (3.17) into account the equation (3.11) reduces to the following one

$$2\xi \frac{\partial}{\partial \xi} \tilde{P}^{(1)} = \hat{L}(Z) \tilde{P}^{(1)}, \quad (3.18)$$

$$\begin{aligned} \hat{L}(Z) = \int dx \bigg[& (Z_1 - 1) \left(\mathcal{A}_\mu^\alpha \frac{\delta}{\delta \mathcal{A}_\mu^\alpha} - \mathcal{A}_\mu^{*\alpha} \frac{\delta}{\delta \mathcal{A}_\mu^{*\alpha}} \right) \\ & + (Z_2 - 1) \left(C^\alpha \frac{\delta}{\delta C^\alpha} - C^{*\alpha} \frac{\delta}{\delta C^{*\alpha}} \right) \\ & + (Z_3 - 1) \left(\psi_j \frac{\delta}{\delta \psi_j} - \psi_j^* \frac{\delta}{\delta \psi_j^*} \right) \\ & + (Z_4 - 1) \left(\bar{\psi}_j \frac{\delta}{\delta \bar{\psi}_j} - \bar{\psi}_j^* \frac{\delta}{\delta \bar{\psi}_j^*} \right) \end{aligned}$$

$$+ (Z_5 - 1) \left(\varphi^\alpha \frac{\delta}{\delta \varphi^\alpha} - \varphi^{*\alpha} \frac{\delta}{\delta \varphi^{*\alpha}} \right) \bigg], \quad (3.19)$$

describing the dependence of renormalization constants on the gauge parameter ξ . We refer to the Eq. (3.10) as the extended master-equation and to (3.18) as the gauge dependence equation.

3.1 Solution to the extended master-equation

Let us consider a solution to the extended master-equation (3.10) for the functional $\tilde{P}^{(1)}$ presented in the form

$$\tilde{P}^{(1)} = \tilde{P}_\theta^{(1)} + \tilde{P}_{\Omega^*}^{(1)} + \tilde{P}_\psi^{(1)} + \tilde{P}_\varphi^{(1)} + \tilde{P}_{\psi\varphi}^{(1)} + \tilde{P}_{\mathcal{A}\mathcal{B}}^{(1)}. \quad (3.20)$$

Functional $\tilde{P}_\theta^{(1)}$ is written as

$$\tilde{P}_\theta^{(1)} = \int dx \theta_\mu^\alpha(x) \tilde{P}_{\theta\mu}^\alpha(x), \quad (3.21)$$

and functionals $\tilde{P}_{\Omega^*}^{(1)}, \tilde{P}_\psi^{(1)}, \tilde{P}_\varphi^{(1)}, \tilde{P}_{\psi\varphi}^{(1)}, \tilde{P}_{\mathcal{A}\mathcal{B}}^{(1)}$ do not depend on the fields θ_μ^α . Taking into account the properties $\dim(\tilde{P}_{\theta\mu}^\alpha) = 2$, $\text{gh}(\tilde{P}_{\theta\mu}^\alpha) = -1$, $\varepsilon(\tilde{P}_{\theta\mu}^\alpha) = 1$, $\varepsilon_f(\tilde{P}_{\theta\mu}^\alpha) = 0$ as well as the Poincare- and T -symmetries of $\tilde{P}_\theta^{(1)}$, we find that

$$\tilde{P}_{\theta\mu}^\alpha(x) = -Z_6 \mathcal{A}_\mu^{*\alpha}(x), \quad \tilde{P}_\theta^{(1)} = Z_6 \Gamma_{0|6}, \quad (3.22)$$

where Z_6 is an arbitrary constant.

Functional $\tilde{P}_{\Omega^*}^{(1)}$ (3.8) is linear in antifields Ω^* and functionals $\tilde{P}_\psi^{(1)}, \tilde{P}_\varphi^{(1)}, \tilde{P}_{\psi\varphi}^{(1)}, \tilde{P}_{\mathcal{A}\mathcal{B}}^{(1)}$ do not depend on antifields Ω^* . The functional $\tilde{P}_{\Omega^*}^{(1)}$ can be presented in the form

$$\tilde{P}_{\Omega^*}^{(1)} = \tilde{P}_{\mathcal{A}^*}^{(1)} + \tilde{P}_{C^*}^{(1)} + \tilde{P}_{\psi^*}^{(1)} + \tilde{P}_{\bar{\psi}^*}^{(1)} + \tilde{P}_{\varphi^*}^{(1)}. \quad (3.23)$$

Using the arguments similar to those that led us to establish the form of the functional $\tilde{P}_\theta^{(1)}$ (3.22), we obtain

$$\tilde{P}_{\mathcal{A}^*}^{(1)} = Z_7 \Gamma_{0|7} + Z_8 \Gamma_{0|8}, \quad \tilde{P}_{C^*}^{(1)} = Z_9 \Gamma_{0|9}, \quad (3.24)$$

$$\tilde{P}_{\psi^*}^{(1)} = Z_{10} \Gamma_{0|10}, \quad \tilde{P}_{\bar{\psi}^*}^{(1)} = Z_{11} \Gamma_{0|11},$$

$$\tilde{P}_{\varphi^*}^{(1)} = Z_{12} \Gamma_{0|12}. \quad (3.25)$$

Functional $\tilde{P}_\psi^{(1)}$ depends on the variables $\Psi, \mathcal{A}, \mathcal{B}$, and is quadratic in Ψ . Taking into account the axial symmetry as well as the Poincare- and $\mathcal{B}$ -gauge invariance, we find the general structure of $\tilde{P}_\psi^{(1)}$,

$$\tilde{P}_\psi^{(1)} = Z_{13} \Gamma_{0|13} + Z_{14} \Gamma_{0|14} + Z_{15} \Gamma_{0|15}. \quad (3.26)$$

Functional $\tilde{P}_\varphi^{(1)}$ depends on the variables $\varphi, \mathcal{A}, \mathcal{B}$, and vanishes when $\varphi = 0$. Taking into account the Poincare- and the $\mathcal{B}$ -gauge invariance, we establish the general form of the functional $\tilde{P}_\varphi^{(1)}$,

$$\begin{aligned} \tilde{P}_\varphi^{(1)} = & Z_{16} \Gamma_{0|16} + Z_{17} \Gamma_{0|17} \\ & + \tilde{P}_{\varphi 18}^{(1)} + Z_{19} \Gamma_{0|19} + Z_{20} \Gamma_{0|20}, \end{aligned} \quad (3.27)$$

where

$$\tilde{P}_{\varphi 18}^{(1)} = \int dx \left[\frac{g^2}{2} Z_{18}^{\alpha\beta\gamma\sigma} \mathcal{A}_\mu^\alpha \varphi^\beta \mathcal{A}_\mu^\gamma \varphi^\sigma \right],$$

$$Z_{18}^{\alpha\beta\gamma\sigma} = Z_{18}^{\gamma\beta\alpha\sigma} = Z_{18}^{\alpha\sigma\gamma\beta}. \quad (3.28)$$

If the condition

$$\int dx \left[Z_{18}^{\alpha\beta\gamma\sigma} \mathcal{A}_\mu^\alpha \varphi^\beta \mathcal{A}_\mu^\gamma \varphi^\sigma \right]$$

$$= Z_{18} \int dx \left[\sum_{\sigma=1}^3 E_{\sigma\beta}^\alpha E_{\sigma\rho}^\gamma \mathcal{A}_\mu^\alpha \varphi^\beta \mathcal{A}_\mu^\gamma \varphi^\rho \right] \quad (3.29)$$

fulfils then the functional $\tilde{P}_{\varphi 18}^{(1)}$ satisfies the Eq. (3.12).

Functional $\tilde{P}_{\varphi\varphi}^{(1)}$ is an interaction vertex of the fields φ and Ψ . Taking into account the axial, Poincare- and T -symmetries, this functional has the following general form:

$$\tilde{P}_{\varphi\varphi}^{(1)} = Z_{21} \Gamma_{0|21}. \quad (3.30)$$

The functional $\tilde{P}_{\varphi\varphi}^{(1)}$ (3.30) satisfies the Eq. (3.12).

Substitute into the Eq. (3.10) the representation for the functional $\tilde{P}^{(1)}$ in the form (3.20) where the terms $\tilde{P}_\theta^{(1)}$, $\tilde{P}_{\Omega^*}^{(1)}$, $\tilde{P}_\psi^{(1)}$, $\tilde{P}_\varphi^{(1)}$ and $\tilde{P}_{\varphi\varphi}^{(1)}$ are described by the relations (3.22), (3.23)–(3.25), (3.26), (3.27)–(2.33) and (3.30), respectively, and take into account that $\tilde{P}_{AB}^{(1)} = \tilde{P}_{AB}^{(1)}(\mathcal{A}, \mathcal{B})$. Then, a solution to the Eq. (3.10) is reduced to solutions to the subequations which follow from the requirement for independent polynomial structures appearing in the left-hand side of the Eq. (3.10), to be equal to zero. In their turn, these subequations are reduced to algebraic equations for coefficients "Z" or, in two cases, to variational differential equations for the functional $\tilde{P}_{AB}^{(1)}(\mathcal{A}, \mathcal{B})$.

We explain the results obtained by using an example for the block

$$\theta \mathcal{A}^* C \Rightarrow \int dx \left[Z_6 Z_8 \Gamma_{0|8} \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^\alpha} \right]$$

$$\frac{\delta}{\delta \mathcal{A}_\mu^{\alpha*}} \Gamma_{0|6} - Z_7 \theta_\mu^\alpha \frac{\delta}{\delta \mathcal{B}_\mu^\alpha} \Gamma_{0|7} \Big] = 0 \Rightarrow \quad (3.31)$$

$$Z_8 = \frac{Z_7}{Z_6}. \quad (3.32)$$

This block should be understood as the following: the requirement for the structure $\theta \mathcal{A}^* C$ to be equal to zero leads to equation (3.31), from which it follows the relation (3.32).

