THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Sum rules for CP asymmetries of charmed baryon decays in the $SU(3)_F$ limit

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Abstract Motivated by the recent LHCb observation of CP violation in charm, we study CP violation in the charmed baryon decays. A simple method to search for the CP violation relations in the flavor $SU(3)$ limit, which is associated with a complete interchange of d and s quarks, is proposed. With this method, hundreds of CP violation sum rules in the doubly and singly charmed baryon decays can be found. As examples, the CP violation sum rules in two-body charmed baryon decays are presented. Some of the CP violation sum rules could help the experiment to find better observables. As byproducts, the branching fraction of $\Xi_c^+ \to pK^-\pi^+$ is predicted to be $(1.7 \pm 0.5)\%$ in the U-spin limit and the fragmentation–fraction ratio is determined to be $f_{\Xi_b}/f_{\Lambda_b} = 0.065 \pm 0.020$ using the LHCb data.

1 Introduction

Very recently, the LHCb Collaboration observed the CP violation in the charm sector [1], with the value of

$$\Delta A_{CP} \equiv A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-)$$
$$= (-1.54 \pm 0.29) \times 10^{-3}, \tag{1}$$

in which the dominated direct CP violation is defined by

$$A_{CP}^{\text{dir}}(i \to f) = \frac{|\mathcal{A}(i \to f)|^2 - |\mathcal{A}(\bar{i} \to \bar{f})|^2}{|\mathcal{A}(i \to f)|^2 + |\mathcal{A}(\bar{i} \to \bar{f})|^2}. \tag{2}$$

It is a milestone of particle physics, since CP violation has been well established in the kaon and B systems for many years [2], while the last piece of the puzzle, CP violation in the charm sector, has not been observed until now. To find CP violation in charm, many theoretical and experimental devoted efforts were made in the past decade. On the other

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hand, with the discovery of doubly charmed baryon [3–5] and the progress of singly charmed baryon measurements [6–22], plenty of theoretical interests focus on charmed baryon decays [23–81]. However, only a few publications studied the CP asymmetries in charmed baryon decays [62,73,82]. The difference between CP asymmetries of $\Lambda_c^+ \to pK^+K^-$ and $\Lambda_c^+ \to p\pi^+\pi^-$ modes has been measured by the LHCb Collaboration [21] and no signal of CP violation is found:

$$\Delta A_{CP}^{\text{baryon}} \equiv A_{CP}(\Lambda_c^+ \to pK^+K^-)$$
$$- A_{CP}(\Lambda_c^+ \to p\pi^+\pi^-)$$
$$= (0.30 \pm 0.91 \pm 0.61)\%. \tag{3}$$

In charmed and bottomed meson decays, some relations for CP asymmetries in (or beyond) the flavor $SU(3)$ limit are found [83–94]. For example, the direct CP asymmetries in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays have following relation in the U-spin limit [86,87]:

$$A_{CP}^{\text{dir}}(D^0 \to K^+K^-) + A_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-) = 0. \tag{4}$$

The two CP asymmetries in ΔA_{CP} have opposite sign and hence are constructive in ΔA_{CP}. But the two CP asymmetries in $\Delta A_{CP}^{\text{baryon}}$, as pointed out in [73], do not have a relation like the ones in ΔA_{CP}. Prospects of measuring the CP asymmetries of charmed baryon decays on LHCb [95], as well as Belle II [96], are bright. It is significative to study the relations for CP asymmetries in the charmed baryon decays and then help to find some promising observables in experiments.

In Ref. [73], three CP violation sum rules associated with a complete interchange of d and s quarks are derived. In this work, we illustrate that the complete interchange of d and s quarks is a universal law to search for the CP violation sum rules of two charmed hadron decay channels in the flavor $SU(3)$ limit. With the universal law, hundreds of CP violation sum rules can be found in the doubly and singly charmed baryon decays. The CP violation sum rules could be tested in future measurements or provide a guide to find

better observables for experiments. Besides, the branching fraction $\mathcal{B}r(\Xi_c^+ \to pK^-\pi^+)$ and fragmentation–fraction ratio f_{Ξ_b}/f_{Λ_b} are estimated in the U-spin limit.

The rest of this paper is organized as follows. In Sect. 2, the effective Hamiltonian of charm decay is decomposed into the $SU(3)$ irreducible representations. In Sect. 3, we derive the CP violation sum rules for charmed meson and baryon decays and sum up a general law for CP violation sum rules in charm. In Sect. 4, we list some results of the CP violation sum rules in charmed baryon decays. Section 5 is a brief summary. The explicit $SU(3)$ decomposition of the operators in charm decays is presented in Appendix A.

2 Effective Hamiltonian of charm decay

The effective Hamiltonian in charm quark weak decay in the Standard Model (SM) can be written as [97]

$$
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=d,s} V_{cq_1}^* V_{uq_2} \left(\sum_{i=1}^{2} C_i(\mu) O_i(\mu) \right) \right.
$$
$$
\left. - V_{cb}^* V_{ub} \left(\sum_{i=3}^{6} C_i(\mu) O_i(\mu) + C_{8g}(\mu) O_{8g}(\mu) \right) \right],
$$
(5)

where G_F is the Fermi coupling constant, C_i is the Wilson coefficient of operator O_i. The tree operators are

$$
O_1 = (\bar{u}_\alpha q_{2\beta})_{V-A}(\bar{q}_{1\beta} c_\alpha)_{V-A},
$$
$$
O_2 = (\bar{u}_\alpha q_{2\alpha})_{V-A}(\bar{q}_{1\beta} c_\beta)_{V-A},
$$
(6)

in which α, β are color indices, $q_{1,2}$ are d and s quarks. The QCD penguin operators are

$$
O_3 = \sum_{q'=u,d,s} (\bar{u}_\alpha c_\alpha)_{V-A}(\bar{q}'_\beta q'_\beta)_{V-A},
$$
$$
O_4 = \sum_{q'=u,d,s} (\bar{u}_\alpha c_\beta)_{V-A}(\bar{q}'_\beta q'_\alpha)_{V-A},
$$
$$
O_5 = \sum_{q'=u,d,s} (\bar{u}_\alpha c_\alpha)_{V-A}(\bar{q}'_\beta q'_\beta)_{V+A},
$$
$$
O_6 = \sum_{q'=u,d,s} (\bar{u}_\alpha c_\beta)_{V-A}(\bar{q}'_\beta q'_\alpha)_{V+A},
$$
(7)

and the chromomagnetic-penguin operator is

$$
O_{8g} = \frac{g}{8\pi^2} m_c \bar{u} \sigma_{\mu\nu} (1 + \gamma_5) T^a G^{a\mu\nu} c.
$$
(8)

The magnetic-penguin contributions can be included into the Wilson coefficients for the penguin operators following the substitutions [98–100]:

$$
C_{3,5}(\mu) \to C_{3,5}(\mu) + \frac{\alpha_s(\mu)}{8\pi N_c} \frac{2m_c^2}{\langle l^2 \rangle} C_{8g}^{\text{eff}}(\mu),
$$

$$
C_{4,6}(\mu) \to C_{4,6}(\mu) - \frac{\alpha_s(\mu)}{8\pi} \frac{2m_c^2}{\langle l^2 \rangle} C_{8g}^{\text{eff}}(\mu),
$$
(9)

with the effective Wilson coefficient $C_{8g}^{\text{eff}} = C_{8g} + C_5$ and $\langle l^2 \rangle$ being the averaged invariant mass squared of the virtual gluon emitted from the magnetic-penguin operator.

