Fermions and vector particles tunnelling from non-rotating weakly isolated horizons

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Abstract Fermions and vector particles tunnelling from non-rotating weakly isolated horizons is investigated in this paper. By applying the WKB approximation to the Dirac equation and Proca equation, we obtain the emission spectrum and Hawking temperature of fermions and vector particles tunnelling from weakly isolated horizons. We consider the back reaction of emitted particles to the space-time, and get the corrected Hawking radiation spectrum. At last we discuss the information recovery of weakly isolated horizons.

1 Introduction

In 1974, Stephen Hawking discovered that black holes can release a black-body radiation due to quantum effects near the event horizon with the help of the Wick Rotation method [1,2] applied for the gravitational collapse. From Hawking's discovery, people know that black holes are not the final state of stars, and, with the emission of Hawking radiation, they could lose energy, and shrink. Later on, several different derivations of Hawking radiation relying on quantum field theory were proposed, which further confirms the existence of Hawking radiation [3–7]. Hawking's discovery that black holes emit Hawking radiation has greatly stimulated the development of black hole physics, and also provides some hints of underlying quantum gravity.

In recent years, semi-classical methods of modelling Hawking radiation as a tunnelling process have garnered lots of interest. Tunnelling provides a useful verification of thermodynamics properties of black holes and an alternate conceptual means for understanding the underlying physical process of black hole radiation. These methods use the

Wentzel–Kramers–Brillouin (WKB) approximation, and the tunnelling probability for the classically forbidden trajectory from inside to outside the horizon is given by

$$\Gamma = \exp \left[-\frac{2}{\hbar} \text{Im} I_0 \right], \quad (1)$$

where I_0 is the classical action of the trajectory to leading order in $\hbar$. There are two kinds of semi-classical approaches, and the difference is in how the action I_0 is calculated. The first black hole tunnelling method – Null Geodesic Method was proposed by Parikh and Wilczek [8,9], which followed from the work of Kraus and Wilczek [10–12]. They found the only part of the action that contributes an imaginary term is $\text{Im} I = \int_{r_{in}}^{r_{out}} p_r dr$, where p_r is the momentum of the emitted null s-wave. Although the original approach was used to calculate the tunnelling of massless particles near spherically symmetric Schwarzschild black holes, Refs. [13,14] extended this method to the tunnelling of charged particles. The other method is the Hamilton–Jacobi Ansatz used by Refs. [15,16], which is an extension of the complex path analysis of Padmanabhan et al. [17–20]. This method applies the WKB approximation to the Klein–Gordon equation, and the lowest order is the Hamilton–Jacobi equation. Then according to the symmetry of the metric, one can pick an appropriate ansatz for the action, and put it into the Hamilton–Jacobi equation to solve for the Hawking radiation of scalar particles. Later, Refs. [21–23] applied the WKB approximation to the Dirac Equation and Rarita–Schwinger equation, and obtained the Hawking radiation of fermions and gravitinos. Recently, Kruglov [24,25] investigated black hole radiation of vector particles by using the WKB approximation to the Proca equation, and first obtained the Hawking radiation of vector particles. In the second tunnelling method, the background of the space-time is fixed, and the standard Hawking purely thermal spectrum is recovered, while the first tunnelling method considers the back reaction of particles to

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the space-time, and obtains the corrected Hawking radiation spectrum. The tunnelling methods have been shown to be very robust, and have been successfully applied to a wide variety of interesting space-times and different kinds of particles [26–44]. However, most of the situations that have been investigated are stationary. Whether such results are right for more general black holes, especially for non-stationary black holes is an interesting question.

In this paper, we will investigate fermions and vector particles tunnelling from non-rotating weakly isolated horizons. Weakly isolated horizon is a new, quasi-local framework which was introduced by Ashtekar and his collaborators [45–48]. Compared with the event horizon, this framework does not need the knowledge of the overall space-time, and only involves quasi-local conditions, so it accords with the practical physical process. In this framework, black holes in equilibrium are described by Weakly Isolated Horizons (WIH). Weakly isolated horizons will cover stationary space-times. So investigations on the tunnelling from weakly isolated horizons are important problems. Reference [49] recovers the semi-classical emission rate in the tunnelling process by applying the Null Geodesic Method of Parikh and Wilczek and Hamilton–Jacobi Ansatz for scalar particles to a weakly isolated horizon. In this paper we investigate the tunnelling effect of fermions and vector particles near non-rotating weakly isolated horizons. We obtain Hawking’s purely thermal spectrum without considering the back reaction of emitted particles to the space-time, which will lead to the information loss puzzle [50–53]. Then we consider the back reaction of emitted particles to the space-time, and get the corrected Hawking radiation spectrum. At last the information recovery of weakly isolated horizons based on the methods in Refs. [54–56] is discussed. Another solution for the black hole information loss problem was developed by Cordain, which is founded on the Bohr-like approach to black hole quantum physics [57,58]. These two solutions are consistent with each other, which has been highlighted in Ref. [59]. Like Ref. [21], we assume that the change of black hole angular momentum due to the spin of the emitted particle is negligible. This is a good approximation for the black hole with mass much larger than the Planck mass.

The remainder of this paper is organized as follows. In Sect. 2, we briefly review the definition of the WIH and the geometry near it. In Sect. 3, we investigate the tunnelling of fermions near non-rotating weakly isolated horizons. In Sect. 4, the tunnelling of vector particles is investigated. In Sect. 5, we discuss the information recovery of weakly isolated horizons. In the last section, discussions and conclusions are given.

In this paper, we take the convenient units of $k = h = c = G = 1$.

2 The near horizon geometry of weakly isolated horizons

In this section we briefly review the geometric properties of WIH [45–47]. Generally speaking, the WIH (Δ) is a non-expansion null hypersurface, with almost stationary inner geometry $[\mathcal{L}, D_a]l^a = 0$ on the horizon, where l^a is the generator of the horizon. $\mathcal{L}$ is Lie derivative, and D_a is the induced derivative operator on the horizon Δ .

For WIH, it is convenient to introduce the Bondi-like coordinates (u, r, θ, φ) , which are well-defined on the horizon, to study the behavior near the neighborhood of WIH [48]. In this coordinate system, we choose a null tetrad $\{n, l, m, \bar{m}\}$ as follows

$$\begin{aligned} l^a &= \frac{\partial}{\partial u} + U \frac{\partial}{\partial r} + X \frac{\partial}{\partial \zeta} + \bar{X} \frac{\partial}{\partial \bar{\zeta}}, \\ n^a &= -\frac{\partial}{\partial r}, \\ m^a &= \omega \frac{\partial}{\partial r} + \xi_3 \frac{\partial}{\partial \zeta} + \xi_4 \frac{\partial}{\partial \bar{\zeta}}, \\ \bar{m}^a &= \bar{\omega} \frac{\partial}{\partial r} + \bar{\xi}_3 \frac{\partial}{\partial \zeta} + \bar{\xi}_4 \frac{\partial}{\partial \bar{\zeta}}, \end{aligned} \quad (2)$$

where $(\zeta, \bar{\zeta})$ are complex coordinates on the section of Δ , and $U \equiv X \equiv \omega \equiv 0$ on Δ . Here, following the notation in Ref. [45], equalities restricted to Δ are denoted by “ $\equiv$ ”. n^a and l^a are future directed.

