Open-string singlet interaction as pomeron in elastic pp , $p\bar{p}$ scattering

Piyabut Burikham^a, Daris Samart^b

High Energy Physics Theory Group, Department of Physics, Faculty of Science, Chulalongkorn University, Phyathai Rd., Bangkok 10330, Thailand

Received: 22 February 2019 / Accepted: 17 May 2019 / Published online: 28 May 2019
© The Author(s) 2019

Abstract We consider contributions of the open-string singlet (OSS) interaction to the proton–proton scattering. Modelling baryon and meson as instanton and open string in the effective QCD string model, the typical pomeron is identified with a massive spin-2 closed string. According to the open-closed string duality, the massive spin-2 closed string is accompanied by the open-string singlet interaction with universal coupling. The flat space 4-point open-string singlet amplitudes and cross sections are computed in the Regge limit. Including both the open and closed string pomerons, we fit the differential cross-section with experimental data of the proton–proton and proton–antiproton scattering from the CDF, E-710, D0, LHC-ATLAS and LHC-TOTEM collaborations. The fitting results show the universality of the string mass scale (or string tension) and consistency of the string model between the open and closed string parameters in describing the pp and $p\bar{p}$ scattering in the Regge limit. The fitting of the string model to experimental data gives the string scale $M_S \simeq 1.7\text{--}1.8$ GeV and the corresponding glueball pole mass $m_g \simeq 1.6\text{--}1.7$ GeV. This lies within experimental values of the observed 2^{++} , $I = 0$ “meson” masses.

1 Introduction

String theory first appeared as the physical interpretation of the Veneziano amplitude describing meson scatterings [1–3]. Tower of resonance states with increasing masses continuing indefinitely explains the zoo of hadronic particles found and at the same time quantitatively accounts for their high energy scattering in the so-called Regge limit (large center of mass energy and small transferred momentum). The string picture of strong interaction eventually fell out of

favour due to the success of the Quantum Chromodynamics (QCD) in the description of the Deep Inelastic scattering (DIS) between leptons and hadrons confirming existence of fractionally charged quarks. QCD with quark model can also describe all known hadronic states and predicts more exotic ones already observed [4]. Moreover, the strongest motivation of string theory as fundamental theory of quantum gravity, originated from the inevitable prediction of the massless spin-2 states that couple universally to other particles [5,6] as a result of the open-closed string duality [7], was ironically the crucial failure of the string theory as the fundamental framework of the strong interaction. Experimentally, there is no massless spin-2 hadronic states.

Despite the success of QCD in the perturbative regime, it fails to give accurate results on the diffractive scattering (mostly small momentum transfer t) and the total cross section of the high-energy hadronic scattering. Experiments found the increase of the total cross sections of the hadronic processes with respect to the scattering energy [8]. Regge theory provides explanation of the rise of the forward amplitude and the total cross section by proposing a trajectory of states with angular momenta $J = \alpha_0 + \alpha' t$ with $\alpha_0 \gtrsim 1$, the so-called pomeron trajectory. In the crossing channel, the trajectory relates the angular momentum of the resonance states with the mass square $J = \alpha_0 + \alpha' M^2$. The pomeron trajectory contains states with vacuum quantum number but the masses are unknown. From the experimental data fittings, it is very likely that the lowest mode of pomeron is the state with spin-2 [9]. In this Regge regime (at high energies and small t , notably the perturbative QCD on the other hand, is applicable for the high-energy scattering with large momentum transfer, i.e. large t), the string description as a flat space realization of the S-matrix formalism of the Regge theory is a better framework to address the scattering processes.

Lattice QCD actually predicts existence of massive spin 0, 1 and 2 colour-singlet glueball states [10]. The conformal symmetry is dynamically broken at low energies by

^a e-mail: piyabut@gmail.com

^b e-mail: dsamart82@gmail.com

the quantum fluctuations and mass gap is generated. Naturally, the spin-2 glueball state is the best candidate of the pomeron with other spin-0,1 states responsible for the daughter trajectories. Revival of the string description of the strong interaction came with the proposal of the AdS/CFT correspondence [11] where it was argued that strongly interacting gauge theory living on the AdS boundary is dual to the weakly coupled gravity in the AdS bulk. Notably, the gauge theory on the boundary does not have gravitons but the correlation $\langle T_{\mu\nu} T^{\mu\nu} \rangle$ can be generated via the exchanges of (massive) gravitons in the bulk. In this way, the problem of existence of massless spin-2 particle in the boundary theory is resolved. Meson as the quark–antiquark bound state can be represented by an open string hanging in the bulk whose ends are living on the boundary. Baryon can be modelled as the open-string instanton with three open strings joining at a baryon vertex with another end of each string living on the boundary [12,13]. Generalized multiquark states can naturally be described as systems of connecting open strings [14]. Scattering of mesons and baryons thus can be thought of as scattering of open strings living close to the AdS boundary. For high energy scattering, each quark effectively appears as open string hanging all the way down into the bulk with only one end living on the boundary and the scattering can be described by the perturbative QCD.

In this unifying framework of “holographic QCD” (or “AdS/QCD”), apart from the missing details of mass-gap generating mechanism, all of the strong interaction processes could be addressed at least in principle (or if ones do not want to take perturbative QCD as a limit of the holographic gauge theory in the string setup, ones can think of the two theories as complementary to one another in describing the strong interaction). The glueball masses or mass gap can be introduced by inserting a cutoff in the AdS coordinate, breaking conformal symmetry of the gauge theory on the boundary. The exchange of glueball as pomeron on the boundary is dual to the exchange of massive graviton (closed string) in the bulk. In this way, glueball is a closed string pomeron. On the other hand, the open-closed string duality implies existence of open-string singlet interaction [15] between gauge singlet states. The graviton exchange diagram of the $2 \rightarrow 2$ process is the nonplanar annulus worldsheet with 2 vertex operators inserted at each boundary [7]. This diagram is dual to the sum of one-loop open-string diagrams containing two twists. Since even the singlet states couple to closed-string graviton via the energy-momentum tensor, they must also inevitably couple through open-string singlet. Interestingly, the 4-point amplitude for the open-string singlets are non-vanishing even though the 3-point is [15–21]. Moreover, the duality of the open-string singlet amplitude naturally eradicates the zeroth mode (as well as the even modes) of the string resonances and its Regge limit behaviour fits excellently with the pomeron as we will see in subsequent sections.

A study of the pp , $p\bar{p}$ scattering in the Regge limit has been investigated in Ref. [22] by using the AdS/QCD framework and its consequences for other scattering processes [23–29]. Authors in Ref. [22] have used massive graviton (closed-string) as exchange particle of the scattering at the tree-level and identified the massive graviton as tensor pomeron or glueball. Although results of the model parameters from AdS/QCD calculations in Refs. [22,23] are in good agreement with the fitting results from the experimental data. However, one must also take into account the effect of open-string singlet interaction in the scattering instead of using only the closed-string exchange. In this work, we investigate the inclusion of open-string pomeron contribution in the pp , $p\bar{p}$ elastic scattering in the Regge regime. In Sect. 2, we calculate the scattering amplitude of the OSS interaction for proton–proton scattering in the Regge limit. Fitting results of the model parameters will be determined in Sect. 3. Section 4 discusses the results and concludes the work.

