Effective action of a dynamical Dp -brane with background fluxes

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Abstract We shall construct the Dirac–Born–Infeld like and the Wess–Zumino like actions for a dynamical Dp -brane with the $U(1)$ gauge potential and the Kalb–Ramond background field. The brane dynamics simultaneously has both tangential and transverse components. Our calculations will be in the context of the type II superstring theory, via the boundary state formalism.

1 Introduction

Since D-branes could be defined as hypersurfaces on which the boundaries of the string worldsheets can end on them, the boundary state formalism elaborates a perfect description of them [1–10]. This method obviously provides a closed string description of the D-branes, and is applicable to various CFTs that have been used for different configurations of the D-branes, e.g. see Refs. [2, 5–21] and references therein.

On the other hand, dynamics of a D-brane is properly described in the field theory by an effective action which is given by the sum of the Dirac–Born–Infeld (DBI) [22–29] and the Wess–Zumino (WZ) [3, 30–32] actions. These are low energy effective actions of the massless fields which are induced on the brane worldvolume. In other words, these actions prominently specify the interactions between the D-brane and the massless fields. However, one of the importance of these actions is that they are indicative of the various dualities of the string theories [33–37].

A D-brane action can be extracted by various rules: the string σ -model approach [28, 29, 38, 39], the background independent open string field theory [24, 40], the scattering amplitude approach [41, 42], and the boundary state method [2, 3, 37, 43–47]. A D-brane couples to the graviton, Dilaton and Kalb–Ramond fields, and since it carries the Ramond–Ramond charges [1], it couples to the R–R fields too. There-

fore, a boundary state, which reveals the couplings of the closed states with the D-brane, gives us a satisfactory method to calculate the D-brane action and its corrections via the boundary actions of the fundamental string.

In this paper we shall obtain the DBI-like and WZ-like actions for a single dynamical Dp -brane with background fields. Our method is the boundary state formalism in the framework of the type IIA and type IIB superstring theories. The brane has been dressed by a $U(1)$ gauge potential A_α and a constant antisymmetric field $B_{\mu\nu}$. Besides, the brane has a uniform rotation inside its volume and a uniform linear motion with both transverse and tangential components. We shall see that the background fields and dynamics of the brane extremely influence the boundary state of the brane, and hence the resultant action. This generalized effective action of the brane, due to the fundamental role of the D-branes, is presumably valuable and may possibly give a deeper understanding of the substantial properties of the D-branes.

This paper is organized as follows. In Sect. 2, we shall introduce the boundary states of the NS–NS and R–R sectors of superstring, corresponding to a dressed-dynamical Dp -brane. In Sect. 3, a DBI-like action for this brane will be constructed. In Sect. 4, a WZ-like action for the same brane will be built. Section 5 is devoted to the conclusions.

2 The boundary state of a dressed-dynamical D-brane

A boundary state is a closed string state that manifestly encodes all properties of the corresponding D-brane such as: the brane couplings with the closed string states, the brane tension, dynamical variables of the brane and internal fields. This adequate state clarifies that a D-brane can emit (absorb) all closed string states. Therefore, a D-brane can be completely described by an appropriate boundary state.

We begin with the following sigma-model action for closed string to compute the boundary state, associated with a dressed-dynamical Dp -brane,

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$$\begin{aligned}
S_{\text{bulk}} &= -\frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma \\
&\quad \left(\sqrt{-g} g^{ab} G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \epsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \right), \\
S_{\text{bdry}} &= \frac{1}{2\pi\alpha'} \int_{\partial\Sigma} d\sigma \left(A_\alpha \partial_\sigma X^\alpha + \omega_{\alpha\beta} J_\tau^{\alpha\beta} \right), \quad (2.1)
\end{aligned}$$

where the total action is $S = S_{\text{bulk}} + S_{\text{bdry}}$. The coordinates $\{x^\alpha | \alpha = 0, 1, \dots, p\}$ indicate the directions along the brane worldvolume. We apply a constant Kalb-Ramond field $B_{\mu\nu}$ and the well-known gauge $A_\alpha = -\frac{1}{2} F_{\alpha\beta} X^\beta$ with a constant field strength $F_{\alpha\beta}$ for the $U(1)$ gauge potential. The string worldsheet and background spacetime are flat with $G_{\mu\nu} = \text{diag}(-1, 1, \dots, 1)$. The tangential dynamics of the brane consists of a constant antisymmetric angular velocity $\omega_{\alpha\beta}$ which represents the tangential linear motion and rotation of the brane, and $J_\tau^{\alpha\beta} = X^\alpha \partial_\tau X^\beta - X^\beta \partial_\tau X^\alpha$ shows the angular momentum density. The parameters $\omega_{\tilde{\alpha}\tilde{\beta}}$ and $\omega_{0\tilde{\alpha}}$, with $\tilde{\alpha}, \tilde{\beta} \in \{1, 2, \dots, p\}$, denote the angular and linear velocities of the brane, respectively. A transverse linear motion will be also added to the brane. Note that presence of the background fields specifies some preferred alignments inside the brane worldvolume. Thus, the Lorentz symmetry in the worldvolume subspace has been explicitly broken. This elucidates that the tangential dynamics of the brane is meaningful.

