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On general spherical fluid collapse

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Abstract A rather general form of spherical fluid collapse is formulated. The fluid is anisotropic, has shear, bulk and shear viscosities and is charged. It radiates energy through a heat flow or null radiation. The exterior solution is the charged Vaidya shining star solution. The general junction conditions are given. The main one represents a Riccati equation for the horizon function. We present a simple neutral solution, where the equation becomes a linear one. The whole mass of the star is radiated away without a remnant and the result is flat spacetime.

1 Introduction

Gravitational collapse is of main importance in relativistic astrophysics. The collapse of a dust cloud, which has energy density but no pressure, was studied first [1]. Then followed studies of collapse of perfect fluids, which have isotropic pressure, see [2] and references within. Spherical symmetry allows unequal anisotropic pressures – radial and tangential one. In 1972 Ruderman [3] argued that nuclear matter at very high densities may have anisotropic features and many such static models were proposed [4]. They probably have appeared as a result of anisotropic collapse. There are many other indications for anisotropy [5]. Evolving spheres without shear [6] or in geodesic flow [7] were found. Expansion-free models were also discussed [8–10]. They necessarily possess shear. Electromagnetic field was added too [11,12].

The process of collapse, however, is highly dissipative in order to account for the enormous binding energy of the resulting object [13]. Thus a more realistic scenario is collapse with heat flow [14] or null radiation [15]. For simplicity, shearless fluid is used quite often. The stability of the shear-free condition during collapse, however, requires fine tuning of one of the structure scalars, which is also the complexity factor [16–18]. Hence, collapse with shear seems to

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be the general case. Spherical, anisotropic and radiating collapse with shear is described by a diagonal metric with three independent components. The exterior solution is the Vaidya shining star [19]. It should be matched to the interior solution at the stellar surface. The main junction condition is a nonlinear differential equation in partial derivatives (along radius and time) for the three metric components and effectively reduces their number to two.

Chan studied numerically its solutions in separated variables [20–23]. The questions of cavity evolution and constant energy density were discussed in [24,25]. Other authors realized that the main junction condition is a Riccati equation in time for the metric component g_{rr} [26–28]. It has no general solution but may be solved in many concrete cases. It also reduces to the integrable Bernoulli or linear equation when some of its coefficients vanish. Analytic solutions were found also by using the Lie symmetry group analysis [29] or separation of variables [30].

Null radiation may be added to the heat flow without changing the main equation [15,31,32]. The same is true when a fluid with shear and bulk viscosity is used. Collapse with shear viscosity was studied in [33,34]. Null radiation was added to it in [35,36]. The effect of bulk viscosity was investigated in [37,38]. Both types of viscosity were introduced in [39–41].

Another direction of research is the study of charged anisotropic radiating fluids. Some models with shear were found [42,43]. Recent shearless models are also worth being mentioned [44,45]. In addition, charged fluid collapse with shear and null radiation was investigated [46], as well as with shear viscosity [47]. The presence of charge changes the main junction equation, but it remains still a Riccati one. The exterior solution becomes the charged Vaidya solution.

It was shown in [48], on the base of previous research, that a spherically symmetric self-gravitating anisotropic fluid model with heat flow can absorb the other characteristics of the fluid like shear and bulk viscosity, null radiation and electric charge. This results in an effective energy-momentum

tensor with effective energy density, pressures and heat flow. This work did not discuss the matching to an exterior solution. As pointed above, the exterior solution must be the charged Vaidya solution and the matching gives a number of junction conditions, the main of which is a Riccati equation for g_{rr}. Recently, this equation was transformed into a Riccati equation with simpler coefficients for the so-called horizon function [32]. This was done for neutral fluid. The horizon function governs the appearance of a horizon and a black hole, as one of the outcomes of collapse. It enters the expressions for observables from the outside like heat flow, surface redshift, total mass and luminosity at infinity. Some solutions of this equation were obtained with the help of the Lie symmetry group method [49]. In the geodesic case the equation leads to a generating function for the solutions [50].

In the present paper we give the Einstein equations in the case of this general type of collapse and generalise the junction equation for the horizon function to the charged case. We also give a simple collapsing solution.

The type of collapse, described above, is not the most general one. There is another approach in which the time evolution of a matter cloud is discussed. No matching to an exterior solution is done and the dissipation of energy is not included. The final result can be a black hole, a naked singularity or a static fluid model. It was developed by Joshi and co-authors (mainly Malafarina). A review of it up to 2011 is given in [51]. More details and recent publications can be found in [52–57]. Important work on spherical collapse has been done also by other authors [58–62].

