




Towards the quantization of the non-relativistic D2-brane in the pure spinor formalism

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Abstract An attempt is made to apply the pure spinor formalism to the non-relativistic IIA D2-brane. The fermionic constraints corresponding to the rescaled fermionic coordinates are given. Two commuting spinor fields are introduced, each one corresponding to a fermionic constraint. A BRST charge is constructed via the ansatz proposed by Berkovits in (JHEP 04:018, [arXiv:hep-th/0001035](#), 2000, JHEP 09:046, [arXiv:hep-th/0006003](#), 2000, JHEP 09:016, [arXiv:hep-th/0105050](#), 2001, ICTP Lect. Notes Ser. 13:57, [arXiv:hep-th/0209059](#), 2003). The nilpotency of the BRST charge leads to a set of constraints for the two spinor fields including pure spinor constraints. A novel non-trivial solution is given for one of the spinor fields which can be written as a sum of two pure spinors.

1 Introduction

The pure spinor formalism introduced by Berkovits [1–4] is a successful attempt to solve the longstanding problem of finding a manifestly supersymmetric and covariant superstring formalism. The basic ingredient is the BRST-like operator $Q = \int dz \lambda^\alpha d_\alpha$ where d_α is the fermionic constraint that appears in the conventional Green-Schwarz formalism and λ_α is a bosonic chiral spinor that plays the role of the associated “ghost”. For Q to be regarded as a BRST operator must be nilpotent and this leads to the relation $\lambda^\alpha \gamma_{\alpha\beta}^m \lambda^\beta = 0$. This in 10 dimensions is the condition for λ^α to be characterized as a pure spinor.

An important property of Q is that its cohomology correctly reproduces the spectrum of the superstring. The pure spinor formalism has been used as well for the covariant

quantization of the superparticle [3] and also to study several aspects of string theory, for example the propagation of strings in curved backgrounds [5–8].

Another important application of the pure spinor formalism is the calculation of scattering amplitudes within the framework of superstring theory [9–13]. The manifest Lorentz covariance and spacetime supersymmetry make the calculation much easier than in other formalisms. Thus, pure spinors play a crucial role within string perturbation theory. However, within the context of D-branes, there are no non-trivial solutions reported in the literature. Moreover, so far it is not known whether this formalism can be consistently extended or generalized to be applied to non-relativistic systems with kappa symmetry, a fact that could allow us to draw some conclusions for the general case too, especially if there are problems in solving the pure spinor constraints in that case.

In this paper we try to extend the pure spinor formalism to the case of the non-relativistic IIA D2-brane. The non-relativistic limit of string theories [14, 15] give us a deeper understanding of string theories themselves. The non-relativistic limit of Dp branes has been studied in [16, 17]. It is important to note that in this limit the kappa symmetry is maintained and this allows us to treat non-relativistic Dp branes in the framework of the pure spinor formalism.

Here we present a novel non-trivial solution for the non-relativistic D2-brane within the pure spinor formalism. This fact could lead to the quantization of branes with interesting and relevant results.

Our starting point is the action of a IIA D2-brane in a flat 10d background. The fields consist of the 10d superspace coordinates (x^m, θ) and an Abelian gauge field A_μ [18–20]:

$$S = -T \int d^3\sigma \sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})} + \int \mathcal{L}_3^{WZ}, \quad (1.1)$$

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where T is the string tension and the Wess-Zumino action reads

$$\mathcal{L}_3^{WZ} = T \left\{ \frac{1}{2} d\bar{\theta} \not{A}^2 \theta - \frac{1}{3} d\bar{\theta} \not{A} \theta_1 + \frac{1}{15} d\bar{\theta} \theta_2 + d\bar{\theta} \Gamma_{11} \theta \mathcal{F} \right\}; \tag{1.2}$$

$G_{\mu\nu} = \eta_{mn} \Pi_\mu^m \Pi_\nu^n$ and here $\Pi_\mu^m = \frac{\partial X^m}{\partial \sigma^\mu} - \bar{\theta} \Gamma^m \frac{\partial \theta}{\partial \sigma^\mu}$.

$$\mathcal{F}_{\mu\nu} = \frac{\partial A_\nu}{\partial \sigma^\mu} - \frac{\partial A_\mu}{\partial \sigma^\nu} + (\bar{\theta} \Gamma_{11} \Gamma_m \partial_\nu \theta) \left(\Pi_\mu^m + \frac{1}{2} \bar{\theta} \Gamma^m \partial_\mu \theta \right) - (\bar{\theta} \Gamma_{11} \Gamma_m \partial_\mu \theta) \left(\Pi_\nu^m + \frac{1}{2} \bar{\theta} \Gamma^m \partial_\nu \theta \right) \tag{1.3}$$

$$\theta_1 = \not{\psi} \theta, \quad \theta_2 = \not{\tilde{\psi}} \psi \theta \tag{1.4}$$

with

$$\not{\psi} = \{ (\bar{\theta} \Gamma^m d\theta) + \Gamma_{11} (\bar{\theta} \Gamma_{11} \Gamma^m d\theta) \} \Gamma_m \tag{1.5}$$

$$\not{\tilde{\psi}} = \{ (\bar{\theta} \Gamma^m d\theta) - \Gamma_{11} (\bar{\theta} \Gamma_{11} \Gamma^m d\theta) \} \Gamma_m \tag{1.6}$$

$$\not{A} = (dX^m + \bar{\theta} \Gamma^m d\theta) \Gamma_m \tag{1.7}$$

$$\mathcal{F} = \frac{1}{2} \mathcal{F}_{\mu\nu} d\sigma^\mu d\sigma^\nu \tag{1.8}$$

and $m, n = 0, \dots, 9/\mu, \nu = 0, 1, 2$.

The action (1.1) has a global supersymmetry and also a local supersymmetry (kappa symmetry).

The action for the non-relativistic D2-brane is obtained from (1.1) by doing the following rescaling [16, 17]:

$$X^\mu = \omega x^\mu \tag{1.9}$$

$$X^a = x^a \tag{1.10}$$

$$T = \frac{1}{\omega} T_{NR} \tag{1.11}$$

$$A_i = \omega \omega_i \tag{1.12}$$

where the subindex NR stands for non-relativistic and $i, \mu = 0, 1, 2/a = 3, \dots, 9$.

$$\theta = \sqrt{\omega} \theta_- + \frac{1}{\sqrt{\omega}} \theta_+ \tag{1.13}$$

where θ_+, θ_- are eigenstates of

$$\Gamma_\star \equiv \Gamma_0 \Gamma_1 \Gamma_2 \tag{1.14}$$

$$\Gamma_\star \theta_\pm = \pm \theta_\pm \tag{1.15}$$

The action of the non-relativistic D2-brane is obtained by expanding (1.1) in powers of ω and keeping the finite part as $\omega \rightarrow \infty$ [17]

$$S_{NR} = \int d^3 \sigma \mathcal{L}_{NR}^{DBI} + \int \mathcal{L}_{NR}^{WZ} \tag{1.16}$$

where

$$\mathcal{L}_{NR}^{DBI} = T_{NR} (\epsilon_{ijk} R_i^0 R_j^1 R_k^2) \left[- \left(\bar{\theta}_+ \widehat{\gamma}^0 \frac{\partial \theta_+}{\partial \sigma^0} + \bar{\theta}_+ \widehat{\gamma}^1 \frac{\partial \theta_+}{\partial \sigma^1} + \bar{\theta}_+ \widehat{\gamma}^2 \frac{\partial \theta_+}{\partial \sigma^2} \right) + \frac{1}{2} \widehat{g}^{il} (\eta_{aa'} u_i^a u_l^{a'}) + \frac{1}{4} \widetilde{\mathcal{F}}_{il}^{(1)} \widetilde{\mathcal{F}}_{jk}^{(1)} \widehat{g}^{ij} \widehat{g}^{lk} \right] \tag{1.17}$$

$$\begin{aligned} \mathcal{L}_{NR}^{WZ} = T_{NR} & \left[\frac{1}{2} (\bar{\theta}_+ \Gamma_{\mu\nu} d\theta_+) \left(dx^\mu + \bar{\theta}_- \Gamma^\mu d\theta_- \right) \right. \\ & \left(dx^\nu + \bar{\theta}_- \Gamma^\nu d\theta_- \right) - (\bar{\theta}_- \Gamma^\mu d\theta_-) \left(dx^\nu + \frac{2}{3} \bar{\theta}_- \Gamma^\nu d\theta_- \right) \\ & + \frac{1}{2} (\bar{\theta}_- \Gamma_{\mu\nu} d\theta_-) (\bar{\theta}_+ \Gamma^\mu d\theta_+) \left(dx^\nu + \frac{2}{3} \bar{\theta}_- \Gamma^\nu d\theta_- \right) \\ & + \frac{1}{2} (\bar{\theta}_- \Gamma_{ab} d\theta_-) \left(dx^a + \bar{\theta}_+ \Gamma^a d\theta_- + \bar{\theta}_- \Gamma^a d\theta_+ \right) \\ & \times \left(dx^b + \bar{\theta}_+ \Gamma^b d\theta_- + \bar{\theta}_- \Gamma^b d\theta_+ \right) \\ & - (\bar{\theta}_+ \Gamma^a d\theta_- + \bar{\theta}_- \Gamma^a d\theta_+) \\ & \times \left(dx^b + \frac{2}{3} (\bar{\theta}_- \Gamma^b d\theta_+ + \bar{\theta}_+ \Gamma^b d\theta_-) \right) \\ & + (\bar{\theta}_+ \Gamma_{\mu a} d\theta_- + \bar{\theta}_- \Gamma_{\mu a} d\theta_+) \left(dx^\mu + \bar{\theta}_- \Gamma^\mu d\theta_- \right) \\ & \times \left(dx^a + \frac{1}{2} (\bar{\theta}_+ \Gamma^a d\theta_- + \bar{\theta}_- \Gamma^a d\theta_+) \right) \\ & - \frac{1}{2} (\bar{\theta}_- \Gamma^\mu d\theta_-) \left(dx^a + \frac{1}{3} (\bar{\theta}_- \Gamma^a d\theta_+ + \bar{\theta}_+ \Gamma^a d\theta_-) \right) \\ & + \frac{1}{2} (\bar{\theta}_+ \Gamma_\mu \Gamma_{11} d\theta_- + \bar{\theta}_- \Gamma_\mu \Gamma_{11} d\theta_+) \\ & \times \left(dx^\mu + \frac{2}{3} \bar{\theta}_- \Gamma^\mu d\theta_- \right) (\bar{\theta}_+ \Gamma_{11} d\theta_- + \bar{\theta}_- \Gamma_{11} d\theta_+) \\ & + \frac{1}{2} (\bar{\theta}_- \Gamma_b \Gamma_{11} d\theta_-) \left(dx^b + \frac{2}{3} (\bar{\theta}_- \Gamma^b d\theta_+ \right. \\ & \left. + \bar{\theta}_+ \Gamma^b d\theta_-) \right) (\bar{\theta}_+ \Gamma_{11} d\theta_- + \bar{\theta}_- \Gamma_{11} d\theta_+) \\ & + (d\bar{\theta}_- \Gamma_{11} \theta_+ + d\bar{\theta}_+ \Gamma_{11} \theta_-) (f - (\bar{\theta}_- \Gamma_\mu \Gamma_{11} d\theta_+ \\ & + \bar{\theta}_+ \Gamma_\mu \Gamma_{11} d\theta_-) \left(dx^\mu + \frac{1}{2} \bar{\theta}_- \Gamma^\mu d\theta_- \right) \\ & \left. - (\bar{\theta}_- \Gamma_a \Gamma_{11} d\theta_-) \left(dx^a + \frac{1}{2} (\bar{\theta}_- \Gamma^a d\theta_+ + \bar{\theta}_+ \Gamma^a d\theta_-) \right) \right) \left. \right] \tag{1.18} \end{aligned}$$

where

$$R_i^\mu = \partial_i x^\mu - \bar{\theta}_- \Gamma^\mu \partial_i \theta_- \tag{1.19}$$

$$u_i^a = \partial_i x^a - \bar{\theta}_- \Gamma^a \partial_i \theta_+ - \bar{\theta}_+ \Gamma^a \partial_i \theta_- \tag{1.20}$$

$i, \mu = 0, 1, 2/a = 3, \dots, 9$; also

$$\widehat{\gamma}^0 = \frac{1}{(\epsilon_{ijk} R_i^0 R_j^1 R_k^2)} (\epsilon_{ijk} \Gamma^i R_1^j R_2^k) \tag{1.21}$$

$$\widehat{\gamma}^1 = - \frac{1}{(\epsilon_{ijk} R_i^0 R_j^1 R_k^2)} (\epsilon_{ijk} \Gamma^i R_0^j R_2^k) \tag{1.22}$$

$$\widehat{\gamma}^2 = \frac{1}{(\epsilon_{ijk} R_i^0 R_j^1 R_k^2)} (\epsilon_{ijk} \Gamma^i R_0^j R_1^k) \tag{1.23}$$

$$\widehat{g}^{jk} = \eta_{\mu\nu} R_j^\mu R_k^\nu \tag{1.24}$$

where we have introduced the following quantities $\epsilon_{012} = 1$, $\eta_{\mu\nu} = \text{diag}(-1, 1, 1)$, $\eta_{aa'} = \text{diag}(1, \dots, 1)$, $\mu, \nu = 0, 1, 2$; $a, a' = 3, \dots, 9$;

$$\begin{aligned} \widetilde{\mathcal{F}}_{ij}^{(1)} = & f_{ij} + \left[(\bar{\theta}_- \Gamma_{11} \Gamma_\mu \partial_j \theta_+ + \bar{\theta}_+ \Gamma_{11} \Gamma_\mu \partial_j \theta_-) \right. \\ & \times \left(R_i^\mu + \frac{1}{2} \bar{\theta}_- \Gamma^\mu \partial_i \theta_- \right) - (i \leftrightarrow j) \Big] \\ & + \left[(\bar{\theta}_- \Gamma_{11} \Gamma_a \partial_j \theta_-) \left(u_i^a + \frac{1}{2} (\bar{\theta}_- \Gamma^a \partial_i \theta_+ \right. \right. \\ & \left. \left. + \bar{\theta}_+ \Gamma^a \partial_i \theta_-) \right) - (i \leftrightarrow j) \right], \end{aligned} \tag{1.25}$$

where $f_{ij} = \partial_i w_j - \partial_j w_i$, $f = \frac{1}{2} f_{ij} d\sigma^i d\sigma^j$, and $\mu, \nu, i, j = 0, 1, 2/a = 3, \dots, 9$.

We denote by $[\mathcal{L}_{NR}^{WZ}]_3$ the 3-form coefficient of \mathcal{L}_{NR}^{WZ} given in (1.18). The Lagrangian density of the non-relativistic D2-brane is given by:

$$\mathcal{L}_{NR} = \mathcal{L}_{NR}^{DBI} + [\mathcal{L}_{NR}^{WZ}]_3 \tag{1.26}$$

The action (1.16) is invariant under the non-relativistic counterpart of the global and local supersymmetric transformations that leave the relativistic D2-brane action (1.1) invariant [16].