Now we impose a transverse velocity to the brane. At first, vanishing the variation of the action (2.1) defines the primary boundary state equations. Then, we introduce a Lorentz boost along the transverse direction x^{i_0} with the velocity $v^{i_0} \equiv v$ into the foregoing boundary state equations. Hence, the boosted boundary state equations possess the features

$$\begin{aligned}
&[\partial_\tau (X^0 - v X^{i_0}) + 4\omega^0_{\tilde{\beta}} \partial_\tau X^{\tilde{\beta}} + \mathcal{F}^0_{\tilde{\beta}} \partial_\sigma X^{\tilde{\beta}}]_{\tau=0} |B_x\rangle = 0, \\
&[\partial_\tau X^{\tilde{\alpha}} + 4\gamma^2 \omega^{\tilde{\alpha}}_0 \partial_\tau (X^0 - v X^{i_0}) + 4\omega^{\tilde{\alpha}}_{\tilde{\beta}} \partial_\tau X^{\tilde{\beta}} \\
&+ \gamma^2 \mathcal{F}^{\tilde{\alpha}}_0 \partial_\sigma (X^0 - v X^{i_0}) + \mathcal{F}^{\tilde{\alpha}}_{\tilde{\beta}} \partial_\sigma X^{\tilde{\beta}}]_{\tau=0} |B_x\rangle = 0, \\
&(X^{i_0} - v X^0 - y^{i_0})_{\tau=0} |B_x\rangle = 0, \\
&(X^i - y^i)_{\tau=0} |B_x\rangle = 0, \quad (2.2)
\end{aligned}$$

where $\gamma = 1/\sqrt{1-v^2}$, $i \in \{p+1, \dots, \hat{i}_0, \dots, 9\}$, i.e. $i \neq i_0$, the set $\{y^i, y^{i_0}\}$ indicates the initial location of the brane, and $\mathcal{F}_{\alpha\beta} = B_{\alpha\beta} - F_{\alpha\beta}$ is the total field strength.

Introducing the mode expansion of the closed string coordinates $X^\mu(\sigma, \tau)$ into Eq. (2.2) gives these equations in terms of the string oscillators and zero-modes α_n^μ , $\tilde{\alpha}_n^\mu$, x^μ and p^μ . The resultant equations can be solved by the coherent state method to produce the boundary state,

$$\begin{aligned}
|B_x\rangle &= \frac{T_p}{2} \sqrt{-\det Q} \\
&\quad \times \exp \left[- \sum_{m=1}^{\infty} \left(\frac{1}{m} \alpha_{-m}^\mu S_{\mu\nu} \tilde{\alpha}_{-m}^\nu \right) \right] |0\rangle_\alpha \otimes |0\rangle_{\tilde{\alpha}}
\end{aligned}$$

$$\begin{aligned}
&\times \delta \left(x^{i_0} - v x^0 - y^{i_0} \right) |p^{i_0} = 0\rangle \\
&\prod_i \left[\delta \left(x^i - y^i \right) |p^i = 0\rangle \right] \prod_\alpha |p^\alpha = 0\rangle, \quad (2.3)
\end{aligned}$$

where $T_p = \sqrt{\pi} (4\pi^2 \alpha')^{(3-p)/2}$ is the D p -brane tension, and the matrices have the following definitions

$$\begin{aligned}
S_{\mu\nu} &= \left((Q^{-1} N)_{\lambda\lambda'}, -\delta_{ij} \right), \\
Q^0_{\lambda} &= \gamma (\delta^0_{\lambda} - v \delta^{i_0}_{\lambda}) + \gamma (4\omega^0_{\tilde{\alpha}} - \mathcal{F}^0_{\tilde{\alpha}}) \delta^{\tilde{\alpha}}_{\lambda}, \\
Q^{\tilde{\alpha}}_{\lambda} &= \delta^{\tilde{\alpha}}_{\lambda} + \gamma^2 (4\omega^{\tilde{\alpha}}_0 - \mathcal{F}^{\tilde{\alpha}}_0) (\delta^0_{\lambda} - v \delta^{i_0}_{\lambda}) \\
&\quad + (4\omega^{\tilde{\alpha}}_{\tilde{\beta}} - \mathcal{F}^{\tilde{\alpha}}_{\tilde{\beta}}) \delta^{\tilde{\beta}}_{\lambda}, \\
Q^{i_0}_{\lambda} &= \delta^{i_0}_{\lambda} - v \delta^0_{\lambda}, \\
N^0_{\lambda} &= \gamma (\delta^0_{\lambda} - v \delta^{i_0}_{\lambda}) + \gamma (4\omega^0_{\tilde{\alpha}} + \mathcal{F}^0_{\tilde{\alpha}}) \delta^{\tilde{\alpha}}_{\lambda}, \\
N^{\tilde{\alpha}}_{\lambda} &= \delta^{\tilde{\alpha}}_{\lambda} + \gamma^2 (4\omega^{\tilde{\alpha}}_0 + \mathcal{F}^{\tilde{\alpha}}_0) (\delta^0_{\lambda} - v \delta^{i_0}_{\lambda}) \\
&\quad + (4\omega^{\tilde{\alpha}}_{\tilde{\beta}} + \mathcal{F}^{\tilde{\alpha}}_{\tilde{\beta}}) \delta^{\tilde{\beta}}_{\lambda}, \\
N^{i_0}_{\lambda} &= -\delta^{i_0}_{\lambda} + v \delta^0_{\lambda}, \quad (2.4)
\end{aligned}$$

where $\lambda, \lambda' \in \{0, 1, \dots, p; i_0\}$.

Since we shall utilize the overall factor of Eq. (2.3), let briefly describe its derivation. The convenient gauge $A_\alpha = -\frac{1}{2} F_{\alpha\beta} X^\beta$, accompanied by the quadratic tangential dynamics term, impose a quadratic form to the boundary action of Eq. (2.1). Path integration on this Gaussian action obviously produces the prefactor $\prod_{n=1}^{\infty} (-\det Q)^{-1}$ in Eq. (2.3). Applying the regularization scheme $\prod_{n=1}^{\infty} a^{-1} \rightarrow \sqrt{a}$ recasts the normalization factor to $\sqrt{-\det Q}$. For such overall factors of the stationary setups e.g. see the Refs. [22–24, 48].

