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Letter

A diquark model for the *d*(2380)* dibaryon resonance?

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Abstract Diquark models have been applied with varying degree of success to tetraquark and pentaquark states involving both light and heavy quark degrees of freedom. We discuss the applicability of such models to light quark dibaryons, viewed as three-diquark objects. Highlighting the case of the $d^*(2380)$ dibaryon resonance, we demonstrate the inapplicability of diquark models in the light quark sector.

1 Introduction

The idea that diquarks ($\mathcal{D}$) play a significant role in hadron spectroscopy was raised by Jaffe to explain the 'inverted' SU(3)-flavor symmetry pattern of the lowest 0^+ scalar-meson nonet in terms of tetraquarks, each made of a $\mathcal{D}\bar{\mathcal{D}}$ pair [1]. Diquarks attracted considerable interest also in trying to understand the structure of the dubious $\Theta^+(1540)$ pentaquark which in some experiments showed up as a narrow KN resonant state [2–4]. More recently, following the discovery of tetraquark and pentaquark structures in the charmed (c) and bottom (b) quark sectors, diquarks have been used in theoretical studies of the structure and decay patterns of such exotic states; for a recent review see, e.g., Refs. [5,6].

A recent attempt to invoke diquarks to the structure of dibaryons, assuming that six-quark ($6q$) dibaryons consist dynamically of three diquarks, was made by Shi et al. [7] for the $d^*(2380)$ dibaryon resonance shown in Fig. 1. This $I(J^P) = 0(3^+)$ fairly narrow resonance, peaked about 80 MeV below the $\Delta\Delta$ threshold, was observed in several two-pion production channels in pn collisions studied by the WASA-at-COSY Collaboration [9]. Its $I = 0$ isospin assignment follows from balancing isospin in the $pn \rightarrow d\pi^0\pi^0$ production reaction, and $J^P = 3^+$ spin-parity follows from the measured deuteron angular distribution. Subsequent measurements of pn scattering and analyzing power [10] led

to a $pn\,^3D_3$ partial-wave Argand diagram that supports the $d^*(2380)$ dibaryon resonance interpretation.

A major problem in understanding the structure of the $d^*(2380)$, viewed as an $L = 0$ $\Delta\Delta$ dibaryon, arises from its relatively small width $\Gamma_{d^*} \approx 70$ MeV, see Fig. 1, which is by far smaller than twice the width of a single Δ baryon. Considering the reduced decay phase space available on average to a single Δ bound in d^*, its width is lowered from the free-space value of ≈ 115 MeV to about 80 MeV, so the problem here is how to account for a width reduction from about 160 MeV to $\Gamma_{d^*} \approx 70$ MeV. This problem was considered in three separate approaches, the most recent of which (third one below) is the one we question in this note.

(i) The $\Delta\Delta$–$\pi N\Delta$ coupled-channel hadronic calculation by Gal and Garcilazo [11,12] finds the $d^*(2380)$ resonance at about the right position in between the corresponding thresholds, and with approximately the observed width. The coupled-channel nature of this description is essential for understanding the relatively small width in simple terms [13].

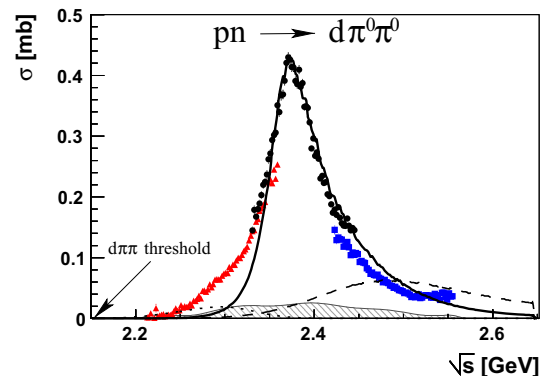(ii) Six-quark resonating-group-method calculations by Dong et al. [14] conclude that $d^*(2380)$ is dominated

Fig. 1 The $d^*(2380)$ dibaryon resonance seen in the $pn \rightarrow d\pi^0\pi^0$ reaction reported by the WASA-at-COSY Collaboration [8]

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by a hidden-color $\Delta_8\Delta_8$ component, roughly 2:1 with respect to a 'normal' $\Delta_1\Delta_1$ component. With color conservation forbidding the decay $\Delta_8 \rightarrow N_1 + \pi_1$ of a color-octet Δ to colorless hadrons, this leads to a substantial reduction of Γ_{d*}, in good agreement with the observed value. However, the compact nature of the decaying $\Delta_1\Delta_1$ component introduces further reduction of the width, thereby resulting in over-suppression of Γ_{d*} [13].