In Sect. 2 the metric and the energy-momentum tensor of the general collapsing fluid is given and the Einstein–Maxwell equations which follow from it. The horizon function is introduced and alternative expressions for the mass, the energy density and the radial pressure are given. The surface characteristics of the star are defined. In Sect. 3 the junction conditions are discussed and the Riccati equation for the horizon function is obtained. The formalism of finding collapsing solutions is explained. Section 4 shows the difference between the neutral and the charged case in integrating the special Riccati equation, which follows from the general one. In Sect. 5 another simplifying assumption is made, which reduces the junction equation to a linear one. Simple realistic solution is given whose mass is radiated away during collapse, leaving flat spacetime, instead of a black hole. The last section contains discussion.

2 General energy-momentum tensor and field equations

The collapse of a fluid sphere with shear is described by the following metric

$$ds^2 = -A^2 dt^2 + B^2 dr^2 + R^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2 \right), \quad (1)$$

where A, B and R are independent functions of the time t and the radius r. The spherical coordinates are numbered as $x^0 = t$, $x^1 = r$, $x^2 = \theta$ and $x^3 = \varphi$. The general energy-momentum tensor referred to in the previous section reads

$$
\begin{aligned}
T_{\alpha\beta} &= (\mu + p_t) u_\alpha u_\beta + p_t g_{\alpha\beta} + (p_r - p_t) \chi_\alpha \chi_\beta \\
&\quad + q_\alpha u_\beta + u_\alpha q_\beta + \varepsilon l_\alpha l_\beta \\
&\quad - \zeta \Theta \left(g_{\alpha\beta} + u_\alpha u_\beta \right) - 2\eta \sigma_{\alpha\beta} + E_{\alpha\beta}.
\end{aligned}
\quad (2)
$$

Here μ is the energy density, p_r is the radial pressure, p_t is the tangential pressure, u^α is the four-velocity of the fluid, χ^α is a unit spacelike vector along the radial direction, q^α is the heat flow vector, also in the radial direction, ε is the energy density of the null fluid with null vector l^α, ζ is the coefficient of the bulk viscosity, η is the coefficient of the shear viscosity, Θ is the expansion scalar, $\sigma_{\alpha\beta}$ is the shear tensor, which reduces to the shear scalar σ, and $E_{\alpha\beta}$ is the electromagnetic energy tensor. In comoving coordinates we have

$$u^\alpha = A^{-1}\delta_0^\alpha, \quad \chi^\alpha = B^{-1}\delta_1^\alpha, \quad q^\alpha = q\chi^\alpha, \quad l^\alpha = u^\alpha + \chi^\alpha. \quad (3)$$

$$a_1 = \frac{A'}{A}, \quad \Theta = \frac{1}{A}\left(\frac{2\dot{R}}{R} + \frac{\dot{B}}{B}\right), \quad \sigma = \frac{1}{3A}\left(\frac{\dot{R}}{R} - \frac{\dot{B}}{B}\right) \quad (4)$$

where a_1 is the four-acceleration. The prime stands for a radial derivative. The dot means a time derivative. We have

$$E_{\alpha\beta} = \frac{1}{4\pi}\left(F_\alpha{}^\gamma F_{\beta\gamma} - \frac{1}{4}g_{\alpha\beta}F^{\gamma\delta}F_{\gamma\delta}\right) \quad (5)$$

where $F_{\alpha\beta}$ is the electromagnetic field tensor. Its only non-trivial component $F_{01} = -F_{10}$ is expressed through the four-potential, which has just a time component Φ:

$$F_{01} = -\Phi'. \quad (6)$$

The Maxwell equations yield

$$\Phi' = \frac{ABl}{R^2}, \quad l(r) = 4\pi \int_0^r sBR^2 dr \quad (7)$$

where s is the charge density and $l(r)$ is the total charge up to radius r. It is time-independent. We use relativistic units with $G = 1, c = 1, k = 8\pi$. It can be shown that the two viscosities, ε and the charge add different pieces to the energy density, the pressures and the heat flow, making them effective.

