



Automatic Peccei–Quinn symmetry

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Abstract We present a dynamical (composite) axion model where the Peccei–Quinn (PQ) symmetry arises automatically as a consequence of chirality and gauge symmetry. The Standard Model is simply extended by a confining and chiral $SU(5)$ gauge symmetry. The PQ symmetry coincides with a $B - L$ symmetry of the exotic sector. The theory is protected by construction from quantum gravitational corrections stemming from operators with mass dimension lower than nine.

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1 Introduction

A most intriguing puzzle of the Standard Model of Particle Physics (SM) is the so-called “strong CP problem”: the extremely small size ($< 10^{-10}$ [1,2]) of the vacuum angle of the strong interactions

$$|\bar{\theta}| = |\theta_{QCD} + \arg \det M|. \quad (1.1)$$

Here, M denotes the quark mass matrix and θ_{QCD} characterises the CP-odd gauge contribution in the $SU(3)_c$ QCD Lagrangian,

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \theta_{QCD}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \bar{q}Mq, \quad (1.2)$$

where α_s denotes the QCD fine structure constant and colour indices have been left implicit.

The most elegant solution to the strong CP problem is to introduce a global chiral $U(1)$ symmetry, usually called Peccei–Quinn (PQ) symmetry [3] $U(1)_{PQ}$, which is exact (and hidden) at the classical level but is anomalous under

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QCD interactions. The latter is the key to solve the problem and also the only source of the mass for the pseudo-Nambu-Goldstone boson of the global $U(1)_{PQ}$ symmetry: the axion.

A simple and most economical implementation would be the $U(1)_{PQ}$ symmetry that would exist if one SM quark were to be massless. The freedom to chirally rotate that fermion would allow to fully reabsorb all contributions to $\bar{\theta}$, making it unphysical. This interesting possibility [4] does not seem to be realized in nature after the constraints stemming from lattice computations and we disregard it, even if the option is not completely excluded [5–11].

It is still possible to solve the strong CP problem with massless quarks, though, if extra exotic massless fermions charged under QCD exist in Nature. As the latter are not observed, the idea [12] is to charge them in addition under a new confining force [13–15], often called “axicolor” [12], whose scale is much larger than that of QCD, Λ_{QCD} . A new spectrum of confined states results composed of those massless fermions, including mesons which play the role of axions. They are often referred to as “dynamical” or composite axions.

In a given theory, when the number of axions—either elementary or composite—outnumbers the total number of distinct instanton-induced scales other than QCD, one (or more) light axions remain. These are called “invisible axions”, whose mass m_a and scale f_a generically obey [16, 17]

$$m_a^2 f_a^2 \sim m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}, \quad (1.3)$$

where m_π , f_π , m_u , m_d denote the pion mass and coupling constant, and the up and down quark masses, respectively.¹ Light enough axions (that is, below $\mathcal{O}(100 \text{ MeV})$) can participate in astrophysical phenomena [30–33]. The constraints that follow from their non-observation in photonic processes lead to very high values for the decay constant, $f_a \geq 10^8 \text{ GeV}$. It follows then from Eq. (1.3) that $m_a \leq 10^{-2} \text{ eV}$. Here we will construct a novel implementation of the invisible axion paradigm via massless exotic quarks, and in consequence Eq. (1.3) will apply.

In the original composite axion proposal [12] the confining sector of the SM was enlarged to $SU(3)_c \times SU(\tilde{N})$, where $SU(\tilde{N})$ is the axicolor group. Two composite axions result, one of which must be invisible and obey Eq. (1.3), as there are only two sources of instantons for three pseudoscalars with anomalous couplings (taking into account the SM η'). The axicolor construction can be seen as a beautiful ultraviolet dynamical completion of the invisible axion paradigm. It has the advantage of being free from the scalar potential fine-

tunings that hinder models of invisible elementary axions à la KSVZ [34, 35] or DFSZ [36, 37].

Dynamical axion constructions often require that the PQ transition predates inflation. This avoids cosmological problems in the form of domain walls (whose accumulated energy could overclose the universe after the QCD phase transition). Our patch of the universe would correspond to a specific initial value of the axion field which determines the axion energy density, because of the misalignment mechanism [38]. In the absence of fine-tuned values of the misalignment angle, if axions were to explain all the dark matter density it is necessary that [39, 40]

$$f_a \simeq 2 \times 10^{10} - 5 \times 10^{12} \text{ GeV}, \quad (1.4)$$

although the axion decay constant could be one order of magnitude smaller if some fine tuning is allowed.

A threat which menaces all types of invisible axion models stems from quantum non-perturbative gravitational corrections [41–49], as f_a is not very far from the Planck scale. These are usually parametrized via effective operators suppressed by powers of the Planck mass, M_{Pl} .² They would explicitly violate the PQ symmetry and can thus spoil the solution to the SM strong CP problem. For instance, Refs. [41–44] concentrated on the simplest (and most dangerous) hypothetical dimension five effective operator

$$g_5 \frac{|\Phi|^4 (\Phi + \Phi^*)}{M_{\text{Pl}}}, \quad (1.5)$$

where g_5 is a dimensionless coefficient and Φ would be a field whose VEV breaks the PQ invariance. In order to avoid that this term moves the minimum of the axion potential unacceptably away from a CP-conserving solution, its coefficient needs a extreme fine-tuning, e.g. $g_5 < 10^{-54}$ for $f_a \sim 10^{12} \text{ GeV}$.³

In this work, the axicolor framework is approached with a novel light: to assume that the $SU(\tilde{N})$ exotic confining gauge sector is chiral. In a minimalistic approach, we require a fermion content such that:

- It confines and renders the theory free from gauge anomalies.
- The exotic fermion representations are chiral, so that fermionic mass terms are automatically forbidden.

² Here, the Planck mass does not denote the reduced Planck scale but the one given by $M_{\text{Pl}} = G^{-1/2}$ with G being the Newton constant.

³ They can be avoided, though, in some invisible axion constructions with a variety of extra assumptions or frameworks [50–57], or be arguably negligible in certain conditions [58]. It is also possible to avoid the dangerous terms in “heavy axion” models [18–29], as their f_a scale can be very low, e.g. not far from the TeV range.

¹ Alternative models with extra sources of instantons may render all axions heavier than the QCD scale, and they are increasingly explored in the last years [18–29].

- Minimality in the specific matter content will be a guideline. Two (or more) different axicolored fermions are present, with at least one of them being QCD colored as well.

In this class of set up, at least two chiral $U(1)$ symmetries emerge in the dynamical sector in the limit of $M_{Pl} \rightarrow \infty$ and nullify the theta angles of the dynamical sector and the QCD sector. It can be checked that it is not possible to obey the three requirements listed above for $SU(3)$, $SU(6)$ or $SU(7)$, at least not with just two exotic fermions in low-dimensional representations of the chiral confining group. It is possible instead for $SU(4)$; nevertheless, this theory would not render an improvement on the gravitational issue, as argued in Appendix A, and it will not be further developed.

We focus here on the case of chiral gauge $SU(5)$, implemented via its lowest dimensional fermion representations, $\bar{\mathbf{5}}$ and $\mathbf{10}$, which together fulfil the conditions above. The $SU(5)$ confinement scale will be assumed to be much larger than that of QCD, $\Lambda_5 \gg \Lambda_{QCD}$. It will be shown that a satisfactory $U(1)_{PQ}$ symmetry is an automatic consequence of the chiral realization of the gauge group. Note that some models have been previously built for which PQ invariance is accidental, that is, not imposed by hand [50–57]. Nevertheless, they all required extra symmetries in addition to axicolor, either gauge or discrete ones. In contrast, axicolor $SU(5)$ will be shown to suffice because of its chiral character, rendering a particularly simple framework.

Relevant aspects to be developed include on one side the identification of the exotic fermion condensates, which in dynamical axion models are the only source of spontaneous symmetry breaking, e.g. for exotic flavour and for the PQ symmetries. Another important question is the impact of $SU(5)$ gauge invariance on the possible non-perturbative gravitational couplings of the theory.

The idea will be implemented in two alternative realizations, selected so as to achieve minimal matter content. They will only differ in the QCD charges of the exotic $\bar{\mathbf{5}}$ and $\mathbf{10}$ fermions present: octets of QCD color in one model, while triplets in a second version.

The structure of the paper can be easily inferred from the table of contents.

2 The $SU(5)$ chiral confining theory

We consider a chiral version of the axicolor model, with $SU(5)$ as an extra confining group, and one set of massless exotic fermions in its five and ten dimensional representations, $\psi_{\bar{\mathbf{5}}}$ and $\psi_{\mathbf{10}}$ (the notation $\psi_{\bar{\mathbf{5}}} \equiv \bar{\mathbf{5}}$, $\psi_{\mathbf{10}} \equiv \mathbf{10}$ will be often used for convenience). Such a set cancels all $SU(5)$ gauge anomalies (as in $SU(5)$ GUT models). The complete

Table 1 Charges of exotic fermions under the confining gauge group $SU(5) \times SU(3)_c$. The left-handed Weyl fermions $\psi_{\bar{\mathbf{5}}}$ and $\psi_{\mathbf{10}}$ are massless and singlets of the SM electroweak gauge group. \mathbf{R} denotes a pseudo-real representation

	$SU(5)$	$SU(3)_c$
$\psi_{\bar{\mathbf{5}}}$	$\bar{\mathbf{5}}$	\mathbf{R}
$\psi_{\mathbf{10}}$	$\mathbf{10}$	\mathbf{R}

gauge group of Nature would then be

$$SU(5) \times SU(3)_c \times SU(2)_L \times U(1). \tag{2.1}$$

An economic implementation is to assume the usual SM fields to be singlets under $SU(5)$, while the exotic chiral fermions in the $\psi_{\bar{\mathbf{5}}}$ and $\psi_{\mathbf{10}}$ representations of $SU(5)$ are singlets under the electroweak SM gauge group.

