Effective theory for a heavy scalar coupled to the SM via vector-like quarks

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Received: 20 February 2019 / Accepted: 6 April 2019 / Published online: 24 April 2019
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Abstract We illustrate the application of the recently developed SCET_{BSM} framework in the context of a specific model, in which the Standard Model (SM) is supplemented by a heavy scalar S and three generations of heavy, vector-like quarks Ψ . We construct the appropriate effective field theory for two-body decays of S into SM particles. We explicitly compute the Wilson coefficients of the SCET_{BSM} operators appearing at leading and next-to-leading order (NLO) in an expansion in powers of v/M_S , as well as for a subset of operators arising at NNLO, retaining the full dependence on the ratio M_S/M_Ψ . For the phenomenologically most relevant decay channels of the heavy scalar, we study the impact of resummation effects of Sudakov logarithms on the decay rates.

1 Introduction

Following the discovery of a new particle with a mass far above the electroweak scale $v \approx 246 \text{ GeV}$, a program for studying its couplings to the Standard Model (SM) would be of highest priority. In the likely situation where the new resonance is the first member of a richer sector of new physics, the appropriate way to study its decay and production processes must rely on an effective field theory (EFT) framework. The main reason is that other, yet undiscovered heavy particles can couple to both the SM and the new resonance S and hence affect its interactions. Secondly, the large scale hierarchy between the mass of the heavy resonance and the weak scale, which (roughly) sets the masses of the SM particles, introduces large Sudakov double logarithms in the calculation of decay rates and production cross sections, which must be resummed to all orders of perturbation theory. Finally, for the most interesting case where the mass of the new reso-

nance is close to the masses M_i of yet undiscovered states, there is short-distance physics associated with both scales, which must be disentangled from the longer-distance physics associated with the electroweak scale.

We have shown in [1] that the appropriate EFT to deal with this scenario must be based on an effective Lagrangian built out of non-local light-ray operators defined in soft-collinear effective theory (SCET) [2–5]. Our theory called SCET_{BSM} provides a systematic expansion of the decay amplitudes of the new heavy particle in powers of $\lambda = v/M_S \ll 1$. For the case of a scalar resonance S transforming as a singlet under the SM gauge group, we have constructed the complete operator basis at leading and subleading order in the expansion, corresponding to operators of $\mathcal{O}(\lambda^2)$ and $\mathcal{O}(\lambda^3)$, respectively.

The leading-order effective Lagrangian for two-body decays of S consists of operators in which S is coupled to two effective bosonic fields, which describe so-called collinear particles moving along directions $\mathbf{n}_1$ and $\mathbf{n}_2$, which point back-to-back in the rest frame of the decaying resonance. One has [1]

$$\mathcal{L}_{\text{eff}}^{(2)} = M C_{\phi\phi}(M, M_S) O_{\phi\phi} + M \sum_A \left[C_{AA}(M, M_S) O_{AA} + \tilde{C}_{AA}(M, M_S) \tilde{O}_{AA} \right]. \quad (1)$$

Here M denotes the characteristic mass scale of unresolved new heavy particles. The sum extends over the three gauge groups of the SM: $A = B$ for $U(1)_Y$, $A = W$ for $SU(2)_L$, and $A = G$ for $SU(3)_C$. The relevant SCET_{BSM} operators have the form (a summation over the group index a is understood for non-abelian fields)

$$\begin{aligned} O_{\phi\phi} &= S_v (\Phi_{n_1}^\dagger \Phi_{n_2} + \Phi_{n_2}^\dagger \Phi_{n_1}), \\ O_{AA} &= S_v g_{\mu\nu}^\perp \mathcal{A}_{n_1}^{\mu,a} \mathcal{A}_{n_2}^{v,a}, \\ \tilde{O}_{AA} &= S_v \epsilon_{\mu\nu}^\perp \mathcal{A}_{n_1}^{\mu,a} \mathcal{A}_{n_2}^{v,a}. \end{aligned} \quad (2)$$

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Here S_v is an effective field for the heavy resonance defined as in heavy-quark effective theory [6–9], with v denoting its 4-velocity. The reference vectors n_1 and n_2 indicate the directions of large momentum flow of the final-state particles. The effective fields consist of so-called “gauge covariant building blocks” [10, 11] Φ and $\mathcal{A}$ containing the Higgs doublet and the transversely polarized gauge fields, respectively, dressed up with Wilson lines in the appropriate representation of the gauge group. The Lorentz indices of the gauge fields can be contracted with either the symmetric tensor $g_{\mu\nu}^\perp$ or the antisymmetric tensor $\epsilon_{\mu\nu}^\perp$, defined in the plane orthogonal to n_1 and n_2 . Note that the different fields in the operators in (2) interact only via soft quanta, since there is only a single collinear field in each sector; hard interactions with virtualities of order M_S^2 or M^2 are integrated out in the construction of the effective Lagrangian and are contained in the Wilson coefficient functions.

Note the important fact that the Wilson coefficients in (1) depend on both, the mass M_S of the scalar resonance and the parameter M representing the typical mass scale of other, yet undiscovered heavy particles. As we shall see below, in this way our effective theory sums infinite towers of local operators in the conventional EFT approach. In some sense, the Wilson coefficients in our Lagrangian can be regarded as form factors depending on the large momentum transfers $q^2 = \mathcal{O}(M_S^2)$ flowing through Feynman diagrams, which can resolve the small non-localities corresponding to exchanges of the heavy VLQs.

At subleading order in power counting the operator basis contains five different types of operators, all of which consist of fermion bilinears along with a Higgs doublet or a gauge field, see Sect. 3. In Sect. 5 we study some aspects of the extension of the effective Lagrangian to $\mathcal{O}(\lambda^4)$, which is necessary to describe the two-body decay $S \rightarrow Zh$.

In this work, we illustrate the SCET_{BSM} approach by considering a concrete extension of the SM featuring a heavy, gauge-singlet scalar field S along with three generations of heavy, vector-like quarks. Vector-like fermions play an important role in models of partial compositeness [12], as realized e.g. in composite-Higgs models (see e.g. [13–15]) and scenarios featuring a warped extra dimension [16–18]. Extensions of the SM featuring both vector-like fermions and a singlet scalar are among the popular simplified models for dark matter (see e.g. [19, 20]).

2 High-energy extension of the SM

The benchmark model we explore in this paper is an extension of the SM by a real scalar S , transforming as a singlet under the SM gauge group, and (three generations of) a vector-like quark (VLQ) doublet Ψ , transforming as

$(\mathbf{3}, \mathbf{2})_{1/6}$. Besides the Higgs portal, the VLQs mediate the renormalizable interactions between the SM and the new sector. We assume that the mass of the scalar and the masses of the VLQs are both much heavier than the electroweak scale $v \approx 246$ GeV. The most general Lagrangian of our model is

$$\begin{aligned} \mathcal{L}_{UV} = & \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - \frac{M_S^2}{2} S^2 - \frac{\lambda_3}{3!} S^3 - \frac{\lambda_4}{4!} S^4 \\ & + \bar{\Psi}(i\not{D} - \mathbf{M})\Psi - (\bar{\Psi}\tilde{\phi} \mathbf{G}_u u_R + \bar{\Psi}\phi \mathbf{G}_d d_R + \text{h.c.}) \\ & - \kappa_1 S \phi^\dagger \phi - \frac{\kappa_2}{2} S^2 \phi^\dagger \phi \\ & - S \bar{\Psi}(\mathbf{X} - i\gamma_5 \tilde{\mathbf{X}})\Psi - S(\bar{\Psi} \mathbf{V}_Q Q_L + \text{h.c.}). \end{aligned} \quad (3)$$

The second line contains the couplings of the VLQs to SM fields, where $\tilde{\phi}_a = \epsilon_{ab}\phi_b^*$. There is no need to include the gauge-invariant terms $\bar{\Psi}i\gamma_5 \tilde{\mathbf{M}}\Psi - (\bar{\Psi} \mathbf{G}_Q Q_L + \text{h.c.})$, since they can be removed by unitary transformations of the quark fields. The terms in the third line contain the portal couplings of the heavy scalar to the Higgs field. Note that the couplings κ_1 and λ_3 have mass dimension 1. The interactions in the last line describe the couplings of S to the VLQs and SM quarks. We assume that the parameters λ_i in the scalar potential are chosen such that the scalar field S does not acquire a vacuum expectation value. For the same reason, we have omitted the tadpole term $\lambda_1 S$ from the potential.