We take the space-time metric g_{ab} to have a signature $(-, +, +, +)$, so the metric can be expressed as

$$g^{ab} = m^a \bar{m}^b + \bar{m}^a m^b - n^a l^b - l^a n^b. \quad (3)$$

The matrix form of the metric is

$$g^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 2(U + |\omega|^2) & X + (\bar{\omega}\xi_3 + \omega\bar{\xi}_4) & \bar{X} + (\bar{\omega}\xi_4 + \omega\bar{\xi}_3) \\ 0 & X + (\bar{\omega}\xi_3 + \omega\bar{\xi}_4) & 2\xi_3\bar{\xi}_4 & |\xi_3|^2 + |\xi_4|^2 \\ 0 & \bar{X} + (\bar{\omega}\xi_4 + \omega\bar{\xi}_3) & |\xi_3|^2 + |\xi_4|^2 & 2\bar{\xi}_3\xi_4 \end{pmatrix}. \quad (4)$$

Choosing Bondi gauge $\nabla_n(l, n, m, \bar{m}) = 0$, which means that the tetrad (2) is parallel transported along n^a in space-time, we have the following gauge conditions on spin coefficients

$$\tau = \nu = \gamma = \alpha + \bar{\beta} - \pi = \mu - \bar{\mu} = 0, \quad \varepsilon - \bar{\varepsilon} \equiv \kappa \equiv 0. \quad (5)$$

The definition of WIH [45–47] implies that there exists a one form ω_a on Δ which satisfies the following relationship, $\mathcal{L}_l \omega^a \equiv 0$ and $D_a l^b \equiv \omega_a l^b$, where D_a is the induced covariant derivative on Δ . In terms of the Newman–Penrose formalism, ω_a can be explicitly expressed as

$$\begin{aligned}\omega_a &= -(\varepsilon + \bar{\varepsilon})n_a + (\alpha + \bar{\beta})\bar{m}_a + (\bar{\alpha} + \beta)m_a \\ &= -(\varepsilon + \bar{\varepsilon})n_a + \pi\bar{m}_a + \bar{\pi}m_a.\end{aligned}\quad (6)$$

The equation $\mathcal{L}_l\omega^a \hat{=} 0$ means that $(\varepsilon + \bar{\varepsilon})$ is constant on horizon Δ . By definition, the surface gravity of the horizon is $\kappa := l^a\omega_a = \varepsilon + \bar{\varepsilon}$.

Reference [45] establishes the first law of weakly isolated horizon thermodynamics,

$$\delta M = \frac{1}{8\pi}\kappa\delta A + \Omega\delta J, \quad (7)$$

where M is the horizon mass, A is the area of the cross section of WIHs, Ω is the angular velocity of the horizon, and $J = -\frac{1}{8\pi}\oint_S(\omega_a\psi^a)dS$ is the angular momentum. The expressions of the surface gravity, angular velocity and horizon energy of weakly isolated horizons are given by

$$\begin{aligned}\kappa &= \frac{R^4 - 4J^2}{2R^3\sqrt{R^4 + 4J^2}}, \quad \Omega = \frac{2J}{R\sqrt{R^4 + 4J^2}}, \\ M &= \frac{\sqrt{R^4 + 4J^2}}{2R},\end{aligned}\quad (8)$$

where R is the horizon radius, and is defined as

$$R \equiv \sqrt{\frac{A}{4\pi}}. \quad (9)$$

A is the area of cross section of the horizon, so the entropy of WIHs is

$$S = \frac{A}{4} = \pi R^2. \quad (10)$$

The commutators of the null tetrad $[l^a, n^a]$ and $[m^a, \bar{m}^a]$ tell us that

$$\begin{aligned}\frac{\partial U}{\partial r} &= (\varepsilon + \bar{\varepsilon}) + \bar{\pi}\bar{\omega} + \pi\omega \hat{=} \varepsilon + \bar{\varepsilon}, \\ \frac{\partial X}{\partial r} &= \bar{\pi}\bar{\xi}_4 + \pi\xi_3, \\ \frac{\partial \omega}{\partial r} &= \bar{\pi} + \bar{\lambda}\bar{\omega} + \mu\omega \hat{=} \bar{\pi}, \\ \frac{\partial \xi_3}{\partial r} &= \bar{\lambda}\bar{\xi}_4 + \mu\xi_3, \\ \frac{\partial \xi_4}{\partial r} &= \bar{\lambda}\bar{\xi}_3 + \mu\xi_4.\end{aligned}\quad (11)$$

The angular momentum of non-rotating weakly isolated horizons is zero, that is, $J = 0$. According to the definition of angular momentum of WIHs and Eq. (6), we find that the spin coefficient

$$\pi \hat{=} 0, \quad (12)$$

where $\pi = \bar{m}^a l^b \nabla_b n_a$. So for non-rotating weakly isolated horizons, the behavior of functions U , X , ω , ξ_3 and ξ_4 near Δ is

$$\begin{aligned}U &= (\varepsilon + \bar{\varepsilon})r + O(r^2), \\ X &= O(r^2), \\ \omega &= O(r^2), \\ \xi_3 &= O(1), \\ \xi_4 &= O(1).\end{aligned}\quad (13)$$

The inverse metric near the horizon then becomes

$$g^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 2\kappa r + O(r^2) & O(r^2) & O(r^2) \\ 0 & O(r^2) & 2\xi_3\bar{\xi}_4 & |\xi_3|^2 + |\xi_4|^2 \\ 0 & O(r^2) & |\xi_3|^2 + |\xi_4|^2 & 2\bar{\xi}_3\xi_4 \end{pmatrix}. \quad (14)$$

The metric near the horizon could be written as

$$g_{\mu\nu} = \begin{pmatrix} -2\kappa r + O(r^2) & 1 & O(r^2) \\ 1 & 0 & 0 \\ O(r^2) & 0 & h_{ab} \end{pmatrix}, \quad (15)$$

where h_{ab} is the intrinsic metric of two-dimensional space section of WIHs.

We introduce Schwarzschild-like coordinates by a coordinate transformation

$$dt = du + \frac{dr}{2\kappa r}, \quad (16)$$

and the asymptotic metric near the horizon becomes

$$\begin{aligned}ds^2 &= [-2\kappa r + O(r^2)]dt^2 + O(r)dt dr \\ &\quad + \left[\frac{1}{2\kappa r} + O(r) \right] dr^2 \\ &\quad + O(r^2)dt dx^i + O(r)dr dx^i + h_{ij}dx^i dx^j.\end{aligned}\quad (17)$$

It turns out that, in order to consider the tunnelling effect near weakly isolated horizons, we need $g_{tt} \sim O(r)$, $g_{tr} \sim O(1)$, $g_{rr} \sim O(r^{-1})$, $g_{ti} \sim O(r)$, and $g_{ri} \sim O(1)$. The asymptotic metric near weakly isolated horizons in Schwarzschild-like coordinate can be expressed as

$$ds^2 = -2\kappa r dt^2 + \frac{1}{2\kappa r} dr^2 + h_{ij}dx^i dx^j. \quad (18)$$

In next section, we use this metric to discuss the tunnelling of emitted particles from non-rotating weakly isolated horizons.

