SU(3) analysis of weak decays of doubly-heavy tetraquarks $b\bar{c}q\bar{q}$

Gang Li¹, Xiao-Feng Wang¹, Ye Xing^{2,a}

¹ School of Physics and Engineering, Qufu Normal University, Qufu 273165, China

² INPAC, SKLPPC, MOE Key Laboratory for Particle Physics, School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China

Received: 15 November 2018 / Accepted: 27 February 2019 / Published online: 8 March 2019

© The Author(s) 2019

Abstract We study the weak decays of exotic tetraquark states $b\bar{c}q\bar{q}$ with two heavy quarks. Under the SU(3) symmetry for light quarks, these tetraquarks can be classified into an octet plus a singlet: $3 \otimes \bar{3} = 1 \oplus 8$. We will concentrate on the octet tetraquarks with $J^P = 0^+$, and study their weak decays, both semileptonic and nonleptonic. Hadron-level effective Hamiltonian is constructed according to the irreducible representations of the SU(3) group. Expanding the Hamiltonian, we obtain the decay amplitudes parameterized in terms of a few irreducible quantities. Based on these amplitudes, relations for decay widths are derived, which can be tested in future. We also give a list of golden channels that can be used to look for these states at various colliders.

1 Introduction

Since the first discovery of X(3872) by Belle in 2003 [1], a large number of charmonium-like and bottomonium-like hadrons have been discovered in the past decade [2]. Many of these discovered states defy a standard quarkonium interpretation and likely have a pair of hidden flavored quarks, with the quark content $Q\bar{Q}q\bar{q}'$ (for a recent review, see Refs. [3, 4]). Here Q represents a heavy bottom/charm quark and $q(q')$ denotes a light u, d, s quark. Extensive theoretical studies have been carried out to explore their structures, productions and decays [5–34]. In 2016, the D0 collaboration has reported an evidence for the open-bottom hadron X(5568) [35], though it has not been confirmed by the other experimental groups [36–39]. Therefore the existence of open-flavored hadrons is an interesting question in hadronic physics, in particular the hadron spectroscopy.

Four-quark states, such as tetraquark states, molecular states, are candidates of the exotic states, by exotic we mean a more complex structure than the simple quark-antiquark scenario for a meson and three-quark structure for a baryon.

The idea of unconventional quark structures dates back to Gellman original proposal of quark model [40]. In general ordinary mesons $Q_i\bar{Q}_j$ could mix with the four-quark states $Q_i\bar{Q}_j q\bar{q}$, by creating and annihilating light quark-antiquark pairs. Therefore, pure physical exotic state can be the mixture between meson and tetraquark (or molecular) or tetraquark and molecular, see some reviews [41–44]. Despite of many progresses, the structure of hadron exotics is still under debate, such as the X(3872), and more work is certainly needed to clarify this issue. In this paper, we choose the tetraquark model and focus on the four-quark states with two different heavy quarks and two light quarks. Since they can provide a unique platform to study strong interactions under two color static sources. In the diquark-antidiquark model [45], the four-quarks system $[bq][\bar{c}\bar{q}]$ with orbital angular momentum $L = 0$ can have $J^P = 0^+$ [46]. Since the 0^+ tetraquarks are lowest lying, their weak decays can provide unique insights to unravel their internal structure. In this paper, we adopt the SU(3) flavor symmetry to handle these weak decays. The SU(3) approach has been successfully applied into the B meson and heavy baryon decays [47–62] and a global picture consistent data has been established. In the SU(3) symmetry, the tetraquarks with two light quarks can form an octet and a singlet. In this work, we will concentrate on the octet, abbreviated as $T_{b\bar{c}8}$.

In the following, we will first construct the hadron-level effective Hamiltonian according to the irreducible representations of the SU(3) group. Expanding these Hamiltonian, we obtain the decay amplitudes parameterized in terms of a few SU(3) irreducible quantities. Based on the expanded amplitudes, relations for decay widths are derived, which can be examined in the future. We also give a list of golden channels that can be used to look for these states at various colliders.

The rest of this paper is organized as follows. In Sect. 2, we give the multiplets expressions under the SU(3) flavor symmetry. From Sects. 3 and 4, we mainly study the semileptonic and non-leptonic weak decays of the $X_{b\bar{c}8}$ states.

^a e-mail: xingye_guang@sjtu.edu.cn

In Sect. 5, we will give a collection of the golden channels that can be used to discover the doubly heavy tetraquarks in future experiments. we make a summary in the last section.