The charm quark decays are categorized into three types, Cabibbo-favored (CF), singly Cabibbo-suppressed (SCS), and doubly Cabibbo-suppressed (DCS) decays, with the flavor structures of

$$
c \to s\bar{d}u, \quad c \to d\bar{d}/s\bar{s}u, \quad c \to d\bar{s}u,
$$
(10)

respectively. In the $SU(3)$ picture, the operators in charm decays embed into the four-quark Hamiltonian,

$$
\mathcal{H}_{\text{eff}} = \sum_{i,j,k=1}^{3} H_{ij}^k O_k^{ij} = \sum_{i,j,k=1}^{3} H_{ij}^k (\bar{q}^i q_k)(\bar{q}^j c).
$$
(11)

Equation (5) implies that the tensor components of H_{ij}^k can be obtained from the map $(\bar{u}q_1)(\bar{q}_2 c) \to V_{cq_2}^* V_{uq_1}$ in current–current operators and $(\bar{q}q)(\bar{u}c) \to -V_{cb}^* V_{ub}$ in penguin operators and the others are zero. The non-zero components of the tensor H_{ij}^k corresponding to tree operators in Eq. (5) are

$$
H_{13}^2 = V_{cs}^* V_{ud}, \quad H_{12}^2 = V_{cd}^* V_{ud},
$$
$$
H_{13}^3 = V_{cs}^* V_{us}, \quad H_{12}^3 = V_{cd}^* V_{us},
$$
(12)

and the non-zero components of the tensor H_{ij}^k corresponding to penguin operators in Eq. (5) are

$$
H_{11}^1 = -V_{cb}^* V_{ub}, \quad H_{21}^2 = -V_{cb}^* V_{ub}, \quad H_{31}^3 = -V_{cb}^* V_{ub}.
$$
(13)

The operator O_k^{ij} is a representation of the $SU(3)$ group, which is decomposed as four irreducible representations: $\bar{3} \otimes 3 \otimes \bar{3} = \bar{3} \oplus \bar{3}' \oplus 6 \oplus \overline{15}$. The explicit decomposition is [86]

$$
O_k^{ij} = \delta_k^j \left(\frac{3}{8} O(\bar{3})^i - \frac{1}{8} O(\bar{3}')^i \right) + \delta_k^i \left(\frac{3}{8} O(\bar{3}')^j - \frac{1}{8} O(\bar{3})^j \right)
$$
$$
+ \epsilon^{ijl} O(6)_{lk} + O(\overline{15})_k^{ij}.
$$
(14)

All components of the irreducible representations are listed in Appendix A. The non-zero components of H_{ij}^k corresponding to tree operators in the $SU(3)$ decomposition are

$$
H(6)^{22} = -\frac{1}{2} V_{cs}^* V_{ud}, \quad H(6)^{23} = \frac{1}{4} (V_{cd}^* V_{ud} - V_{cs}^* V_{us}),
$$
$$
H(6)^{33} = \frac{1}{2} V_{cd}^* V_{us},
$$
$$
H(\overline{15})_{11}^1 = -\frac{1}{4} (V_{cd}^* V_{ud} + V_{cs}^* V_{us}) = \frac{1}{4} V_{cb}^* V_{ub},
$$

$$H(\overline{15})^2_{13} = \frac{1}{2} V^*_{cs} V_{ud},$$

$$H(\overline{15})^3_{12} = \frac{1}{2} V^*_{cd} V_{us},$$

$$H(\overline{15})^2_{12} = \frac{3}{8} V^*_{cd} V_{ud} - \frac{1}{8} V^*_{cs} V_{us},$$

$$H(\overline{15})^3_{13} = \frac{3}{8} V^*_{cs} V_{us} - \frac{1}{8} V^*_{cd} V_{ud},$$

$$H(\overline{3})_1 = V^*_{cd} V_{ud} + V^*_{cs} V_{us} = -V^*_{cb} V_{ub}. \tag{15}$$

The non-zero components of H^k_{ij} corresponding to penguin operators in the $SU(3)$ decomposition are

$$H^P(\overline{3})_1 = -V^*_{cb} V_{ub}, \quad H^P(\overline{3}')_1 = -3V^*_{cb} V_{ub}. \tag{16}$$

Here we use the superscript P to differentiate penguin contributions from tree contributions. Equations (15) and (16) were derived in [86] for the first time. But the non-zero components $H(\overline{15})^1_{11}$ and $H(\overline{3})_1$ in the tree operator contributions are missing in [86].

Recent studies for charmed baryon decays in the $SU(3)$ irreducible representation amplitude (IRA) approach [29,40, 42,54,60,61,64,68,70,71,74,81] do not analyze CP asymmetries because they ignore the two 3-dimensional irreducible representations and make the approximation of $V^*_{cs} V_{us} \simeq -V^*_{cd} V_{ud}$ in the 15- and 6-dimensional irreducible representations, leading to the vanishing of the contributions proportional to $\lambda_b = V^*_{cb} V_{ub}$. If the contributions proportional to λ_b are included, the $SU(3)$ irreducible representation amplitude approach then can be used to investigate CP asymmetries in the charmed baryon decays.

3 CP violation sum rules in charmed meson/baryon decays

In this section, we discuss the method to search for the relations for CP asymmetries of charm decays in the flavor $SU(3)$ limit. We first analyze the CP violation sum rules in charmed meson and baryon decays, respectively, and then sum up a general law for the CP violation sum rules in charm decays.

3.1 CP violation sum rules in charmed meson decays

Two CP asymmetry sum rules for $D \to PP$ decays in the flavor $SU(3)$ limit have been given in [86,87]:

$$A^{\text{dir}}_{CP}(D^0 \to K^+ K^-) + A^{\text{dir}}_{CP}(D^0 \to \pi^+ \pi^-) = 0, \tag{17}$$

$$A^{\text{dir}}_{CP}(D^+ \to K^+ \overline{K}^0) + A^{\text{dir}}_{CP}(D^+_s \to \pi^+ K^0) = 0. \tag{18}$$

To see why the two sum rules are correct, we express the decay amplitudes of $D^0 \to K^+ K^-$, $D^0 \to \pi^+ \pi^-$,

$D^+ \to K^+ \overline{K}^0$ and $D^+_s \to \pi^+ K^0$ modes in the $SU(3)$ irreducible representation amplitude (IRA) approach. The charmed meson anti-triplet is

$$D^i = (D^0, D^+, D^+_s). \tag{19}$$

The pseudoscalar meson nonet is

$$P^i_j = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{2/3}\eta_8 \end{pmatrix}$$
$$+ \frac{1}{\sqrt{3}} \begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_1 \end{pmatrix}. \tag{20}$$

To obtain the $SU(3)$ irreducible representation amplitude of $D \to PP$ decay, one takes various representations in Eqs. (15) and (16) and contracts all indices in D^i and light meson P^i_j with various combinations:

$$\mathcal{A}^{\text{tree}}_{D \to PP} = a_{15} D^i H(\overline{15})^k_{ij} P^j_l P^l_k + b_{15} D^i H(\overline{15})^k_{ij} P^j_k P^l_l$$
$$+ c_{15} D^i H(\overline{15})^k_{jl} P^j_i P^l_k$$
$$+ a_6 D^i H(6)^k_{ij} P^j_l P^l_k + b_6 D^i H(6)^k_{ij} P^j_k P^l_l$$
$$+ c_6 D^i H(6)^k_{jl} P^j_i P^l_k$$
$$+ a_3 D^i H(\overline{3})_i P^j_k P^k_j + b_3 D^i H(\overline{3})_i P^k_k P^j_j$$
$$+ c_3 D^i H(\overline{3})_k P^k_i P^j_j$$
$$+ d_3 D^i H(\overline{3})_k P^j_i P^k_j. \tag{21}$$

$$\mathcal{A}^{\text{penguin}}_{D \to PP} = Pa_3 D^i H^P(\overline{3})_i P^j_k P^k_j + Pb_3 D^i H^P(\overline{3})_i P^k_k P^j_j$$
$$+ Pc_3 D^i H^P(\overline{3})_k P^k_i P^j_j$$
$$+ Pd_3 D^i H^P(\overline{3})_k P^j_i P^k_j + Pa'_3 D^i H^P(\overline{3}')_i P^j_k P^k_j$$
$$+ Pb'_3 D^i H^P(\overline{3}')_i P^k_k P^j_j$$
$$+ Pc'_3 D^i H^P(\overline{3}')_k P^k_i P^j_j + Pd'_3 D^i H^P(\overline{3}')_k P^j_i P^k_j. \tag{22}$$

Notice that only the first components of $\overline{3}$ and $\overline{3}'$ irreducible representations are non-zero. Some amplitudes, for example, a_3, Pa_3 and Pa'_3 are always appear simultaneously since they correspond to the same contraction. Noting that $H^P(\overline{3}')_1 = 3H(\overline{3})_1 = 3H^P(\overline{3})_1 = -3V^*_{cb} V_{ub}$, if we define

$$Pa = a_3 + Pa_3 + 3Pa'_3, \quad Pb = b_3 + Pb_3 + 3Pb'_3, \quad \dots, \tag{23}$$

the amplitude of $D \to PP$ decay will be reduced to

$$\mathcal{A}^{\text{tree+penguin}}_{D \to PP} = a_{15} D^i H(\overline{15})^k_{ij} P^j_l P^l_k + b_{15} D^i H(\overline{15})^k_{ij} P^j_k P^l_l$$
$$+ c_{15} D^i H(\overline{15})^k_{jl} P^j_i P^l_k$$

$$+ a_6 D^i H(6)_{ij}^k P_l^j P_k^l + b_6 D^i H(6)_{ij}^k P_k^j P_l^l$$
$$+ c_6 D^i H(6)_{jl}^k P_i^j P_k^l$$
$$+ Pa D^i H(\bar{3})_i P_k^j P_j^k + Pb D^i H(\bar{3})_i P_k^k P_j^j$$
$$+ Pc D^i H(\bar{3})_k P_i^k P_j^j$$
$$+ Pd D^i H(\bar{3})_k P_i^j P_j^k. \tag{24}$$