Further

$$\theta \bar{\psi} \gamma^\mu \psi \Rightarrow \int dx \left[Z_6 Z_{14} \Gamma_{0|14} \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^\alpha} \frac{\delta}{\delta \mathcal{A}_\mu^{\alpha*}} \right]$$

$$\Gamma_{0|6} - Z_{13} \theta_\mu^\alpha \frac{\delta}{\delta \mathcal{B}_\mu^\alpha} \Gamma_{0|13} \Big] = 0 \Rightarrow$$

$$Z_{14} = \frac{Z_{13}}{Z_6}; \quad (3.33)$$

$$\theta \varphi(D(\mathcal{B})\varphi) \Rightarrow \int dx \left[Z_6 Z_{17} \Gamma_{0|17} \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^\alpha} \frac{\delta}{\delta \mathcal{A}_\mu^{\alpha*}} \right]$$

$$\Gamma_{0|6} - Z_{16} \theta_\mu^\alpha \frac{\delta}{\delta \mathcal{B}_\mu^\alpha} \Gamma_{0|16} \Big] = 0 \Rightarrow$$

$$Z_{17} = \frac{Z_{16}}{Z_6}; \quad (3.34)$$

$$\theta \mathcal{A} \varphi \varphi \Rightarrow \int dx \left[Z_6 \tilde{P}_{\varphi 18}^{(1)} \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^\alpha} \frac{\delta}{\delta \mathcal{A}_\mu^{\alpha*}} \right]$$

$$\Gamma_{0|6} - Z_{17} \theta_\mu^\alpha \frac{\delta}{\delta \mathcal{B}_\mu^\alpha} \Gamma_{0|17} \Big] = 0 \Rightarrow$$

$$Z_{18}^{\alpha\beta\gamma\sigma} \varphi^\beta \mathcal{A}_\mu^\gamma \varphi^\sigma = \frac{Z_{17}}{Z_6} E_{\alpha\beta}^\rho E_{\gamma\sigma}^\rho \varphi^\beta \mathcal{A}_\mu^\gamma \varphi^\sigma. \quad (3.35)$$

Multiplying the equality (3.35) by $\mathcal{A}_\mu^\alpha$ and integrating then over x , we find that the relation (3.29) is satisfied and the functional $\tilde{P}_{\varphi 18}^{(1)}$ reads

$$\tilde{P}_{\varphi 18}^{(1)} = Z_{18} \Gamma_{0|18}, \quad Z_{18} = \frac{Z_{17}}{Z_6} = \frac{Z_{16}}{Z_6^2}. \quad (3.36)$$

$$\theta \mathcal{A}^n \mathcal{B}^k \Rightarrow \int dx \left[Z_6 \tilde{P}_{AB}^{(1)} \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^\alpha} \frac{\delta}{\delta \mathcal{A}_\mu^{\alpha*}} \right]$$

$$\Gamma_{0|6} - \theta_\mu^\alpha \frac{\delta}{\delta \mathcal{B}_\mu^\alpha} \tilde{P}_{AB}^{(1)} \Big] = 0 \Rightarrow$$

$$\left(Z_6 \frac{\delta}{\delta \mathcal{A}_\mu^\alpha} - \frac{\delta}{\delta \mathcal{B}_\mu^\alpha} \right) \tilde{P}_{AB}^{(1)} = 0 \Rightarrow$$

$$\tilde{P}_{AB}^{(1)} = \tilde{P}_{AB}^{(1)}(U), \quad U = U(Z_6) = \mathcal{B} + \frac{1}{Z_6} \mathcal{A}; \quad (3.37)$$

$$\mathcal{A}^* A C C \Rightarrow \int dx \left[Z_8^2 \Gamma_{0|8} \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^\alpha} \frac{\delta}{\delta \mathcal{A}_\mu^{\alpha*}} \right]$$

$$\Gamma_{0|8} + Z_8 Z_9 \Gamma_{0|8} \frac{\overleftarrow{\delta}}{\delta C^\alpha} \frac{\delta}{\delta C^{\alpha*}} \Gamma_{0|9} \Big] = 0 \Rightarrow$$

$$Z_9 = Z_8 = \frac{Z_7}{Z_6}; \quad (3.38)$$

$$\psi^* \psi C C \Rightarrow \int dx \left[Z_{10}^2 \Gamma_{0|10} \frac{\overleftarrow{\delta}}{\delta \psi_j} \frac{\delta}{\delta \psi_j^*} \right]$$

$$\Gamma_{0|10} + Z_{10} Z_9 \Gamma_{0|10} \frac{\overleftarrow{\delta}}{\delta C^\alpha} \frac{\delta}{\delta C^{\alpha*}} \Gamma_{0|9} \Big] = 0 \Rightarrow$$

$$Z_{10} = Z_9 = \frac{Z_7}{Z_6}; \quad (3.39)$$

$$\begin{aligned} \bar{\psi}\psi^*CC &\Rightarrow \int dx \left[Z_{11}^2 \Gamma_{0|11} \frac{\overleftarrow{\delta}}{\delta\psi_j} \frac{\delta}{\delta\psi_j^*} \right. \\ &\quad \left. \Gamma_{0|11} + Z_{11}Z_9\Gamma_{0|11} \frac{\overleftarrow{\delta}}{\delta C^\alpha} \frac{\delta}{\delta C^{*\alpha}} \Gamma_{0|9} \right] = 0 \Rightarrow \\ Z_{11} &= Z_9 = \frac{Z_7}{Z_6}; \end{aligned} \quad (3.40)$$

$$\begin{aligned} \bar{\psi}\psi\varphi C &\Rightarrow \int dx \left[Z_{10}Z_{21}\Gamma_{0|21} \frac{\overleftarrow{\delta}}{\delta\psi_j} \frac{\delta}{\delta\psi_j^*} \right. \\ &\quad \left. \Gamma_{0|10} + Z_{11}Z_{21}\Gamma_{0|21} \frac{\overleftarrow{\delta}}{\delta\psi_j} \frac{\delta}{\delta\psi_j^*} \Gamma_{0|11} \right. \\ &\quad \left. + Z_{12}Z_{21}\Gamma_{0|21} \frac{\overleftarrow{\delta}}{\delta\varphi^\alpha} \frac{\delta}{\delta\varphi^{*\alpha}} \Gamma_{0|12} \right] = 0 \Rightarrow \\ Z_{12} &= Z_{10} = \frac{Z_7}{Z_6}; \end{aligned} \quad (3.41)$$

$$\begin{aligned} \mathcal{A}^n \mathcal{B}^k C &\Rightarrow \int dx \left[\tilde{P}_{AB}^{(1)} \frac{\overleftarrow{\delta}}{\delta\mathcal{A}_\mu^\alpha} \frac{\delta}{\delta\mathcal{A}_\mu^{*\alpha}} \right. \\ &\quad \left. (Z_7\Gamma_{0|7} + Z_8\Gamma_{0|8}) \right] = 0 \\ &\Rightarrow D_\mu^{\alpha\beta}(U) \frac{\delta}{\delta U_\mu^\beta} \tilde{P}_{AB}^{(1)}(U) = 0 \Rightarrow \tilde{P}_{AB}^{(1)} = Z_{22}\Gamma_{0|22}(Z_6), \end{aligned} \quad (3.42)$$

$$\Gamma_{0|22}(Z_6) = -\frac{1}{4} \int dx [G_{\mu\nu}^\alpha(U(Z_6))G_{\mu\nu}^\alpha(U(Z_6))]. \quad (3.43)$$

Requirement for the rest structures in the left-hand side of the Eq. (3.10) to be equal to zero is satisfied identically.

Notice that the functional $\tilde{P}^{(1)}$ can be represented as a linear combination of independent polynomials $\Gamma_{0|k}$,

$$\tilde{P}^{(1)} = \sum_{k=6}^{22} Z_k \Gamma_{0|k}. \quad (3.44)$$

3.2 Solution to the gauge dependence equation

Consider now a solution to the Eq. (3.18) describing the dependence of constants "Z" on the gauge parameter ξ appearing in the general solution to the extended master-equation (3.10) for the functional $\tilde{P}^{(1)}$. For the functional $\tilde{P}^{(1)}$ we will use the representation (3.44). Notice that every functional $\Gamma_{0|k}$, $k = 6, \dots, 21$ is eigen for the operator $\hat{L}$ (3.19),

$$\hat{L}\Gamma_{0|k} = \zeta_k \Gamma_{0|k}, \quad (3.45)$$

where ζ_k are eigenvalues of the operator $\hat{L}$. It means that the Eq. (3.18) reduces to the set of equations of the form

$$2\xi \dot{Z}_k = \zeta_k, \quad k = 6, \dots, 21. \quad (3.46)$$

In relations (3.46) and later on for any quantity $I = I(\xi, \dots)$ we use the notation

$$\dot{I} \equiv \frac{\partial}{\partial \xi} I. \quad (3.47)$$

Equation (3.46) for independent constants Z_k lead to the following relations and consequences,

$$\begin{aligned} k = 6, \quad 2\xi \dot{Z}_6 &= Z_6(1 - Z_1) \\ &\Rightarrow Z_1 = 1 - 2\xi \frac{\dot{Z}_6}{Z_6}, \end{aligned} \quad (3.48)$$

$$\begin{aligned} k = 7, \quad 2\xi \dot{Z}_7 &= Z_7(Z_2 - Z_1) \\ &\Rightarrow Z_2 = 1 + 2\xi \left(\frac{\dot{Z}_7}{Z_7} - \frac{\dot{Z}_6}{Z_6} \right), \end{aligned} \quad (3.49)$$

$$\begin{aligned} k = 13, \quad 2\xi \dot{Z}_{13} &= Z_{13}(Z_3 + Z_4 - 2) \\ &\Rightarrow Z_4 = 2 + 2\xi \frac{\dot{Z}_{13}}{Z_{13}} - Z_3, \end{aligned} \quad (3.50)$$

$$\begin{aligned} k = 15, \quad 2\xi \dot{Z}_{15} &= Z_{15}(Z_3 + Z_4 - 2) \\ &\Rightarrow \dot{Z}_{23} = 0, \quad Z_{15} = Z_{13}Z_{23}, \end{aligned} \quad (3.51)$$

$$\begin{aligned} k = 16, \quad \xi \dot{Z}_{16} &= Z_{16}(Z_5 - 1) \\ &\Rightarrow Z_5 = 1 + \xi \frac{\dot{Z}_{16}}{Z_{16}}, \end{aligned} \quad (3.52)$$

$$\begin{aligned} k = 19, \quad \xi \dot{Z}_{19} &= Z_{19}(Z_5 - 1) \\ &\Rightarrow \dot{Z}_{24} = 0, \quad Z_{19} = Z_{16}Z_{24}, \end{aligned} \quad (3.53)$$

$$\begin{aligned} k = 20, \quad \xi \dot{Z}_{20} &= 2Z_{20}(Z_5 - 1) \\ &\Rightarrow \dot{Z}_{25} = 0, \quad Z_{20} = Z_{16}^2 Z_{25}, \end{aligned} \quad (3.54)$$

$$\begin{aligned} k = 21, \quad 2\xi \dot{Z}_{21} &= Z_{21}(Z_3 + Z_4 + Z_5 - 3) \\ &\Rightarrow \dot{Z}_{26} = 0, \quad Z_{21} = Z_{13}Z_{16}^{1/2}Z_{26}. \end{aligned} \quad (3.55)$$

As to the functional $\Gamma_{0|22}(Z_6)$, let us represent it in the form

$$\Gamma_{0|22}(Z_6) = \sum_{l=0}^4 Z_6^{-l} \Gamma_{0|22,l}. \quad (3.56)$$

Then, from the Eq. (3.18), it follows

$$l = 0, \quad \dot{Z}_{22} = 0, \quad (3.57)$$

$$l = 1, 2, 3, 4, \quad -2\xi \frac{\dot{Z}_6}{Z_6} = Z_1 - 1. \quad (3.58)$$

The last relation is equivalent to (3.48).