Boldface symbols in (3) denote matrices in generation space. The matrices $\mathbf{G}_{u,d}$ and $\mathbf{V}_Q$ are arbitrary complex matrices, while $\mathbf{M}$, $\mathbf{X}$ and $\tilde{\mathbf{X}}$ are hermitian. Without loss of generality we work in the mass basis for the VLQs, where $\mathbf{M}$ is a real, positive diagonal matrix. For simplicity, we assume that the three mass eigenvalues are degenerate, i.e. $\mathbf{M} = M \mathbf{1}$. The common mass of the VLQs is then identified with the “new physics scale” M in (1).

Suppose that the heavy scalar S has been discovered, while the VLQs have not yet been observed experimentally. Our goal is to construct an EFT describing the interactions of S with SM particles. The appropriate EFT in such a scenario is the SCET_{BSM} [1]. It would be straightforward to extend our analysis to the case of vector-like fermions with different quantum numbers. However, in order to keep the presentation as transparent as possible, we find it advantageous to consider the simplest case of a single type of VLQ.

3 Tree-level matching onto SCET_{BSM}

When the full theory in (3) is matched onto the SCET_{BSM} two types of short-distance modes are integrated out: First, one removes virtual exchanges of the VLQs, which do not appear as external states in the EFT (since these particles are assumed to be yet undiscovered). In addition, one integrates out off-shell fluctuations of the SM fields as well as of the scalar field S carrying virtualities of order $q^2 \sim M_S^2$. While

the first step is standard, the second step differentiates the SCET_{B_{SM}} approach from local EFTs such as the SMEFT [21–25].

3.1 Integrating out the vector-like quarks

At tree level, the heavy VLQs can be integrated out by solving their classical equations of motion. This yields the “non-local effective Lagrangian”

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - \frac{M_S^2}{2}S^2 - \frac{\lambda_3}{3!}S^3 - \frac{\lambda_4}{4!}S^4 \\ & - \kappa_1 S \phi^\dagger \phi - \frac{\kappa_2}{2}S^2 \phi^\dagger \phi \\ & - \bar{F} \frac{1}{i\not{D} - M - S(X - i\gamma_5 \tilde{X})} F, \end{aligned} \quad (4)$$

where

$$F = \tilde{\phi} \mathbf{G}_u u_R + \phi \mathbf{G}_d d_R + S \mathbf{V}_Q Q_L. \quad (5)$$

Note that the heavy scalar field S is still a propagating field at this stage, and indeed the last term in (4) contains couplings of SM fields to an arbitrary number of S fields. The terms of zeroth order in S read

$$\mathcal{L}_{\text{eff}}|_{S^0} = \mathcal{L}_{\text{SM}} + \bar{F}_0 \frac{1}{M - i\not{D}} F_0, \quad (6)$$

where $F_0 = \tilde{\phi} \mathbf{G}_u u_R + \phi \mathbf{G}_d d_R$. Expanding the denominator in powers of covariant derivatives would generate an infinite set of higher-dimensional, gauge-invariant operators, which account for the virtual effects of heavy VLQs on the interactions among SM particles in the context of the SMEFT.

For our purposes the most relevant terms in (7) are those linear in S . They are

$$\begin{aligned} \mathcal{L}_{\text{eff}}|_{S^1} = & -\kappa_1 S \phi^\dagger \phi + \left[S \bar{Q}_L \mathbf{V}_Q^\dagger \frac{1}{M - i\not{D}} F_0 + \text{h.c.} \right] \\ & - \bar{F}_0 \frac{1}{M - i\not{D}} S(X - i\gamma_5 \tilde{X}) \frac{1}{M - i\not{D}} F_0. \end{aligned} \quad (7)$$

In order to match this expression onto the SCET_{B_{SM}} effective Lagrangian describing two-body decays of the heavy scalar S , we replace the SM fields by fields in the EFT. The relevant fields are the soft field S_v for the heavy resonance and collinear fields describing particle jets moving along light-like directions $n_1^\mu = (1, \mathbf{n}_1)$ and $n_2^\mu = (1, \mathbf{n}_2)$. The precise definitions of these fields, which include collinear Wilson lines, can be found in [1]. For the special case of the Higgs doublet, the low-energy theory also contains a soft field Φ_0 carrying no 4-momentum. After electroweak symmetry breaking this field is set to the Higgs vacuum expectation value. The relevant replacement rules are extremely simple:

$$\begin{aligned} \phi & \rightarrow \Phi_0 + \Phi_{n_1} + \Phi_{n_2} + \dots, \\ \psi & \rightarrow \psi_{n_1} + \psi_{n_2} + \dots, \\ g A^{\mu,a} & \rightarrow \mathcal{A}_{n_1}^{\mu,a} + \mathcal{A}_{n_2}^{\mu,a} + \dots. \end{aligned} \quad (8)$$

Here $\psi = Q_L, u_R, d_R$ denotes a generic SM quark field, while $A = B, W, G$ is a generic gauge field. The effective gauge fields in SCET_{B_{SM}} include the gauge couplings in their definition. The collinear quark and gauge fields are subject to the constraints $\not{n}_i \psi_{n_i} = 0$ and $\bar{n}_i \cdot \mathcal{A}_{n_i}^a = 0$, where $\bar{n}_i^\mu = (1, -\mathbf{n}_i)$. Note that the components $\bar{n}_i \cdot \mathcal{A}^a$ of the gauge fields are contained in the Wilson lines of the effective theory. The collinear fields satisfy simple power counting rules in the expansion parameter $\lambda = v/M_S$ of SCET_{B_{SM}}: the fields $\Phi_0, \Phi_{n_i}, \psi_{n_i}$ and $\mathcal{A}_{\perp, n_i}^{\mu,a}$ are all of $\mathcal{O}(\lambda)$, whereas the longitudinal gauge fields $n_i \cdot \mathcal{A}_{\perp, n_i}^a$ are of $\mathcal{O}(\lambda^2)$. The subscript $\perp$ refers to the components of an n_i -collinear gauge field perpendicular to the 4-vectors n_i^μ and $\bar{n}_i^\mu$. Derivatives acting on n_i -collinear fields can be decomposed into the components $\bar{n}_i \cdot \partial = \mathcal{O}(\lambda^0)$, $\partial_\perp^\mu = \mathcal{O}(\lambda)$ and $n_i \cdot \partial = \mathcal{O}(\lambda^2)$. The dots in (8) stand for soft fields, which are power-suppressed relative to the collinear fields and will play no role for our discussion.