With Eq. (24), the decay amplitudes of the $D^0 \rightarrow K^+ K^-$, $D^0 \rightarrow \pi^+\pi^-$, $D^+ \rightarrow K^+\overline{K}^0$ and $D_s^+ \rightarrow \pi^+ K^0$ modes read

$$\mathcal{A}(D^0 \rightarrow K^+ K^-) = -\lambda_d \left(\frac{1}{8}a_{15} + \frac{1}{8}c_{15} + \frac{1}{4}a_6 - \frac{1}{4}c_6\right)$$
$$+ \lambda_s \left(\frac{3}{8}a_{15} + \frac{3}{8}c_{15} + \frac{1}{4}a_6 - \frac{1}{4}c_6\right)$$
$$- \lambda_b(2Pa + Pd - a_{15}/4), \tag{25}$$

$$\mathcal{A}(D^0 \rightarrow \pi^+\pi^-) = \lambda_d \left(\frac{3}{8}a_{15} + \frac{3}{8}c_{15} + \frac{1}{4}a_6 - \frac{1}{4}c_6\right)$$
$$- \lambda_s \left(\frac{1}{8}a_{15} + \frac{1}{8}c_{15} + \frac{1}{4}a_6 - \frac{1}{4}c_6\right)$$
$$- \lambda_b(2Pa + Pd - a_{15}/4), \tag{26}$$

$$\mathcal{A}(D^+ \rightarrow K^+\overline{K}^0) = \lambda_d \left(\frac{3}{8}a_{15} - \frac{1}{8}c_{15} - \frac{1}{4}a_6 + \frac{1}{4}c_6\right)$$
$$- \lambda_s \left(\frac{1}{8}a_{15} - \frac{3}{8}c_{15} - \frac{1}{4}a_6 + \frac{1}{4}c_6\right)$$
$$- \lambda_b Pd, \tag{27}$$

$$\mathcal{A}(D_s^+ \rightarrow \pi^+ K^0) = -\lambda_d \left(\frac{1}{8}a_{15} - \frac{3}{8}c_{15} - \frac{1}{4}a_6 + \frac{1}{4}c_6\right)$$
$$+ \lambda_s \left(\frac{3}{8}a_{15} - \frac{1}{8}c_{15} - \frac{1}{4}a_6 + \frac{1}{4}c_6\right)$$
$$- \lambda_b Pd, \tag{28}$$

in which $\lambda_d = V_{cd}^* V_{ud}$, $\lambda_s = V_{cs}^* V_{us}$, $\lambda_b = V_{cb}^* V_{ub}$. Equations (25)–(28) are consistent with [86] except for the last terms in Eqs. (25) and (26) because of the non-vanishing $H(\overline{15})_{11}^1$ component in Eq. (15). From above formulas, the CP violation sum rules listed in Eqs. (17) and (18) are derived if the approximation of

$$\lambda_b\lambda_d = -\lambda_b(\lambda_s + \lambda_b) = -(\lambda_b\lambda_s + \lambda_b^2) \simeq -\lambda_b\lambda_s \tag{29}$$

is used. Besides, the decay amplitude of $D^0 \rightarrow K^0\overline{K}^0$ is expressed as

$$\mathcal{A}(D^0 \rightarrow K^0\overline{K}^0) = -\lambda_b \left(2Pa + \frac{1}{4}a_{15}\right). \tag{30}$$

The direct CP asymmetry in $D^0 \rightarrow K^0\overline{K}^0$ decay is zero in the flavor $SU(3)$ limit:

$$A_{CP}^{\text{dir}}(D^0 \rightarrow K^0\overline{K}^0) = 0. \tag{31}$$

For the CP violation relations (17) and (18), the decay amplitudes of two channels are connected by the interchange of $\lambda_d \leftrightarrow \lambda_s$, and their initial and final states are connected by the interchange of $d \leftrightarrow s$:

$$D^+ \leftrightarrow D_s^+, \quad D^0 \leftrightarrow D^0, \quad K^+ \leftrightarrow \pi^+,$$
$$K^- \leftrightarrow \pi^-, \quad K^0 \leftrightarrow \overline{K}^0. \tag{32}$$

For $D^0 \rightarrow K^0\overline{K}^0$ decay, its corresponding mode in the interchange of $d \leftrightarrow s$ is itself. So all CP violation relations in Eqs. (17), (18) and (31) are associated with the U-spin transformation.

On the other hand, Eqs. (17), (18) and (31) include all the SCS modes without π^0, $\eta^{(\prime)}$ in the final states in $D \rightarrow PP$ decays. Mesons π^0 and $\eta^{(\prime)}$ do not have definite U-spin quantum numbers. Under the interchange of $d \leftrightarrow s$, there are no mesons corresponding to π^0 and $\eta^{(\prime)}$. For example, π^0 has the quark constituent of $(\bar{d}d - \bar{u}u)/\sqrt{2}$. Under the interchange of $d \leftrightarrow s$, $(\bar{d}d - \bar{u}u)/\sqrt{2}$ turns into $(\bar{s}s - \bar{u}u)/\sqrt{2}$. No meson has the quark constituent of $(\bar{s}s - \bar{u}u)/\sqrt{2}$. So those decay channels involving π^0, $\eta^{(\prime)}$ do not have their corresponding modes in the interchange of $d \leftrightarrow s$, and then have no simple CP violation sum rules with two channels.

In fact, not only the $D \rightarrow PP$ decays, there are also some CP violation sum rules in the $D \rightarrow PV$ decays [87]

$$A_{CP}^{\text{dir}}(D^0 \rightarrow \pi^-\rho^+) + A_{CP}^{\text{dir}}(D^0 \rightarrow K^- K^{*+}) = 0, \tag{33}$$
$$A_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+\rho^-) + A_{CP}^{\text{dir}}(D^0 \rightarrow K^+ K^{*-}) = 0, \tag{34}$$
$$A_{CP}^{\text{dir}}(D^+ \rightarrow \overline{K}^0 K^{*+}) + A_{CP}^{\text{dir}}(D_s^+ \rightarrow K^0\rho^+) = 0, \tag{35}$$
$$A_{CP}^{\text{dir}}(D^+ \rightarrow K^+\overline{K}^{*0}) + A_{CP}^{\text{dir}}(D_s^+ \rightarrow \pi^+ K^{*0}) = 0, \tag{36}$$
$$A_{CP}^{\text{dir}}(D^0 \rightarrow K^0\overline{K}^{*0}) + A_{CP}^{\text{dir}}(D^0 \rightarrow \overline{K}^0 K^{*0}) = 0. \tag{37}$$

The detailed derivation of these sum rules is similar to $D \rightarrow PP$ and can be found in Ref. [86]. Again, all the CP violation sum rules in $D \rightarrow PV$ decays are associated with a complete interchange of d and s quarks, and all the singly Cabibbo-suppressed $D \rightarrow PV$ modes with all final states having definite U-spin quantum numbers are included in Eqs. (33)–(37).

3.2 CP violation sum rules in charmed baryon decays

In this subsection, we take charmed baryon decays into one pseudoscalar meson and one decuplet baryon as examples to show the complete interchange of $d \leftrightarrow s$ is still valid for

the CP violation sum rules in charmed baryon decays. The charmed anti-triplet baryon is expressed as

$$\mathcal{B}_{c\bar{3}} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}. \tag{38}$$

The light baryon decuplet is given as

$$\Delta^{++} = \mathcal{B}_{10}^{111}, \quad \Delta^- = \mathcal{B}_{10}^{222}, \quad \Omega^- = \mathcal{B}_{10}^{333},$$

$$\Delta^+ = \frac{1}{\sqrt{3}}(\mathcal{B}_{10}^{112} + \mathcal{B}_{10}^{121} + \mathcal{B}_{10}^{211}),$$

$$\Delta^0 = \frac{1}{\sqrt{3}}(\mathcal{B}_{10}^{122} + \mathcal{B}_{10}^{212} + \mathcal{B}_{10}^{221}),$$

$$\Sigma^{*+} = \frac{1}{\sqrt{3}}(\mathcal{B}_{10}^{113} + \mathcal{B}_{10}^{131} + \mathcal{B}_{10}^{311}),$$

$$\Sigma^{*-} = \frac{1}{\sqrt{3}}(\mathcal{B}_{10}^{223} + \mathcal{B}_{10}^{232} + \mathcal{B}_{10}^{322}),$$

$$\Xi^{*0} = \frac{1}{\sqrt{3}}(\mathcal{B}_{10}^{133} + \mathcal{B}_{10}^{313} + \mathcal{B}_{10}^{331}),$$

$$\Xi^{*-} = \frac{1}{\sqrt{3}}(\mathcal{B}_{10}^{233} + \mathcal{B}_{10}^{323} + \mathcal{B}_{10}^{332}),$$

$$\Sigma^{*0} = \frac{1}{\sqrt{6}}(\mathcal{B}_{10}^{123} + \mathcal{B}_{10}^{132} + \mathcal{B}_{10}^{213} + \mathcal{B}_{10}^{231} + \mathcal{B}_{10}^{312} + \mathcal{B}_{10}^{321}). \tag{39}$$

