Documentation of TauSpinner approach for electroweak corrections in LHC $Z \rightarrow \ell\ell$ observables

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Abstract The LHC Standard Model Z-boson couplings measurements approach the LEP legacy precision. The calculations of electroweak (EW) corrections available for the Monte Carlo generators become of relevance. Predictions of Z-boson production and decay require classes of QED/EW/QCD corrections and separately from the production process QCD dynamics. At the LEP time electroweak form-factors and *Improved Born Approximation* were introduced for non QED genuine weak and line-shape corrections. This formalism was well suited for observables, so-called doubly-deconvoluted the Z-pole region where initial- and final-state QED real and virtual emissions were treated separately or were integrated over. The approach was convenient for implementation into Monte Carlo programs for LEP, Belle-BaBar and other future e^+e^- colliders and for invariant mass of outgoing lepton pair from a few GeV to well above WW and even $t\bar{t}$ threshold. We attempt now to profit from that, for the LHC pp and 70–150 GeV window for the outgoing lepton pair invariant mass. Our technical focus is on the EW corrections for LHC $Z \rightarrow \ell\ell$ observables. For this purpose the TauSpinner package, for the reweighting of previously generated events, is enriched with the genuine EW corrections (QED effects subtracted) of the Dizet electroweak library, taken from the LEP era KKMC Monte Carlo. Complete genuine EW $O(\alpha)$ weak loop corrections and dominant higher-order terms are taken into account. For the efficiency and numerical stability look-up tables are used. For LHC observables: the Z-boson line-shape, the outgoing leptons forward-backward asymmetry, the effective leptonic weak mixing angles and finally for the spherical harmonic expansion coefficients of the lepton distributions, corrections are evaluated. Simplified calculations of *Effective Born* of modified EW couplings are compared with of *Improved Born Approximation* of complete set of EW form-factors. Approach uses LEP precision tests definitions and

thus offers consistency checks. The package can be useful to evaluate of observables precision limits and to determine which corrections are then important for LHC and FCC projects phenomenology.

1 Introduction

A theoretically sound separation of QED/EW effects between the QED emissions and genuine weak effects was essential for the phenomenology of LEP precision physics [1]. It was motivated by the structure of the amplitudes for single Z or (to a lesser degree) WW pairs production in e^+e^- collisions, and by the fact that QED bremsstrahlung occurs at a different energy scale than the electroweak processes. Even more importantly, with this approach multi-loop calculations for complete electroweak sector could be avoided. The QED terms could be resummed in an exclusive exponentiation scheme implemented in Monte Carlo [2]. Note that QED corrections modify the cross-section at the peak by as much as 40%. The details of this paradigm are explained in [3]. It was obtained as a consequence of massive efforts, we will not recall them here. For the present study, the observation that spin amplitudes semi-factorize into a Born-like terms and functional factors responsible for bremsstrahlung [4] was very important.

A similar separation can be also achieved for dynamics of production process in pp collisions, which can be isolated from QED/EW corrections. It was explored recently in the case of configurations with high- p_T jets associated with the Drell-Yan production of Z [5] or W bosons [6] at LHC. The potentially large electroweak Sudakov logarithmic corrections discussed in [7] (absent in our work) represent yet another class of weak effects, separable from those discussed throughout this paper. They are very small for lepton pairs with a virtuality close to the Z-boson pole mass and, if accompanied by the jet when virtuality of $\ell\ell j$ system is not

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much larger than $2 M_W$. Otherwise the Sudakov corrections have to be revisited and calculation of electroweak corrections extended, even if invariant mass of the lepton pair is close to the Z mass.

To assess precisely the size and impact of genuine weak corrections to the Born-like cross section for lepton pair production with a virtuality below threshold for WW pair production, the precision calculations and programs prepared for the LEP era: KKMC Monte Carlo [8] and Dizet electroweak (EW) library, were adapted to provide pre-tabulated EW corrections to be used by LHC specific event reweighting programs like TauSpinner package [9]. Even at present KKMC Monte Carlo use Dizet version 6.21 [10, 11]. We restrict ourselves to that reference version. The TauSpinner package was initially created as a tool to correct with per-event weight longitudinal spin effects in the generated event samples including τ decays. Algorithms implemented there turned out to be of more general usage. The possibility to introduce one-loop electroweak corrections from SANC library [12] in case of Drell-Yan production of the Z -boson became available in TauSpinner since [13]. Pre-tabulation prepared for EW corrections of SANC library, was useful to introduce weights for complete spin effects at each individual event level. However no higher loop contributions were available.

TauSpinner provides a reweighting technique to modify hard process matrix elements (also matrix elements for τ decays) which were used for Monte Carlo generation. For each event no changes of any details for event kinematic configurations are introduced. The reweighting algorithm can be used for events where final state QED bremsstrahlung photons and/or high p_T jets are present. For matrix element calculation used for re-weighting, some contributions such as of QED bremsstrahlung or of jet emissions have to be removed. For that purpose factorization and detailed inspection of fixed order perturbation expansion amplitudes is necessary. The most recent summary on algorithms and their applications is given in [14]. The reference explains in detail how kinematical configurations are reduced to Born-level configurations used for the correcting weights, also for electroweak corrections.¹

Used for both Tauola Univesal Interface and TauSpinner, SANC library [12] of year 2008 calculates one loop i.e. NLO electroweak corrections in two $\alpha(0)$ and

$G\mu$ (G_F) schemes. It was found numerically insufficient for practical applications. For example, it was missing sizable α_s corrections to the calculated Z boson width. Two aspects of EW corrections implementation [13] had to be enhanced.

First, in [5, 6] we have studied separation of QCD higher order corrections and the Born-level spin amplitudes calculated in the adapted *Mustaal* lepton pair rest frame.² It is defined like for QED bremsstrahlung of Ref. [4]. The separation holds to a good approximation for the Drell-Yan processes where one or even two high p_T jets are present. This frame is now used as option for EW weight calculation.

Second, the TauSpinner package and algorithms are now adapted to EW corrections from the Dizet library,³ more accurate than SANC. The EW corrections are introduced with form-factor corrections of Standard Model couplings and propagators which enter spin amplitudes of the *Improved Born Approximation*, used for EW weights calculation. They represent complete $\mathcal{O}(\alpha)$ electroweak corrections with QED contributions removed but augmented with carefully selected dominant higher order terms. This was very successful in analyses of LEP I precision physics. We attempt a similar strategy for the Z -boson pole LHC precision physics; the approach to EW corrections already attracted attention. It was used in the preliminary measurement of effective leptonic weak mixing angle recently published by ATLAS Collaboration [16].

This paper is organized as follows. In Sect. 2 we collect the main formulae of the formalism, in particular we recall the definition of the *Improved Born Approximation*. In Sect. 3 we present numerical results for the electroweak form-factors. Some details on commonly used EW schemes are discussed in Sect. 4, which also recall the definition of the *Effective Born*. In Sect. 5 we comment on the issues of using the Born approximation in pp collisions and in Sect. 6 we give more explanation why the Born approximation of

¹ In Ref. [15] (on Tauola Universal Interface) other than TauSpinner solution was prepared. Then parton level history entries for generated event record were used. For TauSpinner use of history event record entries was abandoned, because of too many variants how corresponding information was required to be interpreted. Instead, contributions from all possible parton level processes, weighted with parton distribution functions are averaged. This could also be used for configurations generated with multi jet matrix elements, when Born level matrix element configurations can not be identified.

² Over the paper we use several variants of coordinate system orientation for the lepton pair rest-frame. The *Mustaal* frame resulted from careful analysis of the cross section for the initial and final state bremsstrahlung that is $e^+e^- \rightarrow \mu^+\mu^-\gamma$. It was found that it can be represented, without any approximation as sum of four incoherently added distributions with well defined probabilities (two for initial and two for final state emission), each factorized into Born cross section calculated in reference frame oriented as required by the form of matrix element and the factor dependent on kinematical variables for the γ . One should keep in mind that the spin carried by the photon cancels out with its orbital momentum. That property of the matrix element originates from the properties of the Lorentz group representations, their combinations for the ultra-relativistic states. That is why it generalizes unchanged to the $q\bar{q} \rightarrow l^+l^-g$ and approximately also to other processes of single or even double jet emissions in a bulk of parton emissions in pp collisions. It was checked numerically in Refs. [5, 6].

³ This legacy library of EW corrections, features numerically important, corrections beyond NLO, in particular to Z and γ^* propagators. Contributions corresponding to QED are carefully removed and left for the independent treatment.

the EW sector is still valid in the presence of NLO QCD matrix elements. In Sect. 7 we define the concept of EW weight which can be applied to introduce EW corrections into already existing samples, generated with Monte Carlo programs with EW LO hard process matrix elements only. In Sect. 8 we discuss, in numerical detail, EW corrections to different observables of interest for precision measurements: Z-boson line-shape, lepton forward-backward asymmetry and for coefficients of lepton spherical harmonic expansion. In this Section we include also a discussion of the effective weak mixing angle in case of pp collision. For results presented in Sect. 8 we use QCD NLO Powheg+MiNLO [17] $Z + j$ Monte Carlo sample, generated for pp collision with $\sqrt{s} = 8$ TeV and EW LO implementation in matrix elements. Section 9 summarizes the paper.

In Appendix A details on the technical implementation of EW weight and how it can be calculated with help of the TauSpinner framework are given. In Appendix B formulae which have been implemented to allow variation of the weak mixing angle parameter of the Born spin amplitudes are discussed. In Appendix C initialization details, and options valuable for future discussions, for the Dizet library are collected.

2 Improved Born Approximation

At LEP times, to match higher order QED effects with the loop corrections of electroweak sector, the concept of electroweak form-factors was introduced [3]. This arrangement was very beneficial and enabled common treatment of one loop electroweak effects with not only higher order QED corrections including bremsstrahlung, but also to incorporate higher order loops into Z and photon propagators, see e.g. documentation of KKMC Monte Carlo [2] or Dizet [11]. Such description has its limitations for the LHC applications, but for the processes of the Drell-Yan type with a moderate virtuality of produced lepton pairs is expected to be useful, even in the case when high p_T jets are present. For the LEP applications [1], the EW form-factors were used together with multi-photon bremsstrahlung amplitudes, but for the purpose of this paper we discuss their use with parton level Born processes only (no QED ISR/FSR).⁴

The terminology *double-deconvoluted observable* was widely used since LEP time and is explained e.g. in [18]. The so called *Improved Born Approximation* (IBA) [11] is employed. It absorbs some of the higher order EW correc-

tions into a redefinition of couplings and propagators of the Born spin amplitude. This allows for straightforward calculation of *doubly-deconvoluted observables* like various cross-sections and asymmetries. QED effects are then removed or integrated over.