It is now straightforward to extract from (7) the terms of leading order in the λ expansion. Obviously, the first term on the right-hand side generates the tree-level contribution

$$C_{\phi\phi} = -\frac{\kappa_1}{M} \quad (9)$$

to the Wilson coefficient of the scalar operator $\mathcal{O}_{\phi\phi}$ in the SCET_{B_{SM}} Lagrangian (1). After the introduction of SCET fields the quantity F_0 is of $\mathcal{O}(\lambda^2)$, while Q_L is of $\mathcal{O}(\lambda)$. Hence the leading terms in the Lagrangian originating from VLQ exchange are of $\mathcal{O}(\lambda^3)$ and arise from the term in brackets in the first line of (7). Since gauge fields in SCET_{B_{SM}} are always power suppressed, we can expand the inverse derivative operator sandwiched between spinor fields of opposite chirality in the form

$$\frac{1}{M - i\not{D}} \rightarrow \frac{M}{M^2 + \square} + \mathcal{O}(\lambda). \quad (10)$$

The Laplace operator in the denominators of these expressions must only be kept if the product of fields on which this operator acts has virtuality of order M_S^2 . We thus obtain

$$\begin{aligned} \mathcal{L}_{\text{eff}}|_{S^1}^{\lambda^3} = & \frac{1}{M} \sum_{q=u,d} \left[S_v \bar{Q}_{L, n_1} \mathbf{V}_Q^\dagger \mathbf{G}_q (\Phi_0 + \Phi_{n_2})_{qR, n_2} \right. \\ & \left. + S_v \bar{Q}_{L, n_1} \mathbf{V}_Q^\dagger \mathbf{G}_q \frac{M^2}{M^2 + \square} \Phi_{n_1 qR, n_2} + \text{h.c.} \right] \\ & + (n_1 \leftrightarrow n_2), \end{aligned} \quad (11)$$

where for $q = u$ the doublet Φ must be replaced by $\tilde{\Phi}$. The first graph in Fig. 1 shows a diagram in the complete theory giving rise to these matching contributions.

Fig. 1 Tree-level diagrams giving rise to the effective Lagrangians (11) [left] and (12) [right]. Thick lines denote S and the VLQs, whereas the thin lines represent SM particles

3.2 Integrating out off-shell fluctuations

If the portal coupling κ_1 in (3) is non-zero, then the second diagram shown in Fig. 1 produces another tree-level matching contribution, in which the propagator for the Higgs doublet carries a virtuality of order $q^2 \sim M_S^2$. The corresponding contribution to the effective Lagrangian can be written in the form

$$\Delta\mathcal{L}_{\text{eff}}|_{S^1}^{\lambda^3} = \sum_{q=u,d} \kappa_1 S_v (\Phi_0^a + \Phi_{n_1}^a + \Phi_{n_2}^a) \frac{1}{\square} \bar{Q}_{L,n_1}^a Y_q q_{R,n_2} + \text{h.c.} + (n_1 \leftrightarrow n_2), \quad (12)$$

where the inverse Laplace operator arises from the Higgs propagator. The sum of (11) and (12) gives the complete tree-level effective Lagrangian at $\mathcal{O}(\lambda^3)$.

3.3 Wilson coefficients

The complete basis of SCET_{BSM} operators at $\mathcal{O}(\lambda^3)$ has been constructed in [1]. The effective Lagrangian at this order can be written in the form (summed over i, j)

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{(3)} = & \frac{1}{M} \sum_{q=u,d} \left[C_{Q_L \bar{q}_R}^{ij}(M, M_S) O_{Q_L \bar{q}_R}^{ij} \right. \\ & + \sum_{k=1,2} \int_0^1 du C_{Q_L \bar{q}_R \phi}^{(k)ij}(u, M, M_S) O_{Q_L \bar{q}_R \phi}^{(k)ij}(u) + \text{h.c.} \left. \right] \\ & + \frac{1}{M} \sum_A \left[\int_0^1 du C_{Q_L \bar{Q}_L A}^{ij}(u, M, M_S) O_{Q_L \bar{Q}_L A}^{ij}(u) \right. \\ & \left. + (Q_L \rightarrow q_R) + \text{h.c.} \right], \quad (13) \end{aligned}$$

where the sum in the last lines runs over the three gauge fields $A = B, W, G$. For simplicity we consider operators containing quark fields only. We have defined the mixed-chirality operators

$$\begin{aligned} O_{Q_L \bar{q}_R}^{ij} &= S_v \bar{Q}_{L,n_1}^i \Phi_0^j q_{R,n_2}^j + (n_1 \leftrightarrow n_2), \\ O_{Q_L \bar{q}_R \phi}^{(1)ij}(u) &= S_v \bar{Q}_{L,n_1}^i \Phi_{n_1}^{(u)} q_{R,n_2}^j + (n_1 \leftrightarrow n_2), \\ O_{Q_L \bar{q}_R \phi}^{(2)ij}(u) &= S_v \bar{Q}_{L,n_1}^i \Phi_{n_2}^{(u)} q_{R,n_2}^j + (n_1 \leftrightarrow n_2), \end{aligned} \quad (14)$$

and the same-chirality operators

$$\begin{aligned} O_{Q_L \bar{Q}_L A}^{ij}(u) &= S_v \bar{Q}_{L,n_1}^i \mathcal{A}_{n_1}^{\perp(u)} Q_{L,n_2}^j + (n_1 \leftrightarrow n_2), \\ O_{q_R \bar{q}_R A}^{ij}(u) &= S_v \bar{q}_{R,n_1}^i \mathcal{A}_{n_1}^{\perp(u)} q_{R,n_2}^j + (n_1 \leftrightarrow n_2), \end{aligned} \quad (15)$$

where i, j are generation indices. When an operator contains more than two collinear fields describing particles moving in the same direction, the total collinear momentum carried by this jet is split up among the fields. Our convention is that in each operator the bosonic field carries the longitudinal momentum fraction $u \in [0, 1]$, while the fermionic field carries momentum fraction $(1 - u)$. From (11) and (12), we obtain for the tree-level matching conditions in matrix notation (with $q = u, d$)

$$\begin{aligned} C_{Q_L \bar{q}_R} &= V_Q^\dagger G_q - \frac{\kappa_1}{M} \frac{Y_q}{\xi}, \\ C_{Q_L \bar{q}_R \phi}^{(1)} &= \frac{V_Q^\dagger G_q}{1 - \xi u - i\epsilon} - \frac{\kappa_1}{M} \frac{Y_q}{\xi(1 - u) + i\epsilon}, \\ C_{Q_L \bar{q}_R \phi}^{(2)} &= V_Q^\dagger G_q - \frac{\kappa_1}{M} \frac{Y_q}{\xi(1 - u) + i\epsilon}, \\ C_{Q_L \bar{Q}_L A} &= C_{q_R \bar{q}_R A} = 0; \quad A = B, W, G, \end{aligned} \quad (16)$$

where we have defined $\xi = M_S^2/M^2$. The $i\epsilon$ prescriptions are those from the Feynman propagators. Note that κ_1 is naturally of order M . The parameter ξ governs the ratio of the mass of the heavy scalar resonance, which we assume has been discovered, and the mass of the VLQs, which we assume have not yet been discovered. This ratio is in principle arbitrary, but in many realistic models is expected to be of $\mathcal{O}(1)$. The fact that SCET_{BSM} correctly captures the dependence on both mass parameters is a unique feature of this EFT [1].

Analogous operators containing lepton fields also exist, and indeed they can be generated at tree level in our model via the Higgs portal interaction proportional to κ_1 . However, the corresponding Wilson coefficients are strongly suppressed by the leptonic Yukawa couplings.

The coefficients in (16) are given in the weak basis. After electroweak symmetry breaking, these coefficients should be transformed to the mass basis of the SM quarks. This transformation diagonalizes the Yukawa matrices Y_q , while $V_Q^\dagger G_q \rightarrow U_{qL}^\dagger V_Q^\dagger G_q W_{qR}$, where U_{qL} and W_{qR} with $q = u, d$ denote the rotation matrices transforming the left-handed and right-handed quark fields from the weak to the mass basis.