(iii) Assuming that $d*(2380)$ consists of three $(\mathbf{6}_f, \bar{\mathbf{3}}_c)$ flavor-color $S_\mathcal{D} = 1$ spin diquarks, Shi et al. [7] argued that the spatial rearrangement involved in transforming three colored diquarks to two color-singlet $3q$ hadrons, with spin and flavor that identify them with two Δ baryons, suppresses the ≈ 160 MeV expected width by a factor of about 0.4. Unfortunately these authors overlooked the rearrangement required also in color-flavor space for a $3\mathcal{D}$ system to become a $\Delta\Delta$ system. This produces another suppression factor of 1/9, as shown in some detail below, so the resulting width is less than 10 MeV.

Apart from demonstrating explicitly, based on the rough width estimate cited above, why a diquark model is not the right model to describe the $d*(2380)$ dibaryon resonance, the present note also discusses other light-quark dibaryon candidates predicted in this diquark model. It is concluded that diquark models in general are inappropriate for describing light quark dibaryons.

2 Classification of nonstrange dibaryon candidates

The quark-quark (qq) interaction is particularly strong in the anti-triplet antisymmetric color state $\bar{\mathbf{3}}_c$ [4]. Hence, we limit the discussion to $\bar{\mathbf{3}}_c$ diquarks. For a nonstrange S-wave diquark, requiring antisymmetry in the combined spin-isospin-color space leaves one with just two spin-isospin options $S_\mathcal{D}, I_\mathcal{D} = 0, 0$ scalar diquarks and $S_\mathcal{D}, I_\mathcal{D} = 1, 1$ vector diquarks.

Consider first a state consisting of three scalar diquark bosons, antisymmetrized in color space to yield a color-confined singlet $\mathbf{1}_c$ wavefunction. Bose-Einstein statistics then imposes antisymmetry on the three-diquark space wavefunction. Based on the experience gained in early triton binding energy calculations [15], an antisymmetric three-body spatial wavefunction is unlikely to support a bound state on its own. This suggests that by trying to construct a dibaryon from three scalar diquarks one overlooks an important aspect of the dynamics. The most likely culprit is the implicit assumption that in order to satisfy spin-statistics one may ignore the diquarks' substructure and treat them all as elementary bosons. This is a rather dubious presumption, because in a

hadron consisting of only light quarks there is a sole dynamical scale – Λ_{QCD}. In the following we limit the discussion of dibaryon candidates to vector $\bar{\mathbf{3}}_c$ diquarks.

Manipulations with $S_\mathcal{D}, I_\mathcal{D} = 1, 1$ vector diquarks are a bit more involved. For a symmetric $3\mathcal{D}$ space wavefunction, with orbital angular momentum $L = 0$ in mind, the spin-isospin degrees of freedom have to be considered explicitly in forming together with a $\mathbf{1}_c$ color wavefunction a totally symmetric $3\mathcal{D}$ wavefunction. This is expressed schematically in terms of a product of two antisymmetric components:

$$\left[\Box_{S,I} \otimes \Box_c \right]_{\Box\Box\Box} . \tag{1}$$

The $(1,1,1)_{S,I}$ Young tableaux stands for the $\mathbf{84}_{S,I}$ antisymmetric representation of SU(9) = SU(3)$_S\otimes$ SU(3)$_I$, where each of the vectors $\mathbf{S}$ and $\mathbf{I}$ is classified in the triplet $\Box$ representation of the respective SU(3). This spin-isospin Young tableaux consists of three direct product terms:

$$\Box\Box\Box_S \otimes \Box_I + \Box\Box_S \otimes \Box\Box_I + \Box_I \otimes \Box\Box\Box_S \tag{2}$$

with S, I values given respectively by

$$1,0 \ 3,0 \quad + \quad 1,1 \ 1,2 \ 2,1 \ 2,2 \quad + \quad 0,1 \ 0,3 . \tag{3}$$

