Investigating tetraquarks composed of $us\bar{d}\bar{b}$ and $ud\bar{s}\bar{b}$

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Abstract In the framework of the quark delocalization color screening model, we investigate tetraquarks composed of $us\bar{d}\bar{b}$ and $ud\bar{s}\bar{b}$ in two structures: meson-meson structure and diquark–antidiquark structure. Neither bound state nor resonance state is found in the system composed of $us\bar{d}\bar{b}$. The reported $X(5568)$ cannot be explained as a molecular state or a diquark–antidiquark resonance of $us\bar{d}\bar{b}$ in present calculation. However, two bound states of the diquark–antidiquark structure are obtained in the tetraquarks system composed of $ud\bar{s}\bar{b}$: an $IJ = 00$ state with the mass of 5701 MeV, and an $IJ = 01$ state with the mass of 5756 MeV, which maybe the better tetraquark states with four different flavors. Our results indicate that the diquark–antidiquark configuration would be a good choice for the tetraquarks $ud\bar{s}\bar{b}$ with quantum numbers $IJ = 00$ and $IJ = 01$. The tetraquarks composed of $ud\bar{s}\bar{b}$ is more possible to form bound states than the one composed of $us\bar{d}\bar{b}$. These bound states are worth investigating in future experiments.

1 Introduction

In the past few decades, the discovery of numbers of exotic states stimulated extensive interest in understanding the structures of the multi-quark hadrons. So far, most tetraquark and pentaquark candidates are composed of hidden charm or bottom quarks. However, the new state $X(5568)$ observed by the D0 collaboration in 2016 [1] was an exception. The $X(5568)$ has a mass $m = 5567.8 \pm 2.9(stat)^{+0.9}_{-1.9}(syst)$ MeV and width $\Gamma = 21.9 \pm 6.4(stat)^{+5.0}_{-2.5}(syst)$ MeV [1]. The decay mode is $X(5568) \rightarrow B_s^0 \pi^\pm$, which indicates that the quark component of the $X(5568)$ should be four different flavors: u, d, s, b . Therefore, the claimed $X(5568)$, if confirmed, would differ from any of the previous observations, as it must be a tetraquark state with $us\bar{d}\bar{b}$ or $ds\bar{u}\bar{b}$ and their

charge-conjugated ones. Unfortunately, this state was not confirmed by other collaborations. The LHCb collaboration [2], the CMS collaboration of LHC [3], the CDF collaboration of Fermilab [4] and the ATLAS Collaboration of LHC [5] all claimed that no evidence for this state was found. Nevertheless, the D0 collaboration's new result still insists on the existence of this tetraquark $X(5568)$ [6]. Clearly, more other measurements are needed.

The discovery of this exotic state $X(5568)$ also stimulates the theoretical interest. Many approaches have been applied to interpret this state, such as the QCD sum rules [7–13], quark models [14–16], the extended light front model [17], rescattering effects [18], and so on. However, several theoretical calculations gave the negative results [19–22]. For example, in Ref. [19], authors investigated two structures, diquark–antidiquark and meson-meson, with all possible color configurations by using the Gaussian expansion method, and they cannot obtain the reported $X(5568)$. Ref. [20] examined the various interpretations of the state $X(5568)$ and found that the threshold, cusp, molecular, and tetraquark models were all unfavored the existence of the $X(5568)$.

To search for the tetraquark states with four different flavors, a better state is $ud\bar{s}\bar{b}$ (or its charge-conjugated one) with replacing the $d/\bar{s}$ in $X(5568)$ by $\bar{d}/s$ [23]. Obviously, such state is a partner of $X(5568)$ under the $SU(3)$ flavor symmetry, and their masses are close to each other. But the threshold of $ud\bar{s}\bar{b}$ is BK , 270 MeV higher than the threshold $B_s \pi$ of $X(5568)$ with $us\bar{d}\bar{b}$. So there is large mass region for this $ud\bar{s}\bar{b}$ state below threshold and being stable. Besides, Ref. [23] pointed out that if the lowest-lying $ud\bar{s}\bar{b}$ state exists below threshold, it can be definitely observed via the weak decay mode $J/\psi K^- K^+ \pi^+$, with the expectation of hundreds of events in the current LHCb data sample but rejecting backgrounds due to its long lifetime. Therefore, the $ud\bar{s}\bar{b}$ state would be a more promising detectable tetraquark state. Ref. [24] investigated such state composed of $ud\bar{s}\bar{b}$ within the chiral quark model, and found the bound state with $IJ^P = 00^+$ was possible. Liu et al. also proposed

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several partner states of $X(5568)$ and estimated the mass difference of these partner states based on the color-magnetic interaction [25], which can provide valuable information on the future experimental search of these states.

It is generally known that quantum chromodynamics (QCD) is the fundamental theory of the strong interaction. Understanding the low-energy behavior of QCD and the nature of the strong interacting matter, however, remains a challenge due to the complexity of QCD. Lattice QCD has provided numerical results describing quark confinement between two static colorful quarks, a preliminary picture of the QCD vacuum and the internal structure of hadrons in addition to a phase transition of strongly interacting matter. But a satisfying description of multi-quark system is out of reach of the present calculation. The QCD-inspired models, incorporating the properties of low-energy QCD: color confinement and chiral symmetry breaking, are also powerful tools to obtain physical insights into many phenomena of the hadronic world. Among many phenomenological models, the quark delocalization color screening model (QDCSM), which was developed in the 1990s with the aim of explaining the similarities between nuclear (hadronic clusters of quarks) and molecular forces [26], has been quite successful in reproducing the energies of the baryon ground states, the properties of deuteron, the nucleon-nucleon (NN) and the hyperon-nucleon (YN) interactions [27–31]. Recently, this model has been used to study the pentaquarks with hidden-strange [32], hidden-charm and hidden-bottom [33]. Therefore, it is interesting to extend this model to the tetraquark system. In present work, the tetraquark state $X(5568)$ with quark contents $us\bar{d}\bar{b}$ and its partner state with $ud\bar{s}\bar{b}$ are investigated. Besides, two structures, meson-meson and diquark-antidiquark, are considered in this work.

