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GUP parameter from maximal acceleration

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Abstract We exhibit a theoretical calculation of the parameter β appearing in the generalized uncertainty principle (GUP) with only a quadratic term in the momentum. A specific numerical value is obtained by comparing the GUP-deformed Unruh temperature with the one predicted within the framework of Caianiello's theory of maximal acceleration. The physical meaning of this result is discussed in connection with constraints on β previously fixed via both theoretical and experimental approaches.

1 Introduction

The task of describing Quantum Mechanics (QM) and General Relativity (GR) in a unified way is the toughest challenge of modern theoretical physics. Indeed, if on the one hand GR seems to predict unphysical results when one tries to apply it to the quantum scale, on the other hand QM is faced with serious problems when extended to cosmic dimensions. In spite of these inconsistencies, however, it is essential to understand to what extent quantum effects and gravitation influence each other, in order to make further progress towards the formulation of a successful theory of quantum gravity. Along this line, it has been argued that, at the quantum gravity scale, the Heisenberg uncertainty principle (HUP)[1]

$$\Delta x\,\Delta p \geq \frac{1}{2}, \tag{1}$$

should be modified [1–4] in order to take into account the existence of a minimal length.

Research on the generalization of the uncertainty principle (GUP) covers a number of domains, ranging from string theory and loop quantum gravity, to deformed special relativity, black hole physics and the Casimir effect [5–20], which has recently been employed as a test bench also in other contexts [21–25]. Many of these studies have converged on the idea that a proper modification of Eq. (1) would be

$$\Delta x\,\Delta p \geq \frac{1}{2}\left(1+4\beta\ell_p^2\Delta p^2\right)=\frac{1}{2}\left[1+\beta\left(\frac{\Delta p}{m_{\rm p}}\right)^2\right], \tag{2}$$

where β is a dimensionless parameter and ℓ_p, m_p are the Planck length and mass, respectively. For mirror-symmetric states (with $\langle\hat{p}\rangle=0$), Eq. (2) is equivalent to the commutator

$$\left[\hat{x},\,\hat{p}\right]=i\left[1+\beta\left(\frac{\hat{p}}{m_{\rm p}}\right)^2\right], \tag{3}$$

since $\Delta x\,\Delta p \geq (1/2)\left|\langle[\hat{x},\,\hat{p}]\rangle\right|$.

We remark that the deformation parameter β in Eq. (2) is not fixed by the theory. In principle, it can be constrained by means of phenomenological approaches (considering, for instance, the spectrum of hydrogen atom [26], gravitational waves [27,28], cold atoms [29], atomic weak equivalence principle tests [30], etc.[2]), or computed on a theoretical basis. In some models of string theory [5–8], for example, β is assumed to be of order unity. This has been confirmed by explicit calculations in the context of Donoghue's effective field theory of gravity [32], noncommutative Schwarzschild geometry [31], and corpuscular gravity [33]. Similar studies in Rindler spacetime have been carried out in Ref. [34], where it has been found that GUP corrections are responsible for a slight shift in the Unruh temperature via both a heuristic and a more rigorous quantum field theoretical treatment. In passing, we mention that deviations from the standard Unruh

[1] Throughout the work, we set $\hbar=c=1$, but we explicitly show the Newton constant G and the Boltzmann constant $k_{\rm B}$. The Planck length is defined as $\ell_p=\sqrt{G}$, the Planck energy as $\mathscr{E}_{\rm p}\,\ell_{\rm p}=1/2$, and the Planck mass as $m_{\rm p}=\mathscr{E}_{\rm p}$, so that $2\,\ell_{\rm p}\,m_{\rm p}=1$.

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[2] For a recent review of the various approaches used to estimate β, see, for example, Ref. [31].

prediction have been recently pointed out also in other scenarios [35–40]. Furthermore, in Ref. [41] it has been argued that the GUP plays a relevant rôle also in cosmology, since the deformation parameter can induce an effective energy density which complies with the holographic principle and allows to introduce IR and UV cut-offs.