4 One-loop matching

With the exception of $O_{\phi\phi}$, the bosonic operators in the SCET_{BSM} Lagrangian receive matching corrections starting at one-loop order. We now discuss the calculation of these corrections for the Wilson coefficients of the leading oper-

Fig. 2 One-loop diagrams contributing to the Wilson coefficients C_{AA} and $\tilde{C}_{AA}$ in (1). We do not show crossed graphs, in which the two boson lines are exchanged. Loops graphs involving SM fermions do not arise at leading order in λ

ators of $\mathcal{O}(\lambda^2)$ in (1). The relevant Feynman diagrams are shown in Fig. 2. The first graph contains a loop of VLQs, while the remaining diagrams feature loops with off-shell Higgs doublets. To perform the matching in the simplest possible way, we calculate these diagrams setting all SM masses to zero. Then loop graphs in the EFT are scaleless and vanish, and hence the Wilson coefficients are given directly in terms of the diagrams shown in the figure. We find (with $A = B, W, G$)

$$C_{AA} = \frac{d_A}{\pi^2} \text{Tr}(X) \left[\frac{4-\xi}{\xi} g^2(\xi) - 1 \right] + \frac{d'_A}{4\pi^2} \frac{\kappa_1}{M}, \quad (17)$$

$$\tilde{C}_{AA} = \frac{d_A}{\pi^2} \text{Tr}(\tilde{X}) g^2(\xi),$$

where $\xi = M_S^2/M^2$ as above, and the group-theory factors d_A are given by

$$d_B = N_c Y_\psi^2 = \frac{1}{12}, \quad d_W = \frac{T_F N_c}{2} = \frac{3}{4}, \quad d_G = T_F = \frac{1}{2},$$

$$d'_B = Y_\phi^2 = \frac{1}{4}, \quad d'_W = \frac{T_F}{2} = \frac{1}{4}, \quad d'_G = 0. \quad (18)$$

The relevant loop function reads

$$g(\xi) = \begin{cases} \arcsin \frac{\sqrt{\xi}}{2}; & \xi \leq 4, \\ \frac{i}{2} \ln \frac{1 + \sqrt{1 - 4/\xi}}{1 - \sqrt{1 - 4/\xi}} + \frac{\pi}{2}; & \xi \geq 4. \end{cases} \quad (19)$$

The ξ -dependent contributions arise from integrating out the VLQs, while the term proportional to the Higgs-portal coupling κ_1 is obtained by integrating out loops of virtual Higgs doublets carrying virtualities of order M_S^2 , in analogy with the discussion in the previous section.

It is instructive to study the ξ -dependent terms in (17) in more detail. Focussing on the case of $\tilde{C}_{AA}$ for concreteness, and assuming that $M_S^2 < 4M^2$, we can expand the Wilson coefficient in powers of the ratio $\xi = M_S^2/M^2$, finding

$$\tilde{C}_{AA} = \frac{d_A}{2\pi^2} \text{Tr}(\tilde{X}) \sum_{k=1}^{\infty} \frac{\Gamma(\frac{1}{2}) \Gamma(k)}{k \Gamma(\frac{1}{2} + k)} \left(\frac{M_S^2}{4M^2} \right)^k. \quad (20)$$

The first term in the sum gives a contribution to (1) which corresponds to the local dimension-5 operator $S F_{\mu\nu} \tilde{F}^{\mu\nu}$, the second term corresponds to local dimension-7 operators such as $S(\partial_\alpha F_{\mu\nu})(\partial^\alpha \tilde{F}^{\mu\nu})$ or $(\square S) F_{\mu\nu} \tilde{F}^{\mu\nu}$, the third

Fig. 3 One-loop diagrams contributing to the coefficient $C_{\phi\phi}^{(1)}$ in (21)

term corresponds to local dimension-9 operators, and so on. Our SCET_BSM approach thus sums up an infinite tower of local operators. In an extension of the SMEFT consisting of local operators built out of SM field and the field S (see e.g. [26,27]), one would typically only include the leading dimension-5 operators. In realistic scenarios where $M_S \sim M$, however, all contributions are of the same order.

The one-loop matching calculation for the coefficient $C_{\phi\phi}$ in (1) is more involved. We write the result in the form

$$C_{\phi\phi} = -\frac{\kappa_1}{M} (1 + \delta_{\kappa_1}) + C_{\phi\phi}^{(1)}. \quad (21)$$

The quantity δ_{κ_1} contains the loop corrections to the tree-level result in (9), while $C_{\phi\phi}^{(1)}$ contains contributions to the Wilson coefficient involving couplings other than κ_1 . The relevant diagrams for the latter terms are shown in Fig. 3. We obtain

$$C_{\phi\phi}^{(1)} = \frac{N_c}{8\pi^2} \text{Tr}[X(G_u G_u^\dagger + G_d G_d^\dagger)]$$

$$\times \left[2 \ln \frac{M^2}{\mu^2} - 3 + 2 \sqrt{\frac{4-\xi}{\xi}} g(\xi) + \frac{4}{\xi} g^2(\xi) \right]$$

$$+ \frac{N_c}{8\pi^2} \text{Re Tr}[V_Q(Y_u G_u^\dagger + Y_d G_d^\dagger)]$$

$$\times \left[2 \ln \frac{M^2}{\mu^2} - 3 - \frac{1-\xi}{\xi} \ln(1-\xi-i\epsilon) \right]$$

$$- \frac{N_c \kappa_2}{2\pi^2 \xi} \text{Tr}(X) \left(\ln \frac{M^2}{\mu^2} - 1 \right)$$

$$- \frac{\kappa_2 \lambda_3}{32\pi^2 M} \left(\frac{\pi}{\sqrt{3}} - 1 \right). \quad (22)$$

The calculation of δ_{κ_1} is discussed in Appendix A. Unlike the results shown in (17), these expressions contain an explicit dependence on the renormalization scale μ , at which the operators and Wilson coefficients are defined (in the $\overline{\text{MS}}$ scheme). The matching results presented here refer to a scale $\mu \sim M$, at which they do not contain any large logarithms; the evolution to lower scales will be discussed later in Sect. 6. The scale dependence of the coefficient $C_{\phi\phi}^{(1)}$ must be compensated by the scale dependence of the portal coupling κ_1 in (21).

5 One-loop matching for $S \rightarrow Zh$

There is one potential two-body decay of a heavy scalar resonance S that cannot be described using the operators arising at leading and subleading order in SCET power counting. This is the mode $S \rightarrow Zh$, where the Z boson is longitudinally polarized. Only the CP-odd component of the scalar can decay to this final state, which makes this channel interesting to study the CP properties of a new scalar resonance [28,29]. The following discussion is significantly more technical than that in the previous two sections and can be skipped in a first reading.

The relevant $\mathcal{O}(\lambda^4)$ operators in the SCET_{BSM} Lagrangian mediating $S \rightarrow Zh$ decay can be written in the form [1]

$$\mathcal{L}_{\text{eff}}^{(4)} = \frac{\tilde{C}_{\phi\phi\phi\phi}(M, M_S)}{M} 2i S_v (\Phi_{n_1}^\dagger \Phi_0 \Phi_{n_2}^\dagger \Phi_0 - \text{h.c.}) + \dots, \quad (23)$$

where the dots stand for fermionic operators, which contribute to the decay amplitude at one-loop order. The operator written out explicitly gives the tree-level contribution

$$\mathcal{M}(S \rightarrow Z_{\parallel} h)|_{\text{tree}} = -i \tilde{C}_{\phi\phi\phi\phi} \frac{v^2}{M}. \quad (24)$$

Since in the VLQ model we consider the Wilson coefficient $\tilde{C}_{\phi\phi\phi\phi}$ is generated starting at one-loop order, it will be necessary to include other loop-suppressed effects for consistency (see below).