The structure of this paper is as follows. A brief introduction of the quark model and wave functions is given in section 2. Section 3 is devoted to the numerical results and discussions. The summary is shown in the last section.

2 Model and wave functions

QDCSM has been described in detail in the literatures [26–31]. Here, we just present the salient features of the model. The Hamiltonian for the tetraquark states is shown below:

$$H = \sum_{i=1}^4 \left(m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1}^4 \left(V_{ij}^{CON} + V_{ij}^{OGE} + V_{ij}^{OBE} \right), \quad (1)$$

$$V_{ij}^{CON} = -a_c \lambda_i^c \cdot \lambda_j^c (f(r_{ij}) + a_{ij}^0), \quad (2)$$

$$f(r_{ij}) = \begin{cases} r_{ij}^2, & i, j \text{ in the same baryon orbit} \\ \frac{1-e^{-\mu_{ij}r_{ij}^2}}{\mu_{ij}}, & \text{otherwise} \end{cases} \quad (3)$$

$$V_{ij}^{OGE} = \frac{1}{4} \alpha_s \lambda_i^c \cdot \lambda_j^c \left[\frac{1}{r_{ij}} - \frac{\pi}{2} \delta r_{ij} \right] \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\sigma_i \cdot \sigma_j}{3m_i m_j} \right) - \frac{3}{4m_i m_j r_{ij}^3} S_{ij} \quad (4)$$

$$V_{ij}^{OBE} = V_\pi(r_{ij}) \sum_{a=1}^3 \lambda_i^a \cdot \lambda_j^a + V_K(r_{ij}) \sum_{a=4}^7 \lambda_i^a \cdot \lambda_j^a + V_\eta(r_{ij}) \left[\lambda_i^8 \cdot \lambda_j^8 \cos \theta_P - \lambda_i^0 \cdot \lambda_j^0 \sin \theta_P \right] \quad (5)$$

$$V_\chi(r_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_\chi^2}{12m_i m_j} \frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2} m_\chi \left\{ \sigma_i \cdot \sigma_j \left[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} Y(\Lambda_\chi r_{ij}) \right] + \left[H(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} H(\Lambda_\chi r_{ij}) \right] S_{ij} \right\}, \quad \chi = \pi, K, \eta, \quad (6)$$

$$S_{ij} = \left\{ 3 \frac{(\sigma_i \cdot r_{ij})(\sigma_j \cdot r_{ij})}{r_{ij}^2} - \sigma_i \cdot \sigma_j \right\}, \quad (7)$$

$$H(x) = (1 + 3/x + 3/x^2)Y(x), \quad Y(x) = e^{-x}/x. \quad (8)$$

where S_{ij} is quark tensor operator; $Y(x)$ and $H(x)$ are standard Yukawa functions; T_c is the kinetic energy of the center of mass; α_s is the quark-gluon coupling constant; g_{ch} is the coupling constant for chiral field, which is determined from the $NN\pi$ coupling constant through

$$\frac{g_{ch}^2}{4\pi} = \left(\frac{3}{5} \right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2}. \quad (9)$$

The other symbols in the above expressions have their usual meanings. All model parameters are determined by fitting the meson spectrum we used in this work and shown in Table 1. The calculated masses of the mesons in comparison with experimental values are shown in Table 2. Besides, a phenomenological color screening confinement potential is used here, and μ_{ij} is the color screening parameter, which is determined by fitting the deuteron properties, NN scattering phase shifts, $N\Lambda$ and $N\Sigma$ scattering phase shifts, respectively, with $\mu_{uu} = 0.45 \text{ fm}^{-2}$, $\mu_{us} = 0.19 \text{ fm}^{-2}$ and $\mu_{ss} = 0.08 \text{ fm}^{-2}$, satisfying the relation, $\mu_{us}^2 = \mu_{uu}\mu_{ss}$ [34]. When extending to the heavy bottom quark case, there is no experimental data available, so we take it as an adjustable parameter $\mu_{bb} = 0.001 \sim 0.0001 \text{ fm}^{-2}$. We find the results are insensitive to the value of μ_{bb} . So in the present work, we take $\mu_{bb} = 0.001 \text{ fm}^{-2}$.

The quark delocalization in QDCSM is realized by specifying the single particle orbital wave function of QDCSM

Table 1 Model parameters: $m_\pi = 0.7 \text{ fm}^{-1}$, $m_K = 2.51 \text{ fm}^{-1}$, $m_\eta = 2.77 \text{ fm}^{-1}$, $\Lambda_\pi = 4.2 \text{ fm}^{-1}$, $\Lambda_K = \Lambda_\eta = 5.2 \text{ fm}^{-1}$, $g_{ch}^2/(4\pi) = 0.54$, $\theta_p = -15^\circ$

b (fm)	m_u (MeV)	m_d (MeV)	m_s (MeV)	m_b (MeV)
0.518	313	313	470	4500
a_c (MeV fm ⁻²)	a_{uu}^0 (fm ²)	a_{us}^0 (fm ²)	a_{ub}^0 (fm ²)	a_{sb}^0 (fm ²)
58.03	-0.733	-0.309	1.701	1.808
α_{suu}	α_{sus}	α_{sub}	α_{sbs}	
1.50	1.46	1.41	1.40	

Table 2 The masses (in MeV) of the mesons obtained from QDCSM. Experimental values are taken from the Particle Data Group (PDG) [35]