It is possible, because the excluded initial/final QCD and QED corrections form separately gauge invariant subsets of diagrams [11]. The QED subset consists of QED-vertices, $\gamma\gamma$ and γZ boxes and bremsstrahlung diagrams. The subset corresponding to the initial/final QCD corrections can be constructed as well. All the remaining corrections contribute to the IBA: purely EW loops, boxes and *internal* QCD corrections for loops (line-shape corrections). They can be split into two more gauge-invariant subsets, giving rise to two *improved (or dressed)* amplitudes: (i) improved γ exchange amplitude with running QED coupling where fermion loops of low Q^2 contribute dominantly and (ii) improved Z-boson exchange amplitude with four complex *EW form-factors*: $\rho_{\ell f}$, $\mathcal{K}_\ell$, $\mathcal{K}_f$, $\mathcal{K}_{\ell f}$. Components of those corrections are as follows:

- Corrections to photon propagator, where fermion loops contribute dominantly the so called vacuum-polarization corrections.
- Corrections to Z-boson propagator and couplings, called EW form-factors.
- Contribution from the purely weak WW and ZZ box diagrams. They are negligible at the Z-peak (suppressed by the factor $(s - M_Z^2)/s$), but very important at higher energies. They enter as corrections to form-factors and introduce non-polynomial dependence on the cos of the scattering angle.
- Mixed $O(\alpha\alpha_s, \alpha\alpha_s^2, \dots)$ corrections which originate from gluon insertions to the fermionic components of bosonic self-energies. They enter as corrections to all form-factors.

Below, to define notation we present the formula of the Born spin amplitude $\mathcal{A}^{\text{Born}}$. We recall conventions from [11]. Let us start with defining the lowest order coupling constants (without EW corrections) of the Z boson to fermions: $s_W^2 = 1 - M_W^2/M_Z^2 = \sin^2\theta_W^2$ defines weak Weinberg angle in the on-mass-shell scheme and $T_3^{\ell, f}$ third component of the isospin. The vector v_ℓ, v_f and axial a_ℓ, a_f couplings for leptons and quarks are defined with the formulae below⁵

$$\begin{aligned} v_\ell &= (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot s_W^2)/\Delta, \\ v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2)/\Delta, \\ a_\ell &= (2 \cdot T_3^\ell)/\Delta, \\ a_f &= (2 \cdot T_3^f)/\Delta. \end{aligned} \quad (1)$$

⁴ Presence in reweighted events of QED initial and final state bremsstrahlung, does not lead to complications of principle, but would obscure presentation. Necessary extensions [14] are technically simple, thanks to properties of QED matrix elements, presented for the first time in [4].

⁵ We will use “ ℓ ” for lepton, and “ f ” for quarks.

where

$$\Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)}, \quad (2)$$

and q_f, q_l denote charge of incoming fermion (quark) and outgoing lepton. With this notation, the $\mathcal{A}^{Born}$ spin amplitude for the $q\bar{q} \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^-$ can be written as:

$$\begin{aligned} \mathcal{A}^{Born} &= \frac{\alpha}{s} \{ [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu u] \cdot (q_\ell \cdot q_f) \cdot \chi_\gamma(s) \\ &\quad + [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu u \cdot (v_\ell \cdot v_f) \\ &\quad + \bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu \gamma^5 u \cdot (v_\ell \cdot a_f) \\ &\quad + \bar{u}\gamma^\mu \gamma^5 v g_{\mu\nu} \bar{v}\gamma^\nu u \cdot (a_\ell \cdot v_f) \\ &\quad + \bar{u}\gamma^\mu \gamma^5 v g_{\mu\nu} \bar{v}\gamma^\nu \gamma^5 u \cdot (a_\ell \cdot a_f)] \cdot \chi_Z(s) \}, \end{aligned} \quad (3)$$

where u, v denote fermion spinors and, α stands for QED coupling constant. The Z-boson and photon propagators are defined respectively as:

$$\chi_\gamma(s) = 1, \quad (4)$$

$$\chi_Z(s) = \frac{G_\mu \cdot M_Z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} \cdot \frac{s}{s - M_Z^2 + i \cdot \Gamma_Z \cdot s/M_Z}. \quad (5)$$

For the IBA, we redefine vector and axial couplings and introduce EW form-factors $\rho_{\ell f}(s, t), \mathcal{K}_\ell(s, t), \mathcal{K}_f(s, t), \mathcal{K}_{\ell f}(s, t)$ as follows:

$$\begin{aligned} v_\ell &= (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot s_W^2 \cdot \mathcal{K}_\ell(s, t)) / \Delta, \\ v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2 \cdot \mathcal{K}_f(s, t)) / \Delta, \\ a_\ell &= (2 \cdot T_3^\ell) / \Delta, \\ a_f &= (2 \cdot T_3^f) / \Delta. \end{aligned} \quad (6)$$

Normalization correction Z_{V_Π} to the Z-boson propagator is defined as

$$Z_{V_\Pi} = \rho_{\ell f}(s, t). \quad (7)$$

Re-summed vacuum polarization corrections Γ_{V_Π} to the γ^* propagator are expressed as

$$\Gamma_{V_\Pi} = \frac{1}{2 - (1 + \Pi_{\gamma\gamma}(s))}, \quad (8)$$

where $\Pi_{\gamma\gamma}(s)$ denotes vacuum polarization loop corrections of virtual photon exchange. Both Γ_{V_Π} and Z_{V_Π} are multiplicative correction factors. The $\rho_{\ell f}(s, t)$ could be also absorbed as multiplicative factor into the definition of vector and axial couplings.

The EW form-factors $\rho_{\ell f}(s, t), \mathcal{K}_\ell(s, t), \mathcal{K}_f(s, t), \mathcal{K}_{\ell f}(s, t)$ depend on two Mandelstam invariants (s, t) due to contributions of the WW and ZZ boxes. The Mandelstam variables satisfy the identity

$$s + t + u = 0 \quad \text{where} \quad t = -\frac{s}{2}(1 - \cos\theta) \quad (9)$$

and $\cos\theta$ is the cosine of the scattering angle, i.e. the angle between incoming and outgoing fermion directions.

Note, that in this approach the mixed EW and QCD loop corrections, originating from gluon insertions to fermionic components of bosonic self-energies, are included in Γ_{V_Π} and Z_{V_Π} .

One has to pay special attention to the angle dependent product of the vector couplings. The corrections break factorization, formula (3), of the couplings into ones associated with either Z boson production or decay. The mixed term has to be added:

$$\begin{aligned} vv_{\ell f} &= \frac{1}{v_\ell \cdot v_f} \left[(2 \cdot T_3^\ell)(2 \cdot T_3^f) - 4 \cdot q_\ell \cdot s_W^2 \cdot \mathcal{K}_f(s, t)(2 \cdot T_3^\ell) \right. \\ &\quad \left. - 4 \cdot q_f \cdot s_W^2 \cdot \mathcal{K}_\ell(s, t)(2 \cdot T_3^f) \right. \\ &\quad \left. + (4 \cdot q_\ell \cdot s_W^2)(4 \cdot q_f \cdot s_W^2) \mathcal{K}_{\ell f}(s, t) \right] \frac{1}{\Delta^2}. \end{aligned} \quad (10)$$

Finally, we can write the spin amplitude for Born with EW corrections, $\mathcal{A}^{Born+EW}$, as:

$$\begin{aligned} \mathcal{A}^{Born+EW} &= \frac{\alpha}{s} \{ [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu u] \cdot (q_\ell \cdot q_f) \cdot \Gamma_{V_\Pi} \cdot \chi_\gamma(s) \\ &\quad + [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu u \cdot (v_\ell \cdot v_f \cdot vv_{\ell f}) \\ &\quad + \bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu \gamma^5 u \cdot (v_\ell \cdot a_f) \\ &\quad + \bar{u}\gamma^\mu \gamma^5 v g_{\mu\nu} \bar{v}\gamma^\nu u \cdot (a_\ell \cdot v_f) \\ &\quad + \bar{u}\gamma^\mu \gamma^5 v g_{\mu\nu} \bar{v}\gamma^\nu \gamma^5 u \cdot (a_\ell \cdot a_f)] \cdot Z_{V_\Pi} \cdot \chi_Z(s) \}. \end{aligned} \quad (11)$$

The EW form-factor corrections: $\rho_{\ell f}, \mathcal{K}_\ell, \mathcal{K}_f, \mathcal{K}_{\ell f}$ can be calculated using the `Dizet` library. This library invokes also calculation of vacuum polarization corrections to the photon propagator $\Pi_{\gamma\gamma}$. For the case of pp collisions we do not introduce QCD corrections to vector and axial couplings of incoming fermions. They are assumed to be included elsewhere as a part of the QCD NLO calculations for the initial parton state, including convolution with proton structure functions.

The *Improved Born Approximation* uses the spin amplitude $\mathcal{A}^{Born+EW}$ of Eq. (11) and $2 \rightarrow 2$ body kinematics to define the differential cross-section with EW corrections for $q\bar{q} \rightarrow Z/\gamma^* \rightarrow ll$ process. The formulae presented above very closely follow the approach taken for implementation⁶ of EW corrections to KKMC Monte Carlo [2].

3 Electroweak form-factors

For the calculation of EW corrections, we use the `Dizet` library, as of the 2010 KKMC Monte Carlo [2] version. For

⁶ Compatibility with this program is also part of the motivation why we leave updates for the `Dizet` library to the forthcoming work. `Dizet 6.21` is also well documented.

this and related projects, massive theoretical effort was necessary. Simultaneous study of several processes, like of $\mu^+\mu^-$, $u\bar{u}$, $d\bar{d}$, $\nu\bar{\nu}$ production in e^+e^- collisions and also in $p\bar{p}$ initiated parton processes, like at Tevatron, was performed. Groups of diagrams for the Z/γ^* propagators, production and decay vertices could be identified and incorporated into form-factors. The core of the Dizet library relies on such separation. It also opened the possibility that for one group of diagrams, such as vacuum polarizations, higher order contributions could be included while for others were not. That was particularly important for quark contributions to vacuum polarizations. Otherwise, the required precision would not be achieved. The above short explanation only indicates fundamental importance of the topic, we delegate the reader to Refs. [2, 19, 20] and experimental papers of LEP and Tevatron experiments quoting these papers.

The interface in KKMC prepares look-up tables with EW form-factors and vacuum polarization corrections. The tabulation grid granularity and ranges of the centre-of-mass energy of outgoing leptons and lepton scattering angle are adapted to variation of the tabulated functions. Theoretical uncertainties on the predictions for EW form-factors have been estimated in times of LEP precision measurements, in the context of either benchmark results like [18] or specific analyses [3]. The predictions are now updated with the known Higgs boson and top-quark masses. In the existing code of the Dizet library, certain types of the corrections or options of the calculations of different corrections can be switched off/on. In Appendix C, we show in Table 11 an almost complete list of options useful for discussions. We do not attempt to estimate the size of theoretical uncertainties, delegating it to the follow up work in the context of LHC EW Precision WG studies. The other versions of electroweak calculations, like of [12, 21], can and should be studied then as well. Already now the precision requirements of LHC experiments [16] are comparable to those of individual LEP measurements, but phenomenology aspects are more involved.