Representative one-loop diagrams contributing to the matching coefficient $\tilde{C}_{\phi\phi\phi\phi}$ are shown in Fig. 4. Evaluating the relevant graphs in the $\overline{\text{MS}}$ scheme, we obtain after a lengthy calculation

$$\begin{aligned} \tilde{C}_{\phi\phi\phi\phi} = & -\frac{N_c}{16\pi^2\xi} \sum_{q=u,d} 2T_3^q \left\{ \text{Tr}(\tilde{X} G_q Y_q^\dagger Y_q G_q^\dagger) [-L_M(\xi) \right. \\ & + (1-\xi) \ln(1-\xi)] + f_1(\xi) \\ & + \text{Tr}(\tilde{X} G_q G_q^\dagger G_q G_q^\dagger) f_2(\xi) \end{aligned}$$

$$\begin{aligned} & + \text{Tr}(\tilde{X} G_q G_q^\dagger) [(g^2 + g'^2) f_3(\xi) + \lambda_H f_4(\xi)] \\ & + \text{Im Tr}(G_q^\dagger V_Q Y_q Y_q^\dagger Y_q) \left[\frac{\xi}{2} L_S^2 \right. \\ & \left. - L_M(\xi + (1+\xi) \ln(1-\xi)) + f_5(\xi) \right] \\ & + \text{Im Tr}(G_q^\dagger V_Q Y_q G_q^\dagger G_q) \left[\xi (L_S \right. \\ & \left. - L_M) \ln(1+\xi) + f_6(\xi) \right] \\ & + \text{Im Tr}(G_q^\dagger V_Q Y_q) \left[(g^2 + g'^2) \left[-L_M \left(1 \right. \right. \right. \\ & \left. \left. + \frac{4-3\xi}{4\xi} \ln(1-\xi) \right) - \frac{\xi}{4} (L_S - L_M) + f_7(\xi) \right] \right. \\ & \left. + \lambda_H \left[2L_M \ln(1-\xi) - 2\xi (L_S - L_M) + f_8(\xi) \right] \right] \\ & - \frac{N_c}{16\pi^2\xi} \sum_{q=u,d} Q_q g'^2 \left\{ \text{Tr}(\tilde{X} G_q G_q^\dagger) \left[L_M \left(\frac{2-\xi}{2} \right. \right. \right. \\ & \left. \left. + \frac{1-\xi}{\xi} \ln(1-\xi) \right) + f_9(\xi) \right] \right. \\ & \left. + \text{Im Tr}(G_q^\dagger V_Q Y_q) \left[L_M \left(\frac{6-\xi}{2} \right. \right. \right. \\ & \left. \left. + \frac{3-2\xi}{\xi} \ln(1-\xi) \right) + f_{10}(\xi) \right] \right\}. \quad (25) \end{aligned}$$

Here $T_3^u = \frac{1}{2}$ and $T_3^d = -\frac{1}{2}$ are the weak isospins of up- and down-type quarks, Q_q denote the quark electric charges in units of e , λ_H is the quartic coupling of the Higgs field, and g, g' are the gauge couplings of $SU(2)_L$ and $U(1)_Y$. The logarithms $L_M = \ln(M^2/\mu^2)$ and $L_S = \ln(M_S^2/\mu^2) - i\pi$ contain the dependence on the factorization scale μ , and we have defined the functions $f_i(\xi)$ collected in Appendix B. For $\xi > 1$, the above expressions must be analytically continued using the prescription $\xi \rightarrow \xi + i\epsilon$. In the limit where $\xi \ll 1$, corresponding to $M_S^2 \ll M^2$, the result (25) can be expanded in powers of ξ . We find that the leading terms of $\mathcal{O}(\xi^0)$ agree with eq. (6.20) in [1], where we had defined the matrices

Fig. 4 Example diagrams contributing to the matching of the Wilson coefficient $\tilde{C}_{\phi\phi\phi\phi}$

$\hat{Y}_q = \mathbf{G}_q^\dagger \mathbf{V}_Q$. Moreover, the terms linear in ξ are consistent with eq. (24) in [28].

An interesting feature of the result (25) is the rather complicated dependence of the terms involving the factorization scale μ , which are contained in the logarithms L_M and L_S , on the mass ratio $\xi = M_S^2/M^2$. In conventional EFT applications the coefficients of the μ -dependent terms in Wilson coefficients and operator matrix elements are functions of the coupling constants of the theory, but they do not depend in a non-trivial way on the masses of heavy particles that have been integrated out. The reason is that the μ -dependence must cancel between Wilson coefficients and matrix elements, and the low-energy theory does not know about the masses of the heavy particles.

In the present case, the μ -dependence of the contribution to the $S \rightarrow Zh$ decay amplitude entering via the Wilson coefficient $\tilde{C}_{\phi\phi\phi}$ in (25) is cancelled by the scale dependence of one-loop matrix elements of operators involving fermion pairs, which are induced by tree-level matching at $\mathcal{O}(\lambda^4)$. Indeed, since in our model $\tilde{C}_{\phi\phi\phi}$ arises at one-loop order, the one-loop matrix elements of other $\mathcal{O}(\lambda^4)$ operators, which appear already at tree level, must be included for consistency. The relevant terms can be extracted from (7). For the purpose of illustration we consider the last operator in this result, which contains the flavor matrices $\mathbf{X}$ and $\tilde{\mathbf{X}}$. At $\mathcal{O}(\lambda^4)$, it gives rise to the structure

$$\mathcal{L}_{\text{eff}}|_{S^1}^{\lambda^4} \ni -\frac{1}{M} \sum_{q=u,d} \left[\bar{q}_{R,n_1} \Phi_0^\dagger S_v \mathbf{G}_q^\dagger (\mathbf{X} - i\gamma_5 \tilde{\mathbf{X}}) \mathbf{G}_q \right. \\ \left. \times \frac{1}{M^2 + \square} i \not{\partial} \Phi_{n_2} q_{R,n_1} + \text{h.c.} \right] + (n_1 \leftrightarrow n_2). \quad (26)$$

For $q = u$ the doublet Φ must be replaced by $\tilde{\Phi}$. We only need to consider operators where both fermions are described by collinear fields moving along the same direction, since later we need to take matrix elements where the fermion pair is converted into a collinear Higgs or Z boson. Between the collinear spinors only the $n_1 \cdot \partial$ component of the derivative survives, and hence the derivative gives zero when acting on the fermions. We now define the following set of SCET_{BSM} hermitian operators (here and below we abbreviate $\bar{u} \equiv 1 - u$):

$$O_{qR\bar{q}R\phi\phi}^{(\pm)ij}(u) = S_v \left[\bar{q}_{R,n_1}^{(u)i} \not{p} q_{R,n_1}^{(\bar{u})j} \mp \bar{q}_{R,n_1}^{(\bar{u})i} \not{p} q_{R,n_1}^{(u)j} \right] \\ \times \left(\Phi_{n_2}^\dagger \Phi_0 \pm \Phi_0^\dagger \Phi_{n_2} \right), \quad (27) \\ \tilde{O}_{qR\bar{q}R\phi\phi}^{(\pm)ij}(u) = i S_v \left[\bar{q}_{R,n_1}^{(u)i} \not{p} q_{R,n_1}^{(\bar{u})j} \pm \bar{q}_{R,n_1}^{(\bar{u})i} \not{p} q_{R,n_1}^{(u)j} \right] \\ \times \left(\Phi_{n_2}^\dagger \Phi_0 \pm \Phi_0^\dagger \Phi_{n_2} \right).$$

Here u denotes the fraction of the total n_1 -collinear momentum carried by the final-state quark. The operators shown in

the first line are CP even, while those in the second line are CP odd. Writing the relevant terms in the Lagrangian in the form

$$\mathcal{L}_{\text{eff}}^{(4)} \ni \frac{1}{M^2} \sum_{q=u,d} \int_0^1 du \left[C_{qR\bar{q}R\phi\phi}^{(\pm)ij}(M, M_S, u) O_{qR\bar{q}R\phi\phi}^{(\pm)ij}(u) \right. \\ \left. + \tilde{C}_{qR\bar{q}R\phi\phi}^{(\pm)ij}(M, M_S, u) \tilde{O}_{qR\bar{q}R\phi\phi}^{(\pm)ij}(u) \right], \quad (28)$$

we obtain the Wilson coefficients (in matrix notation)

$$C_{qR\bar{q}R\phi\phi}^{(+)}(u, M, M_S) = \frac{M_S}{2M} \frac{\mathbf{G}_q^\dagger \mathbf{X} \mathbf{G}_q}{1 - \xi \bar{u}}, \\ C_{qR\bar{q}R\phi\phi}^{(-)}(u, M, M_S) = \frac{M_S}{2M} \frac{2T_3^q \mathbf{G}_q^\dagger \mathbf{X} \mathbf{G}_q}{1 - \xi \bar{u}}, \quad (29) \\ \tilde{C}_{qR\bar{q}R\phi\phi}^{(+)}(u, M, M_S) = \frac{M_S}{2M} \frac{\mathbf{G}_q^\dagger \tilde{\mathbf{X}} \mathbf{G}_q}{1 - \xi \bar{u}}, \\ \tilde{C}_{qR\bar{q}R\phi\phi}^{(-)}(u, M, M_S) = \frac{M_S}{2M} \frac{2T_3^q \mathbf{G}_q^\dagger \tilde{\mathbf{X}} \mathbf{G}_q}{1 - \xi \bar{u}}.$$

The factors $2T_3^q$ arise because for $q = u$ the operators involve the scalar doublets $\tilde{\Phi}$ rather than Φ .