Meson	M_{theo}	M_{exp}
π	140	140
ρ	772	770
K	495	495
K^*	892	892
B	5280	5280
B^*	5319	5325
B_s	5367	5367
B_s^*	5393	5415

as a linear combination of left and right Gaussians, the single particle orbital wave functions used in the ordinary quark cluster model,

$$\begin{aligned}
 \psi_\alpha(\mathbf{s}_i, \epsilon) &= (\phi_\alpha(\mathbf{s}_i) + \epsilon \phi_\alpha(-\mathbf{s}_i)) / N(\epsilon), \\
 \psi_\beta(-\mathbf{s}_i, \epsilon) &= (\phi_\beta(-\mathbf{s}_i) + \epsilon \phi_\beta(\mathbf{s}_i)) / N(\epsilon), \\
 N(\epsilon) &= \sqrt{1 + \epsilon^2 + 2\epsilon e^{-s_i^2/4b^2}}, \\
 \phi_\alpha(\mathbf{s}_i) &= \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2}(\mathbf{r}_\alpha - \mathbf{s}_i/2)^2} \\
 \phi_\beta(-\mathbf{s}_i) &= \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2}(\mathbf{r}_\beta + \mathbf{s}_i/2)^2}.
 \end{aligned} \quad (10)$$

Here $\mathbf{s}_i$, $i = 1, 2, \dots, n$ are the generating coordinates, which are introduced to expand the relative motion wavefunction [27–31]. The mixing parameter $\epsilon(\mathbf{s}_i)$ is not an adjusted one but determined variationally by the dynamics of the multi-quark system itself. In this way, the multi-quark system chooses its favorable configuration in the interacting process. This mechanism has been used to explain the cross-over transition between hadron phase and quark-gluon plasma phase [36].

In this work, the resonating group method (RGM) [37], a well-established method for studying a bound-state or a scattering problem, is used to calculate the energy of all these

states. The wave function of the four-quark system is of the form

$$\Psi = \mathcal{A} \left[[\psi^L \psi^\sigma]_{JM} \psi^f \psi^c \right]. \quad (11)$$

where ψ^L , ψ^σ , ψ^f , and ψ^c are the orbital, spin, flavor and color wave functions, respectively, which are given below. The symbol $\mathcal{A}$ is the anti-symmetrization operator. For the meson-meson structure, $\mathcal{A}$ is defined as

$$\mathcal{A} = 1 - P_{13}. \quad (12)$$

where 1 and 3 stand for the light quarks in two meson clusters respectively; for the diquark–antidiquark structure, $\mathcal{A} = 1$.

The orbital wave function is in the form of

$$\psi^L = \psi_1(\mathbf{R}_1) \psi_2(\mathbf{R}_2) \chi_L(\mathbf{R}). \quad (13)$$

where $\mathbf{R}_1$ and $\mathbf{R}_2$ are the internal coordinates for the cluster 1 and cluster 2. $\mathbf{R} = \mathbf{R}_1 - \mathbf{R}_2$ is the relative coordinate between the two clusters 1 and 2. The ψ_1 and ψ_2 are the internal cluster orbital wave functions of the clusters 1 and 2, and $\chi_L(\mathbf{R})$ is the relative motion wave function between two clusters, which is expanded by gaussian bases

$$\begin{aligned}
 \chi_L(\mathbf{R}) &= \frac{1}{\sqrt{4\pi}} \left(\frac{3}{2\pi b^2} \right) \sum_{i=1}^n C_i \\
 &\times \int \exp \left[-\frac{3}{4b^2} (\mathbf{R} - \mathbf{s}_i)^2 \right] Y_{LM}(\hat{\mathbf{s}}_i) d\hat{\mathbf{s}}_i.
 \end{aligned} \quad (14)$$

where $\mathbf{s}_i$ is called the generate coordinate, n is the number of the gaussian bases, which is determined by the stability of the results. By doing this, the integro-differential equation of RGM can be reduced to an algebraic equation, generalized eigen-equation. Then the energy of the system can be obtained by solving this generalized eigen-equation. The details of solving the RGM equation can be found in Ref. [37]. In our calculation, the maximum generating coordinate s_n is fixed by the stability of the results. The calculated results are stable when the distance between the two clusters is larger than 6 fm. To keep the dimensions of matrix manageable small, the two clusters' separation is taken to be less than 6 fm.

The flavor, spin, and color wave functions are constructed in two steps. First constructing the wave functions for clusters 1 and 2, then coupling the two wave functions of two clusters to form the wave function for tetraquark system. For the meson-meson structure, as the first step, we give the wave functions of the meson cluster. The flavor wave functions of the meson cluster are shown below.

$$\begin{aligned}
 \chi_{I_{11}}^1 &= u\bar{d}, & \chi_{I_{\frac{1}{2}\frac{1}{2}}}^2 &= s\bar{d}, & \chi_{I_{\frac{1}{2}\frac{1}{2}}}^3 &= u\bar{b}, & \chi_{I_{00}}^4 &= s\bar{b}, \\
 \chi_{I_{\frac{1}{2}\frac{1}{2}}}^5 &= u\bar{s}, & \chi_{I_{\frac{1}{2}-\frac{1}{2}}}^6 &= d\bar{s}, & \chi_{I_{\frac{1}{2}-\frac{1}{2}}}^7 &= d\bar{b}.
 \end{aligned} \quad (15)$$

where the superscript of the χ is the index of the flavor wave function for a meson, and the subscript stands for the isospin I and the third component I_z . The spin wave functions of the meson cluster are:

$$\begin{aligned}\chi_{\sigma_{11}}^1 &= \alpha\alpha, & \chi_{\sigma_{10}}^2 &= \sqrt{\frac{1}{2}}(\alpha\beta + \beta\alpha), \\ \chi_{\sigma_{1-1}}^3 &= \beta\beta, & \chi_{\sigma_{00}}^4 &= \sqrt{\frac{1}{2}}(\alpha\beta - \beta\alpha).\end{aligned}\quad (16)$$

and the color wave function of a meson is:

$$\chi_{[111]}^1 = \sqrt{\frac{1}{3}}(r\bar{r} + g\bar{g} + b\bar{b}). \quad (17)$$