In fact, the coherent state method provides the boundary state (2.3) under the condition $SS^T = \mathbf{1}$. This condition introduces the following relations among the input parameters

$$\begin{aligned}
\omega^0_{\tilde{\alpha}} \mathcal{F}^0_{\tilde{\alpha}} &= 0, \\
\omega^{0\tilde{\beta}} \mathcal{F}^{\tilde{\alpha}}_{\tilde{\beta}} + \mathcal{F}^{0\tilde{\beta}} \omega^{\tilde{\alpha}}_{\tilde{\beta}} &= 0, \\
\mathcal{F}^{\tilde{\alpha}}_{\tilde{\kappa}} \omega^{\tilde{\beta}\tilde{\kappa}} + \mathcal{F}^{\tilde{\beta}}_{\tilde{\kappa}} \omega^{\tilde{\alpha}\tilde{\kappa}} - \gamma^2 \\
&\quad \left(\omega^{\tilde{\alpha}}_0 \mathcal{F}^{\tilde{\beta}}_0 + \omega^{\tilde{\beta}}_0 \mathcal{F}^{\tilde{\alpha}}_0 \right) = 0. \quad (2.5)
\end{aligned}$$

Thus, from the total $1 + 3p(p+1)/2$ parameters of the set $\{\omega_{\alpha\beta}, F_{\alpha\beta}, B_{\alpha\beta}, v\}$ only p^2 of them are independent.

The worldsheet supersymmetry guides us to employ the following replacements on the bosonic boundary state equations (2.2) to construct conveniently their fermionic counterparts

$$\begin{aligned}
\partial_+ X^\mu(\sigma, \tau) &\rightarrow -i\eta \psi^\mu_+(\tau + \sigma), \\
\partial_- X^\mu(\sigma, \tau) &\rightarrow -\psi^\mu_-(\tau - \sigma), \quad (2.6)
\end{aligned}$$

where $\partial_\pm = (\partial_\tau \pm \partial_\sigma)/2$, and $\eta = \pm 1$ will be used for the GSO projection. Therefore, the boundary state equations of

the worldsheet fermions, in terms of the fermionic oscillators, find the following features

$$\begin{aligned} (\psi_t^\mu - i\eta S_{\nu}^{\mu} \tilde{\psi}_{-t}^\nu) |B_\psi^{(\text{osc})}; \eta\rangle_{\text{R,NS}} &= 0, \\ (\psi_0^\mu - i\eta S_{\nu}^{\mu} \tilde{\psi}_0^\nu) |B_\psi^{(0)}; \eta\rangle_{\text{R}} &= 0, \end{aligned} \quad (2.7)$$

where the decomposition $|B_\psi; \eta\rangle = |B_\psi^{(\text{osc})}; \eta\rangle \otimes |B_\psi^{(0)}; \eta\rangle$ was applied, and $t \in \mathbb{Z} - \{0\}$ ($t \in \mathbb{Z} + 1/2$) is related to the R–R (NS–NS) sector.

Solutions of Eq. (2.7) are given by

$$\begin{aligned} |B_\psi; \eta\rangle_{\text{NS}} &= -i \exp \left[i\eta \sum_{r=1/2}^{\infty} \psi_{-r}^\mu S_{\mu\nu} \tilde{\psi}_{-r}^\nu \right] |0\rangle_{\text{NS}}, \quad (2.8) \\ |B_\psi; \eta\rangle_{\text{R}} &= -\frac{\gamma}{\sqrt{-\det Q}} \exp \left[i\eta \sum_{n=1}^{\infty} \psi_{-n}^\mu S_{\mu\nu} \tilde{\psi}_{-n}^\nu \right] \\ &\quad \times \left(C(\Gamma^0 + v\Gamma^{i0}) \Gamma^1 \dots \Gamma^p \frac{1+i\eta\Gamma^{11}}{1+i\eta} \mathcal{H} \right)_{AB} |A\rangle \otimes |\tilde{B}\rangle, \\ \mathcal{H} &= \left[1 + v\Gamma^{i0}\Gamma^0 - 2v\Gamma^{i0}\Gamma^0 \right. \\ &\quad \left. \left(1 + (PQ^{-1}N)_{\lambda''}^{i0} \Gamma^{i0}\Gamma^{\lambda''} \right)^{-1} \right]^{-1} \\ &\quad \times : \exp \left(-\frac{1}{2} \Phi_{\lambda\lambda'} \Gamma^\lambda \Gamma^{\lambda'} \right) :, \\ \Phi &= (\phi - \phi^T)/2, \\ \Phi_{\lambda\lambda'} &\equiv \left((PQ^{-1}N + 1)^{-1} (PQ^{-1}N - 1) \right)_{\lambda\lambda'}, \end{aligned} \quad (2.9)$$

where C is the matrix of charge conjugation, and $|A\rangle$ and $|\tilde{B}\rangle$ are spinor vacua. The matrix P is defined by $P_{\lambda'}^\lambda = (\delta_{\beta}^\alpha, -\delta_{i0}^\alpha)$ with $P_{\alpha i0} = P_{i0\alpha} = 0$. The conventional notation $:\dots:$ implies that we should expand the exponential factor with the convention that all Dirac matrices anticommute, thus, a finite number of terms remain. If the dynamical variables vanish we receive $\Phi_{\alpha\beta} = \mathcal{F}_{\alpha\beta}$ which is consistent with the results of the literature.

For eliminating the tachyonic state and preserving the supersymmetry the GSO projection should be applied. The total boundary state of each sector, after the GSO projection, is given by a linear combination of the boundary states with $\eta = \pm 1$,

$$\begin{aligned} |B\rangle_{\text{NS}} &= \frac{1}{2} (|B; +\rangle_{\text{NS}} - |B; -\rangle_{\text{NS}}), \\ |B\rangle_{\text{R}} &= \frac{1}{2} (|B; +\rangle_{\text{R}} + |B; -\rangle_{\text{R}}), \\ |B; \eta\rangle_{\text{NS,R}} &= |B_x\rangle \otimes |B_\psi; \eta\rangle_{\text{NS,R}} \otimes |B_{\text{gh}}\rangle \otimes |B_{\text{sgh}}; \eta\rangle_{\text{NS,R}}, \end{aligned} \quad (2.10)$$

where $|B_{\text{gh}}\rangle$ and $|B_{\text{sgh}}\rangle$ are the known boundary states corresponding to the conformal and superconformal ghosts, respectively. They are independent of the background fields and the brane dynamics.

In the next two sections we shall utilize the GSO-projected boundary states to extend the action of a stationary Dp -brane, i.e. $S_{Dp} = S_{\text{DBI}} + S_{\text{WZ}}$, to our dynamical-dressed Dp -brane. Note that for each setup the corresponding D-brane action accurately represents the interactions of the brane with the massless fields.