3 Fermions tunnelling from non-rotating weakly isolated horizons

In this section, we calculate fermions tunnelling from non-rotating weakly isolated horizons by using the methods in Refs. [21, 22]. The inverse metric of non-rotating weakly isolated horizons can be expressed as from Eq. (18)

$$g^{\mu\nu} = \begin{pmatrix} -\frac{1}{a} & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & c & e \\ 0 & 0 & e & d \end{pmatrix}, \quad (19)$$

where $a = 2\kappa r$, and for convenience we denote c, d, e as elements of the inverse metric of h_{ij} .

The Dirac equation is

$$i\gamma^\mu D_\mu \Psi + \frac{m}{\hbar} \Psi = 0, \quad (20)$$

where m is the rest mass, and

$$\begin{aligned} D_\mu &= \partial_\mu + \Omega_\mu, \\ \Omega_\mu &= \frac{1}{2} i \Gamma_\mu^{\alpha\beta} \Sigma_{\alpha\beta}, \\ \Sigma_{\alpha\beta} &= \frac{1}{4} i [\gamma^\alpha, \gamma^\beta]. \end{aligned}$$

The γ^μ matrices are the γ metric in curved space-time, and satisfy

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \times 1. \quad (21)$$

We choose the γ^μ matrices as follows

$$\begin{aligned} \gamma^0 &\equiv \frac{1}{\sqrt{a}} \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}, \quad \gamma^1 \equiv \sqrt{a} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \\ \gamma^2 &\equiv \sqrt{c} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \\ \gamma^3 &\equiv \sqrt{d - \frac{e^2}{c}} \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} + \frac{e}{\sqrt{c}} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \end{aligned} \quad (22)$$

With the help of the WKB approximation, we employ the following ansatz for the spin-up (i.e. +ve r -direction) Dirac field [21, 22]

$$\begin{aligned} \Psi_\uparrow(t, r, \theta, \phi) &= \begin{pmatrix} A(t, r, \theta, \phi) \xi_\uparrow \\ B(t, r, \theta, \phi) \xi_\uparrow \end{pmatrix} \exp \left[\frac{i}{\hbar} I_\uparrow(t, r, \theta, \phi) \right] \\ &= \begin{pmatrix} A(t, r, \theta, \phi) \\ 0 \\ B(t, r, \theta, \phi) \\ 0 \end{pmatrix} \exp \left[\frac{i}{\hbar} I_\uparrow(t, r, \theta, \phi) \right], \end{aligned} \quad (23)$$

where $I_\uparrow$ is the classical action of the trajectory of spin-up Dirac particles, and is given by

$$I_\uparrow = I_{\uparrow 0} + \hbar I_{\uparrow 1} + \hbar^2 I_{\uparrow 2} + \dots \quad (24)$$

Inserting the ansatz above into the Dirac Equation (20), one gets the equations to the leading order in $\hbar$,

$$\gamma^\mu \partial_\mu I_{\uparrow 0} \begin{pmatrix} A \\ 0 \\ B \\ 0 \end{pmatrix} + \mu \begin{pmatrix} A \\ 0 \\ B \\ 0 \end{pmatrix} = 0. \quad (25)$$

The equations can be expressed in the following explicit form

$$\begin{aligned} -\frac{1}{\sqrt{a}} i A \partial_t I_{\uparrow 0} - \sqrt{a} B \partial_r I_{\uparrow 0} + m A &= 0, \\ \sqrt{c} B \partial_\theta I_{\uparrow 0} + \sqrt{d - \frac{e^2}{c}} i B \partial_\phi I_{\uparrow 0} + \frac{e}{\sqrt{c}} B \partial_\phi I_{\uparrow 0} &= 0, \\ \frac{i B}{\sqrt{a}} \partial_t I_{\uparrow 0} - \sqrt{a} A \partial_r I_{\uparrow 0} + m B &= 0, \\ \sqrt{c} A \partial_\theta I_{\uparrow 0} + \sqrt{d - \frac{e^2}{c}} i A \partial_\phi I_{\uparrow 0} + \frac{e}{\sqrt{c}} A \partial_\phi I_{\uparrow 0} &= 0. \end{aligned} \quad (26)$$

We look for the solution to Eq. (26) in the form

$$I_{\uparrow 0} = -Et + W(r) + J(\theta, \phi) + K, \quad (27)$$

where $E = -\partial_t I_{\uparrow 0}$ is the energy of emitted particles, and K is a complex constant. Putting Eq. (27) into Eq. (26), one obtains

$$\frac{iAE}{\sqrt{a}} - \sqrt{a} B W'(r) + m A = 0, \quad (28)$$

$$\sqrt{c} B J_\theta + \sqrt{d - \frac{e^2}{c}} i B J_\phi + \frac{e}{\sqrt{c}} B J_\phi = 0,$$

$$\frac{iBE}{\sqrt{a}} + \sqrt{a} A W'(r) - m B = 0,$$

$$\sqrt{c} A J_\theta + \sqrt{d - \frac{e^2}{c}} i A J_\phi + \frac{e}{\sqrt{c}} A J_\phi = 0, \quad (29)$$

where $W'(r) = \partial_r I_{\uparrow 0}$, $J_\theta = \partial_\theta I_{\uparrow 0}$ and $J_\phi = \partial_\phi I_{\uparrow 0}$.

In the massless case $m = 0$, Eqs. (28) and (29) have two possible solutions

$$\begin{aligned} A &= -iB, \quad W'(r) = \frac{E}{a} \equiv W'_+, \\ A &= iB, \quad W'(r) = -\frac{E}{a} \equiv W'_-. \end{aligned} \quad (30)$$

Solving for $W_\pm(r)$ yields

$$W_\pm(r) = \pm \int \frac{E}{a} dr = \pm \int \frac{E}{2\kappa r} dr, \quad (31)$$

where $W_+ > 0$ corresponds to fermions moving away from the black hole, and $W_- < 0$ corresponds to fermions moving toward the black hole. The probabilities of crossing the horizon each way are proportional to

$$\begin{aligned}
\text{Prob[out]} &= \exp \left[-\frac{2}{\hbar} \text{Im} I \right] \\
&= \exp \left[-\frac{2}{\hbar} (\text{Im} W_+ + \text{Im} K) \right], \\
\text{Prob[in]} &= \exp \left[-\frac{2}{\hbar} \text{Im} I \right] \\
&= \exp \left[-\frac{2}{\hbar} (\text{Im} W_- + \text{Im} K) \right]. \quad (32)
\end{aligned}$$

The incoming particles crossing the horizon should have a 100% chance of entering the black hole, so we have $\text{Im} K = -\text{Im} W_-$. Considering $W_+ = -W_-$, the probability of a particle tunnelling from inside to outside the horizon is

$$\Gamma = \frac{\text{Prob[out]}}{\text{Prob[in]}} = \exp(-4\text{Im} W_+), \quad (33)$$

where we set $\hbar$ to unity. Integrating Eq. (31) around the pole at the horizon $r_H = 0$, we obtain the imaginary part of W_+ ,

$$\text{Im} W_+ = \frac{\pi}{2\kappa} E. \quad (34)$$

Then the tunnelling probability is

$$\Gamma = \exp \left(-\frac{2\pi}{\kappa} E \right) = \exp \left(-\frac{E}{T} \right), \quad (35)$$

with the Hawking temperature

$$T = \frac{\kappa}{2\pi}. \quad (36)$$

κ is the surface gravity of weakly isolated horizons. The expected Hawking temperature is recovered.