Then, the wave functions for the four-quark system with the meson-meson structure can be obtained by coupling the wave functions of two meson clusters. Every part of wave functions are shown below. The flavor wave functions are:

$$\begin{aligned}\psi_{11}^{f_1} &= \chi_{I_{00}}^4 \chi_{I_{11}}^1, & \psi_{11}^{f_2} &= \chi_{I_{\frac{1}{2}\frac{1}{2}}}^3 \chi_{I_{\frac{1}{2}\frac{1}{2}}}^2, \\ \psi_{00}^{f_3} &= \sqrt{\frac{1}{2}} \left[\chi_{I_{\frac{1}{2}\frac{1}{2}}}^5 \chi_{I_{\frac{1}{2}-\frac{1}{2}}}^7 - \chi_{I_{\frac{1}{2}-\frac{1}{2}}}^7 \chi_{I_{\frac{1}{2}\frac{1}{2}}}^5 \right], \\ \psi_{11}^{f_4} &= \sqrt{\frac{1}{2}} \left[\chi_{I_{\frac{1}{2}\frac{1}{2}}}^5 \chi_{I_{\frac{1}{2}-\frac{1}{2}}}^7 + \chi_{I_{\frac{1}{2}-\frac{1}{2}}}^7 \chi_{I_{\frac{1}{2}\frac{1}{2}}}^5 \right].\end{aligned}\quad (18)$$

The spin wave functions are:

$$\begin{aligned}\psi_{00}^{\sigma_1} &= \chi_{\sigma_{00}}^4 \chi_{\sigma_{00}}^4, \\ \psi_{00}^{\sigma_2} &= \sqrt{\frac{1}{3}} \left[\chi_{\sigma_{11}}^1 \chi_{\sigma_{1-1}}^3 - \chi_{\sigma_{10}}^2 \chi_{\sigma_{10}}^2 + \chi_{\sigma_{1-1}}^3 \chi_{\sigma_{11}}^1 \right], \\ \psi_{11}^{\sigma_3} &= \chi_{\sigma_{00}}^4 \chi_{\sigma_{11}}^1, & \psi_{11}^{\sigma_4} &= \chi_{\sigma_{11}}^1 \chi_{\sigma_{00}}^4, \\ \psi_{11}^{\sigma_5} &= \sqrt{\frac{1}{2}} \left[\chi_{\sigma_{11}}^1 \chi_{\sigma_{10}}^2 - \chi_{\sigma_{10}}^2 \chi_{\sigma_{11}}^1 \right].\end{aligned}\quad (19)$$

The color wave function is:

$$\psi^{c_1} = \chi_{[111]}^1 \chi_{[111]}^1. \quad (20)$$

Finally, multiplying the wave functions ψ^L , ψ^σ , ψ^f , and ψ^c according to the definite quantum number of the system, we can acquire the total wave functions of the system.

For the diquark-antidiquark structure, the orbital and the spin wave functions are the same with those of the meson-meson structure. For the flavor wave functions, we give the functions of the diquark and antidiquark clusters firstly.

$$\begin{aligned}\chi_{I_{10}}^1 &= \frac{1}{\sqrt{2}}(ud + du), & \chi_{I_{00}}^2 &= \frac{1}{\sqrt{2}}(ud - du), \\ \chi_{I_{\frac{1}{2}\frac{1}{2}}}^3 &= \frac{1}{\sqrt{2}}(us + su), & \chi_{I_{\frac{1}{2}\frac{1}{2}}}^4 &= \frac{1}{\sqrt{2}}(us - su), \\ \chi_{I_{\frac{1}{2}\frac{1}{2}}}^5 &= \bar{d}\bar{b}, & \chi_{I_{00}}^6 &= \bar{s}\bar{b}.\end{aligned}\quad (21)$$

Then, the color wave functions of the diquark clusters are:

$$\chi_{[2]}^1 = rr, \quad \chi_{[2]}^2 = \frac{1}{\sqrt{2}}(rg + gr), \quad \chi_{[2]}^3 = gg,$$

$$\begin{aligned}\chi_{[2]}^4 &= \frac{1}{\sqrt{2}}(rb + br), & \chi_{[2]}^5 &= \frac{1}{\sqrt{2}}(gb + bg), \\ \chi_{[2]}^6 &= bb, & \chi_{[11]}^7 &= \frac{1}{\sqrt{2}}(rg - gr), \\ \chi_{[11]}^8 &= \frac{1}{\sqrt{2}}(rb - br), & \chi_{[11]}^9 &= \frac{1}{\sqrt{2}}(gb - bg).\end{aligned}\quad (22)$$

and the color wave functions of the antidiquark clusters are:

$$\begin{aligned}\chi_{[22]}^1 &= \bar{r}\bar{r}, & \chi_{[22]}^2 &= -\frac{1}{\sqrt{2}}(\bar{r}\bar{g} + \bar{g}\bar{r}), & \chi_{[22]}^3 &= \bar{g}\bar{g}, \\ \chi_{[22]}^4 &= \frac{1}{\sqrt{22}}(\bar{r}\bar{b} + \bar{b}\bar{r}), & \chi_{[22]}^5 &= -\frac{1}{\sqrt{2}}(\bar{g}\bar{b} + \bar{b}\bar{g}), \\ \chi_{[22]}^6 &= \bar{b}\bar{b}, & \chi_{[211]}^7 &= \frac{1}{\sqrt{2}}(\bar{r}\bar{g} - \bar{g}\bar{r}), \\ \chi_{[211]}^8 &= -\frac{1}{\sqrt{2}}(\bar{r}\bar{b} - \bar{b}\bar{r}), & \chi_{[211]}^9 &= \frac{1}{\sqrt{2}}(\bar{g}\bar{b} - \bar{b}\bar{g}).\end{aligned}\quad (23)$$