On the other hand, in Ref. [42] it has been shown that a deformation of the Heisenberg uncertainty principle consistent with the GUP (2) is obtained within the quantum geometry model of Caianiello [43–46]. In particular, in such a framework, quantum aspects are embedded into spacetime geometry so that one-particle QM can be reinterpreted in geometric terms. One of the most relevant predictions of this approach is the existence of a maximal value for the acceleration, which can be defined either as the upper limit to the proper acceleration experienced by massive particles along their worldlines [47–49] or as an universal constant depending on the Planck mass [42,47–49]. Note that deformations of the spacetime metric have been addressed also in connection with the spontaneous symmetry breaking of unification theories and gravitational waves. A general way to define such deformations has been described in Ref. [50], where they have been regarded as extended conformal transformations.

The quantum geometry model finds several applications in different sectors of theoretical physics, such as cosmology, dynamics of accelerated strings, black hole physics, neutrino oscillations and relativistic kinematics in non-inertial frames [49,51–62]. Specifically, in Ref. [62] it has been emphasized that modifications of the geometry of Rindler spacetime that include an upper limit on the acceleration have non-trivial implications even on the Unruh effect.

Starting from the outlined scenario, in this paper we evaluate the deformation parameter β by comparing corrections to the Unruh temperature stemming from two different approaches. The first one arises from the GUP (2), and thus explicitly depends on β. In the second case, we consider the correction induced by modifications of the Rindler metric that include the existence of a maximal acceleration. By equating the two terms, we then obtain a numerical estimation for β of the order of unity, as expected from several string theory models. We further discuss our result in connection with the previously obtained bounds on the GUP parameter.

The paper is organized as follows: Sect. 2 is devoted to a heuristic derivation of the Unruh temperature from both the usual and generalized uncertainty principles. In Sect. 3 we review the basics of Caianiello's quantum geometry model, focusing in particular on the emergence of a maximal value for the acceleration. Using the Unruh–DeWitt detector model [63], we show that the Unruh temperature is non-trivially modified in this framework. We then evaluate the deformation GUP parameter β by comparing the GUP-corrected and the geometric-corrected expressions of the Unruh temperature. Conclusions are discussed in Sect. 4.

2 Unruh effect from uncertainty relations

The Unruh effect [64] is one of the most outstanding manifestations of the non-trivial nature of quantum vacuum [63,65–68]. It states that the zero-particle state for an inertial observer in Minkowski spacetime looks like a thermal state for a uniformly accelerating observer, with a temperature given by

$$T_{\mathrm{U}} = \frac{a}{2\pi k_{\mathrm{B}}}\,, \tag{4}$$

where a is the magnitude of the acceleration.

The above relation can be rigorously derived within the framework of Quantum Field Theory [64]. Following Refs. [34,69], however, here we briefly review a heuristic calculation based exclusively on the HUP (for an alternative approach, see for example Ref. [70]). This procedure will be the starting point to compute GUP corrections to the Unruh temperature (4).

Consider a gas of relativistic particles at rest in a uniformly accelerated frame. Assuming that the frame moves a distance δx, the kinetic energy acquired by each of these particles is

$$E_k = m\,a\,\delta x, \tag{5}$$

where m is the mass of the particles and a the acceleration of the frame. Suppose this energy is barely enough to create N particle-antiparticle pairs from the quantum vacuum, i.e. $E_k \simeq 2Nm$. Using Eq. (5), it follows that the minimal distance along which each particle must be accelerated reads

$$\delta x \simeq \frac{2\,N}{a}. \tag{6}$$

Now, since the whole system is localized inside a spatial region of width δx, the energy fluctuation of each single particle can be estimated from the HUP as

$$\delta E \simeq \frac{1}{2\,\delta x}, \tag{7}$$

where we have assumed $\delta E \simeq \delta p$. This gives

$$\delta E \simeq \frac{a}{4\,N}. \tag{8}$$

If we interpret this fluctuation as a thermal agitation effect, from the equipartition theorem we have

$$\frac{3}{2}\,k_{\mathrm{B}}\,T \simeq \delta E \simeq \frac{a}{4\,N}\,, \tag{9}$$

which can be easily inverted for T, yielding

$$T = \frac{a}{6Nk_{\mathrm{B}}}\,. \tag{10}$$

Comparison with the Unruh temperature (4) allows us to set an effective number of pairs $N = \pi/3 \simeq 1$.