Some of these $3\mathcal{D}$ S, I combinations, specifically 1,1 and 2,2, are spurious in terms of the underlying $6q$ wavefunctions which are obtained from the following product:

$$\left[\Box\Box\Box_{S,I} \otimes \Box\Box_c \right]_{\Box}, \tag{4}$$

where the $(3,3)_{S,I}$ Young tableaux stands for the $\mathbf{50}_{S,I}$ representation of the standard SU(4) = SU(2)$_S\otimes$ SU(2)$_I$ for spin-1/2 and isospin-1/2 quarks. The S, I =3,0 dibaryon candidate in this $6q$ scheme was calculated to lie more than 150 MeV above the $d*(2380)$ dibaryon resonance [16][1] which casts doubts on any attempt to ascribe a dominantly hexaquark structure to the observed $d*(2380)$.

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[1] The $6q$ nonstrange dibaryons considered in this work coincide with those predicted long ago by Dyson and Xuong [17] who identified some of them with states observed near the NN and ΔN thresholds, including the deuteron. With this remarkable insight, their predicted S, $I = 3,0$ dibaryon came out just 30 MeV below the $d*(2380)$.

3 Dibaryon masses and rearrangement factors

We focus now on the $I = 0$ $L = 0$ $S = 3$ $3\mathcal{D}$ state identified in Ref. [7] with the $I = 0$ $J^P = 3^+$ $d^*(2380)$ dibaryon resonance. Its mass value was reproduced there by using an effective diquark mass plus color-electric and color-spin interaction matrix elements deduced from applying scalar and vector diquark models in the charmed sector, above 2 GeV. The applicability of these diquark mass and interaction parameters to the light-quark sector is questionable. Nevertheless based on such reproduction of the $d^*(2380)$ mass, we ask where the $I = 0$ $J^P = 1^+$ deuteron-like and the $I = 1$ $J^P = 0^+$ virtual-like NN states are located in this $3\mathcal{D}$ model. Identifying these states with the $I = 0$ $S = 1$ and the $I = 1$ $S = 0$ states of the $\mathbf{84}_{S,I}$ SU(9) representation discussed in the previous section, we evaluate their masses using the same $\mathcal{D}$ mass and $\mathcal{DD}$ interaction parameters used by Shi et al. [7] to evaluate the location of $d^*(2380)$. Details are given here in the Appendix. The deuteron-like state d is found then 263 MeV below the $d^*(2380)$, about 245 MeV above the physical deuteron, with the virtual-like state v further 53 MeV down below d. However, no resonance feature in the corresponding $I = 0$ $J^P = 1^+$ and $I = 1$ $J^P = 0^+$ NN partial-wave phase shifts up to at least $E_{cm} = 2.4$ GeV has ever been observed without any doubt [18].

Next we evaluate the rearrangement factors involved in transforming the $3\mathcal{D}$ model $I = 0$ $L = 0$ $S = 3$ state to a $\Delta\Delta$ $I = 0$ $J^P = 3^+$ $d^*(2380)$. Since the $S = 3$ Pauli spin configuration is fully stretched in both $3\mathcal{D}$ and $\Delta\Delta$ bases, the spin rearrangement factor is simply 1. This is not the case for isospin and for color. Starting with isospin, we write schematically the $3\mathcal{D}$ model couplings in the form

$$\left[(I_1 = 1 \otimes I_2 = 1)_{I_{12}=1} \otimes \left(i_3 = \frac{1}{2} \otimes i_4 = \frac{1}{2} \right)_{I_3=1} \right]_{I=0},$$
(5)

where the isospin structure of the $I_3 = 1$ third diquark is spelled out explicitly in terms of its quark component isospins $i_3 = i_4 = \frac{1}{2}$. We now recouple isospins, so that the quark isospin i_3 joins the diquark isospin $I_1 = 1$ to form a Δ baryon isospin $I_{13} = \frac{3}{2}$, and similarly the quark isospin i_4 joins the diquark isospin $I_2 = 1$ to form another Δ isospin $I_{24} = \frac{3}{2}$, viz.

$$\left[\left(I_1 = 1 \otimes i_3 = \frac{1}{2} \right)_{I_{13}=\frac{3}{2}} \otimes \left(I_2 = 1 \otimes i_4 = \frac{1}{2} \right)_{I_{24}=\frac{3}{2}} \right]_{I=0}.$$
(6)

