Eur. Phys. J. C (2019) 79:604
https://doi.org/10.1140/epjc/s10052-019-7122-8

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Sterile neutrino shortcuts in asymmetrically warped extra dimensions

Dominik Döring[a], **Heinrich Päs**[b]

Fakultät für Physik, Technische Universität Dortmund, 44221 Dortmund, Germany

Received: 18 February 2019 / Accepted: 8 July 2019 / Published online: 18 July 2019

Abstract Light sterile neutrinos are a popular extension of the Standard Model and are being discussed as a possible explanation for various neutrino oscillation anomalies, including the LSND, MiniBooNE, Reactor and Gallium anomalies. In order to avoid inconsistencies with constraints derived from disappearance experiments and cosmology, altered dispersion relations – which may originate from extra dimensions – have been proposed as a possible solution, dubbed as "neutrino shortcuts in the extra dimension". In this paper we develop a neutrino mass model with an asymmetrically warped extra dimension and two additional gauge singlet neutrinos, one being responsible for neutrino mass generation, while the other one is allowed to propagate in the extra dimension, giving rise to the desired change of the dispersion relation on the brane. By compactifying the extra-dimensional theory on an $S^1/\mathbb{Z}_2$ orbifold, deriving the shape of the Kaluza–Klein tower and identifying the effective sterile neutrino dispersion relation on the brane, we can demonstrate that the earlier, phenomenological models are recovered as the 4-dimensional effective field theory limit of the model discussed here.

1 Introduction

Sterile neutrinos are a common prediction in many neutrino mass models and have been proposed as a possible solution for various neutrino anomalies, hints for inconsistencies in cosmological data, and as a possible dark matter candidate. In particular sterile neutrinos with masses in 1 eV mass range are discussed in the context of the LSND, MiniBooNE, Reactor and Gallium anomalies. There exist, however, stringent constraints on light sterile neutrinos, both from neutrino oscillation experiments as well as from cosmology, which rule out the most simple scenarios. A possible way out of this dilemma is the hypothesis that sterile neutrinos may feature effective Lorentz violating corrections to the standard dispersion relation $E^2 = p^2 + m^2$, which leads to an interesting and rich phenomenology. A particularly attractive realization giving rise to such altered dispersion relations (ADRs) are scenarios where the sterile neutrinos can take shortcuts in extra dimensions. It has been conjectured in the past that this phenomenon arises naturally in models where the sterile neutrino propagates in an asymmetrically warped spacetime [1–6]. Since the ADR in this framework is purely geometrically induced and thus does not differentiate between particles and anti-particles, such scenarios provide an explanation for why there are excesses in both neutrino and anti-neutrino channels in the short baseline (SBL) neutrino anomalies.

Models with large extra dimensions became popular in the in the late 1990s, when it was discovered that the hierarchy problem could be resolved or ameliorated by adopting several flat extra dimensions [7] or one compactified, warped extra dimension as in the well-known 'Randall–Sundrum 1 (RS1)' model [8]. In such theories, typically the SM particle content is located on the 3-brane, while gauge singlets (like the graviton or sterile neutrinos) are allowed to propagate in the extra dimension and therefore experience the associated warping.

While the RS model uses symmetrically warped spacetime of the form $\mathrm{d}s^2 = e^{-2kr\phi}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + r^2\mathrm{d}\phi^2$, asymmetrically warped metrics with the form $\mathrm{d}s^2 = -A^2(\phi)\mathrm{d}t^2 + B^2(\phi)\mathrm{d}\mathbf{x}^2 + C^2(\phi)r^2\mathrm{d}\phi^2$ arise from simple bulk sources and are linked to the cosmological constant problem [9] and the horizon problem [10,11]. Whereas these kind of models preserve Lorentz symmetry in the 5D theory, they predict 4D Lorentz violation on the brane via an altered dispersion relation for sterile neutrinos, which proves to be helpful for the solution of the current anomalies in short baseline and reactor neutrino oscillation data. Scenarios with altered dispersion relations adopt additional terms in the usual relation between energy E and momentum $\mathbf{p}$, $E^2 \neq |\mathbf{p}|^2 + m^2$. Energy dependent elements of the mixing matrix and mass squared

[a] e-mail: dominik.doering@tu-dortmund.de
[b] e-mail: heinrich.paes@tu-dortmund.de

differences can be generated by an additional effective potential in the Hamiltonian in flavor space and thus may pose an explanation for the anomalies encountered in short-baseline neutrino oscillation data such as the LSND [12–16] or MiniBooNE [17,18] anomalies. The excess in SBL experiments can, in this framework, be interpreted as a resonance induced by the extra potential and thus circumvent constraints from atmospheric or accelerator experiments on a standard $3+1\nu$ scenario. For the reactor and gallium experiments, which by themselves favor the $3+1\nu$ scenario, the $3+1\nu$ oscillation probabilities are recovered in the low energy limit far below the resonance [6]. Quite recently the MiniBooNE collaboration has reported an evidence of 4.8σ for new physics beyond the Standard Model which, combined with the LSND experiment, increases to 6.1σ [19]. New efforts to clarify this situation are planned or under development [20,21]). In this paper we thus develop a neutrino mass model featuring an asymmetrically-warped extra dimension which justifies the effective 4-dimensional low-energy "sterile neutrino short-cut" phenomenology proposed in [1–6].