Let us now repeat similar calculations in the context of the GUP. From the uncertainty relation (2), we first note that the GUP version of the standard Heisenberg formula (7) is

$$\delta x \simeq \frac{1}{2\,\delta E} + 2\beta\,\ell_{\rm p}^2 \delta E. \tag{11}$$

Upon replacing Eq. (6) into Eq. (11), and using the same thermodynamic argument as in Eq. (9) for δE, we obtain

$$\frac{2N}{a} \simeq \frac{1}{3\,k_{\rm B}\,T} + 3\beta\,\ell_{\rm p}^2\,k_{\rm B}\,T. \tag{12}$$

Once again, by requiring that T equals the Unruh temperature (4) for $\beta \to 0$, we can fix $N = \pi/3$, so that

$$\frac{2\,\pi}{a} \simeq \frac{1}{k_{\rm B}\,T} + 9\beta\,\ell_{\rm p}^2\,k_{\rm B}\,T. \tag{13}$$

Solving for T, we obtain the following expression for the modified Unruh temperature

$$T = \frac{\pi}{9\beta\ell_p^2 k_{\rm B} a}\left(1 \pm \sqrt{1 - 9\beta\ell_p^2 a^2/\pi^2}\right), \tag{14}$$

which agrees with the standard result (4) in the semiclassical limit $\beta\,\ell_p^2\,a^2 \ll 1$, provided that the negative sign is chosen, whereas the positive sign has no evident physical meaning. The above relation will be employed to estimate the deformation parameter β in our subsequent analysis.

3 Maximal acceleration theory

In a series of works [43–46] it has been shown that the one-particle Quantum Mechanics acquires a geometric interpretation if one incorporates quantum aspects into the geometric structure of spacetime. Such an outcome is achieved by treating the momentum and position operators as covariant derivatives with a proper connection in an eight-dimensional manifold. As a result, the usual quantization procedure can be viewed as the curvature of the phase space.

The above geometric picture allows for the emergence of a maximal acceleration A that massive particles can undergo [47–49]. In principle, this new parameter should be regarded as a mass-dependent quantity, since it varies according to

$$A = \frac{2mc^3}{\hbar} \equiv 2m, \tag{15}$$

where m is the rest mass of the particle. On the other side, however, some authors interpret A as a universal constant [42,47–49,51]. In particular, this would happen at energies of the order of Planck scale, where the definition (15) is usually rewritten in terms of the Planck mass as [42,47–49,51]

$$A = \frac{m_p c^3}{\hbar} \equiv m_p. \tag{16}$$

In order to build the aforementioned eight-dimensional manifold, we basically start from the four-dimensional spacetime $\mathscr{M}$ on which the metric tensor $g_{\mu\nu}$ is defined and then enlarge it with the tangent bundle, so that $\mathscr{M}_8 = \mathscr{M} \otimes T\mathscr{M}$. After performing this, the line element on $\mathscr{M}_8$ becomes

$$d\tau^2 = g_{AB} d\xi^A d\xi^B, \quad A, B = 1, \ldots, 8, \tag{17}$$

where the coordinates and the metric in the above equation can be expressed in terms of the corresponding four-dimensional ones by [42]

$$\xi^A = \left(x^\mu, \frac{\dot{x}^\mu}{A}\right), \quad g_{AB} = g_{\mu\nu} \otimes g_{\mu\nu}, \quad \mu, \nu = 1, \ldots, 4. \tag{18}$$

Here, the dot represents a derivative with respect to the proper time s defined on $\mathscr{M}$.