The $SU(3)$ irreducible representation amplitude of $\mathcal{B}_{c\bar{3}} \to \mathcal{B}_{10}M$ decay can be written as

$$\begin{aligned} \mathcal{A}^{\text{tree}}_{\mathcal{B}_{c\bar{3}} \to \mathcal{B}_{10}M} = {} & e_1(\mathcal{B}_{c\bar{3}})_{ij} H(\overline{15})_{kl}^j M_m^i \overline{\mathcal{B}}_{10}^{klm} \\ & + e_2(\mathcal{B}_{c\bar{3}})_{ij} H(\overline{15})_{lm}^k M_k^j \overline{\mathcal{B}}_{10}^{ilm} \\ & + e_3(\mathcal{B}_{c\bar{3}})_{ij} H(\overline{15})_{kl}^j M_m^l \overline{\mathcal{B}}_{10}^{ikm} \\ & + e_4(\mathcal{B}_{c\bar{3}})_{ij} H(6)_{kl}^j M_m^l \overline{\mathcal{B}}_{10}^{ikm} \\ & + e_5(\mathcal{B}_{c\bar{3}})_{ij} H(\overline{15})_{kl}^j M_m^m \overline{\mathcal{B}}_{10}^{ikl} \\ & + e_6(\mathcal{B}_{c\bar{3}})_{ij} H(\bar{3})_k M_m^j \overline{\mathcal{B}}_{10}^{ikm}, \end{aligned} \tag{40}$$

$$\begin{aligned} \mathcal{A}^{\text{penguin}}_{\mathcal{B}_{c\bar{3}} \to \mathcal{B}_{10}M} = {} & Pe_6(\mathcal{B}_{c\bar{3}})_{ij} H^P(\bar{3})_k M_m^j \overline{\mathcal{B}}_{10}^{ikm} \\ & + Pe_7(\mathcal{B}_{c\bar{3}})_{ij} H^P(\bar{3}')_k M_m^j \overline{\mathcal{B}}_{10}^{ikm}. \end{aligned} \tag{41}$$

Similar to $D \to PP$ decay, if we define

$$Pe = e_6 + Pe_6 + 3Pe_7, \tag{42}$$

the amplitude of $\mathcal{B}_{c\bar{3}} \to \mathcal{B}_{10}M$ decay will be reduced to

$$\begin{aligned} \mathcal{A}^{\text{tree+penguin}}_{\mathcal{B}_{c\bar{3}} \to \mathcal{B}_{10}M} = {} & e_1(\mathcal{B}_{c\bar{3}})_{ij} H(\overline{15})_{kl}^j M_m^i \overline{\mathcal{B}}_{10}^{klm} \\ & + e_2(\mathcal{B}_{c\bar{3}})_{ij} H(\overline{15})_{lm}^k M_k^j \overline{\mathcal{B}}_{10}^{ilm} \\ & + e_3(\mathcal{B}_{c\bar{3}})_{ij} H(\overline{15})_{kl}^j M_m^l \overline{\mathcal{B}}_{10}^{ikm} \end{aligned}$$

$$\begin{aligned} & + e_4(\mathcal{B}_{c\bar{3}})_{ij} H(6)_{kl}^j M_m^l \overline{\mathcal{B}}_{10}^{ikm} \\ & + e_5(\mathcal{B}_{c\bar{3}})_{ij} H(\overline{15})_{kl}^j M_m^m \overline{\mathcal{B}}_{10}^{ikl} \\ & + Pe(\mathcal{B}_{c\bar{3}})_{ij} H(\bar{3})_k M_m^j \overline{\mathcal{B}}_{10}^{ikm}. \end{aligned} \tag{43}$$

The first four terms are the same with the formula given in [101]. The fifth term is the decay amplitude associated with singlet η_1, and the six term is the amplitude proportional to λ_b. With Eq. (43), the $SU(3)$ irreducible representation amplitudes of $\mathcal{B}_{c\bar{3}} \to \mathcal{B}_{10}M$ decays are obtained. The results are listed in Table 1.

From Table 1, seven CP violation sum rules in the $SU(3)_F$ limit for the charmed baryon decays into one pseudoscalar meson and one decuplet baryon are found:

$$A_{CP}^{\text{dir}}(\Lambda_c^+ \to \Delta^0 \pi^+) + A_{CP}^{\text{dir}}(\Xi_c^+ \to \Xi^{*0} K^+) = 0, \tag{44}$$

$$A_{CP}^{\text{dir}}(\Lambda_c^+ \to \Sigma^{*+} K^0) + A_{CP}^{\text{dir}}(\Xi_c^+ \to \Delta^+ \overline{K}^0) = 0, \tag{45}$$

$$A_{CP}^{\text{dir}}(\Lambda_c^+ \to \Sigma^{*0} K^+) + A_{CP}^{\text{dir}}(\Xi_c^+ \to \Sigma^{*0} K^+) = 0, \tag{46}$$

$$A_{CP}^{\text{dir}}(\Lambda_c^+ \to \Delta^{++} \pi^-) + A_{CP}^{\text{dir}}(\Xi_c^+ \to \Delta^{++} K^-) = 0, \tag{47}$$

$$A_{CP}^{\text{dir}}(\Xi_c^0 \to \Sigma^{*-} \pi^+) + A_{CP}^{\text{dir}}(\Xi_c^0 \to \Xi^{*-} K^+) = 0, \tag{48}$$

$$A_{CP}^{\text{dir}}(\Xi_c^0 \to \Delta^0 \overline{K}^0) + A_{CP}^{\text{dir}}(\Xi_c^0 \to \Xi^{*0} K^0) = 0, \tag{49}$$

$$A_{CP}^{\text{dir}}(\Xi_c^0 \to \Sigma^{*+} \pi^-) + A_{CP}^{\text{dir}}(\Xi_c^0 \to \Delta^+ K^-) = 0. \tag{50}$$

Similar to the charmed meson decays, all the CP violation sum rules are associated with a complete interchange of d and s quarks in the initial and final states. For charmed anti-triplet baryons,

$$\Lambda_c^+ \leftrightarrow \Xi_c^+, \quad \Xi_c^0 \leftrightarrow \Xi_c^0. \tag{51}$$

For light decuplet baryons,

$$\begin{aligned} &\Delta^0 \leftrightarrow \Xi^{*0}, \quad \Sigma^{*+} \leftrightarrow \Delta^+, \quad \Sigma^{*0} \leftrightarrow \Sigma^{*0}, \quad \Delta^{++} \leftrightarrow \Delta^{++}, \\ &\Xi^{*-} \leftrightarrow \Sigma^{*-}, \quad \Delta^- \leftrightarrow \Omega^-. \end{aligned} \tag{52}$$

Also, Eqs. (44)–(50) include all the SCS modes with all associated particles having definite U-spin quantum numbers in $\mathcal{B}_{c\bar{3}} \to \mathcal{B}_{10}M$ decays.