The conjugate momenta of the variables $x^\mu, x^a, \theta_+, \theta_-$, w_i are given by:

$$\begin{aligned} p_\mu = & \frac{\partial \mathcal{L}_{NR}}{\partial \dot{x}^\mu} \\ = & \frac{\partial \mathcal{L}_{NR}}{\partial R_0^\mu} + \frac{\partial \mathcal{L}_{NR}}{\partial \widetilde{\mathcal{F}}_{0i}^{(1)}} (\bar{\theta}_- \Gamma_{11} \Gamma_\mu \partial_i \theta_+ + \bar{\theta}_+ \Gamma_{11} \Gamma_\mu \partial_i \theta_-) \end{aligned} \tag{1.27}$$

$\mu = 0, 1, 2/i = 1, 2$;

$$\begin{aligned} p_a = & \frac{\partial \mathcal{L}_{NR}}{\partial \dot{x}^a} \\ = & \frac{\partial \mathcal{L}_{NR}}{\partial u_0^a} + \frac{\partial \mathcal{L}_{NR}}{\partial \widetilde{\mathcal{F}}_{0i}^{(1)}} (\bar{\theta}_- \Gamma_{11} \Gamma_a \partial_i \theta_-) \end{aligned} \tag{1.28}$$

$a = 3, \dots, 9$;

$$\begin{aligned} J_{+l} = & \frac{\partial^r \mathcal{L}_{NR}}{\partial \dot{\theta}_{+l}} - \frac{\partial \mathcal{L}_{NR}}{\partial u_0^a} (\bar{\theta}_- \Gamma^a)_l \\ & - \frac{\partial \mathcal{L}_{NR}}{\partial \widetilde{\mathcal{F}}_{0i}^{(1)}} \left[\left(R_i^\mu + \frac{1}{2} \bar{\theta}_- \Gamma^\mu \partial_i \theta_- \right) (\bar{\theta}_- \Gamma_{11} \Gamma_\mu)_l \right. \\ & \left. + \frac{1}{2} (\bar{\theta}_- \Gamma_{11} \Gamma_a \partial_i \theta_-) (\bar{\theta}_- \Gamma^a)_l \right] \end{aligned} \tag{1.29}$$

and

$$\begin{aligned} J_{-l} = & \frac{\partial^r \mathcal{L}_{NR}}{\partial \dot{\theta}_{-l}} = \frac{\partial^r [\mathcal{L}_{NR}^{WZ}]_3}{\partial \dot{\theta}_{-l}} - \frac{\partial \mathcal{L}_{NR}}{\partial R_0^\mu} (\bar{\theta}_- \Gamma^\mu)_l \\ & - \frac{\partial \mathcal{L}_{NR}}{\partial u_0^a} (\bar{\theta}_+ \Gamma^a)_l \\ & - \frac{\mathcal{L}_{NR}}{\partial \widetilde{\mathcal{F}}_{0i}^{(1)}} \left[\frac{1}{2} (\bar{\theta}_- \Gamma_{11} \Gamma_\mu \partial_i \theta_+ + \bar{\theta}_+ \Gamma_{11} \Gamma_\mu \partial_i \theta_-) (\bar{\theta}_- \Gamma^\mu)_l \right. \\ & + \frac{1}{2} (\bar{\theta}_- \Gamma_{11} \Gamma_a \partial_i \theta_-) (\bar{\theta}_+ \Gamma^a)_l \\ & + \left(R_i^\mu + \frac{1}{2} \bar{\theta}_- \Gamma^\mu \partial_i \theta_- \right) (\bar{\theta}_+ \Gamma_{11} \Gamma_\mu)_l \\ & \left. + \left(u_i^a + \frac{1}{2} (\bar{\theta}_- \Gamma^a \partial_i \theta_+ + \bar{\theta}_+ \Gamma^a \partial_i \theta_-) \right) (\bar{\theta}_- \Gamma_{11} \Gamma_a)_l \right], \end{aligned} \tag{1.30}$$

where $l = 1, \dots, 32$; and

$$E^i = \frac{\partial \mathcal{L}_{NR}}{\partial \dot{w}_i} = \frac{\partial \mathcal{L}_{NR}}{\partial \widetilde{\mathcal{F}}_{0i}^{(1)}}. \tag{1.31}$$

The fermionic constraints are given by

$$\begin{aligned} F_{+l} = & J_{+l} + p_a (\bar{\theta}_- \Gamma^a)_l \\ & + E^i \left[\left(R_i^\mu + \frac{1}{2} \bar{\theta}_- \Gamma^\mu \partial_i \theta_- \right) (\bar{\theta}_- \Gamma_{11} \Gamma_\mu)_l \right. \\ & \left. - \frac{1}{2} (\bar{\theta}_- \Gamma_{11} \Gamma_a \partial_i \theta_-) (\bar{\theta}_- \Gamma^a)_l \right] \\ & + T_{NR} \epsilon_{ijk} (\bar{\theta}_+ \Gamma^i)_l R_1^j R_2^k - \frac{\partial^r [\mathcal{L}_{NR}^{WZ}]_3}{\partial \dot{\theta}_{+l}} \end{aligned} \tag{1.32}$$

where the last derivative does not include differentiation with respect to $u_0^a, \widetilde{\mathcal{F}}_{0i}^{(1)}$, and

$$\begin{aligned} F_{-l} = & J_{-l} + p_a (\bar{\theta}_+ \Gamma^a)_l + p_\mu (\bar{\theta}_- \Gamma^\mu)_l \\ & + E^i \left[\left(R_i^\mu + \frac{1}{2} (\bar{\theta}_- \Gamma^\mu \partial_i \theta_-) \right) (\bar{\theta}_+ \Gamma_{11} \Gamma_\mu)_l \right. \\ & + \left(u_i^a + \frac{1}{2} (\bar{\theta}_- \Gamma^a \partial_i \theta_+ + \bar{\theta}_+ \Gamma^a \partial_i \theta_-) \right) (\bar{\theta}_- \Gamma_{11} \Gamma_a)_l \\ & - \frac{1}{2} (\bar{\theta}_- \Gamma_{11} \Gamma_\mu \partial_i \theta_+ + \bar{\theta}_+ \Gamma_{11} \Gamma_\mu \partial_i \theta_-) (\bar{\theta}_- \Gamma^\mu)_l \\ & \left. - \frac{1}{2} (\bar{\theta}_- \Gamma_{11} \Gamma_a \partial_i \theta_-) (\bar{\theta}_+ \Gamma^a)_l \right] - \frac{\partial^r [\mathcal{L}_{NR}^{WZ}]_3}{\partial \dot{\theta}_{-l}}, \end{aligned} \tag{1.33}$$

where the last derivative does not include differentiation with respect to $u_0^a, R_0^\mu, \widetilde{\mathcal{F}}_{0i}^{(1)}$.

We introduce now two commuting spinor fields λ_+, λ_- corresponding to the fermionic constraints F_+, F_- and we write down a BRST charge according to the ansatz proposed in [1–4]:

$$\begin{aligned}
 Q = & \int d^2\sigma \left[\lambda_+^l J_{+l} + \lambda_-^l J_{-l} + p_a (\bar{\theta}_- \Gamma^a \lambda_+ + \bar{\theta}_+ \Gamma^a \lambda_-) \right. \\
 & + p_\mu (\bar{\theta}_- \Gamma^\mu \lambda_-) + E^i \left[\left(R_i^\mu + \frac{1}{2} \bar{\theta}_- \Gamma^\mu \partial_i \theta_- \right) \right. \\
 & \times (\bar{\theta}_+ \Gamma_{11} \Gamma_\mu \lambda_- + \bar{\theta}_- \Gamma_{11} \Gamma_\mu \lambda_+) + \left(u_i^a + \frac{1}{2} (\bar{\theta}_- \Gamma^a \partial_i \theta_+ \right. \\
 & \left. \left. + \bar{\theta}_+ \Gamma^a \partial_i \theta_-) \right) (\bar{\theta}_- \Gamma_{11} \Gamma_a \lambda_-) \right. \\
 & \left. - \frac{1}{2} (\bar{\theta}_- \Gamma_{11} \Gamma_\mu \partial_i \theta_+ + \bar{\theta}_+ \Gamma_{11} \Gamma_\mu \partial_i \theta_-) (\bar{\theta}_- \Gamma^\mu \lambda_-) \right. \\
 & \left. - \frac{1}{2} (\bar{\theta}_- \Gamma_{11} \Gamma_a \partial_i \theta_-) (\bar{\theta}_+ \Gamma^a \lambda_- + \bar{\theta}_- \Gamma^a \lambda_+) \right] \\
 & + T_{NR} \epsilon_{ijk} (\bar{\theta}_+ \Gamma^i \lambda_+) R_1^j R_2^k - \left(\lambda_+^T \frac{\partial^r [\mathcal{L}_{NR}^{WZ}]_3}{\partial \dot{\theta}_+} \right) \\
 & - \left(\lambda_-^T \frac{\partial^r [\mathcal{L}_{NR}^{WZ}]_3}{\partial \dot{\theta}_-} \right) \tag{1.34}
 \end{aligned}$$

with $i, j, k, \mu = 0, 1, 2/a = 3, \dots, 9$.

A set of BRST transformations is the following:

$$\begin{aligned}
 s\theta_+^l &= \lambda_+^l, \\
 s\theta_-^l &= \lambda_-^l, \\
 sx^\mu &= \bar{\theta}_- \Gamma^\mu \lambda_-, \\
 sx^a &= \bar{\theta}_- \Gamma^a \lambda_+ + \bar{\theta}_+ \Gamma^a \lambda_-, \\
 sR_\nu^\mu &= 2(\partial_\nu \bar{\theta}_- \Gamma^\mu \lambda_-), \\
 su_\mu^a &= 2(\partial_\mu \bar{\theta}_- \Gamma^a \lambda_+ + \partial_\mu \bar{\theta}_+ \Gamma^a \lambda_-), \\
 s\tilde{\mathcal{F}}_{ij}^{(1)} &= 2R_j^\mu (\bar{\lambda}_- \Gamma_{11} \Gamma_\mu \partial_i \theta_+ + \bar{\lambda}_+ \Gamma_{11} \Gamma_\mu \partial_i \theta_-) \\
 & + 2u_j^a (\partial_i \bar{\theta}_- \Gamma_{11} \Gamma^a \lambda_-) - (i \leftrightarrow j) \tag{1.35}
 \end{aligned}$$

where again $\mu, \nu = 0, 1, 2/i, j = 1, 2/a = 3, \dots, 9/l = 1, \dots, 32$; from which we obtain

$$\begin{aligned}
 s^2\theta_+^l &= s^2\theta_-^l = 0, \\
 s^2x^\mu &= \bar{\lambda}_- \Gamma^\mu \lambda_-, \\
 s^2x^a &= 2(\bar{\lambda}_- \Gamma^a \lambda_+), \\
 s^2R_i^\mu &= \frac{\partial}{\partial \sigma^i} (\bar{\lambda}_- \Gamma^\mu \lambda_-), \\
 s^2u_i^a &= 2\frac{\partial}{\partial \sigma^i} (\bar{\lambda}_+ \Gamma^a \lambda_-) \tag{1.36}
 \end{aligned}$$

and

$$\begin{aligned}
 s^2\tilde{\mathcal{F}}_{ij}^{(1)} &= 2R_j^\mu \frac{\partial}{\partial \sigma^i} (\bar{\lambda}_+ \Gamma_{11} \Gamma_\mu \lambda_-) + u_j^a \frac{\partial}{\partial \sigma^i} (\bar{\lambda}_- \Gamma_{11} \Gamma_a \lambda_-) \\
 & + 2(\partial_j \bar{\theta}_- \Gamma_{11} \Gamma^\mu \partial_i \theta_-) (\bar{\lambda}_- \Gamma_\mu \lambda_-) \\
 & + 2(\partial_j \bar{\theta}_- \Gamma^a \partial_i \theta_+) (\bar{\lambda}_- \Gamma_{11} \Gamma_a \lambda_-) \\
 & + 2(\partial_j \bar{\theta}_- \Gamma_{11} \Gamma_a \partial_i \theta_-) (\bar{\lambda}_- \Gamma^a \lambda_+) \\
 & + 2(\partial_j \bar{\theta}_- \Gamma^\mu \partial_i \theta_-) (\bar{\lambda}_- \Gamma_{11} \Gamma_\mu \lambda_+) \tag{1.37}
 \end{aligned}$$

where we used the ansatz

$$\begin{aligned}
 \lambda_- &= P_- \lambda, \\
 P_- &= \frac{1}{2} (1 - \Gamma_0 \Gamma_1 \Gamma_2) \tag{1.38}
 \end{aligned}$$

The full set of constraints required for the nilpotency of the BRST charge is obtained by studying the equal time Poisson bracket $\{Q(\sigma), Q(\sigma')\}$ where $Q(\sigma)$ is given in (1.34). A basis for expanding a 32×32 matrix M is given in Appendix A.

Thus, the expression for the equal time Poisson bracket $\{Q(\sigma), Q(\sigma')\}$ where $Q(\sigma)$ is the full BRST charge defined in (1.34) is given by:

$$\begin{aligned}
 \{Q(\sigma), Q(\sigma')\} &= \int d^2\sigma (\partial_i E^i) [(\bar{\theta}_- \Gamma^a \lambda_+ + \bar{\theta}_+ \Gamma^a \lambda_-) (\bar{\theta}_- \Gamma_{11} \Gamma_a \lambda_-) \\
 & + (\bar{\theta}_- \Gamma^\mu \lambda_-) (\bar{\theta}_+ \Gamma_{11} \Gamma_\mu \lambda_- + \bar{\theta}_- \Gamma_{11} \Gamma_\mu \lambda_+)] \\
 & + 2T_{NR} \epsilon_{\mu\nu\rho} \int d^2\sigma R_1^\nu R_2^\rho (\bar{\lambda}_+ \Gamma^\mu \lambda_+) \\
 & + T_{NR} \epsilon_{0jk} \int d^2\sigma R_j^\mu R_k^\nu (\bar{\lambda}_+ \Gamma_{\mu\nu} \lambda_+) \\
 & + \int d^2\sigma [F_\mu (\bar{\lambda}_- \Gamma^\mu \lambda_-) \\
 & + F_a (\bar{\lambda}_- \Gamma^a \Gamma_{11} \lambda_-) + F_{ab} (\bar{\lambda}_- \Gamma^{ab} \lambda_-) \\
 & + F^{vabdf} (\bar{\lambda}_- \Gamma_{vabdf} \lambda_-) + \bar{F} (\bar{\lambda}_- \Gamma_{11} \lambda_+) + \bar{F}_a (\bar{\lambda}_- \Gamma^a \lambda_+) \\
 & + \bar{F}_{vb} (\bar{\lambda}_- \Gamma^{vb} \lambda_+) + \bar{F}_v (\bar{\lambda}_- \Gamma^v \Gamma_{11} \lambda_+) + \bar{F}_{vabc} \\
 & (\bar{\lambda}_- \Gamma^{vabc} \lambda_+) + \bar{F}^{vab} (\bar{\lambda}_+ \Gamma_{vab} \Gamma_{11} \lambda_-) \\
 & + \bar{F}_{fglmn} (\bar{\lambda}_+ \Gamma^{fglmn} \lambda_-) + \bar{F}_{lmn} (\bar{\lambda}_- \Gamma^{lmn} \lambda_+)] \tag{1.39}
 \end{aligned}$$

where now $\epsilon_{012} = 1, i = 1, 2/\mu, \nu, \rho = 0, 1, 2/a, b, c, d, f, g, l, m, n = 3, \dots, 9$; and the ansatz $\lambda_- = P_- \lambda$ was used.