If the exotic fermions carry also QCD color, this theory solves the strong CP problem. Indeed, the presence of (at least) two massless fermions ensures the existence of two distinct $U(1)$ chiral global symmetries, exact at the classical level but explicitly broken by quantum non-perturbative effects. The θ -parameters corresponding to the two confining gauge groups become thus unphysical via chiral rotations of those fermions. Furthermore, the chiral character of the representations forbids fermionic mass terms and thus guarantees that those symmetries are automatic, instead of imposed on a given Lagrangian as customary. Finally, the requirement of a large confining scale $\Lambda_5 \gg \Lambda_{QCD}$ leads to a realistic model, given the non-observation of a spectrum of bound states composed of those massless exotic fermions.

For simplicity, we will consider that the set $\{\psi_{\bar{\mathbf{5}}}, \psi_{\mathbf{10}}\}$ belongs to a (pseudo)real representation \mathbf{R} of color QCD, so as automatically cancel $[SU(3)_c]^3$ anomalies, see Table 1. Later on we will develop in detail two specific choices for \mathbf{R} : the case of the fundamental of QCD with reducible representation $\mathbf{R} = \mathbf{3} + \bar{\mathbf{3}}$ in one case, and the adjoint $\mathbf{R} = \mathbf{8}$ in the second case. In all cases, all mixed gauge anomalies in the confining sector vanish by construction as well, because only non-abelian $SU(N)$ groups are present and the exotic fermions are electroweak singlets.

2.1 Global symmetries

At the scale Λ_5 , $SU(5)$ confines and the massless fermions in Table 1 will form massive bound states including QCD-colored ones. In the limit in which the QCD coupling constant α_s is neglected, the $SU(5)$ gauge Lagrangian exhibits at the classical level a global flavor symmetry

$$U(n)_{\bar{\mathbf{5}}} \times U(n)_{\mathbf{10}} \leftrightarrow SU(n)_{\bar{\mathbf{5}}} \times SU(n)_{\mathbf{10}} \times U(1)_{\bar{\mathbf{5}}} \times U(1)_{\mathbf{10}}, \tag{2.2}$$

Table 2 Global chiral properties at the classical level, in the limit of vanishing α_s

	$SU(n)_{\bar{5}}$	$SU(n)_{10}$	$U(1)_{B-L} \equiv U(1)_{PQ}$
$\psi_{\bar{5}}$	\square	$\mathbf{1}$	-3
ψ_{10}	$\mathbf{1}$	\square	1

where n denotes the dimension of \mathbf{R} , which plays the role of number of exotic flavours,

$$n = \dim\{\mathbf{R}\}. \tag{2.3}$$

The two global $U(1)$ symmetries correspond to independent rotations of the two massless fermion representations. However, they are both broken at the quantum level by anomalous couplings to the $SU(5)$ and QCD field strengths. A generic combination of them will lead to the following anomaly coefficients (see Appendix B):

$$U(1) \times [SU(5)]^2 : n \times (Q_{\bar{5}}T(\bar{\mathbf{5}}) + Q_{10}T(\mathbf{10})) = \frac{n}{2} (Q_{\bar{5}} + 3Q_{10}), \tag{2.4}$$

$$U(1) \times [SU(3)_c]^2 : T(\mathbf{R}) \times (5Q_{\bar{5}} + 10Q_{10}). \tag{2.5}$$

Here, $Q_{\bar{5}}$ and Q_{10} denote arbitrary $U(1)$ charges for $\psi_{\bar{5}}$ and ψ_{10} , respectively, and T 's denote the Dynkin indices of the corresponding representations. It follows from Eq. (2.4) that the charge assignment

$$Q_{\bar{5}} = -3, \quad Q_{10} = 1, \tag{2.6}$$

renders a combination of $U(1)$'s that is free from $SU(5)$ anomaly. The $SU(5)$ anomaly-free combination is analogous to the $B-L$ symmetry in usual $SU(5)$ GUT's. It will play the role of the PQ symmetry in our model, since it is a classically exact symmetry that is only broken by the QCD anomaly. A second combination will remain explicitly broken⁴ by quantum non-perturbative effects of $SU(5)$, so that the classical global symmetry in Eq. (2.2) reduces (for $\alpha_s = 0$) to

$$SU(n)_{\bar{5}} \times SU(n)_{10} \times U(1)_{PQ=B-L}. \tag{2.7}$$

The corresponding global charges of the exotic fermions are shown in Table 2.

Confinement versus chiral symmetry breaking

A first question is whether the confinement of the $SU(5)$ gauge dynamics is accompanied by the spontaneous breaking

⁴ This can be for instance, the orthogonal combination corresponding to $\{Q_{\bar{5}} = 1, Q_{10} = 3\}$, although any combination different from that free from anomalous $SU(5)$ couplings can play this role.

of the associated chiral global symmetries. Two alternative realizations are possible:

- The global symmetries can be spontaneously broken via fermion condensates. As a result, (almost) massless (pseudo)Goldstone bosons (pGBs) will be present in the low energy theory.
- Conversely, they could remain unbroken and the spectrum of bound states would explicitly reflect those global symmetries via multiplets of degenerate states. In particular, massless baryons are then needed in order to fulfil the 't Hooft anomaly consistency conditions [59] to match the anomalies of the high and low energy theories.

It can be shown that it is not possible to comply with the 't Hooft consistency conditions for the complete flavour group. That is, it is impossible to match the $[SU(n)_{\bar{5}}]^3$ and $[SU(n)_{10}]^3$ anomalies before confinement – and thus in terms of quarks – with the anomalies after confinement in terms of massless “baryons”. The demonstration can be found in Appendix D. The confinement of gauge $SU(5)$ is thus necessarily accompanied by the spontaneous breaking of the chiral global $SU(n)_{\bar{5}} \times SU(n)_{10}$ symmetry, and associated (pseudo)Goldstone bosons (pGBs) will be present in the low-energy spectrum.

In contrast, for $U(1)_{PQ}$ it is possible to fulfil 't Hooft anomaly conditions [60,61]. At high energies and in terms of quarks, the spectrum in Table 2 contributes to the global anomalies as

$$[U(1)_{PQ}]^3 : n (5(Q_{\bar{5}})^3 + 10(Q_{10})^3) = -125n, \tag{2.8}$$

$$U(1)_{PQ} \times [SU(3)_c]^2 : N \equiv 2(5Q_{\bar{5}}T(\mathbf{R}) + 10Q_{10}T(\mathbf{R})) = -10T(\mathbf{R}), \tag{2.9}$$

$$U(1)_{PQ} \times [\text{grav}]^2 : n(5Q_{\bar{5}} + 10Q_{10}) = -5n. \tag{2.10}$$

where N denotes as customary the QCD anomaly factor. The low-energy spectrum admits in turn a massless baryon composed by three fermions,

$$\chi \equiv \mathbf{10}\bar{\mathbf{5}}\bar{\mathbf{5}}, \tag{2.11}$$

which has PQ charge $Q_\chi = -5$ and can belong to the \mathbf{R} representation of $SU(3)_c$. Its contribution to the anomaly equations matches the anomalies at the quark level in Eqs. (2.8) and (2.10):

$$[U(1)_{PQ}]^3 : nQ_\chi^3 = -125n, \tag{2.12}$$

$$U(1)_{PQ} \times [SU(3)_c]^2 : N \equiv 2Q_\chi T(\mathbf{R}) = -10T(\mathbf{R}), \tag{2.13}$$

$$U(1)_{PQ} \times [\text{grav}]^2 : nQ_\chi = -5n, \tag{2.14}$$

In consequence, the chiral confining $SU(5)$ theory would be a priori perfectly consistent even if the $U(1)_{PQ}$ were to remain unbroken after confinement. Nevertheless, this is not phenomenologically viable since (almost) massless QCD colored fermions are not observed in Nature (other than the light SM quarks).

To sum up, parts of the global symmetries in Eq. (2.7) with the field content in Table 2 need to be spontaneously broken by fermion condensates upon $SU(5)$ confinement.

2.2 Fermion condensates: chiral-breaking versus PQ-breaking

It will be assumed that Λ_5 settles the overall scale for all dynamical breaking mechanisms in the $SU(5)$ sector, which will take place through fermion condensates.