For other types of charm baryon decay, for example $\mathcal{B}_{c\bar{3}} \to \mathcal{B}_8 M$ decay, multi-body decay and doubly charmed baryon decay, the treatments of their $SU(3)$ irreducible representation amplitudes are similar to $\mathcal{B}_{c\bar{3}} \to \mathcal{B}_{10}M$. Related discussions can be found in Refs. [29,40,42,101]. But notice

Table 1 $SU(3)$ irreducible representation amplitudes in $\mathcal{B}_{c\bar{3}} \to \mathcal{B}_{10}M$ decays, in which only those modes that all initial and final states have definite U-spin quantum numbers are listed

Channel	Amplitude
$\Lambda_c^+ \to \Delta^0 \pi^+$	$\frac{1}{8\sqrt{3}}\lambda_d(6e_1 - 6e_2 + 5e_3 - 2e_4) - \frac{1}{8\sqrt{3}}\lambda_s(2e_1 - 2e_2 - e_3 - 2e_4) + \frac{1}{\sqrt{3}}\lambda_b Pe$
$\Lambda_c^+ \to \Sigma^{*+} K^0$	$\frac{1}{8\sqrt{3}}\lambda_d(2e_1 - 2e_2 + 3e_3 + 2e_4) + \frac{1}{8\sqrt{3}}\lambda_s(2e_1 + 6e_2 - e_3 - 2e_4) - \frac{1}{\sqrt{3}}\lambda_b Pe$
$\Lambda_c^+ \to \Sigma^{*0} K^+$	$\frac{1}{8\sqrt{6}}\lambda_d(6e_1 + 2e_2 + 5e_3 - 2e_4) - \frac{1}{8\sqrt{6}}\lambda_s(2e_1 + 6e_2 - e_3 - 2e_4) + \frac{1}{\sqrt{6}}\lambda_b Pe$
$\Lambda_c^+ \to \Delta^{++} \pi^-$	$\frac{1}{8}\lambda_d(2e_1 - 2e_2 + 3e_3 + 2e_4) + \frac{1}{8}\lambda_s(2e_1 - 2e_2 - e_3 - 2e_4) - \lambda_b Pe$
$\Xi_c^+ \to \Xi^{*0} K^+$	$-\frac{1}{8\sqrt{3}}\lambda_d(2e_1 - 2e_2 - e_3 - 2e_4) + \frac{1}{8\sqrt{3}}\lambda_s(6e_1 - 6e_2 + 5e_3 - 2e_4) + \frac{1}{\sqrt{3}}\lambda_b Pe$
$\Xi_c^+ \to \Delta^+ \overline{K}^0$	$\frac{1}{8\sqrt{3}}\lambda_d(2e_1 + 6e_2 - e_3 - 2e_4) + \frac{1}{8\sqrt{3}}\lambda_s(2e_1 - 2e_2 + 3e_3 + 2e_4) - \frac{1}{\sqrt{3}}\lambda_b Pe$
$\Xi_c^+ \to \Sigma^{*0} \pi^+$	$-\frac{1}{8\sqrt{6}}\lambda_d(2e_1 + 6e_2 - e_3 - 2e_4) + \frac{1}{8\sqrt{6}}\lambda_s(6e_1 + 2e_2 + 5e_3 - 2e_4) + \frac{1}{\sqrt{6}}\lambda_b Pe$
$\Xi_c^+ \to \Delta^{++} K^-$	$\frac{1}{8}\lambda_d(2e_1 - 2e_2 - e_3 - 2e_4) + \frac{1}{8}\lambda_s(2e_1 - 2e_2 + 3e_3 + 2e_4) - \lambda_b Pe$
$\Xi_c^0 \to \Sigma^{*-} \pi^+$	$-\frac{1}{2\sqrt{3}}\lambda_d(e_3 - e_4) + \frac{1}{2\sqrt{3}}\lambda_s(e_3 - e_4)$
$\Xi_c^0 \to \Xi^{*-} K^+$	$-\frac{1}{2\sqrt{3}}\lambda_d(e_3 - e_4) + \frac{1}{2\sqrt{3}}\lambda_s(e_3 - e_4)$
$\Xi_c^0 \to \Delta^0 \overline{K}^0$	$-\frac{1}{8\sqrt{3}}\lambda_d(6e_1 - 6e_2 + e_3 + 2e_4) + \frac{1}{8\sqrt{3}}\lambda_s(2e_1 - 2e_2 + 3e_3 + 2e_4) - \frac{1}{\sqrt{3}}\lambda_b Pe$
$\Xi_c^0 \to \Xi^{*0} K^0$	$-\frac{1}{8\sqrt{3}}\lambda_d(2e_1 - 2e_2 + 3e_3 + 2e_4) + \frac{1}{8\sqrt{3}}\lambda_s(6e_1 - 6e_2 + e_3 + 2e_4) + \frac{1}{\sqrt{3}}\lambda_b Pe$
$\Xi_c^0 \to \Sigma^{*+} \pi^-$	$-\frac{1}{8\sqrt{3}}\lambda_d(2e_1 - 2e_2 + 3e_3 + 2e_4) + \frac{1}{8\sqrt{3}}\lambda_s(6e_1 + 2e_2 + e_3 + 2e_4) + \frac{1}{\sqrt{3}}\lambda_b Pe$
$\Xi_c^0 \to \Delta^+ K^-$	$-\frac{1}{8\sqrt{3}}\lambda_d(6e_1 + 2e_2 + e_3 + 2e_4) + \frac{1}{8\sqrt{3}}\lambda_s(2e_1 - 2e_2 + 3e_3 + 2e_4) - \frac{1}{\sqrt{3}}\lambda_b Pe$

that the contributions proportional to λ_b are neglected in this literature. To get a complete expression of decay amplitude and then analyze the CP asymmetries, the neglected terms must be found back, just like we have done in this work. One can check that the CP violation sum rules associated with the complete interchange of d and s quarks works in various types of decay.

3.3 A universal law for CP violation sum rules in the charm sector

From the above discussions, one can find the CP violation sum rules in the $SU(3)_F$ limit are always associated with a complete interchange of d and s quarks. In this subsection, we illustrate that it is a universal law in the charm sector.

Firstly, the complete interchange of $d \leftrightarrow s$ quarks in initial and final states leads to the interchange of $d \leftrightarrow s$ in operators O_k^{ij}. It can be understood in following argument. In the IRA approach, each decay amplitude connects to one invariant tensor [in which all covariant indices are contracted with contravariant indices; see Eq. (24) for example], no matter charm meson or baryon decays and two- or multi-body decays. If a complete interchange of d and s quarks is performed in the tensors corresponding to initial and final states, the complete interchange of $d \leftrightarrow s$ must be performed in tensor H_{ij}^k in order to keep all covariant and contravariant indices contracted. From Eq. (11), one can find H_{ij}^k corresponds to O_k^{ij} one by one. So the d and s quark constituents in opera-

tors O_k^{ij} must be interchanged. In physics, if the quark constituents of all initial and final particles in one decay channel are replaced by $d \to s$ and $s \to d$, the quark constituents in the effective weak vertexes should be replaced by $d \to s$ and $s \to d$ also. The operators O_k^{ij} are abstracted from the effective weak vertexes, so the quark constituents of operators O_k^{ij} transform as a complete interchange of $d \leftrightarrow s$.

Secondly, the interchange of $d \leftrightarrow s$ in operators O_k^{ij} leads to the decay amplitudes proportional to λ_d/λ_s are connected by the interchange of $\lambda_d \leftrightarrow \lambda_s$ and the decay amplitudes proportional to λ_b are the same in the flavor $SU(3)$ symmetry. The contributions proportional to λ_d/λ_s in SCS decays are induced by following operators in the $SU(3)$ irreducible representation:

$$O(6)_{23}, \quad O(\overline{15})_2^{12}, \quad O(\overline{15})_3^{13}. \tag{53}$$

Under the interchange of $d \leftrightarrow s$, these operators are transformed as

$$O(6)_{23} \leftrightarrow -O(6)_{23}, \quad O(\overline{15})_2^{12} \leftrightarrow O(\overline{15})_3^{13}. \tag{54}$$

These properties can be read from the explicit $SU(3)$ decomposition of O_k^{ij}; see Appendix A. The corresponding CKM matrix elements then transform as

$$H(6)^{23} \leftrightarrow -H(6)^{23}, \quad H(\overline{15})_{12}^2 \leftrightarrow H(\overline{15})_{13}^3. \tag{55}$$

According to Eq. (15), Eq. (55) equals

$$\frac{1}{4}(\lambda_d - \lambda_s) \leftrightarrow -\frac{1}{4}(\lambda_d - \lambda_s), \quad \frac{3}{8}\lambda_d - \frac{1}{8}\lambda_s \leftrightarrow \frac{3}{8}\lambda_s - \frac{1}{8}\lambda_d.$$
(56)

One can find that Eq. (56) is equivalent to the interchange of $\lambda_d \leftrightarrow \lambda_s$. The contributions proportional to λ_b in the SCS decays are induced by the following operators:

$$O(\overline{15})_1^{11}, \quad O(\overline{3})^1, \quad O(\overline{3}')^1.$$
(57)

Form Appendix A, it is found that these operators are invariable under the interchange of $d \leftrightarrow s$, as are the corresponding CKM matrix elements.