2 Formalism

2.1 Scattering amplitude from the open-string singlet interaction

We start with the OSS amplitude for elastic proton–proton scattering. In holographic QCD, proton can be described by three open strings joining at a baryon vertex and hanging close to the boundary of an asymptotically AdS space. We can imagine when two protons collide, a string from each baryon vertex will scatter off one another. Instead of considering detailed scattering dynamics of the strings and smeared baryon vertices in the curved AdS space and also motivated by success of the dual resonance model in the Regge regime, we will assume the scattering can be described by simple flat-space open-string amplitude. The general form of the four-fermion interaction of the open string can be written as [18]

$$\begin{aligned} \mathcal{A}_{\text{string}} = & -g_S (t \bar{\psi}_2 \gamma_\mu \psi_1 \bar{\psi}_4 \gamma^\mu \psi_3 - s \bar{\psi}_4 \gamma_\mu \psi_1 \bar{\psi}_2 \gamma^\mu \psi_3) \\ & \times \left(\frac{S(s, t)}{s t} T(1234) \right. \\ & \left. + (1 \leftrightarrow 4, s \leftrightarrow u) + (1 \leftrightarrow 2, t \leftrightarrow u) \right), \end{aligned} \quad (1)$$

where g_S is the dimensionless string coupling and the particles (fermions, ψ_i) in the two-body scattering are labeled by $12 \rightarrow 34$. The function $S(x, y)$ is the Veneziano-like amplitude defined by [30]

$$S(x, y) = \frac{\Gamma(1 - \alpha' x) \Gamma(1 - \alpha' y)}{\Gamma(1 - \alpha' x - \alpha' y)}, \quad (2)$$

where $\alpha' \equiv M_S^{-2}$ and M_S is the string mass scale. The $T(ijkl)$ are the Chan–Paton factors of the matrices $\lambda^a \in U(n)$ and they are defined by

$$T(1234) = \text{tr}(\lambda^1 \lambda^2 \lambda^3 \lambda^4) + \text{tr}(\lambda^4 \lambda^3 \lambda^2 \lambda^1), \quad (3)$$

with the normalization $\text{tr}(\lambda^a \lambda^b) = \delta^{ab}$. The Mandelstam variables are defined as

$$s = -(p_1 + p_2)^2, \quad t = -(p_1 + p_4)^2, \quad u = -(p_1 + p_3)^2, \quad (4)$$

where all momenta are directed inward. As shown in Refs. [15, 16], the stringy gauge singlet or the OSS interaction occurs between all gauge singlet particles in the $U(n)$ group that give $T(1234) = T(1324) = T(1243) \equiv T$. One then can rewrite the four-fermion amplitude for the OSS as

$$\mathcal{A}_{\text{OSS}} = -g_S T \left(t \bar{\psi}_2 \gamma_\mu \psi_1 \bar{\psi}_4 \gamma^\mu \psi_3 - s \bar{\psi}_4 \gamma_\mu \psi_1 \bar{\psi}_2 \gamma^\mu \psi_3 \right) \times \frac{1}{s t u} f(s, t, u), \quad (5)$$

where the $f(s, t, u)$ function is

$$f(s, t, u) = u S(s, t) + t S(s, u) + s S(u, t). \quad (6)$$

Applying the Fierz transformation [31], we obtain

$$\begin{aligned} 4 \bar{\psi}_i \gamma_\mu \psi_j \bar{\psi}_k \gamma^\mu \psi_l &= 4 \bar{\psi}_k \psi_j \bar{\psi}_i \psi_l - 2 \bar{\psi}_k \gamma_\mu \gamma_5 \psi_j \bar{\psi}_i \gamma^\mu \gamma_5 \psi_l \\ &\quad - 2 \bar{\psi}_k \gamma_\mu \psi_j \bar{\psi}_i \gamma^\mu \psi_l \\ &\quad - 4 \bar{\psi}_k \gamma_5 \psi_j \bar{\psi}_i \gamma_5 \psi_l. \end{aligned} \quad (7)$$

Then, the $\mathcal{A}_{\text{OSS}}$ amplitude reads,

$$\begin{aligned} \mathcal{A}_{\text{OSS}} &= -\frac{1}{4} g_S T \left[t \left(4 \bar{\psi}_4 \psi_1 \bar{\psi}_3 \psi_2 \right. \right. \\ &\quad - 2 \bar{\psi}_4 \gamma_\mu \gamma_5 \psi_1 \bar{\psi}_3 \gamma^\mu \gamma_5 \psi_2 \\ &\quad - 2 \bar{\psi}_4 \gamma_\mu \psi_1 \bar{\psi}_3 \gamma^\mu \psi_2 - 4 \bar{\psi}_4 \gamma_5 \psi_1 \bar{\psi}_3 \gamma_5 \psi_2 \Big) \\ &\quad - u \left(4 \bar{\psi}_3 \psi_1 \bar{\psi}_4 \psi_2 \right. \\ &\quad - 2 \bar{\psi}_3 \gamma_\mu \gamma_5 \psi_1 \bar{\psi}_4 \gamma^\mu \gamma_5 \psi_2 - 2 \bar{\psi}_3 \gamma_\mu \psi_1 \bar{\psi}_4 \gamma^\mu \psi_2 \\ &\quad \left. \left. - 4 \bar{\psi}_3 \gamma_5 \psi_1 \bar{\psi}_4 \gamma_5 \psi_2 \right) \right] \frac{1}{s t u} f(s, t, u). \end{aligned} \quad (8)$$

Note that the amplitude is naturally Reggeized by construction. To see the Regge behaviour in the limit s large and fixed small t , we recall some relevant properties of the gamma function,

$$\Gamma(1-x) = \frac{\pi}{\Gamma(x) \sin(\pi x)}, \quad (9)$$

$$\frac{\Gamma(x+a)}{\Gamma(x+b)} \stackrel{\text{large } x}{\simeq} x^{a-b} \left(1 + \mathcal{O}\left(\frac{1}{x}\right) \right). \quad (10)$$

In case of $|s| \rightarrow \infty$ along the complex plane on the real axis of s , we will encounter an infinite number of poles in the $S(s, t)$ function. To avoid singularities, we consider the

imaginary part of the s variable by adding $i\epsilon$ to skip the poles on the real axis, this gives

$$S(s, t) \stackrel{\text{large } s}{\simeq} \Gamma(1-\alpha't) e^{-i\pi\alpha't} (\alpha's)^{\alpha't}. \quad (11)$$

By using the same trick for $S(t, u)$, we obtain

$$S(t, u) \stackrel{\text{large } s}{\simeq} \Gamma(1-\alpha't) (\alpha's)^{\alpha't}. \quad (12)$$

Lastly, the $S(s, u)$ function in the Regge limit reads,

$$S(s, u) \stackrel{\text{large } s}{\simeq} 0. \quad (13)$$

This term rapidly vanishes for $s \rightarrow \infty$ and there is no pole in the t channel. We refer detailed derivation of the Veneziano amplitude in the Regge limit in [32–34]. The amplitude function $f(s, t, u)$ in the Regge limit is then

$$\begin{aligned} f(s, t, u) &= u S(s, t) + t S(s, u) + s S(u, t) \\ &\stackrel{\text{large } s}{\simeq} \frac{1}{\alpha'} \Gamma(1-\alpha't) \left(1 - e^{-i\pi\alpha't} \right) (\alpha's)^{\alpha't+1}, \end{aligned} \quad (14)$$

where $u \simeq -s$ has been used in the Regge limit. The stringy s, t, u symmetric function in the amplitude becomes

$$\frac{f(s, t, u)}{s t u} \rightarrow -\frac{\alpha'}{t} \Gamma(1-\alpha't) \left(1 - e^{-i\pi\alpha't} \right) (\alpha's)^{\alpha't-1}. \quad (15)$$