3 The DBI-like action

In one hand we have the DBI action which reveals the couplings of the brane with the graviton, dilaton and Kalb–Ramond fields. On the other hand, since the boundary state encodes all properties of the brane, it also reproduces the same couplings between the brane and the massless states of closed string. Meanwhile, the disk partition function [22–24, 48], which is proportional to the inner product $\langle \text{vacuum} | B \rangle_{\text{NS}}$, elucidates that the normalization factor of the NS–NS boundary state nearly defines a DBI-like Lagrangian. Thus, the DBI-like action for our brane is given by

$$S_{\text{DBI}}^{(\omega, v)} = -\frac{T_p}{\kappa} \int d^{p+1} \xi \sqrt{-\det \tilde{Q}_{\lambda\lambda'}}, \quad (3.1)$$

where $\kappa = (2\pi)^{7/2} (\alpha')^2 g_s / \sqrt{2}$ is the gravitational constant and g_s is the string coupling. The matrix $\tilde{Q}_{\lambda\lambda'}$ is closely related to $Q_{\lambda\lambda'}$, i.e., we should apply the pull-back of the metric and Kalb–Ramond field to the matrix Q to obtain $\tilde{Q}$. However, this is a generalized DBI action which is corresponding to a dynamical Dp -brane with background fields. The explicit form of the matrix Q , i.e. Eq. (2.4), shows the combination $(4\omega - \mathcal{F})_{\alpha\beta}$ in the action (3.1). This clarifies that the tangential dynamics and the internal parts of the background fields appear in a similar fashion in the DBI-like action. However, for the stationary branes, i.e. by quenching ω and v , the DBI-like action (3.1) reduces to the conventional DBI action, as expected.

As a special case, by stopping the transverse motion we obtain the action

$$\begin{aligned} S_{\text{DBI}}^{(\omega, 0)} &= -\frac{T_p}{\kappa} \int d^{p+1} \xi \sqrt{-\det [\tilde{G}_{\alpha\beta} + \tilde{B}_{\alpha\beta} - 2\pi\alpha' (F_{\alpha\beta} - 4\omega_{\alpha\beta})]}, \end{aligned} \quad (3.2)$$

where $\tilde{G}_{\alpha\beta}$ and $\tilde{B}_{\alpha\beta}$ are pull-back of $G_{\mu\nu}$ and $B_{\mu\nu}$ on the brane worldvolume, respectively. In fact, we applied the static gauge $\xi^{i0} = X^{i0}$ which simplified the elements of the induced metric $\tilde{G}_{\lambda\lambda'}$ as $\tilde{G}_{i0i0} = 1$, $\tilde{G}_{\alpha i0} = 0$ and $\tilde{G}_{\alpha\beta}$. Expansion of this action for $\omega_{\alpha\beta} \ll 1$ yields the usual DBI action and its corrections due to the tangential dynamics. Therefore, by using the formula

$$\sqrt{\det(M_0 + M)} = \sqrt{\det M_0} \left[1 + \frac{1}{2} \text{Tr} \left(M_0^{-1} M \right) - \frac{1}{4} \text{Tr} \left(M_0^{-1} M \right)^2 + \frac{1}{4} \left[\text{Tr} \left(M_0^{-1} M \right) \right]^2 + \dots \right], \quad (3.3)$$

we acquire the following ω -corrections

$$S_{\text{DBI}}^{(\omega,0)} = -\frac{T_p}{\kappa} \int d^{p+1} \xi \sqrt{-\det \left(\tilde{G}_{\alpha\beta} + \tilde{B}_{\alpha\beta} - 2\pi\alpha' F_{\alpha\beta} \right)} \times \left\{ 1 + 4\pi\alpha' \text{Tr} \left[\left(\tilde{G} + 2\pi\alpha' \tilde{F} \right)^{-1} \omega \right] - 16\pi^2 \alpha'^2 \text{Tr} \left[\left(\tilde{G} + 2\pi\alpha' \tilde{F} \right)^{-1} \omega \right]^2 + 16\pi^2 \alpha'^2 \left(\text{Tr} \left[\left(\tilde{G} + 2\pi\alpha' \tilde{F} \right)^{-1} \omega \right] \right)^2 + \dots \right\}, \quad (3.4)$$

where $2\pi\alpha' \tilde{F} = \tilde{B}_{\alpha\beta} - 2\pi\alpha' F_{\alpha\beta}$.

As another special case, quench the tangential dynamics. In this case working with an arbitrary Dp -brane does not give the explicit form of the action. Hence, we consider a D3-brane with the velocity v along the x^4 -direction. Furthermore, let the 4×4 antisymmetric matrix $\mathcal{F}_{\alpha\beta}$ be skew-diagonal, i.e. block-diagonal with two 2×2 antisymmetric matrices. The nonzero elements of the blocks are $2\pi\alpha' f$, $-2\pi\alpha' f$, $2\pi\alpha' g$ and $-2\pi\alpha' g$. Thus, the matrix $\tilde{Q}_{\lambda\lambda'}$ possesses the structure

$$\tilde{Q}_{\lambda\lambda'}^{\lambda} = \begin{pmatrix} \gamma & -2\pi\alpha' \gamma f & 0 & 0 & -v\gamma \\ -2\pi\alpha' \gamma^2 f & 1 & 0 & 0 & 2\pi\alpha' \gamma^2 v f \\ 0 & 0 & 1 & -2\pi\alpha' g & 0 \\ 0 & 0 & 2\pi\alpha' g & 0 & 0 \\ -\gamma v & 0 & 0 & 0 & \gamma \end{pmatrix}. \quad (3.5)$$

For constructing the action we apply $\tilde{Q}_{\lambda\lambda'} = \tilde{G}_{\lambda\lambda''} \tilde{Q}_{\lambda'}^{\lambda''}$ and the static gauge. In this case the generalized action (3.1) reduces to

$$S_{\text{DBI}}^{(0,v)} = -\frac{\gamma T_3}{\kappa} \int d^4 \xi \sqrt{-\det \left(\tilde{G}_{\alpha\beta} + \tilde{B}_{\alpha\beta} - 2\pi\alpha' F_{\alpha\beta} \right) + v^2 \left(1 + (2\pi\alpha' g)^2 \right) \det \tilde{G}_{\alpha\beta}}. \quad (3.6)$$