2 An asymmetrically warped neutrino mass model

Orbifolding the fifth dimension on $S^1/\mathbb{Z}_2$ allows to parameterize the extra dimension by an angular coordinate ϕ and an extra dimensional radius r. It also ensures that the ϕ-coordinate satisfies the periodic boundary conditions $\phi = \phi + 2\pi$ and Z_2-symmetry $\phi = -\phi$. Hence, the extra dimension can be entirely described with values for ϕ in the range $0 \leq \phi \leq \pi$. Just like in the RS-model, orbifold fixed points will be populated by 3-branes, corresponding to standard (3+1) Minkowskian spactime parametrized by the coordinates x^μ. For a sufficiently general ansatz we use a metric tensor G_{MN} of the form

$$(G_{MN}) = \begin{pmatrix} -A^2 & & & & \\ & B^2 & & & \\ & & B^2 & & \\ & & & B^2 & \\ & & & & (rC)^2 \end{pmatrix}, \tag{1}$$

where where the metric elements $A = A(\phi)$, $B = B(\phi)$, $C = C(\phi)$ are functions of the extra dimensional angular coordinate ϕ and the latin indices $M, N = 0, 1, 2, 3, 4$ imply a five-dimensional metric, and are chosen to recover Minkowskian space $\eta_{\mu\nu}$ on the 3-branes. This class of non-factorizable spacetimes is called 'asymmetrically warped'. Note that such a metric tensor does not represent a vacuum solution of Einstein's equations, but can be achieved e.g. by introducing simple bulk sources [9]. We adopt a single fermionic SM singlet field Ψ to be allowed to enter the extra dimension, thus the general action for such a Dirac fermion is [22]

$$S = \int \mathrm{d}^4x \int \mathrm{d}\phi \sqrt{\det G} \left\{ E_a^A \left[\frac{i}{2} \overline{\Psi} \gamma^a \left(\partial_A - \overleftarrow{\partial_A} \right) \Psi + \frac{\omega_{bcA}}{8} \overline{\Psi} \left\{ \gamma^a, \sigma^{bc} \right\} \Psi \right] - m\, \mathrm{sgn}(\phi) \overline{\Psi} \Psi \right\}, \tag{2}$$

where E_a^A denotes the inverse Vielbein, ω_{bcA} is the spin connection, $\sigma^{bc} = [\gamma^b, \gamma^c]$ is the commutator of the Dirac matrices and m is the fermion's fundamental Dirac mass. The Vielbein is defined via $G^{MN} = \eta^{mn} E_m^M E_n^N$ as the transformation of a coordinate basis of basis vectors ∂_a into another, equivalent basis $e_A = E_A^a \partial_a$. This allows for a conversion of spacetime indices A to Lorentz indices a in the local tangent space. For this to be true, the Vielbein has to be non-singular. For the metric 1 we obtain $\left(E_a^A\right) = \mathrm{diag}\left(\frac{1}{A}, \frac{1}{B}, \frac{1}{B}, \frac{1}{B}, \frac{1}{rC}\right)$.

The gamma-matrices γ^a obey the Clifford-Algebra $\left\{\gamma^a, \gamma^b\right\} = 2\eta^{ab}$, where η^{ab} is the Minkowski-metric, with $\gamma^{a=4} = i\gamma^5$. The mass term of Eq. (2) contains the sign of the extra dimensional coordinate in order to preserve Z_2-symmetry. It can be shown that the spin connection term $E_a^A \frac{\omega_{bcA}}{8} \overline{\Psi} \left\{\gamma^a, \sigma^{bc}\right\} \Psi$ vanishes so that we are left with

$$S = \int \mathrm{d}^4x \int \mathrm{d}\phi \sqrt{\det G} \left\{ E_a^A \left[\frac{i}{2} \overline{\Psi} \gamma^a \left(\partial_A - \overleftarrow{\partial_A} \right) \Psi \right] - m\, \mathrm{sgn}(\phi) \overline{\Psi} \Psi \right\}. \tag{3}$$

We now decompose the Dirac spinor Ψ using the chiral operator $\Psi_{1/2} = \frac{1 \mp \gamma^5}{2} \Psi$. Note that we do not explicitly call these spinors left- or right-handed, since in five dimensions this concept cannot be applied. This is because γ^5 is part of the Clifford Algebra in 5D and therefore cannot serve as a chiral projector. This is true for any odd dimensional spacetime.