2 Particle multiplets

Based on the light flavor SU(3) symmetry, open-flavor tetraquark with the quark constituents $b\bar{c}q\bar{q}$ can form an octet and a singlet, $3 \otimes \bar{3} = 1 \oplus 8$, of which the octet and singlet can be represented as

$$T_{b\bar{c}8} = \begin{pmatrix} T_{\pi^0}^{Bc} + \frac{T_{\eta_8}^{Bc}}{\sqrt{2}} & T_{\pi^+}^{Bc} & T_{K^+}^{Bc} \\ T_{\pi^-}^{Bc} & -\frac{T_{\pi^0}^{Bc}}{\sqrt{2}} + \frac{T_{\eta_8}^{Bc}}{\sqrt{6}} & T_{K^0}^{Bc} \\ T_{K^-}^{Bc} & T_{\bar{K}^0}^{Bc} & -\frac{2}{\sqrt{6}}T_{\eta_8}^{Bc} \end{pmatrix},$$

$$T_{b\bar{c}1} = \begin{pmatrix} \frac{T_{\eta_1}^{Bc}}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{T_{\eta_1}^{Bc}}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{T_{\eta_1}^{Bc}}{\sqrt{3}} \end{pmatrix}. \quad (1)$$

In the light meson sector, pseudoscalar mesons or vector mesons can also form an octet plus a singlet, generally, the octet and singlet are written as

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}. \quad (2)$$

In the flavor space, $\eta_8 = (\bar{u}u + \bar{d}d - 2\bar{s}s)/\sqrt{6}$, and $\eta_1 = (\bar{u}u + \bar{d}d + \bar{s}s)/\sqrt{3}$. With the same quark content, the vector meson octet and singlet will take a similar structure:

$$V = \begin{pmatrix} \frac{\rho^0 + \omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0 + \omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}, \quad (3)$$

where ω and ϕ are the mixtures of V_8 and V_1 :

$$\begin{pmatrix} \omega \\ \phi \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} V_8 \\ V_1 \end{pmatrix}. \quad (4)$$

An ideal mixing exists for ω and ϕ and thus $\theta \simeq 54.74^\circ$. Besides, we need the representation of bottom mesons which form an SU(3) anti-triplet given as: $B_i = (B^-, \bar{B}^0, \bar{B}_s^0)$, and the representation of anti-triplet charmed mesons given as: $D_i = (D^0, D^+, D_s^+)$, $\bar{D}^i = (\bar{D}^0, D^-, D_s^-)$. Actually, in the SU(3) symmetry, one should add a minus sign to D^+ and $\bar{K}^0$, but this will not affect the total decay widths.

3 Semi-leptonic $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays

In this section, we will discuss the possible semi-leptonic weak decay modes of the tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$. At quark level, both b -quark and $\bar{c}$ -quark in $T_{b\bar{c}8}/T_{b\bar{c}1}$ can decay. For the b -quark, semi-leptonic weak decays are governed by

$$b \rightarrow c/u\ell^-\bar{\nu}_\ell. \quad (5)$$

For the $\bar{c}$ -quark, semi-leptonic decays are induced by

$$\bar{c} \rightarrow \bar{d}/\bar{s}\ell^-\bar{\nu}_\ell. \quad (6)$$

3.1 $b \rightarrow c/u\ell^-\bar{\nu}_\ell$: decays into a meson and $\ell^-\bar{\nu}_\ell$

The general electro-weak Hamiltonian of the $b \rightarrow c/u\ell^-\bar{\nu}_\ell$ transition is given as

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} [V_{q'b}\bar{q}'\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell] + h.c., \quad (7)$$

with $q' = u, c$, in which the electro-weak vertex denotes the $V - A$ structure. For $b \rightarrow u\ell^-\bar{\nu}_\ell$, the electro-weak vertex forms a triplet H'_3 within SU(3) flavor symmetry, specifically $(H'_3)^1 = 1$ and $(H'_3)^{2,3} = 0$. At the hadron level, the transition can be included into the process that $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays to a charmed meson and $\ell\bar{\nu}_\ell$. The lepton pair does not carry any SU(3) index, and thus is an SU(3) singlet. Following the SU(3) analysis, the Hamiltonian at the hadronic level is constructed as

$$\mathcal{H}_{eff} = a_1(T_{b\bar{c}8})_j^i(H'_3)^j D_i \bar{\ell}\nu + a'_1(T_{b\bar{c}1})_i^i(H'_3)^j D_j \bar{\ell}\nu, \quad (8)$$

here, the coefficients a_1 and a'_1 represent the non-perturbative parameters. For completeness, we give the corresponding topological diagram at quark level shown in Fig. 1a, b. The diagrams given in Fig. 1 are topological diagrams, which are collections of Feynman diagrams with the same SU(3) topology. It is convenient to achieve the decay amplitudes given in Table 1 by expanding the Hamiltonian constructed above, in which all amplitudes are represented as a_1 and a'_1 . Therefore, we can directly obtain the relations between different decay channels given as follows:

$$\begin{aligned} \Gamma(T_{\pi^-}^{Bc} \rightarrow D^- l^- \bar{\nu}) &= 2\Gamma(T_{\pi^0}^{Bc} \rightarrow \bar{D}^0 l^- \bar{\nu}) \\ &= \Gamma(T_{K^-}^{Bc} \rightarrow D_s^- l^- \bar{\nu}) \\ &= 6\Gamma(T_{\eta_8}^{Bc} \rightarrow \bar{D}^0 l^- \bar{\nu}). \end{aligned}$$

It should be noted that the phase space difference will provide corrections to these relations.