The expressions for $F_\mu, F_a, F_{ab}, F_{vabdf}, \bar{F}, \bar{F}_a, \bar{F}_v, \bar{F}_{vb}, \bar{F}_{vabc}, \bar{F}_{vab}, \bar{F}_{fglm}, \bar{F}_{lmn}$ are given in Appendix B. These expressions are different from zero, so the following constraints guarantee the nilpotency of the BRST charge:

$$\begin{aligned}
 \partial_i E^i &= 0 \quad (\text{Gauss law}), \\
 \bar{\lambda}_+ \Gamma_\mu \lambda_+ &= 0, \quad \bar{\lambda}_+ \Gamma_{\mu\nu} \lambda_+ = 0, \\
 \bar{\lambda}_- \Gamma_\mu \lambda_- &= 0, \quad \bar{\lambda}_- \Gamma_a \Gamma_{11} \lambda_- = 0, \\
 \bar{\lambda}_- \Gamma_{ab} \lambda_- &= 0, \quad \bar{\lambda}_- \Gamma_{\mu abcd} \lambda_- = 0, \\
 \bar{\lambda}_- \Gamma_{11} \lambda_+ &= 0, \quad \bar{\lambda}_- \Gamma_a \lambda_+ = 0, \\
 \bar{\lambda}_- \Gamma_{\mu a} \lambda_+ &= 0, \quad \bar{\lambda}_- \Gamma_\mu \Gamma_{11} \lambda_+ = 0, \\
 \bar{\lambda}_- \Gamma_{\mu abc} \lambda_+ &= 0, \quad \bar{\lambda}_+ \Gamma_{\mu ab} \Gamma_{11} \lambda_- = 0, \\
 \bar{\lambda}_+ \Gamma_{fglmn} \lambda_- &= 0, \quad \bar{\lambda}_- \Gamma_{lmn} \lambda_+ = 0 \tag{1.40}
 \end{aligned}$$

with the ansatz $\lambda_- = P_- \lambda, (i = 1, 2/\mu, \nu = 0, 1, 2/a, b, c, d, f, g, l, m, n = 3, \dots, 9)$.

2 Solving the constraints

In order to solve the constraints (1.40) we use the methodology used in [21,22]. The Dirac matrices Γ^m ($m = 0, 1, \dots, 9$) are combined into five creation operators a^i ($i = 1, \dots, 5$) and five annihilation operators a_i ($i = 1, \dots, 5$) as follows:

$$\begin{aligned} a^1 &= \frac{1}{2}(\Gamma^1 - i\Gamma^2), & a^2 &= \frac{1}{2}(\Gamma^3 - i\Gamma^4), \\ a^3 &= \frac{1}{2}(\Gamma^5 - i\Gamma^6), \\ a^4 &= \frac{1}{2}(\Gamma^7 - i\Gamma^8), & a^5 &= \frac{1}{2}(\Gamma^9 + \Gamma^0), \\ a_1 &= \frac{1}{2}(\Gamma^1 + i\Gamma^2), & a_2 &= \frac{1}{2}(\Gamma^3 + i\Gamma^4), \\ a_3 &= \frac{1}{2}(\Gamma^5 + i\Gamma^6), \\ a_4 &= \frac{1}{2}(\Gamma^7 + i\Gamma^8), & a_5 &= \frac{1}{2}(\Gamma^9 - \Gamma^0) \end{aligned} \tag{2.1}$$

The following identities hold:

$$\begin{aligned} \{a_i, a^j\} &= \delta_{ij}, \{a^i, a^j\} = 0, & \{\Gamma_{11}, a^i\} &= 0, \\ \{\Gamma_{11}, a_j\} &= 0, & a_i &= a^{i\dagger}, \\ & & \text{with now } i, j &= 1, \dots, 5. \end{aligned} \tag{2.2}$$

Here we introduce the vacuum state $|0\rangle$ and the state $\langle 0|$ with the equations:

$$a_i |0\rangle = 0, \quad \langle 0| a^i = 0, \quad i = 1, \dots, 5. \tag{2.3}$$

We decompose a 32-component Dirac spinor into a sum of a positive chirality 16-component part and a negative chirality 16-component part as follows:

$$\begin{aligned} |\lambda\rangle &= \left(\lambda_+ |0\rangle + \frac{1}{2} \lambda_{ij} a^j a^i |0\rangle \right. \\ &\quad \left. + \frac{1}{4!} \lambda^i \epsilon_{ijklm} a^j a^k a^l a^m |0\rangle \right) \\ &\quad + \left(\frac{1}{5!} \lambda'_+ \epsilon_{ijklm} a^i a^j a^k a^l a^m |0\rangle \right. \\ &\quad \left. + \frac{1}{3!} \lambda'^{ij} \epsilon_{ijklm} a^k a^l a^m |0\rangle + \lambda'_i a^i |0\rangle \right) \\ &= (1 + \overline{10} + 5) + (1 + 10 + \overline{5}), \end{aligned} \tag{2.4}$$

where $\lambda_{ij} = -\lambda_{ji}$, $\lambda'^{ij} = -\lambda'^{ji}$.

Let us try to solve from (1.40) the constraints $\bar{\lambda}_+ \Gamma_\mu \lambda_+ = 0$ and $\bar{\lambda}_+ \Gamma_{\mu\nu} \lambda_+ = 0$ (where $\mu, \nu = 0, 1, 2$).

It is interesting to note that λ_+ can be written as a sum of two pure spinors $\lambda_{1+} = P_+ \lambda_+$ and $\lambda_{2+} = P_- \lambda_+$ where $P_\pm = \frac{1}{2}(1 \pm \Gamma_0 \Gamma_1 \Gamma_2)$ and the constraints $\bar{\lambda}_+ \Gamma_\mu \lambda_+ = 0$, $\bar{\lambda}_+ \Gamma_{\mu\nu} \lambda_+ = 0$, ($\mu, \nu = 0, 1, 2$) can be expressed equivalently as two pure spinor constraints: $\bar{\lambda}_{1+} \Gamma_m \lambda_{1+} = 0$, $\bar{\lambda}_{2+} \Gamma_m \lambda_{2+} = 0$, ($m = 0, \dots, 9$).

The constraints $\bar{\lambda}_+ \Gamma_\mu \lambda_+ = 0$ can be written as follows:

$$\begin{aligned} \langle \lambda_+ | C a_1 | \lambda_+ \rangle &= 0, \\ \langle \lambda_+ | C a^1 | \lambda_+ \rangle &= 0, \\ \langle \lambda_+ | C a_5 | \lambda_+ \rangle &= \langle \lambda_+ | C a^5 | \lambda_+ \rangle, \end{aligned} \tag{2.5}$$

whereas the constraints $\bar{\lambda}_+ \Gamma_{\mu\nu} \lambda_+ = 0$, ($\mu, \nu = 0, 1, 2$) can be expressed as:

$$\begin{aligned} \langle \lambda_+ | C(a^5 - a_5)(a^1 + a_1) | \lambda_+ \rangle &= 0, \\ \langle \lambda_+ | C(a^1 + a_1)(a^1 - a_1) | \lambda_+ \rangle &= 0, \\ \langle \lambda_+ | C(a^5 - a_5)(a^1 - a_1) | \lambda_+ \rangle &= 0. \end{aligned} \tag{2.6}$$

For a general $|\lambda_+\rangle$ we can write

$$\begin{aligned} |\lambda_+\rangle &= \lambda_{++} |0\rangle + \frac{1}{2} \lambda_{+ij} a^j a^i |0\rangle \\ &\quad + \frac{1}{4!} \lambda^i \epsilon_{ijklm} a^j a^k a^l a^m |0\rangle \\ &\quad + \lambda'_{+i} a^i |0\rangle + \frac{1}{3!} \lambda'^{ij} \epsilon_{ijklm} a^l a^k a^m |0\rangle \\ &\quad + \frac{1}{5!} \lambda'_{++} \epsilon_{ijklm} a^i a^j a^k a^l a^m |0\rangle, \end{aligned} \tag{2.7}$$

where now $\epsilon_{12345} = 1$.

Thus we have the following relations:

$$\begin{aligned} \langle \lambda_+ | C a^1 | \lambda_+ \rangle &= 2 \left(\lambda_{++} \lambda^1_+ + \frac{1}{8} \epsilon_{1ijkl} \lambda_{+ij} \lambda_{+lk} \right) - 4 \lambda'_{+i} \lambda'^{i1}_+ = 0, \end{aligned} \tag{2.8}$$

$$\begin{aligned} \langle \lambda_+ | C a_1 | \lambda_+ \rangle &= 2 \lambda^i \lambda_{+i1} - 2 \lambda'_{+1} \lambda'_{++} + \epsilon_{1ijj'j'} \lambda'^{ij}_+ \lambda'^{i'j'}_+ = 0, \end{aligned} \tag{2.9}$$

$$\begin{aligned} \langle \lambda_+ | C a^5 | \lambda_+ \rangle &= \langle \lambda_+ | C a_5 | \lambda_+ \rangle \\ &\Rightarrow 2(\lambda_{++} \lambda^5_+ + \frac{1}{8} \epsilon_{5ijkl} \lambda_{+ij} \lambda_{+lk}) - 4 \lambda'_{+i} \lambda'^{i5}_+ \\ &= 2 \lambda^i \lambda_{+i5} - 2 \lambda'_{+5} \lambda'_{++} + \epsilon_{5ijj'j'} \lambda'^{ij}_+ \lambda'^{i'j'}_+. \end{aligned} \tag{2.10}$$

On the other hand the constraints $\bar{\lambda}_+ \Gamma^{\mu\nu} \lambda_+ = 0$ can be rewritten as follows:

$$\begin{aligned} \frac{1}{2} \langle \lambda_+ | C(a_1 a^1 - a^1 a_1) | \lambda_+ \rangle &= -\lambda_{++} \lambda'_{++} + 4 \lambda_{+i1} \lambda'^{i1}_+ - \lambda_{+ij} \lambda'^{ij}_+ \\ &\quad + \lambda^i \lambda'_{+i} - 2 \lambda^1_+ \lambda'_{+1} = 0, \\ \langle \lambda_+ | C(a^5 - a_5) a^1 | \lambda_+ \rangle &= 4 \lambda_{++} \lambda'^{51}_+ + 4 \lambda'^{i1}_+ \lambda_{+i5} + 2 \lambda'_{+5} \lambda^1_+ \\ &\quad - \epsilon_{15ijj'} \lambda_{+ij} \lambda'_{+i'} = 0, \\ \frac{1}{2} \langle \lambda_+ | C(a^5 - a_5) a_1 | \lambda_+ \rangle &= -2 \lambda_{+i1} \lambda'^{i5}_+ + \lambda_{+51} \lambda'_{++} + \lambda^5_+ \lambda'_{+1} \\ &\quad - \epsilon_{ijk15} \lambda'^{ij}_+ \lambda^k_+ = 0. \end{aligned} \tag{2.11}$$

From (2.8)–(2.11) we obtain the non-trivial solution

$$\begin{aligned} \lambda_+^1 &= \frac{1}{\lambda_{++}} \left(2\lambda'_{+i}\lambda_{+i}^1 + \frac{1}{8}\epsilon_{ijkl}\lambda_{+ij}\lambda_{+kl} \right), \\ \lambda'_{+1} &= \frac{1}{2\lambda'_{++}} \left(2\lambda_{+i}^1\lambda_{+i+1} + \epsilon_{ijkl}\lambda_{+ij}\lambda_{+kl}^1 \right), \\ \lambda_+^5 &= \frac{1}{\lambda_{++}} \left(\lambda_{+i}\lambda_{+i5} - \lambda'_{+5}\lambda'_{++} + 2\lambda'_{+i}\lambda_{+i}^5 \right. \\ &\quad \left. + \frac{1}{2}\epsilon_{5ijkl}\lambda_{+ij}\lambda_{+kl} + \frac{1}{8}\epsilon_{5ijkl}\lambda_{+ij}\lambda_{+kl} \right), \\ \lambda_{+5}^1 &= \frac{1}{4\lambda_{++}} \left(-4\lambda_{+i}^1\lambda_{+i5} - 2\lambda'_{+5}\lambda_{+i}^1 + \epsilon_{15ijk}\lambda_{+ij}\lambda_{+k}^1 \right), \\ \lambda_{+51} &= \frac{1}{\lambda'_{++}} \left(2\lambda_{+i1}\lambda_{+i5}^1 - \lambda_{+i}^5\lambda'_{+1} + \epsilon_{15ijk}\lambda_{+ij}\lambda_{+k}^5 \right), \\ \lambda'_{++} &= \frac{1}{\lambda_{++}} \left(4\lambda_{+i1}\lambda_{+i}^1 - \lambda_{+ij}\lambda_{+i}^1 + \lambda_{+i}^1\lambda'_{+i} - 2\lambda_{+i}^1\lambda'_{+1} \right). \end{aligned} \tag{2.12}$$

Now from (1.40) we deal with the following constraints:

$$\begin{aligned} \bar{\lambda}_-\Gamma^\mu\lambda_- &= 0, & \bar{\lambda}_-\Gamma^a\Gamma_{11}\lambda_- &= 0, \\ \bar{\lambda}_-\Gamma^{ab}\lambda_- &= 0, & \bar{\lambda}_-\Gamma^{\mu abcd}\lambda_- &= 0, \end{aligned} \tag{2.13}$$

where $\mu = 0, 1, 2/a, b, c, d = 3, \dots, 9$ with $\lambda_- = P_-\lambda$ (P_- is given in (1.38)). Thus, we can write

$$P_- = \frac{1}{2} \left[1 + i(a^5 - a_5)(a_1 a^1 - a^1 a_1) \right]. \tag{2.14}$$

We have that $P_-^T C = C P_-$ where C is the charge conjugation matrix. So for example the constraints $\bar{\lambda}_-\Gamma^\mu\lambda_- = 0$ can be rewritten as follows:

$$\begin{aligned} \langle \lambda | C P_- a^1 P_- | \lambda \rangle &= 0, \\ \langle \lambda | C P_- a_1 P_- | \lambda \rangle &= 0, \\ \langle \lambda | C P_- a^5 P_- | \lambda \rangle &= \langle \lambda | C P_- a_5 P_- | \lambda \rangle. \end{aligned} \tag{2.15}$$