After that, the wave functions for the four-quark system with the diquark-antidiquark structure can be obtained by coupling the wave functions of two clusters. Every part of wave functions are shown below. The flavor wave functions are:

$$\begin{aligned}\psi_{11}^{f_1} &= \chi_{I_{\frac{1}{2}\frac{1}{2}}}^3 \chi_{I_{\frac{1}{2}\frac{1}{2}}}^5, & \psi_{11}^{f_2} &= \chi_{I_{\frac{1}{2}\frac{1}{2}}}^4 \chi_{I_{\frac{1}{2}\frac{1}{2}}}^5, \\ \psi_{11}^{f_3} &= \chi_{I_{11}}^1 \chi_{I_{00}}^6, & \psi_{00}^{f_4} &= \chi_{I_{00}}^2 \chi_{I_{00}}^6.\end{aligned}\quad (24)$$

The color wave functions are:

$$\begin{aligned}\psi^{c_1} &= \sqrt{\frac{1}{6}} [\chi_{[2]}^1 \chi_{[22]}^1 - \chi_{[2]}^2 \chi_{[22]}^2 + \chi_{[2]}^3 \chi_{[22]}^3 \\ &\quad + \chi_{[2]}^4 \chi_{[22]}^4 - \chi_{[2]}^5 \chi_{[22]}^5 + \chi_{[2]}^6 \chi_{[22]}^6], \\ \psi^{c_2} &= \sqrt{\frac{1}{3}} [\chi_{[11]}^7 \chi_{[211]}^7 - \chi_{[11]}^8 \chi_{[211]}^8 + \chi_{[11]}^9 \chi_{[211]}^9].\end{aligned}\quad (25)$$

Finally, we can acquire the total wave functions by substituting the wave functions of the orbital, the spin, the flavor and the color parts into the Eq. (10) according to the given quantum number of the system.

3 The results and discussions

In present work, we investigate tetraquarks with quark components: $us\bar{d}\bar{b}$ and $ud\bar{s}\bar{b}$ in two structures, meson-meson and diquark-antidiquark. The quantum numbers of the tetraquarks we study here are $I = 0, 1$, $J = 0, 1$ and the parity is $P = +$. The orbital angular momenta are set to zero because we are interested in the ground states. To check whether or not there is any bound state in such tetraquark system, we do a dynamic bound-state calculation. Both the single-channel and channel-coupling calculations are carried out in this work. All the general features of the calculated results are as follows.

Table 3 The energies (in MeV) of the meson-meson structure for tetraquarks $us\bar{d}\bar{b}$

IJ	$[\psi^{f_i} \psi^{\sigma_j} \psi^{c_k}]$	Channel	E_{th}	E_{sc}	E_{cc}
10	$[\psi^{f_1} \psi^{\sigma_1} \psi^{c_1}]$	$B_s^0 \pi^+$	5506.9	5514.0	5513.1
	$[\psi^{f_1} \psi^{\sigma_2} \psi^{c_1}]$	$B_s^{*0} \rho^+$	6165.4	6169.2	
	$[\psi^{f_2} \psi^{\sigma_1} \psi^{c_1}]$	$B^+ \bar{K}^0$	5774.9	5782.6	
	$[\psi^{f_2} \psi^{\sigma_2} \psi^{c_1}]$	$B^{*+} \bar{K}^{*0}$	6212.6	6217.5	
11	$[\psi^{f_1} \psi^{\sigma_3} \psi^{c_1}]$	$B_s^0 \rho^+$	6139.4	6143.9	5539.3
	$[\psi^{f_1} \psi^{\sigma_4} \psi^{c_1}]$	$B_s^{*0} \pi^+$	5532.9	5539.7	
	$[\psi^{f_2} \psi^{\sigma_3} \psi^{c_1}]$	$B^+ \bar{K}^{*0}$	6172.3	6179.1	
	$[\psi^{f_2} \psi^{\sigma_4} \psi^{c_1}]$	$B^{*+} \bar{K}^0$	5814.2	5821.5	

Table 4 The energies (in MeV) of the diquark–antidiquark structure for tetraquarks $us\bar{d}\bar{b}$

IJ	$[\psi^{f_i} \psi^{\sigma_j} \psi^{c_k}]$	E_{th}	E_{sc}	E_{cc}
10	$[\psi^{f_1} \psi^{\sigma_1} \psi^{c_1}]$	5506.9	6283.1	5551.8
	$[\psi^{f_1} \psi^{\sigma_2} \psi^{c_2}]$		6186.9	
	$[\psi^{f_2} \psi^{\sigma_1} \psi^{c_2}]$		6096.1	
	$[\psi^{f_2} \psi^{\sigma_2} \psi^{c_1}]$		5846.6	
11	$[\psi^{f_1} \psi^{\sigma_4} \psi^{c_2}]$	5532.9	6308.8	5613.4
	$[\psi^{f_1} \psi^{\sigma_5} \psi^{c_2}]$		6261.9	
	$[\psi^{f_1} \psi^{\sigma_3} \psi^{c_1}]$		6276.0	
	$[\psi^{f_2} \psi^{\sigma_4} \psi^{c_1}]$		6216.5	
	$[\psi^{f_2} \psi^{\sigma_5} \psi^{c_1}]$		6059.4	
	$[\psi^{f_2} \psi^{\sigma_3} \psi^{c_2}]$		6118.7	