For a very small speed, i.e. $v \ll 1$, the action (3.6) is decomposed into the conventional DBI action and its velocity corrections

$$S_{\text{DBI}}^{(0,v)} = -\frac{T_3}{\kappa} \int d^4 \xi \sqrt{-\det \left(\tilde{G}_{\alpha\beta} + \tilde{B}_{\alpha\beta} - 2\pi\alpha' F_{\alpha\beta} \right)} \times \left[1 + \frac{v^2}{2} \left(1 - \frac{1 + (2\pi\alpha' g)^2}{\det(1 + 2\pi\alpha' \tilde{G}^{-1} \tilde{F})} \right) + \dots \right]. \quad (3.7)$$

We observe that the lowest order correction is the second order of v , while Eq. (3.4) demonstrates that the corrections due to the tangential dynamics begin with the first order of ω . The generalization (3.6) and the velocity correction (3.7) are

compatible with the literature, specially for the D0-branes, e.g. see [26,49–51].

3.1 Some note on the DBI-like action

3.1.1 The Yang–Mills theory

It is well known that the $(p+1)$ -dimensional Yang–Mills theory can be extracted from the Dp -brane effective action. For example, Eq. (3.7) illustrates that the Yang–Mills theory, which lives on the worldvolume of the D3-brane with a slow transverse motion, possesses the coupling constant $g_{\text{YM}} = \frac{\sqrt{\kappa}}{\pi\alpha'\sqrt{2T_3}} \left(1 - \frac{v^2}{4} \right)$. In fact, the Yang–Mills theory, similar to the effective action of the brane, includes information about the brane. Therefore, various properties of the branes can be reliably described in the language of the Yang–Mills theory [51].

3.1.2 The brane cosmology

Here we give a brief speculation around the brane cosmology corresponding to a dynamical brane. For example, we consider the action (3.2). Quench the B -field and the $U(1)$ gauge potential. By introducing a DBI field ϕ and an appropriate potential $V(\phi)$ we receive the effective action

$$I = -\int d^4 x \left[\frac{1}{f(\phi)} \sqrt{-\det \left(\tilde{G}_{\alpha\beta} + 8\pi\alpha' \omega_{\alpha\beta} + f(\phi) \partial_\alpha \phi \partial_\beta \phi \right)} + \sqrt{-\det \tilde{G}_{\alpha\beta}} \left(-\frac{1}{f(\phi)} + V(\phi) \right) \right], \quad (3.8)$$

where the free functional $f(\phi)$ is related to the inverse of the D3-brane tension, and is specified by fixing the cosmological model, e.g. see [52,53]. The reparametrization symmetry induces a gauge freedom, which was fixed by selecting the static gauge $\{x^\alpha = \xi^\alpha | \alpha = 0, 1, 2, 3\}$. For the D3-brane with the low-energy tangential dynamics this action takes the form

$$I' = -\int d^4 x \sqrt{-\det \tilde{G}_{\alpha\beta}} \left[\frac{1}{f(\phi)} \sqrt{1 + 32(\pi\alpha')^2 \omega^{\alpha\beta} \omega_{\alpha\beta} + f(\phi) \tilde{G}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi} - \frac{1}{f(\phi)} + V(\phi) \right]. \quad (3.9)$$

For a stationary brane this action reduces to the conventional action of the literature, e.g. see [52]. By ignoring the spatial derivatives of ϕ , i.e. for an approximately homogeneous DBI field, the third term under the square root finds the feature $-f(\phi) \dot{\phi}^2$.

Now let us define the Lorentz-like factor

$$\Gamma_\omega = 1 / \sqrt{1 + 32(\pi\alpha')^2 \omega^{\alpha\beta} \omega_{\alpha\beta} - f(\phi) \dot{\phi}^2}. \quad (3.10)$$

For a constant DBI field and in the absence of the tangential rotation, i.e. for $\omega_{\tilde{\alpha}\tilde{\beta}} = 0$ with $\tilde{\alpha}, \tilde{\beta} \in \{1, 2, \dots, p\}$, this Lorentz-like factor manifestly reduces to the usual Lorentz factor of special relativity with the velocity components $V_{\tilde{\alpha}} = 4\pi\alpha'\sqrt{2\omega_{0\tilde{\alpha}}}$. We observe that the scalar field cannot roll down arbitrarily fast. Its rolling is controlled by the positivity of the phrase under the square root in Eq. (3.10), the linear velocity $\omega_{0\tilde{\alpha}}$ and the angular velocity $\omega_{\tilde{\alpha}\tilde{\beta}}$ of the brane.

According to the action (3.9) the energy density and pressure of the DBI field possess the forms

$$\begin{aligned}\rho_\phi &= \rho_\phi^{(0)} + (\Gamma_\omega - \Gamma_0) \frac{1}{f(\phi)}, \\ p_\phi &= p_\phi^{(0)} + \left(\frac{1}{\Gamma_0} - \frac{1}{\Gamma_\omega} \right) \frac{1}{f(\phi)},\end{aligned}\quad (3.11)$$

where $\rho_\phi^{(0)}$ and $p_\phi^{(0)}$ exhibit the energy density and pressure of the DBI field, associated with the stationary brane. It is usually assumed that $f(\phi)$ to be non-negative. Therefore, since there is $\Gamma_\omega < \Gamma_0$, we acquire $\rho_\phi < \rho_\phi^{(0)}$ and $p_\phi < p_\phi^{(0)}$. In the same way, the modification of the energy-momentum tensor is given by

$$T_{\alpha\beta} = T_{\alpha\beta}^{(0)} + \left(\frac{1}{\Gamma_\omega} - \frac{1}{\Gamma_0} \right) \frac{1}{f(\phi)} \tilde{G}_{\alpha\beta}. \quad (3.12)$$

Equations (3.11) and (3.12) imply that the brane dynamics extremely modifies the main quantities of the brane cosmology. However, by applying the action (3.9) and Eqs. (3.11) and (3.12) one may perform the principal equations to investigate the behavior of the corresponding brane cosmology. For example, the inflation and dark energy solutions, extracted from the DBI models [54,55], will be obviously improved by the brane dynamics.