Below, in Sect. 5, we find that all constants "Z" can be interpreted as renormalization constants which are uniquely defined from the conditions of reducing divergences.

Now we formulate the results obtained in this section as the following lemma.

Lemma: Let

$$P = \int dx P(Q, Q^*, \bar{C}, B, \mathcal{B}, \xi, \theta, \chi), \quad (3.59)$$

be a local functional of all variables with quantum numbers of the action S_{ext} and satisfies all Eqs. (2.19)–(2.24) and

additional symmetries (the axial-, Poincare-invariance and so on) which were used in solving the Eqs. (2.19)–(2.24) (with substitution $S_{ext} \rightarrow P$). Then the functional P has the form

$$P = \Gamma_{0|0} + \sum_{k=6}^{22} Z_k \Gamma_{0|k} + \chi \sum_{k=1}^5 Z_k \Gamma_{0|k}, \quad (3.60)$$

$$\begin{aligned} Z_8 = Z_9 = Z_{10} = Z_{11} = Z_{12} &= \frac{Z_7}{Z_6}, \\ Z_{14} &= \frac{Z_{13}}{Z_6}, \quad Z_{15} = Z_{13} Z_{23}, \quad Z_{17} = \frac{Z_{16}}{Z_6}, \\ Z_{18} &= \frac{Z_{16}}{Z_6^2}, \quad Z_{19} = Z_{16} Z_{24}, \quad Z_{20} = Z_{16}^2 Z_{25}, \\ Z_{21} &= Z_{13} Z_{16}^{1/2} Z_{26}, \\ Z_1 &= 1 - 2\xi \frac{\dot{Z}_6}{Z_6}, \quad Z_2 = 1 + 2\xi \left(\frac{\dot{Z}_7}{Z_7} - \frac{\dot{Z}_6}{Z_6} \right), \end{aligned} \quad (3.61)$$

$$\begin{aligned} Z_4 &= 2 + 2\xi \frac{\dot{Z}_{13}}{Z_{13}} - Z_3, \\ Z_5 &= 1 + 2\xi \frac{\dot{Z}_{16}}{Z_{16}}, \quad \dot{Z}_{22} = \dot{Z}_{23} = \dot{Z}_{24} = \dot{Z}_{25} = \dot{Z}_{26} = 0, \end{aligned} \quad (3.62)$$

where $Z_6, Z_7, Z_{13}, Z_{15}, Z_{16}, Z_{19}, Z_{20}, Z_{21}, Z_{22}$ are arbitrary constants depending perhaps on ξ , the constants $Z_{22}, Z_{23}, Z_{24}, Z_{25}, Z_{26}$ do not depend on ξ . Among the set of indices of all constants " Z'' ", $\{k\} = 1, \dots, 26$ we will highlight the three groups: the first is indices of independent constants,

$$\{k_{ind}\} = 3, 6, 7, 13, 16, 22, 23, 24, 25, 26; \quad (3.63)$$

the second is indices of constants independent of ξ ,

$$\{k_{con}\} = 22, 23, 24, 25, 26 \subset \{k_{ind}\}; \quad (3.64)$$

and the third is indices of dependent constants,

$$\{k_{dep}\} = 1, 2, 4, 5, 8, 9, 10, 11, 12, 14, 15, 17, 18, 19, 20, 21. \quad (3.65)$$

It should be noted that the functional P does not contain vertices additional to that from which the action S_{ext} (2.9) is built up. Namely, the obvious relation is valid,

$$S_{ext} = P|_{Z''=1}. \quad (3.66)$$

In its turn, the functional P (3.60) can be represented in the form analogous to (2.9) for the action S_{ext} ,

$$\begin{aligned} P = \int dx \Bigg\{ & -\frac{Z_{22}}{4} G_{\mu\nu}^\alpha(U) G_{\mu\nu}^\alpha(U) + Z_{13} i \bar{\psi} \gamma^\mu \\ & \times D_{\psi\mu}(U) \psi + \frac{Z_{16}}{2} (D_{\varphi\mu}^{\alpha\beta}(U) \varphi^\beta) D_{\varphi\mu}^{\alpha\gamma}(U) \varphi^\gamma \\ & - Z_{15} m \bar{\psi} \psi - Z_{19} \frac{M^2}{2} \varphi^2 - Z_{20} \frac{\lambda}{4} \varphi^4 \\ & + Z_{21} i \vartheta \varphi^\alpha \bar{\psi} t^\alpha \psi + Z_6 \theta_\mu^\alpha [D_\mu^{\alpha\beta}(U) \bar{C}^\beta - A_\mu^{*\alpha}] \\ & + \frac{Z_7}{Z_6} Q^* \mathcal{T}_Q + \chi [Z_1 (A_\mu^{*\alpha} - D_\mu^{\alpha\beta}(B) \bar{C}^\beta) A_\mu^\alpha \\ & + Z_2 C^{*\alpha} C^\alpha + Z_3 \psi^* \psi + Z_4 \bar{\psi}^* \bar{\psi} + Z_5 \varphi^{*\alpha} \varphi^\alpha] \\ & + (\xi/2) B^\alpha B^\alpha + B^\alpha D_\mu^{\alpha\beta}(B) A_\mu^\beta \\ & + Z_7 \bar{C}^\alpha D_\mu^{\alpha\beta}(B) D_\mu^{\beta\gamma}(U) C^\gamma \Bigg\}, \end{aligned} \quad (3.67)$$

$$\mathcal{T}_{A_\mu^\alpha} = Z_6 D_\mu^{\alpha\beta}(U) C^\beta, \quad \mathcal{T}_{\bar{\psi}, \bar{\psi}, \varphi, C} = \mathcal{R}_{\bar{\psi}, \bar{\psi}, \varphi, C}. \quad (3.68)$$

Representation (3.67) for the functional P shows that it can be considered as (renormalized) action of the Yang–Mills theory, and from representations (3.60) and (3.66) of functionals P and S_{ext} it follows that the renormalization is multiplicative.

The inverse (maybe trivial but important) statement is valid: if the functional P has the form (3.60) and the relations (3.61), (3.62) are satisfied then this functional satisfies the Eqs. (2.20)–(2.24).

4 Effective action

It is useful to define the generating functional of Green functions with the help of the functional P constructed in the previous section, because it allows one to obtain a finite theory from the beginning. In what follows we redefine the functional P , $P \equiv S_R$, and, respectively, $P^{(k)} \equiv S_R^{(k)}$, $\tilde{P}^{(k)} \equiv \tilde{S}_R^{(k)}$, $k = 1, 2$.

The generating functional of Green functions is given by the following functional integral

$$\begin{aligned} Z(J_\Phi, L) &= \int d\Phi \exp \left(\frac{i}{\eta} [S_R + J_\Phi \Phi] \right) \\ &= \exp \left\{ \frac{i}{\eta} W(J_\Phi, L) \right\}, \end{aligned} \quad (4.1)$$

where η is the parameter of a loop expansion of expression in the exponent (4.1), $W(J_\Phi, L)$ is the generating functional of connected Green functions, J_Φ is the set of sources to fields Φ and the notations $\Phi = \{Q, \bar{C}, B\}$ and $L = \{L^A\} = \{B, Q^*, \xi, \theta, \chi\}$ are introduced. We suppose also that all " Z'' " are functions of η , " $Z'' = Z(\eta)$ " expanding in the Taylor series, $Z_k(0) = 1$, $\dot{Z}_k = O(\eta)$, $k = 1, \dots, 26$. In that case the functional S_R becomes a function of η ,

$$S_R = S_R(\eta) = \sum_{l=0}^{\infty} \eta^l S_{R,l}, \quad S_R^{[k]} = \sum_{l=0}^k \eta^l S_{R,l}, \quad (4.2)$$

and all functionals $S_{R,l}$ are linear combinations of the same set of polynomials $\{\Gamma_{0|k}, k = 0, 1, \dots, 22\}$ with the help of which the functionals S_{ext} and P are presented.

The generating functional of vertex functions (effective action) is defined with the help of the Legendre transformation,

$$\Gamma(\Phi_m|, L) = W(J_\Phi, L) - J_\Phi \Phi_m|, \quad \Phi_m| = \frac{\delta}{\delta J_\Phi} W(J_\Phi, L), \quad (4.3)$$

with quantum numbers $\varepsilon(\Gamma) = 0$, $\text{gh}(\Gamma) = 0$, $\dim(\Gamma) = 0$, $\varepsilon_f(\Gamma) = 0$ and satisfies the relations

$$\begin{aligned} \Gamma(\Phi_m|, L) \frac{\overleftarrow{\delta}}{\delta \Phi_m|} &= -J_\Phi(\Phi_m|, L), \\ \Gamma(\Phi_m|, L) \frac{\overleftarrow{\delta}}{\delta L^A} &= W(J_\Phi, L) \frac{\overleftarrow{\delta}}{\delta L^A} \end{aligned} \quad (4.4)$$