The solution of the constraints in (2.13) is the trivial one. This can be shown as follows. Let us consider the following 16 constraints:

$$\begin{aligned} \bar{\lambda}_-\Gamma^{13579}\lambda_- &= 0, & \bar{\lambda}_-\Gamma^{13589}\lambda_- &= 0, & \bar{\lambda}_-\Gamma^{13679}\lambda_- &= 0, \\ \bar{\lambda}_-\Gamma^{13689}\lambda_- &= 0, & & & & \\ \bar{\lambda}_-\Gamma^{14579}\lambda_- &= 0, & \bar{\lambda}_-\Gamma^{14679}\lambda_- &= 0, & \bar{\lambda}_-\Gamma^{14589}\lambda_- &= 0, \\ \bar{\lambda}_-\Gamma^{14689}\lambda_- &= 0, & & & & \\ \bar{\lambda}_-\Gamma^{23579}\lambda_- &= 0, & \bar{\lambda}_-\Gamma^{23589}\lambda_- &= 0, & \bar{\lambda}_-\Gamma^{23679}\lambda_- &= 0, \\ \bar{\lambda}_-\Gamma^{23689}\lambda_- &= 0, & & & & \\ \bar{\lambda}_-\Gamma^{24579}\lambda_- &= 0, & \bar{\lambda}_-\Gamma^{24589}\lambda_- &= 0, & \bar{\lambda}_-\Gamma^{24679}\lambda_- &= 0, \\ \bar{\lambda}_-\Gamma^{24689}\lambda_- &= 0. \end{aligned} \tag{2.16}$$

which can be rewritten as:

$$\begin{aligned} \langle \lambda | C a^1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a^2 a^3 a^4 | \lambda \rangle \\ = -(\lambda_+)^2 + (\lambda'_5)^2 + 2i(\lambda_+ \lambda'_5) = 0 \end{aligned} \tag{2.17}$$

$$\begin{aligned} \langle \lambda | C a^1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a^2 a^3 a_4 | \lambda \rangle \\ = -(\lambda_{45})^2 + (\lambda'_4)^2 - 2i(\lambda_4 \lambda_{45}) = 0 \end{aligned} \tag{2.18}$$

$$\begin{aligned} \langle \lambda | C a^1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a^2 a_3 a^4 | \lambda \rangle \\ = -(\lambda_{35})^2 + (\lambda'_3)^2 - 2i(\lambda_3 \lambda_{35}) = 0 \end{aligned} \tag{2.19}$$

$$\begin{aligned} \langle \lambda | C a^1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a_2 a^3 a^4 | \lambda \rangle \\ = -(\lambda_{52})^2 + (\lambda'_2)^2 - 2i(\lambda_2 \lambda_{52}) = 0 \end{aligned} \tag{2.20}$$

$$\begin{aligned} \langle \lambda | C a^1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a^2 a_3 a_4 | \lambda \rangle \\ = -(\lambda_{34})^2 + 4(\lambda'^{12})^2 - 4i(\lambda_{34} \lambda'^{12}) = 0 \end{aligned} \tag{2.21}$$

$$\begin{aligned} \langle \lambda | C a^1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a_2 a^3 a_4 | \lambda \rangle \\ = -(\lambda_{42})^2 + 4(\lambda'^{13})^2 - 4i(\lambda_{42} \lambda'^{13}) = 0 \end{aligned} \tag{2.22}$$

$$\begin{aligned} \langle \lambda | C a^1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a_2 a_3 a^4 | \lambda \rangle \\ = -(\lambda_{23})^2 + 4(\lambda'^{14})^2 - 4i(\lambda_{23} \lambda'^{14}) = 0 \end{aligned} \tag{2.23}$$

$$\begin{aligned} \langle \lambda | C a^1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a_2 a_3 a_4 | \lambda \rangle \\ = -(\lambda^1)^2 + 4(\lambda'^{15})^2 - 4i(\lambda^1 \lambda'^{15}) = 0 \end{aligned} \tag{2.24}$$

$$\begin{aligned} \langle \lambda | C a_1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a^2 a^3 a^4 | \lambda \rangle \\ = -(\lambda_{15})^2 + (\lambda'_1)^2 + 2i(\lambda'_1 \lambda_{15}) = 0 \end{aligned} \tag{2.25}$$

$$\begin{aligned} \langle \lambda | C a_1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a^2 a^3 a_4 | \lambda \rangle \\ = -(\lambda_{14})^2 + 4(\lambda'^{23})^2 + 4i(\lambda_{14} \lambda'^{23}) = 0 \end{aligned} \tag{2.26}$$

$$\begin{aligned} \langle \lambda | C a_1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a^2 a_3 a^4 | \lambda \rangle \\ = -(\lambda_{13})^2 + 4(\lambda'^{24})^2 - 4i(\lambda_{13} \lambda'^{24}) = 0 \end{aligned} \tag{2.27}$$

$$\begin{aligned} \langle \lambda | C a_1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a_2 a^3 a^4 | \lambda \rangle \\ = -(\lambda_{12})^2 + 4(\lambda'^{34})^2 + 4i(\lambda_{12} \lambda'^{34}) = 0 \end{aligned} \tag{2.28}$$

$$\begin{aligned} \langle \lambda | C a_1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a^2 a_3 a_4 | \lambda \rangle \\ = -(\lambda^2)^2 + 4(\lambda'^{52})^2 - 4i(\lambda^2 \lambda'^{52}) = 0 \end{aligned} \tag{2.29}$$

$$\begin{aligned} \langle \lambda | C a_1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a_2 a^3 a_4 | \lambda \rangle \\ = -(\lambda^3)^2 + 4(\lambda'^{53})^2 + 4i(\lambda^3 \lambda'^{53}) = 0 \end{aligned} \tag{2.30}$$

$$\begin{aligned} \langle \lambda | C a_1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a_2 a_3 a^4 | \lambda \rangle \\ = -(\lambda^4)^2 + 4(\lambda'^{45})^2 + 4i(\lambda^4 \lambda'^{45}) = 0 \end{aligned} \tag{2.31}$$

$$\begin{aligned} \langle \lambda | C a_1((a^5 + a_5) - i(a_5 a^5 - a^5 a_5)) a_2 a_3 a_4 | \lambda \rangle \\ = -(\lambda^5)^2 + (\lambda'_+)^2 - 2i(\lambda^5 \lambda'_+) = 0 \end{aligned} \tag{2.32}$$

and the expression for $|\lambda\rangle$ is given in (2.4). According to Comment 2 in [21,22] we choose all the λ s and λ' s to be real. However the Dirac spinor becomes complex in a general Lorentz frame. Then the relations (2.17)–(2.32) imply that all the components of $|\lambda\rangle$ are zero and the solution of the constraints in (2.16) is the trivial one. It is interesting to note that λ_- corresponds to θ_- which is a gauge degree of freedom as pointed out in [16].

Thus, the novel nontrivial solution (2.12) found for one of the spinor fields actually proves that the pure spinor formalism can be successfully applied to non-relativistic systems with kappa symmetry as is the case of the IIA D2-brane.

3 Conclusions

This paper is the first attempt to extend the application of the pure spinor formalism to non-relativistic systems with kappa symmetry expecting to draw some conclusions for the general case as well. This can be very useful especially in cases where it is difficult to solve the pure spinor constraints in the general case. We treated the non-relativistic IIA D2 brane in the framework of this formalism [23] and we derived the fermionic constraints corresponding to the rescaled fermionic coordinates. We introduced two commuting spinor fields each one corresponding to a fermionic coordinate. The nilpotency of the BRST charge leads to a set of constraints for the two spinor fields including pure spinor constraints. Nontrivial solutions are found for the spinor field λ_+ which corresponds to the fermionic coordinate θ_+ . It is interesting to note that this solution can be written as the sum of two pure spinors $\lambda_{1+} = P_+\lambda_+$ and $\lambda_{2+} = P_-\lambda_+$ where $P_{\pm} = \frac{1}{2}(1 \pm \Gamma_0\Gamma_1\Gamma_2)$. The solution for the spinor field λ_- corresponding to θ_- which, according to the proof given in [16] constitutes a gauge degree of freedom, is the trivial one. So for example the expression of the BRST charge for $\theta_- = 0$ in the Schrödinger representation is given by $Q = \int d^2\sigma [-i(\lambda_{1+} + \lambda_{2+})^\alpha \frac{\partial}{\partial \theta_+^\alpha} + 2T_{NR}\epsilon_{ijk}(\bar{\theta}_+\Gamma^i\lambda_{1+})\partial_1x^j\partial_2x^k]$. Cohomological issues concerning the BRST charge are currently under investigation [26–29]. This study can also be performed for more general manifolds and for general dimensions as well. We would like to finally mention that the treatment of the relativistic Dp-brane in the framework of the pure spinor formalism has been reported in [24].

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4 Appendix A

For the matrices Γ^m ($m = 0, \dots, 9$) we use the Majorana representation (Γ^0 is real antisymmetric, Γ^i ($i = 1, \dots, 9$) are real symmetric)

$$\{\Gamma^m, \Gamma^n\} = 2\eta^{mn} \tag{4.1}$$

where $\eta^{mn} = \text{diag}(-1, 1, \dots, 1)$ and $m, n = 0, \dots, 9$; and

$$C = \Gamma_0, \tag{4.2}$$

$$(\Gamma^m)^T = -\Gamma_0\Gamma^m\Gamma_0^{-1}.$$

A basis for the 32×32 matrices is given by (see Appendix B in [25] for instance):

$$B = \left\{ I, \Gamma^A\Gamma^{AB}, \Gamma^{ABC}, \Gamma^{ABCD}, \Gamma^{ABCDE}, \Gamma^{ABCD}\Gamma_{11}, \Gamma^{AB}\Gamma_{11}, \Gamma^A\Gamma_{11}, \Gamma_{11} \right\}, \tag{4.3}$$

with $A < B < C < D < E$.

For the differential form $d\theta$ we use the convention given in [18]:

$$d\theta = d\sigma^\mu\partial_\mu\theta = -\partial_\mu\theta d\sigma^\mu. \tag{4.4}$$

The following identities hold for $\lambda_- = P_-\lambda$:

$$\Gamma_{\mu\nu}\lambda_- = -\epsilon_{\mu\nu\rho}\Gamma^\rho\lambda_-$$

$$\Gamma_{\mu\nu\rho}\lambda_- = -\epsilon_{\mu\nu\rho}\lambda_- \tag{4.5}$$

$$\epsilon_{012} = 1.$$

5 Appendix B

$$F_\mu = 2[p_\mu - E^i(\bar{\theta}_-\Gamma_{11}\Gamma_\mu\partial_i\theta_+ + \bar{\theta}_+\Gamma_{11}\Gamma_\mu\partial_i\theta_-)]$$

$$+ 2T_{NR}\epsilon_{0jk}[-2(\bar{\theta}_+\Gamma_{\mu\nu}\partial_j\theta_+)R_k^\nu$$

$$+ (\bar{\theta}_+\Gamma_{a\mu}\partial_j\theta_- + \bar{\theta}_-\Gamma_{a\mu}\partial_j\theta_+)u_k^a]$$

$$+ \frac{T_{NR}}{96}\epsilon_{0jk}[(\bar{\theta}_-\Gamma^\nu\partial_k\theta_-)(-89(\bar{\theta}_+\Gamma_{\mu\nu}\partial_j\theta_+)$$

$$+ 22(\bar{\theta}_+\partial_j\theta_+)\eta_{\mu\nu}) + (\bar{\theta}_+\Gamma^\nu\partial_k\theta_+)(83(\bar{\theta}_-\Gamma_{\mu\nu}\partial_j\theta_-)$$

$$+ 6(\bar{\theta}_-\partial_j\theta_-)\eta_{\mu\nu}) - 2(\bar{\theta}_-\Gamma^a\partial_k\theta_+$$

$$+ \bar{\theta}_+\Gamma^a\partial_k\theta_-)(47(\bar{\theta}_+\Gamma_{\mu a}\partial_j\theta_-) + 23(\bar{\theta}_-\Gamma_{\mu a}\partial_j\theta_+))$$

$$- 5(\bar{\theta}_+\Gamma_\mu\Gamma_{\nu a}\partial_j\theta_-)(\partial_k\bar{\theta}_+\Gamma^{\nu a}\theta_-)$$

$$- \frac{1}{3}(\partial_k\bar{\theta}_+\Gamma^a\theta_-)(37(\partial_j\bar{\theta}_-\Gamma_{\mu a}\theta_+)$$

$$- 66(\partial_j\bar{\theta}_+\Gamma_{\mu a}\theta_-)) - (\partial_j\theta_-\Gamma^a\theta_+)(21(\partial_k\bar{\theta}_+\Gamma_{\mu a}\theta_-)$$

$$- 2(\partial_k\theta_-\Gamma_{\mu a}\theta_+)) - 2(\bar{\theta}_+\Gamma_{11}\partial\theta_- + \bar{\theta}_-\Gamma_{11}\partial\theta_+)$$

$$\times (25(\bar{\theta}_+\Gamma_{11}\Gamma_\mu\partial_j\theta_-) + 31(\bar{\theta}_-\Gamma_{11}\Gamma_\mu\partial_j\theta_+))$$

$$+ 2(\bar{\theta}_-\Gamma_{11}\Gamma^\nu\partial_k\theta_+ + \bar{\theta}_+\Gamma_{11}\Gamma^\nu\partial_k\theta_-)$$

$$\times (\bar{\theta}_+\Gamma_{11}\Gamma_\mu\Gamma_\nu\partial_j\theta_- + 2(\bar{\theta}_-\Gamma_{11}\partial_j\theta_+)\eta_{\mu\nu})$$

$$- (\partial_j\bar{\theta}_-\Gamma_{11}\Gamma^\nu\theta_-)(64(\partial_k\bar{\theta}_+\Gamma_{11}\theta_-))$$

$$\begin{aligned}
 & -11(\partial_k \bar{\theta}_+ \Gamma_{11} \Gamma_{\mu\nu} \theta_-) + 46(\partial_k \bar{\theta}_- \Gamma_{11} \theta_+) \eta_{\mu\nu} \\
 & -2(\partial_k \bar{\theta}_- \Gamma_{11} \Gamma_\nu \Gamma_\mu \theta_+) + (\bar{\theta}_- \Gamma_{11} \Gamma_\nu \partial_k \theta_+) \\
 & \times (9(\bar{\theta}_- \Gamma_{11} \Gamma_\nu \Gamma_\mu \theta_+) + 34(\bar{\theta}_- \Gamma_{11} \partial_j \theta_+) \eta_{\mu\nu} \\
 & + 23(\bar{\theta}_+ \Gamma_{11} \partial_j \theta_-) \eta_{\mu\nu}) - 7(\partial_j \bar{\theta}_- \Gamma_{11} \Gamma_a \theta_-) \\
 & \times (\partial_k \bar{\theta}_+ \Gamma_{11} \Gamma_{\mu a} \theta_+) + 5(\partial_k \bar{\theta}_+ \Gamma_{11} \Gamma^{\nu a} \Gamma_\mu \theta_+) \\
 & (\bar{\theta}_- \Gamma_{\nu a} \Gamma_{11} \partial_j \theta_-) + (\bar{\theta}_- \Gamma^{\nu a} \Gamma_{11} \Gamma_\mu \partial_j \theta_-) \\
 & \times (\partial_k \bar{\theta}_+ \Gamma_{\nu a} \Gamma_{11} \theta_+) - (\partial_k \bar{\theta}_+ \Gamma_{11} \Gamma^a \theta_+) \\
 & (\partial_j \bar{\theta}_- \Gamma_{11} \Gamma_{\mu a} \theta_-) + (\bar{\theta}_+ \Gamma_{\nu a} \partial_j \theta_-) (\bar{\theta}_- \Gamma^{\nu a} \Gamma_\mu \partial_k \theta_+)] \\
 \end{aligned} \tag{5.1}$$