In the massive case that $m \neq 0$, solving Eqs. (28) and (29) for A and B leads to the result:

$$\begin{aligned}
B \left[a W'(r) W'(r) - \frac{E^2}{a} - m^2 \right] &= 0, \\
A \left[a W'(r) W'(r) - \frac{E^2}{a} - m^2 \right] &= 0. \quad (37)
\end{aligned}$$

The solution is

$$W'(r) = \pm \frac{\sqrt{E^2 + am^2}}{a}. \quad (38)$$

Solving for $W(r)$ yields

$$W_{\pm}(r) = \pm \int \frac{\sqrt{E^2 + am^2}}{a} dr = \pm \int \frac{\sqrt{E^2 + 2\kappa r m^2}}{2\kappa r} dr. \quad (39)$$

Similarly, integrating around the pole at the horizon, we obtain

$$\text{Im} W_+ = \frac{\pi}{2\kappa} E, \quad (40)$$

which is the same as Eq. (34). So the result in the massive case is the same as the massless case. The spin-down case proceeds in a manner fully analogous to the spin-up case

discussed above, and has the same tunnelling probability and Hawking temperature.

We have discussed the tunnelling of spin-up fermions from non-rotating weakly isolated horizons by following the methods in Refs. [21, 22], and get the standard Hawking temperature. However, there is naturally a question, how about fermions tunnelling with arbitrary spin directions? At last of this section, we investigate the tunnelling of fermions with arbitrary spin directions. We make the ansatz as follows

$$\Psi(t, r, \theta, \phi) = \begin{pmatrix} A(t, r, \theta, \phi) \\ B(t, r, \theta, \phi) \\ C(t, r, \theta, \phi) \\ D(t, r, \theta, \phi) \end{pmatrix} \exp \left[\frac{i}{\hbar} I(t, r, \theta, \phi) \right], \quad (41)$$

where

$$I = I_0 + \hbar I_1 + \hbar^2 I_2 + \dots \quad (42)$$

When $B = D = 0$, Eq. (41) returns to the spin-up case (23).

Putting Eqs. (41) and (42) into Eq. (20), the equations to the leading order in $\hbar$ are

$$\begin{aligned}
&\left(-\frac{i}{\sqrt{a}} \partial_t I_0 + m \right) A + (-\sqrt{a} \partial_r I_0) C \\
&+ \left(-\sqrt{c} \partial_2 I_0 + i \sqrt{d - \frac{e^2}{c}} \partial_3 I_0 - \frac{e}{\sqrt{c}} \partial_3 I_0 \right) D = 0, \\
&\left(-\frac{i}{\sqrt{a}} \partial_t I_0 + m \right) B \\
&+ \left(-\sqrt{c} \partial_2 I_0 - i \sqrt{d - \frac{e^2}{c}} \partial_3 I_0 - \frac{e}{\sqrt{c}} \partial_3 I_0 \right) C \\
&+ \sqrt{a} (\partial_1 I_0) D = 0, \\
&(-\sqrt{a} \partial_1 I_0) A \\
&+ \left(-\sqrt{c} \partial_2 I_0 + i \sqrt{d - \frac{e^2}{c}} \partial_3 I_0 - \frac{e}{\sqrt{c}} \partial_3 I_0 \right) B \\
&+ \left(\frac{i}{\sqrt{a}} \partial_0 I_0 + m \right) C = 0, \\
&\left(-\sqrt{c} \partial_2 I_0 - i \sqrt{d - \frac{e^2}{c}} \partial_3 I_0 - \frac{e}{\sqrt{c}} \partial_3 I_0 \right) A \\
&+ \sqrt{a} \partial_1 I_0 B + \left(\frac{i}{\sqrt{a}} \partial_0 I_0 + m \right) D = 0. \quad (43)
\end{aligned}$$

There exists a solution of the form

$$I_0 = -Et + W(r) + J(\theta, \phi) + K, \quad (44)$$

where $E = -\partial_t I_0$ is the energy of emitted particles, and K is a complex constant.

Inserting Eq. (44) into Eq. (43), one obtains the following matrix equation

$$\Lambda (A, B, C, D)^T = 0, \quad (45)$$

where Λ is a 4×4 matrix, and the components are expressed as

$$\begin{aligned} \Lambda_{11} &= \frac{iE}{\sqrt{a}} + m, \quad \Lambda_{12} = 0, \quad \Lambda_{13} = -\sqrt{a}W', \\ \Lambda_{14} &= -\left(\sqrt{c}J_\theta - i\sqrt{d - \frac{e^2}{c}}J_\phi + \frac{e}{\sqrt{c}}J_\phi\right), \quad \Lambda_{21} = 0, \\ \Lambda_{22} &= \left(\frac{iE}{\sqrt{a}} + m\right), \quad \Lambda_{23} = -\left(\sqrt{c}J_\theta + i\sqrt{d - \frac{e^2}{c}}J_\phi + \frac{e}{\sqrt{c}}J_\phi\right), \\ \Lambda_{24} &= \sqrt{a}W', \quad \Lambda_{31} = -\sqrt{a}W', \\ \Lambda_{32} &= -\left(\sqrt{c}J_\theta - i\sqrt{d - \frac{e^2}{c}}J_\phi + \frac{e}{\sqrt{c}}J_\phi\right), \\ \Lambda_{33} &= \frac{-iE}{\sqrt{a}} + m, \quad \Lambda_{34} = 0, \\ \Lambda_{41} &= -\left(\sqrt{c}J_\theta + i\sqrt{d - \frac{e^2}{c}}J_\phi + \frac{e}{\sqrt{c}}J_\phi\right), \quad \Lambda_{42} = \sqrt{a}W', \\ \Lambda_{43} &= 0, \quad \Lambda_{44} = \frac{-iE}{\sqrt{a}} + m, \end{aligned} \quad (46)$$

where $W' = \frac{\partial W}{\partial r}$, $J_\theta = \frac{\partial J}{\partial \theta}$, and $J_\phi = \frac{\partial J}{\partial \phi}$.