The Einstein field equations are:

$$
\begin{aligned}
8\pi \tilde{\mu} = 8\pi \left(\mu + \varepsilon + \frac{l^2}{8\pi R^4}\right) &= \frac{1}{A^2}\left(\frac{2\dot{B}}{B} + \frac{\dot{R}}{R}\right)\frac{\dot{R}}{R} \\
&\quad - \frac{1}{B^2}\left(\frac{2R''}{R} + \frac{R'^2}{R^2} - \frac{2B'R'}{BR} - \frac{B^2}{R^2}\right),
\end{aligned}
\quad (8)
$$

$$8\pi \tilde{p}_r = 8\pi \left(p_r - \zeta\Theta + 4\eta\sigma + \varepsilon - \frac{l^2}{8\pi R^4}\right)$$

$$-\frac{1}{A^2}\left[\frac{2\ddot{R}}{R}-\left(\frac{2\dot{A}}{A}-\frac{\dot{R}}{R}\right)\frac{\dot{R}}{R}\right]$$
$$+\frac{1}{B^2}\left(\frac{2A'}{A}+\frac{R'}{R}\right)\frac{R'}{R}-\frac{1}{R^2}, \tag{9}$$

$$8\pi\tilde{p}_t = 8\pi\left(p_t-\zeta\Theta-2\eta\sigma+\frac{l^2}{8\pi R^4}\right)$$
$$=-\frac{1}{A^2}\left[\frac{\ddot{B}}{B}+\frac{\ddot{R}}{R}-\frac{\dot{A}}{A}\left(\frac{\dot{B}}{B}+\frac{\dot{R}}{R}\right)+\frac{\dot{B}\dot{R}}{BR}\right]$$
$$+\frac{1}{B^2}\left[\frac{A''}{A}+\frac{R''}{R}-\frac{A'B'}{AB}+\left(\frac{A'}{A}-\frac{B'}{B}\right)\frac{R'}{R}\right] \tag{10}$$

$$8\pi\tilde{q}\tilde{B} = 8\pi\left(qB+\varepsilon\right)=-\frac{2}{AB}\left(\frac{\dot{B}R'}{BR}-\frac{\dot{R}'}{R}+\frac{\dot{R}A'}{RA}\right), \tag{11}$$

Next, we introduce the horizon function H [32]

$$H=\frac{R'}{B}+\frac{\dot{R}}{A}. \tag{12}$$

It replaces the metric component B. The latter becomes

$$B=\frac{R'}{H-\dot{R}/A}. \tag{13}$$

During the collapse the radius of the star shrinks in general, hence, $\dot{R}<0$. Since B is positive, we need $R'>0$. This is exactly the condition for the absence of shell crossing singularities [60,61].

The mass m, entrapped within radius r is given by the expression

$$m=\frac{R}{2}\left[1+\left(\frac{\dot{R}}{A}\right)^2-\left(\frac{R'}{B}\right)^2\right]+\frac{l^2}{2R}. \tag{14}$$

On the stellar surface Σ it becomes the mass of the star. The compactness parameter is $u=m/R$. Equation (14) can be rewritten using H

$$\frac{2m}{R}=1-H^2+\frac{2\dot{R}}{A}H+\frac{l^2}{R^2}. \tag{15}$$

Comparing Eqs. (8, 11) one can deduce the formula

$$8\pi\left(\mu+\varepsilon\right)=\frac{2m'}{R^2R'}-\frac{8\pi\left(qB+\varepsilon\right)B\dot{R}}{AR'}-\frac{2ll'}{R^3R'} \tag{16}$$

When $\varepsilon=q=l=0$ this is the formula for μ from the mass formalism. It was generalised for $q\neq0$ in [16]. There is a similar formula for p_r. However, one can derive another formula for the radial pressure without invoking the mass function. Comparing Eqs. (9, 11) we find

$$4\pi AR\left(p_r-\zeta\Theta+4\eta\sigma-qB\right)$$
$$=-\dot{H}+\left(\frac{A}{2R}+\frac{A'}{R'}\right)H^2$$
$$-\left(\frac{A}{R}+\frac{A'}{R'}\right)\frac{\dot{R}}{A}H-\frac{A}{2R}+\frac{Al^2}{2R^3} \tag{17}$$

Some important stellar characteristics are defined on the surface of the star. These are the surface redshift z_Σ

$$z_\Sigma=\frac{1}{H_\Sigma}-1, \tag{18}$$

the surface luminosity Λ_Σ and the luminosity at infinity Λ_∞

$$\Lambda_\Sigma=\left(4\pi qBR^2\right)_\Sigma, \tag{19}$$
$$\Lambda_\infty=H_\Sigma^2\Lambda_\Sigma. \tag{20}$$

The temperature at the surface is given by

$$T_\Sigma^4=\frac{(qB)_\Sigma}{\delta}, \tag{21}$$

where δ is some constant.