From the above considerations, it is straightforward to check that

$$d\tau^2 = \left(1 - \frac{|\ddot{x}^\mu \ddot{x}_\mu|}{A^2}\right) ds^2 \equiv \left(1 - \frac{a^2}{A^2}\right) ds^2, \tag{19}$$

with a being the squared length of the spacelike four-acceleration.

With the aid of Eq. (19), in what follows we derive the modification to the Unruh temperature due to the presence of an upper limit for the acceleration. To this aim, we employ the Unruh–DeWitt particle detector method as explained in Ref. [63].

3.1 Unruh temperature from maximal acceleration

Consider a massless scalar field ϕ interacting with a particle detector with internal energy levels by means of a monopole interaction. The Lagrangian related to this process can be sketched as [63]

$$\mathscr{L}_{int} = \chi M(s)\phi(x(s)), \tag{20}$$

where χ is a small coupling constant and M is the monopole moment operator of the detector, which travels along a world line with proper time s. Let us further assume that the scalar field is initially in the Minkowski vacuum $|0_M\rangle \equiv |0\rangle$ and the detector in its ground state with energy E_0. Since we do not impose any restriction to the detector's trajectory, it is

possible that these initial conditions vary along the world line due to the interaction, thus allowing the scalar field to reach an excited state $|\lambda\rangle$ and the detector to undergo a transition to an energy level $E > E_0$.

By resorting to a first order perturbation theory, the transition amplitude for the process $|E_0, 0\rangle \to |E, \lambda\rangle$ reads [63]

$$\mathscr{A} = i\chi \langle E, \lambda | \left(\int M(s)\phi(x(s))ds \right) |E_0, 0\rangle, \tag{21}$$

or

$$\mathscr{A} = i\chi \langle E|M(0)|E_0\rangle \int e^{i(E-E_0)s} \langle \lambda|\phi(x(s))|0\rangle ds. \tag{22}$$

where the integral extends over all the real axis.

We stress that the equality between the above relations is guaranteed by the time evolution equation of the operator $M(s)$. By squaring the modulus of $\mathscr{A}$ and summing over all the complete set of values for E and λ, we obtain the transition probability $\mathscr{P}$ related to any possible excitation of the analyzed system. In the case of a trajectory lying on Minkowski background, it is possible to write the transition probability per unit proper time, $\Gamma \equiv \mathscr{P}/T$, as follows

$$\Gamma = -\frac{\chi^2 \sum_E |\langle E|M(0)|E_0\rangle|^2}{4\pi^2} \times \int \frac{e^{-i(E-E_0)\Delta s}\, d(\Delta s)}{(t - t' - i\varepsilon)^2 - |\mathbf{x} - \mathbf{x}'|^2}. \tag{23}$$

At this point, we must select the parameterization for the trajectory we mean to study. In order to derive the modified expression of the Unruh temperature, we require the particle detector to move along a hyperbola in the (t, x) plane. This indeed corresponds to the characteristic worldline of a relativistic uniformly accelerated (Rindler) motion with proper acceleration a. Such a trajectory can be parameterized as[3]

$$t = 1/a \sinh(as), \tag{24}$$
$$x = 1/a \cosh(as). \tag{25}$$

Using Eq. (19), we can now rewrite the above relations in terms of the parameter τ, so as to make the dependence on the maximal acceleration A manifest. We then obtain

$$t = 1/a \sinh(a\gamma\tau), \tag{26}$$
$$x = 1/a \cosh(a\gamma\tau), \tag{27}$$

where we have defined $\gamma \equiv 1/\sqrt{1 - a^2/A^2}$.