$$\begin{aligned}
 F_a = & -2E_i u_a^i + 2T_{NR} \epsilon_{0jk} (\bar{\theta}_+ \Gamma_{11} \partial_j \theta_- + \bar{\theta}_- \Gamma_{11} \partial_j \theta_+) u_{ak} \\
 & + \frac{T_{NR}}{3} \epsilon_{0jk} [-\frac{1}{16} (\bar{\theta}_+ \Gamma^b \partial_j \theta_-) \\
 & \times (10(\bar{\theta}_- \Gamma_{ab} \Gamma_{11} \partial_k \theta_+) - (\bar{\theta}_+ \Gamma_{ab} \Gamma_{11} \partial_k \theta_-) \\
 & + 3(\partial_k \theta_- \Gamma_{11} \theta_+) \eta_{ab} + 45(\partial_k \bar{\theta}_+ \Gamma_{11} \theta_-) \eta_{ab}) \\
 & + \frac{3}{16} (\bar{\theta}_- \Gamma^b \partial_j \theta_+) (-8(\partial_k \bar{\theta}_+ \Gamma_{ab} \Gamma_{11} \theta_-) \\
 & + 12(\bar{\theta}_- \Gamma_{11} \partial_k \theta_+) \eta_{ab} - (\bar{\theta}_+ \Gamma_{11} \partial_k \theta_-) \eta_{ab}) \\
 & + \frac{7}{8} (\bar{\theta}_+ \Gamma^\mu \partial_k \theta_+) (\partial_j \bar{\theta}_- \Gamma_{\mu a} \Gamma_{11} \theta_-) \\
 & + \frac{1}{48} (\bar{\theta}_+ \Gamma_{11} \Gamma^\mu \partial_k \theta_-) (5(\partial_j \bar{\theta}_+ \Gamma_{\mu a} \theta_-) - 6(\partial_j \bar{\theta}_- \Gamma_{\mu a} \theta_+)) \\
 & - \frac{1}{16} (\partial_j \bar{\theta}_- \Gamma_{11} \Gamma^b \theta_-) (13(\partial_k \bar{\theta}_+ \Gamma_{ab} \theta_+) \\
 & + 3(\bar{\theta}_+ \partial_k \theta_+) \eta_{ab}) - \frac{1}{32} (\partial_j \bar{\theta}_- \Gamma_{abc} \theta_+) \\
 & \times (2(\partial_k \bar{\theta}_+ \Gamma^{bc} \Gamma_{11} \theta_-) + \partial_k \bar{\theta}_- \Gamma^{bc} \Gamma_{11} \theta_+) \\
 & + \frac{1}{16} (\partial_j \bar{\theta}_- \Gamma^{\mu b} \theta_+) (-2(\partial_k \bar{\theta}_- \Gamma_{\mu ab} \Gamma_{11} \theta_+) \\
 & - 6(\partial_k \bar{\theta}_+ \Gamma_{11} \Gamma_{\mu ab} \theta_-) + 9(\bar{\theta}_- \Gamma_{11} \Gamma_\mu \partial_k \theta_+) \eta_{ab} \\
 & + (\bar{\theta}_+ \Gamma_{11} \Gamma_\mu \partial_k \theta_-) \eta_{ab}) - \frac{9}{16} (\bar{\theta}_- \Gamma^\mu \partial_j \theta_-) \\
 & \times (\bar{\theta}_+ \Gamma_{\mu a} \Gamma_{11} \partial_k \theta_+) + \frac{3}{8} (\bar{\theta}_+ \Gamma^{\mu b} \Gamma_{11} \partial_k \theta_+) (\partial_j \bar{\theta}_- \Gamma_{\mu ab} \theta_-) \\
 & + \frac{117}{64} (\bar{\theta}_- \partial_j \theta_-) (\bar{\theta}_+ \Gamma_{11} \Gamma_a \partial_k \theta_+) \\
 & + \frac{137}{64} (\bar{\theta}_- \Gamma_{ab} \partial_j \theta_-) (\bar{\theta}_+ \Gamma_{11} \Gamma^b \partial_k \theta_+) \\
 & + \frac{1}{16} (\bar{\theta}_- \Gamma^{\mu b} \partial_j \theta_+) (\partial_k \bar{\theta}_- \Gamma_{11} \Gamma_{\mu ab} \theta_+) \\
 & - \frac{1}{16} (\bar{\theta}_- \Gamma_{11} \Gamma^{\mu b} \partial_j \theta_-) \\
 & \times (\partial_k \bar{\theta}_+ \Gamma_{\mu ab} \theta_+) - \frac{1}{16} (\bar{\theta}_- \Gamma^{bc} \partial_k \theta_-) (\bar{\theta}_+ \Gamma_{abc} \Gamma_{11} \partial_j \theta_+)] \\
 \end{aligned} \tag{5.2}$$

$$\begin{aligned}
 F_{ab} = & T_{NR} \epsilon_{0jk} u_{aj} u_{bk} + \frac{T_{NR}}{16} \epsilon_{0jk} [(\bar{\theta}_+ \Gamma^d \partial_j \theta_-) \\
 & \times (-\frac{14}{3} (\partial_k \bar{\theta}_- \Gamma_{ab} \Gamma_d \theta_+) \\
 & + \frac{14}{3} (\partial_k \bar{\theta}_- \Gamma_d \Gamma_{ab} \theta_+) + \frac{35}{6} (\partial_k \bar{\theta}_+ \Gamma_{ab} \Gamma_d \theta_-) \\
 & - \frac{15}{2} (\partial_k \bar{\theta}_+ \Gamma_d \Gamma_{ab} \theta_-) + \frac{16}{3} (\bar{\theta}_+ \Gamma_b \partial_k \theta_-) \eta_{ad} \\
 & + \frac{32}{3} (\bar{\theta}_- \Gamma_b \partial_k \theta_+) \eta_{ad}] + \frac{1}{6} (\bar{\theta}_- \Gamma^d \partial_j \theta_+) (7(\partial_k \bar{\theta}_+ \Gamma_d \Gamma_{ab} \theta_-) \\
 \end{aligned}$$

$$\begin{aligned}
 & + 35(\partial_k \bar{\theta}_+ \Gamma_{ab} \Gamma_d \theta_-) \\
 & + 4(\partial_k \bar{\theta}_- \Gamma_d \Gamma_{ab} \theta_+) - 33(\partial_k \bar{\theta}_- \Gamma_{ab} \Gamma_d \theta_+) \\
 & - 32(\bar{\theta}_- \Gamma_a \partial_k \theta_+) \eta_{bd} + \frac{1}{6} (\bar{\theta}_- \Gamma_{11} \Gamma^\mu \partial_k \theta_+) \\
 & \times (-4(\partial_j \bar{\theta}_+ \Gamma_{11} \Gamma_{\mu ab} \theta_-) + 37(\partial_j \bar{\theta}_- \Gamma_{11} \Gamma_{\mu ab} \theta_-)) \\
 & + \frac{15}{4} (\bar{\theta}_+ \Gamma^{\mu d} \Gamma_{11} \partial_k \theta_+) (\partial_j \bar{\theta}_- \Gamma_{11} \Gamma_{\mu d} \Gamma_{ab} \theta_-) \\
 & + \frac{25}{12} (\partial_j \bar{\theta}_- \Gamma^{\mu d} \theta_+) (\partial_k \bar{\theta}_+ \Gamma_{\mu d} \Gamma_{ab} \theta_-) \\
 & - \frac{77}{12} (\bar{\theta}_+ \Gamma^d \Gamma_{11} \partial_k \theta_+) (\partial_j \bar{\theta}_- \Gamma_{11} \Gamma_d \Gamma_{ab} \theta_-) \\
 & - \frac{35}{4} (\partial_j \bar{\theta}_- \Gamma_{11} \theta_+) (\partial_k \bar{\theta}_+ \Gamma_{11} \Gamma_{ab} \theta_-) \\
 & - \frac{45}{4} (\bar{\theta}_- \Gamma_{ab} \partial_j \theta_-) (\bar{\theta}_+ \partial_k \theta_+) + \frac{37}{4} (\bar{\theta}_+ \Gamma^\mu \partial_k \theta_+) \\
 & \times (\partial_j \bar{\theta}_- \Gamma_{\mu ab} \theta_-) + \frac{1}{6} (\bar{\theta}_- \Gamma_{11} \Gamma^d \partial_j \theta_-) (5(\partial_k \bar{\theta}_+ \Gamma_{11} \Gamma_{ab} \Gamma_d \theta_+) \\
 & + 39(\partial_k \bar{\theta}_+ \Gamma_d \Gamma_{ab} \Gamma_{11} \theta_+)) - 3(\bar{\theta}_- \Gamma_{11} \partial_k \theta_+) (\partial_j \bar{\theta}_- \Gamma_{11} \Gamma_{ab} \theta_+) \\
 & + (\bar{\theta}_- \Gamma^{\mu d} \partial_j \theta_+) (\partial_k \bar{\theta}_- \Gamma_{ab} \Gamma_{\mu d} \theta_+) - (\bar{\theta}_- \Gamma_{df} \partial_k \theta_-) \\
 & \times (\bar{\theta}_+ \Gamma^d \Gamma_{ab} \Gamma^f \partial_j \theta_+) - \frac{1}{4} (\bar{\theta}_+ \Gamma_{11} \Gamma^\mu \partial_k \theta_-) (\partial_j \bar{\theta}_+ \Gamma_{11} \Gamma_{\mu ab} \theta_-) \\
 & - \frac{1}{2} (\bar{\theta}_- \partial_j \theta_-) (\partial_k \bar{\theta}_+ \Gamma_{ab} \theta_+) \\
 & - \frac{1}{6} (\bar{\theta}_- \Gamma_{11} \Gamma^{\mu d} \partial_j \theta_-) (\partial_k \bar{\theta}_+ \Gamma_{11} \Gamma_{\mu ab} \Gamma_d \theta_+) \\
 & + \frac{1}{6} (\partial_k \bar{\theta}_+ \Gamma_{df} \Gamma_{11} \theta_-) (\partial_j \bar{\theta}_- \Gamma_{11} \Gamma^d \Gamma_{ab} \Gamma^f \theta_+) \\
 & - 4(\bar{\theta}_- \Gamma^\mu \partial_j \theta_-) (\partial_k \bar{\theta}_+ \Gamma_{\mu ab} \theta_+)] \\
 \end{aligned} \tag{5.3}$$

$$\begin{aligned}
 \bar{F} = & 2T_{NR} \epsilon_{0jk} [\tilde{\mathcal{F}}_{jk}^{(1)} - \frac{1}{3} ((\bar{\theta}_- \Gamma^\mu \partial_k \theta_-) (\bar{\theta}_+ \Gamma_\mu \Gamma_{11} \partial_j \theta_-) \\
 & + \bar{\theta}_- \Gamma_\mu \Gamma_{11} \partial_j \theta_+) \\
 & + (\bar{\theta}_- \Gamma^a \Gamma_{11} \partial_j \theta_-) (\bar{\theta}_- \Gamma_a \partial_k \theta_+ + \bar{\theta}_+ \Gamma_a \partial_k \theta_-)] \\
 \end{aligned} \tag{5.4}$$

$$\begin{aligned}
 \bar{F}_\nu = & -4E^i R_{\nu i} + 4T_{NR} \epsilon_{0jk} R_{\nu k} (\bar{\theta}_+ \Gamma_{11} \partial_j \theta_- + \bar{\theta}_- \Gamma_{11} \partial_j \theta_+) \\
 & + \frac{1}{48} T_{NR} \epsilon_{0jk} [-\bar{\theta}_- \Gamma^{\mu a} \partial_j \theta_+) \\
 & \times (16(\partial_k \bar{\theta}_- \Gamma_{11} \Gamma_a \theta_-) \eta_{\mu\nu} - 5(\partial_k \bar{\theta}_- \Gamma_{\mu\nu a} \Gamma_{11} \theta_-)) \\
 & - 12(\partial_k \bar{\theta}_- \Gamma_{11} \Gamma^a \theta_-) (\bar{\theta}_+ \Gamma_{\nu a} \partial_j \theta_-) \\
 & - (\bar{\theta}_- \Gamma^\mu \partial_j \theta_-) (-37(\partial_k \bar{\theta}_+ \Gamma_{\mu\nu} \Gamma_{11} \theta_-) \\
 & + 41(\partial_k \bar{\theta}_+ \Gamma_{11} \theta_-) \eta_{\mu\nu} + 18(\partial_k \bar{\theta}_- \Gamma_{11} \theta_+) \eta_{\mu\nu}) \\
 & + (\partial_j \bar{\theta}_- \Gamma_{\nu a} \Gamma_b \theta_-) (-5(\partial_k \bar{\theta}_+ \Gamma^b \Gamma^a \Gamma_{11} \theta_-) \\
 & + 2(\partial_k \bar{\theta}_- \Gamma^b \Gamma^a \Gamma_{11} \theta_+)) - (\bar{\theta}_- \Gamma^{ab} \partial_k \theta_-) \\
 & \times (7(\bar{\theta}_- \Gamma_{\nu ab} \Gamma_{11} \partial_j \theta_+) + 2(\bar{\theta}_+ \Gamma_{\nu ab} \Gamma_{11} \partial_j \theta_-)) \\
 & + (\partial_j \bar{\theta}_+ \Gamma_{11} \Gamma^\mu \theta_-) (70(\bar{\theta}_- \partial_k \theta_-) \eta_{\mu\nu} \\
 & + 59(\partial_k \bar{\theta}_- \Gamma_{\mu\nu} \theta_-) + (\partial_j \bar{\theta}_- \Gamma_{\nu a} \Gamma_{11} \theta_-) \\
 & \times (49(\bar{\theta}_- \Gamma^a \partial_k \theta_+) + 24(\bar{\theta}_+ \Gamma^a \partial_k \theta_-)) \\
 & - 5(\partial_k \bar{\theta}_+ \Gamma^{\mu a} \Gamma_\nu \theta_-) (\bar{\theta}_- \Gamma_{11} \Gamma_{\mu a} \partial_j \theta_-)] \\
 \end{aligned} \tag{5.5}$$