After an integration by parts we obtain expressions for the action, which can be associated with a mass-term and a kinetic-term respectively, because of their spinor structure (for details see appendix A). The expressions yield

$$S_{\mathrm{kin}} = \int \mathrm{d}^4x \int \mathrm{d}\phi \sqrt{\det G} \left\{ \overline{\Psi}_1 i \left[\frac{\gamma^0}{A} \partial_0 + \frac{\gamma^k}{B} \partial_k \right] \Psi_1 + \overline{\Psi}_2 i \left[\frac{\gamma^0}{A} \partial_0 + \frac{\gamma^k}{B} \partial_k \right] \Psi_2 \right\} \tag{4}$$

and

$$S_{\mathrm{mass}} = \int \mathrm{d}^4x \int \mathrm{d}\phi \left\{ - \left[\overline{\Psi}_1 \left(\frac{\sqrt{\det G}}{2rC} \partial_\phi + \partial_\phi \frac{\sqrt{\det G}}{2rC} \right) \Psi_2 - \overline{\Psi}_2 \left(\frac{\sqrt{\det G}}{2rC} \partial_\phi + \partial_\phi \frac{\sqrt{\det G}}{2rC} \right) \Psi_1 \right] - m\, \mathrm{sgn}(\phi) \left[\overline{\Psi}_1 \Psi_2 + \overline{\Psi}_2 \Psi_1 \right] \right\}. \tag{5}$$

We apply a Kaluza–Klein (KK) decomposition, i.e. we expand the 5D spinors $\Psi_{1/2}(x,\phi)$ in a in a series of a product of functions $\psi_n^{1/2}(x)$ and $\hat{f}_n^{1/2}(\phi)$

$$\Psi_{1/2} = \sum_n \psi_n^{1/2}(x) \frac{1}{\sqrt{2r\xi}} \hat{f}_n^{1/2}(\phi) \quad \text{with} \quad \xi = \xi(\phi) = \frac{\sqrt{\det G}}{2rC}, \tag{6}$$

where $\hat{f}_n^{1/2}(\phi)$ will be constructed as eigenfunctions of a Hermitian operator. This operator arises as we compare the decomposed action to the standard Dirac action in 4D. It can be shown that the kinetic part of the action is actually able to recover the 4D Dirac actions kinetic part

$$S_{\text{DiracKin}} = \int \mathrm{d}^4x \left\{ \overline{\psi_n}(x) \left(i\partial\!\!\!/ + \hat{\Omega} \right) \psi_n(x) \right\} \tag{7}$$

up to some correction Ω by choosing the scalar product

$$\int \mathrm{d}\phi \, \hat{f}_n^{1/2} \frac{C}{A} \hat{f}_m^{1/2\dagger} := \delta_{nm}, \tag{8}$$

for the functions $\hat{f}_n^{1/2}(\phi)$. The correction term can be identified as

$$\Omega = \int \mathrm{d}\phi \sum_{mn} \sum_{j=1}^{2} \left[\overline{\psi_n^j} \hat{f}_n^{j\dagger} \frac{C(A-B)}{AB} \, i\gamma^k \partial_k \, \psi_m^j \hat{f}_m^j \right]. \tag{9}$$

While the theory is Lorentz invariant in the full 5D picture, the Ω-term induces Lorentz violation (LV) in the 4D projection after integrating out the extra dimension ϕ. This can lead to a different interplay between the momentum and the energy of a particle on the brane (see e.g. [23–25]). In other words, the operator changes the dispersion relation $E^2 = \mathbf{p}^2 + m^2$ experienced by an observer on the brane. Applying Eq. (8) to the decomposed S_{mass} and matching it to the standard mass term of the Dirac action, we can derive another condition

$$\left(\mp \frac{\partial_\phi}{r} - mC \right) \hat{f}_k^{2/1} = -M_k \frac{C}{A} \hat{f}_k^{1/2}, \tag{10}$$

for the extra dimensional function $\hat{f}_k^{1/2}$. This is a system of first order, coupled, eigenvalue-like equations, which determine the behavior of the extra dimensional function $\hat{f}_{1/2}$ and therefore the shape of the KK spectrum of masses. From this expression we can derive that the shape of the KK tower in the asymmetrically warped case does not differ from symmetric warping scenarios ($\mathrm{d}s^2 = F(\phi)\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + r^2 \mathrm{d}\phi$). This is due to the non-dependence on the metric element $B(\phi)$. In an RS-like set-up, the KK spectrum is shaped like the roots of Bessel's function. The order of this function is determined by the extra dimensional fermion's fundamental mass and the inverse radius of the extra dimension (for further reading see [22]). The only difference between symmetric and asymmetric scenarios is therefore the induced LV on the brane, which is dependent on the difference $A(\phi) - B(\phi)$. Obviously, the LV vanishes in the symmetric limit, recovering the results of [22].