Functional averaging of Eqs. (2.20)–(2.24) with substitution $S_{ext} \rightarrow S_R$ leads to the equations for the functional $\Gamma = \Gamma(\Phi_m|, L)$ copying the equations for S_R ,

$$\begin{aligned} \int dx \left(\Gamma \frac{\overleftarrow{\delta}}{\delta Q_m|} \frac{\delta}{\delta Q^*} \Gamma - B_m| \frac{\delta}{\delta \bar{C}_m|} \Gamma - \theta \frac{\delta}{\delta \beta} \Gamma \right) \\ + 2\chi \xi \frac{\partial}{\partial \xi} \Gamma \\ + \chi \int dx \left[\left(Q_m| \frac{\delta}{\delta Q_m|} - Q^* \frac{\delta}{\delta Q^*} \right. \right. \\ \left. \left. - \bar{C}_m| \frac{\delta}{\delta \bar{C}_m|} - B_m| \frac{\delta}{\delta B_m|} \right) \Gamma \right] = 0, \end{aligned} \quad (4.5)$$

$$\Gamma \overleftarrow{H}_m^\alpha \omega_\alpha = 0, \quad (4.6)$$

where $\overleftarrow{H}_m^\alpha \omega_\alpha$ is given by expression (2.21) with substitution $\Phi \rightarrow \Phi_m|$,

$$\frac{\delta}{\delta B_m^\alpha} \Gamma = D_\mu^{\alpha\beta}(B) A_{m|\mu}^\beta + \xi B_{m|}^\alpha, \quad (4.7)$$

$$D_\mu^{\alpha\beta}(B) \frac{\delta}{\delta A_{m|\mu}^{*\beta}} \Gamma - \frac{\delta}{\delta \bar{C}_m^\alpha} \Gamma = -g \varepsilon^{\alpha\beta\gamma} A_{m|\mu}^\beta \theta_\mu^\gamma. \quad (4.8)$$

Represent the functional Γ in the form

$$\Gamma = \Gamma_{0|0} + \Gamma^{(1)} + \chi \Gamma^{(2)}, \quad (4.9)$$

where the functionals $\Gamma^{(1)}$ and $\Gamma^{(2)}$ do not depend on the parameter χ . Thanks to the structure chosen for the functional (4.9), from the Eqs. (4.7) and (4.8), it follows that the functionals $\Gamma^{(1)}$ and $\Gamma^{(2)}$ do not depend on the fields $B_{m|}^\alpha$,

$$\begin{aligned} \frac{\delta}{\delta B_m|} \Gamma^{(k)} &= 0, \quad \Gamma^{(k)} \\ &= \Gamma^{(k)}(Q_m|, \bar{C}_m|, B, Q^*, \xi, \theta), \quad k = 1, 2, \end{aligned} \quad (4.10)$$

and satisfy the equations

$$\left(D_\mu^{\alpha\beta}(B) \frac{\delta}{\delta A_{m|\mu}^{*\beta}} - \frac{\delta}{\delta \bar{C}_m^\alpha} \right) \Gamma^{(k)} = 0, \quad k = 1, 2. \quad (4.11)$$

In its turn, the Eq. (4.5) splits into two ones, one of them is closed with respect to $\Gamma^{(1)}$,

$$\begin{aligned} \int dx \left[\Gamma^{(1)} \frac{\overleftarrow{\delta}}{\delta Q_m|} \frac{\delta}{\delta Q^*} \Gamma^{(1)} - g \theta_\mu^\alpha \varepsilon^{\alpha\beta\gamma} \bar{C}_m|^\beta \right. \\ \left. \frac{\delta}{\delta A_{m|\mu}^{*\gamma}} \Gamma^{(1)} - \theta_\mu^\alpha \frac{\delta}{\delta \beta} \Gamma^{(1)} \right] = 0, \end{aligned} \quad (4.12)$$

and the second includes both functionals and describes their dependence on the gauge parameter ξ ,

$$\begin{aligned} 2\xi \frac{\partial}{\partial \xi} \Gamma^{(1)} &= \int dx \left[\Gamma^{(1)} \left(\frac{\overleftarrow{\delta}}{\delta Q_m|} \frac{\delta}{\delta Q^*} - \frac{\overleftarrow{\delta}}{\delta Q^*} \frac{\delta}{\delta Q_m|} \right) \Gamma^{(2)} \right. \\ &\quad \left. - \left(g \theta_\mu^\alpha \varepsilon^{\alpha\beta\gamma} \bar{C}_m|^\beta \frac{\delta}{\delta A_{m|\mu}^{*\gamma}} + \theta_\mu^\alpha \frac{\delta}{\delta \beta} \right) \Gamma^{(2)} \right] \\ &\quad + \int dx \left[\left(\bar{C}_m| \frac{\delta}{\delta \bar{C}_m|} - Q_m| \frac{\delta}{\delta Q_m|} + Q^* \frac{\delta}{\delta Q^*} \right) \Gamma^{(1)} \right]. \end{aligned} \quad (4.13)$$

Now, the Eq. (4.6) rewrites in the form of the two equations for the functionals $\Gamma^{(1)}$ and $\Gamma^{(2)}$,

$$\Gamma^{(k)} \overleftarrow{h}_m^\alpha \omega_\alpha = 0, \quad k = 1, 2, \quad (4.14)$$

where

$$\begin{aligned} \overleftarrow{h}_m^\alpha \omega_\alpha &= \int dx \left\{ \left[\frac{\overleftarrow{\delta}}{\delta B_\mu^\beta} D_\mu^{\beta\alpha}(B) \right. \right. \\ &\quad + g \varepsilon^{\beta\gamma\alpha} \left(\frac{\overleftarrow{\delta}}{\delta A_{m|\mu}^\beta} A_{m|\mu}^\gamma + \frac{\overleftarrow{\delta}}{\delta \varphi_{m|}^\beta} \varphi_{m|}^\gamma + \frac{\overleftarrow{\delta}}{\delta C_{m|}^\beta} C_{m|}^\gamma \right) \\ &\quad + g \varepsilon^{\beta\gamma\alpha} \left(\frac{\overleftarrow{\delta}}{\delta \bar{C}_m^\beta} \bar{C}_m|^\gamma + \frac{\overleftarrow{\delta}}{\delta A_{m|\mu}^{*\beta}} A_{m|\mu}^{*\gamma} + \frac{\overleftarrow{\delta}}{\delta C^{*\beta}} C^{*\gamma} \right. \\ &\quad \left. + \frac{\overleftarrow{\delta}}{\delta \varphi^{*\beta}} \varphi^{*\gamma} + \frac{\overleftarrow{\delta}}{\delta \theta_\mu^\beta} \theta_\mu^\gamma \right) \\ &\quad \left. - g t_{jk}^\alpha \left(\frac{\overleftarrow{\delta}}{\delta \psi_{m|j}} \psi_{m|k} + \frac{\overleftarrow{\delta}}{\delta \bar{\psi}_j^*} \bar{\psi}_k^* \right) + g \right. \\ &\quad \left. \left(\frac{\overleftarrow{\delta}}{\delta \bar{\psi}_{m|j}} \bar{\psi}_{m|k} + \frac{\overleftarrow{\delta}}{\delta \psi_j^*} \psi_k^* \right) t_{kj}^\alpha \right\} \omega_\alpha. \end{aligned} \quad (4.15)$$

Due to the Eq. (4.11) it is useful to introduce new variables $\mathcal{A}_\mu^{*\alpha} = \mathcal{A}_\mu^{*\alpha}(x)$, $\mathcal{A}_{m|\mu}^{*\alpha} = \mathcal{A}_{m|\mu}^{*\alpha}(x)$,

$$\begin{aligned}\mathcal{A}_\mu^{*\alpha} &= A_\mu^{*\alpha} - D_\mu^{\alpha\beta}(\mathcal{B})\bar{C}^\beta, \\ \mathcal{A}_{m|\mu}^{*\alpha} &= A_{m|\mu}^{*\alpha} - D_\mu^{\alpha\beta}(\mathcal{B})\bar{C}_{m|}^\beta,\end{aligned}\quad (4.16)$$

and to use the following agreement for the sake of uniformity of further notations

$$\mathcal{A}_\mu^\alpha = A_\mu^\alpha. \quad (4.17)$$

Also, let us introduce new functionals $\tilde{\Gamma}^{(k)}$ by the rule

$$\begin{aligned}\tilde{\Gamma}^{(k)}(\mathcal{B}, \bar{C}_{m|}, \mathcal{A}_{m|}^*, \Lambda_{m|}) &= \Gamma^{(k)}(\mathcal{B}, \bar{C}_{m|}, A^*, \Lambda_{m|}) \\ &\Big|_{A^* \rightarrow \mathcal{A}_{m|}^* + D_\mu^{\alpha\beta}(\mathcal{B})\bar{C}_{m|}^\beta},\end{aligned}\quad (4.18)$$

where the following notations

$$\Lambda = \{Q, \psi^*, \bar{\psi}^*, C^*, \xi, \theta\}, \quad \Lambda_{m|} = \{Q_{m|}, \psi^*, \bar{\psi}^*, C^*, \xi, \theta\} \quad (4.19)$$

were used. Taking into account the definitions (4.16)–(4.18) we have

$$\frac{\delta}{\delta A_\mu^{*\alpha}} \Gamma^{(k)} = \frac{\delta}{\delta \mathcal{A}_{m|\mu}^{*\alpha}} \tilde{\Gamma}^{(k)}, \quad (4.20)$$

$$\frac{\delta}{\delta \bar{C}_{m|}^\alpha} \Gamma^{(k)} = \frac{\delta}{\delta \bar{C}_{m|}^\alpha} \tilde{\Gamma}^{(k)} + D_\mu^{\alpha\beta}(\mathcal{B}) \frac{\delta}{\delta \mathcal{A}_{m|\mu}^{*\beta}} \tilde{\Gamma}^{(k)}, \quad (4.21)$$

$$\frac{\delta}{\delta \mathcal{B}_\mu^\alpha} \Gamma^{(k)} = \frac{\delta}{\delta \mathcal{B}_\mu^\alpha} \tilde{\Gamma}^{(k)} - g f^{\alpha\beta\gamma} \bar{C}_{m|}^\beta \frac{\delta}{\delta \mathcal{A}_{m|\mu}^{*\gamma}} \tilde{\Gamma}^{(k)}, \quad k = 1, 2. \quad (4.22)$$

Then, from the Eqs. (4.11), (4.18), (4.20), (4.21), we find that

$$\frac{\delta}{\delta \bar{C}_{m|}^\alpha} \tilde{\Gamma}^{(k)} = 0, \quad (4.23)$$

the functionals $\tilde{\Gamma}^{(k)}$, $k = 1, 2$ do not depend on the fields $\bar{C}_{m|}^\alpha$,

$$\tilde{\Gamma}^{(k)} = \tilde{\Gamma}^{(k)}(\Omega_{m|}, \Omega_{m|}^*, \mathcal{B}, \xi, \theta). \quad (4.24)$$