3.1 Input parameters to Dizet

The Dizet package relies on the so called *on-mass-shell* (OMS) normalization scheme [19, 20] but modifications are present. The OMS uses the masses of all fundamental particles, both fermions and bosons, the electromagnetic coupling constant $\alpha(0)$ and the strong coupling $\alpha_s(M_Z^2)$. The dependence on the ill-defined masses of the light quarks u , d , c , s and b is solved by dispersion relations, for details see [11]. Another exception is the W -boson mass M_W , which still can be predicted with better theoretical accuracy than experimentally measured. The Fermi constant G_μ is precisely known from μ -decay. For this reason, M_W was usually, in time of LEP analyses, replaced by G_μ as an input.

The knowledge about the hadronic vacuum polarization is contained in $\Delta\alpha_h^{(5)}(s)$, which is used as external, easy to change, parametrization. It can be either computed from quark masses or, preferably, fitted to experimental low energy $e^+e^- \rightarrow \text{hadrons}$ data.

The M_W is calculated iteratively from the equation

$$M_W = \frac{M_Z}{\sqrt{2}} \sqrt{1 + \sqrt{1 - \frac{4A_0^2}{M_Z^2(1 - \Delta r)}}}, \quad (12)$$

where

$$A_0 = \sqrt{\frac{\pi\alpha(0)}{\sqrt{2}G_\mu}}. \quad (13)$$

The Sirlin's parameter Δr [22]

$$\Delta r = \Delta\alpha(M_Z^2) + \Delta r_{EW} \quad (14)$$

is also calculated iteratively, and the definition of Δr_{EW} involves re-summation and higher order corrections. This term implicitly depends on M_W and M_Z , and the iterative procedure is needed. The re-summation term in formula (14) is not formally justified by renormalisation group arguments, the correct generalization is to compute higher order corrections, see discussion in [11].

Note that once the M_W is recalculated from formula (12), the lowest order Standard Model relationship between the weak and electromagnetic couplings

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2 \theta_W} \quad (15)$$

is not fulfilled anymore, unless the G_μ is redefined away from the measured value. This is an approach of some EW LO schemes, but not the one used by Dizet. It requires therefore the complete expression for $\chi_Z(s)$ propagator in spin amplitude of Eq. (11), as defined by formula (5).

In the OMS renormalisation scheme the weak mixing angle is defined uniquely through the gauge-boson masses:

$$\sin^2 \theta_W = s_W^2 = 1 - \frac{M_W^2}{M_Z^2}. \quad (16)$$

With this scheme, measuring $\sin^2 \theta_W$ will be equivalent to indirect measurement of M_W^2 through the relation (16).

In Table 1 we collect numerical values for all parameters used in the presented below evaluations. Note that formally they are not representing EW LO scheme, as the relation (15) is not obeyed. The M_W in (16) is recalculated with (12) but G_μ , M_Z remain unchanged.

3.2 The EW form-factors

Real parts of the $\rho_{\ell f}(s, t)$, $\mathcal{K}_f(s, t)$, $\mathcal{K}_\ell(s, t)$, $\mathcal{K}_{\ell f}(s, t)$ EW form-factors are shown in Fig. 1 for a few values of $\cos \theta$,

Table 1 The `Dizet` initialization: masses and couplings. The calculated M_W and s_W^2 are shown also

Parameter	Value	Description
M_Z	91.1876 GeV	Mass of Z boson
M_H	125.0 GeV	Mass of Higgs boson
m_t	173.0 GeV	Mass of top quark
m_b	4.7 GeV	Mass of b quark
$1/\alpha(0)$	137.0359895(61)	QED coupling
G_μ	$1.166389(22) \cdot 10^{-5} \text{GeV}^{-2}$	Fermi constant in μ -decay
M_W	80.353 GeV	Formula (12)
s_W^2	0.22351946	Formula (16)

the angle between directions of the incoming quark and the outgoing lepton, calculated in the outgoing lepton pair centre-of-mass frame. Eq. (9) relates Mandelstam variables (s, t) to the invariant mass and $\cos\theta$. The $\cos\theta$ dependence of the box correction is more sizable for the up-quarks.

Note, that at the Z-boson peak, Born-like couplings are only weakly modified; form-factors are close to 1 and of no numerically significant angular dependence. At lower virtualities corrections are relatively larger because the Z-boson contributions are non resonant and thus smaller. In this phase-space region the Z-boson is itself dominated by the virtual photon contribution. Above the peak, the WW and later also ZZ boxes contributions become sizable, the dependence on the $\cos\theta$ appears; contributions become gradually doubly resonant and sizable.

3.3 Running $\alpha(s)$

Fermionic loop insertion of the photon propagator, i.e. vacuum polarization corrections, are summed together as a multiplicative factor $\Gamma_{V\Pi}$, Eq. (8), for the photon exchange in Eq. (11). But it can be interpreted as the *running QED coupling*:

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha_h^{(5)}(s) - \Delta\alpha_\ell(s) - \Delta\alpha_t(s) - \Delta\alpha^{\alpha\alpha_s}(s)}. \quad (17)$$

The hadronic contribution at M_Z is a significant [11] correction: $\Delta\alpha_h^{(5)}(M_Z^2) = 0.0280398$. It is calculated in the five flavour scheme with use of dispersion relation and input from low energy experiments. We will continue to use LEP times parametrization, while the most recent measured $\Delta\alpha_h^{(5)}(M_Z^2) = 0.02753 \pm 0.00009$ [23]. The changed value modifies predicted form-factors, in particular the effective leptonic mixing angle $\sin^2\theta_{eff}^{lep}(M_Z^2) = \text{Re}(\mathcal{K}_l(M_Z^2))s_W^2$ is shifted by almost $20 \cdot 10^{-5}$ closer to the measured LEP value. This is not included in the numerical results presented as we consistently remain with the defaults used in `KKMC`.

The leptonic loop contribution $\Delta\alpha_\ell(s)$ is calculated analytically up to the 3-loops, and is a comparably significant

correction, $\Delta\alpha_\ell(M_Z^2) = 0.0314976$. The other contributions are very small.

Figure 2 shows the vacuum polarization corrections to the $\chi_V(s)$ propagator, directly representing the ratio $\alpha(s)/\alpha(0)$ of Eq. (17).

4 EW input schemes and effective born

Formally, at the lowest EW order, only three independent parameters can be set, other are calculated following the structure of $SU(2) \times U(1)$ group from Standard Model constraints. Formula (15) represents one of such constraints. Following report [24], the most common choices at hadron colliders are: G_μ scheme (G_μ, M_Z, M_W) and $\alpha(0)$ scheme ($\alpha(0), M_Z, M_W$). There exists by now a family of different modifications of the G_μ scheme, see discussion in [24], and they are considered as preferred schemes for hadron collider physics.⁷

Let us recall, that the calculations of EW corrections available in `Dizet` work with a variant of the $\alpha(0)$ scheme. It is defined by the input parameters ($\alpha(0), G_\mu, M_Z$). Then M_W is calculated iteratively from formula (12) and s_W^2 of Eq. (16) uses that value of M_W . This formally brings it beyond EW LO scheme. The numerical value of s_W^2 calculated from (16) does not fulfill the EW LO relation (15) anymore.

At this point we introduce two options for the *Effective Born* spin amplitudes parametrization, which works well for

⁷ The Monte Carlo generators usually allow user to define set of input parameters (α, M_Z, M_W), (α, M_Z, G_μ) or (α, M_Z, s_W^2). However, within this flexibility, formally multiplicative factor $\chi_Z(s)$ in the Z-boson propagator, see formula (5), is always kept to be equal to 1:

$$\frac{G_\mu \cdot M_Z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} = 1, \quad (18)$$

where Δ is given by Eq. (2). The multiplicative factor of (18) in the definition of $\chi_Z(s)$ is quite often absent in the programs code. With the choice of primary parameters, the others are adjusted to match the constraint Eq. (18), regardless if they fall outside their measurement uncertainty window or not.

Fig. 1 Real parts of the $\rho_{e,up}$, K_e , K_{up} and $K_{e,up}$ EW form-factors of $q\bar{q} \rightarrow Z \rightarrow ee$ process, as a function of $\sqrt{s}$ and for the few values of $\cos\theta$. For the up-type quark flavour, left side plots are collected and for the down-type the right side plots. Note, that K_e depends on the flavour of incoming quarks

Fig. 2 The vacuum polarization ($\alpha(s)/\alpha(0)$) correction of γ propagator, Eq. (17)

parametrizing EW corrections near the Z-pole and denote them respectively as *LEP* and *LEP with improved norm.*:

- The *LEP* parametrization uses formula (11) for spin amplitude but with $\alpha(s) = \alpha(M_Z^2) = 1./128.8667$, $s_W^2 = \sin^2 \theta_W^{eff}(M_Z^2) = 0.23152$, i.e. as measured at the Z-pole and reported in [25]. All form-factors are set to 1.0.
- The *LEP with improved norm.* parametrization also uses formula (11) for spin amplitude with parameters set as for *LEP* parametrization. All form-factors are set to 1, but $\rho_{\ell f} = 1.005$. This corresponds to the measured $\rho(M_Z^2) = 1.005$, as reported in [25].

Table 2 collects initialization constants of EW schemes relevant for our discussion. We specify parameters which enter formula (11) for Born spin amplitudes used for: (i) actual MC events generation,⁸ (ii) the EW LO $\alpha(0)$ scheme, (iii) effective Born (*LEP*) parametrization and (iv) effective Born (*LEP with improved norm.*). In each case parameters are chosen such that the SM relation, formula (18), is obeyed.

In the *Improved Born Approximation* complete $O(\alpha)$ EW corrections, supplemented by selected higher order terms, are handled thanks to s-, t-dependent form-factors, which multiply couplings and propagators of the usual Born expressions. Instead, the *Effective Born* absorbs the bulk of EW corrections into a redefinition of a few fixed parameters (i.e. couplings).

In the following, we will systematically compare predictions obtained with the EW corrections and those calculated with *LEP* or *LEP with improved norm.* approximations. As we will see, effective Born with *LEP with improved norm.*

⁸ The EW LO initialization is consistent with PDG $\sin^2 \theta_{eff}^{lep} = 0.23113$, but commonly used G_μ scheme, ($G_\mu = 1.1663787 \cdot 10^{-5} \text{ GeV}^{-2}$, $M_Z = 91.1876 \text{ GeV}$, $M_W = 80.385 \text{ GeV}$) correspond to $s_W^2 = 0.2228972$.

works very well around Z-pole both for the line-shape and forward-backward asymmetry.