Finally, the OSS amplitude of proton–proton scattering, $\mathcal{A}_{\text{OSS}}$ in the Regge limit is given by

$$\begin{aligned} \mathcal{A}_{\text{OSS}} &= \frac{1}{4} g_S T \frac{\alpha'}{t} \Gamma(1-\alpha't) \left(1 - e^{-i\pi\alpha't} \right) (\alpha's)^{\alpha't-1} \\ &\quad \times \left[t \left(4 \bar{\psi}_4 \psi_1 \bar{\psi}_3 \psi_2 - 2 \bar{\psi}_4 \gamma_\mu \gamma_5 \psi_1 \bar{\psi}_3 \gamma^\mu \gamma_5 \psi_2 \right. \right. \\ &\quad - 2 \bar{\psi}_4 \gamma_\mu \psi_1 \bar{\psi}_3 \gamma^\mu \psi_2 - 4 \bar{\psi}_4 \gamma_5 \psi_1 \bar{\psi}_3 \gamma_5 \psi_2 \Big) \\ &\quad \left. - u (3 \leftrightarrow 4) \right]. \end{aligned} \quad (16)$$

In $pp \rightarrow pp$ elastic process at high energies, the t -channel reggeon-pomeron exchange is dominant. To take into account effects of the internal structure of the proton, we introduce the dipole form-factors, $A(t)$ [22, 35] phenomenologically to the vector–vector amplitude and the effective proton–proton–Reggeon vertex in the following form

$$A(t) = \left(\frac{1}{1-t/\Lambda^2} \right)^2. \quad (17)$$

The Λ parameter is the IR cutoff for the amplitude and we will treat it as a free parameter in this work. The final form of OSS amplitude in the Regge limit reads,

$$\begin{aligned} \mathcal{A}_{\text{OSS}} &= \frac{1}{4} g_S T \frac{\alpha'}{t} \Gamma(1-\alpha't) \left(1 - e^{-i\pi\alpha't} \right) (\alpha's)^{\alpha't-1} \\ &\quad \times \left[t \left(4 \bar{\psi}_4 \psi_1 \bar{\psi}_3 \psi_2 - 2 \bar{\psi}_4 \gamma_\mu \gamma_5 \psi_1 \bar{\psi}_3 \gamma^\mu \gamma_5 \psi_2 \right. \right. \end{aligned}$$

$$-2\bar{\psi}_4\gamma_\mu\psi_1\bar{\psi}_3\gamma^\mu\psi_2-4\bar{\psi}_4\gamma_5\psi_1\bar{\psi}_3\gamma_5\psi_2) \\ -u(3\leftrightarrow 4)\left[\left(\frac{1}{1-t/\Lambda^2}\right)^4\right]. \quad (18)$$

There are three free parameters in the amplitude of the OSS interaction for proton–proton scattering; the open-string coupling (g_S), the string mass scale ($M_S = 1/\sqrt{\alpha'}$) and the dipole IR-cutoff (Λ). As shown in Appendix 1, the OSS cross sections of the pp and $p\bar{p}$ collisions are identical. Therefore we will evaluate these parameters by fitting with the pp and $p\bar{p}$ scattering data from several collaborations in subsequent sections. To prepare observable for determining the parameters of the model from the experimental data, we will calculate the differential cross-section as function of t in the next section.

2.2 Differential cross-section

In this section, we calculate the absolute square of the amplitude $|\mathcal{A}_{\text{OSS}}|^2$ analytically by using FeynCalc [36,37] package of MATHEMATICA. The unpolarized spin sum of the amplitude, $|\mathcal{A}_{\text{OSS}}|^2$ is given by,

$$\sum_{\text{spin}} |\mathcal{A}_{\text{OSS}}|^2 = 8g_S^2 T^2 \frac{\alpha'^2}{t^2} \left| 1 - e^{-i\pi\alpha't} \right|^2 \\ \times \Gamma^2(1 - \alpha't) (\alpha's)^{2\alpha't-2} A^4(t) \\ \times \left(8m^4(7s^2 + 7st + 5t^2) - 8m^2s^3 \right. \\ \left. + 2s^4 + 4s^3t + 2s^2t^2 + t^4 \right), \quad (19)$$

where m is proton mass. In the $s \gg t$, m limit, the s^4 term dominates in the last bracket and we have

$$\frac{1}{4} \sum_{\text{spin}} |\mathcal{A}_{\text{OSS}}|^2 \stackrel{s \gg t}{\simeq} 8g_S^2 T^2 (1 - \cos(\pi\alpha't)) \\ \times \left(\frac{\Gamma(1 - \alpha't)}{\alpha't} \right)^2 (\alpha's)^{2\alpha't+2} A^4(t). \quad (20)$$

The differential cross-section $d\sigma/dt$ is given by

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} \left(\frac{1}{4} \sum_{\text{spin}} |\mathcal{A}_{\text{OSS}}|^2 \right) \\ = \frac{1}{2\pi} g_S^2 T^2 \alpha'^2 (1 - \cos(\pi\alpha't)) \\ \times \left(\frac{\Gamma(1 - \alpha't)}{\alpha't} \right)^2 (\alpha's)^{2\alpha't} A^4(t). \quad (21)$$

Finally, one obtains the differential cross-section formula of the OSS model for pp , $p\bar{p}$ scattering and we will use the formula in Eq. (21) to fit with the experimental data.

3 Fitting model parameters with experimental data

3.1 Fitting the parameters

We will use the experimental data from the Measurement of Elastic and Total Cross Sections in pp and $p\bar{p}$ interactions from The Durham Hep database [38]. The data for the first fitting is from the E-710 collaboration at $\sqrt{s} = 1800$ GeV [39]. The fitting to the data at other scattering energies will be presented in subsequent section.