Here and below we will use the notations

$$\begin{aligned}\Omega_{m|} &= \{\mathcal{A}_{m|}, \psi_{m|}, \bar{\psi}_{m|}, \varphi_{m|}, C_{m|}\}, \\ \Omega_{m|}^* &= \{\mathcal{A}_{m|}^*, \psi^*, \bar{\psi}^*, \varphi^*, C^*\}.\end{aligned}\quad (4.25)$$

Now, thanks to (4.18), (4.24), (4.25), the equations (4.12) and (4.13) rewrite as

$$\frac{1}{2}(\tilde{\Gamma}^{(1)}, \tilde{\Gamma}^{(1)}) - \int dx \left(\theta \frac{\delta}{\delta \mathcal{B}} \right) \tilde{\Gamma}^{(1)} = 0, \quad (4.26)$$

$$\begin{aligned}2\xi \frac{\partial}{\partial \xi} \tilde{\Gamma}^{(1)} &= (\tilde{\Gamma}^{(1)}, \tilde{\Gamma}^{(2)}) \\ &+ \int dx \left(\Omega_{m|}^* \frac{\delta}{\delta \Omega_{m|}^*} - \Omega_{m|} \frac{\delta}{\delta \Omega_{m|}} \right) \tilde{\Gamma}^{(1)} \\ &- \int dx \left(\theta \frac{\delta}{\delta \mathcal{B}} \right) \tilde{\Gamma}^{(2)},\end{aligned}\quad (4.27)$$

where the notation for antibracket [3, 4]

$$(F, G) = \frac{1}{2} F \int dx \left(\frac{\overleftarrow{\delta}}{\delta \Omega_{m|}} \frac{\delta}{\delta \Omega_{m|}^*} - \frac{\overleftarrow{\delta}}{\delta \Omega_{m|}^*} \frac{\delta}{\delta \Omega_{m|}} \right) G \quad (4.28)$$

is used. Further, taking into account (4.18), (4.24) and

$$\begin{aligned}\Gamma^{(k)} \int dx \left[\frac{\overleftarrow{\delta}}{\delta \mathcal{B}_\mu^\beta} D_\mu^{\beta\alpha}(\mathcal{B}) + g \varepsilon^{\beta\gamma\alpha} \right. \\ \left. \times \left(\frac{\overleftarrow{\delta}}{\delta \bar{C}_{m|}^\beta} \bar{C}_{m|}^\gamma + \frac{\overleftarrow{\delta}}{\delta A_{m|\mu}^{*\beta}} A_{m|\mu}^{*\gamma} \right) \right] \\ = \tilde{\Gamma}^{(k)} \int dx \left[\frac{\overleftarrow{\delta}}{\delta \mathcal{B}_\mu^\beta} D_\mu^{\beta\alpha}(\mathcal{B}) + g \varepsilon^{\beta\gamma\alpha} \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_{m|\mu}^{*\beta}} \mathcal{A}_{m|\mu}^{*\gamma} \right],\end{aligned}\quad (4.29)$$

we find that

$$\Gamma^{(k)} \overleftarrow{h}_{m|}^\alpha \omega_\alpha = \tilde{\Gamma}^{(k)} \overleftarrow{h}_{m|}^\alpha \omega_\alpha = 0, \quad k = 1, 2, \quad (4.30)$$

where the operator $\overleftarrow{h}_{m|}^\alpha \omega_\alpha$ is defined in the equality (3.13) with the substitution $\Omega \rightarrow \Omega_{m|}$, $\Omega^* \rightarrow \Omega_{m|}^*$.

Later, on when studying the tensor structure of divergent parts of the generating functional of vertex functions, it is useful to use the consequence of the Eq. (4.30) corresponding to the particular case of $T_{m|}$ -symmetry when $\omega_\alpha(x) = \text{const}$,

$$\tilde{\Gamma}^{(k)} \overleftarrow{T}_{m|}^\alpha = 0, \quad k = 1, 2, \quad (4.31)$$

where the operators $\overleftarrow{T}_{m|}^\alpha$ are defined by the equalities (3.15) with substitution $\Omega \rightarrow \Omega_{m|}$, $\Omega^* \rightarrow \Omega_{m|}^*$.

5 Multiplicative renormalization

In this section, we study a structure of renormalization in the model under consideration, and find multiplicative character of renormalizability. The main role is played by solving the extended master-equation (3.10) and the equation describing the gauge dependence (3.18). We will show that the renormalized quantum action and the effective action satisfy this

equation exactly to every order of loop expansions. The structure of the renormalized action is determined by the same monomials in fields and antifields as the non-renormalized quantum action does but with constants defined by the divergences of the effective action. For simplicity of notations, we will often omit the lower index m in arguments of functional Γ .

5.1 Tree approximation ($\eta = 0$)

Consider the tree approximation for the functional Γ , $\Gamma_0 = S_{\text{ext}}$, which in new variables reads

$$\Gamma_0 = \Gamma_{0|0} + \Gamma_0^{(1)} + \chi \Gamma_0^{(2)}, \quad (5.1)$$

$$\Gamma_0^{(1)} = \tilde{\Gamma}_0^{(1)}, \quad \Gamma_0^{(2)} = \tilde{\Gamma}_0^{(2)}, \quad (5.2)$$

Represent the functional $\tilde{\Gamma}_0^{(1)}$ in the form

$$\tilde{\Gamma}_0^{(1)} = \Gamma_{0\theta} + \Gamma_{0\Omega^*} + \Gamma_{0\psi} + \Gamma_{0\varphi} + \Gamma_{0\psi\varphi} + \Gamma_{0\mathcal{A}\mathcal{B}}, \quad (5.3)$$

$$\Gamma_{0\Omega^*} = \Gamma_{0\mathcal{A}^*} + \Gamma_{0C^*} + \Gamma_{0\psi^*} + \Gamma_{0\bar{\psi}^*} + \Gamma_{0\varphi^*}, \quad (5.4)$$

where the following notations useful for further calculations will be used,

$$\Gamma_{0\theta} = \Gamma_{0|6}, \quad \Gamma_{0\mathcal{A}^*} = \Gamma_{0|7} + \Gamma_{0|8}, \quad \Gamma_{0C^*} = \Gamma_{0|9}, \quad (5.5)$$

$$\Gamma_{0\psi^*} = \Gamma_{0|10}, \quad \Gamma_{0\bar{\psi}^*} = \Gamma_{0|11}, \quad \Gamma_{0\varphi^*} = \Gamma_{0|12}, \quad (5.6)$$

$$\Gamma_{0\psi} = \Gamma_{0|13} + \Gamma_{0|14} + \Gamma_{0|15}, \quad (5.7)$$

$$\Gamma_{0\varphi} = \Gamma_{0|16} + \Gamma_{0|17} + \Gamma_{0|18} + \Gamma_{0|19} + \Gamma_{0|20}, \quad (5.8)$$

$$\Gamma_{0\psi\varphi} = \Gamma_{0|21}, \quad (5.8)$$

$$\Gamma_{0\mathcal{A}\mathcal{B}} = \Gamma_{0|22}. \quad (5.9)$$

The functional $\tilde{\Gamma}_0^{(2)}$ has the form

$$\tilde{\Gamma}_0^{(2)} = \Gamma_{0|1} + \Gamma_{0|2} + \Gamma_{0|3} + \Gamma_{0|4} + \Gamma_{0|5}. \quad (5.10)$$

Remind that the functional Γ_0 satisfies the Eq. (2.20)–(2.24).

5.2 (l+1)-loop approximation (order η^{l+1})

The proof of multiplicative renormalizability will be given by the method of mathematical induction in loop expansions of the effective action applying the scheme of minimal subtractions. To this end, let us suppose we have found the parameters $Z_{k_{ind}}$,

$$\begin{aligned} Z_{k_{ind}} &= Z_{k_{ind}}^{[l]} + O(\eta^{l+1}), \\ Z_{k_{ind}}^{[l]} &= 1 + \sum_{n=1}^l \eta^n z_{k_{ind},n}, \quad \forall k_{ind}, \\ \dot{z}_{k_{con},n} &= 0, \quad \forall k_{con}, \quad 1 \leq n \leq l, \end{aligned} \quad (5.11)$$

so that the l -loop approximation for Γ , $\Gamma^{[l]} = \sum_{n=0}^l \eta^n \Gamma_n$, is a finite functional. We will show that the $(l+1)$ -loop

approximation for $Z_{k_{ind}}$ can be picked up so that,

$$\begin{aligned} Z_{k_{ind}} &= Z_{k_{ind}}^{[l+1]} + O(\eta^{l+2}), \\ Z_{k_{ind}}^{[l+1]} &= Z_{k_{ind}}^{[l]} + z_{k_{ind},l+1}, \end{aligned} \quad (5.12)$$

which compensates divergences of the $(l+1)$ -loop approximation of the functional Γ .

Represent the action S_R in the form

$$S_R = S_R^{[l]} + \eta^{l+1} s_{l+1} + O(\eta^{l+2}), \quad (5.13)$$

where $S_R^{[l]}$ is the action S_R with independent parameters $Z_{k_{ind}}$ replaced by $Z_{k_{ind}}^{[l]}$ satisfying the Eqs. (2.20)–(2.24) and functional s_{l+1} is equal to

$$s_{l+1} = s_{l+1}^{(1)} + \chi s_{l+1}^{(2)}. \quad (5.14)$$

In this section we will use the short notations for variational derivatives of the type

$$\frac{\delta}{\delta \mathcal{A}} \rightarrow \partial_{\mathcal{A}}, \quad \mathcal{A} \partial_{\mathcal{A}} = \int dx \mathcal{A} \frac{\delta}{\delta \mathcal{A}}, \quad (5.15)$$

when it will not lead to uncertainties.

Now let us study the structure of the functional Γ with taking into account the $(l+1)$ -loop approximation. It describes by diagrams with vertices of the action S_R with parameters $z_{k_{ind},n}$, $0 \leq n \leq l+1$, $z_{k_{ind},0} = 0$, i.e. with vertices of action $S_R^{[l]}$ and with vertices of s_{l+1} . Because we are interested in diagrams of loop order not higher than $l+1$, the vertices from s_{l+1} cannot appear in loop diagrams, i.e. the vertices from s_{l+1} give "tree" contribution to Γ equal to $\eta^{l+1} s_{l+1}$. The rest diagrams are generated by the action $S_R^{[l]}$. Denote the contribution of these diagrams to functional Γ as $\Gamma(S_R^{[l]})$, i.e.,

$$\Gamma = \Gamma(S_R^{[l]}) + \eta^{l+1} s_{l+1} + O(\eta^{l+2}). \quad (5.16)$$

Because the action $S_R^{[l]}$ satisfies the Eqs. (2.20)–(2.24), the functional $\Gamma(S_R^{[l]})$ satisfies these equations too (with substitutions $Q, \bar{C}, B \rightarrow Q_m, \bar{C}_m, B_m$).