Homogeneous system of linear equations (45) possesses nontrivial solutions if the determinant of the matrix Λ equals zero, that is, $\det \Lambda = 0$. After the calculation, we have

$$\det \Lambda = \frac{[-E^2 + a(cJ_\theta^2 + dJ_\phi^2 - m^2 + 2eJ_\theta J_\phi) + a^2W'^2]^2}{a^2} = 0. \quad (47)$$

We get immediately

$$(W')^2 = \frac{E^2 - acJ_\theta^2 - adJ_\phi^2 - 2aeJ_\theta J_\phi + am^2}{a^2}, \quad (48)$$

and

$$\begin{aligned} W_\pm &= \pm \int \frac{\sqrt{E^2 - acJ_\theta^2 - adJ_\phi^2 - 2aeJ_\theta J_\phi + am^2}}{a} dr \\ &= \pm \int \frac{\sqrt{E^2 - 2\kappa rcJ_\theta^2 - 2\kappa rdJ_\phi^2 - 4\kappa reJ_\theta J_\phi + 2\kappa rm^2}}{2\kappa r} dr. \end{aligned} \quad (49)$$

Integrating W_+ around the pole at the horizon $r_H = 0$, we obtain

$$\text{Im} W_+ = \frac{\pi}{2\kappa} E, \quad (50)$$

which is the same as Eq. (34). So fermions tunnelling with arbitrary spin directions has the same probability of radiation and Hawking temperature.

4 Vector particles tunnelling from non-rotating weakly isolated horizons

In this section, we investigate vector particles tunnelling from non-rotating weakly isolated horizons. The Proca equations for vector particles are [24, 25]

$$D_\mu \psi^{\nu\mu} + \frac{m^2}{\hbar^2} \psi^\nu = 0, \quad (51)$$

$$\psi_{\nu\mu} = D_\nu \psi_\mu - D_\mu \psi_\nu = \partial_\nu \psi_\mu - \partial_\mu \psi_\nu, \quad (52)$$

where D_μ are covariant derivatives, and $\psi_\nu = (\psi_0, \psi_1, \psi_2, \psi_3)$. From the definition, $\psi^{\nu\mu}$ is an anti-symmetrical tensor, so using the equation

$$D_\mu \psi^{\nu\mu} = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \psi^{\nu\mu}), \quad (53)$$

the Proca equations become

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \psi^{\nu\mu}) + \frac{m^2}{\hbar^2} \psi^\nu = 0. \quad (54)$$

According to the WKB approximation, the solution exists in the form

$$\psi_\mu = c_\mu \exp \left[\frac{i}{\hbar} I(t, r, \theta, \varphi) \right], \quad (55)$$

where I is the classical action of the trajectory, and is defined as

$$I = I_0 + \hbar I_1 + \hbar^2 I_2 + \dots \quad (56)$$

Substituting Eqs. (55) and (56) into the Proca equation (54), to the leading order in $\hbar$, one gets equations as follows

$$g^{\rho\mu} g^{\nu\lambda} (-c_\lambda \partial_\mu I_0 \partial_\rho I_0 + c_\rho \partial_\mu I_0 \partial_\lambda I_0) + m^2 g^{\nu\lambda} c_\lambda = 0. \quad (57)$$

Inserting the inverse metric (19) into Eq. (57), one can obtain

$$\begin{aligned} &c_0 \partial_1 I_0 \partial_1 I_0 + \frac{c}{a} c_0 \partial_2 I_0 \partial_2 I_0 + 2 \frac{e}{a} c_0 \partial_3 I_0 \partial_2 I_0 \\ &+ \frac{d}{a} c_0 \partial_3 I_0 \partial_3 I_0 - c_1 \partial_1 I_0 \partial_0 I_0 \\ &- \frac{c}{a} c_2 \partial_2 I_0 \partial_0 I_0 - \frac{e}{a} c_2 \partial_3 I_0 \partial_0 I_0 - \frac{e}{a} c_3 \partial_2 I_0 \partial_0 I_0 \\ &- \frac{d}{a} c_3 \partial_3 I_0 \partial_0 I_0 - \frac{m^2}{a} c_0 = 0, \\ &c_1 \partial_0 I_0 \partial_0 I_0 - acc_1 \partial_2 I_0 \partial_2 I_0 - 2eac_1 \partial_3 I_0 \partial_2 I_0 \\ &- dac_1 \partial_3 I_0 \partial_3 I_0 - c_0 \partial_0 I_0 \partial_1 I_0 \\ &+ cac_2 \partial_2 I_0 \partial_1 I_0 + eac_2 \partial_3 I_0 \partial_1 I_0 \\ &+ eac_3 \partial_2 I_0 \partial_1 I_0 + dac_3 \partial_3 I_0 \partial_1 I_0 + m^2 ac_1 = 0, \\ &\frac{c}{a} c_2 \partial_0 I_0 \partial_0 I_0 + \frac{e}{a} c_3 \partial_0 I_0 \partial_0 I_0 - cac_2 \partial_1 I_0 \partial_1 I_0 \\ &- aec_3 \partial_1 I_0 \partial_1 I_0 - e^2 c_3 \partial_3 I_0 \partial_2 I_0 \\ &- dcc_2 \partial_3 I_0 \partial_3 I_0 - \frac{c}{a} c_0 \partial_0 I_0 \partial_2 I_0 \\ &- \frac{e}{a} c_0 \partial_0 I_0 \partial_3 I_0 + acc_1 \partial_1 I_0 \partial_2 I_0 + aec_1 \partial_1 I_0 \partial_3 I_0 \end{aligned}$$

$$\begin{aligned}
& + e^2 c_2 \partial_3 I_0 \partial_3 I_0 + d c c_3 \partial_3 I_0 \partial_2 I_0 + m^2 c c_2 + m^2 e c_3 = 0, \\
& \frac{e}{a} c_2 \partial_0 I_0 \partial_0 I_0 + \frac{d}{a} c_3 \partial_0 I_0 \partial_0 I_0 - a e c_2 \partial_1 I_0 \partial_1 I_0 \\
& - a d c_3 \partial_1 I_0 \partial_1 I_0 \\
& - c d c_3 \partial_2 I_0 \partial_2 I_0 \\
& - e^2 c_2 \partial_3 I_0 \partial_2 I_0 - \frac{e}{a} c_0 \partial_0 I_0 \partial_2 I_0 - \frac{d}{a} c_0 \partial_0 I_0 \partial_3 I_0 \\
& + a e c_1 \partial_1 I_0 \partial_2 I_0 + a d c_1 \partial_1 I_0 \partial_3 I_0 \\
& + c d c_2 \partial_2 I_0 \partial_3 I_0 + e^2 c_3 \partial_2 I_0 \partial_2 I_0 \\
& + m^2 e c_2 + m^2 d c_3 = 0.
\end{aligned} \quad (58)$$