3 Junction conditions

The exterior spacetime of the stellar model is given by the charged Vaidya solution

$$ds^2=-\left[1-\frac{2M(v)}{\rho}+\frac{Q^2}{\rho^2}\right]dv^2-2dvd\rho$$
$$+\rho^2\left(d\theta^2+\sin^2\theta d\varphi^2\right), \tag{22}$$

where $M(v)$ is the mass of the star, measured at the exterior time v by an observer at infinity, Q is the total charge, while ρ is the exterior radius. Both solutions should be joined smoothly at Σ, which leads to the following junction conditions [63,64]:

$$R_\Sigma=\rho_\Sigma(v), \tag{23}$$
$$m_\Sigma=M_\Sigma, \tag{24}$$
$$l_\Sigma=Q \tag{25}$$
$$(p_r-\zeta\Theta+4\eta\sigma)_\Sigma=(qB)_\Sigma. \tag{26}$$

Equation (26) should be satisfied by A, H and R, while the other equations are definitions of different stellar characteristics. Replacing Eq. (17) in Eq. (26) we obtain on the surface of the star Σ

$$\dot{H}=\left(\frac{A}{2R}+\frac{A'}{R'}\right)H^2-\left(\frac{A}{R}+\frac{A'}{R'}\right)\frac{\dot{R}}{A}H-\frac{A}{2R}+\frac{AQ^2}{2R^3}. \tag{27}$$

In the uncharged case $Q=0$ this is exactly Eq. (30) from [32], derived there by transforming the boundary condition for B. In both cases it is a Riccati equation, but for H it is much simpler. Here we have derived it from Eq. (17), which also holds in the bulk of the star. The charge enters the free term.

It is clear that the two viscosities and the null radiation ε do not alter this Riccati equation, only the charge does. Equation

(27) is the universal boundary condition for the general fluid model, described by the energy-momentum tensor in Eq. (2).

It is seen that the star properties have simpler expressions when written in terms of H. The redshift is positive during collapse. Then Eq. (18) shows that $0 \le H_\Sigma(t) \le 1$. When $H_\Sigma = 0$ we obtain from Eq. (15) and the junction conditions

$$\left(1 - \frac{2m}{R} + \frac{l^2}{R^2}\right)_\Sigma = \left(1 - \frac{2M(v)}{\rho} + \frac{Q^2}{\rho^2}\right)_\Sigma = 0. \quad (28)$$

This is true if $\dot{R}H$ vanishes at $H = 0$. A special case, when this is not so because $\dot{R} \to \infty$, will be discussed in Sect. 5. Equations (14, 15) show that, at first sight, the equality $R'/B = \dot{R}/A$ also leads to Eq. (28). However, $R' > 0$, while $\dot{R} \le 0$ due to collapse, so this possibility is discarded on physical grounds.

Like in the neutral case, Eq. (28) signals the appearance of a horizon and a black hole within it, which is the typical end of gravitational collapse. The redshift becomes infinite, while the luminosity at infinity drops to zero. The point in time when collapse starts is taken as t_i. There H_Σ should have some positive initial value $H_{\Sigma i}$, less or equal to 1. During the collapse to a black hole the horizon function decreases to zero and hence $\dot{H}_\Sigma \le 0$.

There is no general solution of the Riccati equation, although it is integrable in many concrete cases. Furthermore, when the coefficient in front of H^2 vanishes it becomes a linear equation, which is solvable for any given A and R. When the free term vanishes it becomes a Bernoulli equation, which is also solvable. Finally, when the coefficient before the linear term vanishes, we get another Riccati equation, which sometimes is simpler. The function A_Σ is related to the four-acceleration, R_Σ is the physical radius of the star as seen from the outside, while H_Σ has a lot of physical applications, so it's important to have a simple equation for it. Equation (18) shows that the surface redshift also satisfies a Riccati equation. However, it is more complicated and inconvenient to work with than Eq. (27).