This recoupling is given by a unitary operator U with matrix elements proportional to SU(2) 9j symbols [19]:[2]

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[2] The proportionality constant $= \sqrt{(2I_{12}+1)(2I_3+1)(2I_{13}+1)(2I_{24}+1)}$ $= 12$.

$$U \begin{pmatrix} I_1 = 1 & I_2 = 1 & I_{12} = 1 \\ i_3 = \frac{1}{2} & i_4 = \frac{1}{2} & I_3 = 1 \\ I_{13} = \frac{3}{2} & I_{24} = \frac{3}{2} & I = 0 \end{pmatrix} = -\sqrt{\frac{1}{3}}.$$
(7)

Recoupling in color space is done by generalizing from SU(2)-isospin to SU(3)-color. The corresponding unitary operator matrix element is given by [20]:

$$U \begin{pmatrix} \bar{\mathbf{3}}_c & \bar{\mathbf{3}}_c & \mathbf{3}_c \\ \mathbf{3}_c & \mathbf{3}_c & \bar{\mathbf{3}}_c \\ \mathbf{1}_c & \mathbf{1}_c & \mathbf{1}_c \end{pmatrix} = \sqrt{\frac{\dim(\mathbf{3}_c)}{\dim(\bar{\mathbf{3}}_c) \times \dim(\bar{\mathbf{3}}_c)}} = \sqrt{\frac{1}{3}},$$
(8)

where the notation 'dim' stands for the dimension (=3) of the marked SU(3)$_c$ representations.

The combined recoupling coefficient in both isospin and color spaces is given by a product of the values noted in Eqs. (7) and (8) which amounts to $-1/3$. It enters quadratically in the evaluation of the $d^*(2380)$ decay width to nucleons and pions via a $\Delta\Delta$ hadronic doorway state, hence the width suppression factor 1/9 overlooked in Ref. [7].

In a similar way, rearrangement factors for d and v to go into the corresponding NN doorway states can also be evaluated, yielding somewhat smaller values of less than 0.1. This means that the widths involved in decays of such hypothetical dibaryons should be in the range of tens of MeV at most. Therefore, if the d and v $3\mathcal{D}$ dibaryon states exist, they should have been already observed in NN scattering experiments.

4 Discussion and summary

In this brief note we discussed the applicability of $\bar{\mathbf{3}}_c$ diquark models to light-quark nonstrange dibaryons, following a suggestion made by Shi et al. [7] that the observed $d^*(2380)$ dibaryon is dominantly of a $3\mathcal{D}$ structure. A useful test of any dibaryon model is provided by the extent to which it describes well the low lying hadronic spectrum. In this respect, we found that the $3\mathcal{D}$ $I = 0$ $J^P = 1^+$ deuteron-like and the $I = 1$ $J^P = 0^+$ virtual-like states in the particular diquark model suggested by these authors are located some 200–250 MeV above the physical deuteron, where no hint of irregularities in the corresponding NN phase-shift analyses exist. This demonstrates that diquark models are not physically appropriate models for binding six quarks into a dibaryon. Hadronic sizes that are relevant for binding together two baryons, particularly through pion exchange, are of order 1–2 fm and are considerably larger than the sub-fermi sizes expected for deeply bound $3\mathcal{D}$ structures. This results in extremely small $6q$ admixtures in the deuteron, see e.g. Ref. [21] for a recent discussion.

As for the $d^*(2380)$ dibaryon specifically, which is observed through decay modes involving nucleons and pions that are consistent with a size of 1–2 fm [13], we noted that if it were dominated by a $3\mathcal{D}$ structure, its decay width would

Table 1 $\mathcal{DD}$ color-spin and color interaction contributions to the listed total mass values $M_{3\mathcal{D}}$ of selected $3\mathcal{D}$ dibaryons, using Eq. (A.3) with $\tilde{M}_{\mathcal{D}}$ from Eq. (A.2). The qq interaction parameters from Ref. [7] are $\alpha = -39.5$ MeV, $\beta = 32.15$ MeV. Masses are given in MeV

$3\mathcal{D}$	(I, J^P)	$3\tilde{M}_{\mathcal{D}}$	$V_{\text{color-spin}}$	V_{color}	$M_{3\mathcal{D}}$
d^*	$(0, 3^+)$	2740	-4α	-16β	2383
d	$(0, 1^+)$	2740	$+\frac{8}{3}\alpha$	-16β	2120
v	$(1, 0^+)$	2740	$+4\alpha$	-16β	2067

have been suppressed by at least an isospin-color recoupling factor of 1/9 with respect to the initial $\Delta\Delta$ hadronic estimate of 160 MeV width. We conclude that assigning a $3\mathcal{D}$ structure to the $d^*(2380)$ dibaryon is in serious disagreement with its total width $\Gamma_{d*} \approx 70$ MeV.