With the above setting, one can check that Eq. (23) takes the form

$$\Gamma = \gamma\chi^2 \sum_E |\langle E|M(0)|E_0\rangle|^2 \times \int e^{-i\gamma(E-E_0)\Delta\tau}\, W(\Delta\tau)\, d(\Delta\tau), \tag{28}$$

where $\Delta\tau \equiv \tau - \tau' > 0$, and

$$W = -\left\{ \frac{16\pi^2}{a^2} \left[\sinh^2\left(a\frac{\gamma\Delta\tau}{2}\right) - i\varepsilon a \sinh\left(a\frac{\gamma\Delta\tau}{2}\right) \right] \right\}^{-1} = -\left[\frac{16\pi^2}{a^2} \sinh^2\left(a\frac{\gamma\Delta\tau - 2i\varepsilon}{2}\right) \right]^{-1}, \tag{29}$$

is the positive frequency Wightman Green function [63]

$$W(s, s') = \langle 0|\phi(x(s))\phi(x(s'))|0\rangle. \tag{30}$$

Note that, in the second step of Eq. (29), we have redefined ε by extracting the positive function $2\cosh(a\gamma\Delta\tau/2)$. We further emphasize that the particular dependence of W on $\Delta\tau$ (rather than τ and τ' separately) reflects the fact that our system is invariant under time translations in the reference frame of the detector.[4]

Now, using for $W(\Delta\tau)$ the identity

$$\mathrm{cosec}^2(\pi x) = \pi^{-2} \sum_{k=-\infty}^{\infty} (x - k)^{-2}, \tag{31}$$

and replacing into Eq. (28), we obtain

$$\Gamma = \frac{\chi^2}{2\pi} \sum_{\tilde{E}} \frac{\left(\tilde{E} - \tilde{E}_0\right) \left|\langle \tilde{E}|M(0)|\tilde{E}_0\rangle\right|^2}{e^{2\pi(\tilde{E}-\tilde{E}_0)/a\gamma} - 1}, \tag{32}$$

where the Fourier transform has been performed by means of a contour integral [63]. Moreover, we have absorbed a factor γ into the definition of ε introduced in Eq. (29) and $\tilde{E} \equiv \gamma E$ is the energy defined with respect to the detector proper time τ.

Because of the appearance of the Planck factor in Eq. (32), the rate of absorption of the accelerated detector due to the interaction with the field in its ground state is the same as the one we would obtain if the detector were static, but immersed in a thermal bath at the temperature

$$T = \frac{a\gamma}{2\pi k_B} \equiv T_{\mathrm{U}} \left(1 - \frac{a^2}{A^2}\right)^{-1/2}. \tag{33}$$

[3] For simplicity, we assume that the acceleration is directed along the x-axis.

[4] In other terms, we can say that the detector is in equilibrium with the field ϕ, so that the rate of absorbed quanta is constant.

We remark that this result is in agreement with the one of Ref. [63], where the correction induced by the existence of a maximal acceleration has been derived by employing the time-dependent Doppler effect approach proposed in Ref. [71].

3.2 GUP parameter from maximal acceleration

In Ref. [42], it was argued that the geometrical interpretation of QM through a quantization model that implies the existence of a maximal acceleration naturally leads to a generalization of the uncertainty principle similar to the one in Eq. (2). Thus, one may wonder which is the value of the parameter β that allows the GUP-deformed and the metric-deformed Unruh temperatures in Eqs. (14) and (33) to coincide. Clearly, given that the regime of validity of Eq. (2) is at Planck scale, we should consider the maximal acceleration A as depending on the quantity m_p (see Eq. (16)) in order to compare the two expressions.

Since we are only interested in small (i.e. linear in β) corrections to the Unruh temperature, we can expand Eq. (14) as

$$T \simeq T_{\rm U}\left(1+\frac{9\,\beta}{4}\,\frac{\ell_{\rm p}^2\,a^2}{\pi^2}\right), \tag{34}$$

which obviously recovers the standard Unruh result (4) for $\beta \to 0$.