4 The Wess–Zumino like action

It is known that a D-brane carries an R–R charge [1]. This implies that there are couplings between the massless R–R fields and the brane. The effective action which accurately specifies these interactions is the Wess–Zumino action [3,30–32]. The corresponding Lagrangian can be naturally obtained by computing the inner product between the states $|C_n\rangle$, representing the massless R–R states, and the boundary state of the R–R sector [3], i.e.,

$$\mathcal{L}_{\text{WZ}} \propto \langle C_n | B \rangle_{\text{R}}, \quad (4.1)$$

where n is odd (even) for the type IIA (type IIB) theory. The massless R–R states $|C_n\rangle$, in the picture $(-1/2, -3/2)$, can be expressed as [3],

$$\begin{aligned}|C_n\rangle &= \frac{1}{2\sqrt{2}n!} C_{\mu_1 \dots \mu_n} \left[(C\Gamma^{\mu_1 \dots \mu_n} \Pi_+)_{AB} \cos(\gamma_0 \tilde{\beta}_0) \right. \\ &\quad \left. + (C\Gamma^{\mu_1 \dots \mu_n} \Pi_-)_{AB} \sin(\gamma_0 \tilde{\beta}_0) \right] \\ &|A; k/2\rangle_{-1/2} \otimes |\tilde{B}; \tilde{k}/2\rangle_{-3/2},\end{aligned}\quad (4.2)$$

where $\Gamma^{\mu_1 \dots \mu_n}$ is the antisymmetrized product of the matrices $\{\Gamma^{\mu_1}, \Gamma^{\mu_2}, \dots, \Gamma^{\mu_n}\}$, β_0 and γ_0 are the superghost zero-modes, and $\Pi_\pm = (1 \pm \Gamma_{11})/2$. The state $|C_n\rangle$ is directly associated with the n -form R–R potential C_n .

Now we can compute a Wess–Zumino like action for the dressed-dynamical brane. The coupling between the brane and the R–R potential C_n is explicitly given by the overlap between the states (4.2) and the boundary state of the R–R sector

$$\begin{aligned}\langle C_n | B \rangle_{\text{R}} &= -\frac{T_p}{16\sqrt{2}n!} \frac{V_p}{v} C_{\mu_1 \dots \mu_n} \\ &\text{Tr} \left(\Gamma^{\mu_n \dots \mu_1} \left(\Gamma^0 + v \Gamma^{i_0} \right) \Gamma^1 \dots \Gamma^p \mathcal{H} : e^{-\frac{1}{2} \Phi_{\lambda\lambda'} \Gamma^\lambda \Gamma^{\lambda'}} : \right), \\ \mathcal{H} &= \left[\mathbf{1} + v \Gamma^{i_0} \Gamma^0 - 2v \Gamma^{i_0} \Gamma^0 \right. \\ &\quad \left. \left(\mathbf{1} + (PQ^{-1}N)^{i_0}_{\lambda} \Gamma^{i_0} \Gamma^\lambda \right)^{-1} \right]^{-1}.\end{aligned}\quad (4.3)$$

Calculation of the above trace for an arbitrary velocity is very complicated. For simplification we assume that the brane moves with a small velocity, i.e. $v \ll 1$. Furthermore, we consider the indices $\mu_1, \dots, \mu_n$ along the worldvolume of the brane. Consequently, the exponential part and the matrix $\mathcal{H}$ reduce to

$$\begin{aligned}&: e^{-\frac{1}{2} \Phi_{\lambda\lambda'} \Gamma^\lambda \Gamma^{\lambda'}} : \\ &:= \left[1 - \frac{v}{2} \left(\frac{\partial \Phi_{\lambda\lambda'}}{\partial v} \right)_{v=0} : \Gamma^\lambda \Gamma^{\lambda'} : + \mathcal{O}(v^2) \right] : e^{-\frac{1}{2} \Phi_{\lambda_1 \lambda_2}^{(0)} \Gamma^{\lambda_1} \Gamma^{\lambda_2}} :, \\ \mathcal{H} &= \mathbf{1} + v \Gamma^{i_0} \Gamma^0 - 2v(PQ_{(0)}^{-1} N_{(0)})^{i_0}_{\lambda} \Gamma^0 \Gamma^\lambda + \mathcal{O}(v^2),\end{aligned}\quad (4.4)$$

where $\Phi_{\lambda_1 \lambda_2}^{(0)}$, $Q_{(0)}$ and $N_{(0)}$ are $\Phi_{\lambda_1 \lambda_2}$, Q and N with $v = 0$, respectively. Now by expanding the exponential factor of the right-hand side of Eq. (4.4), due to the antisymmetrization symbol $:$, different exponents of $\Phi^{(0)}$ of the rank $l \in \{0, 1, \dots, l_{\text{max}}\}$ will appear in the first equation of (4.3). For receiving a nonzero trace we determine l_{max} via the velocity independent term inside the trace part. Therefore, we acquire $l_{\text{max}} = p/2$ ($l_{\text{max}} = (p+1)/2$) for the type IIA theory (type IIB theory), and we should use $n = p+1-2l$.