3 Altered dispersion relation and connections to the shortcut parameter

To study the LV on the brane quantitatively, we extract the aforementioned altered dispersion relation (ADR) on the brane from the correction term (Eq. (9)) of the underlying action S. By using Eq. (8), we can express this correction term as

$$S \supset \int \mathrm{d}^4x \sum_{n,m} \sum_{j=1}^{2} \left[\overline{\psi_n^j} \, \tilde{I}_{nm}^j \, i\gamma^k \partial_k \, \psi_m^j \right], \tag{11}$$

where $\tilde{I}_{nm} = \int \mathrm{d}\phi \, \hat{f}_n^{j\dagger} \frac{C}{B} \hat{f}_m^j - \delta_{nm}$ is the correction parameter. To study neutrino oscillation properties, we now introduce an active, brane-bound, lefthanded neutrino state ν_L to the action and consider only the lefthanded zero-mode Ψ_L^0 of the extra dimensional singlet state, without taking into account the Kaluza–Klein excitations (this can be justified by adopting a sufficiently small extra dimension). A righthanded zero-mode is forbidden because of $\mathbb{Z}_2$ symmetry in the $S^1/\mathbb{Z}_2$ orbifolding. To generate the active neutrino masses, we have to introduce another righthanded neutrino N, which is not allowed to propagate in the extra dimension.[1] This righthanded state couples to the active states via tiny Yukawa couplings y_0, whereas the extra dimensional gauge singlet Ψ_L^0 couples to N via dimensionful couplings κ. This way ν_L gets indirectly coupled to the extra dimensional Ψ_L^0. The corresponding, CP conserving action yields

$$S = \int \mathrm{d}^4x \left(\overline{\nu_L}, \overline{\Psi_L^0}, \overline{N} \right) \begin{pmatrix} i\partial\!\!\!/ & 0 & y_0 v \\ 0 & i\partial\!\!\!/ + i\tilde{I}_{00}\partial_k\gamma^k & \kappa \\ y_0 v & \kappa & i\partial\!\!\!/ \end{pmatrix} \times \begin{pmatrix} \nu_L \\ \Psi_L^0 \\ N \end{pmatrix}, \tag{12}$$

[1] In extra-dimensional scenarios like this, assigning gauge charges to fields is a common tool to localize those on a brane [26] In this case, N can, for instance, be in a non-trivial representation of some larger symmetry (e.g. an $SO(10)$ GUT), which is broken at a higher energy scale. After symmetry breaking, N is in the singlet representation of the remaining subgroup $SU(3)_C \times SU(2)_L \times U(1)_Y$. This mechanism fixes N on the brane along with the other SM particles.

where $\tilde{I}_{00}$ is the mode diagonal correction parameter for the zero mode. This parameter can be calculated analytically for the metric $\mathrm{d}s^2 = \mathrm{d}t^2 + \exp(2kr\phi)\mathrm{d}\mathbf{x}^2 + r^2\mathrm{d}\phi^2$ chosen here. We obtain

$$\tilde{I}_{00} = \frac{1 - \exp(-4\pi kr)}{4\pi kr}, \tag{13}$$

in the case where the fundamental Dirac mass m is much smaller than the warping scale parameter k. These parameters have to be chosen this way, since we want the left- and righthanded correction integrals to be approximately equal to one another.

To discuss the dispersion relations, we perform a rotation from the interaction basis $\left(\nu_L\,,\,\Psi_L^0,\,N\right)$ to the propagation basis $\left(\phi\,,\,\chi,\,\xi\right)$. For the propagations eigenstates ϕ, χ and ξ the dispersion relations can be calculated by variation $\delta S = 0$ of the action, leading to the Euler–Lagrange equations for this particular problem (see Appendix C). The solutions for these ADR are given by

$$E_\phi^2 = \mathbf{p}^2, \tag{14}$$

$$E_{\chi/\xi}^2 = \kappa^2 + \mathbf{p}^2 \underbrace{\left[\left(1 + \frac{\tilde{I}_{00}}{2}\right)^2 - \frac{\tilde{I}_{00}^2}{4}\right]}_{f(\tilde{I}_{00})} \tag{15}$$

where $\mathbf{p}$ is the 3-momentum on the brane. As expected, the decoupled dispersion relation (14) does not get affected at zeroth order approximation for $y_0 v$, while the relations for the other two eigenstates are altered by a factor of $f(\tilde{I}_{00})$. This can be interpreted as an altered dispersion relation allowing for sterile neutrino shortcuts in the extra dimension as suggested in the phenomenological approach of [1]. Rearranging Eq. (15) and taking into account that $\tilde{I}_{00} \ll 1$ holds, we can expand the ADR in a Taylor series up to first order in $\tilde{I}_{00}$ additionally to the high energy limit $E \gg \kappa$, yielding

$$p_{\chi/\xi} \approx E - \frac{\kappa^2}{2E} - \frac{E^2}{2E}\tilde{I}_{00} + \mathcal{O}(\tilde{I}_{00}^2) + \mathcal{O}(\kappa^2\tilde{I}_{00}) \tag{16}$$