5 Born kinematic approximation and pp scattering

The solution to define Born-like parton level kinematics for pp scattering process is encoded in the *TauSpinner* package [14]. It does not exploit hard-process, so-called history entries which only sometimes are stored for the generated events. In particular, the flavour and momenta of the incoming partons have to be emulated from the kinematics of final states and incoming protons momenta. Probabilities calculated from parton level cross-sections and PDFs weight all possible contributions. Let us now recall briefly principles and choices for optimization.

5.1 Average over incoming partons flavour

The parton level Born cross-section $\sigma_{Born}^{q\bar{q}}(\hat{s}, \cos \theta)$ has to be convoluted with the structure functions, and summed over all possible flavours of incoming partons and all possible helicity states of outgoing leptons. The lowest order formula⁹ is given below

$$\begin{aligned} d\sigma_{Born}(x_1, x_2, \hat{s}, \cos \theta) &= \sum_{qf, \bar{q}f} [f^{qf}(x_1, \dots) f^{\bar{q}f}(x_2, \dots) d\sigma_{Born}^{qf\bar{q}f}(\hat{s}, \cos \theta) \\ &+ f^{\bar{q}f}(x_1, \dots) f^{qf}(x_2, \dots) d\sigma_{Born}^{qf\bar{q}f}(\hat{s}, -\cos \theta)], \end{aligned} \quad (19)$$

where x_1, x_2 denote fractions of incoming protons momenta carried by the corresponding parton, $\hat{s} = x_1 x_2 s$ and $f/\bar{f}$ denotes parton (quark-/anti-quark) density functions. We assume that kinematics is reconstructed from four-momenta of the outgoing leptons. The incoming quark and anti-quark may come respectively either from the first and second proton or reversely from the second and first. Both possibilities are taken into account¹⁰ by the two terms of (19). The sign in front of $\cos \theta$, the cosine of the scattering angle, is negative

⁹ Valid for the ultra-relativistic leptons.

¹⁰ One should mention photon induced contributions. They are of the same coupling order as electroweak corrections. For production of the lepton pairs in pp collisions, contributions were evaluated e.g. in [26]. In general, for the calculation of *TauSpinner* weights, sum over partons is not restricted as in Eq. (19) to the quarks and anti-quarks only. Gluon PDF's are used when weight calculation with matrix elements for lepton pair with two jets in final state is used [27]. The $\gamma\gamma \rightarrow l^+l^-$ contributions can be then taken into account as a part of the $2 \rightarrow 4$ matrix elements.

Photon induced processes are however usually generated and stored separately. That is why our reweighting algorithm for EW corrections does not need to take such (rather small) contributions into account in Eq. (19).

Table 2 The EW parameters used for: (i) MC events generation, (ii) the EW LO $\alpha(0)$ scheme, (iii) effective Born spin amplitude around the Z-pole and (iv) effective Born with improved normalization. In eachcase parameters are chosen such that the SM relation, formula (18), is obeyed. The $G_\mu = 1.166389 \cdot 10^{-5} \text{ GeV}^{-2}$, $M_Z = 91.1876 \text{ GeV}$ and $\mathcal{K}_f, \mathcal{K}_e, \mathcal{K}_{\ell f} = 1$ are taken

EW LO MC generator	EW LO $\alpha(0)$ scheme	Effective born <i>LEP</i>	Effective born <i>LEP with improved norm.</i>
$\alpha = 1/128.8886$	$\alpha = 1/137.3599$	$\alpha = 1/128.8667$	$\alpha = 1/128.8667$
$s_W^2 = 0.23113$	$s_W^2 = 0.21215$	$s_W^2 = 0.23152$	$s_W^2 = 0.23152$
$\rho_{\ell f} = 1.0$	$\rho_{\ell f} = 1.0$	$\rho_{\ell f} = 1.0$	$\rho_{\ell f} = 1.005$

for the second term. Then the parton of the first incoming proton which carries x_1 and follows the direction of the z -axis is an anti-quark, not a quark. The formula is used for calculating the differential cross-section $d\sigma_{\text{Born}}(x_1, x_2, \hat{s}, \cos\theta)$ of each analyzed event, regardless if its kinematics and flavours of incoming partons may be available from the event history entries or not. The formula can be used to a good approximation in case of NLO QCD spin amplitudes. The momenta of outgoing leptons are used to construct *effective* kinematics of the Drell-Yan production process and decay, without the need of information on parton-level hard-process itself. Born-like kinematics can be constructed, as we will see later, even for events of quark-gluon or gluon-gluon parton level collisions (as inspected for test in the event history entries) too.

5.2 Effective beams kinematics

The x_1, x_2 are calculated from the kinematics of outgoing leptons, following formulae of [15]

$$x_{1,2} = \frac{1}{2} \left(\pm \frac{p_z^{\ell\ell}}{E} + \sqrt{\left(\frac{p_z^{\ell\ell}}{E}\right)^2 + \frac{m_{\ell\ell}^2}{E^2}} \right), \quad (20)$$

where E denotes energy of the proton beam and $p_z^{\ell\ell}$ denotes z -axis momentum of outgoing lepton pair in the laboratory frame and $m_{\ell\ell}$ lepton pair virtuality. Note that this formula can be used, as approximation, for the events with hard jets too.

5.3 Definition of the polar angle

For the polar angle $\cos\theta$, of factorized Born level $q\bar{q} \rightarrow Z \rightarrow \ell\ell$ process, weighted average of the outgoing leptons angles with respect to the beams' directions, denoted as $\cos\theta^*$, was used. In [28] it was found helpful to compensate the effect of initial state hard bremsstrahlung photons of $e^+e^- \rightarrow Zn\gamma$, $Z \rightarrow \ell\ell m\gamma$, where m, n denote the number of accompanying photons. Extension to pp collisions required to take both options in Eq. (19) into account; when the z -axis is parallel- and anti-parallel to the incoming quark.

For the further calculation, boost of all four-momenta (also of incoming beams) into the rest frame of the lepton pair need to be performed. The $\cos\theta^*$ is then calculated from

$$\begin{aligned} \cos\theta_1 &= \frac{\tau_x^{(1)}b_x^{(1)} + \tau_y^{(1)}b_y^{(1)} + \tau_z^{(1)}b_z^{(1)}}{|\boldsymbol{\tau}^{(1)}||\mathbf{b}^{(1)}|}, \\ \cos\theta_2 &= \frac{\tau_x^{(2)}b_x^{(2)} + \tau_y^{(2)}b_y^{(2)} + \tau_z^{(2)}b_z^{(2)}}{|\boldsymbol{\tau}^{(2)}||\mathbf{b}^{(2)}|}, \end{aligned} \quad (21)$$

as follows:

$$\cos\theta^* = \frac{\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1}{\sin\theta_1 + \sin\theta_2} \quad (22)$$

where $\boldsymbol{\tau}^{(1)}, \boldsymbol{\tau}^{(2)}$ denote 3-vectors of outgoing leptons and $\mathbf{b}^{(1)}, \mathbf{b}^{(2)}$ denote 3-vectors of incoming beams' four-momenta.

The polar angle definition, Eq. (22), is at present the TauSpinner default. For tests we have used variants; *Mustaal* [4] and *Collins-Soper* [29] frames, which differ when high p_T jets are present. We will return later to the frame choice, best suitable when NLO QCD corrections are included in the production process of generated events.

6 QCD corrections and angular coefficients

For the Drell-Yan production [30] one can separate QCD and EW components of the fully differential cross-section and describe the $Z/\gamma^* \rightarrow \ell\ell$ sub-process with lepton angular (θ, ϕ) dependence

$$\frac{d\sigma}{dp_T^2 dY d\Omega} = \sum_{\alpha=1}^9 g_\alpha(\theta, \phi) \frac{3}{16\pi} \frac{d\sigma^\alpha}{dp_T^2 dY}, \quad (23)$$

where the $g_\alpha(\theta, \phi)$ denotes second order spherical harmonics, multiplied by normalization constants and $d\sigma^\alpha$ denotes helicity cross-sections, for each of nine helicity configurations of $q\bar{q} \rightarrow Z/\gamma^* \rightarrow \ell\ell$. The polar and azimuthal $(\theta$ and $\phi)$ angles of $d\Omega = d\cos\theta d\phi$ are defined in the Z -boson rest-frame. The p_T, Y denote laboratory frame transverse momenta and rapidity of the intermediate Z/γ^* -boson. Thanks to the effort [31–33] from the early 90's

one expects such factorization to break with non-logarithmic $\mathcal{O}(\alpha_s^2) \sim 0.01$ QCD corrections¹¹ only.

There is some flexibility for the Z-boson rest frame z -axis choice. The most common, so called *helicity frame*, is to take the Z-boson laboratory frame momentum. For the *Collins-Soper* frame it is defined from directions of the two beams in the Z-boson rest frame and is signed with the Z-boson p_z laboratory frame sign.

Equation (23) with explicit spherical harmonics and coefficients reads

$$\begin{aligned} \frac{d\sigma}{dp_T^2 dY d\cos\theta d\phi} = & \frac{3}{16\pi} \frac{d\sigma^{U+L}}{dp_T^2 dY} [(1 + \cos^2\theta) \\ & + 1/2 A_0(1 - 3\cos^2\theta) + A_1 \sin 2\theta \cos\phi \\ & + 1/2 A_2 \sin^2\theta \cos(2\phi) + A_3 \sin\theta \cos\phi + A_4 \cos\theta \\ & + A_5 \sin^2\theta \sin(2\phi) + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta \sin\phi], \end{aligned} \quad (24)$$

where $d\sigma^{U+L}$ denotes the unpolarised differential cross-section (notation used in several papers of the 80's). The coefficients $A_i(p_T, Y)$ are related to ratios of definite intermediate state helicity contributions to the $d\sigma^{U+L}$ cross-sections. The first term of the polynomial expansion is $(1 + \cos^2\theta)$ because intermediate boson is of the spin 1.

The dynamics of the production process is hidden in the angular coefficients $A_i(p_T, Y)$. In particular, all the hadronic physics is described implicitly by the angular coefficients and it decouples from the well understood leptonic and intermediate boson physics.

For the present paper, of particular interest are coupling constants present in coefficients A_i of Eq. (24) representing ratios of the so-called helicity cross sections [31–33]:

$$\begin{aligned} \sigma^{U+L} & \sim (v_\ell^2 + a_\ell^2)(v_q^2 + a_q^2), \\ A_0, A_1, A_2 & \sim 1, A_3, A_4 \sim \frac{v_\ell a_\ell v_q a_q}{(v_\ell^2 + a_\ell^2)(v_q^2 + a_q^2)}, \\ A_5, A_6 & \sim \frac{(v_\ell^2 + a_\ell^2)(v_q a_q)}{(v_\ell^2 + a_\ell^2)(v_q^2 + a_q^2)}, \\ A_7 & \sim \frac{v_\ell a_\ell (v_q^2 + a_q^2)}{(v_\ell^2 + a_\ell^2)(v_q^2 + a_q^2)}. \end{aligned} \quad (25)$$

Integration¹² over the azimuthal angle ϕ reduces Eq. (24) to

$$\begin{aligned} \frac{d\sigma}{dp_T^2 dY d\cos\theta} = & \frac{3}{8\pi} \frac{d\sigma^{U+L}}{dp_T^2 dY} [(1 + \cos^2\theta) \\ & + 1/2 A_0(1 - 3\cos^2\theta) + A_4 \cos\theta]. \end{aligned} \quad (26)$$

¹¹ Also the impact of final state QED bremsstrahlung can be overcome with a proper definition of frames. The solution is available thanks to Ref. [4]. We use it with the definition of frames A and A' ; Section 3.1 of [14].