Thirdly, if two decay channels have the relations that their decay amplitudes proportional to λ_d/λ_s are connected by the interchange of $\lambda_d \leftrightarrow \lambda_s$ and the decay amplitudes proportional to λ_b are the same, the sum of their direct CP asymmetries is zero in the $SU(3)_F$ limit under the approximation in Eq. (29). For one decay mode with amplitude of

$$\begin{aligned}\mathcal{A}(i \rightarrow f) &= \lambda_d A + \lambda_s B + \lambda_b C \\ &= -(\lambda_s + |\lambda_b| e^{i\phi}) |A| e^{i\delta_A} + \lambda_s |B| e^{i\delta_B} \\ &\quad + |\lambda_b| e^{i\phi} |C| e^{i\delta_C},\end{aligned}$$
(58)

its CP asymmetry in the order of $\mathcal{O}(\lambda_b)$ is derived as

$$\begin{aligned}A_{CP}^{\text{dir}}(i \rightarrow f) &= \frac{|\lambda_d A + \lambda_s B + \lambda_b C|^2 - |\lambda_d^* A + \lambda_s^* B + \lambda_b^* C|^2}{|\lambda_d A + \lambda_s B + \lambda_b C|^2 + |\lambda_d^* A + \lambda_s^* B + \lambda_b^* C|^2} \\ &\simeq 2\frac{|\lambda_b|}{\lambda_s}\frac{|AB|\sin(\delta_A - \delta_B) - |AC|\sin(\delta_A - \delta_C) + |BC|\sin(\delta_B - \delta_C)}{|A|^2 + |B|^2 - 2|AB|\cos(\delta_A - \delta_B)} \sin\phi.\end{aligned}$$
(59)

For one decay mode with amplitude of

$$\mathcal{A}(i' \rightarrow f') = \lambda_d B + \lambda_s A + \lambda_b C,$$
(60)

which is connected to Eq. (58) by $\lambda_d \leftrightarrow \lambda_s$, its CP asymmetry in the order of $\mathcal{O}(\lambda_b)$ is derived as

$$\begin{aligned}&A_{CP}^{\text{dir}}(i' \rightarrow f') \\ &\simeq -2\frac{|\lambda_b|}{\lambda_s}\frac{|AB|\sin(\delta_A - \delta_B) - |AC|\sin(\delta_A - \delta_C) + |BC|\sin(\delta_B - \delta_C)}{|A|^2 + |B|^2 - 2|AB|\cos(\delta_A - \delta_B)} \sin\phi.\end{aligned}$$
(61)

It is apparent that

$$A_{CP}^{\text{dir}}(i \rightarrow f) + A_{CP}^{\text{dir}}(i' \rightarrow f') \simeq 0.$$
(62)

Based on the above analysis, a useful method to search for the CP violation sum rules with two charmed hadron decay channels is proposed:

- For one type of charmed hadron decay, write down all the SCS decay modes in which the associated hadrons have definite U-spin quantum numbers;
- For each decay mode, find the corresponding decay mode in the complete interchange of $d \leftrightarrow s$;
- If there are two decay modes connected by the interchange of $d \leftrightarrow s$, the sum of their direct CP asymmetries is zero in the $SU(3)_F$ limit;
- If the corresponding decay mode is itself, the direct CP asymmetry in this mode is zero in the $SU(3)_F$ limit.

4 Results and discussions

With the method proposed in Sect. 3, one can find many sum rules for CP asymmetries in charm meson/baryon decays. There are hundreds of sum rules for CP asymmetries in the singly and doubly charmed baryon decays. We are not going to list all the CP violation sum rules, but only present some of them as examples.

Under the complete interchange of $d \leftrightarrow s$, the light octet baryons are interchanged as

$$p \leftrightarrow \Sigma^+, \quad n \leftrightarrow \Xi^0, \quad \Sigma^- \leftrightarrow \Xi^-.$$
(63)

The sum rules for CP asymmetries in charmed baryon decays into one pseudoscalar meson and one octet baryon are

$$A_{CP}^{\text{dir}}(\Lambda_c^+ \rightarrow \Sigma^+ K^0) + A_{CP}^{\text{dir}}(\Xi_c^+ \rightarrow p\overline{K}^0) = 0,$$
(64)

$$A_{CP}^{\text{dir}}(\Lambda_c^+ \rightarrow n\pi^+) + A_{CP}^{\text{dir}}(\Xi_c^+ \rightarrow \Xi^0 K^+) = 0,$$
(65)

$$A_{CP}^{\text{dir}}(\Xi_c^0 \rightarrow \Sigma^- \pi^+) + A_{CP}^{\text{dir}}(\Xi_c^0 \rightarrow \Xi^- K^+) = 0,$$
(66)

$$A^{\text{dir}}_{CP}(\Xi^0_c \to n\overline{K}^0) + A^{\text{dir}}_{CP}(\Xi^0_c \to \Xi^0 K^0) = 0, \quad (67)$$

$$A^{\text{dir}}_{CP}(\Xi^0_c \to \Sigma^+\pi^-) + A^{\text{dir}}_{CP}(\Xi^0_c \to pK^-) = 0. \quad (68)$$

Under the complete interchange of $d \leftrightarrow s$, the doubly charmed baryons are interchanged as

$$\Xi^{++}_{cc} \leftrightarrow \Xi^{++}_{cc}, \quad \Xi^+_{cc} \leftrightarrow \Omega^+_{cc}. \quad (69)$$

The sum rules for CP asymmetries in doubly charmed baryon decays into one pseudoscalar meson and one charmed triplet baryon are

$$A^{\text{dir}}_{CP}(\Xi^{++}_{cc} \to \Lambda^+_c\pi^+) + A^{\text{dir}}_{CP}(\Xi^{++}_{cc} \to \Xi^+_c K^+) = 0, \quad (70)$$

$$A^{\text{dir}}_{CP}(\Xi^+_{cc} \to \Xi^+_c K^0) + A^{\text{dir}}_{CP}(\Omega^+_{cc} \to \Lambda^+_c \overline{K}^0) = 0, \quad (71)$$

$$A^{\text{dir}}_{CP}(\Xi^+_{cc} \to \Xi^0_c K^+) + A^{\text{dir}}_{CP}(\Omega^+_{cc} \to \Xi^0_c \pi^+) = 0. \quad (72)$$

Under the complete interchange of $d \leftrightarrow s$, the charmed sextet baryons are interchanged as

$$\Sigma^+_c \leftrightarrow \Xi^{*+}_c, \quad \Sigma^{++}_c \leftrightarrow \Sigma^{++}_c,$$
$$\Xi^{*0}_c \leftrightarrow \Xi^{*0}_c, \quad \Sigma^0_c \leftrightarrow \Omega^0_c. \quad (73)$$

The sum rules for CP asymmetries in doubly charmed baryon decays into one pseudoscalar meson and one charmed sextet baryon are

$$A^{\text{dir}}_{CP}(\Xi^{++}_{cc} \to \Sigma^+_c\pi^+) + A^{\text{dir}}_{CP}(\Xi^{++}_{cc} \to \Xi^{*+}_c K^+) = 0, \quad (74)$$

$$A^{\text{dir}}_{CP}(\Xi^+_{cc} \to \Sigma^{++}_c\pi^-) + A^{\text{dir}}_{CP}(\Omega^+_{cc} \to \Sigma^{++}_c K^-) = 0, \quad (75)$$

$$A^{\text{dir}}_{CP}(\Xi^+_{cc} \to \Sigma^0_c\pi^+) + A^{\text{dir}}_{CP}(\Omega^+_{cc} \to \Omega^0_c K^+) = 0, \quad (76)$$

$$A^{\text{dir}}_{CP}(\Xi^+_{cc} \to \Xi^{*+}_c K^0) + A^{\text{dir}}_{CP}(\Omega^+_{cc} \to \Sigma^+_c \overline{K}^0) = 0, \quad (77)$$

$$A^{\text{dir}}_{CP}(\Xi^+_{cc} \to \Xi^{*0}_c K^+) + A^{\text{dir}}_{CP}(\Omega^+_{cc} \to \Xi^{*0}_c \pi^+) = 0. \quad (78)$$

With the interchange rules mentioned above, the CP violation sum rules in doubly charmed baryon decays into one charmed meson and one octet baryon are

$$A^{\text{dir}}_{CP}(\Xi^{++}_{cc} \to \Sigma^+ D^+_s) + A^{\text{dir}}_{CP}(\Xi^{++}_{cc} \to pD^+) = 0, \quad (79)$$

$$A^{\text{dir}}_{CP}(\Xi^+_{cc} \to pD^0) + A^{\text{dir}}_{CP}(\Omega^+_{cc} \to \Sigma^+ D^0) = 0, \quad (80)$$

$$A^{\text{dir}}_{CP}(\Xi^+_{cc} \to nD^+) + A^{\text{dir}}_{CP}(\Omega^+_{cc} \to \Xi^0 D^+_s) = 0. \quad (81)$$