for two of the propagation eigenstates χ and ξ. The contributions to the mass-squared-difference of the order $\mathcal{O}(\kappa^2\tilde{I}_{00})$ are neglected, since it is only a renormalization of the coupling κ. Therefore we effectively end up with an additional, energy dependent potential $V_+ = E^2\tilde{I}_{00}$, which induces new resonance phenomena in neutrino oscillations. Just as in [1], the potential V_+ has some properties, which are different from the standard matter potential induced from elastic forward scattering of active neutrinos and matter, being non-discriminatory between neutrinos and anti neutrinos and possessing a stronger energy dependence $V_+ \sim E^2$ instead of a linear dependence. The correction term $\tilde{I}_{00}$ and the shortcut parameter ϵ proposed in [1] can be identified, when the mixing between active and sterile states is small. In this context we cannot confirm a hint towards baseline dependence of the resonant behaviour as suggested in [2] and consider it to be an artifact of the semi-classical approach adopted in that work. In order to account for the correct resonance energy while neglecting the effect of heavy KK excitations, one needs to vary the warp factor k and extra dimensional radius r independently. In order to explain the hierarchy problem one might be forced to invoke more than one extra dimension.

4 Conclusion

In this paper we have developed a neutrino mass model giving rise to sterile neutrino shortcuts in an asymmetrically warped extra dimension.

In this context we have derived the shape of the KK tower of an additional fermionic singlet in a general extra dimensional asymmetric warping framework and have demonstrated that this shape does not differ from symmetric warping scenarios (where the warp factors of time and 3-space are the same). The main difference between both warping scenarios is the emergence of effective Lorentz-violation on the 3-brane and a resulting altered dispersion relation of the fermionic singlet and any particles mixing with it.

Moreover, we have developed a concrete mechanism of neutrino mass generation, based on an additional particle content consisting of a gauge singlet neutrino Ψ, which is able to propagate in the extra dimension, and an SM singlet neutrino N confined to the 3-brane. The SM singlet brane neutrino mixes with the gauge singlet Ψ and the active neutrinos ν_L which conveys the effects of asymmetric warping to the active neutrinos, whose masses are generated by Yukawa interaction with the standard Higgs field. We have shown that such a model features an effective potential $V_+ \sim E^2$ during propagation, leading to resonant active-sterile neutrino oscillations.

The sterile neutrino shortcut scenario proposed in [1,3–6] can thus be understood as the four-dimensional effective field theory limit of the model presented here.

Acknowledgements HP thanks Andre de Gouvea for constant encouragement. This work was supported by DFG Grant no. PA 803/10-1.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: There is no data deposited, since this is a theoretical study and no data was generated.]

Appendix A: Calculation of the KK spectrum

From all the terms that are contained in Eq. (3), the ones containing Greek indices are analyzed first. This is because of their similarities regarding their spinor structure $\overline{\Psi}_{1/2}\mathcal{O}(\partial_\mu)\Psi_{1/2}$, making them comparable to a kinetic term. They are

$$S \supseteq S_{\text{kin}} = \int \mathrm{d}^4x \int \mathrm{d}\phi \sqrt{\det G} \left\{ \frac{i}{2A} \left[\overline{\Psi}_1 \gamma^0 \left(\partial_0 - \overleftarrow{\partial_0} \right) \Psi_1 + \overline{\Psi}_2 \gamma^0 \left(\partial_0 - \overleftarrow{\partial_0} \right) \Psi_2 \right] + \frac{i}{2B} \left[\overline{\Psi}_1 \gamma^k \left(\partial_k - \overleftarrow{\partial_k} \right) \Psi_1 + \overline{\Psi}_2 \gamma^k \left(\partial_k - \overleftarrow{\partial_k} \right) \Psi_2 \right] \right\}. \tag{A.1}$$

To convert the left-bound derivatives into standard (right-bound) derivatives, an integration-by-parts is used:

$$\begin{aligned} S_{\text{kin}} = &\int \mathrm{d}^4x \int \mathrm{d}\phi\, i \left\{ +\overline{\Psi}_1 \gamma^0 \frac{\sqrt{\det G}}{2A} \partial_0 \Psi_1 + \overline{\Psi}_2 \gamma^0 \frac{\sqrt{\det G}}{2A} \partial_0 \Psi_2 + \overline{\Psi}_1 \gamma^k \frac{\sqrt{\det G}}{2B} \partial_k \Psi_1 + \overline{\Psi}_2 \gamma^k \frac{\sqrt{\det G}}{2B} \partial_k \Psi_2 \right\} \\ &- \int \mathrm{d}^3x^k \int \mathrm{d}\phi\, i \left\{ \underbrace{\left[\overline{\Psi}_1 \gamma^0 \frac{\sqrt{\det G}}{2A} \Psi_1 \right]_{\partial V}}_{=0} - \underbrace{\left[\overline{\Psi}_2 \gamma^0 \frac{\sqrt{\det G}}{2A} \Psi_2 \right]_{\partial V}}_{=0} \right\} \\ &- \int \mathrm{d}^{4-k}x \int \mathrm{d}\phi\, i \left\{ \underbrace{\left[\overline{\Psi}_1 \gamma^k \frac{\sqrt{\det G}}{2B} \Psi_1 \right]_{\partial V}}_{=0} - \underbrace{\left[\overline{\Psi}_2 \gamma^k \frac{\sqrt{\det G}}{2B} \Psi_2 \right]_{\partial V}}_{=0} \right\} \\ &+ \int \mathrm{d}\phi\, i \left\{ \overline{\Psi}_1 \gamma^0 \partial_0 \frac{\sqrt{\det G}}{2A} \Psi_1 + \overline{\Psi}_2 \gamma^0 \partial_0 \frac{\sqrt{\det G}}{2A} \Psi_2 + \overline{\Psi}_1 \gamma^k \partial_k \frac{\sqrt{\det G}}{2B} \Psi_1 + \overline{\Psi}_2 \gamma^k \partial_k \frac{\sqrt{\det G}}{2B} \Psi_2 \right\}. \end{aligned} \tag{A.2}$$