Appendix A: Color-spin matrix elements

Masses of nonstrange light-quark dibaryons in the diquark model of Ref. [7] were given in Eq. (8) there by

$$M_{3\mathcal{D}} = 3M_{\mathcal{D}} + \sum_{i \neq j} (\alpha \langle \lambda_i \cdot \lambda_j \, \mathbf{s}_i \cdot \mathbf{s}_j \rangle + \beta \langle \lambda_i \cdot \lambda_j \rangle), \quad \text{(A.1)}$$

where λ denotes collectively the eight Gell–Mann SU(3) 3×3 matrices in color space and the sum on $i \neq j$ runs over all quark pairs, in the same diquark $\mathcal{D}$ as well as in different ones. For a diquark model, it is more appropriate to absorb same-diquark interaction terms into an effective diquark mass $\tilde{M}_{\mathcal{D}}$. For such quarks, $\langle \lambda_i \cdot \lambda_j \rangle = -\frac{8}{3}$ and $\langle \mathbf{s}_i \cdot \mathbf{s}_j \rangle = \frac{1}{4}$, hence

$$\tilde{M}_{\mathcal{D}} = M_{\mathcal{D}} + 2\left(-\frac{2}{3}\alpha - \frac{8}{3}\beta\right) = 913.2 \text{ MeV}, \quad \text{(A.2)}$$

where the values of $M_{\mathcal{D}} = 1032$ MeV, α and β were taken from Ref. [7]. Expression (A.1) is rewritten then in the form

$$M_{3\mathcal{D}} = 3\tilde{M}_{\mathcal{D}} + \sum_{m \neq n}\left(\frac{1}{4}\alpha \langle \lambda_m \cdot \lambda_n \, \mathbf{S}_m \cdot \mathbf{S}_n \rangle + \beta \langle \lambda_m \cdot \lambda_n \rangle\right), \quad \text{(A.3)}$$

where the sum on $m \neq n$ runs on the three vector diquarks of which d^*, d and v are composed. To evaluate Eq. (A.3) we note that by coupling $\bar{\mathbf{3}}_c$ diquarks m and n to a $\mathbf{3}_c$ $\mathcal{DD}$

configuration, the color $\mathcal{DD}$ interaction is determined by a single matrix element $\langle \lambda_m \cdot \lambda_n \rangle = -\frac{8}{3}$, independently of the $3\mathcal{D}$ dibaryon considered. Furthermore, for a spin-symmetric or antisymmetric $3\mathcal{D}$ wavefunction

$$\mathbf{S}^2 = (\mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3)^2 = 6 + 6 \langle \mathbf{S}_m \cdot \mathbf{S}_n \rangle, \quad \text{(A.4)}$$

so $\langle \mathbf{S}_m \cdot \mathbf{S}_n \rangle = 1, -2/3, -1$ for d^*, d, v, respectively. The resulting color-spin contributions in Eq. (A.3) are repulsive for d^*, about 160 MeV, becoming attractive for the other two dibaryon candidates, whereas the color-electric contributions are attractive, about -500 MeV independently of which dibaryon as long as all three of them are in the same $3\mathcal{D}$ $\mathbf{1}_c$ color representation. The mass values calculated in this $3\mathcal{D}$ model are listed in Table 1.

Taken at face value, the physical implications of Table 1 are quite striking: the effective diquark mass in the model of Ref. [7] is 913.2 MeV. There are three of them, with a total $3\mathcal{D}$ mass of 2740 MeV, so the model implies their binding energy into $d^*(2383)$ is about 360 MeV. This is a huge binding energy for a system containing light quarks only. It entails a tiny radius ~ 0.4 fm for the $3\mathcal{D}$ system, much smaller than anything known to occur in light quark systems. This raises further doubts regarding the physical basis of the model proposed in Ref. [7].

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