The CP violation sum rules in doubly charmed baryon decays into one charmed meson and one decuplet baryon are

$$A^{\text{dir}}_{CP}(\Xi^{++}_{cc} \to \Delta^+ D^+) + A^{\text{dir}}_{CP}(\Xi^{++}_{cc} \to \Sigma^{*+} D^+_s) = 0, \quad (82)$$

$$A^{\text{dir}}_{CP}(\Xi^+_{cc} \to \Delta^+ D^0) + A^{\text{dir}}_{CP}(\Omega^+_{cc} \to \Sigma^{*+} D^0) = 0, \quad (83)$$

$$A^{\text{dir}}_{CP}(\Xi^+_{cc} \to \Delta^0 D^+) + A^{\text{dir}}_{CP}(\Omega^+_{cc} \to \Xi^{*0} D^+_s) = 0, \quad (84)$$

$$A^{\text{dir}}_{CP}(\Xi^+_{cc} \to \Sigma^{*0} D^+_s) + A^{\text{dir}}_{CP}(\Omega^+_{cc} \to \Sigma^{*0} D^+) = 0. \quad (85)$$

For three-body decays, we only list the CP violation sum rules in charmed baryon decays into one octet baryon and two pseudoscalar mesons as examples:

$$A^{\text{dir}}_{CP}(\Lambda^+_c \to pK^-K^+) + A^{\text{dir}}_{CP}(\Xi^+_c \to \Sigma^+\pi^-\pi^+) = 0, \quad (86)$$

$$A^{\text{dir}}_{CP}(\Lambda^+_c \to p\pi^-\pi^+) + A^{\text{dir}}_{CP}(\Xi^+_c \to \Sigma^+ K^-K^+) = 0, \quad (87)$$

$$A^{\text{dir}}_{CP}(\Lambda^+_c \to \Sigma^+\pi^-K^+) + A^{\text{dir}}_{CP}(\Xi^+_c \to pK^-\pi^+) = 0, \quad (88)$$

$$A^{\text{dir}}_{CP}(\Lambda^+_c \to \Sigma^-\pi^+K^+) + A^{\text{dir}}_{CP}(\Xi^+_c \to \Xi^-K^+\pi^+) = 0, \quad (89)$$

$$A^{\text{dir}}_{CP}(\Lambda^+_c \to nK^+\overline{K}^0) + A^{\text{dir}}_{CP}(\Xi^+_c \to \Xi^0\pi^+K^0) = 0, \quad (90)$$

$$A^{\text{dir}}_{CP}(\Xi^0_c \to \Sigma^+ K^-K^0) + A^{\text{dir}}_{CP}(\Xi^0_c \to p\pi^-\overline{K}^0) = 0, \quad (91)$$

$$A^{\text{dir}}_{CP}(\Xi^0_c \to \Sigma^- K^+\overline{K}^0) + A^{\text{dir}}_{CP}(\Xi^0_c \to \Xi^-\pi^+K^0) = 0, \quad (92)$$

$$A^{\text{dir}}_{CP}(\Xi^0_c \to \Xi^0\pi^-K^+) + A^{\text{dir}}_{CP}(\Xi^0_c \to nK^-\pi^+) = 0. \quad (93)$$

The first three sum rules are the same as in [73]. In all above sum rules, the pseudoscalar mesons can be replaced by vector mesons by the following correspondence:

$$\pi^+ \to \rho^+, \quad \pi^- \to \rho^-, \quad K^+ \to K^{*+}, \quad K^- \to K^{*-},$$
$$K^0 \to K^{*0}, \quad \overline{K}^0 \to \overline{K}^{*0}. \quad (94)$$

The CP violation sum rules are derived in the U-spin limit. Considering the U-spin breaking, the CP violation sum rules are no longer valid, as pointed out in [73]. Since the U-spin breaking is sizable in the charm sector, the CP violation sum rules might not be reliable. But they indicate that the CP asymmetries in some decay modes have opposite sign and then can be used to find some promising observables in experiments. In charmed meson decays, Eq. (4) makes the

two CP asymmetries in observable $\Delta A_{CP} \equiv A_{CP}(D^0 \to K^+ K^-) - A_{CP}(D^0 \to \pi^+\pi^-)$ constructive. Similarly, one can use the CP violation sum rules in charmed baryon decays to construct some observables in which two CP asymmetries are constructive. Some observables are selected for experimental discretion:

$$\Delta A_{CP}^{\text{baryon},1} = A_{CP}(\Lambda_c^+ \to \Sigma^+ K^{*0}) - A_{CP}(\Xi_c^+ \to p\overline{K}^{*0}), \tag{95}$$

$$\Delta A_{CP}^{\text{baryon},2} = A_{CP}(\Xi_c^0 \to \Sigma^+ \pi^-) - A_{CP}(\Xi_c^0 \to pK^-), \tag{96}$$

$$\Delta A_{CP}^{\text{baryon},3} = A_{CP}(\Lambda_c^+ \to \Delta^{++} \pi^-) - A_{CP}(\Xi_c^+ \to \Delta^{++} K^-), \tag{97}$$

$$\Delta A_{CP}^{\text{baryon},4} = A_{CP}(\Lambda_c^+ \to \Sigma^+ \pi^- K^+) - A_{CP}(\Xi_c^+ \to pK^-\pi^+). \tag{98}$$

If the contributions proportional to λ_b are neglected, the decay amplitudes of the two channels connected by the interchange of $d \leftrightarrow s$ are the same (except for a minus sign) in the $SU(3)_F$ limit [see Eqs. (58) and (60)]. One can use this relation to predict the branching fractions. As an example, we estimate the branching fraction of $\Xi_c^+ \to pK^-\pi^+$. The integration over the phase space of the three-body decay $\mathcal{B}_c \to \mathcal{B} M_1 M_2$ relies on the equation of [2]

$$\Gamma(\mathcal{B}_c \to \mathcal{B} M_1 M_2)$$
$$= \int_{m_{12}^2} \int_{m_{23}^2} \frac{|\mathcal{A}(\mathcal{B}_c \to \mathcal{B} M_1 M_2)|^2}{32 m_{\mathcal{B}_c}^3} \mathrm{d}m_{12}^2 \mathrm{d}m_{23}^2, \tag{99}$$

where $m_{12}^2 = (p_{M_1} + p_{M_2})^2$ and $m_{23}^2 = (p_{M_2} + p_{\mathcal{B}})^2$. With the experimental data given in [2],

$$Br(\Lambda_c^+ \to \Sigma^+ \pi^- K^+) = (2.1 \pm 0.6) \times 10^{-3}, \tag{100}$$

and the relation

$$|\mathcal{A}(\Xi_c^+ \to pK^-\pi^+)| \simeq |\mathcal{A}(\Lambda_c^+ \to \Sigma^+ \pi^- K^+)|, \tag{101}$$

the branching fraction of $\Xi_c^+ \to pK^-\pi^+$ decay is predicted to be

$$Br(\Xi_c^+ \to pK^-\pi^+) = (1.7 \pm 0.5)\%. \tag{102}$$

One can find the branching fraction $Br(\Xi_c^+ \to pK^-\pi^+)$ is larger than $Br(\Lambda_c^+ \to \Sigma^+ \pi^- K^+)$ because of the larger phase space and the longer lifetime of Ξ_c^+. But it is still smaller than the predictions given in [66,70]. In the above estimation, only the decay amplitude is obtained by the U-spin symmetry. The phase space is calculated without approximation. It is plausible since the global fit in Refs. [60,61,64,68,70,74,81] give the reasonable estimations for branching fractions of charmed baryon decays. The uncertainty in Eq. (102) is dominated by the branching fraction of

$\Lambda_c^+ \to \Sigma^+ \pi^- K^+$ decay and does not include the U-spin breaking effects. It is not available to estimate the U-spin breaking effects at the current stage since the understanding of the dynamics of charmed baryon decay is still a challenge. Some discussions of the uncertainty induced by U-spin breaking can be found in [66].

With the method introduced in [66], and the LHCb data [102]

$$\frac{f_{\Xi_b}}{f_{\Lambda_b}} \cdot \frac{Br(\Xi_b^0 \to \Xi_c^+ \pi^-)}{Br(\Lambda_b^0 \to \Lambda_c^+ \pi^-)} \cdot \frac{Br(\Xi_c^+ \to pK^-\pi^+)}{Br(\Lambda_c^+ \to pK^-\pi^+)}$$
$$= (1.88 \pm 0.04 \pm 0.03) \times 10^{-2}, \tag{103}$$

the fragmentation–fraction ratio f_{Ξ_b}/f_{Λ_b} is determined to be

$$f_{\Xi_b}/f_{\Lambda_b} = 0.065 \pm 0.020. \tag{104}$$

Recent measurement confirmed this result [103]. Our result is consistent with the one obtained via $\Lambda_b^0 \to J/\psi \Lambda^0$ [104], $f_{\Xi_b}/f_{\Lambda_b} = 0.11 \pm 0.03$, the one via $\Xi_b^- \to J/\psi \Xi^-$ [105], $f_{\Xi_b}/f_{\Lambda_b} = 0.108 \pm 0.034$, and the one via the diquark model for $\Xi_b^- \to \Lambda_b^0 \pi^-$ [106] using the LHCb data [107], $f_{\Xi_b}/f_{\Lambda_b} = 0.08 \pm 0.03$. The detailed comparison for different methods of estimating f_{Ξ_b}/f_{Λ_b} can be found in [66].