Chiral condensate

The lowest dimension fermionic condensate which is gauge invariant and breaks the non-abelian chiral symmetries in Eq. (2.7) is a dimension six operator:

$$\mathbf{101010\bar{5}}, \tag{2.15}$$

with vacuum expectation value (VEV) and breaking pattern expected to obey

$$\langle \mathbf{101010\bar{5}} \rangle \sim \Lambda_5^6 \implies SU(n)_{\bar{5}} \times SU(n)_{10} \longrightarrow G \supset SU(3)_c. \tag{2.16}$$

On the right-hand side of this expression, it has been assumed that the QCD gauge group is contained in the unbroken subgroup G of $SU(n)_{\bar{5}} \times SU(n)_{10}$. This is possible as the product of four \mathbf{R} representations contains an $SU(3)$ singlet since \mathbf{R} is (pseudo)real. It should be noted that the unbroken subgroup G which contains $SU(3)$ is not necessarily aligned with the one which contains $SU(3)_c$ for $\alpha_s = 0$. Once α_s is turned on, on the other hand, the QCD interaction forces the condensates to preserve color, which implies that only the QCD invariant condensates will form (see also [51]).⁵

If \mathbf{R} is an irreducible representation of $SU(3)_c$, then the only part of the non-abelian chiral symmetry in Eq. (2.16) that is expected to remain unbroken is $SU(3)_c$. If \mathbf{R} is reducible instead, some $U(1)$'s can remain exact (see Sect. 3 where $\mathbf{R} = \mathbf{3} + \bar{\mathbf{3}}$). Therefore, irrespective of G , most generators of $SU(n)_{\bar{5}} \times SU(n)_{10}$ other than those of $SU(3)_c$ would be

explicitly broken by QCD interactions,

$$SU(n)_{\bar{5}} \times SU(n)_{10} \xrightarrow{\langle \mathbf{101010\bar{5}} \rangle} G \xrightarrow{\alpha_s \neq 0} SU(3)_c. \tag{2.17}$$

In consequence, most of the pGBs associated to the broken generators of the non-abelian chiral symmetry are necessarily colored under QCD. Their masses m are quadratically sensitive to large scales via gluon loops and thus safely large,

$$m^2(\mathbf{R}) \sim \frac{3\alpha_s}{4\pi} C(\mathbf{R}) \Lambda_5^2, \tag{2.18}$$

where $C(\mathbf{R})$ is the quadratic Casimir of the QCD representation \mathbf{R} to which a given pGB belongs, $T_R^a T_R^a = C(\mathbf{R}) \mathbb{1}$.

The chiral condensate in Eq. (2.16) is $U(1)_{PQ}$ invariant, though, since its PQ charge is vanishing. The spontaneous breaking of the PQ symmetry (which is phenomenologically the only viable option as earlier explained) can only be achieved via higher dimensional fermionic condensates.

PQ condensate

The lowest dimensional operator which is gauge invariant but has non-vanishing PQ-charge is

$$\bar{\mathbf{5}}\bar{\mathbf{5}}\mathbf{10}\bar{\mathbf{5}}\mathbf{10}, \tag{2.19}$$

which has mass dimension nine and PQ-charge -10. In order to achieve spontaneous $U(1)_{PQ}$ symmetry breaking, we assume that this operator obtains a non-vanishing VEV,⁶

$$\langle \bar{\mathbf{5}}\bar{\mathbf{5}}\mathbf{10}\bar{\mathbf{5}}\mathbf{10} \rangle \sim \Lambda_5^9, \tag{2.20}$$

which is associated with the QCD axion as a composite field.

In summary, the combined action of the two condensates in Eqs. (2.16) and (2.20) induces a breaking pattern of the global symmetries of the exotic $SU(5)$ sector of the form

$$SU(n)_{\bar{5}} \times SU(n)_{10} \times U(1)_{PQ} \xrightarrow{\langle \mathbf{101010\bar{5}} \rangle} G \times U(1)_{PQ} \xrightarrow{\langle \bar{\mathbf{5}}\bar{\mathbf{5}}\mathbf{10}\bar{\mathbf{5}}\mathbf{10} \rangle} G' \xrightarrow{\alpha_s \neq 0} SU(3)_c. \tag{2.21}$$

For later use, it is convenient to parametrize the field combination in Eq. (2.20) as

$$\bar{\mathbf{5}}\bar{\mathbf{5}}\mathbf{10}\bar{\mathbf{5}}\mathbf{10} \sim \Lambda_5^9 e^{-i10a/f_{PQ}}, \tag{2.22}$$

where the radial degrees of freedom are left implicit, a denotes the dynamical axion that corresponds to the axial

⁵ In the thermal bath, for example, the QCD breaking vacua have higher energy density than the QCD preserving one due to the thermal potential proportional to $m_{\text{gluon}}^2 T^2$, where m_{gluon} denotes the gluon mass on the QCD breaking vacua.

⁶ Its VEV also breaks the non-abelian chiral symmetry, but this effect should be subdominant with respect to that of the lower dimension operator in Eq. (2.16).

excitation of the operator, and the PQ charge of the condensate resulting from Table 2 is explicitly shown. The PQ scale f_{PQ} associated to the pGB nature of the axion obeys

$$f_{PQ} \propto \Lambda_5. \tag{2.23}$$

It should be noted that the PQ charges of the $SU(5)$ invariant states are multiples of 5, and hence, the PQ symmetry in the broken phase is realized by a shift of the axion given by

$$\frac{5a}{f_{PQ}} \rightarrow \frac{5a}{f_{PQ}} + \alpha, \quad \alpha = [0, 2\pi), \tag{2.24}$$

see also Appendix C.

2.3 The axion Lagrangian

In order to obtain the low-energy effective Lagrangian for the axion, the conservation of the PQ current will be studied next. The current at high energies can be computed in terms of the fundamental fermions by applying Noether’s formula:

$$\begin{aligned} j_{PQ}^\mu &= Q_5 \psi_5^\dagger \bar{\sigma}^\mu \psi_5 + Q_{10} \psi_{10}^\dagger \bar{\sigma}^\mu \psi_{10} \\ &= -3 \psi_5^\dagger \bar{\sigma}^\mu \psi_5 + \psi_{10}^\dagger \bar{\sigma}^\mu \psi_{10} = f_{PQ} \partial^\mu a. \end{aligned} \tag{2.25}$$

At energies below $SU(5)$ confinement, the current can be expressed in terms of the composite fermions (i.e. the composite baryons that will be generically denoted by χ_i) and the composite scalar (the dynamical axion a),

$$j_{PQ}^\mu = f_{PQ} \partial^\mu a + \sum_i Q_{\chi_i} (\chi_i^\dagger \bar{\sigma}^\mu \chi_i). \tag{2.26}$$

This current is classically conserved but it has a QCD anomaly,

$$\partial_\mu j_{PQ}^\mu = N \frac{\alpha_s}{8\pi} G\tilde{G}. \tag{2.27}$$

This ward identity is reproduced by the following effective Lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{1}{2} \partial^\mu a \partial_\mu a + \frac{\partial_\mu a}{f_{PQ}} \sum_i Q_{\chi_i} (\chi_i^\dagger \bar{\sigma}^\mu \chi_i) \\ &+ N \frac{\alpha_s}{8\pi} \frac{a}{f_{PQ}} G\tilde{G}, \end{aligned} \tag{2.28}$$

where the PQ symmetry is realized by the shift of the axion in Eq. (2.24) with χ_i ’s kept invariant.

Relation between f_{PQ} and Λ_5 in naive dimensional analysis

The effective Lagrangian obtained above can be rewritten in terms of a complex field satisfying $U U^\dagger = 1$,

$$U = e^{i5a/f_{PQ}}, \tag{2.29}$$

where the factor 5 is introduced to take into account that the physical domain of the axion field is $a/f_{PQ} \in [0, 2\pi/5)$, as shown in Appendix C. The result is

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \left(\frac{f_{PQ}}{5} \right)^2 \partial^\mu U^* \partial_\mu U - 5 \frac{\partial_\mu a}{f_{PQ}} (\chi^\dagger \bar{\sigma}^\mu \chi) + \dots \tag{2.30}$$

where the kinetic term is canonically normalized. In this equation, the sum over composite baryons only shows explicitly the unique type of baryon made out of three fermions, which happens to be the baryon χ with PQ charge $Q_\chi = -5$ defined in Eq. (2.11), albeit now being massive.

Applying Naïve Dimensional Analysis (NDA) [62,63] to the Lagrangian in Eq. (2.30), it follows that

$$\mathcal{L}_{\text{eff}} = \left(\frac{\Lambda_5}{4\pi} \right)^2 \partial^\mu U^* \partial_\mu U + \left(\frac{4\pi}{\Lambda_5} \right) \partial_\mu a (\chi^\dagger \bar{\sigma}^\mu \chi) + \dots \tag{2.31}$$

leading to the identification

$$\Lambda_5 \simeq \frac{4\pi f_{PQ}}{5}. \tag{2.32}$$

Customarily, the axion scale f_a is defined reabsorbing in it the QCD anomaly factor N ,

$$f_a \equiv \frac{f_{PQ}}{N}. \tag{2.33}$$

Coupling to gluons and domain walls

Because of the periodicity of the instanton potential, the anomalous coupling of the axion to gluons breaks explicitly $U(1)_{PQ}$ to a discrete symmetry $S(m)$,

$$S(m) : a \longrightarrow a + \frac{2\pi m}{N} f_{PQ}, \quad m \in \mathbb{Z}. \tag{2.34}$$

Nevertheless, not all $S(m)$ transformations are nontrivial, as some of them are equivalent via gauge transformations (see Appendix C). The physical discrete symmetry corresponds to the quotient $S_{\text{phys}} = S/\mathbb{Z}_5$, where \mathbb{Z}_5 is the center of the $SU(5)$ group [64]. This implies that the QCD potential has $\dim[S_{\text{phys}}]$ degenerate minima and therefore a number of domain walls $N_{DW} = \dim[S_{\text{phys}}]$ will be generated when the axion field takes a VEV, as this breaks spontaneously the discrete symmetry,

$$N_{DW} = \frac{|N|}{5}. \tag{2.35}$$

Any theory with $N_{DW} > 1$ has a domain wall problem: domain walls could dominate the energy density of the universe and overclose it. It will be seen further below that in our theory indeed $N_{DW} > 1$, and in consequence a pre-inflationary PQ-transition will be assumed to avoid this issue (see e.g. [65] and references therein). Besides, we also assume that the highest temperature after inflation is lower than Λ_5 to avoid the production of massive particles in the dynamical sector, as some of them are stable due to the \mathbb{Z}_2 unbroken subgroup of the PQ symmetry, leading to an unacceptably large relic density.