¹² One can easily check that A_{FB} of Eq. (28) equals to $\frac{3}{8} A_4$.

Both Eqs. (24) and (26) are valid in any rest frame of the outgoing lepton pairs, however the $A_i(p_T, Y)$ are frame dependent. The *Collins-Soper* frame is the most convenient and usual choice for the analyses dedicated to QCD dynamics. In this frame, in the low p_T limit, A_4 is the only non-zero coefficient. It carries direct information on the EW couplings, as can be concluded from formulae (25). All other coefficients depart from zero with increasing p_T while at the same time A_4 gradually decreases.

Due to different transfer dependence of the Z and γ^* propagators, the A_i vary with m_{ll} . The A_i dependence on (p_T, Y) , expressing production dynamics, differ with the frame definition variants of distinct coordinate system orientations. For the studies of EW couplings, it is convenient when the lepton-pair rest-frame definition absorbs effects of production dynamics partly into the z -axis choice. Then, those A_i coefficients which are proportional to the product of EW vector and axial couplings remain non-zero over the full range of p_T . Promising for that purpose frame was developed at LEP times for the *Mustaal* Monte Carlo program [4]. Recently, an extension of this *Mustaal* frame, for the case of hadron-hadron collisions, was introduced and discussed in [5]. As shown in that paper, both *Collins-Soper* and *Mustaal* frames are equivalent in the $p_T = 0$ limit. Then A_4 is the only non-zero coefficient for both frames and is also numerically very close. With increasing p_T , in the *Mustaal* frame A_4 remains as the only sizably non-zero coefficient, while several A_i coefficients depart from zero with the *Collins-Soper* frame.

In the collision of the same-charge protons the careful choice for the z -axis orientation is necessary for the A_4 coefficient to remain non-zero. For the *Collins-Soper* frame, the z -axis follows the direction of the intermediate Z-boson in the laboratory frame. In case of the *Mustaal* frame the choice of the sign is made stochastically using information of the system of leptons and outgoing accompanying visible jets. For details see [5], alternatively the same sign choice for the z -axis as in the *Collins-Soper* case, can be used.

The shape of A_i coefficients as a function of laboratory frame Z-boson transverse momenta p_T depends on the choice of lepton pairs rest-frame. In Fig. 3, A_i coefficients of the *Collins-Soper* and *Mustaal* frames are shown. As intended, even for large p_T , with this frame, only A_4 coefficient is sizably non-zero.

7 Concept of the EW weight

The EW corrections enter the $\sigma_{Born}(\hat{s}, \cos\theta)$ through the definition of the vector and axial couplings, also photon and Z-boson propagators. They modify normalization of the cross-sections, the line-shape of the Z-boson peak, polarization of the outgoing leptons and asymmetries.

Fig. 3 The A_i coefficients for $Z \rightarrow e^+e^-$ in lepton pair invariant mass range $80 < m_{ee} < 100$ GeV. The $Z + j$ production process in pp collisions at 8 TeV centre-of-mass energy, was used for the sample

generation with Powheg+MiNLO Monte Carlo. The A_i coefficients are calculated in the *Collins-Soper* and *Mustraal* frames with moments method [32]

Given that, we were able to factorize QCD and EW components of the cross-section to a good approximation and define per-event weights which specifically correct for EW effects. Such a weight may modify events generated with EW LO to the ones including the EW corrections. This is very much the same idea as already implemented in TauSpinner for introducing corrections for other effects: spin correlations, production process, etc.

The per-event wt^{EW} is defined as ratio of the Born-level cross-sections with and without EW corrections

$$wt^{EW} = \frac{d\sigma_{Born+EW}(s, \cos\theta)}{d\sigma_{Born}(s, \cos\theta)}, \quad (27)$$

where $\cos\theta$ can be taken according to $\cos\theta^*$, $\cos\theta^{Mustraal}$ (*Mustraal* frame) or $\cos\theta^{CS}$ (*Collins-Soper* frame) prescription. For most events, the three choices will lead to numerically very close values for $\cos\theta$ and thus resulting wt^{EW} . The difference originates from distinct $\cos\theta$ dependence of Z and γ^* exchange amplitudes and not only from electroweak boxes. The wt^{EW} allows for flexible implementation of the EW corrections using TauSpinner framework and form-factors calculated e.g. with Dizet.

The formula for wt^{EW} can be used to re-weight from one EW LO scheme to another too. In that case, both the

numerator and denominator of Eq. (27) will use lowest order $d\sigma_{Born}$, calculated in different EW schemes¹³ though.

8 EW corrections to doubly-deconvoluted observables

Now that all components needed for calculation of wt^{EW} are explained, we can present results for selected examples of doubly-deconvoluted observables around the Z-pole.

The Powheg+MiNLO Monte Carlo, with NLO QCD and LO EW matrix elements, was used to generate $Z + j$ events with $Z \rightarrow e^+e^-$ decays in pp collisions at 8 TeV. No selection was applied to generated events, except for an outgoing electron pair invariant mass range of $70 < m_{ee} < 150$ GeV. For events generation, the EW parameters as shown in left-most column of Table 2 were used. It is often used as a default for phenomenological studies at LHC. The α and s_W^2 close to the ones of $\overline{MS}$ scheme discussed in [25] were taken. Note that they do not coincide accurately with the precise LEP experiments measurements at the Z-pole [1].

To quantify the effect of the EW corrections, we re-weight events generated, to EW LO with the scheme used by the Dizet: Table 2 second column. Only then we gradually introduce EW corrections and form-factors calculated with that library. For each step, the appropriate numerator of the wt^{EW} is calculated, while for the denominator the EW LO $\mathcal{A}^{Born}$ matrix element Eq. (3) is used; parameters as in the left-most column of Table 2. The sequential steps, in which we illustrate effects of EW corrections are given below:

1. Re-weight with wt^{EW} , from EW LO scheme used for MC events generation to EW LO scheme with $s_W^2 = 0.21215$, Table 2 second column. The $\mathcal{A}^{Born}$ matrix element, Eq. (3), is used¹⁴ for calculating numerator of wt^{EW} .
2. As in step (1), but include EW corrections to M_W , effectively changing to $s_W^2 = 0.22352$ in calculation of wt^{EW} . Relation, formula (15), is not obeyed anymore.
3. As in step (2), but include EW loop corrections to the normalization of Z-boson and γ^* propagators, i.e. QCD/EW corrections to $\alpha(0)$ and $\rho_{\ell f}(s)$ form-factor calculated without box corrections. The $\mathcal{A}^{Born+EW}$, Eq. (11), is used for calculating numerator of wt^{EW} .
4. As in step (3), but include EW corrections to Z-boson vector couplings: $\mathcal{K}_f, \mathcal{K}_l, \mathcal{K}_{\ell f}$, calculated without box corrections. The $\mathcal{A}^{Born+EW}$ is used for calculating numerator of wt^{EW} .

5. As in step (4), but $\rho_{\ell f}, \mathcal{K}_f, \mathcal{K}_l, \mathcal{K}_{\ell f}$ form-factors include box corrections. The $\mathcal{A}^{Born+EW}$ is used for calculating numerator of wt^{EW} .

After step (1) the sample is EW LO and QCD NLO, but with different EW scheme than used originally for events generation. Then steps (2)–(5) introduce EW corrections. Step (3) effectively changes α back to be close to $\alpha(M_Z^2)$, while steps (4)–(5) effectively shift back v_f, v_l close to the values used in generation. Parameters for EW LO scheme used for event generation are already close to measured at the Z-pole. That is why we expect the total EW corrections to the generated sample to be roughly at the percent level only.

In the following, we will estimate how precise it would be to use effective Born approximation with *LEP* or *LEP with improved norm.* parametrisations instead of complete EW corrections. To obtain those predictions, re-weighting similar to step (1) listed above is needed, but in the numerator of wt^{EW} the $\mathcal{A}^{Born}$ parametrisations as specified in the right two columns of Table 2 are used. For *LEP with improved norm.* the $\rho_{\ell, f} = 1.005$ has to be included as well.

The important flexibility of the proposed approach is that wt^{EW} can be calculated using $d\sigma_{Born}$ in different frames: $\cos\theta^*$, *Mustaal* or *Collins-Soper*. For some observables, frame choice used for wt^{EW} calculation is not numerically relevant at all; the simplest $\cos\theta^*$ frame can be used. We show later an example, where only the *Mustaal* frame for the wt^{EW} calculation leads to correct results.

8.1 The Z-boson line-shape

In the EW LO, the Z-boson line-shape, assuming that the constraint (15) holds, depends predominantly on M_Z and Γ_Z . The effects on the line-shape from EW loop corrections are due to corrections to the propagators: vacuum polarization corrections (running α) and ρ form-factor, which change relative contributions of the Z to γ^* and, the Z-boson vector to axial coupling ratio ($\sin^2\theta_{eff}$). The above affects not only shape but also normalization of the cross-section. In the formulae (27) we do not use running Z-boson width, which remains fixed.

In Fig. 4 (top-left) distributions of generated and EW corrected line-shapes are shown. With the logarithmic scale, a difference is barely visible. With the following plots of the same Figure we study details. The ratios of the line-shape distributions with gradually introduced EW corrections are shown. For the reference distributions (ratio-histograms denominators) for the following three plots: (i) EW LO $\alpha(0)$ scheme, (ii) effective Born (*LEP*) and (iii) effective Born (*LEP with improved norm.*) are used. At the Z-pole, complete EW corrections contribute about 0.1% with respect

¹³ In this way, in particular, the fixed width description for the Z-boson propagator can be replaced with the s dependent one.

¹⁴ The MC sample is generated with fixed width propagator. We remain with this convention. This could also be changed with the help of wt^{EW} .

Fig. 4 Top-left: line-shape distribution as generated with Powheg+MiNLO (blue triangles) and after reweighting introducing all EW corrections (red triangles). The two choices are barely distinguishable. Ratios of the line-shapes with gradually introduced

to the one of effective Born (*LEP with improved norm.*). A use of events generated with EW LO matrix element but of different parametrisations significantly reduce the numerical size of missing EW corrections.