3.1 Tetraquarks composed of $us\bar{d}\bar{b}$

For tetraquarks composed of $us\bar{d}\bar{b}$, the isospin is $I = 1$. The energies of the states with $J = 0, 1$ are calculated and the results are listed in Tables 3 and 4. In the tables, the second column gives the index of the wave functions of each channel. The columns headed with E_{sc} and E_{cc} represent the energies of the single-channel and channel-coupling calculation respectively. E_{th} denotes the theoretical threshold of the channel. For meson-meson structure, there is an additional column headed with “Channel” which denotes the physical contents of the channel. From the Table 3, we can see that the energies of every single channel approach to the corresponding theoretical threshold. The channel-coupling cannot help too much. Energies are still above the threshold of the lowest channel ($B_s^0 \pi^+$ for $IJ = 10$ and $B_s^{*0} \pi^+$ for $IJ = 11$), which indicates that no bound $us\bar{d}\bar{b}$ state with meson-meson structure is formed in our quark model calculation.

With regard to the diquark–antidiquark structure, the energies are listed in Table 4. The channels with different flavor-spin-color configurations have different energies and the coupling of them is rather stronger than that of the meson-meson

Fig. 1 The effective potentials as a function of the distance between the diquark (qq) and antidiquark ($\bar{q}\bar{q}$) for the $IJ = 10$ and $IJ = 11$ states of $us\bar{d}\bar{b}$

structure. However, the energy of the $IJ = 10$ state is still higher than the theoretical threshold of the lowest channel $B_s^0 \pi^+$, 5506.9 MeV. Similarly, the energy of the $IJ = 11$ state is higher than the theoretical threshold of the lowest channel $B_s^{*0} \pi^+$, 5532.9 MeV. Thus, there is no bound state with diquark–antidiquark structure in the present calculation.

Nevertheless, the colorful subclusters diquark (qq) and antidiquark ($\bar{q}\bar{q}$) cannot fall apart because of the color confinement, so there may be a resonance state with diquark–antidiquark structure. To check the possibility, we perform an adiabatic calculation of the effective potentials for both the $IJ = 10$ and $IJ = 11$ states. The effective potentials are obtained by $V_E(S) = E(S) - E_{th}$, where E_{th} is the threshold of the corresponding lowest channel. $E(S)$ is the energy at each S , which is the distance between two subclusters, and $E(S)$ can be obtained by the formula:

$$E(S) = \frac{\langle \Psi(S) | H | \Psi(S) \rangle}{\langle \Psi(S) | \Psi(S) \rangle}.$$

where $\langle \Psi(S) | H | \Psi(S) \rangle$ and $\langle \Psi(S) | \Psi(S) \rangle$ are the Hamiltonian matrix element and the overlap of the state. The results are shown in Fig. 1. It is obvious in Fig. 1 that the effective potentials of both the $IJ = 10$ and $IJ = 11$ states are increasing when the two subclusters fall apart, which indicates that the two subclusters tend to clump together. In this case, the odds are the same for the states being meson-meson structure, diquark–antidiquark structure or other structures. Besides, from the Tables 3 and 4, the energy of the diquark–antidiquark structure is always higher than the one of the meson-meson structure. So the unstable states will decay through the corresponding meson-meson channel. Then the system prefers to be meson-meson structure, which can fall apart. Therefore neither the state of $IJ=10$ nor the state of $IJ=11$ is a resonance state in QDCSM.

Fig. 2 The effective potentials as a function of the distance between two mesons for the $IJ = 10$ and $IJ = 11$ states of $us\bar{d}\bar{b}$

Besides, we can also calculate the effective potentials for the meson-meson structure, which are shown in Fig. 2. We find the potentials for both the $IJ = 10$ and $IJ = 11$ states are repulsive, which further shows that it is difficult to form a molecular bound state for either $IJ = 10$ or $IJ = 11$ system.

Based on the discussion above, the $X(5568)$ cannot be explained as a molecular state or a diquark–antidiquark resonance of $us\bar{d}\bar{b}$ in the present calculation. Our results are consistent with the analysis of Refs. [19, 20, 22]. In Ref. [19], the four-quark system $us\bar{d}\bar{b}$ with both meson-meson structure and diquark–antidiquark structure was studied in the framework of the chiral quark model by using the Gaussian expansion method, and no candidate of $X(5568)$ was found. In Ref. [20], Burns and Swanson explored a lot of possible explanations of the $X(5568)$ signal, a tetraquark, a hadronic molecule or a threshold effect and found that none of them can be a candidate of the observed state. In Ref. [22], they concluded that the $X(5568)$ cannot be generated by the $\bar{B}_s\pi - \bar{B}K$ rescattering.

3.2 Tetraquarks composed of $us\bar{d}\bar{b}$

For tetraquarks composed of $us\bar{d}\bar{b}$, four states with the quantum numbers $IJ = 00, 01, 10$ and 11 are studied. The energies of the meson-meson structure and the diquark–antidiquark structure are listed in Tables 5 and 6, respectively. For the meson-meson structure, the results are similar to that of the tetraquarks of $us\bar{d}\bar{b}$. Table 5 shows that the energies of every single channel are above the corresponding theoretical threshold. The effect of channel-coupling is very small except for the $IJ = 01$ state. For the states with $IJ = 00, 10$, and $IJ = 11$, all energies are above the threshold of the lowest channel (B^0K^+ for $IJ = 00$, B^0K^+ for $IJ = 10$, and $B^{*0}K^+$ for $IJ = 11$) even by the channel-coupling calculation.