At first we apply $n = p+1$, which indicates the coupling of the Dp -brane with the $(p+1)$ -form potential C_{p+1} ,

$$\begin{aligned}\langle C_{p+1} | B \rangle_{\text{R}} &= \frac{\sqrt{2}T_p}{(p+1)!} \frac{V_p}{v} \tilde{C}_{\alpha_0 \dots \alpha_p} \varepsilon^{\alpha_0 \dots \alpha_p} \\ &\left[1 + 2v(PQ_{(0)}^{-1} N_{(0)})^{i_0}_0 + 2v(PQ_{(0)}^{-1} N_{(0)})^{i_0}_{\lambda} \Phi_{0\lambda}^{(0)} \right. \\ &\quad \left. - \frac{v}{4} \left(\frac{\partial \Phi_{\alpha\beta}}{\partial v} \right)_{v=0} \Phi_{\alpha\beta}^{(0)} + \mathcal{O}(v^2) \right],\end{aligned}\quad (4.5)$$

where $\bar{\lambda} \in \{1, 2, \dots, p, i_0\}$, V_p is the brane volume, $\varepsilon^{\alpha_0 \dots \alpha_p}$ is the Levi-Civita tensor, and the tensor $\tilde{C}_{\alpha_0 \dots \alpha_p}$ is the pull-back of $C_{\mu_0 \dots \mu_p}$ on the brane worldvolume.

The next case is $n = p - 1$, which defines the following interaction terms

$$\begin{aligned} \langle C_{p-1}|B\rangle_R = & -\frac{\sqrt{2}T_p}{2(p-1)!} \frac{V_p}{v} \left\{ \tilde{C}_{\alpha_0 \dots \alpha_{p-2}} \varepsilon^{\alpha_0 \dots \alpha_p} \right. \\ & \left[\Phi_{\alpha_{p-1}\alpha_p}^{(0)} \left(1 + 2v(PQ_{(0)}^{-1}N_{(0)})_{i_0}^{i_0} \right. \right. \\ & + 2v(PQ_{(0)}^{-1}N_{(0)})_{\bar{\lambda}}^{i_0} \Phi_{0\bar{\lambda}}^{(0)} - \frac{v}{4} \left(\frac{\partial \Phi_{\alpha\beta}}{\partial v} \right)_{v=0} \Phi_{\alpha\beta}^{(0)} \\ & \left. \left. - \frac{v}{4} \Phi_{i_0\alpha_{p-1}}^{(0)} \left(\frac{\partial \Phi_{i_0\alpha_p}}{\partial v} \right)_{v=0} + v \left(\frac{\partial \Phi_{\alpha_{p-1}\alpha_p}}{\partial v} \right)_{v=0} \right] \right. \\ & \left. + 4v\tilde{C}_{\bar{\alpha}_1 \dots \bar{\alpha}_{p-1}} \varepsilon^{\bar{\alpha}_1 \dots \bar{\alpha}_{p-1} \bar{\alpha}_p} (PQ_{(0)}^{-1}N_{(0)})_{\bar{\alpha}_p}^{i_0} \right\}. \quad (4.6) \end{aligned}$$

The first R–R interaction represents the coupling of the potential C_{p-1} with the Dp -brane. This coupling clearly comprises all components of the pull-back tensor $\tilde{C}_{\alpha_0 \dots \alpha_{p-2}}$. The second R–R interaction reveals the coupling of the Dp -brane with the same potential C_{p-1} . This coupling includes only the pure spatial components of $\tilde{C}_{\alpha_0 \dots \alpha_{p-2}}$, i.e. $\bar{\alpha}_1, \dots, \bar{\alpha}_{p-1} \neq 0$.

The coupling of the brane with the potential C_{p-3} is given by

$$\begin{aligned} \langle C_{p-3}|B\rangle_R = & \frac{\sqrt{2}T_p}{4(p-3)!} \frac{V_p}{v} \left\{ \tilde{C}_{\alpha_0 \dots \alpha_{p-4}} \varepsilon^{\alpha_0 \dots \alpha_p} \Phi_{\alpha_{p-3}\alpha_{p-2}}^{(0)} \right. \\ & \left[\Phi_{\alpha_{p-1}\alpha_p}^{(0)} \left(1 + 2v(PQ_{(0)}^{-1}N_{(0)})_{i_0}^{i_0} \right. \right. \\ & + 2v(PQ_{(0)}^{-1}N_{(0)})_{\bar{\lambda}}^{i_0} \Phi_{0\bar{\lambda}}^{(0)} - \frac{v}{4} \left(\frac{\partial \Phi_{\alpha\beta}}{\partial v} \right)_{v=0} \Phi_{\alpha\beta}^{(0)} \\ & \left. \left. - \frac{v}{2} \Phi_{i_0\alpha_{p-1}}^{(0)} \left(\frac{\partial \Phi_{i_0\alpha_p}}{\partial v} \right)_{v=0} + v \left(\frac{\partial \Phi_{\alpha_{p-1}\alpha_p}}{\partial v} \right)_{v=0} \right] \right. \\ & \left. + 4v\tilde{C}_{\bar{\alpha}_1 \dots \bar{\alpha}_{p-3}} \varepsilon^{\bar{\alpha}_1 \dots \bar{\alpha}_{p-3} \bar{\alpha}_{p-2} \bar{\alpha}_{p-1}} (PQ_{(0)}^{-1}N_{(0)})_{\bar{\alpha}_{p-1}}^{i_0} \right\}, \quad (4.7) \end{aligned}$$

The first R–R interaction shows the coupling of the potential C_{p-3} with the Dp -brane, which contains all components of the pull-back tensor $\tilde{C}_{\alpha_0 \dots \alpha_{p-4}}$. The second R–R interaction clarifies the coupling of the brane with the same potential C_{p-3} . This coupling only consists of the pure spatial components of $\tilde{C}_{\alpha_0 \dots \alpha_{p-4}}$.