2.4 Planck suppressed operators

It has been argued that quantum gravity may violate all global symmetries. In particular, Planck suppressed operators which are not PQ invariant could be dangerous for axion solutions to the strong CP problem, since they can unacceptably displace the minimum of the axion potential from the CP conserving point.

Within our model, because of gauge invariance and chirality, the lowest dimensional operator of this type has mass dimension nine, as previously argued: it is the operator in Eq. (2.20), whose VEV breaks PQ spontaneously. This significantly strong Planck suppression suggest that our model can be protected from those gravitational issues. This is to be contrasted with the usual expectation in axion models which allow lower dimension effective operators of gravitational origin, e.g. dimension five couplings as in Eq. (1.5).

The prefactors of the effective operator are relevant and they can be settled using NDA [62,63], resulting in:

$$\mathcal{L}_{\mathcal{P}\mathcal{Q}} = c \frac{1}{4\pi} \frac{1}{M_{\text{Pl}}^5} \frac{1}{2!4!} \bar{5} \bar{5} 10 \bar{5} \bar{5} 10, \tag{2.36}$$

at around the Planck scale. Here, c would be generically of order one and a combinatorial factor due to the presence of identical fields has been explicitly included in the definition of the operator.⁷ In order to quantify its impact on the location of the minimum of the axion potential, it is necessary to express it in terms of the low-energy composite fields. NDA leads to

$$\mathcal{L}_{\mathcal{P}\mathcal{Q}} = c \frac{(4\pi)^2}{2!4!} \left(\frac{N}{5}\right)^9 \frac{f_a^9}{M_{\text{Pl}}^5} e^{-i\frac{10}{N}a/f_a} + \text{h.c.} \tag{2.37}$$

⁷ Consistently, this would correspond to a combinatorial factor of 1 in the corresponding Feynman rules.

The resulting axion potential, including as well the QCD contribution reads⁸

$$V(a) = -m_a^2 f_a^2 \cos\left(\frac{a}{f_a}\right) - c \frac{(4\pi)^2}{4!} \left(\frac{N}{5}\right)^9 \frac{f_a^9}{M_{\text{Pl}}^5} \cos\left(\frac{10}{N} \frac{a}{f_a} + \delta\right), \tag{2.38}$$

where δ is the relative phase between the Planck-suppressed operator in Eq. (2.37) and the QCD vacuum parameter. The displacement of the axion VEV with respect to the CP conserving minimum is then given by

$$|\Delta\bar{\theta}_{eff}| = |c \sin(\delta)| \frac{2(4\pi)^2}{4!} \left(\frac{N}{5}\right)^8 \frac{f_a^7}{M_{\text{Pl}}^5 m_a^2}, \tag{2.39}$$

which is strongly constrained by the experimental limit on the neutron electric dipole moment (EDM). For a given implementation of the $SU(5)$ theory, this indicates an upper bound on the f_a value needed to avoid to fine-tune the coefficient of the gravitationally induced effective operator.

There is a certain degree of uncertainty when using power counting arguments in the present context, though, which may change the prefactors significantly. As illustration, if f_a is taken as the PQ physics scale (instead of saturating it by $\Lambda_5 \sim 4\pi f_{PQ}/5$ as in NDA), the operator in Eq. (2.36) would translate into

$$\mathcal{L}_{\mathcal{P}\mathcal{Q}} = c \frac{1}{2!4!} \frac{f_a^9}{M_{\text{Pl}}^5} e^{-i\frac{10}{N}a/f_a} + \text{h.c.}, \tag{2.40}$$

instead of Eq. (2.37). The displacement induced on the effective QCD vacuum angle would then be significantly smaller, depending on the value of the anomaly factor N in a given realization of the chiral confining $SU(5)$ theory.

We will apply next the analysis above to two examples of the confining chiral $SU(5)$ theory, which differ in the QCD charges of the exotic fermions $\{\psi_{\bar{5}}, \psi_{10}\}$, corresponding respectively to a reducible and irreducible QCD representation \mathbf{R} . In the first model $R = 3 + \bar{3}$, while $R = 8$ will be assumed in the second model. While the former requires four exotic fermions (instead of just two for the second option), its matter content is smaller in terms of number of degrees of freedom.

⁸ The QCD axion potential is approximated here by a cosine dependence, since we are only interested in the displacement of the minimum where that approximation is perfectly valid. For the correct dependence using chiral Lagrangians at NLO see Ref. [66].

Planck suppressed operators and neutrino masses

In addition to the dimension nine operator in Eq. (2.36), other operators with lower dimensionality may be present, e.g.⁹:

$$\mathcal{L}_{PQ} = c \frac{1}{4\pi} \frac{1}{M_{\text{Pl}}^5} \frac{1}{2!} \bar{\mathbf{5}} \bar{\mathbf{5}} \mathbf{10} LH + \frac{c_v}{M_{\text{Pl}}} (LH)^2, \quad (2.41)$$

where L and H denote the lepton and Higgs doublets, respectively. The combination of these two operators breaks the PQ symmetry,¹⁰ modifying the axion potential and thus displacing θ_{eff} . However, for pseudo-Dirac neutrinos (where the size of the observed neutrino masses is dominated by a Dirac Yukawa coupling), the impact of these operators is a negligible correction to the coefficient of the dimension nine operator discussed above in Eq. (2.36). Nevertheless, depending on the mechanism responsible for realistic neutrino masses, the presence of fields beyond the SM ones may or may not allow for additional dangerous operators. For instance, for the seesaw type I mechanism the following terms should be considered

$$\mathcal{L} = \frac{1}{M_{\text{Pl}}^3} (\bar{\mathbf{5}} \bar{\mathbf{5}} \mathbf{10} \bar{N}) + M \bar{N} N + y_\nu \bar{N} LH, \quad (2.42)$$

where N denotes a singlet fermion, M is the Majorana scale, and y_ν its Yukawa coupling. The combination of the couplings present in Eq. (2.42) generates the dimension nine operator discussed above,

$$\mathcal{L} \propto \frac{1}{M_{\text{Pl}}^2} \frac{1}{M_{\text{Pl}}^2} \frac{1}{M} (\bar{\mathbf{5}} \bar{\mathbf{5}} \mathbf{10} \bar{\mathbf{5}} \bar{\mathbf{5}} \mathbf{10}), \quad (2.43)$$

albeit with a milder suppression by a factor M_{Pl}/M . In consequence, the gravity-induced operator $(\bar{\mathbf{5}} \bar{\mathbf{5}} \mathbf{10} \bar{N})$ can be dangerous and result in a larger displacement of θ_{eff} than that considered in this paper. A simple option to avoid this type of operators would be to gauge B-L in the SM sector with three right-handed neutrinos. In this case, the gauged B-L symmetry would be spontaneously broken the vev of a scalar field, leaving a residual gauge Z_2 (under which N and LH are odd) that forbids the two Planck-suppressed operators considered above, see Eqs. (2.41–2.42).

3 Model I: color-triplet fermions

In the first model, the exotic $\{\psi_{\bar{\mathbf{5}}}, \psi_{\mathbf{10}}\}$ fermions appear in the fundamental representation of QCD, alike to SM quarks,

⁹ We thank the referee for pointing out this coupling.

¹⁰ Because any global symmetry is expected to be broken by gravitational effects, B-L may not be an exact low energy symmetry and a Planck suppressed Majorana contribution to neutrino masses may be present, although numerically negligible.

with

$$\mathbf{R} = \mathbf{3} + \bar{\mathbf{3}}, \quad (3.1)$$

as shown in Table 3. The $[SU(3)_c]^3$ anomalies are then automatically cancelled due to the the four distinct $SU(5)$ fermions present. Being the latter massless, at the classical level this spectrum has four independent $U(1)$ global chiral symmetries. One combination is broken by non-perturbative $SU(5)$ effects, and three would remain unbroken for vanishing α_s , one of them being the PQ symmetry. The dimension of the (pseudo)real representation is then

$$n = 6. \quad (3.2)$$

As indicated in Eq. (2.7), the global chiral symmetries correspond to $SU(6)_{\bar{\mathbf{5}}} \times SU(6)_{\mathbf{10}} \times U(1)_{PQ}$ for $\alpha_s = 0$, which is explicitly broken by QCD down to

$$\begin{aligned} &SU(6)_{\bar{\mathbf{5}}} \times SU(6)_{\mathbf{10}} \times U(1)_{PQ} \\ &\xrightarrow{\alpha_s \neq 0} SU(3)_c \times U(1)_{V, \bar{\mathbf{5}}} \times U(1)_{V, \mathbf{10}}. \end{aligned} \quad (3.3)$$

That is, only QCD plus two global $U(1)$ symmetries remain unbroken for $\alpha_s \neq 0$, while $U(1)_{PQ}$ is broken by the non-perturbative QCD effects. The two surviving $U(1)$ symmetries are the left-over of the four classical $U(1)$ symmetries associated to the four independent massless fermions of this model (see Table 3), as two were explicitly broken by anomalous couplings at the quantum level: respectively $SU(5)$ and QCD interactions.