Represent the functional $\Gamma(S_R^{[l]})$ in the form

$$\Gamma(S_R^{[l]}) = \Gamma_{0|0} + \Gamma^{(1)}(S_R^{[l]}) + \chi \Gamma^{(2)}(S_R^{[l]}), \quad (5.17)$$

Repeating calculations made in Sect. 3 we find that

$$\begin{aligned} \Gamma^{(k)}(S_R^{[l]}|Q_m, \bar{C}_m, \mathcal{B}, Q^*, B_m, \xi, \theta) \\ = \tilde{\Gamma}^{(k)}(S_R^{[l]}|\Omega_m, \Omega_m^*, \mathcal{B}, \xi, \theta), \quad k = 1, 2, \end{aligned} \quad (5.18)$$

and functionals $\tilde{\Gamma}^{(k)}(S_R^{[l]})$ satisfy the Eqs. (4.26), (4.27) and (4.30).

Represent the functionals $\tilde{\Gamma}^{(k)}(S_R^{[l]})$ in the sum of divergent and finite (after removing a regularization) terms. Taking into account that the functionals $\tilde{\Gamma}^{(k)}(S_R^{[l]})$ are finite to n -loop

approximations, $0 \leq n \leq l$, by proposition, we obtain

$$\tilde{\Gamma}^{(k)}(S_R^{[l]}) = \tilde{\Gamma}^{(k)}(S_R^{[l]})_{\text{fin}} + \eta^{l+1} \tilde{\Gamma}^{(k)}(S_R^{[l]})_{l+1, \text{div}} + O(\eta^{l+2}). \quad (5.19)$$

To simplify the presentations, we introduce the notations

$$\tilde{\Gamma}^{(k)}(S_R^{[l]})_{l+1, \text{div}} = u_{l+1}^{(k)}, \quad u_{l+1}^{(1)} + \chi u_{l+1}^{(2)} = u_{l+1}, \quad k = 1, 2. \quad (5.20)$$

Then,

$$\Gamma = \Gamma(S_R^{[l]})_{\text{fin}} + \eta^{l+1} (u_{l+1} + s_{l+1}) + O(\eta^{l+2}), \quad (5.21)$$

and functionals $u_{l+1}^{(k)}$ are local polynomials of their arguments with quantum numbers of the action S_{ext} and contain only divergent summands (formalism of minimal subtraction). Then, they satisfy the equations which follow from the Eqs. (4.26), (4.27) and (4.30),

$$\left(\tilde{\Gamma}_0^{(1)}, u_{l+1}^{(1)} \right) - \int dx \left(\theta \frac{\delta}{\delta \mathcal{B}} \right) u_{l+1}^{(1)} = 0, \quad (5.22)$$

$$2\xi \frac{\partial}{\partial \xi} u_{l+1}^{(1)} = (\tilde{\Gamma}_0^{(1)}, u_{l+1}^{(2)}) - (\tilde{\Gamma}_0^{(2)}, u_{l+1}^{(1)}) - \int dx \left(\theta \frac{\delta}{\delta \mathcal{B}} \right) u_{l+1}^{(2)} + \int dx \left(\Omega_m^* \frac{\delta}{\delta \Omega_m^*} - \Omega_m \frac{\delta}{\delta \Omega_m} \right) u_{l+1}^{(1)}, \quad (5.23)$$

$$u_{l+1}^{(k)} \overleftarrow{h}^\alpha \omega_\alpha = 0, \quad (5.24)$$

$$u_{l+1}^{(k)} \overleftarrow{T}^\alpha = 0, \quad k = 1, 2. \quad (5.25)$$

Notice that the form of Eqs. (5.22)–(5.25) does not depend on index l .

Taking into account the quantum numbers, the axial-, Poincare- and T-symmetries we find general expression for local functional $u_{l+1}^{(2)}$,

$$u_{l+1}^{(2)} = \sum_{k=1}^5 q_{k,l+1} \Gamma_{0|k} + q'_{1,l+1} \mathcal{A}^* \mathcal{B}, \quad (5.26)$$

where $q_{k,l+1}, k = 1, \dots, 5$, and $q'_{1,l+1}$ are arbitrary constants. Further, using the Eq. (5.24) for $u_{l+1}^{(2)}$, we find $q'_{1,l+1} = 0$. Final expression for $q'_{1,l+1} = 0$ reads

$$u_{l+1}^{(2)} = \sum_{k=1}^5 q_{k,l+1} \Gamma_{0|k}. \quad (5.27)$$

Notice that the functional $u_{l+1}^{(2)}$ does not depend on the fields θ and $\mathcal{B}$.

Due to the expression (5.27) for $u_{l+1}^{(2)}$, the Eq. (5.23) reduces to

$$2\xi \frac{\partial}{\partial \xi} u_{l+1}^{(1)} = \hat{L}(1 + q_{l+1}) \tilde{\Gamma}_0^{(1)}, \quad (5.28)$$

where the operator $\hat{L}$ is defined in (3.18) with substitution $Z_k \rightarrow 1 + q_{k,l+1}, k = 1, \dots, 5$.

5.2.1 Solution to equation (5.22)

Consider a solution to the Eq. (5.22) for the functional $\tilde{\Gamma}_{l+1, \text{div}}^{(1)} = \tilde{\Gamma}(S_R^{[l]})_{l+1, \text{div}}^{(1)} = u_{l+1}^{(1)}$ using the presentation

$$u_{l+1}^{(1)} = M_{\theta, l+1} + M_{\Omega^*, l+1} + M_{\psi, l+1} + M_{\varphi, l+1} + M_{\psi\varphi, l+1} + M_{\mathcal{AB}, l+1}(\mathcal{A}, \mathcal{B}), \quad M_{\Omega^*, l+1} = M_{\mathcal{A}^*, l+1} + M_{C^*, l+1} + M_{\psi^*, l+1} + M_{\overline{\psi}^*, l+1} + M_{\varphi^*, l+1}. \quad (5.29)$$

To this end, we will find the general form of the functional $\tilde{\Gamma}_{l+1, \text{div}}^{(1)}$ following from locality, quantum numbers, axial-, Poincare-, T -symmetries and, partially, of the gauge symmetry with respect to the external field $\mathcal{B}$. In fact, all needed calculations copy calculations made in Sect. 3 when constructing the general form of the functional $\tilde{P}^{(1)}$ (see formulas (3.21)–(3.30)) with evident substitutions like $\tilde{P}_\theta^{(1)} \rightarrow M_\theta$. Here we give the final results only. The functional $M_{\theta, l+1}$ reads

$$M_{\theta, l+1} = q_{6, l+1} \Gamma_{0|6}. \quad (5.30)$$

For the functionals linear in antifields we find

$$M_{\mathcal{A}^*, l+1} = q_{7, l+1} \Gamma_{0|7} + q_{8, l+1} \Gamma_{0|8}, \quad (5.31)$$

$$M_{C^*, l+1} = q_{9, l+1} \Gamma_{0|9}, \quad (5.32)$$

$$M_{\psi^*, l+1} = q_{10, l+1} \Gamma_{0|10}, \quad M_{\overline{\psi}^*, l+1} = q_{11, l+1} \Gamma_{0|11}, \quad M_{\varphi^*, l+1} = q_{12, l+1} \Gamma_{0|12}. \quad (5.33)$$

For the functionals $M_{\psi, l+1}, M_{\varphi, l+1}, M_{\psi\varphi, l+1}$, we obtain

$$M_{\psi, l+1} = q_{13, l+1} \Gamma_{0|13} + q_{14, l+1} \Gamma_{0|14} + q_{15, l+1} \Gamma_{0|15}, \quad (5.34)$$

$$M_{\varphi, l+1} = q_{16, l+1} \Gamma_{0|16} + q_{17, l+1} \Gamma_{0|17} + M_{\varphi 18, l+1} + q_{19, l+1} \Gamma_{0|19} + q_{20, l+1} \Gamma_{0|20}, \quad (5.35)$$

$$M_{\psi\varphi, l+1} = q_{21, l+1} \Gamma_{0|21}, \quad (5.36)$$

$$M_{\varphi 18, l+1} = \int dx \left[\frac{g^2}{2} q_{18}^{\alpha\beta\gamma\sigma} \mathcal{A}_\mu^\alpha \varphi^\beta \mathcal{A}_\mu^\gamma \varphi^\sigma \right], \quad q_{18}^{\alpha\beta\gamma\sigma} = q_{18}^{\gamma\beta\alpha\sigma} = q_{18}^{\alpha\sigma\gamma\beta}. \quad (5.37)$$

Consider the consequences of the equality to zero for contributions proportional to θ .

$$\begin{aligned}\theta \bar{\psi} \psi &\Rightarrow (q_{6,l+1} + q_{14,l+1}) \partial_{\mathcal{A}_\mu^\alpha} \\ &\times \Gamma_{0|14} - q_{13,l+1} \partial_{\mathcal{B}_\mu^\alpha} \Gamma_{0|13} = 0 \\ &\Rightarrow q_{14,l+1} = q_{13,l+1} - q_{6,l+1};\end{aligned}\quad (5.38)$$

$$\begin{aligned}\theta \mathcal{A}^* C &\Rightarrow (q_{6,l+1} + q_{8,l+1}) \partial_{\mathcal{A}_\mu^\alpha} \\ &\times \Gamma_{0|8} - q_{7,l+1} \partial_{\mathcal{B}_\mu^\alpha} \Gamma_{0|7} = 0 \\ &\Rightarrow q_{8,l+1} = q_{7,l+1} - q_{6,l+1};\end{aligned}\quad (5.39)$$

$$\begin{aligned}\theta \left(D_\varphi(\mathcal{B})\varphi \right) &\Rightarrow (q_{6,l+1} + q_{17,l+1}) \partial_{\mathcal{A}_\mu^\alpha} \\ &\times \Gamma_{0|17} - q_{16,l+1} \partial_{\mathcal{B}_\mu^\alpha} \Gamma_{0|16} = 0 \\ &\Rightarrow q_{17,l+1} = q_{16,l+1} - q_{6,l+1};\end{aligned}\quad (5.40)$$

$$\begin{aligned}\theta \mathcal{A} \varphi &\Rightarrow q_{6,l+1} \partial_{\mathcal{A}_\mu^\alpha} \\ &\times \Gamma_{0|18} + \partial_{\mathcal{A}_\mu^\alpha} M_{\varphi 18,l+1} - q_{17,l+1} \partial_{\mathcal{B}_\mu^\alpha} \Gamma_{0|17} = 0 \\ &\Rightarrow M_{\varphi 18,l+1} \\ &= q_{18,l+1} \Gamma_{0|18}, \quad q_{18,l+1} = q_{16,l+1} - 2q_{6,l+1};\end{aligned}\quad (5.41)$$

$$\begin{aligned}\theta \mathcal{A}^n \mathcal{B}^m &\Rightarrow q_{6,l+1} \partial_{\mathcal{A}_\mu^\alpha} \\ &\times \Gamma_{0|22}(V) + \partial_{\mathcal{A}_\mu^\alpha} M_{\mathcal{A}\mathcal{B},l+1} - \partial_{\mathcal{B}_\mu^\alpha} M_{\mathcal{A}\mathcal{B},l+1} = 0 \\ &\Rightarrow M_{\mathcal{A}\mathcal{B},l+1} = -q_{6,l+1} \mathcal{A} \partial_{\mathcal{A}} \Gamma_{0|22}(V) \\ &+ M'_{\mathcal{A}\mathcal{B},l+1}(V),\end{aligned}\quad (5.42)$$

where $V = \mathcal{A} + \mathcal{B}$ and $M'_{\mathcal{A}\mathcal{B},l+1}(V)$ is an arbitrary functional of V .