Likewise, for realistic values of the acceleration, we have $a << A \sim 10^{51}\,{\rm m/s^2}$, so that Eq. (33) becomes (to the leading order)

$$T \approx T_{\rm U}\left(1+\frac{1}{2}\,\frac{a^2}{A^2}\right) = T_{\rm U}\left(1+2\,\ell_p^2 a^2\right), \tag{35}$$

where we have used the definition (16) of the maximal acceleration. By requiring the GUP-deformed Unruh temperature to be equal to the corresponding geometric-corrected formula, we then obtain

$$\beta = \frac{8\pi^2}{9}, \tag{36}$$

which is of the order of unity, in agreement with the general belief and with several models of string theory. We stress that such a result is perfectly consistent with the outcome of Ref. [42], where it has been shown that the generalized uncertainty principle of string theory is recovered (up to a free parameter) by taking into account the existence of an upper limit on the acceleration.

In the next section, we discuss the physical meaning of Eq. (36) in connection with other bounds on β present in literature.

4 Conclusion

In this work, we have calculated the deformation parameter β appearing in the GUP with a quadratic term in the momentum. A specific numerical value has been obtained by computing the Unruh temperature for a uniformly accelerated observer in two different ways. In the first case, the GUP (instead of the usual HUP) has been used to derive the Unruh formula. The resulting temperature (34) exhibits a (first-order) correction that explicitly depends on β. The second calculation has been performed within the framework of Caianiello's quantum geometry model. By rewriting the line element of a uniformly accelerated observer in such a way to include an upper limit on the acceleration, the Unruh temperature turns out to be accordingly modified (see Eq. (33)). Then, if we demand the GUP-deformed and the metric-deformed Unruh temperatures to be equal, we obtain the numerical value $\beta = 8\pi^2/9$ for the GUP parameter.

In this connection, we emphasize that, although a variety of experiments have been proposed to test GUP effects in laboratory [72–75], to the best of our knowledge there are only few theoretical studies which aim to fix the deformation parameter β in contexts other than string theory. In this regard, the pioneering analysis has been carried out in Ref. [32], where the conjecture that the GUP-deformed temperature of a Schwarzschild black hole coincides with the modified Hawking temperature of a quantum-corrected Schwarzschild black hole yields $\beta = 82\pi/5$. Developments of this result have been obtained in Ref. [76], where the parameter α_0 appearing in the GUP with both a linear and quadratic term in momentum has been expressed in terms of the dimensionless ratio m_p/M, with M being the mass of the considered black hole. Along this line, in Ref. [31] a possible link between the GUP parameter β and the deformation parameter Υ arising in the framework of noncommutative geometry has been discussed in Schwarzschild spacetime. In particular, it has been argued that setting Υ of the order of Planck scale would lead to $|\beta| = 7\pi^2/2$.

In line with these findings, our result corroborates string theory's prediction of $\beta \sim \mathcal{O}(1)$ on the basis of field theoretical (rather than gravitational) considerations in non-inertial frames. However, we should also note that the current experimental constraints on β are by far less stringent than the value exhibited here. For instance, gravitational tests give $\beta < 10^{78}$ from light deflection experiments [77], $\beta < 10^{60}$ from the spectrum of GW 150914 [78], and $\beta < 10^{21}$ from violation of equivalence principle on Earth [79]. Likewise, tests which do not involve the gravitational interaction lead to $\beta < 10^{39}$ from ^{87}Rb cold-atom-recoil experiments [29], $\beta < 10^{34}$ from electroweak measurement [80,81], $\beta < 10^{20}$ from Lamb shift experiments [80,81], and $\beta < 10^{18}$ from the evolution of micro and nano mechanical oscillators at Planck mass [74].

A possible matching between theoretical and experimental studies on GUP would inevitably require the development of more advanced techniques suitable to test modifications of the canonical commutator in novel parameter regimes. More work is clearly needed along this direction.

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