Similarly, there exists a solution of the form

$$I_0 = -Et + W(r) + J(\theta, \phi) + K, \quad (59)$$

where $E = -\partial_0 I_0 = -\partial_t I_0$ is the energy of emitted particles. Putting Eq. (59) into Eq. (58), we obtain

$$\begin{aligned}
& \left[W'(r) W'(r) + \frac{c}{a} J_\theta J_\theta + 2 \frac{e}{a} J_\theta J_\phi + \frac{d}{a} J_\phi J_\phi - \frac{m^2}{a} \right] c_0 \\
& + W'(r) E c_1 \\
& + \left[\frac{c}{a} J_\theta E + \frac{e}{a} E J_\phi \right] c_2 + \left[\frac{e}{a} E J_\theta + \frac{d}{a} E J_\phi \right] c_3 = 0,
\end{aligned} \quad (60)$$

$$\begin{aligned}
& E W'(r) c_0 + \left[E^2 - a c J_\theta J_\theta - 2 e a J_\theta J_\phi - d a J_\phi J_\phi + m^2 a \right] c_1 \\
& + \left[c a J_\theta W'(r) + e a J_\phi W'(r) \right] c_2 \\
& + \left[e a J_\theta W'(r) + d a J_\phi W'(r) \right] c_3 = 0,
\end{aligned} \quad (61)$$

$$\begin{aligned}
& \left[\frac{c}{a} E J_\theta + \frac{e}{a} E J_\phi \right] c_0 \\
& + \left[a c W'(r) J_\theta + a e W'(r) J_\phi \right] c_1 \\
& + \left[\frac{c}{a} E^2 - a c W'(r) W'(r) - d c J_\phi J_\phi + e^2 J_\phi J_\phi + m^2 c \right] c_2 \\
& + \left[\frac{e}{a} E^2 - a e W'(r) W'(r) - e^2 J_\theta J_\phi \right. \\
& \left. + d c J_\theta J_\phi + m^2 e \right] c_3 = 0,
\end{aligned} \quad (62)$$

$$\begin{aligned}
& \left[\frac{e}{a} E J_\theta + \frac{d}{a} E J_\phi \right] c_0 + \left[a e W'(r) J_\theta + a d W'(r) J_\phi \right] c_1 \\
& + \left[\frac{e}{a} E^2 - a e W'(r) W'(r) - e^2 J_\theta J_\phi + c d J_\theta J_\phi + m^2 e \right] c_2 \\
& + \left[\frac{d}{a} E^2 - a d W'(r) W'(r) \right. \\
& \left. - c d J_\theta J_\phi + e^2 J_\theta J_\phi + m^2 d \right] c_3 = 0,
\end{aligned} \quad (63)$$

where $W'(r) = \frac{\partial W}{\partial r}$, $J_\theta = \frac{\partial J}{\partial \theta}$ and $J_\phi = \frac{\partial J}{\partial \phi}$. One can treat the equations above as a matrix equation $\Lambda(c_0, c_1, c_2, c_3)^T = 0$, where Λ is a 4×4 matrix. The matrix equation possesses nontrivial solution if the determinant of the matrix Λ equals zero, that is, $\det \Lambda = 0$. After the calculation, we have

$$\begin{aligned}
& \det \Lambda \\
& = \frac{m^2 (c d - e^2) [-E^2 + a (c J_\theta^2 + d J_\phi^2 - m^2 + 2 e J_\theta J_\phi) + a^2 W'^2]^3}{a^3} \\
& = 0.
\end{aligned} \quad (64)$$

We obtain immediately

$$(W')^2 = \frac{E^2 - a (c J_\theta^2 + d J_\phi^2 - m^2 + 2 J_\theta J_\phi)}{a^2}, \quad (65)$$

and

$$\begin{aligned}
W_+(r) &= \int \frac{\sqrt{E^2 - a (c J_\theta^2 + d J_\phi^2 - m^2 + 2 J_\theta J_\phi)}}{a} dr \\
&= \int \frac{\sqrt{E^2 - 2 \kappa r (c J_\theta^2 + d J_\phi^2 - m^2 + 2 J_\theta J_\phi)}}{2 \kappa r} dr,
\end{aligned} \quad (66)$$

where we have used the relationship $a = 2 \kappa r$ in the second equation.

Integrating Eq. (66) around the pole at the horizon $r_H = 0$, we obtain

$$\text{Im} W_+ = \frac{\pi}{2 \kappa} E. \quad (67)$$

Therefor the tunnelling probability is

$$\begin{aligned}
\Gamma &= \exp \left(-\frac{2 \pi}{\kappa} E \right) \\
&= \exp \left(-\frac{E}{T} \right),
\end{aligned} \quad (68)$$

where $T = \frac{\kappa}{2 \pi}$ is the Hawking temperature.

We get the emission spectrum and Hawking temperature of vector particles tunnelling from weakly isolated horizons. The result is the same as the tunnelling of Dirac particles and scalar particles [49].

5 Back reaction to the space-time and the information loss recovery of weakly isolated horizons

In the last two sections, we fix the background of the space-time, and obtain Hawking's purely thermal spectrum of fermions and vector particles tunnelling from weakly isolated horizons, which will lead to the information loss puzzle [50–53].

As a matter of fact, when a particle with energy E_i radiates from the black hole with mass M , the mass of the black hole M should reduce to $M - E_i$, and the emission rate should be

$$\Gamma_i = e^{\frac{-2 \pi E_i}{\kappa_i}} = e^{\frac{-E_i}{T_i}}, \quad (69)$$

where κ_i and T_i are the surface gravity and Hawking temperature of the black hole respectively after emitting this particle.

For many particles' emission, assuming that they radiate one by one, we have

$$\Gamma = \prod_i \Gamma_i = e^{\sum_i \frac{-2 \pi E_i}{\kappa_i}} = e^{\sum_i \frac{-E_i}{T_i}}. \quad (70)$$

We consider the emission as a continuous procession [60–62], so the sum in Eq. (70) should be replaced by integration

$$\Gamma = e^{\int_0^E -\frac{dE}{T}} \quad (71)$$

When the back reaction of emitted particles to the space-time is considered, we have the conservation relationship $dM = -dE$. Then the tunnelling probability is

$$\Gamma = e^{\int_M^{M-E} \frac{dM}{T}}. \quad (72)$$

The first law of black hole thermodynamics for non-rotating and uncharged weakly isolated horizons is $dS = \frac{dM}{T}$, so the emission rate can be expressed as

$$\Gamma = e^{\int_M^{M-E} dS} = e^{S(M-E)-S(M)} = e^{\Delta S}, \quad (73)$$

where ΔS is the difference between the entropies of the weakly isolated horizon before and after emission. Thus, when we consider the back reaction of emitted particles, we get the un-thermal spectrum.

For non-rotating WIH, $\Omega = 0$ and $J = 0$, we obtain from Eq. (8)

$$\kappa = \frac{1}{4M}. \quad (74)$$

According to Eqs. (8) and (10), the entropy of non-rotating WIHs can be expressed as

$$S = \pi R^2 = 4\pi M^2. \quad (75)$$

The change of the entropy after emitting a particle with energy E is

$$\Delta S = 4\pi(M-E)^2 - 4\pi M^2, \quad (76)$$

and the tunnelling rate is

$$\begin{aligned} \Gamma &= \exp(-2\text{Im}I) = \exp(\Delta S) \\ &= \exp[4\pi(M-E)^2 - 4\pi M^2] \\ &= \exp\left[\frac{-E + \frac{E^2}{2M}}{T}\right] \\ &= \exp\left[-\frac{E}{T} + O(E^2)\right]. \end{aligned} \quad (77)$$

This is the corrected Hawking radiation when the back reaction of emitted particles to the background of space-time is considered. The leading-order term gives the purely thermal spectrum $\Gamma = \exp(-\frac{E}{T})$, and when the higher-order term of E is considered, we have the corrected spectrum $\Gamma = \exp(\Delta S) = \exp[-\frac{E}{T} + O(E^2)]$ which deviates from the purely thermal spectrum. We will show that the corrected spectrum leads to the information conservation during the process of black hole evaporation based on the methods in Refs. [54–56].