When $\theta = 0$, the Eq. (5.22) reduces to

$$\begin{aligned}\int dx (\Gamma_{0\Omega^*} + \Gamma_{0\psi} + \Gamma_{0\varphi} + \Gamma_{0\psi\varphi} + \Gamma_{0\mathcal{A}\mathcal{B}}) \\ \times \left(\overleftarrow{\partial}_{\Omega} \partial_{\Omega^*} - \overleftarrow{\partial}_{\Omega^*} \partial_{\Omega} \right) \\ \times (M_{\Omega^*,l+1} + M_{\psi,l+1} + M_{\varphi,l+1} \\ + M_{\psi\varphi,l+1} + M_{\mathcal{A}\mathcal{B},l+1}) = 0,\end{aligned}\quad (5.43)$$

which is not more than linear in antifields.

We have

$$\begin{aligned}\mathcal{A}^* \mathcal{A} C C &\Rightarrow 2q_{8,l+1} \Gamma_{0|8} \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^\alpha} \frac{\delta}{\delta \mathcal{A}_\mu^{*\alpha}} \\ &\times \Gamma_{0|8} + (q_{8,l+1} + q_{9,l+1}) \Gamma_{0|8} \frac{\overleftarrow{\delta}}{\delta C^\alpha} \frac{\delta}{\delta C^{*\alpha}} \Gamma_{0|9} = 0 \\ &\Rightarrow q_{9,l+1} = q_{8,l+1} = q_{7,l+1} - q_{6,l+1};\end{aligned}\quad (5.44)$$

$$\begin{aligned}\psi^* \psi C C &\Rightarrow (q_{9,l+1} + q_{10,l+1}) \Gamma_{0|10} \frac{\overleftarrow{\delta}}{\delta C^\alpha} \frac{\delta}{\delta C^{*\alpha}} \\ &\times \Gamma_{0|9} + 2q_{10,l+1} \Gamma_{0|10} \frac{\overleftarrow{\delta}}{\delta \psi_j} \frac{\delta}{\delta \psi_j^*} \Gamma_{0|10} = 0 \\ &\Rightarrow q_{10,l+1} = q_{9,l+1} = q_{7,l+1} - q_{6,l+1};\end{aligned}\quad (5.45)$$

$$\bar{\psi} \psi^* C C \Rightarrow (q_{9,l+1} + q_{11,l+1}) \Gamma_{0|11} \frac{\overleftarrow{\delta}}{\delta C^\alpha} \frac{\delta}{\delta C^{*\alpha}}$$

$$\begin{aligned}&\times \Gamma_{0|9} + 2q_{11,l+1} \Gamma_{0|11} \frac{\overleftarrow{\delta}}{\delta \psi_j} \frac{\delta}{\delta \psi_j^*} \Gamma_{0|10} = 0 \\ &\Rightarrow q_{11,l+1} = q_{9,l+1} = q_{7,l+1} - q_{6,l+1};\end{aligned}\quad (5.46)$$

$$\begin{aligned}\bar{\psi} \psi \varphi C &\Rightarrow (q_{12,l+1} + q_{21,l+1}) \Gamma_{0|21} \frac{\overleftarrow{\delta}}{\delta \varphi^\alpha} \frac{\delta}{\delta \varphi^{*\alpha}} \\ &\times \Gamma_{0|12} + (q_{10,l+1} + q_{21,l+1}) \Gamma_{0|21} \frac{\overleftarrow{\delta}}{\delta \psi_j} \frac{\delta}{\delta \psi_j^*} \Gamma_{0|10} \\ &+ (q_{11,l+1} + q_{21,l+1}) \Gamma_{0|21} \frac{\overleftarrow{\delta}}{\delta \psi_j} \frac{\delta}{\delta \psi_j^*} \Gamma_{0|11} = 0 \Rightarrow \\ &\times q_{12,l+1} = q_{10,l+1} = q_{7,l+1} - q_{6,l+1},\end{aligned}\quad (5.47)$$

where the relation $q_{11,l+1} = q_{10,l+1}$ was used.

$$\begin{aligned}\mathcal{A}^n \mathcal{B}^k C &\Rightarrow \Gamma_{0|22} \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^\alpha} \frac{\delta}{\delta \mathcal{A}_\mu^{*\alpha}} (q_{7,l+1} \Gamma_{0|7} + q_{8,l+1} \Gamma_{0|8}) \\ &- (\Gamma_{0|7} + \Gamma_{0|8}) \frac{\overleftarrow{\delta}}{\delta \mathcal{A}_\mu^{*\alpha}} \frac{\delta}{\delta \mathcal{A}_\mu^\alpha} M_{\mathcal{A}\mathcal{B},l+1} = 0 \Rightarrow \\ M'_{\mathcal{A}\mathcal{B},l+1} &= q_{22,l+1} \Gamma_{0|22}(V), \quad M_{\mathcal{A}\mathcal{B},l+1} \\ &= -q_{6,l+1} \mathcal{A} \partial_{\mathcal{A}} \Gamma_{0|22}(V) + q_{22,l+1} \Gamma_{0|22}(V).\end{aligned}\quad (5.48)$$

Thus, the divergences $u_{l+1}^{(1)}$ can be represented in the form

$$u_{l+1}^{(1)} = \sum_{k=6}^{21} q_{k,l+1} \Gamma_{0|k} + M_{\mathcal{A}\mathcal{B},l+1},\quad (5.49)$$

and the functional $M_{\mathcal{A}\mathcal{B},l+1}$ is given by the formula (5.48). As it was noted above, all functionals considered as polynomials are independent, and polynomials $\Gamma_{0|6} - \Gamma_{0|21}$ are homogeneous with respect to the fields Ω and the antifields Ω^* .

5.2.2 Solution to equation (5.28)

Taking into consideration that the polynomials $\Gamma_{0|k}$, $k = 6, \dots, 21$, are eigen for operator $\hat{L}$, we find that Eq. (5.28) is reduced to the set of equations

$$2\xi \frac{\partial}{\partial \xi} q_{k,l+1} = \varrho_k, \quad k = 6, \dots, 21,\quad (5.50)$$

$$2\xi \frac{\partial}{\partial \xi} M_{\mathcal{A}\mathcal{B},l+1} = q_{1,l+1} \mathcal{A} \partial_{\mathcal{A}} \Gamma_{0|22}(V),\quad (5.51)$$

where ϱ_k are the eigenvalues of the operator $\hat{L}$.

We have

$$k = 6 \Rightarrow 2\xi \dot{q}_{6,l+1} = -q_{1,l+1} \Rightarrow q_{1,l+1} = -2\xi \dot{q}_{6,l+1}; \quad (5.52)$$

$$k = 7 \Rightarrow 2\xi \dot{q}_{7,l+1} = -q_{1,l+1} + q_{2,l+1} \Rightarrow q_{2,l+1} = 2\xi \dot{q}_{7,l+1} - 2\xi \dot{q}_{6,l+1}; \quad (5.53)$$

$$k = 13 \Rightarrow 2\xi \dot{q}_{13,l+1} = q_{3,l+1} + q_{4,l+1} \Rightarrow q_{4,l+1} = 2\xi \dot{q}_{13,l+1} - q_{3,l+1}; \quad (5.54)$$

$$k = 15 \Rightarrow 2\xi \dot{q}_{15,l+1} = q_{3,l+1} + q_{4,l+1} \Rightarrow \dot{q}_{23,l+1} = 0, \quad q_{15,l+1} = q_{13,l+1} + q_{23,l+1}; \quad (5.55)$$

$$k = 16 \Rightarrow 2\xi \dot{q}_{16,l+1} = 2q_{5,l+1} \Rightarrow q_{5,l+1} = \xi \dot{q}_{16,l+1}; \quad (5.56)$$

$$k = 19 \Rightarrow 2\xi \dot{q}_{19,l+1} = 2q_{5,l+1} \Rightarrow \dot{q}_{24,l+1} = 0, \quad q_{19,l+1} = q_{16,l+1} + q_{24,l+1}; \quad (5.57)$$

$$k = 20 \Rightarrow 2\xi \dot{q}_{20,l+1} = 4q_{5,l+1} \Rightarrow \dot{q}_{25,l+1} = 0, \quad q_{20,l+1} = 2q_{16,l+1} + q_{25,l+1}; \quad (5.58)$$

$$k = 21 \Rightarrow 2\xi \dot{q}_{21,l+1} = q_{3,l+1} + q_{4,l+1} + q_{5,l+1} \Rightarrow \dot{q}_{26,l+1} = 0, \quad q_{21,l+1} = q_{13,l+1} + \frac{1}{2}q_{16,l+1} + q_{26,l+1}; \quad (5.59)$$

Finally, from (5.51) it follows

$$2\xi \dot{q}_{22,l+1} \Gamma_{0|22} - (2\xi \dot{q}_{6,l+1} + q_{1,l+1}) \mathcal{A} \partial_{\mathcal{A}} \Gamma_{0|22}(V) = 0 \Rightarrow 2\xi \dot{q}_{22,l+1} \Gamma_{0|22} = 0 \Rightarrow \dot{q}_{22,l+1} = 0, \quad (5.60)$$

where we used the result of the block $k = 6$.