Table 3 details numerically EW corrections to the normalization (ratio of the cross-sections) integrated in the range $80 < m_{ee} < 100$ GeV and $89 < m_{ee} < 93$ GeV. Results from EW weight with the $\cos\theta^*$ definition of the scattering angle are shown. The total EW correction factor is about 0.965 for cross-section normalization and EW LO $\alpha(0)$, while the total correction for the effective Born (*LEP with improved norm.*) is of about 1.001. In Table 4 results with w_t^{EW} calculated with different frames are compared. If *Mu*-*straal* or *Collins-Soper* frames are used instead of $\cos\theta^*$ for weight calculations, the differences are at most at the 5-th significant digit.

8.2 The A_{FB} distribution

The forward-backward asymmetry for pp collisions reads

$$A_{FB} = \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma(\cos\theta > 0) + \sigma(\cos\theta < 0)}, \quad (28)$$

EW corrections are shown in consecutive plots, where as a reference (black dashed line) respectively: (i) EW LO $\alpha(0)$ scheme (top-right), (ii) effective Born (*LEP*) (bottom-left) and, (iii) effective Born (*LEP with improved norm.*) (bottom-right), was used

where $\cos\theta$ of the *Collins-Soper* frame is used.

The EW corrections change A_{FB} , particularly around the Z-pole. In Fig. 5 (top-left), the A_{FB} as generated (EW LO) and EW corrected is shown as a function of m_{ee} . In the following plots of this Figure, we study details. The $\Delta A_{FB} = A_{FB} - A_{FB}^{ref}$, with gradually introduced EW corrections to A_{FB} is shown and compared with the following reference choices for A_{FB}^{ref} : (i) EW LO $\alpha(0)$ scheme, (ii) effective Born (*LEP*) and (iii) effective Born (*LEP with improved norm.*).

Complete EW corrections to predictions of EW LO $\alpha(0)$ scheme for A_{FB} integrated around Z-pole give $\Delta A_{FB} = -0.03534$. The EW correction ΔA_{FB} to prediction of effective Born (*LEP with improved norm.*), is only -0.00005. We observe that effective Born (*LEP improved norm.*) reproduces EW loop corrections precision better and $\Delta A_{FB} = -0.0001$ in the full presented mass range. The remaining box corrections contribute around $m_{ee} = 150$ GeV about -0.002 to ΔA_{FB} .

Table 5 details numerically EW corrections, for A_{FB} integrated over the $80 < m_{ee} < 100$ GeV and $89 < m_{ee} < 93$ GeV ranges. For calculating EW weight, the $\cos\theta^*$ definition of the scattering angle was used. In Table 6 results

Table 3 EW corrections for cross-sections integrated over the specified mass windows. The EW weight is calculated with $\cos \theta^*$

Corrections to cross-section	$89 < m_{ee} < 93 \text{ GeV}$	$80 < m_{ee} < 100 \text{ GeV}$
$\sigma(\text{EW corr. to } m_W)/\sigma(\text{EW LO } \alpha(0))$	0.97114	0.97162
$\sigma(\text{EW corr. to } \chi(Z), \chi(\gamma))/\sigma(\text{EW LO } \alpha(0))$	0.98246	0.98346
$\sigma(\text{EW/QCD FF no boxes})/\sigma(\text{EW LO } \alpha(0))$	0.96469	0.96602
$\sigma(\text{EW/QCD FF with boxes})/\sigma(\text{EW LO } \alpha(0))$	0.96473	0.96607
$\sigma(\text{LEP})/\sigma(\text{EW/QCD FF with boxes})$	1.01102	1.01093
$\sigma(\text{LEP with improved norm.})/\sigma(\text{EW/QCD FF with boxes})$	1.00100	1.00098

Table 4 EW corrections for cross-sections integrated over the mass window around Z-pole; $89 < m_{ee} < 93 \text{ GeV}$. The EW weight is calculated with $\cos \theta^*$, $\cos \theta^{\text{Mustraal}}$ or $\cos \theta^{\text{CS}}$

Corrections to cross-section ($89 < m_{ee} < 93 \text{ GeV}$)	$wt^{\text{EW}}(\cos \theta^*)$	$wt^{\text{EW}}(\cos \theta^{\text{Mustraal}})$	$wt^{\text{EW}}(\cos \theta^{\text{CS}})$
$\sigma(\text{EW corr. to } m_W)/\sigma(\text{EW LO } \alpha(0))$	0.97114	0.97115	0.97114
$\sigma(\text{EW corr. to } \chi(Z), \chi(\gamma))/\sigma(\text{EW LO } \alpha(0))$	0.98246	0.98247	0.98246
$\sigma(\text{EW/QCD FF no boxes})/\sigma(\text{EW LO } \alpha(0))$	0.96469	0.96471	0.96470
$\sigma(\text{EW/QCD FF with boxes})/\sigma(\text{EW LO } \alpha(0))$	0.96473	0.96475	0.96474
$\sigma(\text{LEP})/\sigma(\text{EW/QCD FF with boxes})$	1.01102	1.01103	1.01102
$\sigma(\text{LEP with improved norm.})/\sigma(\text{EW/QCD FF with boxes})$	1.00100	1.00102	1.00100

obtained with wt^{EW} calculated in different frames are compared. When the *Mustraal* or *Collins-Soper* frame is used instead of $\cos \theta^*$, the differences are at most at the 5-th significant digit, similar as for the line-shape.

8.3 Effective weak mixing angles

The forward-backward asymmetry A_{FB} at the Z-pole can be used as an observable for effective weak mixing Weinberg angles, dependent on the invariant mass of lepton pairs. We extend standard LEP definition of effective weak mixing angles to

$$\sin^2 \theta_{eff}^f(s, t) = \text{Re}(\mathcal{K}^f(s, t))s_W^2 + I_f^2(s, t), \quad (29)$$

which is more suitable for LHC and for the off Z-pole regions. The flavour dependent effective weak mixing angles, calculated using: Eq. (29), EW form-factors of `Dizet` library, and $s_W^2 = 0.22352$ are shown on Fig. 6 as a function of the invariant mass of outgoing lepton pair and for $\cos \theta = 0.5$. The imaginary part of $I_f^2(s, t)$ is about 10^{-4} only. In Table 8 we display effective weak mixing angles averaged over specified mass windows.

The effective $\sin \theta_{eff}^f$ on the Z-pole, printed by `Dizet` is shown in Table 7. It is numerically slightly different than of Table 8, which is an average over mass window close to Z-pole. Note, that the observed very good agreement at the Z-pole between A_{FB} predictions of effective Born with (*LEP*) or (*LEP with improved norm.*) parametrisations and fully EW corrected is not reflected for predictions of flavour dependent effective weak Weinberg angles. Effective Born (*LEP*) and (*LEP with improved norm.*) are parametrised with $s_W^2 = 0.23152$, while `Dizet` library predicts leptonic effec-

tive weak mixing angle $\sin^2 \theta_{eff}^\ell(M_Z^2) = 0.23176$ which is about $20 \cdot 10^{-5}$ different. Why then such a good agreement on ΔA_{FB} as seen on Fig. 5 bottom plots? Certainly this requires further attention.

8.4 The A_4 , A_3 angular coefficients

To complete the discussion on doubly-deconvoluted observables, we turn our attention to angular coefficients A_4 and A_3 (proportional to product of vector and axial couplings) and to EW corrections. The coefficients are calculated from the event sample with the moments methods [32] and in the *Collins-Soper* frame. The EW weight wt^{EW} is used to introduce EW corrections and is calculated with the help of $\cos \theta^*$, $\cos \theta^{\text{Mustraal}}$ or $\cos \theta^{\text{CS}}$ angles.

Similarly as for A_{FB} , the EW corrections change overall size and the shape of A_4 as a function of m_{ee} ; particularly around the Z-pole. In Fig. 7 (top-right), the A_4 for generated sample (EW LO) and EW corrected is shown as a function of m_{ee} . In the following plots of the figure details are studied. The $\Delta A_4 = A_4 - A_4^{\text{ref}}$ with gradually introduced EW corrections is shown and compared with the following reference choices for A_4^{ref} : (i) EW LO $\alpha(0)$ scheme, (ii) effective Born (*LEP*) and (iii) effective Born (*LEP with improved norm.*). Conclusions are very similar as for previous ΔA_{FB} discussion. Note that ΔA_4 and ΔA_{FB} scale approximately with the relation $A_4 = 8/3 A_{FB}$.

The analogous set of plots, Fig. 8, is prepared for A_3 . In this case, only the *Mustraal* frame turned out to be adequate for wt^{EW} calculation. Both the $\cos \theta^*$ and $\cos \theta^{\text{CS}}$ were unable to fully capture the effects of EW corrections.

Fig. 5 Top-left: the A_{FB} as generated with Powheg+MINLO (blue triangles) and after reweighting introducing all EW corrections (red triangles). The two choices are barely distinguishable. The differences $\Delta A_{FB} = A_{FB} - A_{FB}^{ref}$, due to gradually introduced EW corrections

are shown in consecutive plots, where as a reference (black dashed line) respectively: (i) EW LO $\alpha(0)$ scheme (top-right), (ii) effective Born (LEP) (bottom-left) and, (iii) effective Born (LEP with improved norm.) (bottom-right), was used

Table 5 The difference ΔA_{FB} in forward-backward asymmetry calculated in the specified mass window. The $\cos \theta^{CS}$ is used to define forward and backward hemispheres. The EW weight is calculated from θ^* definition of the scattering angle

Corrections to A_{FB}	$89 < m_{ee} < 93$ GeV	$80 < m_{ee} < 100$ GeV
$A_{FB}(\text{EW corr. } m_W) - A_{FB}(\text{EW LO } \alpha(0))$	-0.02097	-0.02103
$A_{FB}(\text{EW corr. prop. } \chi(Z), \chi(\gamma)) - A_{FB}(\text{EW LO } \alpha(0))$	-0.02066	-0.02098
$A_{FB}(\text{EW/QCD FF no boxes}) - A_{FB}(\text{EW LO } \alpha(0))$	-0.03535	-0.03569
$A_{FB}(\text{EW/QCD FF with boxes}) - A_{FB}(\text{EW LO } \alpha(0))$	-0.03534	-0.03567
$A_{FB}(\text{LEP}) - A_{FB}(\text{EW/QCD FF with boxes})$	-0.00006	-0.00001
$A_{FB}(\text{LEP with improved norm.}) - A_{FB}(\text{EW/QCD FF with boxes})$	-0.00005	-0.00002

Table 6 The difference ΔA_{FB} in forward-backward asymmetry around Z-pole, $m_{ee} = 89\text{--}93$ GeV. The $\cos \theta^{CS}$ is used to define forward and backward hemispheres. The EW weight is calculated respectively from $\cos \theta^*$, $\cos \theta^{Mu\text{straal}}$ or $\cos \theta^{CS}$