According to Eq. (77), the probability for the emission of a particle with energy E_1 is

$$\Gamma(E_1) = \exp[4\pi(M-E_1)^2 - 4\pi M^2]. \quad (78)$$

And the probability for the emission of a particle with energy E_2 is

$$\Gamma(E_2) = \exp[4\pi(M-E_2)^2 - 4\pi M^2]. \quad (79)$$

E_1 and E_2 represent two independent emitted particles, so Eqs. (78) and (79) should have the same form.

Let us consider a process as follows. One particle with energy E_1 emits, then another particle with energy E_2 radiates. The probability for the emission of the second particle is

$$\Gamma(E_2|E_1) = \exp\left[4\pi(M-E_1-E_2)^2 - 4\pi(M-E_1)^2\right], \quad (80)$$

which is the conditional probability, and is different from the independent probability (79). The probability for the two successive emissions with energy E_1 and E_2 can be calculated as

$$\begin{aligned} \Gamma(E_1, E_2) &\equiv \Gamma(E_1)\Gamma(E_2|E_1) \\ &= \exp[4\pi(M-E_1)^2 - 4\pi M^2] \\ &\quad \times \exp[4\pi(M-E_1-E_2)^2 - 4\pi(M-E_1)^2] \\ &= \exp[4\pi(M-E_1-E_2)^2 - 4\pi M^2] \\ &= \Gamma(E_1+E_2). \end{aligned} \quad (81)$$

This is an important relationship which tells us that the probability for two particles emitted successively with energy E_1 and E_2 is the same as the probability for one particle emitted with energy $E_1 + E_2$. It is easy to see that

$$\begin{aligned} \Gamma(E_1, E_2, \dots, E_i) &= \Gamma(E_1)\Gamma(E_2|E_1) \times \dots \\ &\quad \times \Gamma(E_i|E_1, \dots, E_{i-1}) \\ &= \Gamma(E_1 + \dots + E_i). \end{aligned} \quad (82)$$

This is an important relationship we will use later.

The correlation between two sequential Hawking radiations can be calculated as [54–56]

$$\begin{aligned} &\ln \Gamma(E_1 + E_2) - \ln[\Gamma(E_1)\Gamma(E_2)] \\ &= \ln \frac{\Gamma(E_1 + E_2)}{\Gamma(E_1)\Gamma(E_2)} \\ &= \ln \frac{\Gamma(E_1)\Gamma(E_2|E_1)}{\Gamma(E_1)\Gamma(E_2)} \\ &= \ln \frac{\Gamma(E_2|E_1)}{\Gamma(E_2)} \neq 0. \end{aligned} \quad (83)$$

The nonzero correlation function shows that the two sequential emissions are statistically dependent; that is to say, there exist correlations between sequential Hawking radiations from WIHs.

Similar to Eq. (80), the conditional probability $\Gamma(E_i|E_1, \dots, E_{i-1})$ is the tunnelling probability for a particle emitted with energy E_i after a sequence of radiations from the first particle to the $(i-1)$ th particle, so conditional entropy taken away by the i th particle is

$$S(E_i|E_1, \dots, E_{i-1}) = -\ln \Gamma(E_i|E_1, \dots, E_{i-1}).$$

The mutual information between sequential emissions of two particles with energy E_1 and E_2 is defined as [54–56]

$$\begin{aligned} S(E_2 : E_1) &\equiv S(E_2) - S(E_2|E_1) \\ &= -\ln \Gamma(E_2) + \ln \Gamma(E_2|E_1) \\ &= \ln \frac{\Gamma(E_2|E_1)}{\Gamma(E_2)}, \end{aligned} \quad (84)$$

which shows that mutual information equals the correlation between sequential emissions; that is to say, the information is encoded in the correlations between Hawking radiations.

Let us calculate the entropy carried out by Hawking radiations. The entropy carried out by the first emitted particle with energy E_1 is

$$S(E_1) = -\ln \Gamma(E_1). \quad (85)$$

The conditional entropy carried out by the second emitted particle with energy E_2 after the first emission is

$$S(E_2|E_1) = -\ln \Gamma(E_2|E_1). \quad (86)$$

So the total entropy carried by the two sequential emissions is

$$S(E_1, E_2) = S(E_1) + S(E_2|E_1). \quad (87)$$

Assuming the black hole exhausts after radiating n particles, we have the following relationship

$$\sum_i^n E_i = M, \quad (88)$$

where M is the mass of WIHs. The entropy carried out by all the emitted particles is

$$\begin{aligned} S(E_1, \dots, E_n) &= S(E_1) + S(E_2|E_1) + \dots \\ &\quad + S(E_n|E_1, \dots, E_{n-1}) \\ &= -\ln \Gamma(E_1) - \ln \Gamma(E_2|E_1) - \dots \\ &\quad - \ln \Gamma(E_n|E_1, \dots, E_{n-1}) \\ &= -\ln[\Gamma(E_1) \times \Gamma(E_2|E_1) \\ &\quad \times \dots \times \Gamma(E_n|E_1, \dots, E_{n-1})] \\ &= -\ln \Gamma(E_1 + E_2 + \dots + E_n) \\ &= -\ln \Gamma(M) = 4\pi M^2 = S_{WIH}, \end{aligned} \quad (89)$$

where we use the Eq. (82) in the fourth equation and the Eq. (75) in the last equation. The result shows that the entropy carried out by all the emitted particles equals the original

black hole entropy, so the total entropy is conserved during the radiation process.

In this section, we find that there exist correlations between sequential Hawking radiations from the locally defined black holes—weakly isolated horizons, information can be carried out by such correlations, and the entropy of weakly isolated horizons is conserved during the radiation process.

6 Discussions and conclusions

In this paper we investigate the tunnelling of fermions and vector particles from a non-rotating weakly isolated horizon. Weakly isolated horizon is a new, quasi-local framework which was introduced by Ashtekar and his collaborators. Compared with the event horizon, this framework does not need the knowledge of the overall space-time, and only involves quasi-local conditions, so it accords with the practical physical process. Weakly isolated horizons not only cover all stationary space-times, but also many non-stationary cases, so the investigation on the Hawking radiation of weakly isolated horizons is a quite significant issue.

We calculate fermions and vector particles tunnelling for non-rotating weakly isolated horizons. Although we discuss non-rotating weakly isolated horizons for simplicity, the shape of the horizon may not have spherical symmetry. In the practical physical process, black holes are often distorted by surrounding matters, so our discussion accords with the practical physical process.

In the discussion of fermions tunnelling, we first follow Kerner and Mann's methods, and calculate the tunnelling of the Dirac particles with spin-up and spin-down. Then we extend the method to investigate the tunnelling of Dirac particles with arbitrary spin directions, and obtain the expected Hawking temperature.

At last, we consider the back reaction of emitted particles to the space-time, and get the corrected Hawking radiation spectrum. The leading-order term gives the purely thermal spectrum, and when the higher-order term is considered, we have the corrected emission spectrum. We discuss the information recovery of the corrected emission spectrum from weakly isolated horizons based on the methods in Refs. [54–56]. We find that there exist correlations between sequential Hawking radiations from the locally defined black holes—weakly isolated horizons, information can be carried out by such correlations, and the entropy of weakly isolated horizons is conserved during the radiation process.