In the same way one can obtain couplings of the brane with the other R–R potentials C_n for $n \leq p - 5$. These couplings, accompanied by Eqs. (4.5)–(4.7), establish the following Wess–Zumino like action

$$\begin{aligned} S_{\text{WZ}}^{(\omega, v)} = & \frac{\mu_p V_p}{v V_{p+1}} \int_{V_{p+1}} \left\{ \left[\sum_{l=0}^{l_{\max}} C_{p+1-2l} \wedge e^{2\pi\alpha'\Phi^{(0)}} \right]_{p+1} \right. \\ & + vC_{p+1} \left[2(PQ^{-1}N)_{i_0}^{i_0} + 2(PQ^{-1}N)_{\bar{\lambda}}^{i_0} \Phi_{0\bar{\lambda}}^{(0)} \right. \\ & \left. \left. - \frac{1}{4} \frac{\partial \Phi_{\alpha\beta}}{\partial v} \Phi_{\alpha\beta} \right]_{v=0} \right. \\ & - 2\pi\alpha' vC_{p-1} \wedge \left[\Phi \left(2(PQ^{-1}N)_{i_0}^{i_0} + 2(PQ^{-1}N)_{\bar{\lambda}}^{i_0} \Phi_{0\bar{\lambda}}^{(0)} \right. \right. \\ & \left. \left. - \frac{1}{4} \frac{\partial \Phi_{\alpha\beta}}{\partial v} \Phi_{\alpha\beta} \right) - \frac{1}{4} \Omega + \frac{\partial \Phi}{\partial v} \right]_{v=0} \\ & + \frac{1}{2} (2\pi\alpha')^2 vC_{p-3} \wedge \Phi^{(0)} \\ & \wedge \left[\Phi \left(2(PQ^{-1}N)_{i_0}^{i_0} - \frac{1}{4} \frac{\partial \Phi_{\alpha\beta}}{\partial v} \Phi_{\alpha\beta} + 2(PQ^{-1}N)_{\bar{\lambda}}^{i_0} \Phi_{0\bar{\lambda}}^{(0)} \right. \right. \\ & \left. \left. - \frac{1}{2} \Omega + \frac{\partial \Phi}{\partial v} \right]_{v=0} + \dots \right\} \\ & - 8\pi\alpha' \mu_p v \int_{V_p} \left[\left(\tilde{C}_{p-1} - \frac{1}{2} \pi\alpha' \tilde{C}_{p-3} \wedge \bar{\Phi}^{(0)} \right) \right. \\ & \left. \wedge \bar{W} + \dots \right], \quad (4.8) \end{aligned}$$

where $\mu_p = \sqrt{2}T_p$ is the R–R charge of the brane because of the potential C_{p+1} . The differential forms Φ , $\bar{\Phi}^{(0)}$, Ω and $\bar{W}$ have the following definitions

$$\begin{aligned} \Phi = & \frac{1}{2} \Phi_{\alpha\beta} d\xi^\alpha \wedge d\xi^\beta, \\ \bar{\Phi}^{(0)} = & \frac{1}{2} \Phi_{\bar{\alpha}\bar{\beta}}^{(0)} d\xi^{\bar{\alpha}} \wedge d\xi^{\bar{\beta}}, \\ \Omega = & \frac{1}{2} \left(\Phi_{i_0\alpha} \frac{\partial \Phi_{i_0\beta}}{\partial v} - \Phi_{i_0\beta} \frac{\partial \Phi_{i_0\alpha}}{\partial v} \right)_{v=0} d\xi^\alpha \wedge d\xi^\beta, \\ \bar{W} = & (PQ_{(0)}^{-1}N_{(0)})_{\bar{\alpha}}^{i_0} d\xi^{\bar{\alpha}}. \quad (4.9) \end{aligned}$$

The R–R forms in the last integral possess only the pure spatial components, and are defined by

$$\bar{C}_m = \frac{1}{m!} \tilde{C}_{\bar{\alpha}_1 \bar{\alpha}_2 \dots \bar{\alpha}_m} d\xi^{\bar{\alpha}_1} \wedge d\xi^{\bar{\alpha}_2} \wedge \dots \wedge d\xi^{\bar{\alpha}_m}, \quad (4.10)$$

where $m \in \{p-1, p-3, p-5, \dots\}$.

Note that the matrices Q , $Q_{(0)}$, N , $N_{(0)}$, Φ and $\Phi^{(0)}$ explicitly depend on the potential $A_\alpha(\xi^0, \xi^1, \dots, \xi^p)$, via its field strength $F_{\alpha\beta}$, and the worldvolume coordinates $X^\mu(\xi^0, \xi^1, \dots, \xi^p)$. Since the gauge field and worldvolume coordinates are the main degrees of freedom, the WZ-like and the DBI-like actions exhibit a generalized effective action for the dressed-dynamical brane. However, the effective Lagrangian is a very complicated functional of the foregoing degrees of freedom.

Finally, by stopping the brane, i.e. by setting ω and v to zero, we receive $\Phi_{\alpha\beta} = \mathcal{F}_{\alpha\beta}$, and hence the WZ-like action (4.8) reduces to the conventional WZ action, as expected.

We extended the effective action of a Dp -brane via its tangential and transverse dynamics. In fact, there are various extensions for the brane effective action: derivative corrections [46], α' -corrections [56], tachyonic extension [57] and curvature corrections [58]. Accordingly, for a given setup of a Dp -brane one may combine some of these modifications to construct a suitable action. For example, for a dynamical Dp -brane with the tachyon field in a curved background one should add an appropriate tachyon potential and the curvature improvements to our action.

5 Conclusions

We obtained the effective action of a dynamical Dp -brane with background fields. This generalized action consists of the DBI-like and WZ-like parts. To obtain the effective action we applied the boundary state formalism with the following background fields: a constant Kalb-Ramond field and a $U(1)$ internal gauge potential. The dynamics of the brane includes a tangential rotation and a linear motion with both tangential and transverse components. For slow motion of the brane we decomposed the DBI-like action into a pure DBI one and its corrections due to the brane dynamics. We acquired the WZ-like action by computing the couplings of the R–R sector boundary state with the massless states of the R–R sector.

The effective action of the brane depends on the brane velocities v and $\omega_{\alpha\beta}$, and the fields $F_{\alpha\beta}$ and $B_{\mu\nu}$. The variety of the variables $\{F_{\alpha\beta}, B_{\mu\nu}, \omega_{\alpha\beta}, v; p\}$ dedicated a generalized feature to the action. However, by expanding the determinant and then square root in the DBI-like part, and also Q^{-1} and exponential in the WZ-like part, one can read the coupling constants. These constants depend on the input parameters $\omega_{\alpha\beta}$ and v . Thus, by adjusting the values of these parameters, the values of the coupling constants can be accurately adjusted to any desirable values.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This is a theoretical work. No experimental data were used.]

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