To convert to correct units, we multiply $(0.3894)^2$ to the right-hand side of Eq. (21) since $1 \text{ GeV}^{-2} = 0.3894 \text{ mbarn}$ and all of phase convention from [4] is implied. We use input values of the other parameters in Eq. (21) as

$$m = 0.938 \text{ GeV}, \quad \sqrt{s} = 1800 \text{ GeV}, \quad T = 1. \quad (22)$$

Fitting the data with (21) gives:

$$g_S = 88.316, \quad M_S = 1.351 \text{ GeV}, \quad \Lambda = 6.570 \text{ GeV}. \quad (23)$$

In addition, if we perform the fitting by dropping the form factor i.e. $A(t) = 1$. The fitting results are:

$$g_S = 87.400, \quad M_S = 1.344 \text{ GeV}. \quad (24)$$

The plot of the best-fit parameters (23) is shown in Fig. 1. We found that the fitting plots with and without the form-factor are almost identical. We can examine this situation by considering the logarithm of the differential cross section in Eq. (21). Considering the series expansion of Eq. (21) and keeping t at leading order, one gets

$$\log\left(\frac{d\sigma}{dt}\right) = \log\left(\frac{1}{2\pi} g_S^2 T^2 \alpha'^2 (1 - \cos(\pi\alpha't)) \right. \\ \left. \times \left(\frac{\Gamma(1 - \alpha't)}{\alpha't} \right)^2 (\alpha's)^{2\alpha't} A^4(t) \right) \\ \approx m_1 t + C_1, \quad (25)$$

Fig. 1 A log-linear plot of the differential cross section at $\sqrt{s} = 1800$ GeV from the E-710 collaboration, as a function of $-t$ (in GeV^2). Dotted line is the experimental data. Red line is Eq. (21) with $g_S = 88.316$, $M_S = 1/\sqrt{\alpha'} = 1.351 \text{ GeV}$, $\Lambda = 6.570 \text{ GeV}$ and input parameters in Eq. (22)

where

$$m_1 = 2\alpha' \log(\alpha' s) + 2\gamma_E \alpha' + \frac{8}{\Lambda^2}, \quad (26)$$

$$C_1 = \log\left(\frac{\pi}{4} g_s^2 T^2 \alpha'^2\right), \quad (27)$$

and $\gamma_E \simeq 0.5772$ is the Euler's constant. Here m_1 and C_1 are the slope and vertical intercept of the log-linear plot respectively. In the Regge limit i.e. large s , the slope m_1 can be approximated for large $\Lambda \gg 1$ as

$$m_1 \approx 2\alpha' \log(\alpha' s), \quad (28)$$

mostly insensitive to Λ . However, for small $\Lambda \simeq 1$ GeV, the slope is affected significantly from the term $8/\Lambda^2$ as will be shown in the next section.

3.2 Universality of the string mass scale in the OSS interaction

In the last section, we have performed the fitting of elastic proton–proton scattering using OSS interaction in the Regge limit. Next, we would like to see if the string mass scale M_S is universal for other sets of the experimental data at high energies. We will use data from the LHC-ATLAS [40], D0 [41] and CDF [42] collaborations at $\sqrt{s} = 7000, 1960, 1800$ and 546 GeV, respectively.

In the following, the input parameters are fixed to

$$m = 0.938 \text{ GeV}, \quad \Lambda = 1.17 \text{ GeV}, \quad T = 1, \quad (29)$$

and consider scattering at $\sqrt{s} = 7000, 1960, 1800$ and 546 GeV. The value $\Lambda = 1.17$ GeV is estimated by using the Skyrme model [43] and it is a good approximation up to $|t| < 0.8 \text{ GeV}^2$ [22]. More importantly, the data will be fit in the log-linear regime i.e., only up to $-t \simeq 0.6 \text{ GeV}^2$ for $\sqrt{s} = 1960$ GeV as shown in Fig. 2.

The fitting is performed in two cases; with the dipole form-factor $A(t) = 1/(1 - t/\Lambda^2)^2$ and without form-factor i.e., $A(t) = 1$.

With the dipole form-factor, the fitting results are given by

$$\begin{aligned} g_s &= 156.887, & M_S &= 1.576 \text{ GeV}, & \sqrt{s} &= 7000 \text{ GeV}, \\ g_s &= 138.728, & M_S &= 1.576 \text{ GeV}, & \sqrt{s} &= 1960 \text{ GeV}, \\ g_s &= 125.535, & M_S &= 1.585 \text{ GeV}, & \sqrt{s} &= 1800 \text{ GeV}, \\ g_s &= 100.867, & M_S &= 1.564 \text{ GeV}, & \sqrt{s} &= 546 \text{ GeV}. \end{aligned} \quad (30)$$

Without form-factor, the fitting results are given by

$$\begin{aligned} g_s &= 114.452, & M_S &= 1.355 \text{ GeV}, & \sqrt{s} &= 7000 \text{ GeV}, \\ g_s &= 90.441, & M_S &= 1.357 \text{ GeV}, & \sqrt{s} &= 1960 \text{ GeV}, \\ g_s &= 87.400, & M_S &= 1.344 \text{ GeV}, & \sqrt{s} &= 1800 \text{ GeV}, \\ g_s &= 66.574, & M_S &= 1.272 \text{ GeV}, & \sqrt{s} &= 546 \text{ GeV}. \end{aligned} \quad (31)$$

According to the results in both cases, the values of the string mass, M_S are almost identical within each case. In contrast to the previous section where both cases give roughly the same fitting values of g_s and M_S , the fitting values of M_S for both cases are different by roughly 15% with $\Lambda = 1.17$ GeV. When $\Lambda \lesssim 1$, the term $8/\Lambda^2$ in Eq. (26) becomes larger and contributes more to the slope of the differential cross-section resulting in different fitting values. Without form factor, smaller M_S (larger α') is required to compensate for the contribution from the Λ .

The fitting results demonstrate the universality of the string mass, M_S for the OSS interaction in the elastic pp , $p\bar{p}$ scattering in the Regge limit (the low energy scattering e.g. at $\sqrt{s} = 546$ GeV gives a slightly different value). While the slope of the log-linear plot is determined by M_S and s , the vertical intercept is determined also by strength of the coupling $g_s^2 T^2$ in addition to the M_S (from Eqs. (26) and (27)). The fitting values of the string coupling are alarmingly large and non-perturbative. Using holographic model we could possibly attribute a fraction of the large value of g_s to the warp factor and integral of the overlapping wave function in the holographic coordinate in order to keep the “bare” value in the perturbative regime. However, in the next

Fig. 2 A log-linear plot of the differential cross section as a function of $-t$ (in GeV^2) at $\sqrt{s} = 13000, 8000, 7000, 2760, 1960, 1800$ and 546 GeV from the LHC-TOTEM [46–48], LHC-ATLAS [40], D0 [41], E-710 [39] and CDF [42] collaborations

Section it turns out that once the closed string pomeron is included, the fitting values of the coupling reduce to fall in the perturbative regime and almost unchanged with respect to the collision energies, implying that both open and closed string pomerons must coexist in a consistent framework.

3.3 Inclusion of the closed-string interaction

A fascinating aspect of the string model is the open-closed string duality that connects loop diagrams of open string to the existence of closed string diagram by the worldsheet duality. It is thus inevitable to include t -channel closed string diagram with vacuum quantum number together with the contributions from the open-string singlet.