The CP-odd operators in (27) contribute at one-loop order to the $S \rightarrow Zh$ decay amplitude via the diagrams shown in Fig. 5. Working in the fermion mass basis, we find in the $\overline{\text{MS}}$ scheme

$$i \langle Z \| h | \tilde{O}_{qR\bar{q}R\phi\phi}^{(+ij)}(u) | S \rangle \\ = \frac{\delta^{ij} N_c}{8\pi^2} v^2 M_S (y_q^2 T_3^q - Q_q g'^2 u \bar{u}) \ln \frac{\mu^2}{m_q^2 - u \bar{u} m_Z^2 - i\epsilon}, \\ i \langle Z \| h | \tilde{O}_{qR\bar{q}R\phi\phi}^{(-ij)}(u) | S \rangle \\ = \frac{\delta^{ij} N_c}{16\pi^2} v^2 M_S y_q^2 (u - \bar{u}) \ln \frac{\mu^2}{m_q^2 - u \bar{u} m_h^2 - i\epsilon}. \quad (30)$$

Multiplying these expressions with the corresponding Wilson coefficients in (29) and integrating the result over u we obtain the contribution to the $S \rightarrow Zh$ decay amplitude, which must be added to the one in (24).

Here we are mainly concerned with the cancellation of the μ -dependent terms in the final expression for the decay amplitude. Note that the scale-dependent terms in (30) have simple coefficients involving coupling constants and some

Fig. 5 One-loop contributions of the operators $\tilde{O}_{qR\bar{q}R\phi\phi}^{(+ij)}$ (left) and $\tilde{O}_{qR\bar{q}R\phi\phi}^{(-ij)}$ (right) to the $S \rightarrow Zh$ decay amplitude

factors of v and M_S needed for dimensional reasons. The non-trivial dependence on the mass ratio ξ arises when these matrix elements are multiplied by the corresponding Wilson coefficients and integrated over the variable u . To display our results we use the Z -boson mass in the denominator of the corresponding logarithms, and we omit the remaining terms that are scale independent and free of large logarithms. Combining the contributions in (24) and (30), we find

$$\begin{aligned} \langle Z \| h | \mathcal{L}_{\text{eff}}^{(4)} | S \rangle &= -i \tilde{C}_{\phi\phi\phi\phi} \frac{v^2}{M} \\ &\quad - i \frac{N_c}{16\pi^2 \xi} \frac{v^2}{M} \sum_{q=u,d} \left\{ 2T_3^q \text{Tr}(\tilde{X} G_q Y_q^\dagger Y_q G_q^\dagger) \ln \frac{\mu^2}{m_Z^2} (\xi) \right. \\ &\quad + (1 - \xi) \ln(1 - \xi) \\ &\quad \left. - Q_q g'^2 \text{Tr}(\tilde{X} G_q G_q^\dagger) \ln \frac{\mu^2}{m_Z^2} \left(\frac{2 - \xi}{2} + \frac{1 - \xi}{\xi} \ln(1 - \xi) \right) \right\} \\ &\quad + \text{terms involving } V_Q \} + \text{scale-independent terms.} \quad (31) \end{aligned}$$

We have transformed the expressions (30) back to the weak basis by replacing $y_q^2 \delta_{ij} \rightarrow (Y_q^\dagger Y_q)_{ij}$. Inspection of (25) shows that the μ -dependent terms indeed cancel out in this result.

6 Resummation of large logarithms

SCET_{BSM} offers a systematic framework for expanding the decay amplitudes for the heavy resonance S into SM particles in powers of v/M_S and resumming large logarithms of this scale ratio. (As before, we assume that the scales M and M_S are of similar magnitude.) Since the rates are affected by Sudakov double logarithms, resummation is important even in cases where the logarithms arise from electroweak interactions [30–32]. These logarithms suppress the decay rates and hence should be taken into account when deriving bounds on the masses and couplings of hypothetical new heavy particles. We now illustrate this point by focussing on a few important two-body decay modes of a heavy scalar resonance S .

For the purposes of illustration, we assume $M_S = 2\text{TeV}$ and $M = 2.5\text{TeV}$ for the masses of S and of the VLQs, respectively. We calculate the Wilson coefficients in the effective Lagrangians (1) and (13) at the high scale $\mu = M$ and evolve them down to a characteristic scale for the process of interest. This evolution is governed by renormalization-group (RG) equations derived in [1]. As long as the characteristic scale is of the order of the weak scale, it is appropriate to include all SM particles in the anomalous dimensions and β -functions of the EFT. A consistent approximation is obtained by including the leading terms in the matching coefficients at the high scale and using two-loop approximations for the

cuspidal anomalous dimension and β -functions as well as one-loop approximations for all other anomalous dimensions in the evolution to low energies (see below).

6.1 $S \rightarrow 2$ jets decay

At lowest order in perturbation theory the process $S \rightarrow 2$ jets proceeds primarily via the decay $S \rightarrow gg$, whose rate is enhanced by a factor M_S^2/v^2 relative to the $S \rightarrow q\bar{q}$ decay rate. Also, in many models the latter rate is suppressed by the light quark masses. We thus obtain $\Gamma(S \rightarrow 2 \text{ jets}) \approx \Gamma(S \rightarrow gg)$ with

$$\Gamma(S \rightarrow gg) = \frac{M^2}{M_S} 8\pi\alpha_s^2(\mu_j) (|C_{GG}(\mu_j)|^2 + |\tilde{C}_{GG}(\mu_j)|^2). \quad (32)$$

Here μ_j is the characteristic scale inherent in the definition of the jets, such as an upper bound on the jet invariant mass. At the high matching scale $\mu_h = M$ the relevant Wilson coefficients have been given in (17). The two coefficients obey the same RG equation [1]

$$\mu \frac{d}{d\mu} C_{GG}(\mu) = \left[3\gamma_{\text{cusp}}^{(3)} \left(\ln \frac{M_S^2}{\mu^2} - i\pi \right) + 2\gamma^G \right] C_{GG}(\mu). \quad (33)$$

Note the important fact that for Sudakov problems the anomalous dimensions themselves contain a (so-called “cusp”) logarithm, and that they have non-zero imaginary parts. At leading logarithmic order, we need $\gamma_{\text{cusp}}^{(3)}$ to two-loop and γ^G to one-loop order. The relevant expressions are

$$\gamma_{\text{cusp}}^{(3)} = \frac{\alpha_s}{\pi} + \left(\frac{47}{12} - \frac{\pi^2}{4} \right) \left(\frac{\alpha_s}{\pi} \right)^2 + \mathcal{O}(\alpha_s^3), \quad (34)$$

and $\gamma^G = 0 + \mathcal{O}(\alpha_s^2)$. Solving the RG equation, we find $C_{GG}(\mu)/C_{GG}(M) = \tilde{C}_{GG}(\mu)/\tilde{C}_{GG}(M) = U_{GG}(\mu, M)$, where [33,34]

$$U_{GG}(\mu, M) = \exp \left[\frac{6}{49} g(\mu, M) + \frac{6}{7} (i\pi - \ln \xi) \ln r \right], \quad (35)$$

with $r = \alpha_s(\mu)/\alpha_s(M)$ and

$$\begin{aligned} g(\mu, M) &= -\frac{4\pi}{\alpha_s(M)} \left(\frac{1}{r} - 1 + \ln r \right) \\ &\quad - \left(\frac{251}{21} - \pi^2 \right) (r - 1 - \ln r) + \frac{13}{7} \ln^2 r. \end{aligned} \quad (36)$$

An analogous relation holds for $\tilde{C}_{GG}$. If we assume that the characteristic jet scale is $\mu_j = 100\text{GeV}$, then

$$U_{GG}(\mu_j, M) \approx 0.38 e^{0.98i}. \quad (37)$$

The decay rate in (32) is suppressed by the factor $|U_{GG}(\mu_j, M)|^2 \approx 0.147$. Not including these resummation effects would vastly overestimate the decay rate.