The terms on the border vanish since it is assumed that all quantum fields vanish in infinity. Since the entries of the metric G are not dependent on the brane coordinates, the derivatives commute with the operator expression $\partial_\mu \sqrt{\det G(\phi)}/2f(\phi) = \sqrt{\det G(\phi)}/2f(\phi)\,\partial_\mu$ and the action can be written as in Eq. (4). This form allows for a 'smooth' KK decomposition.

In the second analysis of the five dimensional action S, the terms containing derivatives with respect to the extra dimension ∂_ϕ are under examination. Due to the configuration of their spinors, these terms are connected to the mass-term. The relevant terms are

$$S \supseteq S_{\text{mass}} = \int \mathrm{d}^4x \int \mathrm{d}\phi \sqrt{\det G} \left\{ -\frac{1}{2rC} \left[\overline{\Psi}_1 \gamma^5 \left(\partial_\phi - \overleftarrow{\partial_\phi} \right) \Psi_2 + \overline{\Psi}_1 \gamma^5 \left(\partial_\phi - \overleftarrow{\partial_\phi} \right) \Psi_2 \right] - m\,\mathrm{sgn}(\phi) \left[\overline{\Psi}_1 \Psi_2 + \overline{\Psi}_2 \Psi_1 \right] \right\}. \tag{A.3}$$

In analogy to the calculations in the terms with indices μ, an integration-by-parts is conducted and using the relationship

$$\overline{\Psi}_{1/2} \gamma^5 \Psi_{2/1} = \pm \overline{\Psi}_{1/2} \Psi_{2/1} \tag{A.4}$$

we arrive at Eq. (5). which can be decomposed via Eq. (6). The kinetic part after decomposition reads

$$S_{\text{kin}} = \int \mathrm{d}^4x \int \mathrm{d}\phi \sum_n \sum_m \left\{ \frac{C}{A} \left[\overline{\psi^1_n} \hat{f}^{1\dagger}_n\, i \left(\partial\!\!\!/ + \frac{(A-B)\gamma^k}{B} \partial_k \right) \psi^1_m \hat{f}^1_m + \overline{\psi^2_n} \hat{f}^{2\dagger}_n\, i \left(\partial\!\!\!/ + \frac{(A-B)\gamma^k}{B} \partial_k \right) \psi^2_m \hat{f}^2_m \right] \right\}, \tag{A.5}$$

from which we can infer the scalar product in Eq. (8) and the correction term (9) by matching it to the corrected Dirac action (7).

With these conditions set, we decompose the mass term of the action and find

$$\begin{aligned} S_{\text{mass}} = \int \mathrm{d}^4x \int \mathrm{d}\phi \sum_n \sum_m \Big\{ &- \Big[\overline{\psi^1_n} \frac{1}{\sqrt{2r\xi}} \hat{f}^{1\dagger}_n \left(\xi\, \partial_\phi + \partial_\phi \xi \right) \psi^2_m \frac{1}{\sqrt{2r\xi}} \hat{f}^2_m \\ &- \overline{\psi^2_n} \frac{1}{\sqrt{2r\xi}} \hat{f}^{2\dagger}_n \left(\xi\, \partial_\phi + \partial_\phi \xi \right) \psi^1_m \frac{1}{\sqrt{2r\xi}} \hat{f}^1_m \Big] \\ &- 2rC\xi\, m \frac{\mathrm{sgn}(\phi)}{2r\xi} \left[\overline{\psi^1_n} \hat{f}^{1\dagger}_n \cdot \psi^2_m \hat{f}^2_m + \overline{\psi^2_n} \hat{f}^{2\dagger}_n \cdot \psi^1_m \hat{f}^1_m \right] \Big\}. \end{aligned} \tag{A.6}$$

Carrying out derivatives leads to terms, which cancel in a nice way because of the carefully chosen ansatz (6) and we obtain

$$\begin{aligned} S_{\text{mass}} = \int \mathrm{d}^4x \int \mathrm{d}\phi \sum_n \sum_m \Big\{ &\overline{\psi^1_n} \hat{f}^{1\dagger}_n \left(-\frac{\partial_\phi}{r} - m\,\mathrm{sgn}(\phi) C \right) \psi^2_m \hat{f}^2_m \\ &+ \overline{\psi^2_n} \hat{f}^{2\dagger}_n \left(+\frac{\partial_\phi}{r} - m\,\mathrm{sgn}(\phi) C \right) \psi^1_m \hat{f}^1_m \Big\}. \end{aligned} \tag{A.7}$$

To make use of the already determined scalar product (8), the functions $\hat{f}^{1/2}_n$ are constructed as eigenfunctions of the

Hermitian operator $\left(\pm\frac{\partial_\phi}{r} - mC'\right)$. This condition is a system of coupled, first order differential equations (10), which fixes the shape of the KK spectrum, while Eq. (8) fixes normalization.