5 Summary

In summary, we find that if two singly Cabibbo-suppressed decay modes of charmed hadrons are connected by a complete interchange of d and s quarks, the sum of their direct CP asymmetries is zero in the flavor $SU(3)$ limit. According to this conclusion, many CP violation sum rules can be found in the doubly and singly charmed baryon decays. Some of them could help to find better observables in experiments. As byproducts, the branching fraction $Br(\Xi_c^+ \to pK^-\pi^+)$ is predicted to be $(1.7 \pm 0.5)\%$ in the U-spin limit, and the fragmentation–fraction ratio is determined as $f_{\Xi_b}/f_{\Lambda_b} = 0.065 \pm 0.020$.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This manuscript is a theoretical analysis without available data.]

Appendix A: $SU(3)$ decomposition of operator O_k^{ij}

All components of the $SU(3)$ decomposition in Eq. (14) are listed in the following.

$\bar{3}$ presentation:

$$O(\bar{3})^1 = (\bar{u}u)(\bar{u}c) + (\bar{u}d)(\bar{d}c) + (\bar{u}s)(\bar{s}c),$$
$$O(\bar{3})^2 = (\bar{d}u)(\bar{u}c) + (\bar{d}d)(\bar{d}c) + (\bar{d}s)(\bar{s}c),$$
$$O(\bar{3})^3 = (\bar{s}u)(\bar{u}c) + (\bar{s}d)(\bar{d}c) + (\bar{s}s)(\bar{s}c). \tag{A1}$$

$\bar{3}'$ presentation:

$$O(\bar{3}')^1 = (\bar{u}u)(\bar{u}c) + (\bar{d}d)(\bar{u}c) + (\bar{s}s)(\bar{u}c),$$
$$O(\bar{3}')^2 = (\bar{u}u)(\bar{d}c) + (\bar{d}d)(\bar{d}c) + (\bar{s}s)(\bar{d}c),$$
$$O(\bar{3}')^3 = (\bar{u}u)(\bar{s}c) + (\bar{d}d)(\bar{s}c) + (\bar{s}s)(\bar{s}c). \tag{A2}$$

6 presentation:

$$O(6)_{11} = \frac{1}{2}[(\bar{d}u)(\bar{s}c) - (\bar{s}u)(\bar{d}c)],$$
$$O(6)_{22} = \frac{1}{2}[(\bar{s}d)(\bar{u}c) - (\bar{u}d)(\bar{s}c)],$$
$$O(6)_{33} = \frac{1}{2}[(\bar{u}s)(\bar{d}c) - (\bar{d}s)(\bar{u}c)],$$
$$O(6)_{12} = \frac{1}{4}[(\bar{s}u)(\bar{u}c) - (\bar{u}u)(\bar{s}c) + (\bar{d}d)(\bar{s}c) - (\bar{s}d)(\bar{d}c)],$$
$$O(6)_{23} = \frac{1}{4}[(\bar{u}d)(\bar{d}c) - (\bar{d}d)(\bar{u}c) + (\bar{s}s)(\bar{u}c) - (\bar{u}s)(\bar{s}c)],$$
$$O(6)_{31} = \frac{1}{4}[(\bar{d}s)(\bar{s}c) - (\bar{s}s)(\bar{d}c) + (\bar{u}u)(\bar{d}c) - (\bar{d}u)(\bar{u}c)]. \tag{A3}$$

$\overline{15}$ presentation:

$$O(\overline{15})_1^{11} = \frac{1}{2}(\bar{u}u)(\bar{u}c) - \frac{1}{4}[(\bar{u}s)(\bar{s}c) + (\bar{u}d)(\bar{d}c) + (\bar{d}d)(\bar{u}c) + (\bar{s}s)(\bar{u}c)],$$
$$O(\overline{15})_2^{22} = \frac{1}{2}(\bar{d}d)(\bar{d}c) - \frac{1}{4}[(\bar{d}u)(\bar{u}c) + (\bar{d}s)(\bar{s}c) + (\bar{u}u)(\bar{d}c) + (\bar{s}s)(\bar{d}c)],$$
$$O(\overline{15})_3^{33} = \frac{1}{2}(\bar{s}s)(\bar{s}c) - \frac{1}{4}[(\bar{s}u)(\bar{u}c) + (\bar{s}d)(\bar{d}c) + (\bar{u}u)(\bar{s}c) + (\bar{d}d)(\bar{s}c)],$$
$$O(\overline{15})_1^{23} = \frac{1}{2}[(\bar{d}u)(\bar{s}c) + (\bar{s}u)(\bar{d}c)],$$
$$O(\overline{15})_2^{13} = \frac{1}{2}[(\bar{s}d)(\bar{u}c) + (\bar{u}d)(\bar{s}c)],$$
$$O(\overline{15})_3^{12} = \frac{1}{2}[(\bar{u}s)(\bar{d}c) + (\bar{d}s)(\bar{u}c)],$$
$$O(\overline{15})_2^{11} = (\bar{u}d)(\bar{u}c), \quad O(\overline{15})_3^{11} = (\bar{u}s)(\bar{u}c),$$

$$O(\overline{15})_1^{22} = (\bar{d}u)(\bar{d}c),$$
$$O(\overline{15})_3^{22} = (\bar{d}s)(\bar{d}c), \quad O(\overline{15})_1^{33} = (\bar{s}u)(\bar{s}c),$$
$$O(\overline{15})_2^{33} = (\bar{s}d)(\bar{s}c),$$
$$O(\overline{15})_1^{21} = \frac{3}{8}[(\bar{u}u)(\bar{d}c) + (\bar{d}u)(\bar{u}c)] - \frac{1}{4}(\bar{d}d)(\bar{d}c) - \frac{1}{8}[(\bar{d}s)(\bar{s}c) + (\bar{s}s)(\bar{d}c)],$$
$$O(\overline{15})_2^{12} = \frac{3}{8}[(\bar{u}d)(\bar{d}c) + (\bar{d}d)(\bar{u}c)] - \frac{1}{4}(\bar{u}u)(\bar{u}c) - \frac{1}{8}[(\bar{u}s)(\bar{s}c) + (\bar{s}s)(\bar{u}c)],$$
$$O(\overline{15})_1^{31} = \frac{3}{8}[(\bar{u}u)(\bar{s}c) + (\bar{s}u)(\bar{u}c)] - \frac{1}{4}(\bar{s}s)(\bar{s}c) - \frac{1}{8}[(\bar{s}d)(\bar{d}c) + (\bar{d}d)(\bar{s}c)],$$
$$O(\overline{15})_3^{13} = \frac{3}{8}[(\bar{u}s)(\bar{s}c) + (\bar{s}s)(\bar{u}c)] - \frac{1}{4}(\bar{u}u)(\bar{u}c) - \frac{1}{8}[(\bar{u}d)(\bar{d}c) + (\bar{d}d)(\bar{u}c)],$$
$$O(\overline{15})_2^{32} = \frac{3}{8}[(\bar{d}d)(\bar{s}c) + (\bar{s}d)(\bar{d}c)] - \frac{1}{4}(\bar{s}s)(\bar{s}c) - \frac{1}{8}[(\bar{s}u)(\bar{u}c) + (\bar{u}u)(\bar{s}c)],$$
$$O(\overline{15})_3^{23} = \frac{3}{8}[(\bar{d}s)(\bar{s}c) + (\bar{s}s)(\bar{d}c)] - \frac{1}{4}(\bar{d}d)(\bar{d}c) - \frac{1}{8}[(\bar{d}u)(\bar{u}c) + (\bar{u}u)(\bar{d}c)]. \tag{A4}$$

The above results are consistent with Ref. [86]. The operators $O(6)_{ij}$ are symmetric in the interchange of their two indices and can be written as $O(6)_k^{ij}$ by contracting with the totally antisymmetric Levi–Civita tensor, $O(6)_k^{ij} = \epsilon^{ijl}O(6)_{lk}$. There are 18 operators in the irreducible representation $\overline{15}$, but only 15 of them are independent because of the following equations:

$$O(\overline{15})_1^{11} = -[O(\overline{15})_2^{12} + O(\overline{15})_3^{13}],$$
$$O(\overline{15})_2^{22} = -[O(\overline{15})_1^{21} + O(\overline{15})_3^{23}],$$
$$O(\overline{15})_3^{33} = -[O(\overline{15})_1^{31} + O(\overline{15})_2^{32}]. \tag{A5}$$

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