$$\begin{aligned}
 \bar{F}_a = & 4(p_a - E^i (\bar{\theta}_- \Gamma_{11} \Gamma_a \partial_i \theta_-)) \\
 & - 2T_{NR} \epsilon_{0jk} [(\bar{\theta}_- \Gamma_{\mu a} \partial_k \theta_+ + \bar{\theta}_+ \Gamma_{\mu a} \partial_k \theta_-) R_j^\mu \\
 \end{aligned}$$

$$\begin{aligned}
 & -(\bar{\theta} - \Gamma_{ab} \partial_k \theta_-) u_j^b + \frac{2}{3} T_{NR} \epsilon_{0jk} \left[\frac{1}{16} (\bar{\theta} - \Gamma^\mu \partial_j \theta_-) \right. \\
 & \times (25(\partial_k \bar{\theta} + \Gamma_{\mu a} \theta_-) + 19(\partial_k \theta - \Gamma_{\mu a} \theta_+)) \\
 & + \frac{1}{8} (\bar{\theta} - \partial_j \theta_-) (21(\partial_k \bar{\theta} + \Gamma_a \theta_-) + 9(\partial_k \bar{\theta} - \Gamma_a \theta_+)) \\
 & \left. - \frac{1}{32} (\partial_j \bar{\theta} - \Gamma_{\mu ab} \theta_-) (5(\bar{\theta} - \Gamma^{\mu b} \partial_k \theta_+)) \right. \\
 & - 2(\bar{\theta} + \Gamma^{\mu b} \partial_k \theta_-) + \frac{1}{32} (\bar{\theta} - \Gamma_{ab} \partial_k \theta_-) (7(\bar{\theta} - \Gamma^b \partial_j \theta_+)) \\
 & + 12(\bar{\theta} + \Gamma^b \partial_j \theta_-) + \frac{1}{16} (\partial_j \bar{\theta} - \Gamma_{11} \Gamma_{\mu a} \theta_-) \\
 & \times (3(\partial_k \bar{\theta} - \Gamma_{11} \Gamma^\mu \theta_+) + 10(\partial_k \bar{\theta} + \Gamma_{11} \Gamma^\mu \theta_-)) \\
 & + \frac{1}{16} (\bar{\theta} - \Gamma_a \Gamma_{11} \partial_j \theta_-) (29(\bar{\theta} + \Gamma_{11} \partial_k \theta_-) \\
 & + 20(\bar{\theta} - \Gamma_{11} \partial_k \theta_+)) + \frac{1}{32} (\bar{\theta} - \Gamma_{11} \Gamma^b \partial_k \theta_-) \\
 & \times (2(\bar{\theta} + \Gamma_{11} \Gamma_{ab} \partial_j \theta_-) - \bar{\theta} - \Gamma_{11} \Gamma^{ab} \partial_j \theta_+) \\
 & + \frac{1}{32} (\partial_j \bar{\theta} - \Gamma_{11} \Gamma_{abc} \theta_-) (\bar{\theta} + \Gamma_{11} \Gamma^{bc} \partial_k \theta_- \\
 & - 5(\bar{\theta} - \Gamma_{11} \Gamma^{bc} \partial_k \theta_+)) + \frac{1}{32} (\bar{\theta} - \Gamma^{bc} \partial_j \theta_-) \\
 & \times (7(\partial_k \bar{\theta} + \Gamma_{abc} \theta_-) + 2(\partial_k \bar{\theta} - \Gamma_{abc} \theta_+)) \\
 & \left. - \frac{5}{32} (\partial_k \bar{\theta} + \Gamma_{11} \Gamma_{\mu ab} \theta_-) (\bar{\theta} - \Gamma_{11} \Gamma^{\mu b} \partial_j \theta_-) \right] \tag{5.6}
 \end{aligned}$$

$$\begin{aligned}
 \bar{F}_{va} = & 4T_{NR} \epsilon_{0jk} R_{vj} u_{ak} + \frac{1}{48} T_{NR} \epsilon_{0jk} [(\bar{\theta} - \Gamma^\mu \partial_j \theta_+) \\
 & \times (37(\bar{\theta} - \Gamma_a \partial_k \theta_+) \eta_{\mu\nu} \\
 & + 34(\bar{\theta} + \Gamma_a \partial_k \theta_-) \eta_{\mu\nu} - 5(\bar{\theta} - \Gamma_{\mu\nu a} \partial_k \theta_+) \\
 & + 6(\bar{\theta} + \Gamma_{\mu\nu a} \partial_k \theta_-) + (\bar{\theta} - \Gamma_{11} \Gamma^b \partial_j \theta_-) \\
 & \times (5(\partial_k \bar{\theta} + \Gamma_{11} \Gamma_\nu (\eta_{ab} + \Gamma_{ba}) \theta_-) \\
 & - 2(\partial_k \bar{\theta} - \Gamma_{11} \Gamma_{vba} \theta_+)) - (\bar{\theta} - \Gamma^b \partial_k \theta_+) \\
 & \times (-19(\partial_j \bar{\theta} - \Gamma_{vba} \theta_-) + 9(\partial_j \bar{\theta} - \Gamma_\nu \theta_-) \eta_{ab}) \\
 & + (\bar{\theta} - \Gamma^{b\mu} \Gamma_{11} \partial_j \theta_-) (\partial_k \bar{\theta} + \Gamma_\mu \Gamma_{11} \Gamma_{va} \Gamma_b \theta_-) \\
 & + 5(\bar{\theta} - \Gamma^\mu \Gamma_{11} \partial_k \theta_+) (\partial_j \bar{\theta} - \Gamma_\mu \Gamma_{11} \Gamma_{va} \theta_-) \\
 & - (\bar{\theta} - \Gamma^{bc} \Gamma_{11} \partial_k \theta_+) (\partial_j \bar{\theta} - \Gamma_{11} \Gamma_{vabc} \theta_-) \\
 & + (\partial_k \bar{\theta} + \Gamma^{b\mu} \theta_-) (\partial_j \bar{\theta} - \Gamma_\mu \Gamma_{va} \Gamma_b \theta_-) \\
 & - 5(\bar{\theta} - \Gamma_{11} \partial_k \theta_+) (\bar{\theta} - \Gamma_{va} \Gamma_{11} \partial_j \theta_-) - 22(\bar{\theta} + \Gamma^b \partial_j \theta_-) \\
 & \times (\bar{\theta} - \Gamma_{vba} \partial_k \theta_-) + 2(\bar{\theta} + \Gamma_{11} \Gamma^\mu \partial_k \theta_-) (-\partial_j \bar{\theta} - \Gamma_{11} \Gamma_{\mu\nu a} \theta_- \\
 & + 2(\partial_j \bar{\theta} - \Gamma_{11} \Gamma_a \theta_-) \eta_{\mu\nu}) \\
 & + 2(\partial_k \bar{\theta} - \Gamma^{b\mu} \theta_+) (2(\partial_j \bar{\theta} - \Gamma_{\mu\nu} \theta_-) \eta_{ab} + (\partial_j \bar{\theta} - \Gamma_{\mu\nu ba} \theta_-) \\
 & - 2(\partial_j \bar{\theta} - \Gamma_{ba} \theta_-) \eta_{\mu\nu} \\
 & - (\partial_j \bar{\theta} - \theta_-) \eta_{\mu\nu} \eta_{ab}) - 2(\partial_j \bar{\theta} - \Gamma_{11} \theta_+) (\partial_k \bar{\theta} - \Gamma_{11} \Gamma_{va} \theta_-) \\
 & + (\partial_k \bar{\theta} - \Gamma^{bc} \theta_-) \\
 & \times (\partial_j \bar{\theta} - \Gamma_{vabc} \theta_+ - 2(\partial_j \bar{\theta} + \Gamma_{vabc} \theta_-))] \tag{5.7}
 \end{aligned}$$

$$\begin{aligned}
 \bar{F}^{vab} = & \frac{1}{96} T_{NR} \epsilon_{0jk} [(\partial_k \bar{\theta} - \Gamma^\mu \Gamma_{11} \Gamma^{vab} \Gamma^d \theta_-) \\
 & \times (4(\partial_j \bar{\theta} - \Gamma_{d\mu} \theta_+) - \partial_j \bar{\theta} + \Gamma_{d\mu} \theta_-) \\
 & - 6(\partial_k \bar{\theta} - \Gamma^d \theta_+) (\partial_j \bar{\theta} - (\Gamma_d \Gamma^{vab} + 3\Gamma^{vab} \Gamma_d) \Gamma_{11} \theta_-)
 \end{aligned}$$

$$\begin{aligned}
 & + 3(\bar{\theta} - \Gamma^d \partial_k \theta_+) \\
 & \times (\partial_j \bar{\theta} - (\Gamma_d \Gamma^{vab} + 6\Gamma^{vab} \Gamma_d) \Gamma_{11} \theta_-) \\
 & + (\partial_k \bar{\theta} + \Gamma_{dc} \Gamma_{11} \theta_-) (\partial_j \bar{\theta} - \Gamma^d \Gamma^{vab} \Gamma^c \theta_-) \\
 & - 3(\partial_k \bar{\theta} + \Gamma_{11} \theta_-) (\partial_j \bar{\theta} - \Gamma^{vab} \theta_-) \\
 & + 3(\partial_j \bar{\theta} - \Gamma^\mu \theta_-) (\partial_k \bar{\theta} + \Gamma_\mu \Gamma_{11} \Gamma^{vab} \theta_-) \\
 & - (\bar{\theta} - \Gamma_{cd} \partial_j \theta_-) (\partial_k \bar{\theta} + \Gamma^c \Gamma_{11} \Gamma^{vab} \Gamma^d \theta_-) \\
 & + (\bar{\theta} - \Gamma_{d\mu} \Gamma_{11} \partial_j \theta_-) (\partial_k \bar{\theta} + \Gamma^\mu \Gamma^{vab} \Gamma^d \theta_-) \\
 & + 3(\bar{\theta} - \Gamma^\mu \Gamma_{11} \partial_k \theta_+) (\partial_j \bar{\theta} - \Gamma_\mu \Gamma^{vab} \theta_-) \\
 & + 3(\bar{\theta} - \partial_j \theta_-) (\partial_k \bar{\theta} + \Gamma_{11} \Gamma^{vab} \theta_-) \\
 & - 3(\bar{\theta} - \Gamma_{11} \Gamma^d \partial_j \theta_-) (\bar{\theta} - \Gamma^{vab} \Gamma_d \partial_k \theta_-) \tag{5.8}
 \end{aligned}$$

$$\begin{aligned}
 \bar{F}^{lmn} = & \frac{1}{48} T_{NR} \epsilon_{0jk} \left[-\frac{1}{4} (\bar{\theta} + \Gamma^d \partial_k \theta_-) \right. \\
 & \times (\bar{\theta} - \Gamma^{lmn} \Gamma_d \partial_j \theta_- + \bar{\theta} - \Gamma_d \Gamma^{lmn} \partial_j \theta_-) \\
 & - \frac{1}{32} (\bar{\theta} - \Gamma^d \partial_k \theta_+) (5(\bar{\theta} - \Gamma^{lmn} \Gamma_d \partial_j \theta_-) \\
 & + 6(\bar{\theta} - \Gamma_d \Gamma^{lmn} \partial_j \theta_-)) + \frac{1}{32} (\bar{\theta} - \Gamma_\mu \partial_j \theta_-) \\
 & \times (\partial_k \bar{\theta} + \Gamma^{\mu lmn} \theta_-) + \frac{1}{32} (\bar{\theta} - \Gamma_{\mu d} \partial_j \theta_+) \\
 & \times (\bar{\theta} - \Gamma^d \Gamma^{\mu lmn} \partial_k \theta_-) + \frac{1}{32} (\partial_k \bar{\theta} + \Gamma_{11} \Gamma_\mu \theta_-) \\
 & \times (\partial_j \bar{\theta} - \Gamma_{11} \Gamma^{\mu lmn} \theta_-) - \frac{1}{32} (\bar{\theta} - \Gamma_{11} \Gamma_d \partial_k \theta_-) \\
 & (\bar{\theta} - \Gamma_{11} \Gamma^{lmnd} \partial_j \theta_+) + \frac{1}{32} (\bar{\theta} - \Gamma_{11} \partial_k \theta_+) \\
 & \times (\bar{\theta} - \Gamma^{lmn} \Gamma_{11} \partial_j \theta_-) - \frac{1}{32} (\partial_k \bar{\theta} + \Gamma^{df} \Gamma_{11} \theta_-) \\
 & \times (\partial_j \bar{\theta} - \Gamma_{11} \Gamma_d \Gamma^{lmn} \Gamma_f \theta_-) + \frac{1}{16} (\partial_j \bar{\theta} - \Gamma^{df} \Gamma_{11} \theta_+) \\
 & \times (\partial_k \bar{\theta} - \Gamma_{11} \Gamma_d \Gamma^{lmn} \Gamma_f \theta_-) - \frac{3}{32} (\bar{\theta} - \Gamma_{11} \Gamma^l \partial_k \theta_-) \\
 & \times (\bar{\theta} - \Gamma_{11} \Gamma^{mn} \partial_j \theta_+ - 4(\bar{\theta} + \Gamma_{11} \Gamma^{mn} \partial_j \theta_-)) \\
 & - \frac{1}{32} ((\bar{\theta} - \Gamma_{11} \Gamma^{d\mu} \partial_j \theta_-) (\partial_k \bar{\theta} + \Gamma_{11} \Gamma^{lmn} \Gamma_{d\mu} \theta_-) \\
 & + (\bar{\theta} - \Gamma_{df} \partial_j \theta_-) (\partial_k \bar{\theta} + \Gamma^d \Gamma^{lmn} \Gamma_f \theta_-) \\
 & + (\bar{\theta} - \partial_j \theta_-) (\partial_k \bar{\theta} + \Gamma^{lmn} \theta_-)) \\
 & - \frac{1}{16} (\bar{\theta} - \Gamma^{df} \partial_j \theta_-) (\bar{\theta} + \Gamma_d \Gamma^{lmn} \Gamma_f \partial_k \theta_-) \\
 & - \frac{1}{96 \times 4! \times 5!} \epsilon_{defglmn} \delta_{defg}^{[abcd]} [-5(\bar{\theta} - \partial_j \theta_-) \\
 & (\partial_k \bar{\theta} + \Gamma_{11} \Gamma^{abcd} \theta_-) + 3(\partial_j \bar{\theta} - \Gamma_\mu \theta_-) \\
 & \times (\bar{\theta} - \Gamma_{11} \Gamma^{\mu abcd} \partial_k \theta_+) + (\partial_j \theta - \Gamma_{11} \Gamma^f \theta_-) \\
 & \times (7(\bar{\theta} - \Gamma^{abcd} \Gamma_f \partial_k \theta_+) - 2(\bar{\theta} + (\Gamma^{abcd} \Gamma_f \\
 & + \Gamma_f \Gamma^{abcd}) \partial_k \theta_-)) - (\partial_k \bar{\theta} + \Gamma_f \theta_-) (\bar{\theta} - \Gamma^{abcd} \Gamma_{11} \Gamma^f \partial_j \theta_- \\
 & + 6(\bar{\theta} - \Gamma^f \Gamma^{abcd} \Gamma_{11} \partial_j \theta_-)) \\
 & - 8(\partial_k \bar{\theta} - \Gamma_f \theta_+) (\bar{\theta} - (\Gamma^{abcd} \Gamma^f + \Gamma^f \Gamma^{abcd}) \Gamma_{11} \partial_j \theta_-) \\
 & + (\bar{\theta} - \Gamma_{\mu f} \partial_j \theta_+) (\partial_k \bar{\theta} - \Gamma_{11} \Gamma^{abcd} \Gamma^{\mu f} \theta_- \\
 & + 2(\partial_k \bar{\theta} - \Gamma^{\mu f} \Gamma^{abcd} \Gamma_{11} \theta_-)) \\
 & - (\bar{\theta} - \Gamma_{11} \Gamma_{\mu f} \partial_j \theta_-) (\partial_k \bar{\theta} + \Gamma^{abcd} \Gamma^{\mu f} \theta_- \\
 & + 2(\partial_k \bar{\theta} + \Gamma^{\mu f} \Gamma^{abcd} \theta_-))
 \end{aligned}$$