Let us summarise. In the general formalism, described above, we find collapsing solutions along the following chain. First we take some positive functions $l(r)$, $A(t, r)$ and $R(t, r)$, satisfying $\dot{R} < 0$ and $R' > 0$ and set $r = r_\Sigma$. Then a positive solution for $H_\Sigma(t) \in [0, 1]$ is found from Eq. (27), which must be a decreasing function, starting from some $H_{\Sigma i}$. Next we continue it in the interior for any r. Then we find B from Eq. (13) and the mass m from Eq. (14). Then a_1, σ and Θ follow from Eq. (4). Now it is possible to obtain μ, p_r, p_t, q from the Einstein field equations (8-11). The characteristics of the electromagnetic field s, Φ, $F_{\alpha\beta}$ are found from the Maxwell equations (5, 6, 7). The redshift, the two luminosities and the temperature on the star surface are given by Eqs. (18–21). Some of these quantities are defined on the surface, some throughout the star. For realistic solutions

we must have A, B, R, R', m, m', q, H, μ, p_r, l, $Q > 0$ and $\dot{R}$, $\dot{H}$, $\dot{m} < 0$.

The general formalism contains many special cases. We can turn off the bulk viscosity ($\zeta = 0$), the shear viscosity ($\eta = 0$), or the null radiation ($\varepsilon = 0$) without changing the junction conditions. The expressions for the energy density, the pressures and the heat flow do change. Turning off the charge Q, however, changes Eq. (27), eliminates the Maxwell sector and leads to the neutral case. The passage to shear-free fluids is done by the metric constraint $R = rB$, which follows from Eq. (4). Due to Eq (13), it becomes a relation between A, H and R. Euclidean stars are defined by the constraint $R' = B$ [65]. Some relations between A and R simplify Eq. (27) as was mentioned above. Geodesic collapse is characterised by $A = 1$. One can find a solution generating function in the neutral [50], as well as in the charged case [66]. An equation of state $p_r = f(\mu)$ implies another constraint. Perfect fluids are obtained by imposing the constraint $p_r = p_t = p$, which sets the anisotropy to zero. Dust collapse takes place when $p = 0$. Collapse without heat radiation is obtained by the constraint $q = 0$.

4 Special Riccati equation

We shall write the functions on the stellar surface without the index Σ. They depend on t only. Functions in the interior depend on t and r. Let A and R satisfy the equation

$$\frac{A}{R} + \frac{A'}{R'} = 0. \quad (29)$$

This eliminates the linear term in Eq. (27) and it becomes

$$\dot{H} = \left(\frac{A}{2R} + \frac{A'}{R'}\right)H^2 - \frac{A}{2R} + \frac{AQ^2}{2R^3}. \quad (30)$$

Equation (30) provides a relation between A and R:

$$R = \frac{K(t)}{A}, \quad (31)$$

where $K(t)$ is an arbitrary positive function. Unlike the neutral case, due to the additional charge term, Eq. (30) is not integrable.

5 Linear equation

The quadratic term in Eq. (27) disappears and it becomes a linear differential equation when

$$\frac{A}{2R} + \frac{A'}{R'} = 0. \quad (32)$$

This constraint leads to the relation

$$R = \frac{K(t)}{A^2}, \quad (33)$$

where, as before, $K(t)$ is an arbitrary positive function. This case was discussed in Ref. [26] and for B it leads to a Bernoulli equation with a very complicated solution. Here Eq. (27) simplifies considerably

$$2R\dot{H} = -\dot{R}H - A + \frac{AQ^2}{R^2} \quad (34)$$

and is linear in H. In the neutral case it is linear in R too. Usually it is assumed that collapse starts at $t_i = -\infty$. Then its duration is always infinite. To allow the option for collapse in finite time, we set $t_i = 0$ when a non-zero heat flow q appears, due to instability of the star, being static before that time.

In [32] we have given a solution, where the star becomes a black hole because energy is pumped from outside ($q < 0$, $\dot{m} > 0$). Now we shall present a more realistic concrete solution for collapse of a compact star. We set $Q = 0$ for the purpose of comparison.

Instead of using the general solution of the linear Eq. (34) we choose

$$H = aR \quad (35)$$

where a is some small positive constant. Both functions are positive and decreasing, but while $H \leq 1$, R is of the order of 10 km for a compact star. The main junction condition on Σ becomes

$$\left(R^2\right)^{\cdot} = -\frac{2A}{3a} \quad (36)$$

We assume that $A = A(r)$ in the interior, hence on the surface A is constant. The substitution of Eq. (33) leads to

$$R = \sqrt{R_0^2 - \frac{2A}{3a}t} \quad (37)$$

where $R_0 = K_0/A^2$, $K_0 = K(0)$. H is obtained from Eq. (35) and thus a simple solution of Eq. (34) is found. It should be noted that when the set A, B, R is used in separated variables, the resulting differential equation has to be solved numerically.