The question of whether the QCD group $SU(3)_c$ is indeed the surviving unbroken group after chiral symmetry breaking, as indicated in Eqs. (2.16), (2.17) and (2.21), deserves a specific discussion. To see this, let us note that an $SO(6)$ subgroup of the global symmetry $SU(6)_{\bar{\mathbf{5}}} \times SU(6)_{\mathbf{10}}$ satisfies the 't Hooft anomaly consistency conditions. Besides, the condensates $(\mathbf{10} \mathbf{10} \mathbf{10} \bar{\mathbf{5}})$ and $(\bar{\mathbf{5}} \bar{\mathbf{5}} \mathbf{10} \bar{\mathbf{5}} \bar{\mathbf{5}} \mathbf{10})$ can be $SO(6)$ singlets. This means that the unbroken subgroup G of the global symmetry $SU(6)_{\bar{\mathbf{5}}} \times SU(6)_{\mathbf{10}}$ contains $SO(6)$, i.e.

Table 3 Model I: charges of exotic fermions under the confining gauge group $SU(5) \times SU(3)_c$, the PQ symmetry and the spontaneously broken global $U(1)$ symmetries. The left-handed Weyl fermions $\psi_{\bar{\mathbf{5}}}$ and $\psi_{\mathbf{10}}$ are massless and singlets of the SM electroweak gauge group; their QCD representation has been indicated as an additional subscript

	$SU(5)$	$SU(3)_c$	$U(1)_{PQ}$	$U(1)_{V, \bar{\mathbf{5}}}$	$U(1)_{V, \mathbf{10}}$
$\psi_{(\bar{\mathbf{5}}, 3)}$	$\bar{\mathbf{5}}$	$\mathbf{3}$	−3	1	0
$\psi_{(\bar{\mathbf{5}}, \bar{\mathbf{3}})}$	$\bar{\mathbf{5}}$	$\bar{\mathbf{3}}$	−3	−1	0
$\psi_{(\mathbf{10}, 3)}$	$\mathbf{10}$	$\mathbf{3}$	+1	0	1
$\psi_{(\mathbf{10}, \bar{\mathbf{3}})}$	$\mathbf{10}$	$\bar{\mathbf{3}}$	+1	0	−1

$G \supset SO(6)$.¹¹ The $SU(3)$ subgroup of $SO(6)$ is then obtained by identifying the vector representation of $SO(6)$ to be $\mathbf{3} + \bar{\mathbf{3}}$. Therefore, it is clear that an $SU(3)$ global symmetry remains unbroken below the confinement scale.

It should be noted that an $SO(6)$ subgroup of $SU(6)_{\bar{5}} \times SU(6)_{10}$ is not uniquely determined, and hence, the unbroken $SO(6)$ is not in general aligned to the one which contains $SU(3)_c$ for $\alpha_s = 0$. However, it has been argued that, among the possible condensate channels, the minimum of the potential corresponds to the one preserving QCD for $\alpha_s \neq 0$ [51]. Thus, we find that it is most likely that the $SU(5)$ dynamics with the non-vanishing chiral and PQ condensates in Eqs. (2.16) and (2.20) preserves $SU(3)_c$.

The $U(1)_{V,\bar{5}}$ and $U(1)_{V,10}$ symmetries are generically broken by those condensates. In fact, the chiral condensate in Eq. 2.16 breaks spontaneously $U(1)_{\bar{5}} \times U(1)_{10}$ down to a $U(1)$, where the number of positive and negative charges with respect to this $U(1)$ is balanced at the QCD preserving vacuum. The PQ condensate could also break this remaining $U(1)$ if the quarks in the condensates are all either in the $\mathbf{3}$ or in the $\bar{\mathbf{3}}$ representation of QCD. Accordingly, the model predicts one or two additional pGBs which obtain tiny masses from the higher dimensional gravitational operators in Eq. (2.36). As those pGBs decouple from the thermal bath at a temperature much higher than the weak scale, the contribution of each pGB to the effective number of relativistic species is suppressed, i.e. $\Delta N_{\text{eff}} \simeq 0.03$, and hence the model is consistent with the current constraint $N_{\text{eff}} = 2.99^{+0.34}_{-0.33}$ [67].

In this model, the PQ current in Eq. (2.25) takes the form

$$j_{\text{PQ}}^\mu = -3\psi_{(\bar{5},3)}^\dagger \bar{\sigma}^\mu \psi_{(\bar{5},3)} - 3\psi_{(\bar{5},3^*)}^\dagger \bar{\sigma}^\mu \psi_{(\bar{5},3^*)} \quad (3.4)$$

$$+ \psi_{(10,3)}^\dagger \bar{\sigma}^\mu \psi_{(10,3)} + \psi_{(10,3^*)}^\dagger \bar{\sigma}^\mu \psi_{(10,3^*)} = f_{\text{PQ}} \partial^\mu a. \quad (3.5)$$

For fermions in the fundamental of QCD ($T(\bar{\mathbf{3}}) = T(\mathbf{3}) = 1/2$), the QCD anomaly factor and the domain wall number in Eqs. (2.10) and (2.35) are then, respectively,

$$N = -10, \quad N_{\text{DW}} = 2. \quad (3.6)$$

The resulting domain wall problem is avoided here by the assumption of pre-inflationary PQ transition, as earlier explained.

Planck suppressed operators

For the value of N in Eq. (3.6), the displacement induced on the QCD $\bar{\theta}$ parameter by the NDA estimation of the

¹¹ Our arguments do not depend on whether $G = SO(6)$ or $G \supseteq SO(6)$, although we expect that $G = SO(6)$.

Planck suppressed operator in Eq. (2.37) is illustrated in Fig. 1 (left panel). The figure also depicts the stringent constraint imposed by the experimental bound on the neutron EDM [1], which for the most conservative estimates [2] translates into the requirement

$$\mathbf{3} + \bar{\mathbf{3}} \text{ Model: } f_a \lesssim (4.5 \times 10^8, 1 \times 10^9) \text{ GeV}, \quad \text{for } |c \sin(\delta)| \in (0.001, 1). \quad (3.7)$$

The softer constraint that follows if NDA is disregarded and substituted by the estimation stemming from Eq. (2.40) is also depicted.¹² The degree of tuning of the operator coefficient is illustrated in Fig. 2 (left panel).

Axion dark matter

In the misalignment mechanism, the relic axion abundance Ω_a depends then on two variables: the axion decay constant f_a , and the initial misalignment angle $\theta_i = a_i/f_a$. For $|\theta_i| \ll \pi$ it reads [40]

$$\Omega_a h^2 = 0.35 \left(\frac{\theta_i}{0.001} \right)^2 \left(\frac{f_a}{3 \times 10^{17} \text{ GeV}} \right)^{1.17}, \quad (3.8)$$

where h is the present Hubble parameter. If axions were to explain the total relic dark matter density $\Omega_{\text{DM}} h^2 \simeq 0.12$ [67], the f_a value required for an initial misalignment angle in the range $\theta_i \in (0.1, 3)$ is

$$f_a \simeq 2 \times 10^{10} - 5 \times 10^{12} \text{ GeV}. \quad (3.9)$$

However, for values of $\theta_i \sim \pi$, the anharmonicities of the QCD potential are important and f_a can be as low as [39, 40, 68]

$$f_a \sim 2 \times 10^9 \text{ GeV}. \quad (3.10)$$

These two estimations of the f_a values that allow axions to constitute all the dark matter of the universe have been depicted in Fig. 1 by green bands dubbed, respectively, “preferred” and “correct” densities. The bounds on f_a stemming from measurements of the neutrino burst duration of SN1987a are depicted in red.¹³

The lower f_a value in Eq. (3.10) is about a factor of two too large to be compatible with that required in Eq. (3.7) by the neutron EDM bounds. This option requires a fine-tuning of the coefficient c of the Planck suppressed operator

¹² The explicit breaking can be further suppressed if, for example, we assume supersymmetry with R-symmetry. In such cases, f_a in the preferred value for the DM relic density is also allowed, though we do not pursue such possibilities further in this paper.

¹³ Assuming the fiducial density profile for the proto-neutron star in Ref. [31], the bound reads $f_a > 10^8 \text{ GeV}$.