Appendix B: Calculation of the zero mode correction integral

Beginning with Eq. (10), we make the zeroth KK mass M_0 to vanish. This way, the equations decouple and we are left with a simple equation

$$\left(\mp\frac{\partial_\phi}{r} - m\right)\hat{f}_0^{2/1} = 0. \tag{B.8}$$

This equation can be solved via the separation of variables method, yielding

$$\hat{f}_0^{2/1}(\phi) = \hat{K}_0^{2/1}\exp\left(\mp 2rm\phi\right). \tag{B.9}$$

The second condition in Eq. (8), the scalar product of the defined functions, gives us the normalization

$$\int \mathrm{d}\phi\, \hat{f}_0^{2/1}(\phi)\hat{f}_0^{2/1\,\dagger}(\phi) = 1 \tag{B.10}$$

$$\Rightarrow \left(\hat{K}_0^{2/1}\right)^2 = \frac{\mp 2rm}{\exp\left(\mp 4\pi rm\right) - 1} \tag{B.11}$$

Using the definition of the correction integral $\tilde{I}_{00}^{2/1} = \int \mathrm{d}\phi\, \hat{f}_0^{2/1\,\dagger}\frac{C}{B}\hat{f}_0^{2/1} - 1$, we obtain

$$\tilde{I}_{00}^{2/1} = \frac{\frac{m}{k}}{\frac{m}{k}\pm 1}\,\frac{\exp\left(4\pi kr\left(\mp\frac{m}{k} - 1\right)\right) - 1}{\exp\left(\mp 4\pi kr\frac{m}{k}\right) - 1}. \tag{B.12}$$

In the limit of $\frac{m}{k} \to 0$ an equality between $\tilde{I}_{00}^1$ and $\tilde{I}_{00}^2$ is achieved. In this case the correction integral breaks down to the value in Eq. (13).

Appendix C: Calculation of the ADR

Starting from the 4D-Lagrangian $\mathcal{L}_{\text{int}} = \overline{v}\underline{\underline{\mathcal{L}_{\text{int}}}}v$ in Eq. (12) in interaction space, we rotate to propagation space via a unitary transformation U so that

$$\mathcal{L}_{\text{int}} = \left(\overline{\phi},\, \overline{\chi},\, \overline{\xi}\right)\underbrace{U^\dagger \underline{\underline{\mathcal{L}_{\text{int}}}}\, U}_{=:\underline{\underline{\mathcal{L}_{\text{pro}}}}}\begin{pmatrix}\phi\\ \chi\\ \xi\end{pmatrix} \tag{C.13}$$

holds. The eigenvalue equation

$$\det\left(\underline{\underline{\mathcal{L}_{\text{int}}}} - \mathbb{1}_3 \otimes \lambda\right) = 0 \tag{C.14}$$

of the external structure can be written in Fourier space as

$$\begin{aligned}\det\begin{pmatrix}\mathbb{V} & \mathbb{W}\\ \mathbb{X} & \mathbb{Y}\end{pmatrix} &= \det\left[\underbrace{\begin{pmatrix}\mathbb{V} & 0\\ \mathbb{X} & \mathbb{1}\end{pmatrix}}_{:=\mathbb{A}}\underbrace{\begin{pmatrix}\mathbb{1} & \mathbb{V}^{-1}\mathbb{W}\\ 0 & \mathbb{Y} - \mathbb{X}\mathbb{V}^{-1}\mathbb{W}\end{pmatrix}}_{:=\mathbb{B}}\right], \\ &\quad \text{if } \mathbb{V} \text{ is invertible} \\ &= \det\left(\mathbb{A}\right)\det\left(\mathbb{B}\right) \\ &= \det\left(\mathbb{V}\right)\det\left(\mathbb{Y} - \mathbb{X}\mathbb{V}^{-1}\mathbb{W}\right).\end{aligned} \tag{C.15}$$

where

$$\begin{aligned}\mathbb{V} &= \not{p} & \mathbb{W} &= \left(0 \;\; y_0 v\right) \\ \mathbb{X} &= \left(0 \;\; y_0 v\right)^T & \mathbb{Y} &= \begin{pmatrix}\not{p} + \tilde{I}_{00}\, p_k\gamma^k & \kappa\\ \kappa & \not{p}\end{pmatrix}.\end{aligned} \tag{C.16}$$