$$\begin{aligned}
 &+(\partial_j \bar{\theta} - \Gamma^f \Gamma^{abcd} \Gamma^e \theta_-)(\partial_k \bar{\theta} + \Gamma_{fe} \Gamma_{11} \theta_- \\
 &+ 2(\partial_k \bar{\theta} - \Gamma_{fe} \Gamma_{11} \theta_+)) - 5(\bar{\theta} - \Gamma^{abcd} \partial_j \theta_-) \\
 &\times (\bar{\theta} - \Gamma_{11} \partial_k \theta_+) - (\bar{\theta} - \Gamma_{ef} \partial_k \theta_-)(\bar{\theta} - \Gamma^e \Gamma^{abcd} \Gamma^f \partial_j \theta_+ \\
 &+ 2(\bar{\theta} + \Gamma^e \Gamma^{abcd} \Gamma^f \Gamma_{11} \partial_j \theta_-)) \\
 &+ 3(\partial_j \bar{\theta} - \Gamma^{\mu abc d} \theta_-)(\partial_k \bar{\theta} + \Gamma_{11} \Gamma_\mu \theta_-) \tag{5.9}
 \end{aligned}$$

$$\begin{aligned}
 \bar{F}_{vabc} = & \frac{1}{288} T_{NR} \epsilon_{0jk} [(\bar{\theta} - \Gamma^\mu \partial_j \theta_-)(3(\bar{\theta} - \Gamma_{\mu vabc} \partial_k \theta_+) \\
 &- 23(\bar{\theta} - \Gamma_{abc} \partial_k \theta_+) \eta_{\mu\nu}) \\
 &+ (\bar{\theta} - \Gamma^d \partial_j \theta_+)(5(\bar{\theta} - \Gamma_{vabcd} \partial_k \theta_-) \\
 &+ 3(\bar{\theta} - \Gamma_{vbc} \partial_k \theta_-) \eta_{ad}) - 24(\bar{\theta} + \Gamma_a \partial_j \theta_-) \\
 &\times (\bar{\theta} - \Gamma_{vbc} \partial_k \theta_-) + (\bar{\theta} - \Gamma_{11} \Gamma^\mu \partial_k \theta_+) \\
 &\times (-3(\bar{\theta} - \Gamma_{11} \Gamma_{abc} \partial_j \theta_-) \eta_{\mu\nu} + 5(\partial_j \bar{\theta} - \Gamma_{11} \Gamma_{\mu vabc} \theta_-)) \\
 &+ (\bar{\theta} - \Gamma_{11} \Gamma^d \partial_j \theta_-)(\bar{\theta} - \Gamma_{11} \Gamma_{vabc} \Gamma_d \partial_k \theta_+ \\
 &- 4(\bar{\theta} - \Gamma_d \Gamma_{vabc} \Gamma_{11} \partial_k \theta_+) \\
 &+ 12(\bar{\theta} + \Gamma_{vbc} \Gamma_{11} \partial_k \theta_-) \eta_{ad}) + (\bar{\theta} - \Gamma^{\mu d} \partial_k \theta_+) \\
 &\times (\partial_j \bar{\theta} - \Gamma_\mu \Gamma_{vabc} \Gamma_d \theta_- - 4(\partial_j \bar{\theta} - \Gamma_d \Gamma_{vabc} \Gamma_\mu \theta_-)) \\
 &- (\bar{\theta} - \Gamma^{df} \partial_k \theta_-)(\bar{\theta} - \Gamma_d \Gamma_{vabc} \Gamma_f \partial_j \theta_+ \\
 &+ 2(\bar{\theta} + \Gamma_d \Gamma_{vabc} \Gamma_f \partial_j \theta_-)) + (\partial_k \bar{\theta} - \Gamma_d \Gamma_{11} \Gamma_{vabc} \Gamma_f \theta_-) \\
 &\times (3(\partial_j \bar{\theta} + \Gamma^{df} \Gamma_{11} \theta_-) + 2(\partial_j \bar{\theta} - \Gamma^{df} \Gamma_{11} \theta_+)) \\
 &+ 5(\partial_j \bar{\theta} + \Gamma_{11} \theta_-)(\partial_k \bar{\theta} - \Gamma_{11} \Gamma_{vabc} \theta_-) \\
 &+ 3(\bar{\theta} - \Gamma_{11} \Gamma_{\mu d} \partial_j \theta_-)(\partial_k \bar{\theta} + \Gamma^\mu \Gamma_{11} \Gamma_{vabc} \Gamma^d \theta_-) \\
 &- 3(\bar{\theta} - \partial_j \theta_-)(\partial_k \bar{\theta} + \Gamma_{vabc} \theta_-) \tag{5.10}
 \end{aligned}$$

$$\begin{aligned}
 F^{vabdf} = & \frac{1}{96} T_{NR} \epsilon_{0jk} [6(\bar{\theta} - \Gamma^c \partial_j \theta_+)(\bar{\theta} - \Gamma^{vabdf} \Gamma_c \partial_k \theta_+ \\
 &- \bar{\theta} - \Gamma_c \Gamma^{vabdf} \partial_k \theta_+) \\
 &- 2(\bar{\theta} - \Gamma_{11} \Gamma_\mu \partial_k \theta_+)(\partial_j \bar{\theta} + \Gamma_{11} \Gamma^\mu \Gamma^{vabdf} \theta_- \\
 &+ \partial_j \bar{\theta} + \Gamma_{11} \Gamma^{vabdf} \Gamma^\mu \theta_-) - 2(\bar{\theta} - \Gamma_{11} \Gamma_{\rho\sigma} \partial_k \theta_+) \\
 &\times (\partial_j \bar{\theta} + \Gamma_{11} \Gamma^\rho \Gamma^{vabdf} \Gamma^\sigma \theta_-) + 2(\bar{\theta} - \Gamma^c \partial_j \theta_+) \\
 &\times (\partial_k \bar{\theta} - \Gamma^{vadf} \Gamma_c \theta_+ + \partial_k \bar{\theta} - \Gamma_c \Gamma^{vabdf} \theta_+) \\
 &+ 3(\bar{\theta} + \Gamma^c \partial_j \theta_-)(-2(\partial_k \bar{\theta} + \Gamma^{vabdf} \Gamma_c \theta_-) \\
 &+ \bar{\theta} - \Gamma^{vabdf} \Gamma_c \partial_k \theta_+) - 4(\bar{\theta} - \Gamma_{11} \Gamma_\mu \partial_k \theta_+) \\
 &\times (\partial_j \bar{\theta} - \Gamma_{11} \Gamma^\mu \Gamma^{vabdf} \theta_+) \\
 &+ (\bar{\theta} + \Gamma_{11} \Gamma_\mu \partial_k \theta_-)(\partial_j \bar{\theta} + \Gamma_{11} \Gamma^\mu \Gamma^{vabdf} \theta_- \\
 &- 2(\partial_j \bar{\theta} + \Gamma_{11} \Gamma^{vabdf} \Gamma^\mu \theta_-)) \\
 &- 2(\partial_j \bar{\theta} - \Gamma^\mu \theta_-)(\partial_k \bar{\theta} + \Gamma_\mu \Gamma^{vabdf} \theta_+) \\
 &+ 2(\partial_k \bar{\theta} + \Gamma_{\mu c} \theta_-)(\bar{\theta} + \Gamma^c \Gamma^{vabdf} \Gamma^\mu \partial_j \theta_-) \\
 &+ 2(\bar{\theta} - \Gamma^c \Gamma_{11} \partial_j \theta_-)(\partial_k \bar{\theta} + \Gamma_{11} \Gamma^{vabdf} \Gamma_c \theta_+ \\
 &- 2(\partial_k \bar{\theta} + \Gamma_{11} \Gamma_c \Gamma^{vabdf} \theta_+)) \\
 &+ 2(\partial_k \bar{\theta} + \Gamma_{11} \theta_-)(\bar{\theta} + \Gamma_{11} \Gamma^{vabdf} \partial_j \theta_-) + (\bar{\theta} + \Gamma^\mu \partial_k \theta_+) \\
 &\times (6(\partial_j \bar{\theta} - \Gamma_\mu \Gamma^{vabdf} \theta_-) + 5(\partial_j \theta_- \Gamma^{vabdf} \Gamma_\mu \theta_-)) \\
 &+ 7(\bar{\theta} + \Gamma_{\mu\sigma} \partial_k \theta_+)(\partial_j \bar{\theta} - \Gamma^\sigma \Gamma^{vabdf} \Gamma^\mu \theta_-) \\
 &- 3(\bar{\theta} + \Gamma_{\mu c} \partial_j \theta_-)(\partial_k \bar{\theta} + \Gamma^c \Gamma^{vabdf} \Gamma^\mu \theta_-) \\
 &- 4(\bar{\theta} + \Gamma_{11} \partial_j \theta_-)(\partial_k \bar{\theta} + \Gamma_{11} \Gamma^{vabdf} \theta_-)
 \end{aligned}$$

$$\begin{aligned}
 &+(\partial_k \bar{\theta} + \Gamma_{\mu c} \Gamma_{11} \theta_+)(\partial_j \bar{\theta} - \Gamma^c \Gamma_{11} \Gamma^{vabdf} \Gamma^\mu \theta_-) \\
 &+ (\partial_k \bar{\theta} + \Gamma_\mu \Gamma_\sigma \theta_+) \\
 &\times (\partial_j \bar{\theta} - \Gamma^\sigma \Gamma^{vabdf} \Gamma^\mu \theta_-) + (\bar{\theta} + \Gamma_{11} \Gamma_\sigma \Gamma_\mu \partial_j \theta_-) \\
 &\times (\partial_k \bar{\theta} + \Gamma^\sigma \Gamma_{11} \Gamma^{vabdf} \Gamma^\mu \theta_-) \\
 &- (\partial_k \bar{\theta} + \Gamma_{11} \Gamma_c \theta_+)(\partial_j \bar{\theta} - \Gamma_{11} \Gamma^c \Gamma^{vabdf} \theta_-) \\
 &+ 2(\bar{\theta} - \Gamma_{ce} \partial_k \theta_-)(\bar{\theta} + \Gamma^c \Gamma^{vabdf} \Gamma^e \partial_j \theta_+) \\
 &- \frac{1}{4!} \epsilon_{cgeabdf} \delta_{cge}^{[lmn]} \frac{T_{NR}}{576} [-36(\bar{\theta} - \Gamma^l \partial_j \theta_+) \\
 &\times (\partial_k \bar{\theta} + \Gamma^{vmn} \Gamma_{11} \theta_-) + 2(\bar{\theta} - \Gamma^r \partial_j \theta_+) \\
 &\times (\partial_k \bar{\theta} - \Gamma_{11} \Gamma^{vlmn} \Gamma_r \theta_+ + \partial_k \bar{\theta} - \Gamma_{11} \Gamma_r \Gamma^{vlmn} \theta_+) \\
 &+ 3(\bar{\theta} + \Gamma^r \partial_j \theta_-) \\
 &\times (\partial_k \bar{\theta} + \Gamma_r \Gamma^{vlmn} \Gamma_{11} \theta_- + 2(\partial_k \bar{\theta} + \Gamma_{11} \Gamma^{vlmn} \Gamma_r \theta_-)) \\
 &- 4(\bar{\theta} - \Gamma_{11} \Gamma_\mu \partial_k \theta_+) \\
 &\times (\partial_j \bar{\theta} - \Gamma^\mu \Gamma^{vlmn} \theta_+) - (\bar{\theta} + \Gamma_{11} \Gamma_\mu \partial_k \theta_-) \\
 &\times (2(\partial_j \bar{\theta} + \Gamma^{vlmn} \Gamma^\mu \theta_-) + \partial_j \bar{\theta} + \Gamma^\mu \Gamma^{vlmn} \theta_-) \\
 &+ 2(\bar{\theta} - \Gamma^\mu \partial_j \theta_-)(\partial_k \bar{\theta} + \Gamma_{11} \Gamma_\mu \Gamma^{vlmn} \theta_+) \\
 &- 2(\partial_k \bar{\theta} + \Gamma_r \mu \theta_-)(\bar{\theta} + \Gamma^r \Gamma^{vlmn} \Gamma_{11} \Gamma^\mu \partial_j \theta_-) \\
 &+ 2(\bar{\theta} - \Gamma^r \Gamma_{11} \partial_j \theta_-)(-2(\partial_k \bar{\theta} + \Gamma_r \Gamma^{vlmn} \theta_+) \\
 &+ \partial_k \bar{\theta} + \Gamma^{vlmn} \Gamma_r \theta_+) + 2(\partial_k \bar{\theta} + \Gamma_{11} \theta_-) \\
 &\times (\bar{\theta} + \Gamma^{vlmn} \partial_j \theta_-) + 6(\bar{\theta} + \Gamma_\sigma \Gamma_\mu \partial_k \theta_+) \\
 &\times (\partial_j \bar{\theta} - \Gamma^\mu \Gamma^{vlmn} \Gamma_{11} \Gamma^\sigma \theta_-) - (\partial_k \bar{\theta} + \theta_+) \\
 &\times (\partial_j \bar{\theta} - \Gamma^{vlmn} \Gamma_{11} \theta_-) + (\partial_k \bar{\theta} + \Gamma_{\mu r} \Gamma_{11} \theta_+) \\
 &(\partial_j \bar{\theta} - \Gamma^r \Gamma^{vlmn} \Gamma^\mu \theta_-) + (\partial_j \bar{\theta} - \Gamma_{11} \Gamma_\mu \sigma \theta_+) \\
 &\times (\partial_k \bar{\theta} + \Gamma^\mu \Gamma^{vlmn} \Gamma^\sigma \theta_-) + 3(\partial_j \bar{\theta} - \Gamma_{11} \theta_+) \\
 &\times (\partial_k \bar{\theta} + \Gamma^{vlmn} \theta_-) + (\partial_k \bar{\theta} + \Gamma_{11} \Gamma^r \theta_+) \\
 &\times (\partial_j \bar{\theta} - \Gamma_r \Gamma^{vlmn} \theta_-) \\
 &+ 3(\bar{\theta} + \Gamma_{rs} \partial_j \theta_-)(\partial_k \bar{\theta} + \Gamma^r \Gamma^{vlmn} \Gamma_{11} \Gamma^\sigma \theta_-) \\
 &+ (\bar{\theta} + \Gamma^\mu \partial_k \theta_+)(6(\partial_j \bar{\theta} - \Gamma_\mu \Gamma^{vlmn} \Gamma_{11} \theta_-) \\
 &+ 5(\partial_j \bar{\theta} - \Gamma^{vlmn} \Gamma_{11} \Gamma_\mu \theta_-)) + 2(\bar{\theta} - \Gamma_{rs} \partial_k \theta_-) \\
 &\times (\bar{\theta} + \Gamma^r \Gamma^{vlmn} \Gamma_{11} \Gamma^s \partial_j \theta_+) \tag{5.11}
 \end{aligned}$$