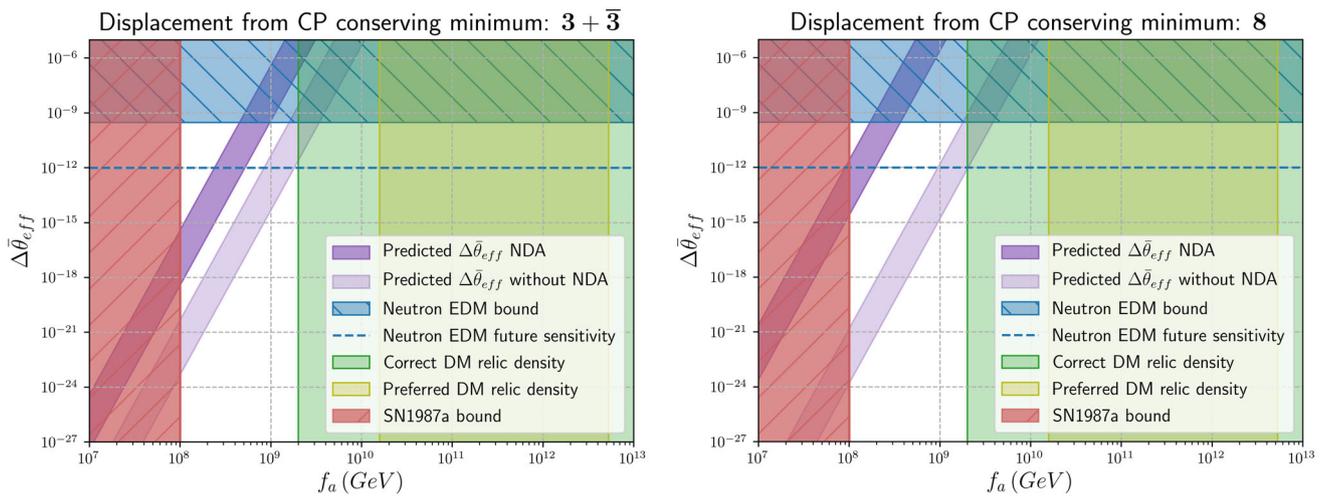


Fig. 1 Displacement of the CP conserving minimum due to the presence of the Planck suppressed operator, for $|c \sin(\delta)| \in (0.001, 1)$ and assuming NDA. The regions excluded by the experimental limits on the neutron EDM are depicted in blue, while future prospects are indicated

by a dashed blue line. The red band corresponds to the SN1987a bounds axion-nucleon couplings [31]. The f_a values that suffice to account for the full content of dark matter in the pre-inflationary scenario are depicted in green

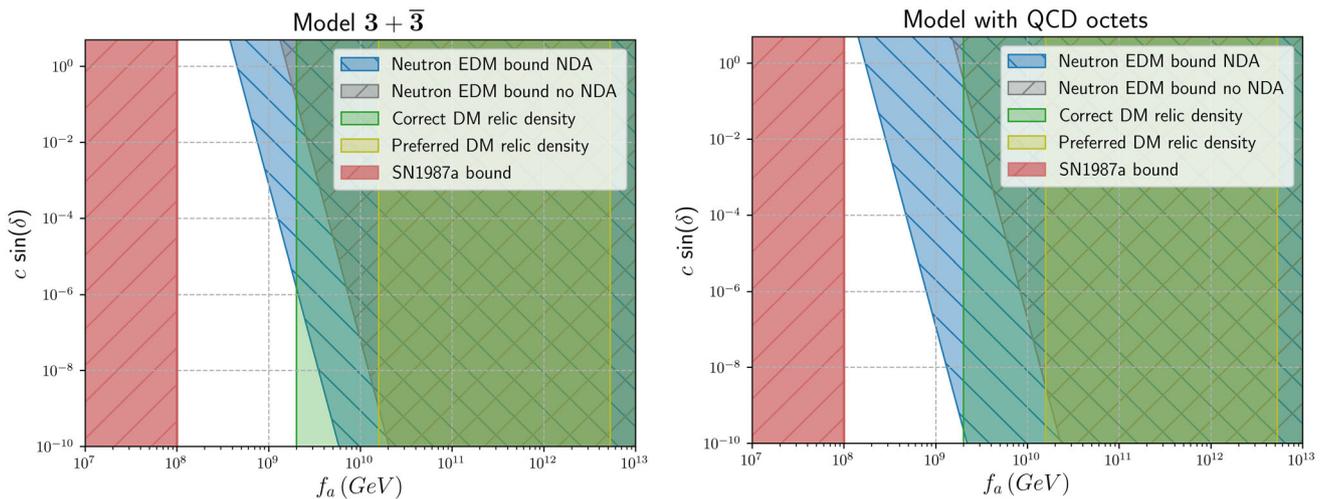


Fig. 2 Allowed values for the Planck suppressed operator coefficient $|c \sin(\delta)|$, for axion dark matter compatible with neutron EDM and SN1987a bounds

of $\mathcal{O}(10^{-7})$, to be compared with the typical adjustment by 54 orders of magnitude in axion models with dimension five Planck-suppressed operators. Furthermore, for a misalignment angle close to π and low inflation scales, lower values of f_a are possible and the fine-tuning of c could be avoided altogether, even in this most conservative case of the NDA estimate of the effect. Conversely, would the NDA prefactors be disregarded, $\mathcal{O}(1)$ coefficients for the Planck suppressed operator are seen to be allowed in a large fraction of the parameter space.

4 Model II: color-octet fermions

We consider here an alternative realization with only one $\{\psi_{\bar{5}}, \psi_{10}\}$ set of two fermions charged under $SU(5)$ and belonging to the adjoint representation of QCD, see Table 4. All gauge anomalies cancel then automatically. This model is less economical than Model I, though, from the point of view of the total number of exotic degrees of freedom.

In the limit of vanishing α_s the number of flavours of the $SU(5)$ fermionic sector is

Table 4 Model II: charges of exotic fermions under the confining gauge group $SU(5) \times SU(3)_c$. Their PQ charges are shown as well. The left-handed Weyl fermions $\psi_{\bar{5}}$ and ψ_{10} are massless and singlets of the SM electroweak gauge group

	$SU(5)$	$SU(3)_c$	$U(1)_{PQ}$
$\psi_{\bar{5}}$	$\bar{\mathbf{5}}$	$\mathbf{8}$	-3
ψ_{10}	$\mathbf{10}$	$\mathbf{8}$	$+1$

$$n = 8, \tag{4.1}$$

and thus the global chiral symmetries of the $SU(5)$ Lagrangian correspond to

$$SU(8)_{\bar{5}} \times SU(8)_{10} \times U(1)_{PQ} \xrightarrow{\alpha_s \neq 0} SU(3)_c. \tag{4.2}$$

In consequence, in this model only QCD remains unbroken for $\alpha_s \neq 0$, and hence no light pNGs appear associated with the spontaneous breaking of the non-abelian global symmetries.

To see whether the QCD gauge group remains ultimately unbroken, note that an $SO(8)$ subgroup of the global symmetry $SU(8)_{\bar{5}} \times SU(8)_{10}$ satisfies the 't Hooft anomaly consistency conditions, while the condensates $\langle \mathbf{10} \mathbf{10} \mathbf{10} \bar{\mathbf{5}} \rangle$ and $\langle \bar{\mathbf{5}} \bar{\mathbf{5}} \mathbf{10} \bar{\mathbf{5}} \bar{\mathbf{5}} \mathbf{10} \rangle$ can be $SO(8)$ singlets. In this case, we find that the unbroken subgroup G contains $SO(8)$, i.e. $G \supset SO(8)$. The $SU(3)$ subgroup of $SO(8)$ is realized as the special maximal embedding where the vector representation of $SO(8)$ is identified with the octet of $SU(3)$ (see e.g. [69]). Thus, it is again clear that an $SU(3)$ global symmetry remains unbroken below the confinement scale, with non-vanishing $\langle \mathbf{10} \mathbf{10} \mathbf{10} \bar{\mathbf{5}} \rangle$ and $\langle \bar{\mathbf{5}} \bar{\mathbf{5}} \mathbf{10} \bar{\mathbf{5}} \bar{\mathbf{5}} \mathbf{10} \rangle$ condensates. Finally, the $SO(8)$ symmetry is aligned with that containing $SU(3)_c$ once $\alpha_s \neq 0$ is taken into account. This shows that, also in this model, it is most likely for the $SU(5)$ dynamics to preserve $SU(3)_c$.

For fermions in the adjoint of QCD ($T(\mathbf{R}) = 3$), the QCD anomaly factor and the domain wall number in Eqs. (2.10) and (2.35) are, respectively,

$$N = -30, \quad N_{\text{DW}} = 6. \tag{4.3}$$

Planck suppressed operators

Figure 1 (right panel) shows the displacement induced by the operator in Eq. (2.37) on the QCD vacuum parameter, for the value of N expected from NDA, see Eq. (4.3), which implies the requirement

$$\begin{aligned} \mathbf{8} \text{ Model: } f_a &\lesssim (1.7 \times 10^8, 3.7 \times 10^8) \text{ GeV,} \\ \text{for } |c \sin(\delta)| &\in (0.001, 1), \end{aligned} \tag{4.4}$$

to comply with the experimental bound on the neutron EDM. This constraint is stronger than that for Model I for QCD-triplet exotic fermions, Eq. (3.7). A softer constraint if NDA was disregarded in the estimation is also illustrated.

Axion dark matter

The comparison between Eq. (4.4) and the f_a ranges in Eqs. (3.9) and (3.10) shows that this model with exotic fermions in the adjoint of QCD is more in tension than model I, if axions are to explain all the dark matter of the universe without recurring to fine tunings. Figure 1 (right panel) illustrates this situation. For the NDA estimation of Planck suppressed couplings, f_a as required by dark matter is a factor of five too large with respect to the neutron EDM constraint; this translates into the requirement of a $\mathcal{O}(10^{-10})$ fine-tuning of the coefficient c of the Planck suppressed operator, as illustrated in Fig. 2 (right panel). Alternatively, the present model could explain a subdominant fraction of the dark matter content.