6.2 $S \rightarrow t\bar{t}$ decay

The largest two-body decay rate into quark-antiquark final states is likely to be that into top quarks. At leading order in perturbation theory the corresponding decay rate is given by

$$\Gamma(S \rightarrow t\bar{t}) = \frac{3}{16\pi} \frac{v^2 M_S}{M^2} \sqrt{1 - \frac{4m_t^2}{M_S^2}} |(C_{Q_L \bar{u}_R})_{33}(m_t)|^2. \quad (38)$$

At the high matching scale $\mu_h = M$ the coefficient $C_{Q_L \bar{u}_R}$ has been given in (16). The related coefficient $\mathbf{C}_{Q_L \bar{u}_R}$ (with a straight letter “C”) is obtained by transforming this expression to the quark mass basis. Including only QCD effects, it obeys the RG equation [1]

$$\mu \frac{d}{d\mu} \mathbf{C}_{Q_L \bar{u}_R}(\mu) = \left[\frac{4}{3} \gamma_{\text{cusp}}^{(3)} \left(\ln \frac{M_S^2}{\mu^2} - i\pi \right) + 2\gamma^q \right] \mathbf{C}_{Q_L \bar{u}_R}(\mu), \quad (39)$$

where $\gamma^q = -\alpha_s/\pi + \mathcal{O}(\alpha_s^2)$. Solving this equation, we obtain $\mathbf{C}_{Q_L \bar{u}_R}(\mu) = U_{q\bar{q}}(\mu, M) \mathbf{C}_{Q_L \bar{u}_R}(M)$ with

$$U_{q\bar{q}}(\mu, M) = \exp \left[\frac{8}{147} g(\mu, M) + \frac{8}{21} \left(i\pi + \frac{3}{2} - \ln \xi \right) \ln r \right]. \quad (40)$$

Evolving the coefficient down to the scale of the top-quark mass, we find

$$U_{q\bar{q}}(m_t, M) \approx 0.90 e^{0.31i}. \quad (41)$$

The decay rate in (38) is suppressed by the factor $|U_{q\bar{q}}(m_t, M)|^2 \approx 0.81$. In this case, resummation effects have a more modest impact on the decay rate.

6.3 $S \rightarrow \gamma\gamma$ decay

It is instructive to also consider an example where only electroweak Sudakov logarithms contribute. The diphoton decay mode has a very similar structure as the $S \rightarrow gg$ mode discussed above. At leading order in perturbation theory the decay rate is given by

$$\Gamma(S \rightarrow \gamma\gamma) = \frac{M^2}{M_S} \pi \alpha^2 \left(|C_{WW}(m_W) + C_{BB}(m_W)|^2 + |\tilde{C}_{WW}(m_W) + \tilde{C}_{BB}(m_W)|^2 \right). \quad (42)$$

Here $\alpha \approx 1/137.036$ is the fine-structure constant. The Wilson coefficients need to be evolved down to the scale of electroweak symmetry breaking, which we identify with the mass of the W boson. Below the weak scale the running stops. At the high matching scale $\mu_h = M$ the relevant coefficients have been given in (17). The coefficients C_{WW} and $\tilde{C}_{WW}$ obey the same RG equation [1]

$$\mu \frac{d}{d\mu} C_{WW}(\mu) = \left[2\gamma_{\text{cusp}}^{(2)} \left(\ln \frac{M_S^2}{\mu^2} - i\pi \right) + 2\gamma^W \right] C_{WW}(\mu). \quad (43)$$

The relevant cusp anomalous dimension is

$$\gamma_{\text{cusp}}^{(2)} = \frac{\alpha_2}{\pi} + \left(2 - \frac{\pi^2}{6} \right) \left(\frac{\alpha_2}{\pi} \right)^2 + \dots, \quad (44)$$

whereas γ^W vanishes at one-loop order. Here $\alpha_2 = g^2/(4\pi)$ is the coupling constant of $SU(2)_L$. The Wilson coefficients C_{BB} and $\tilde{C}_{BB}$, on the other hand, are scale independent at leading logarithmic order. It follows that

$$\begin{aligned} C_{WW}(m_W) + C_{BB}(m_W) \\ = U_{WW}(m_W, M) C_{WW}(M) + U_{BB}(m_W, M) C_{BB}(M) \end{aligned} \quad (45)$$

and similarly for the other two coefficients in (42), where $U_{BB}(m_W, M) \approx 1$, while $U_{WW}(\mu, M)$ is given by an expression similar to (35), but with different numerical coefficients and with $\alpha_s(\mu)$ replaced by the coupling $\alpha_2(\mu)$. Numerically, we obtain

$$U_{WW}(m_W, M) \approx 0.80 e^{0.23i}, \quad U_{BB}(m_W, M) \approx 1. \quad (46)$$

The impact of these resummation effects on the diphoton decay rate depends on the values of κ_1/M and $\text{Tr}(X)$ in (17). In the limit where the term proportional to κ_1 can be neglected, the decay rate is suppressed by the factor $|0.9 U_{WW}(m_W, M) + 0.1|^2 \approx 0.67$. The resummation of electroweak Sudakov logarithms thus has a sizable impact on the rate.

6.4 $S \rightarrow hh$ decay

As a final example we consider the decay mode $S \rightarrow hh$, whose rate is given by

$$\Gamma_{S \rightarrow hh} = \frac{M^2}{32\pi M_S} \sqrt{1 - \frac{4m_h^2}{M_S^2}} |C_{\phi\phi}(m_h)|^2. \quad (47)$$

The Wilson coefficient satisfies the RG equation [1]

$$\begin{aligned} \mu \frac{d}{d\mu} C_{\phi\phi}(\mu) &= \left[\left(\frac{1}{4} \gamma_{\text{cusp}}^{(1)} + \frac{3}{4} \gamma_{\text{cusp}}^{(2)} \right) \left(\ln \frac{M_S^2}{\mu^2} - i\pi \right) + 2\gamma^\phi \right] C_{\phi\phi}(\mu), \end{aligned} \quad (48)$$

where

$$\begin{aligned} \gamma_{\text{cusp}}^{(1)} &= \frac{\alpha_1}{\pi} - \frac{17}{6} \left(\frac{\alpha_1}{\pi} \right)^2 + \dots, \\ \gamma^\phi &= -\frac{\alpha_1}{4\pi} - \frac{3\alpha_2}{4\pi} + \frac{3y_t^2}{8\pi^2} + \dots, \end{aligned} \quad (49)$$

and $\gamma_{\text{cusp}}^{(2)}$ has been given in (44). Here α_1 is the coupling constant of $U(1)_Y$ (not rescaled by a factor 5/3). Since there are now three different couplings involved, it is easiest to integrate the RG equation (48) numerically, using the one-loop β -functions for the various couplings. Writing the solution in the form $C_{\phi\phi}(m_h) = U_{\phi\phi}(m_h, M) C_{\phi\phi}(M)$, we find

$$U_{\phi\phi}(m_h, M) \approx 0.79 e^{0.08i}. \quad (50)$$

It follows that the di-Higgs decay rate is suppressed by the factor $|U_{\phi\phi}(m_h, M)|^2 \approx 0.62$, which is once again a significant correction.