$$\begin{aligned}
 \bar{F}_{fglmn} = & \frac{1}{96 \times 5!} T_{NR} \epsilon_{0jk} [3(\bar{\theta} - \Gamma^\mu \partial_j \theta_-)(-5(\bar{\theta} - \Gamma_\mu fglmn \partial_k \theta_+) \\
 &+ 4(\bar{\theta} + \Gamma_\mu fglmn \partial_k \theta_-)) + (\bar{\theta} - \Gamma^a \partial_j \theta_+) \\
 &\times (17(\bar{\theta} - \Gamma_{fglmn} \Gamma_a \partial_k \theta_-) - 22(\bar{\theta} - \Gamma_a \Gamma_{fglmn} \partial_k \theta_-)) \\
 &+ 4(\bar{\theta} + \Gamma^a \partial_j \theta_-)(\bar{\theta} - \Gamma_{fglmn} \Gamma_a \partial_k \theta_- \\
 &- 5(\bar{\theta} - \Gamma_a \Gamma_{fglmn} \partial_k \theta_-)) + 9(\bar{\theta} - \Gamma_{11} \Gamma^\mu \partial_k \theta_+) \\
 &\times (\partial_j \bar{\theta} - \Gamma_{11} \Gamma_\mu fglmn \theta_-) \\
 &+ 12(\bar{\theta} + \Gamma_{11} \Gamma^\mu \partial_k \theta_-)(\partial_j \bar{\theta} - \Gamma_\mu fglmn \Gamma_{11} \theta_-) \\
 &+ (\partial_j \bar{\theta} - \Gamma_{11} \Gamma^a \theta_-) \\
 &\times (-4(\partial_k \bar{\theta} + \Gamma_{11} \Gamma_{fglmn} \Gamma_a \theta_-) \\
 &+ 13(\partial_k \bar{\theta} + \Gamma_{11} \Gamma_a \Gamma_{fglmn} \theta_-) + 6(\partial_k \bar{\theta} - \Gamma_{11} \Gamma_a \Gamma_{fglmn} \theta_+) \\
 &+ 2(\partial_k \bar{\theta} - \Gamma_{11} \Gamma_{fglmn} \Gamma_a \theta_+) \\
 &- (\partial_k \bar{\theta} - \Gamma_{11} \Gamma^a \Gamma_{fglmn} \Gamma^b \theta_-)(4(\partial_j \bar{\theta} - \Gamma_b \Gamma_a \Gamma_{11} \theta_+) \\
 &+ 3(\partial_j \bar{\theta} + \Gamma_b \Gamma_a \Gamma_{11} \theta_-)) + 3(\bar{\theta} - \Gamma_b \Gamma_a \partial_j \theta_-)
 \end{aligned}$$

$$\begin{aligned}
 & \times (\partial_k \bar{\theta}_+ \Gamma^b \Gamma_{fglmn} \Gamma^a \theta_-) + 2(\bar{\theta}_- \Gamma^{ab} \partial_k \theta_-) \\
 & \times (\bar{\theta}_- \Gamma_a \Gamma_{fglmn} \Gamma_b \partial_j \theta_+ + \bar{\theta}_+ \Gamma_a \Gamma_{fglmn} \Gamma_b \partial_j \theta_-) \\
 & - 6(\bar{\theta}_- \Gamma_{11} \partial_k \theta_+) (\partial_j \bar{\theta}_- \Gamma_{11} \Gamma_{fglmn} \theta_-) \\
 & + 12(\partial_j \bar{\theta}_- \Gamma_{11} \theta_+) (\partial_k \bar{\theta}_- \Gamma_{11} \Gamma_{fglmn} \theta_-) \\
 & - 3(\partial_k \bar{\theta}_+ \Gamma_{11} \Gamma_{fglmn} \Gamma^{\mu a} \theta_-) (\bar{\theta}_- \Gamma_{11} \Gamma_{\mu a} \partial_j \theta_-) \\
 & - (\bar{\theta}_- \Gamma^{\mu a} \partial_k \theta_+) (4(\partial_j \bar{\theta}_- \Gamma_{\mu a} \Gamma_{fglmn} \theta_-) \\
 & - \partial_j \bar{\theta}_- \Gamma_{fglmn} \Gamma_{\mu a} \theta_-) - 2(\bar{\theta}_+ \Gamma^{\mu a} \partial_k \theta_-) \\
 & \times (\bar{\theta}_- \Gamma_{\mu a} \Gamma_{fglmn} \partial_j \theta_- + \partial_j \Gamma_{\mu a} \Gamma_{fglmn} \theta_-) \\
 & - \frac{1}{192 \times 5!} \epsilon_{0j k \epsilon d e f g l m n} \delta_{d e}^{b a} [(16(\bar{\theta}_+ \Gamma_c \partial_j \theta_-) \\
 & + 21(\bar{\theta}_- \Gamma_c \partial_j \theta_+)) (\bar{\theta}_- \Gamma^{abc} \Gamma_{11} \partial_k \theta_-) \\
 & + 8(7(\bar{\theta}_+ \Gamma^a \partial_j \theta_-) + 3(\bar{\theta}_- \Gamma^a \partial_j \theta_+)) (\bar{\theta}_- \Gamma^b \Gamma_{11} \partial_k \theta_-) \\
 & - (\bar{\theta}_- \Gamma_{\mu} \partial_j \theta_-) \\
 & \times (12(\bar{\theta}_+ \Gamma^{\mu ab} \Gamma_{11} \partial_k \theta_-) + 11(\bar{\theta}_- \Gamma_{11} \Gamma^{\mu ab} \partial_k \theta_+)) \\
 & + 8(\bar{\theta}_- \Gamma_{\mu} \Gamma^b \Gamma_{11} \partial_j \theta_-) (\bar{\theta}_+ \Gamma^{\mu a} \partial_k \theta_- \\
 & + \bar{\theta}_- \Gamma^{\mu a} \partial_k \theta_+) + (\partial_k \bar{\theta}_+ \Gamma_{\mu c} \theta_-) (\partial_j \bar{\theta}_- \Gamma^{\mu abc} \Gamma_{11} \theta_- \\
 & + 2(\bar{\theta}_- \Gamma^{\mu b} \Gamma_{11} \partial_j \theta_-) \eta^{ac}) \\
 & + (\bar{\theta}_- \Gamma_{11} \Gamma_c \partial_k \theta_-) (-8(\bar{\theta}_+ \Gamma^{abc} \partial_j \theta_-) \\
 & + 3(\bar{\theta}_- \Gamma^{abc} \partial_j \theta_+)) + (\bar{\theta}_- \Gamma^{\mu ab} \partial_j \theta_-) \\
 & \times (12(\bar{\theta}_+ \Gamma_{11} \Gamma_{\mu} \partial_k \theta_-) - 17(\bar{\theta}_- \Gamma_{11} \Gamma_{\mu} \partial_k \theta_+)) \\
 & - 4(\partial_j \bar{\theta}_- \Gamma_{cd} \Gamma_{11} \theta_+) (\partial_k \bar{\theta}_- \Gamma^c \Gamma^{ab} \Gamma^d \theta_-) \\
 & + (\partial_j \bar{\theta}_+ \Gamma_{cd} \Gamma_{11} \theta_-) (2(\partial_k \bar{\theta}_- \Gamma^{cd} \Gamma^{ab} \theta_-) \\
 & - 17(\bar{\theta}_- \partial_k \theta_-) \eta^{ac} \eta^{bd}) + (\bar{\theta}_- \Gamma_{cd} \partial_k \theta_-) \\
 & \times (11(\partial_j \bar{\theta}_+ \Gamma_{11} \theta_-) \eta^{ac} \eta^{bd} + 2(-\partial_j \bar{\theta}_+ \Gamma^{cd} \Gamma^{ab} \Gamma_{11} \theta_- \\
 & + \bar{\theta}_- \Gamma^c \Gamma^{ab} \Gamma^d \Gamma_{11} \partial_j \theta_+ \\
 & - \bar{\theta}_+ \Gamma^c \Gamma^{ab} \Gamma^d \Gamma_{11} \partial_j \theta_-) + 3(\bar{\theta}_- \Gamma_{\mu c} \Gamma_{11} \partial_j \theta_-) \\
 & \times (\partial_k \bar{\theta}_+ \Gamma^{\mu abc} \theta_- + 2(\partial_k \bar{\theta}_+ \Gamma^{\mu b} \theta_-) \eta^{ac})] \tag{5.12}
 \end{aligned}$$

References

1. N. Berkovits, Super-Poincaré covariant quantization of the superstring. *JHEP* **04**, 018 (2000). [arXiv:hep-th/0001035](#)
2. N. Berkovits, Cohomology in the pure spinor formalism for the superstring. *JHEP* **09**, 046 (2000). [arXiv:hep-th/0006003](#)
3. N. Berkovits, Covariant quantization of the superparticle using pure spinors. *JHEP* **09**, 016 (2001). [arXiv:hep-th/0105050](#)
4. N. Berkovits, ICTP lectures on covariant quantization of the superstring. *ICTP Lect. Notes Ser.* **13**, 57 (2003). [arXiv:hep-th/0209059](#)
5. O. Chandía, B.C. Vallilo, Conformal invariance of the pure spinor superstring in a curved background. *JHEP* **0404**, 041 (2004). [arXiv:hep-th/0401226](#)
6. O. Chandía, A note on the classical BRST symmetry of the pure spinor string in a curved background. *JHEP* **0607**, 019 (2006). [arXiv:hep-th/0604115](#)

7. O.A. Bedoya, O. Chandía, One loop conformal invariance of the type II pure spinor superstring in a curved background. *JHEP* **0701**, 042 (2007). [arXiv:hep-th/0609161](#)
8. J. Kluson, Note about redefinition of BRST operator for pure spinor strings in general background. *Phys. Rev. D* **78**, 066011 (2008). [arXiv:0803.4390](#) [hep-th]
9. C. Mafra, Superstring scattering amplitudes with the pure spinor formalism. PhD Thesis, [arXiv:0902.1552](#) [hep-th]
10. C. Mafra, C. Stahn, The one-loop open superstring massless five-point amplitude with the non-minimal pure spinor formalism. *JHEP* **0903**, 126 (2009). [arXiv:0902.1539](#) [hep-th]
11. N. Berkovits, C. Mafra, Equivalence of the two-loop superstring amplitudes in the pure spinor and RNS formalisms. *Phys. Rev. Lett.* **96**, 011602 (2006). [arXiv:hep-th/0509234](#)
12. N. Berkovits, C. Mafra, Some superstring amplitude computations with the non-minimal pure spinor formalism. *JHEP* **0611**, 079 (2006). [arXiv:hep-th/0607187](#)
13. C. Mafra, O. Schlotterer, Cohomology foundations of one-loop amplitudes in pure spinor superspace. [arXiv:1408.3605](#) [hep-th]
14. J. Gomis, H. Ooguri, Nonrelativistic closed string theory. *J. Math. Phys.* **42**, 3127 (2001). [arXiv:hep-th/0009181](#)
15. J. Brugues, T. Curtright, J. Gomis, L. Mezincescu, Non-relativistic strings and branes as non-linear realizations of Galilei groups. *Phys. Lett. B* **594**, 227 (2004). [arXiv:hep-th/0404175](#)
16. J. Gomis, F. Passerini, T. Ramirez, A. van Proeyen, Non relativistic Dp branes. *JHEP* **0510**, 007 (2005). [arXiv:hep-th/0507135](#)
17. K. Kamimura, T. Ramirez, Brane dualities in non-relativistic limit. *JHEP* **0603**, 058 (2006). [arXiv:hep-th/0512146](#)
18. M. Aganagic, C. Popescu, J.H. Schwarz, D-brane actions with local kappa symmetry. *Phys. Lett. B* **353**, 311 (1997). [arXiv:hep-th/9610249](#)
19. M. Aganagic, C. Popescu, J.H. Schwarz, Gauge invariant and gauge fixed D-brane actions. *Nucl. Phys. B* **495**, 99 (1997). [arXiv:hep-th/9612080](#)
20. M. Hatsuda, K. Kamimura, Wess–Zumino actions for IIA D-branes and their supersymmetries. *Nucl. Phys. B* **535**, 499 (1998). [arXiv:hep-th/9804087](#)
21. P.A. Grassi, G. Policastro, P. van Nieuwenhuisen, An introduction to the covariant quantization of the superstring. *Class. Quant. Gravit.* **20**, S395 (2003). [arXiv:hep-th/0302147](#)
22. P.A. Grassi, G. Policastro, P. van Nieuwenhuisen, Yang–Mills theory as an illustration of the covariant quantization of superstrings. In 3rd International Sakharov Conference on Physics, 24–29 June (Russia, Moscow, 2002). [arXiv:hep-th/0211095](#)
23. J. Hoogeveen, *Fundamentals of the Pure Spinor Formalism* (Amsterdam University Press, Amsterdam, 2010)
24. L. Anguelova, P.A. Grassi, Super D-branes from BRST symmetry. *JHEP* **0311**, 010 (2003). [arXiv:hep-th/0307260](#)
25. L. Brink, M. Henneaux, *Principles of String Theory* (Plenum Press, New York, 1988)
26. N. Berkovits, T. Fleury, Harmonic superspace from the $AdS_5 \times S^5$ pure spinor formalism. *JHEP* **03**, 022 (2013). [arXiv:1212.3296](#) [hep-th]
27. A. Mikhailov, A. Schwarz, R. Xu, Cohomology ring of the BRST operator associated to the sum of two pure spinors. *Mod. Phys. Lett. A* **28**(23), 1350107 (2013). [arXiv:1305.0071](#) [hep-th]
28. R. Xu, Cohomological Problems arising in Quantum Field Theory and String Theory. Ph.D. thesis, University of California, DAVIS
29. A. Mikhailov, R. Xu, BRST cohomology of the sum of two pure spinors. [arXiv:1301.3353](#) [hep-th]