The closed-string pomeron contribution to the pp , $p\bar{p}$ scattering has been explored in Ref. [22]. Massive graviton (closed-string) is coupled to the stress tensor of the protons (fermions) and identified as the spin-2 glueball state leading to the pomeron Regge trajectory. The holographic Sakai-Sugimoto (SS) model [44] is used to calculate the relevant parameters and compared with fitting values from the experimental data. The differential cross-section of the closed-string pomeron contribution from Ref. [22] is given by

$$\frac{d\sigma}{dt} = \frac{\lambda^4}{16\pi} \left(\frac{\Gamma(-\chi)\Gamma(1-\alpha_c(t)/2)}{\Gamma(\alpha_c(t)/2-1-\chi)} \right)^2 \times \left(\frac{\alpha'_c s}{2} \right)^{2\alpha_c(t)-2} A^4(t), \quad (32)$$

where λ is the (graviton) glueball–proton–proton coupling constant which has dimension $[\text{mass}]^{-1}$ and

$$\chi = 2\alpha'_c m^2 + 3\alpha_c(0)/2 - 3, \quad \alpha_c(t) = \alpha_c(0) + \alpha'_c t, \\ A(t) = \left(\frac{1}{1-t/\Lambda^2} \right)^2. \quad (33)$$

Here m is proton mass and Λ is the dipole mass. In addition, the glueball mass, m_g is defined from the Regge trajectory ($J = \alpha_c(0) + \alpha'_c t$) by,

$$m_g^2 = \frac{2 - \alpha_c(0)}{\alpha'_c}, \quad (34)$$

for spin $J = 2$ and at the pole $t = m_g^2$. There are four free parameters in this model, λ , $\alpha_c(0)$, α'_c and Λ . By using the SS-model, one finds [23]

$$\lambda = 9.02 \text{ GeV}^{-1}, \quad m_g = 1.49 \text{ GeV}, \quad \Lambda = 1.17 \text{ GeV}. \quad (35)$$

We note that $\Lambda = 1.17 \text{ GeV}$ is estimated from the four-dimensional Skyrme model [43]. By using formula in Eq. (32) and fitting with the E-710 collaboration scattering data at $\sqrt{s} = 1800 \text{ GeV}$, we obtain,

$$\lambda = 8.62 \text{ GeV}^{-1}, \quad \alpha_c(0) = 1.074,$$

$$\alpha'_c = 0.286 \text{ GeV}^{-2}, \quad \Lambda = 0.970 \text{ GeV}. \quad (36)$$

The fitting values of λ and other parameters are consistent with the results of the latest version of Ref. [22] and Ref. [23]. The value of $\alpha_c(0) \sim 1.06 - 1.08$ is also required in order to obtain a slowly increasing total cross section observed [8, 45]. Next, we consider the log-linear profile of the differential cross-section from the closed-string interaction. One finds,

$$\log \left(\frac{d\sigma}{dt} \right) = \log \left(\frac{\lambda^4}{16\pi} \left(\frac{\Gamma(-\chi)\Gamma(1-\alpha_c(t)/2)}{\Gamma(\alpha_c(t)/2-1-\chi)} \right)^2 \times \left(\frac{\alpha'_c s}{2} \right)^{2\alpha_c(t)-2} A^4(t) \right) \\ \approx m_2 t + C_2, \quad (37)$$

where

$$m_2 = 2\alpha'_c \log \left(\frac{\alpha'_c s}{2} \right) + \frac{8}{\Lambda^2} - \alpha'_c \psi^{(0)}(1 - \alpha_c(0)/2) \\ - \alpha'_c \psi^{(0)}(\alpha_c(0)/2 - \chi - 1), \quad (38)$$

$$C_2 = 2(\alpha_c(0) - 1) \log \left(\frac{\alpha'_c s}{2} \right) \\ + 2 \log \left(\frac{\lambda^2}{4\sqrt{\pi}} \frac{\Gamma(-\chi)\Gamma(1-\alpha_c(0)/2)}{\Gamma(\alpha_c(0)/2-\chi-1)} \right) \quad (39)$$

where $\psi^{(0)}(x) \equiv \frac{1}{\Gamma(x)} \frac{d\Gamma(x)}{dx}$ is the polygamma function. Once again, the slope of log-linear of differential cross-section in the Regge limit for $\Lambda \gg 1 \text{ GeV}$ is given by

$$m_2 \approx 2\alpha'_c \log \left(\frac{\alpha'_c s}{2} \right). \quad (40)$$

Similar to the OSS amplitude case, only α'_c and s determine the slope of the logarithmic plot for large Λ and yet the effect of $\Lambda \simeq 1 \text{ GeV}$ cannot be neglected at the collision energies under consideration.

Although, as mentioned in Ref. [22], the closed-string (pomeron) interaction gives quite good agreement between the SS-model predictions and fitting results from experimental data but the closed-string contribution is not the only pomeron in the pp , $p\bar{p}$ elastic scattering. As demonstrated in Ref. [16] in the stringy toy model of $U(2)$ QED and $U(4)$ QCD and more generically in Refs. [15, 17], open-string singlet interaction in the $2 \rightarrow 2$ process also contribute (more dominantly since the OSS leading diagram is proportional to g_S while the closed string diagram is proportional to g_S^2) as a gauge-singlet interaction, i.e., as a pomeron. Reference [16] estimates the graviton–proton–proton coupling parameter λ in the order

$$\frac{\lambda^2}{m_g^2} \simeq \frac{\pi^2}{32} \frac{g_S^2}{M_S^4}, \quad (41)$$

i.e., the λ is proportional to the string coupling g_S . To extend and improve the scattering model on the theoretical side, one

has to take both the open-string and closed string contributions into account of the scattering process. The central result of this work is to consider the pp , $p\bar{p}$ elastic scattering and demonstrate the validity of the model. In the following, we recall amplitudes from both of OSS from Eq. (18) and closed string interactions in the Regge limit, they read,

$$\begin{aligned} \mathcal{A}_{\text{OSS}} &= \frac{1}{4} g_S T \frac{\alpha'}{t} \Gamma(1 - \alpha' t) \left(1 - e^{-i\pi\alpha' t}\right) (\alpha' s)^{\alpha' t - 1} \\ &\quad \times \left[t \left(4 \bar{u}(p_4) \bar{u}(p_1) \bar{u}(p_3) u(p_2) \right. \right. \\ &\quad \left. \left. - 2 \bar{u}(p_4) \gamma_\mu \gamma_5 u(p_1) \bar{u}(p_3) \gamma^\mu \gamma_5 u(p_2) \right. \right. \\ &\quad \left. \left. - 2 \bar{u}(p_4) \gamma_\mu u(p_1) \bar{u}(p_3) \gamma^\mu u(p_2) \right. \right. \\ &\quad \left. \left. - 4 \bar{u}(p_4) \gamma_5 u(p_1) \bar{u}(p_3) \gamma_5 u(p_2) \right) \right. \\ &\quad \left. - u(3 \leftrightarrow 4) \right] A^2(t) \\ \mathcal{A}_{\text{closed}} &= \frac{1}{8} \lambda^2 \frac{\Gamma[-\chi] \Gamma[1 - \alpha_c(t)/2]}{\Gamma[\alpha_c(t)/2 - 1 - \chi]} \frac{\alpha'_c}{2} \\ &\quad \times e^{-i\pi\alpha_c(t)/2} \left(\frac{\alpha'_c s}{2} \right)^{\alpha_c(t) - 2} \\ &\quad \times \left[2 s \bar{u}(p_4) \gamma_\mu u(p_1) \bar{u}(p_3) \gamma^\mu u(p_2) \right. \\ &\quad \left. - 4 p_1^\mu p_2^\nu \bar{u}(p_4) \gamma_\nu u(p_1) \bar{u}(p_3) \gamma_\mu u(p_2) \right] A^2(t), \end{aligned} \quad (42)$$

where the $u(p)$ is standard positive energy Dirac spinor with normalization $\bar{u}(p) u(p) = 2m$ and the $\mathcal{A}_{\text{closed}}$ is the closed string amplitude consistent with Ref. [22]. The absolute square of the total amplitude, i.e. $\mathcal{A}_{\text{tot}} = \mathcal{A}_{\text{OSS}} + \mathcal{A}_{\text{closed}}$ at the leading order terms of s is given by