A comparison without NDA power counting estimates is also illustrated: non-fine tuned values of the coefficient c are then compatible with the axion accounting for the ensemble of dark matter, while complying with EDM limits. Overall, the uncertainty on the estimations of non-perturbative gravitational effects, and on the f_a values required to account for dark matter, is large enough to still consider this model as a candidate scenario for purely axionic dark matter.

5 Conclusions

We have presented a novel composite axion theory that solves the strong CP problem and has as singular features:

- A gauge confining symmetry which is chiral, unlike usual axicolor models which use vectorial fermions. In consequence, the PQ symmetry is automatic, without any need to invoke extra symmetries.
- Exotic $SU(5)$ fermions in (pseudo)real representations of QCD.
- Inherent protection from dangerous quantum non-perturbative gravitational effects.

The gauge group selected and illustrated here is chiral $SU(5)$ with two massless fermions in its $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations and a confining scale much higher than that of QCD. A new spectrum of composite bound states is expected.

We showed that the 't Hooft anomaly conditions for the global symmetries of the exotic fermionic sector imply that the non-abelian global symmetries must be spontaneously broken. The global abelian symmetries, e.g. the PQ symmetry, must also be spontaneously broken for the theory to be

phenomenologically viable, resulting in a dynamical invisible axion. Furthermore, the PQ invariance is the analogous of the $B - L$ symmetry in $SU(5)$ Grand Unified Theory (GUT).

We have determined the fermionic operators with lowest dimension which may condense and induce spontaneous breaking. Because of $SU(5)$ gauge invariance, six is the minimal dimension for the operator whose VEV may break the exotic flavour symmetries. An even higher dimensional condensate is needed in order to break PQ invariance: the VEV of a dimension nine operator. The latter is also the lowest dimensional effective operator which could result from gravitational quantum contributions, breaking explicitly the PQ symmetry, as these effects must respect gauge invariance. Its high dimensionality is at the heart of the inherent protection of this theory with respect to the gravitational issue.

We have developed two complete ultraviolet completions of the chiral confining $SU(5)$ theory, which only differ in the (pseudo)real QCD representations chosen for the exotic fermions: a reducible $\mathbf{3} + \bar{\mathbf{3}}$ representation for Model I, and the irreducible adjoint in model II. The former is more economical in terms of the total number of degrees of freedom. Both models are phenomenological viable and largely protected from quantum gravitational concerns. Remarkably, in the case of exotic fermions in the fundamental of QCD, the f_a range allowed if axions are to explain the full dark matter content of the universe can be compatible with that required to avoid a fine-tuned coefficient for the Planck suppressed operator. For octet-colour fermions the compatibility is marginal but still possible.

The basic novel idea of the construction is to use a chiral confining group, which provides an automatic implementation of PQ invariance. The most economic avenue is to implement it via just two exotic fermions in (pseudo)real representations of QCD. In this perspective, we have briefly explored other confining groups as well. For instance, a chiral and confining gauge $SU(4)$ symmetry would be a viable alternative, although it does not enjoy a sufficient protection from gravitational issues, at least in the case of only two exotic fermions. Even the smaller chiral confining $SU(3)$ symmetry is possible, although the versions with only two exotic fermions require very high-dimensional representations of the confining group and, again, they are less protected from gravitational issues than the $SU(5)$ case (see Appendix A). Nevertheless, as the estimation of gravitational effects is somehow uncertain, it may be pertinent to dedicate specific studies to these alternative directions.

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A Alternative confining groups: $SU(3)$ and $SU(4)$

$SU(4)$ Model

It is also possible to construct a chiral axicolor model that fulfills the requirements explained in the introduction (see Sect. 1) with an $SU(4)$ gauge group (Table 5).

It is easy to check that this theory is free from gauge anomalies¹⁴ and that the global $U(1)_{PQ}$ in Table 6 is exact at the classical level but explicitly broken by $SU(3)_c$ instantons, solving therefore the strong CP problem à la Peccei–Quinn.

However we will not study this model further since it lacks special protection against PQ-violating gravity operators. Indeed the lowest dimensional non-renormalizable operators that break PQ and could be generated by quantum gravity effects are

$$\mathcal{L}_{Planck} \propto \frac{c}{M_{Pl}^2} \frac{1}{4!} \bar{\mathbf{4}}\bar{\mathbf{4}}\bar{\mathbf{4}}\bar{\mathbf{4}} + \frac{c}{M_{Pl}^2} \frac{1}{4!} \mathbf{10}\mathbf{10}\mathbf{10}\mathbf{10}, \quad (\text{A.1})$$

and would lead to unacceptable deviations from the CP-conserving minimum (barring a fine-tuning of c by several tens of orders of magnitude) and thus spoil the solution of the strong CP problem.

¹⁴ $[SU(4)]^3$ anomaly: $8A(\bar{\mathbf{4}}) + A(\mathbf{10}) = 0$, since $A(\bar{\mathbf{4}}) = -1$, and $A(\mathbf{10}) = 8$.

Table 5 Charges of exotic fermions under the confining gauge group $SU(4) \times SU(3)_c$. The left-handed Weyl fermions $\psi_{\bar{4}}$ and ψ_{10} are massless and singlets of the SM electroweak gauge group

	$SU(4)$	$SU(3)_c$	$U(1)_{PQ}$
$\psi_{\bar{4}}$	$\bar{\mathbf{4}}$	$\mathbf{8}$	-3
ψ_{10}	$\mathbf{10}$	$\mathbf{1}$	4

Table 6 Charges of exotic fermions under the confining gauge group $SU(4) \times SU(3)_c$. The left-handed Weyl fermions $\psi_{\bar{35}}$ and ψ_{70} are massless and singlets of the SM electroweak gauge group

	$SU(4)$	$SU(3)_c$	$U(1)_{PQ}$
$\psi_{\bar{35}}$	$\bar{\mathbf{35}}$	$\mathbf{8}$	-98
ψ_{70}	$\mathbf{70}$	$\mathbf{8}$	56

Table 7 Charges of exotic fermions under the confining gauge group $SU(3) \times SU(3)_c$. The left-handed Weyl fermions $\psi_{\bar{15}'}$ and ψ_{42} are massless and singlets of the SM electroweak gauge group

	$SU(3)$	$SU(3)_c$	$U(1)_{PQ}$
$\psi_{\bar{15}'}$	$\bar{\mathbf{15}'}$	\mathbf{R}	-119
ψ_{42}	$\mathbf{42}$	\mathbf{R}	35

Alternative SU(4)

It is possible to implement the confining gauge $SU(4)$ solution in a setup in which two exotic fermions belong to the adjoint of QCD, by considering higher $SU(4)$ representations, e.g. $\bar{\mathbf{35}}$ and $\mathbf{70}$ since $A(\bar{\mathbf{35}}) = -112$, $A(\mathbf{70}) = +112$, see Table 6.

SU(3) Model

The idea of using a chiral confining theory as solution to the strong CP problem can also be implemented with a confining $SU(3)$ gauge group, for instance via the fermionic content in Table 7.

This theory is anomaly free since $A(\bar{\mathbf{15}'}) = -A(\mathbf{42}) = 77$ and again the exotic fermions transform as pseudoreal representations \mathbf{R} of the QCD group. However, the theory is not as protected against PQ breaking gravitational effect as the $SU(5)$ case, since the corresponding effective operators can appear at dimension six,

$$\mathcal{L}_{Planck} \propto \frac{c}{M_{Pl}^2} \frac{1}{2!2!} \bar{\mathbf{15}'} \bar{\mathbf{15}'} \mathbf{42} \mathbf{42}, \tag{A.2}$$

and in consequence we will not further elaborate on this model.

B Anomaly factors

In this appendix we review the group theoretical factors that are relevant when computing the global or gauge anomalies in our theory. Let us consider a given conserved current j_μ^a that corresponds to the symmetry associated to the generator T^a . In the presence of the gauge field F_b the divergence of the current reads,

$$\partial^\mu j_\mu^a = \frac{\alpha_i}{8\pi} C_{group}^{abc} F_{b\mu\nu} \tilde{F}_c^{\mu\nu}, \tag{B.1}$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\sigma\rho} F_{\sigma\rho}$, the fine structure constant of the corresponding gauge interaction is denoted by $\alpha_i = \frac{g_i^2}{4\pi}$ and the group theoretical factor C_{group} is given by

$$C_{group}^{abc} = \sum Tr [T^a \{t_R^b, t_R^c\}], \tag{B.2}$$

where the sum runs over all fermionic representations \mathbf{R} of the gauge group t_R^a . Throughout the paper the fermionic degrees of freedom will be expressed in terms of left-handed Weyl fermions.

This formula is used for three different cases, depending on whether the groups are abelian or non-abelian and whether the anomaly is cubic or mixed.

- Non-abelian cubic anomalies:

$$[SU(N)]^3 : C_{group}^{abc} = \sum_R Tr [t_R^a \{t_R^b, t_R^c\}] \equiv d^{abc} \sum_R A(\mathbf{R}), \tag{B.3}$$

where $A(\mathbf{R})$ denotes anomaly coefficient or triality of the representation \mathbf{R} .

- Abelian cubic anomalies:

$$[U(1)]^3 : C_{group} = \sum_R Tr [Q_R \{Q_R, Q_R\}] = 2 \sum_R Q_R^3, \tag{B.4}$$

where Q_R denotes the $U(1)$ charge of the corresponding fermion.