Table 5 The energies (in MeV) of the meson-meson structure for tetraquarks $us\bar{d}\bar{b}$

IJ	$[\psi^{f_i}\psi^{\sigma_j}\psi^{c_k}]$	Channel	E_{th}	E_{sc}	E_{cc}
00	$[\psi^{f_3}\psi^{\sigma_1}\psi^{c_1}]$	B^0K^+	5774.9	5781.4	5779.9
	$[\psi^{f_3}\psi^{\sigma_2}\psi^{c_1}]$	$B^{*0}K^{*+}$	6212.6	6218.1	
01	$[\psi^{f_3}\psi^{\sigma_3}\psi^{c_1}]$	B^0K^{*+}	6172.3	6176.2	5813.1
	$[\psi^{f_3}\psi^{\sigma_4}\psi^{c_1}]$	$B^{*0}K^+$	5814.2	5820.5	
	$[\psi^{f_3}\psi^{\sigma_5}\psi^{c_1}]$	$B^{*0}K^{*+}$	6212.6	6216.1	
	$[\psi^{f_4}\psi^{\sigma_1}\psi^{c_1}]$	B^0K^+	5774.9	5783.2	5783.0
10	$[\psi^{f_4}\psi^{\sigma_2}\psi^{c_1}]$	$B^{*0}K^{*+}$	6212.6	6218.1	
	$[\psi^{f_4}\psi^{\sigma_3}\psi^{c_1}]$	B^0K^{*+}	6172.3	6180.3	5821.5
	$[\psi^{f_4}\psi^{\sigma_4}\psi^{c_1}]$	$B^{*0}K^+$	5814.2	5822.0	
	$[\psi^{f_4}\psi^{\sigma_5}\psi^{c_1}]$	$B^{*0}K^{*+}$	6212.6	6219.1	

Table 6 The energies (in MeV) of the diquark–antidiquark structure for tetraquarks $us\bar{d}\bar{b}$

IJ	$[\psi^{f_i}\psi^{\sigma_j}\psi^{c_k}]$	E_{th}	E_{sc}	E_{cc}
00	$[\psi^{f_1}\psi^{\sigma_1}\psi^{c_1}]$	5774.9	5867.2	5701.1
	$[\psi^{f_1}\psi^{\sigma_2}\psi^{c_2}]$		6058.2	
01	$[\psi^{f_1}\psi^{\sigma_1}\psi^{c_1}]$	5814.2	6334.7	5756.3
	$[\psi^{f_2}\psi^{\sigma_1}\psi^{c_2}]$		6213.3	
	$[\psi^{f_2}\psi^{\sigma_2}\psi^{c_1}]$		5881.4	
	$[\psi^{f_1}\psi^{\sigma_1}\psi^{c_1}]$	5774.9	6452.1	6103.8
11	$[\psi^{f_1}\psi^{\sigma_2}\psi^{c_2}]$		6200.8	
	$[\psi^{f_1}\psi^{\sigma_1}\psi^{c_1}]$	5814.2	6289.1	6130.1
	$[\psi^{f_2}\psi^{\sigma_1}\psi^{c_2}]$		6253.4	
	$[\psi^{f_2}\psi^{\sigma_2}\psi^{c_1}]$		6447.9	

tion. While for the state with $IJ = 01$, the energy is about 1.0 MeV lower than the threshold of the lowest channel $B^{*0}K^+$ after channel-coupling. However, the binding energy is not very large, so there maybe a weak molecular bound state of $us\bar{d}\bar{b}$ with quantum numbers of $IJ = 01$, and the mass of this state is about 5813 MeV.

For the diquark–antidiquark structure, all the possible channels are shown in Table 6. One can see that the energy of each single channel is higher than the theoretical threshold of the corresponding channel, which are shown in Table 5. Although the effect of the channel-coupling is much stronger than that of the meson-meson structure, the energy of the $IJ = 10$ and $IJ = 11$ states are still above the theoretical threshold of the corresponding channel. So there is no any bound states for the $IJ = 10$ or $IJ = 11$ state. In order to check if there is any resonance state, we also perform the adiabatic calculation of effective potentials for both the $IJ = 10$ and $IJ = 11$ states. The results are shown in Fig. 3. The case is similar to the tetraquarks composed of $us\bar{d}\bar{b}$. The effective potentials of both the $IJ = 10$ and $IJ = 11$ states are

Fig. 3 The effective potentials as a function of the distance between the diquark (qq) and antiquark ($\bar{q}\bar{q}$) for the $IJ = 10$ and $IJ = 11$ states of $ud\bar{s}\bar{b}$

increasing when the two subclusters diquark (qq) and antiquark ($\bar{q}\bar{q}$) fall apart, which indicates that the two subclusters tend to clump together. Hence, the $IJ = 10$ and $IJ = 11$ states are not stable within the diquark–antiquark configuration, and no resonance state with quantum numbers $IJ = 10$ and $IJ = 11$ exists.

However, things are different for the $IJ = 00$ state and the $IJ = 01$ state. The energy of the $IJ = 00$ state is about 5701 MeV, 74 MeV lower than the theoretical threshold of the $B^0 K^+$, which indicates that the $IJ = 00$ state of the diquark–antiquark structure can be a bound state. Ref. [24] also found that the bound state with $IJ = 00$ was possible. Meanwhile, the energy of the $IJ = 01$ state is 58 MeV lower than the theoretical threshold of the $B^{*0} K^+$, so the $IJ = 01$ state is also bound here. Thus, both the $IJ = 00$ state and the $IJ = 01$ state of diquark–antiquark structure can form bound states.