The mass M of the star is found from Eq. (15) on Σ

$$2M = R\left(\frac{1}{3} - H^2\right) \quad (38)$$

It is positive when $H_i < 1/\sqrt{3} = 0.577$. Its time derivative is

$$2\dot{M} = \dot{R}\left(\frac{1}{3} - 3H^2\right) \quad (39)$$

The mass should decrease because of the energy dissipation, hence, $H_i = aR_0 < 1/3 = 0.333$, which is the final constraint on the horizon function, that is on aR_0. Equations (35, 37) show that the collapse ends when $R_f = 0 = H_f$. Thus, in accord with Eq. (38), the compactness changes in the range

$$\frac{2}{9} < \frac{2M}{R} < \frac{1}{3} \quad (40)$$

and Eq. (28) (with $l = 0$) shows that no horizon and black hole are formed. Hence, the solution given by Eq. (35) illustrates the special case, mentioned after Eq. (28). We have in the end $H_f = 0$, but no horizon and black hole because $\dot{R}H$ is finite and even constant as seen from Eqs. (35, 36). Equation (38) shows that the stellar mass also vanishes at the endpoint, $M_f = 0$ and the final result is Minkowski spacetime. The star burns without a remnant. Similar results were found in [21,23,28,31]. This happens because the radiation of mass away is too strong and never allows a critical mass to be formed to create a black hole.

Let us find the evolution of the heat flux next. The mass formalism provides a formula for $\dot{m}$ in the bulk

$$\frac{\dot{m}}{A} = -4\pi\left(p_r\frac{\dot{R}}{A} + qB\frac{R'}{B}\right)R^2 \quad (41)$$

Taking account of the junction condition we obtain on the surface Σ

$$\dot{M} = -4\pi qBHAR^2 \quad (42)$$

We find from it

$$8\pi qB = \frac{a^2\left(1 - 9H^2\right)}{9H^4} \quad (43)$$

It is positive and grows to infinity at the collapse's end. The same is true for the surface redshift from Eq. (18)

$$Z_\Sigma = \frac{1 - H}{H} \quad (44)$$

The two luminosities follow from Eqs. (19, 20):

$$\Lambda_\Sigma = \frac{1 - 9H^2}{9H^2} \quad (45)$$

$$\Lambda_\infty = \frac{1}{9} - H^2 \quad (46)$$

The luminosity at the surface goes to infinity, but the luminosity seen by a distant observer is finite, starts from zero, reaches it maximum of 1/9 at the end of collapse and then drops suddenly to zero.

The above formulas show that collapse ends in a kind of explosion to flat spacetime. Its duration t_f is given by Eq. (37)

$$t_f = \frac{3a}{2A}R_0^2 \quad (47)$$

We would like to express R_0 through M_0. Equation (38) shows that R_0 satisfies an algebraic equation of third degree, which is solvable, but the solution is complicated. Having in mind the narrow range of the compactness, given by Eq. (40), we can use instead the approximate formula

$$R_0 \approx 6M_0 \quad (48)$$

Finally

$$t_f \approx \frac{54a}{A} M_0^2 \tag{49}$$

The smaller the initial stellar mass, the quicker the evaporation.

6 Discussion

The main purpose of this paper is to examine a rather general form of spherical collapse. The fluid is anisotropic, has shear, bulk and shear viscosities and is charged. It radiates energy through a heat flow or null radiation. The Einstein equations express the energy density, the pressures and the heat flow in an effective form. The exterior solution is the charged Vaidya shining star solution. The general junction conditions are given. The main one represents in principle a Riccati equation for the metric component B. It has been rewritten as a Riccati equation for the horizon function H. We have done this with the help of Eq. (17), which holds in the interior of the star.

The resulting equation is simpler and directly guides the behaviour of H and its zero, where a horizon and a black hole appear. Equation (18) gives a simple relation between the observable redshift and the horizon function H. It represents a fractional linear transformation of H. Hence, the redshift also satisfies a Riccati equation. H transforms the surface luminosity (depending mainly on the heat flow) into the luminosity at infinity and serves as a measure of its faintness. The horizon function also enters the mass formula, the compactness parameter and the heat flow. In this approach we again start with explicit expressions for A and the luminous radius of the star R and then solve the equation for H. This formalism is especially useful for solutions with shear. Special cases are obtained by setting some constants to zero or by imposing constraints on the metric functions.

Setting certain coefficient to zero changes the Riccati equation into an integrable one. This requires a simple relation between A and R. We present a simple neutral solution, where H satisfies a linear equation. The whole mass of the star is radiated away without a remnant and the result is flat spacetime.

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