Corrections to A_{FB} ($89 < m_{ee} < 93$ GeV)	$wt^{EW}(\cos \theta^*)$	$wt^{EW}(\cos \theta^{ML})$	$wt^{EW}(\cos \theta^{CS})$
$A_{FB}(\text{EW/QCD corr. to } m_W) - A_{FB}(\text{EW LO } \alpha(0))$	-0.02097	-0.02112	-0.02101
$A_{FB}(\text{EW/QCD corr. to } \chi(Z), \chi(\gamma)) - A_{FB}(\text{EW LO } \alpha(0))$	-0.02066	-0.02081	-0.02070
$A_{FB}(\text{EW/QCD FF no boxes}) - A_{FB}(\text{EW LO } \alpha(0))$	-0.03535	-0.03560	-0.03542
$A_{FB}(\text{EW/QCD FF with boxes}) - A_{FB}(\text{EW LO } \alpha(0))$	-0.03534	-0.03559	-0.03541
$A_{FB}(\text{LEP}) - A_{FB}(\text{EW/QCD FF with boxes})$	-0.00006	-0.00005	-0.00006
$A_{FB}(\text{LEP with improved norm.}) - A_{FB}(\text{EW/QCD FF with boxes})$	-0.00005	-0.00005	-0.00005

Fig. 6 Effective weak mixing angles $\sin^2 \theta_{eff}^f(s, t)$ as a function of m_{ee} and $\cos \theta = 0$, without (left-hand plot) and with (right-hand plot) box corrections. The $\mathcal{K}^f(s, t)$ form-factor calculated using Dizet

library and on-mass-shell $s_W^2 = 0.22352$ were used. Only the real part is shown, imaginary part of $I_f^2(s, t)$ is only about 10^{-4}

Table 7 From the Dizet library printout: effective weak mixing angles and $\alpha(M_Z^2)$. For details of ZPAR parameter matrix definition see technical documentation of KKMC interface and DIZET library itself [2, 11]

Parameter	Value	Description
$\alpha(M_Z^2)$	0.00775995	From Eq. (17)
$1/\alpha(M_Z^2)$	128.86674	
$ZPAR(6) - ZPAR(8)$	0.23176	$\sin^2 \theta_{eff}^\ell(M_Z^2) (\ell = e, \mu, \tau)$
$ZPAR(9)$	0.23165	$\sin^2 \theta_{eff}^{up}(M_Z^2)$
$ZPAR(10)$	0.23152	$\sin^2 \theta_{eff}^{down}(M_Z^2)$

Table 8 The effective weak mixing angles $\sin^2 \theta_{eff}^f$, for different mass windows with/without box corrections. The form-factor corrections are averaged with realistic line-shape and $\cos \theta$ distribution

Parameter [GeV]	$\sin^2 \theta_{eff}^\ell$	$\sin^2 \theta_{eff}^{up}$	$\sin^2 \theta_{eff}^{down}$
EW loops without box corrections			
$80 < m_{ee} < 100$	0.23171	0.23171	0.23146
$78 < m_{ee} < 82$	0.23179	0.23172	0.23159
$89 < m_{ee} < 93$	0.23170	0.23169	0.23147
$108 < m_{ee} < 112$	0.23168	0.23175	0.23137
EW loops with box corrections			
$80 < m_{ee} < 100$	0.23171	0.23171	0.23146
$78 < m_{ee} < 82$	0.23136	0.23167	0.23158
$89 < m_{ee} < 93$	0.23168	0.23169	0.23147
$108 < m_{ee} < 112$	0.23246	0.23174	0.23130

The results for ΔA_3 are collected in Table 9. The mass window $80 < m_{ee} < 100$ GeV and $p_T^{ee} < 30$ GeV are chosen. The estimation for ΔA_4 differ little if $\cos \theta^*$, $\cos \theta^{CS}$ or $\cos \theta^{Mustraal}$ is used for calculations of EW corrections. The ΔA_3 is non-zero, as it should be, only if the $\cos \theta^{Mustraal}$ is used in wt^{EW} calculation. For A_4 , multiplied by $\frac{8}{3}$ entries of Table 5 are good enough.

9 Summary

In this paper we have shown how the EW corrections for double-deconvoluted observables at LHC can be evaluated

using *Improved Born Approximation*. We have exploited a wealth of the LEP era results encapsulated in the Dizet library developed at that time. We have used that formalism to calculate and present numerically EW corrections for doubly-deconvoluted observables, such as Z-boson line-shape, forward-backward asymmetry A_{FB} , effective weak mixing angles or lepton direction angular coefficients.

We have followed largely discussions available in Dizet documentation. We have introduced the notion of the effective Born and explained how Monte Carlo events generated at NLO QCD can be transformed to reduced kinematics, of strong interaction lowest order, for the calculation of spin amplitudes $q\bar{q} \rightarrow Z/\gamma^* \rightarrow \ell\ell$. This could be achieved

Fig. 7 Top-left: the A_4 as function of m_{ee} . Overlaid are generated and EW corrected A_4 predictions. These results are barely distinguishable. The differences $\Delta A_4 = A_4 - A_4^{ref}$ due to gradually introduced EW

corrections are shown in consecutive plots, where as a reference A_4^{ref} (black dashed line) respectively (i) EW LO $\alpha(0)$ scheme (top-right), (ii) effective Born (LEP) (bottom-left) and (iii) effective Born (LEP with improved norm.) (bottom-right) was used

thanks to properties of spin amplitudes discussed in [5,6]. We explained how per-event weight wt^{EW} , can be build and used to attribute EW corrections to already generated events.

We have re-visited the notion of *Effective Born* with *LEP* (or *with LEP of improved norm.*) parametrisations where dominant parts of EW corrections are taken into account with a redefinition of coupling constants. We have evaluated how well it works for observables of the paper. The discussed approach for treating EW corrections for Drell-Yan process in pp collisions has been implemented in the Tauola/TauSpinner package [9,15] to be available starting from the forthcoming release.

Once the formalism was explained, numerical results of EW corrections to the Z-boson line-shape, forward-backward asymmetries, lepton angular coefficients were presented. Results were obtained using Dizet for calculating EW form-factors and Tauola/TauSpinner for calculating respective EW weights of *Improved Born Approximation* or *Effective Born* with *LEP* (or *with LEP improved norm.*) parametrisations.

The choice of the version of EW library was dictated by the compatibility with the KKMC Monte Carlo [2], the pro-

gram widely used at the LEP times. It relies on a published version of Dizet, thus suits the purposes of a reference point well. Also, omitted effects are rather small. In the future, the algorithm of TauSpinner can be useful to quantify the differences among distinct implementations of the electroweak sector.

The numerical studies with the updates to Dizet version 6.42 [12,21] and with other, sometimes unpublished electroweak codes are left for the future work. One should stress the necessity of such future numerical discussion and updates, in particular due to the photonic vacuum polarization, e.g. as provided in Refs. [34,35] but absent in the last published (or presently public) version of Dizet 6.42. This update is required already at LHC precision of Z-boson couplings measurements.

In many applications focused on challenges of strong interactions, electroweak corrections are receiving rather minimal attention and in particular Z boson fixed value width, or running only in proportion to the energy transfer, is used. This may be inappropriate for large s as found e.g. in [36]. TauSpinner can be used to evaluate numerical consequences of such approximation. Finally let us mention

Fig. 8 Top-left: the A_3 as function of m_{ee} . Overlaid are generated and EW corrected A_3 predictions. These results are barely distinguishable. The differences $\Delta A_3 = A_3 - A_3^{ref}$ due to gradually introduced EW corrections are shown in consecutive plots, where as a reference A_3^{ref}

(black dashed line) respectively (i) EW LO $\alpha(0)$ scheme (top-right), (ii) effective Born (*LEP*) (bottom-left) and (iii) effective Born (*LEP with improved norm.*) (bottom-right) was used. In this case, the EW weight is calculated with $\cos \theta^{Mu\text{straal}}$

Table 9 The ΔA_3 shift of the A_3 , due to EW corrections, averaged over $p_T^{ee} < 30$ GeV and $80 < m_{ee} < 100$ GeV ranges. The $\cos \theta^{CS}$ is used for angular polynomials but for the EW weight calculation $\cos \theta^*$, $\cos \theta^{Mu\text{straal}}$ or $\cos \theta^{CS}$ are used respectively

Corrections to A_3 ($p_T^{ee} < 30$ GeV)	$wt^{EW}(\cos \theta^*)$	$wt^{EW}(\cos \theta^{Mu\text{straal}})$	$wt^{EW}(\cos \theta^{CS})$
$A_3(\text{EW/QCD corr. to } m_W) - A_3(\text{EW LO } \alpha(0))$	-0.00060	-0.00321	-0.00060
$A_3(\text{EW/QCD corr. to } \chi(Z), \chi(\gamma)) - A_3(\text{EW LO } \alpha(0))$	-0.00061	-0.00322	-0.00061
$A_3(\text{EW/QCD FF no boxes}) - A_3(\text{EW LO } \alpha(0))$	-0.00103	-0.00546	-0.00102
$A_3(\text{EW/QCD FF with boxes}) - A_3(\text{EW LO } \alpha(0))$	-0.00103	-0.00545	-0.00102
$A_3(\text{LEP}) - A_3(\text{EW/QCD FF with boxes})$	0.00000	0.00000	0.00000
$A_3(\text{LEP with improved norm.}) - A_3(\text{EW/QCD FF with boxes})$	0.00000	0.00000	0.00000

that presented implementation of EW corrections as per-even weight, was already found useful for experimental measurements [16] at LHC and for discussions during recent workshops, see e.g. Ref. [37].

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Data Availability Statement This manuscript has associated data in a data repository. [Authors' comment: The code for presented algorithm is available as part of the code which can be downloaded from <http://taulapp.web.cern.ch/taulapp/> versions later than of Tauola release 1.1.6c (and of TauSpinner release 2.0.3).]