$$\begin{aligned} \sum_{\text{spin}} |\mathcal{A}_{\text{tot}}|^2 &= \sum_{\text{spin}} |\mathcal{A}_{\text{OSS}} + \mathcal{A}_{\text{closed}}|^2 \\ &= 32 g_S^2 T^2 (1 - \cos(\pi\alpha' t)) \\ &\quad \times \left(\frac{\Gamma(1 - \alpha' t)}{\alpha' t} \right)^2 (\alpha' s)^{2\alpha' t + 2} A^4(t) \\ &\quad + 4 s^4 \lambda^4 \left(\frac{\Gamma(-\chi) \Gamma(1 - \alpha_c(t)/2)}{\Gamma(\alpha_c(t)/2 - 1 - \chi)} \right)^2 \\ &\quad \times \frac{\alpha_c'^2}{4} \left(\frac{\alpha'_c s}{2} \right)^{2\alpha_c(t) - 4} A^4(t) \\ &\quad + 8 g_S T \lambda^2 s^2 \left(\frac{\Gamma(-\chi) \Gamma(1 - \alpha_c(t)/2)}{\Gamma(\alpha_c(t)/2 - 1 - \chi)} \right) \\ &\quad \times \frac{\alpha'_c}{2} \left(\frac{\Gamma(1 - \alpha' t)}{\alpha' t} \right) (\alpha' s)^{\alpha' t + 1} \left(\frac{\alpha'_c s}{2} \right)^{\alpha_c(t) - 2} \\ &\quad \times \left((1 - e^{i\pi\alpha' t}) e^{-i\pi\alpha_c(t)/2} + \text{c.c.} \right) A^4(t). \end{aligned} \quad (44)$$

And the differential cross-section for the total amplitude in Eq. (44) is given by,

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} \left(\frac{1}{4} \sum_{\text{spin}} |\mathcal{A}_{\text{tot}}|^2 \right). \quad (45)$$

This formula will be used for fitting the model parameters with the LHC-TOTEM [46–48], LHC-ATLAS [40], D0 [41], E-710 [39] and CDF [42] scattering data in the small ($-t$) region where single (soft) pomeron exchange is dominant, see Fig. 2. Larger ($-t$) scattering are complicated by other hard processes such as the hard (BFKL) pomeron and perturbative QCD of the quarks and gluon. Notably, both soft and hard pomerons can be described by a single holographic string model in curved space [49].

According to the open-closed string duality, we set the string tension $\alpha' = \alpha'_c$. And fix $\Lambda = 1.17$ GeV, $\alpha_c(0) = 1.08$ in order to obtain slowly increasing total cross section $\sim s^{0.08}$ (we have also performed the fitting by taking $\alpha_c(0)$ as free parameter and the best-fit values are very close to 1.08). Since the closed string tree-level diagram is proportional to g_S^2 , the graviton–proton–proton coupling is then rewritten using the string coupling as $\lambda = \alpha g_S$. The fitting results for various center of mass energies of the pp , $p\bar{p}$ collisions are given in Table 1 and Fig. 3.

Remarkably, the fitting values of M_S among all scattering data are very close to one another implying the universal value of the mass scale in the QCD-string model of the pomeron. Interestingly, the fitting values of the coupling g_S could remain perturbative, i.e. $g_S/4\pi < 1$, and yet take a wide range of values (compensated by varying values of the corresponding α) while the best-fit parameter of the effective closed string coupling hardly vary within the narrow range, $\lambda (= \alpha g_S) \simeq 12 - 13 \text{ GeV}^{-1}$.

Moreover, the tensor glueball mass m_g can be determined from the following formula [22]

$$m_g = M_S \sqrt{(2 - \alpha_c(0))}. \quad (46)$$

All data universally gives $m_g \simeq 1.6$ GeV. This glueball mass value is quite close to the estimation by the Sakai-Sugimoto

Table 1 The best-fit string parameters from both open and closed string pomerons

$\sqrt{s}/\text{GeV}$	g_S	α/GeV^{-1}	M_S/GeV	λ/GeV^{-1}
13,000	1.39–10.94	9.09–1.18	1.65	12.61–12.86
8,000	1.37–10.86	9.13–1.18	1.66	12.54–12.81
7,000	1.37–10.85	8.95–1.15	1.66	12.24–12.52
2,760	1.48–11.77	8.46–1.09	1.68	12.49–12.81
1,960	1.64–13.18	7.42–0.95	1.68–1.69	12.15–12.48
1,800	1.36–10.96	8.45–1.08	1.70–1.71	11.53–11.87
546	1.34–10.61	9.04–1.18	1.66–1.67	12.08–12.48

Fig. 3 The logarithmic fit of the pp , $p\bar{p}$ elastic collisions at $\sqrt{s} = 13000, 8000, 7000, 2760, 1800, 546$ GeV by the combination of the open and closed string pomeron. The fits are performed with data in the small ($-t$) region where linearity from single pomeron contribution is apparent

model [22]. However, this tensor glueball mass value is quite lower than lattice QCD estimation i.e. $m_g^{\text{lattice}} = 2.40$ GeV [50]. Interestingly, conformal symmetry breaking on one hand, generates mass of the glueball (making graviton massive in the holographic picture) and at the same time shifts the intercept $\alpha_c(0)$ so that $J = 2 = \alpha_c(0) + \alpha' m_g^2$ is still preserved. The tiny mismatch between the string mass M_S and the glueball mass m_g ($< M_S$) results in a slightly larger than 1 value of the intercept $\alpha_c(0) = 1.08$.

3.4 Total cross-section from the OSS and closed string amplitudes

The total cross-section of the pp , $p\bar{p}$ scattering from both the OSS and massive graviton amplitudes can be calculated by the optical theorem,

$$\sigma_{\text{tot}} = \frac{1}{s} \text{Im} \left(\mathcal{A}_{\text{OSS}}(s, t) + \mathcal{A}_{\text{closed}}(s, t) \right) \Big|_{t=0}. \quad (47)$$

By using the invariant amplitudes in Eqs. (A3, A7), the total cross-section of the closed string and the OSS amplitudes reads,

$$\sigma_{\text{tot}} = \frac{\pi \lambda^2 \Gamma[-\chi] (\alpha'_c s/2)^{\alpha_c(0)-1}}{\Gamma[\alpha_c(0)/2] \Gamma[\alpha_c(0)/2 - 1 - \chi]} + \pi g_S T \alpha', \quad (48)$$

where only $f_\alpha f_\alpha \rightarrow f_\alpha f_\alpha$, $\alpha = L, R$ contribute to the OSS amplitude at $t = 0$ in the Regge limit. Notably, we obtain the

constant total cross-section in the Regge limit i.e. $\sigma_{\text{tot}}^{\text{OSS}} \propto s^0$ for the open-string singlet contribution and almost constant $\sim s^{0.08}$ for $\alpha_c(0) = 1.08$ from the closed-string amplitude. While the Regge theory gives general result of the total cross-section as $\sigma_{\text{tot}} \propto s^{\alpha(0)-1}$ where $\alpha(0)$ is the intercept of the angular momentum of any Regge trajectory. The pomeron trajectory with $\alpha(0) = 1$ is proposed by Chew [51] and Gribov [52]. It has the vacuum quantum number, even signatures (charge and parity) and also satisfies Froissart bound i.e. $\sigma_{\text{tot}} \leq (\ln s)^2$ at $s \rightarrow \infty$. The universality of the OSS is revealed by (48), the effective size of each particle seen by another is proportional to the area of circle with radius equal to the string length $\ell_S \equiv \sqrt{\alpha'}$ and the string coupling g_S .