- Mixed anomalies:

$$[SU(N)]^2 \times U(1) : C_{group}^{bc} = \sum_R Tr [Q_R \{t_R^b, t_R^c\}] \equiv \delta^{bc} \sum_R Q_R 2T(\mathbf{R}), \tag{B.5}$$

where $T(\mathbf{R})$ is the Dynkin index of the representation \mathbf{R} .

These group theoretical factors are tabulated [70] and can also be computed with the Mathematica package LieART [71].

Table 8 Global chiral properties at the classical level, in the limit of vanishing α_s

	$SU(8)_{\bar{5}}$	$SU(8)_{10}$	$U(1)_{PQ}$
$\psi_{\bar{5}}$	\square	$\mathbf{1}$	-3
ψ_{10}	$\mathbf{1}$	\square	1

C Axion field domain

Our definition of the PQ symmetry according to the charges in Table 1 corresponds to the following transformations:

$$\begin{aligned} \psi_{10} &\longrightarrow e^{i\alpha} \psi_{10}, \\ \psi_{\bar{5}} &\longrightarrow e^{-3i\alpha} \psi_{\bar{5}}, \end{aligned} \tag{C.1}$$

where α is the rotation angle. However, the domain of α does not correspond to the full range $[0, 2\pi)$ since some of these rotations are equivalent due to gauge transformations. In particular, the center of $SU(5)$ is the discrete symmetry $Z[SU(5)] = \mathbb{Z}_5$, that corresponds to the following gauge transformations:

$$\begin{aligned} \psi_{10} &\longrightarrow e^{2\pi i k/5} \psi_{10} e^{2\pi i k/5} = e^{4\pi i k/5} \psi_{10}, \\ \psi_{\bar{5}} &\longrightarrow e^{-2\pi i k/5} \psi_{\bar{5}}, \end{aligned} \tag{C.2}$$

for $k = \{0, 1, 2, 3, 4\}$. It is easy to see now that a PQ transformation with angle $\alpha = 2\pi/5$ is gauge equivalent to $\alpha = 2\pi$ with $k = 2$. As a consequence, the axion transforms under PQ as

$$\frac{a}{f_{PQ}} \longrightarrow \frac{a}{f_{PQ}} + \alpha \tag{C.3}$$

and its physical domain is

$$\frac{a}{f_{PQ}} \in [0, 2\pi/5). \tag{C.4}$$

D ‘t Hooft anomaly matching conditions: is $SU(8)_{\bar{5}} \times SU(8)_{10} \times U(1)_{PQ}$ spontaneously broken?

If the $SU(5)$ group confines without breaking the chiral symmetries in Table 8, the consistency of the theory implies the existence of massless baryons in the low energy that match the global anomalies of the high-energy theory. However, for some theories these ‘t Hooft anomaly matching conditions cannot be satisfied as a consequence of the properties of the fermionic representations. It must be then concluded that these theories can only be realized via spontaneous breaking of its chiral symmetries. This will be the case for the $SU(8)_{\bar{5}} \times SU(8)_{10}$ chiral symmetry of our $SU(5)$ model.

Let us first compute the global anomalies in the high energy theory (in terms of the fundamental quarks $\psi_{\bar{5}}$ and ψ_{10}):

$$[SU(8)_{\bar{5}}]^3 : 5 \times A(\square) = 5, \tag{D.1}$$

$$[SU(8)_{10}]^3 : 10 \times A(\square) = 10, \tag{D.2}$$

$$U(1)_{PQ} \times [SU(8)_{\bar{5}}]^2 : 5 \times 2 T(\square) Q_{\bar{5}} = -15, \tag{D.3}$$

$$U(1)_{PQ} \times [SU(8)_{10}]^2 : 10 \times 2 T(\square) Q_{10} = 10, \tag{D.4}$$

$$[U(1)_{PQ}]^3 : 8 \left(5 (Q_{\bar{5}})^3 + 10 (Q_{10})^3 \right) = -1000. \tag{D.5}$$

If chiral symmetries remain unbroken these anomalies will match those in the low energy theory in terms of the bound states. The simplest $SU(5)$ singlet that can be formed in this theory consists of three fundamental quarks, $\chi \equiv 10 \bar{5} \bar{5}$. Can it match the previous anomalies? The transformation properties of χ under the global symmetries are

$$SU(8)_{\bar{5}} : \mathbf{8} \times \mathbf{8} = \mathbf{28} + \mathbf{36}, \tag{D.6}$$

$$SU(8)_{10} : \mathbf{8}, \tag{D.7}$$

$$U(1)_{PQ} : -3 - 3 + 1 = -5. \tag{D.8}$$

In consequence, there are two possible representations for the baryon χ under $SU(8)_{\bar{5}} \times SU(8)_{10} \times U(1)_{PQ}$: $(\mathbf{28}, \mathbf{8}, -5)$ and $(\mathbf{36}, \mathbf{8}, -5)$. If the low energy contains a number n_{28} and n_{36} of baryons transforming under each representation respectively, then the anomalies are given by

$$\begin{aligned} [SU(8)_{\bar{5}}]^3 : & 8 (n_{28} A(\mathbf{28}) + n_{36} A(\mathbf{36})) \\ & = 32(n_{28} + 3n_{36}), \end{aligned} \tag{D.9}$$

$$\begin{aligned} [SU(8)_{10}]^3 : & 28 n_{28} A(\mathbf{8}) + 36 n_{36} A(\mathbf{8}) \\ & = 4(7 n_{28} + 9 n_{36}), \end{aligned} \tag{D.10}$$

$$\begin{aligned} U(1)_{PQ} \times [SU(8)_{\bar{5}}]^2 : & 8 Q_{\chi} (n_{28} 2 T(\mathbf{28}) \\ & + n_{36} 2 T(\mathbf{36})) = -80(3 n_{28} + 5 n_{36}), \end{aligned} \tag{D.11}$$

$$\begin{aligned} U(1)_{PQ} \times [SU(8)_{10}]^2 : & Q_{\chi} (28 n_{28} 2 T(\mathbf{8}) \\ & + 36 n_{36} 2 T(\mathbf{8})) = -20(7 n_{28} + 9 n_{36}), \end{aligned} \tag{D.12}$$

$$\begin{aligned} [U(1)_{PQ}]^3 : & 8 (28 n_{28} + 36 n_{36}) (Q_{\chi})^3 \\ & = -4000 (7 n_{28} + 9 n_{36}). \end{aligned} \tag{D.13}$$

It is easy to see that there is no way of matching these anomalies with $n_{28}, n_{36} \in \mathbb{N}$. If we would alternatively consider 5-quark bound states, there are two options: $\bar{5} \bar{5} \bar{5} \bar{5} \bar{5}$ and $\mathbf{10} \mathbf{10} \mathbf{10} \mathbf{10} \mathbf{10}$.

For the first one, $\bar{5} \bar{5} \bar{5} \bar{5} \bar{5}$, the transformation properties are:

$$SU(8)_{\bar{5}} : \mathbf{8} \times \mathbf{8} \times \mathbf{8} \times \mathbf{8} \times \mathbf{8} = (\bar{\mathbf{56}})$$

Table 9 Dynkin index $T(\mathbf{R})$ and anomaly factor $A(\mathbf{R})$ of the different representations of $SU(8)_5$ that are contained in $[8]^5$

R	$2T(\mathbf{R})$	$A(\mathbf{R})$
$\overline{56}$	13	-5
$\overline{504}$	213	75
$\overline{792}$	713	1287
$\overline{1008}$	524	294
$\overline{1680}$	1088	1066
$\overline{1512}'$	883	777

$$+ 4(\overline{504}) + (\overline{792}) + 5(\overline{1008}) + 6(\overline{1512}') \tag{D.14}$$

$$+ 5(\overline{1680}) + 4(\overline{1848}), \tag{D.15}$$

$$SU(8)_{10} : \mathbf{1}, \tag{D.16}$$

$$U(1)_{PQ} : 5(-3) = -15. \tag{D.17}$$

For **10 10 10 10 10** the transformation properties are:

$$SU(8)_5 : \mathbf{1}, \tag{D.18}$$

$$SU(8)_{10} : \mathbf{8} \times \mathbf{8} \times \mathbf{8} \times \mathbf{8} \times \mathbf{8} \\ = (\overline{56}) + 4(\overline{504}) + (\overline{792}) + 5(\overline{1008}) + 6(\overline{1512}') \tag{D.19}$$

$$+ 5(\overline{1680}) + 4(\overline{1848}), \tag{D.20}$$

$$U(1)_{PQ} : 5(+1) = +5. \tag{D.21}$$

Repeating the analogous exercise to that in Eqs. (D.9)–(D.13), and using the properties of the representations of the 5-quark bound states in Table 9, it follows the same conclusion as before: the chiral symmetry must necessarily be spontaneously broken due to the impossibility of satisfying ‘t Hooft anomaly matching conditions.

This does not mean, however, that the full $SU(8)_5 \times SU(8)_{10} \times U(1)_{PQ}$ is completely spontaneously broken. Some subgroup can remain unbroken. In particular, it is shown in the body of the paper that it is possible to leave unbroken the $U(1)_{PQ}$ with the baryon in Eq. (2.11) satisfying the anomaly matching conditions. Nevertheless, this possibility is phenomenologically excluded due to the absence of coloured massless quarks in Nature.

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