Finally, the solution to the set of Eqs. (5.22)–(5.25) for $u_{l+1}^{(k)}$, $k = 1, 2$, can be represented in the form

$$u_{l+1}^{(1)} = \sum_{k=6}^{21} q_{k,l+1} \Gamma_{0|k} - q_{6,l+1} \mathcal{A} \partial_{\mathcal{A}} \Gamma_{0|22}(V) + q_{22,l+1} \Gamma_{0|22}(V), \quad (5.61)$$

$$u_{l+1}^{(2)} = \sum_{k=1}^5 q_{k,l+1} \Gamma_{0|k}. \quad (5.62)$$

Let $\{k_{ind}\} = \{\{K_a\}, 22\}$, $\{K_a\} = \{3, 6, 7, 13, 16, 23, \dots, 26\}$. Then we have

$$q_{k_{dep},l+1} = \sum_a \left([X_{q|k_{dep},a} + Y_{q|k_{dep},a} \xi \partial_{\xi}] q_{K_a,l+1} \right), \quad (5.63)$$

where the set $\{q_{k_{dep},l+1}\} = \{\{q_{K_a,l+1}\}, q_{22}\}$ is the set of arbitrary numbers, 5 of them (with indexes 3, 6, 7, 13, 16) can depend on ξ and $\dot{q}_{22,l+1} = \dot{q}_{23,l+1} = \dot{q}_{24,l+1} = \dot{q}_{25,l+1} = \dot{q}_{26,l+1} = 0$. Numerical matrices $X_{q|k_{dep},a}$ and $Y_{q|k_{dep},a}$ in Eq. (5.63) can be restored by using the relations (5.38)–(5.41), (5.44)–(5.47), (5.52)–(5.59).

5.3 Finiteness of effective action

Now, let us prove that one can choose the renormalization constants in such a way as to make the effective action finite to the $(l+1)$ -loop approximation. To this end, we consider the divergent part of the effective action Γ , $\Gamma_{l+1,\text{div}}$, described by the Eq. (5.21),

$$\Gamma_{l+1,\text{div}} = u_{l+1} + s_{l+1} = u_{l+1}^{(1)} + s_{l+1}^{(1)} + \chi[u_{l+1}^{(2)} + s_{l+1}^{(2)}]. \quad (5.64)$$

It is easy to see that the functional s_{l+1} satisfies the same Eqs. (5.22)–(5.25) as u_{l+1} does (with substitution $u_{l+1}^{(k)} \rightarrow s_{l+1}^{(k)}$, $k = 1, 2$), such that we have

$$s_{l+1}^{(1)} = \sum_{k=6}^{21} z_{k,l+1} \Gamma_{0|k} - z_{6,l+1} \mathcal{A} \partial_{\mathcal{A}} \times \Gamma_{0|22}(V) + z_{22,l+1} \Gamma_{0|22}(V), \quad (5.65)$$

$$s_{l+1}^{(2)} = \sum_{k=1}^5 z_{k,l+1} \Gamma_{0|k}, \quad \dot{z}_{22,l+1} = \dot{z}_{23,l+1} = \dot{z}_{24,l+1} = \dot{z}_{25,l+1} = \dot{z}_{26,l+1} = 0, \quad (5.66)$$

$$z_{k_{dep},l+1} = \sum_a \left([X_{z|k_{dep},a} + Y_{z|k_{dep},a} \xi \partial_{\xi}] z_{K_a,l+1} \right), \quad (5.67)$$

$$X_{z|k_{dep},a} = X_{q|k_{dep},a}, \quad Y_{z|k_{dep},a} = Y_{q|k_{dep},a}. \quad (5.68)$$

Due to (5.61), (5.62), (5.65) and (5.66) the functional $\Gamma_{l+1,\text{div}}$ (5.64) is written in the form

$$\Gamma_{l+1,\text{div}} = \sum_{k=6}^{21} (q_{k,l+1} + z_{k,l+1}) \Gamma_{0|k} - (q_{6,l+1} + z_{6,l+1}) \mathcal{A} \partial_{\mathcal{A}} \Gamma_{0|22}(V) + (q_{22,l+1} + z_{22,l+1}) \Gamma_{0|22}(V) + \chi \sum_{k=1}^5 [(q_{k,l+1} + z_{k,l+1}) \Gamma_{0|k}]. \quad (5.69)$$

Now, let us take $z_{k_{ind},l+1}$ as

$$z_{k_{ind},l+1} = -q_{k_{ind},l+1}. \quad (5.70)$$

Then, due to (5.67), (3.49) and (5.63), we obtain that

$$z_{k_{dep},l+1} = -q_{k_{dep},l+1}, \quad (5.71)$$

and as a consequence

$$\Gamma_{l+1,\text{div}} = 0. \quad (5.72)$$

Thus we have found the parameters $Z_{k_{ind}}$,

$$Z_{k_{ind}} = Z_{k_{ind}}^{[l+1]} + O(\eta^{l+2}), \quad Z_{k_{ind}}^{[l+1]} = Z_{k_{ind}}^{[l]} + z_{k_{ind},l+1}, \quad \dot{Z}_{k_{con}}^{[l+1]} = 0 \quad (5.73)$$

so that the $(l+1)$ -loop approximation for Γ , $\Gamma^{[l+1]} = \sum_{n=0}^{l+1} \eta^n \Gamma_n$, is a finite functional. Note that the parameters

$z_{k_{ind},l+1}$ and $z_{k_{dep},l+1}$ given by Eqs. (5.70) and (5.67), respectively, are defined unambiguously by the divergences.

It is evident that this method works for any l , in particular for $l = 0$, so that by using the loop induction method in Feynman diagrams for the functional Γ , we arrive at the following statement: for the l -loop approximation $\Gamma^{[l]}$ of the functional Γ defined by the relations (4.2), (4.3) with arbitrary positive integer l ,

$$\Gamma^{[l]} = \sum_{n=0}^l \eta^n \Gamma_n, \quad (5.74)$$

there exist the uniquely defined parameters $Z_{k_{ind}}^{[l]}$, and parameters $z_{k_{dep}}^{[l]}$ defining by Eq. (5.67),

$$\dot{Z}_{k_{con}}^{[l]} = 0, \quad (5.75)$$

such that the functional $\Gamma^{[l]}$ does not contain divergences and Γ satisfies the Eqs. (4.5)–(4.8).

6 Gauge independence of physical parameters

In that section, we find relations between some parameters of the action S_R and the standard renormalization constants. Within the expression for S_R , we restrict ourselves only by desired vertexes in symbolic notation

$$S_R = \int dx \left(Z_{22} Z_6^{-2} \partial A \partial A + g Z_{22} Z_6^{-3} A^2 \partial A + Z_{13} \bar{\psi} \partial \psi + m Z_{15} \bar{\psi} \psi + Z_{16} \partial \varphi \partial \varphi + M^2 Z_{19} \varphi^2 + \lambda Z_{20} \varphi^4 + \vartheta Z_{21} \varphi \bar{\psi} \psi \dots \right), \quad (6.1)$$

where the ellipsis means the rest vertexes. As the propagators of the fields A , ψ and φ are finite, they should be considered as renormalized fields. Then, we find:

$$Z_A = Z_{22}^{1/2} Z_6^{-1}, \quad Z_\psi = Z_{13}^{1/2}, \quad Z_\varphi = Z_{16}^{1/2}, \quad (6.2)$$

where Z_A , Z_ψ and Z_φ are the renormalization constants of the bare fields A_0 , ψ_0 and φ_0 . The coefficient of the second vertex in the expression (6.1) gives the renormalization for vertex $g A^2 \partial A$,

$$Z_{g A^2 \partial A} = Z_{22} Z_6^{-3}. \quad (6.3)$$

Analogously,

$$Z_{m \bar{\psi} \psi} = Z_{15}, \quad Z_{M^2 \varphi^2} = Z_{19}, \quad Z_{\lambda \varphi^4} = Z_{20}, \quad Z_{\vartheta \varphi \bar{\psi} \psi} = Z_{21}. \quad (6.4)$$

Then we find

$$g_0 = Z_g g, \quad Z_g = Z_{g A^2 \partial A} Z_A^{-3} = Z_{22}^{-1/2}, \quad (6.5)$$

$$m_0 = Z_m m = Z_{m \bar{\psi} \psi} Z_{13}^{-1} = Z_{23}, \quad (6.6)$$

$$M_0^2 = Z_{M^2} M^2, \quad Z_{M^2} = Z_{19} Z_{16}^{-1} = Z_{24}, \quad (6.7)$$

$$\lambda_0 = Z_\lambda \lambda, \quad Z_\lambda = Z_{20} Z_\varphi^{-4} = Z_{25}, \quad (6.8)$$

$$\vartheta_0 = Z_\vartheta \vartheta, \quad Z_\vartheta = Z_{21} Z_\varphi^{-1/2} Z_\psi^{-1} = Z_{26}. \quad (6.9)$$

It follows from the equations (5.75) that the renormalization constants of physical parameters g , m , M^2 , λ and ϑ do not depend on gauge,

$$\partial_\xi Z_g = \partial_\xi Z_m = \partial_\xi Z_{M^2} = \partial_\xi Z_\lambda = \partial_\xi Z_\vartheta = 0. \quad (6.10)$$

7 Summary

In the present paper, within the background field formalism [21, 22, 24], we have studied the renormalization and the gauge dependence of the $SU(2)$ Yang–Mills theory with the multiplets of massive spinor and scalar fields. The corresponding master-action of the BV-formalism [3, 4] has been extended with the help of additional fermion vector field θ and fermion constant parameter χ . The action introduced is invariant under global supersymmetry and gauge transformations caused by the background vector field $\mathcal{B}$ appearing in the background field formalism. These symmetries allowed one to reduce, at the quantum level, the analysis of the renormalization and the gauge dependence problem for solutions to the extended master-equation and the gauge dependence equation. In comparison with our previous investigations of the multiplicative renormalization of the Yang–Mills theories [34], recent study involves the scalar fields which can be responsible for generating masses to physical particles through the mechanism of spontaneous symmetry breaking [36].

The proofs of multiplicative renormalizability and gauge independence of renormalization constants are based on the possibility to expand the extended effective action in loops, as well as to use the minimal subtraction scheme as to eliminate divergences. In addition, we propose the existence of a regularization preserving the used symmetries. Among the results obtained, we emphasize the rigorous proof of the gauge independence of all the physical parameters of the theory under consideration, to any order of loop expansions.

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