4 Discussions and conclusions

The open and closed string pomerons are fit with the high energy pp , $p\bar{p}$ scattering in the Regge regime (large s , small fixed t) at various energies. The fitting string parameters show remarkable universality in the values of the string mass scale M_S . The fitting coupling parameters g_S and λ also take almost universal values among scattering at various energies as shown in Table 1. The fitting values of g_S can vary in a wide range given that it is compensated by the closed-string coupling parameter α while keeping the product $\lambda = \alpha g_S$ mostly unchanged.

Table 2 List of light $J^{PC} = 2^{++}$ and $I = 0$ mesons as glueball candidate [4]

2^{++} meson	Mass (GeV)	Width (GeV)
$f_2(1270)$	1.276	0.190
$f_2(1430)$	1.430	Undetermined
$f_2'(1525)$	1.525	0.073
$f_2(1565)$	1.562	0.134
$f_2(1640)$	1.639	0.100
$f_2(1810)$	1.815	0.200
$f_2(1910)$	1.910	0.160
$f_2(2010)$	2.011	0.200
$f_2(2150)$	2.160	0.150
$f_2(2220)$	2.231	0.023
$f_2(2300)$	2.300	0.150
$f_2(2340)$	2.345	0.322

An insight obtained from the stringy picture of the pomeron is the relationship between the ratio of the two mass scales, $M_S = 1/\sqrt{\alpha'}$ and m_g , and the pomeron intercept $\alpha_c(0)$,

$$\alpha_c(0) = 2 - \frac{m_g^2}{M_S^2}. \quad (49)$$

The slowly increasing total cross section $\sim s^{1-(\frac{m_g}{M_S})^2}$, requires that $m_g \lesssim M_S$. For $\alpha_c(0) = 1.08$, the ratio is $m_g/M_S = 0.96$. Since our fitting value $M_S \simeq 1.7$ GeV is less than the lattice value of $m_g^{\text{lattice}} = 2.4$ GeV, the string model is in tension with the lattice results. However, the lattice calculation ignored the light quarks and meson mixing so the actual pole mass value of the 2^{++} glueball could be quite different. The fitting value of M_S also depends on the choice of the form factor scale Λ . For $\Lambda = 0.94$ GeV (proton mass), the fitting value becomes $M_S \simeq 1.8$ GeV (still relatively universal among all data). Taken as a prediction, the string model prefers glueball pole mass around $1.6 - 1.7$ GeV. Remarkably, this lies within the values of various 2^{++} (“meson”) candidates observed in experiments, see Table 2. With the same quantum numbers, glueball will mix with mesons as mass eigenstates listed among the 2^{++} candidates.

From the amplitudes in the Regge limit of the OSS and the closed string formulae (A3) and (A7), we can experimentally verify the existence of the OSS contribution by considering the scattering with polarized beams of proton and/or antiproton. The closed string pomeron will give the same cross sections for any polarization while the OSS will give different results according to Eq. (A3).

Acknowledgements P.B. is supported in part by the Thailand Research Fund (TRF), Office of Higher Education Commission (OHEC) and Chulalongkorn University under Grant RSA6180002. D.S. is supported

by Rachadapisek Sompote Fund for Postdoctoral Fellowship, Chulalongkorn University.

Data Availability Statement This manuscript has associated data in a data repository. [Authors’ comment: All of the data used in this paper are available in Refs. [38–42, 46–48].]

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

Appendix A: Helicity amplitudes of open and closed string pomerons in the Regge limit

In this appendix, we present the helicity amplitude of the proton–proton scattering for OSS and close string amplitudes. The A_{OSS} amplitudes are given by,

$$A_{\text{OSS}} = g_S T K_{\text{OSS}}(s, t, u) f(s, t, u) A^2(t), \quad (A1)$$

where $K_{\text{OSS}}(s, t, u)$ is kinematics function and characterizes the helicity structures of the amplitudes. It is given by

$$K_{\text{OSS}}(s, t, u) = \begin{cases} -\frac{2s}{ut} & \text{for } f_\alpha f_\alpha \rightarrow f_\alpha f_\alpha, \\ -\frac{2t}{su} & \text{for } f_\alpha f_\beta \rightarrow f_\alpha f_\beta, \end{cases} \quad (A2)$$

where $\alpha, \beta = L, R$, the helicity of the fermion. The amplitudes for $f\bar{f} \rightarrow f\bar{f}$ can be obtained by crossing $s \leftrightarrow t$. Observe that in crossing, each helicity combination of (A2) turn into one another and thus the OSS differential cross section (containing all helicity combinations) between ff and $f\bar{f}$ are naturally identical.

In the Regge limit i.e., $u \rightarrow -s$, we find,

$$A_{\text{OSS}} = \begin{cases} 2g_S T \frac{1}{t} f(s, t, u) A^2(t) & \text{for } f_\alpha f_\alpha \rightarrow f_\alpha f_\alpha, \\ 2g_S T \frac{t}{s^2} f(s, t, u) A^2(t) & \text{for } f_\alpha f_\beta \rightarrow f_\alpha f_\beta, \end{cases} \quad (A3)$$

where the function $f(s, t, u)$ in the Regge limit is given by Eq. (14). This means that different structures of the helicity amplitudes have different forms in the Regge limit. For the closed-string amplitudes in Eq. (43), we obtain,

$$A_{\text{closed}} = \lambda^2 K_{\text{closed}}(s, t, u) \mathcal{P}_g A^2(t), \quad (A4)$$

where $K_{\text{closed}}(s, t, u)$ is the helicity dependent kinematic function and

$$\mathcal{P}_g = \frac{\Gamma[-\chi] \Gamma[1 - \alpha_c(t)/2]}{\Gamma[\alpha_c(t)/2 - 1 - \chi]} (\alpha'_c/2) e^{-i\pi\alpha_c(t)/2} (\alpha'_c s/2)^{\alpha_c(t)-2}. \quad (A5)$$

The $K_{\text{closed}}(s, t, u)$ function in the Regge limit can be written as

$$K_{\text{closed}}(s, t, u) = \begin{cases} \frac{1}{2} s^2 \left(1 - \frac{u}{s}\right) & \text{for } f_{\alpha} f_{\alpha} \rightarrow f_{\alpha} f_{\alpha}, \\ -s u & \text{for } f_{\alpha} f_{\beta} \rightarrow f_{\alpha} f_{\beta}. \end{cases} \quad (\text{A6})$$

In the Regge limit i.e., $u \approx -s$, one finds,

$$A_{\text{closed}} = \begin{cases} \lambda^2 s^2 \mathcal{P}_g A^2(t) & \text{for } f_{\alpha} f_{\alpha} \rightarrow f_{\alpha} f_{\alpha}, \\ \lambda^2 s^2 \mathcal{P}_g A^2(t) & \text{for } f_{\alpha} f_{\beta} \rightarrow f_{\alpha} f_{\beta}. \end{cases} \quad (\text{A7})$$

The helicity amplitudes of the A_{closed} have the same form for all helicity configurations in the Regge limit.