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Fig. 9 The A_4 variation due to shifts induced with the presented in Appendix B options; as a function of s_W^2 (left-hand side) and as a function of $\sin^2 \theta_{eff}^l$ (right-hand side). The “FF G_μ varied”, FF m_t varied” correspond to the case when form-factors were recalculated. Otherwise they were kept at nominal values

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A Comment on technical details of TauSpinner EW effects implementation

Although the framework of Tauola/TauSpinner package [9, 15] has been used for numerical results presented in this paper, the code is not yet available with the public release but only in the private distribution and only partly in development version [38] which updates itself daily from our work repository. Tests and some of the code developments need to be completed. Once we achieve confidence the official stable version of the code will become public at [38]. Let us nonetheless list main points of the implementation which was already used to obtain numerical results:

- Pre-tabulated EW corrections: form-factors, vacuum polarization corrections in form of 2D root histograms or alternatively ASCII files of the KMMC project [2] were used to assure modularity and to enable graphic tests.
- Functions to calculate $\cos \theta^*$, $\cos \theta^{Mu\text{straal}}$, $\cos \theta^{CS}$ from kinematics of outgoing final state (leptons and partons/jets) used for numerical results are already in part available in `TAUOLA/TauSpinner/examples/Dizet-example` directory. The README file of that directory is gradually filled with technical details.
- Routine to initialize parameters of the Born function is provided. The SUBROUTINE `INITWK` of `TAUOLA/src/tauolaFortranInterfaces/tauola_extras.f` has been copied and extended. It is available under the name `INITWKSDELTA`, with the following input:
 - G_μ, α, M_Z, s ,

Table 10 Dizet initialization parameters: masses and couplings

Parameter	Value	Description
M_Z	91.1876 GeV	Mass of Z boson
M_h	125.0 GeV	Mass of Higgs boson
m_t	173.0 GeV	Mass of top quark
$1/\alpha(0)$	137.0359895(61)	$\alpha_{QED}(0)$
G_μ	$1.166389(22) \cdot 10^{-5}$ GeV^{-2}	Fermi constant In μ -decay

- EW form-factors and vacuum polarization corrections,
- s_W^2 and parameters for couplings variations δ_{s2W}, δ_V , see Section B for details.
- To calculate $d\sigma_{Born}$ and the wt^{EW} the `t_bornew` function with flexible options for EW scheme and δ_{s2W}, δ_V , is prepared. It is used by TauSpinner library function `default_nonSM_born(ID, S, cost, H1, H2, key)` now.
- It is premature for complete documentation, but comments on the software used to obtain numerical results are in place.

B How to vary s_W^2 beyond the EW LO schemes.

In the discussed EW scheme ($\alpha(0), G_\mu, M_Z$), the s_W^2 is not directly available for fits. It is calculated from relation (16) of the Standard Model. One possibility to vary s_W^2 , but stay within Standard Model framework is to vary some other constants which impact s_W^2 . The candidates within Standard Model, which are also inputs to the Dizet library, are G_μ or m_t . From the simple estimates, to allow $\pm 100 \cdot 10^{-5}$ varia-

Table 11 Dizet initialization flags. Unmodified comments taken from the KKMC code

Internal flag	Default value	Optional values	Description
ibox	1	0,1	EW boxes on/off
lhyp	1	1,2,3	Jegerlehner/Eidelman, Jegerlehner(1988), Burkhardt et al.
lamt4	4	0,1,2,3,4	=4 the best, Degrassi/Gambino
lqcd	3	1,2,3	approx/fast/lep1, exact/Slow!/Bardin/, exact/fast/Kniehl
lmoms	1	0,1	=1 W mass recalculated
lmass	0	0,1	=1 test only, effective quark masses
lscre	0	0,1,2	Remainder terms
lalem	3	1,3 or 0,2,	for 1,3 DALH5 not input
lmask	0	0,1	=0: Quark masses everywhere; =1 Phys. threshold in the ph.sp.
lscal	0	0,1,2,3	Kniehl=1,2,3, Sirlin=4
lbarb	2	-1,0,1,2	Barbieri???
lftjr	1	0,1	FTJR corrections
lfacr	0	0,1,2,3	Expansion of δ_r ; =0 none; =3 fully, unrecorred.
lfact	0	0,1,2,3,4,5	Expansion of kappa; =0 none
lhigs	0	0,1	Leading Higgs contribution re-summation
lafmt	1	0,1	=0 for old ZF
lewlc	1	0,1	???
lczak	1	0,1	Czarnecki/Kuehn corrections
lhig2	1	0,1	Two-loop higgs corrections off,on
lale2	3	1,2,3	Two-loop constant corrections in δ_α
lgfer	2	0,1,2	QED corrections for fermi constant
lddzz	1	0,1	??? DD-ZZ game, internal flag

Table 12 Dizet recalculated quantities available for the TauSpinner use. For details of the ZPAR table see Refs. [2, 11]

Parameter	Value	Description
$\alpha_{QED}(M_Z^2)$	0.007759	Calculated from $\Delta\alpha_h^{(5)}(M_Z)$ by Dizet
$1/\alpha_{QED}(M_Z^2)$	128.882588	
$\alpha_s(M_Z^2)$	0.1250	Recalculated by Dizet
$\alpha_s(m_t^2)$	0.1134	Recalculated by Dizet
$ZPAR(1) = \delta r$	0.03694272	The loop corrections to G_μ
$ZPAR(2) = \delta r_{rem}$	0.01169749	The remainder contribution $O(\alpha)$
$ZPAR(3) = s_W^2$	0.22352	Weak mixing angle defined by weak masses
$ZPAR(4) = G_\mu$	$1.166370 \cdot 10^{-5}$	Muon decay constant
$ZPAR(6) - ZPAR(14)$	0.23176–0.23152	Effective weak mixing angles
$ZPAR(15) = \alpha_s(M_Z^2)$	0.12500	Recalculated by Dizet
$ZPAR(16) - ZPAR(30)$		QCD corrections

tion of s_W^2 , those parameter will have to be varied far beyond their experimental ambiguities.¹⁵

One can extend formulae for $\mathcal{A}^{Born+EW}$ (11) beyond the Standard Model too. Additional v-like contribution to Z-boson v_ℓ , v_f couplings can be introduced with δ_{S2W} or δ_V as presented later. Below few details and options on implementation into $\mathcal{A}^{Born+EW}$ amplitudes are given:

– optME = 1: introduce unspecified heavy particle coupling to the Z-boson, to modify fermions vector couplings

$$\begin{aligned}
 v_\ell &= (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot (s_W^2 + \delta_{S2W}) \cdot \mathcal{K}_\ell(s, t)) / \Delta, \\
 v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot (s_W^2 + \delta_{S2W}) \cdot \mathcal{K}_f(s, t)) / \Delta, \\
 vv_{\ell f} &= \frac{1}{v_\ell \cdot v_f} [(2 \cdot T_3^\ell)(2 \cdot T_3^f) \\
 &\quad - 4 \cdot q_\ell \cdot (s_W^2 + \delta_{S2W}) \cdot \mathcal{K}_f(s, t)(2 \cdot T_3^\ell)
 \end{aligned}$$

¹⁵ Range would be ± 10 GeV for m_t or $\pm 4 \cdot 10^{-8} GeV^{-2}$ for G_μ .

$$\begin{aligned}
& -4 \cdot q_f \cdot (s_W^2 + \delta_{S2W}) \cdot \mathcal{K}_\ell(s, t) (2 \cdot T_3^f) \\
& + (4 \cdot q_\ell \cdot s_W^2) (4 \cdot q_f \cdot s_W^2) \mathcal{K}_{\ell f}(s, t) \\
& + 2 \cdot (4 \cdot q_\ell) (4 \cdot q_f) \cdot s_W^2 \cdot \delta_{S2W} \Big] \frac{1}{\Delta^2} \quad (30)
\end{aligned}$$

but do not alter

$$\Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)} \quad (31)$$

or any other $\mathcal{A}^{Born+EW}$ (11) couplings or calculations of the EW form-factors.

- `optME = 2`: recalculate M_W for numerically modified m_t or G_μ and modify accordingly Standard Model $s_W^2 = 1 - M_Z^2/M_W^2$, for s_W^2 present in $\mathcal{A}^{Born+EW}$. The form-factors are (are not) recalculated.¹⁶ In total, 3 variants of this option were used for Fig. 9.
- `optME = 3`: similar as `optME = 1` but redefine directly fermions vector couplings with δ_V . We keep relative normalization (charge structure) of δ_V similar to δ_{S2W} , to facilitate comparisons. Then

$$\begin{aligned}
v_\ell &= (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot (s_W^2 \cdot \mathcal{K}_\ell(s, t) + \delta_V)) / \Delta, \\
v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot (s_W^2 \cdot \mathcal{K}_f(s, t) + \delta_V)) / \Delta, \\
vv_{\ell f} &= \frac{1}{v_\ell \cdot v_f} [(2 \cdot T_3^\ell) (2 \cdot T_3^f) \\
& - 4 \cdot q_\ell \cdot (s_W^2 \cdot \mathcal{K}_f(s, t) + \delta_V) (2 \cdot T_3^\ell) \\
& - 4 \cdot q_f \cdot (s_W^2 \cdot \mathcal{K}_\ell(s, t) + \delta_V) (2 \cdot T_3^f) \\
& + (4 \cdot q_\ell \cdot s_W^2) (4 \cdot q_f \cdot s_W^2) \mathcal{K}_{\ell f}(s, t) \\
& + 2 \cdot (4 \cdot q_\ell) (4 \cdot q_f) \cdot s_W^2 \cdot \delta_V] \frac{1}{\Delta^2}. \quad (32)
\end{aligned}$$

The δ_V shift is almost equivalent to δ_{S2W} shift, but affects couplings in a (s, t) independent manner.

Even though discussion of s_W^2 variation necessary for fits, is generally out of scope of the present paper and it can not be now exhausted, let us provide some numerical results to illustrate stability of the method.¹⁷ The variations for $A_4(M_Z)$ are presented in Figure 9: as a function of s_W^2 on the left-hand side plot and as a function of $\sin^2 \theta_{eff}^l$ on the right-hand side

plot. It is very reassuring, that all presented `optME` methods lead to the same slope of the $A_4(M_Z)$ as a function of $\sin^2 \theta_{eff}^l$. Very similar curve could be presented for A_{FB} , which would be scaled by $\frac{3}{8}$ with respect to A_4 only.

C Initialization of the `Dizet` library

There is a wealth of initialization constants and options available for `Dizet` library. The documentation of that program and of its interface for `KKMC`, explains options available for the `TauSpinner` users as well. Tables 10 and 11 recall available `Dizet` initialization, Table 12 lists calculated by `Dizet` quantities for the use in `TauSpinner` library.

In the present work, we have relied on the `Dizet` library version as installed in the `KKMC` Monte Carlo [8] and used at a time of LEP 1 in detector simulations. Already for the data analysis and in particular for final fits [1], further effects of minor, but non-negligible numerical impact were taken into account. Gradually, effects such as improved top contributions [39] or better photonic vacuum polarization [23], were taken into account. This has to be updated for `Dizet` library too.

Such update is of importance also for the `KKMC` project itself because of forthcoming applications for the Future Circular Collider or for LHC [40].

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¹⁶ The `optME = 1, 2`, if form-factors are not recalculated, formally differ by the term proportional to δ_{S2W}^2 and only in the expression for $vv_{\ell f}$. Change of input parameters G_μ or m_t as a source for s_W^2 variations in `optME = 2`, implies changes of the couplings and thus for consistency, recalculation of form-factors. All these options can be realized with the `Tauola/TauSpinner` package, of the development version.

¹⁷ For `optME=2`, the m_t (or G_μ) have been shifted to move s_W^2 by $\pm 100 \cdot 10^{-5}$. Then the form-factors were recalculated, or optionally kept at nominal values.

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