This leads to the solution

$$\begin{aligned}0 &= \left(\not{p} - \lambda\right)^3 + \tilde{I}_{00}\, p_k\gamma^k\left[\left(\not{p} - \lambda\right)^2 - y_0^2 v^2\right] \\ &\quad - \left[y_0^2 v^2 - \kappa^2\right]\left(\not{p} - \lambda\right)\end{aligned} \tag{C.17}$$

$$\overset{y^2 \ll 1}{\Rightarrow} \lambda \approx \not{p} \vee \lambda \approx \not{p} + \frac{\tilde{I}_{00}\, p_k\gamma^k}{2} \pm \frac{\sqrt{4\kappa^2 + \left(\tilde{I}_{00}\, p_k\gamma^k\right)^2}}{2}. \tag{C.18}$$

These correspond to the eigenvalues of propagation.

References

1. H. Paes, S. Pakvasa, T.J. Weiler, Phys. Rev. D **72**, 095017 (2005). https://doi.org/10.1103/PhysRevD.72.095017
2. S. Hollenberg, O. Micu, H. Pas, T.J. Weiler, Phys. Rev. D **80**, 093005 (2009). https://doi.org/10.1103/PhysRevD.80.093005
3. E. Aeikens, H. Päs, S. Pakvasa, T.J. Weiler, Phys. Rev. D **94**(11), 113010 (2016). https://doi.org/10.1103/PhysRevD.94.113010
4. E. Aeikens, H. Päs, S. Pakvasa, P. Sicking, JCAP **1510**(10), 005 (2015). https://doi.org/10.1088/1475-7516/2015/10/005
5. D. Marfatia, H. Pas, S. Pakvasa, T.J. Weiler, Phys. Lett. B **707**, 553 (2012). https://doi.org/10.1016/j.physletb.2012.01.028
6. D. Döring, H. Päs, P. Sicking, T.J. Weiler (2018)
7. N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B **429**, 263 (1998). https://doi.org/10.1016/S0370-2693(98)00466-3
8. L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999). https://doi.org/10.1103/PhysRevLett.83.3370
9. C. Csaki, J. Erlich, C. Grojean, Nucl. Phys. B **604**, 312 (2001). https://doi.org/10.1016/S0550-3213(01)00175-4
10. D.J.H. Chung, K. Freese, Phys. Rev. D **62**, 063513 (2000). https://doi.org/10.1103/PhysRevD.62.063513

11. D.J.H. Chung, K. Freese, Phys. Rev. D **61**, 023511 (2000). https://doi.org/10.1103/PhysRevD.61.023511
12. C. Athanassopoulos et al., Phys. Rev. Lett. **75**, 2650 (1995). https://doi.org/10.1103/PhysRevLett.75.2650
13. C. Athanassopoulos et al., Phys. Rev. Lett. **77**, 3082 (1996). https://doi.org/10.1103/PhysRevLett.77.3082
14. C. Athanassopoulos et al., Phys. Rev. Lett. **81**, 1774 (1998). https://doi.org/10.1103/PhysRevLett.81.1774
15. C. Athanassopoulos et al., Phys. Rev. C **58**, 2489 (1998). https://doi.org/10.1103/PhysRevC.58.2489
16. A. Aguilar-Arevalo et al., Phys. Rev. D **64**, 112007 (2001). https://doi.org/10.1103/PhysRevD.64.112007
17. A.A. Aguilar-Arevalo et al. (2012). http://lss.fnal.gov/archive/2012/pub/fermilab-pub-12-394-ad-ppd.pdf. Accessed 15 July 2019
18. AAea Aguilar-Arevalo, Phys. Rev. Lett. **110**, 161801 (2013). https://doi.org/10.1103/PhysRevLett.110.161801
19. A.A. Aguilar-Arevalo et al., Phys. Rev. D **64**, 112007 (2001). https://doi.org/10.1103/PhysRevD.64.112007
20. M. Antonello et al. (2015). arXiv:1503.01520
21. K. Terao, J.P.S. Conf, Proc. **8**, 023014 (2015). https://doi.org/10.7566/JPSCP.8.023014
22. Y. Grossman, M. Neubert, Phys. Lett. B **474**, 361 (2000). https://doi.org/10.1016/S0370-2693(00)00054-X
23. V.A. Kostelecky, M. Mewes, Phys. Rev. D **70**, 076002 (2004). https://doi.org/10.1103/PhysRevD.70.076002
24. A. Kostelecky, M. Mewes, Phys. Rev. D **85**, 096005 (2012). https://doi.org/10.1103/PhysRevD.85.096005
25. V. Antonelli, L. Miramonti, M.D.C. Torri (2018), Eur. Phys. J. C 78, 667 (2018). https://doi.org/10.1140/epjc/s10052-018-6124-2
26. A. Donini, S. Rigolin, Nucl. Phys. B **550**, 59 (1999). https://doi.org/10.1016/S0550-3213(99)00207-2