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References

1. S. Hawking, *Nature* **248**, 30 (1974)
2. S. Hawking, *Commun. Math. Phys.* **43**, 199 (1975)
3. N.D. Birrel, P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1982)
4. S.A. Fulling, *Aspects of Quantum Field Theory in Curved Space-Time* (Cambridge University Press, Cambridge, 1989)
5. V.P. Frolov, I.D. Novikov, *Black Hole Physics* (Kluwer Academic Publishers, Dordrecht, 1998)
6. R. Wald, *Quantum Field Theory in Curved Space-Time and Black Hole Thermodynamics*, Chicago Lectures in Physics (The University of Chicago Press, Chicago, 1994)
7. T. Damour, R. Ruffini, *Phys. Rev. D* **14**, 332 (1976)
8. M.K. Parikh, F. Wilczek, *Phys. Rev. Lett.* **85**, 5042 (2000)
9. M.K. Parikh, *Int. J. Mod. Phys. D* **13**, 2351 (2004)
10. P. Kraus, F. Wilczek, *Nucl. Phys. B* **433**, 403 (1995)
11. P. Kraus, F. Wilczek, *Nucl. Phys. B* **437**, 231 (1995)
12. P. Kraus, E. Keski-Vakkuri, *Nucl. Phys. B* **491**, 249 (1997)
13. Jingyi Zhang, Zheng Zhao, *Phys. Lett. B* **638**, 110 (2006)
14. Qing-Quan Jiang, Wu Shuang-Qing, Xu Cai, *Phys. Rev. D* **73**, 064003 (2006)
15. M. Angheben, M. Nadalini, L. Vanzo, S. Zerbini, *JHEP* **05**, 014 (2005)
16. R. Kerner, R.B. Mann, *Phys. Rev. D* **73**, 104010 (2006)
17. K. Srinivasan, T. Padmanabhan, *Phys. Rev. D* **60**, 24007 (1999)
18. S. Shankaranarayanan, K. Srinivasan, T. Padmanabhan, *Mod. Phys. Lett. A* **16**, 571 (2001)
19. S. Shankaranarayanan, T. Padmanabhan, K. Srinivasan, *Class. Quantum Gravity* **19**, 2671 (2002)
20. S. Shankaranarayanan, *Phys. Rev. D* **67**, 084026 (2003)
21. R. Kerner, R.B. Mann, *Class. Quantum Gravity* **25**, 095014 (2008)
22. R. Kerner, R.B. Mann, *Phys. Lett. B* **665**, 277 (2008)
23. Alexandre Yale, Robert B. Mann, *Phys. Lett. B* **673**, 168 (2008)
24. S.I. Kruglov, *Int. J. Mod. Phys. A* **29**, 1450118 (2014)
25. S.I. Kruglov, *Mod. Phys. Lett. A* **29**, 1450203 (2014)
26. A.J.M. Medved, E.C. Vagenas, *Mod. Phys. Lett. A* **20**, 2449 (2005)
27. Shuang-Qing Wu, Qing-Quan Jiang, *JHEP* **0603**, 079 (2006)
28. Q.Q. Jiang, S.Q. Wu, X. Cai, *Acta. Phys. Sin.* **56**, 3083 (2007)
29. H.L. Li, S.Z. Yang, *Europhys. Lett.* **79**, 20001 (2007)
30. S.Z. Yang, Q.Q. Jiang, *Int. J. Theor. Phys.* **46**, 2138 (2007)
31. R. Li, J.R. Ren, *Phys. Lett. B* **661**, 370 (2008)
32. R. Di Criscienzo, L. Vanzo, *Europhys. Lett.* **82**, 60001 (2008)
33. R. Li, J.R. Ren, S.W. Wei, *Class. Quantum Gravity* **25**, 125016 (2008)
34. D.Y. Chen, Q.Q. Jiang, X.T. Zu, *Phys. Lett. B* **665**, 106 (2008)
35. D.Y. Chen, Q.Q. Jiang, S.Z. Yang, X.T. Zu, *Class. Quantum Gravity* **25**, 205022 (2008)
36. Q.Q. Jiang, *Phys. Rev. D* **78**, 044009 (2008)
37. Hui-Ling Li, Shu-Zheng Yang, Teng-Jiao Zhou, Rong Lin, *EPL* **84**, 20003 (2008)
38. De-You Chen, Qing-Quan Jiang, Zu Xiao-Tao, *Phys. Lett. B* **665**, 106 (2008)
39. Ge-Rui Chen, Shiwei Zhou, *Int. J. Mod. Phys. D* **24**, 1550005 (2014)
40. I. Sakalli, A. Ovgun, *Gen. Relativ. Gravit.* **48**, 1 (2016)
41. Ran Li, Junkun Zhao, *Eur. Phys. J. Plus* **131**, 249 (2016)
42. Ganim Gecim, Yusuf Sucu, *Gen. Relativ. Gravit.* **50**, 152 (2018)
43. Zhong-Wen Feng, *Int. J. Theor. Phys.* **58**, 143 (2019)
44. De-Wen Song, Hui-Ling Li, *Int. J. Theor. Phys.* **58**, 656 (2019)
45. A. Ashtekar, B. Krishnan, *Living Rev. Relativ.* **7**, 10 (2004)
46. A. Ashtekar, C. Beetle, S. Fairhurst, *Class. Quantum Gravity* **17**, 253 (2000)
47. A. Ashtekar, C. Beetle, J. Lewandowski, *Phys. Rev. D* **64**, 044016 (2001)
48. B. Krishnan, *Class. Quantum Gravity* **29**, 205006 (2012)
49. Wu Xiaoning, Sijie Gao, *Phys. Rev. D* **75**, 044027 (2007)
50. S.W. Hawking, *Phys. Rev. D* **14**, 2460 (1976)
51. S.W. Hawking, *Phys. Rev. D* **72**, 084013 (2005)
52. C.G. Callan, J.M. Maldacena, *Nucl. Phys. B* **472**, 591 (1996)
53. Samir D. Mathur, *Class. Quantum Gravity* **26**, 224001 (2009)
54. B.C. Zhang, Q-Yu. Cai, L. You, M.-S. Zhan, *Phys. Lett. B* **675**, 98 (2009)
55. B.C. Zhang, Q-Yu. Cai, L. You, M.-S. Zhan, L. You, *Ann. Phys.* **326**, 350 (2011)
56. B.C. Zhang, Q-Yu. Cai, L. You, M.-S. Zhan, *Int. J. Mod. Phys. D* **22**, 1341014 (2013)
57. C. Corda, *Adv. High Energy Phys.* **2015**, 867601 (2015)
58. C. Corda, *Ann. Phys.* **353**, 71 (2015)
59. X.-K. Guo, Q.-Y. Cai, *Int. J. Theor. Phys.* **53**, 2980 (2014)
60. P. Kraus, F. Wilczek, *Nucl. Phys. B* **433**, 403 (1995)
61. X. He, W. Liu, *Phys. Lett. B* **653**, 330 (2007)
62. S. Zhou, W. Liu, *Phys. Rev. D* **77**, 104021 (2008)