7 Conclusions

When a new heavy resonance beyond the SM is discovered, it will be important to have an effective field-theory description of its decay and production modes, in which the new-physics scale M is disentangled from the electroweak scale. This effective theory should be able to deal with the situation that the new state is a member of a larger sector of new physics. In this paper we have illustrated the recently developed SCET_{BSM} approach [1] to solve this problem in the context of an extension of the SM by a heavy scalar singlet S and a set of vector-like heavy quarks. We have performed the matching calculation for the Wilson coefficients in the effective Lagrangian both at tree level and including the leading one-loop corrections. These coefficients are in general non-trivial functions of the mass ratio $\xi = M_S^2/M^2$, where M_S is the mass of the scalar resonance while M sets the masses of the vector-like quarks. In this way, our effective theory resums an infinite tower of local operators in the conventional effective field-theory approach to describe the interactions of S with SM fields. For the special case of the decay $S \rightarrow Zh$, the Wilson coefficient of the relevant operator contains logarithms of the form $\ln(M_S^2/\mu^2)$ and $\ln(M^2/\mu^2)$ with coefficients that depend in a non-polynomial way on the ratio ξ . We have explained the origin of this effect and demonstrated how the scale dependence is cancelled in the effective theory.

The SCET_{BSM} framework allows one to resum large Sudakov logarithms affecting the decay rates of S into SM particles. We have explicitly performed the resummation at leading logarithmic order for the decays $S \rightarrow 2 \text{ jets}$, $S \rightarrow t\bar{t}$, $S \rightarrow \gamma\gamma$ and $S \rightarrow hh$, finding that in all cases the decay rates are significantly reduced. It is important to take these resummation effects into account when placing bounds on the masses and couplings of hypothetical new heavy particles. Possible avenues worthy to pursue in the future include extensions of our work to resonances of non-zero spin as well as particles that are not singlets under the SM gauge group. In this way, the SCET_{BSM} approach can be applied to collider searches for heavy particles proposed in many extensions of the SM.

Acknowledgements We are grateful to Florian Goertz for useful discussions. The work of S. A. and M. N. was supported by the DFG Clusters of Excellence *Precision Physics, Fundamental Interactions and Structure of Matter* PRISMA (EXC 1098) and PRISMA⁺ (EXC 2118). The research of S. A. was also supported by the DFG Research Training Group *Symmetry Breaking in Fundamental Interactions* (GRK 1581). M. K. gratefully acknowledges support by the Swiss National Science Foundation (SNF) under contract 200021-175940. Funding was provided by Deutsche Forschungsgemeinschaft (Project ID 39083149).

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This paper is based on theoretical work, and hence there is no data associated with it.]

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Appendix A: Calculation of the quantity δ_{κ_1}

Here we report our result for the one-loop coefficient δ_{κ_1} in (21). It receives contributions from the vertex-correction diagrams shown in the first line of Fig. 6 as well as from hard matching corrections to the wave-function renormalization constants of the scalar fields. We find

$$\begin{aligned} \delta_{\kappa_1} &= \frac{3\lambda_H}{8\pi^2} (L_S - 2) \\ &\quad - \frac{3g^2 + g'^2}{64\pi^2} \left(L_S^2 - L_S + 2 - \frac{\pi^2}{6} \right) \\ &\quad + \frac{\kappa_1^2}{16\pi^2 M_S^2} \left(\frac{1}{2} + \frac{\pi^2}{12} + i\pi \ln 2 \right) \\ &\quad + \frac{\kappa_1 \lambda_3}{16\pi^2 M_S^2} \frac{\pi^2}{9} - \frac{\lambda_3^2}{64\pi^2 M_S^2} \left(\frac{2\pi}{3\sqrt{3}} - 1 \right) \\ &\quad + \frac{N_c}{8\pi^2} \text{Tr}(V_Q^\dagger V_Q) \left[L_M - 1 - \frac{1}{\xi} - \frac{1 - \xi^2}{\xi^2} \ln(1 - \xi) \right] \end{aligned}$$

Fig. 6 One-loop diagrams contributing to the parameter δ_{k_1} . The graphs in the last two lines show the matching corrections to the wave-function renormalization constants of the heavy resonance S and the Higgs scalar ϕ

$$\begin{aligned}
 & + \frac{N_c}{8\pi^2} \text{Tr}(X^2) \left[L_M - 1 - \frac{4}{\xi} + \frac{2(2+\xi)}{\xi} \sqrt{\frac{4-\xi}{\xi}} g(\xi) \right] \\
 & + \frac{N_c}{8\pi^2} \text{Tr}(\tilde{X}^2) \left[L_M - 1 + \frac{2(2-\xi)}{\sqrt{\xi(4-\xi)}} g(\xi) \right] \\
 & + \frac{N_c}{8\pi^2} \text{Tr}(G_u^\dagger G_u + G_d^\dagger G_d) \left(L_M - \frac{1}{2} \right), \quad (\text{A1})
 \end{aligned}$$

where $L_M = \ln(M^2/\mu^2)$ and $L_S = \ln(M_S^2/\mu^2) - i\pi$, and the function $g(\xi)$ has been given in (19).

Appendix B: Coefficient functions $f_i(\xi)$

The explicit expressions for the functions $f_i(\xi)$ entering the result for $\tilde{C}_{\phi\phi\phi}$ in (25) are

$$\begin{aligned}
 f_1(\xi) &= \xi - (1-\xi) \ln^2(1-\xi) - (1-\xi) \text{Li}_2(\xi) + 4\xi g^2(\xi), \\
 f_2(\xi) &= -\xi + (1-\xi) \text{Li}_2(\xi) + 2\xi g^2(\xi), \\
 f_3(\xi) &= -\frac{5-\xi}{4} - \frac{3-4\xi+\xi^2}{4\xi} \ln(1-\xi) \\
 & \quad + \frac{1}{2\xi} \text{Li}_2(\xi) + \frac{1}{2} \sqrt{\xi(4-\xi)} g(\xi) - g^2(\xi), \\
 f_4(\xi) &= -2 + \xi - \frac{2(1-\xi)^2}{\xi} \ln(1-\xi) \\
 & \quad + 4\sqrt{\xi(4-\xi)} g(\xi) - 8g^2(\xi), \\
 f_5(\xi) &= \xi - (1+\xi) \ln^2(1-\xi) - (1+\xi) \text{Li}_2(\xi) - \xi \frac{\pi^2}{12}, \\
 f_6(\xi) &= -\xi + \xi \text{Li}_2(-\xi) + (1+\xi) \text{Li}_2(\xi), \\
 f_7(\xi) &= -\frac{5-\xi}{4} - \frac{3-2\xi-\xi^2}{4\xi} \ln(1-\xi) \\
 & \quad - \frac{4-3\xi}{4\xi} \ln^2(1-\xi) + \frac{2+5\xi}{4\xi} \text{Li}_2(\xi), \\
 f_8(\xi) &= -2 + \xi - \frac{2(1-\xi^2)}{\xi} \ln(1-\xi) \\
 & \quad + 2 \ln^2(1-\xi) + 2 \text{Li}_2(\xi),
 \end{aligned}$$

$$\begin{aligned}
 f_9(\xi) &= \frac{1-\xi}{\xi} \ln^2(1-\xi) - \text{Li}_2(\xi), \\
 f_{10}(\xi) &= \frac{3-2\xi}{\xi} \ln^2(1-\xi) - 3 \text{Li}_2(\xi). \quad (\text{B1})
 \end{aligned}$$

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