References

1. Y. Nambu, Quark model and the factorization of the Veneziano amplitude, In *Detroit 1969, Proceedings, International Conference on Symmetries and Quark Models, Gordon and Breach, pp. 269–277
2. Z. Koba, H.B. Nielsen, Nucl. Phys. B **10**, 633 (1969)
3. L. Susskind, Phys. Rev. D **1**, 1182 (1970)
4. M. Tanabashi et al. [Particle Data Group], Phys. Rev. D **98**(3), 030001 (2018)
5. T. Yoneya, Prog. Theor. Phys. **51**, 1907 (1974)
6. J. Scherk, J.H. Schwarz, Nucl. Phys. B **81**, 118 (1974)
7. M.B. Green, J.H. Schwarz, E. Witten, *Superstring Theory*, vol. 1 and 2 (Cambridge University Press, Cambridge, 1987)
8. A. Donnachie, P.V. Landshoff, Phys. Lett. B **296**, 227 (1992). [arXiv:hep-ph/9209205](#)
9. C. Ewerz, P. Lebiedowicz, O. Nachtmann, A. Szczurek, Phys. Lett. B **763**, 382 (2016). [arXiv:1606.08067](#) [hep-ph]
10. W. Ochs, J. Phys. G **40**, 043001 (2013). [arXiv:1301.5183](#) [hep-ph]
11. J.M. Maldacena, Int. J. Theor. Phys. **38**, 1113 (1999). [Adv. Theor. Math. Phys. **2**, 231 (1998)]. [arXiv:hep-th/9711200](#)
12. E. Witten, JHEP **9807**, 006 (1998). [arXiv:hep-th/9805112](#)
13. D.J. Gross, H. Ooguri, Phys. Rev. D **58**, 106002 (1998). [arXiv:hep-th/9805129](#)
14. P. Burikham, A. Chatrabhuti, E. Hirunsirisawat, JHEP **0905**, 006 (2009). [https://doi.org/10.1088/1126-6708/2009/05/006](#). [arXiv:0811.0243](#) [hep-ph]
15. P. Burikham, Int. J. Mod. Phys. A **22**, 29 (2007)
16. S. Cullen, M. Perelstein, M.E. Peskin, Phys. Rev. D **62**, 055012 (2000). [arXiv:hep-ph/0001166](#)
17. P. Burikham, PhD thesis, UMI-31-86286
18. P. Burikham, T. Han, F. Hussain, D.W. McKay, Phys. Rev. D **69**, 095001 (2004). [arXiv:hep-ph/0309132](#)
19. P. Burikham, T. Figy, T. Han, Phys. Rev. D **71**, 016005 (2005). Erratum: [Phys. Rev. D **71**, 019905 (2005)]. [arXiv:hep-ph/0411094](#)
20. P. Burikham, JHEP **0407**, 053 (2004). [arXiv:hep-ph/0407271](#)
21. P. Burikham, Phys. Rev. D **73**, 055006 (2006). [arXiv:hep-ph/0601142](#)
22. S.K. Domokos, J.A. Harvey, N. Mann, Phys. Rev. D **80**, 126015 (2009). [arXiv:0907.1084](#) [hep-ph]
23. S.K. Domokos, J.A. Harvey, N. Mann, Phys. Rev. D **82**, 106007 (2010). [arXiv:1008.2963](#) [hep-th]
24. E. Avsar, Y. Hatta, T. Matsuo, JHEP **1003**, 037 (2010). [arXiv:0912.3806](#) [hep-th]
25. N. Anderson, S.K. Domokos, J.A. Harvey, N. Mann, Phys. Rev. D **90**(8), 086010 (2014). [arXiv:1406.7010](#) [hep-ph]
26. I. Iatrakis, A. Ramamurti, E. Shuryak, Phys. Rev. D **94**(4), 045005 (2016). [arXiv:1602.05014](#) [hep-ph]
27. N. Anderson, S. Domokos, N. Mann, Phys. Rev. D **96**(4), 046002 (2017). [arXiv:1612.07457](#) [hep-ph]
28. Z. Hu, B. Maddock, N. Mann, JHEP **1808**, 093 (2018). [arXiv:1710.02463](#) [hep-ph]
29. W. Xie, A. Watanabe, M. Huang. [arXiv:1901.09564](#) [hep-ph]
30. G. Veneziano, Nuovo Cim. A **57**, 190 (1968)
31. C. Itzykson, J.B. Zuber, *International Series in Pure and Applied Physics* (McGraw-Hill, New York, 1980), pp. 160–162
32. P.H. Frampton, *Dual Resonance Models and Superstrings* (World Scientific, Singapore, 1986)
33. S. Donnachie, H.G. Dosch, O. Nachtmann, P. Landshoff, *Pomeron Physics and QCD* (Cambridge University Press, Cambridge, 2002)
34. C.Y. Wong, *Introduction to High-Energy Heavy Ion Collisions* (World Scientific, Singapore, 1994)
35. H. Pagels, Phys. Rev. **144**, 1250 (1966)
36. V. Shtabovenko, R. Mertig, F. Orellana, Comput. Phys. Commun. **207**, 432 (2016). [arXiv:1601.01167](#) [hep-ph]
37. R. Mertig, M. Bohm, A. Denner, Comput. Phys. Commun. **64**, 345 (1991)
38. The Durham HEP database. <http://durpdg.dur.ac.uk/>. Accessed 20 Sept 2018
39. N.A. Amos et al. [E-710 Collaboration], Phys. Lett. B **247**, 127 (1990)
40. G. Aad et al. [ATLAS Collaboration], Nucl. Phys. B **889**, 486 (2014). [arXiv:1408.5778](#) [hep-ex]
41. V.M. Abazov et al. [D0 Collaboration], Phys. Rev. D **86**, 012009 (2012). [arXiv:1206.0687](#) [hep-ex]
42. F. Abe et al. [CDF Collaboration], Phys. Rev. D **50**, 5518 (1994)
43. C. Cebulla, K. Goeke, J. Ossmann, P. Schweitzer, Nucl. Phys. A **794**, 87 (2007). [arXiv:hep-ph/0703025](#) [hep-ph]
44. T. Sakai, S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005). [arXiv:hep-th/0412141](#)
45. P.D.B. Collins, F.D. Gault, A.D. Martin, Nucl. Phys. B **80**, 135 (1974)
46. G. Antchev et al. [TOTEM Collaboration], [arXiv:1812.08610](#) [hep-ex]
47. G. Antchev et al. [TOTEM Collaboration], Eur. Phys. J. C **76**(12), 661 (2016). [arXiv:1610.00603](#) [nucl-ex]
48. G. Antchev et al. [TOTEM Collaboration]. [arXiv:1812.08283](#) [hep-ex]
49. R.C. Brower, J. Polchinski, M.J. Strassler, C.I. Tan, JHEP **0712**, 005 (2007). [arXiv:hep-th/0603115](#)
50. C.J. Morningstar, M.J. Peardon, Phys. Rev. D **60**, 034509 (1999). [arXiv:hep-lat/9901004](#)
51. G.F. Chew, S.C. Frautschi, Phys. Rev. Lett. **7**, 394 (1961)
52. V.N. Gribov, JETP **14**, 478 (1962)