By comparing with the results of the meson-meson structure, we note that the energy of the $IJ = 01$ state of the diquark–antiquark structure is about 5756 MeV, which is much lower than that of the meson-meson structure shown in Table 5. This shows that the $IJ = 01$ state prefers to be a bound state of the diquark–antiquark structure. Moreover, the $IJ = 00$ state of the diquark–antiquark structure is easier to form the bound state than the one of the meson-meson structure. All these indicate that the diquark–antiquark configuration may be a good choice for some tetraquarks. Some work has been done to explain the exotic XYZ states depending on the diquark–antiquark configuration. Reference [38] proposed the hypothesis that the diquarks and antiquarks in tetraquarks were separated by a potential barrier to explain the properties of exotic resonances such as X and Z . Ref. [39] presented a dynamical picture to explain the nature of some exotic XYZ states based on a diquark–

antiquark open-string configuration. The picture combined the advantages of diquark-based models, which can accommodate much of the known XYZ spectrum, with the experimental fact that such states are both relatively narrow and are produced promptly. Thus both the $IJ = 00$ and $IJ = 01$ states of the diquark–antiquark structure we obtain here may be the narrow resonance states. The study of the decay width of these states is our further work.

Contrasting with the tetraquarks composed of $us\bar{d}\bar{b}$, we find the tetraquarks composed of $ud\bar{s}\bar{b}$ are more likely to form bound states. The reasons are as follows. First, the diquark pair of two light quarks (ud) or two heavier quarks (sb) is usually more stable than the one of two quarks with larger mass difference like us or db pair. Our results show that the tetraquarks composed of $ud\bar{s}\bar{b}$ of the diquark–antiquark structure are most possible to form bound states, which just supports this point. Secondly, the lowest threshold of $ud\bar{s}\bar{b}$ is BK , 270 MeV higher than the threshold $B_s\pi$ of $us\bar{d}\bar{b}$. So there is a large mass region for this $ud\bar{s}\bar{b}$ state below threshold and being stable. This conclusion also supports the assumption of Ref. [23], which proposed such a particle with the quark component of $ud\bar{s}\bar{b}$ (or its charge-conjugated one) as a partner of $X(5568)$ of $us\bar{d}\bar{b}$ under the $SU(3)$ flavor symmetry. Besides, Ref. [40] also stressed the possible existence of a near-threshold bound or virtual state both for $J^P = 0^+$ and 1^+ sectors, with quark content $sb\bar{u}\bar{d}$.

Because of the heavy flavor symmetry, we also extend the study to the above tetraquarks by considering a c quark instead of a b quark, which are composed of $us\bar{d}\bar{c}$ or $ud\bar{s}\bar{c}$. No bound state or resonance state is found in these systems. This is possible in our quark model calculation. In the heavy-quark sector, the large masses of the heavy quarks reduce the kinetic energy of the system, which makes them easier to form bound states. So it is more difficult for the tetraquark in the charm sector than the one in the bottom sector to form bound states. However, some other tetraquarks in the charm sector were studied before. Ref. [40] found a two-pole structure of the $D_0^*(2400)$, with the largest couplings to the $D\pi$ and $D_s\bar{K}$ channels. They showed the higher pole was a threshold enhancement in the $D_s\bar{K}$ invariant mass distribution, and the lower pole belongs to the same $SU(3)$ multiplet as the $D_{s0}^*(2317)$ state. Such tetraquark system is also worth investigating in our quark model calculation, which will be our further work.

4 Summary

In summary, we investigate tetraquarks composed of $us\bar{d}\bar{b}$ and $ud\bar{s}\bar{b}$ in the framework of QDCSM. Two structures, meson-meson and diquark–antiquark, are considered. Our results show that there is no bound state or resonance state composed of $us\bar{d}\bar{b}$. The reported $X(5568)$ cannot be

explained as a molecular state or a diquark–antidiquark resonance of $us\bar{d}\bar{b}$ in present calculation. In contrast, two bound states are obtained for the tetraquarks system composed of $ud\bar{s}\bar{b}$: an $IJ = 00$ state with the mass of 5701 MeV, and an $IJ = 01$ state with the mass of 5756 MeV, which maybe the better tetraquark candidates with four different flavors. These two bound states are of the diquark–antidiquark structure. For the system with $IJ = 00$ and $IJ = 01$, it is obvious that the state of the diquark–antidiquark structure is more likely to form bound state than that of the meson–meson structure, which indicates that the diquark–antidiquark configuration would be a good choice for the tetraquarks $ud\bar{s}\bar{b}$ with $IJ = 00$ and $IJ = 01$. During the calculation, we find that the effect of the channel-coupling in the diquark–antidiquark structure is much stronger than that in the meson–meson structure, and the channel-coupling plays an important role in forming bound states in the diquark–antidiquark structure.

Meanwhile, our results also show that the tetraquarks composed of $ud\bar{s}\bar{b}$ is more possible to form bound states than the one composed of $us\bar{d}\bar{b}$. Thus, if the $X(5568)$ does exist, the tetraquarks composed of $ud\bar{s}\bar{b}$ must be a more stable state. If the $X(5568)$ is proved to be nonexistent, it is still possible for the existence of such tetraquarks with $ud\bar{s}\bar{b}$ components. This conclusion is in accordance with the point of Ref. [23], which proposed the state composed of $ud\bar{s}\bar{b}$ (or its charge-conjugated one) as a partner of $X(5568)$ of $us\bar{d}\bar{b}$. Ref. [23] also pointed out that if the lowest-lying $ud\bar{s}\bar{b}$ state exists below threshold, it can be definitely observed via the weak decay mode $J/\psi K^- K^- \pi^+$, with the expectation of hundreds of events in the current LHCb data sample but rejecting backgrounds due to its long lifetime. Therefore, the $ud\bar{s}\bar{b}$ state would be a promising detectable tetraquark state. We hope that experiments will help to discover these interesting tetraquark states.

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