Fig. 1 Topological diagrams for semileptonic decays of tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$. **a–c** Correspond to the b quark decay and **d, e** denotes the $\bar{c}$ quark decay. It is necessary to point out that any gluon exchange can

introduce new Feynman diagrams, but these Feynman diagrams have the same SU(3) topology, and thus can be classified as the same topological diagram

Table 1 Amplitudes for tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into anti-charmed meson, a light meson or J/Ψ

Channel	Amplitude	Channel	Amplitude
$T_{\pi^-}^{Bc} \rightarrow D^- l^- \bar{\nu}$	$a_1 V_{ub}$	$T_{\pi^0}^{Bc} \rightarrow \bar{D}^0 l^- \bar{\nu}$	$\frac{a_1 V_{ub}}{\sqrt{2}}$
$T_{K^-}^{Bc} \rightarrow D_s^- l^- \bar{\nu}$	$a_1 V_{ub}$	$T_{\eta_8}^{Bc} \rightarrow \bar{D}^0 l^- \bar{\nu}$	$\frac{a_1 V_{ub}}{\sqrt{6}}$
$T_{\eta_1}^{Bc} \rightarrow \bar{D}^0 l^- \bar{\nu}$	$\sqrt{3} a_1' V_{ub}$		
$T_{\pi^-}^{Bc} \rightarrow \pi^- l^- \bar{\nu}$	a_2	$T_{\pi^0}^{Bc} \rightarrow \pi^0 l^- \bar{\nu}$	a_2
$T_{\pi^+}^{Bc} \rightarrow \pi^+ l^- \bar{\nu}$	a_2	$T_{K^-}^{Bc} \rightarrow K^- l^- \bar{\nu}$	a_2
$T_{\bar{K}^0}^{Bc} \rightarrow \bar{K}^0 l^- \bar{\nu}$	a_2	$T_{K^0}^{Bc} \rightarrow K^0 l^- \bar{\nu}$	a_2
$T_{K^+}^{Bc} \rightarrow K^+ l^- \bar{\nu}$	a_2	$T_{\eta_8}^{Bc} \rightarrow \eta_8 l^- \bar{\nu}$	a_2
$T_{\eta_1}^{Bc} \rightarrow \eta_1 l^- \bar{\nu}$	$3a_2'$	$T_{\eta_1}^{Bc} \rightarrow J/\psi l^- \bar{\nu}$	$\sqrt{3} a_3'$

For the SU(3) singlet $b \rightarrow c$ transition, the process at the hadron level is that $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into a light meson octet or J/ψ and $\ell \bar{\nu}_\ell$. Consequently, the Hamiltonian at the hadron level is constructed as

$$\mathcal{H}_{eff} = a_2 (T_{b\bar{c}8})_j^i M_j^i \bar{\ell} \nu + a_2' (T_{b\bar{c}1})_i^j M_j^i \bar{\ell} \nu + a_3' (T_{b\bar{c}1})_i^j J/\psi \bar{\ell} \nu. \quad (9)$$

One then obtains the amplitudes of different decay channels listed in Table 1, from which we derive that all the channels of the octet tetraquark in the transition give equal decay widths.

3.2 $\bar{c} \rightarrow \bar{d}/\bar{s} \ell^- \bar{\nu}$ decay into B meson and $\ell^- \bar{\nu}$

The electro-weak effective Hamiltonian for $\bar{c}$ quark decay is given as

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} [V_{cq} \bar{c} \gamma^\mu (1 - \gamma_5) q \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell] + h.c., \quad (10)$$

where $q = d, s$, V_{cd} and V_{cs} are CKM matrix elements. Under the SU(3) symmetry, the $\bar{c} \rightarrow \bar{q} \ell^- \bar{\nu}$ transition can form an SU(3) triplet, denoted as H_3 with $(H_3)^1 =$

Table 2 Amplitudes for tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into a B meson

Channel	Amplitude	Channel	Amplitude
$T_{\pi^-}^{Bc} \rightarrow B^- l^- \bar{\nu}$	$c_1 V_{cd}$	$T_{\pi^0}^{Bc} \rightarrow \bar{B}^0 l^- \bar{\nu}$	$-\frac{c_1 V_{cd}}{\sqrt{2}}$
$T_{K^-}^{Bc} \rightarrow B^- l^- \bar{\nu}$	$c_1 V_{cs}$	$T_{\bar{K}^0}^{Bc} \rightarrow \bar{B}^0 l^- \bar{\nu}$	$c_1 V_{cs}$
$T_{K^0}^{Bc} \rightarrow \bar{B}_s^0 l^- \bar{\nu}$	$c_1 V_{cd}$	$T_{\eta_8}^{Bc} \rightarrow \bar{B}^0 l^- \bar{\nu}$	$\frac{c_1 V_{cd}}{\sqrt{6}}$
$T_{\eta_8}^{Bc} \rightarrow \bar{B}_s^0 l^- \bar{\nu}$	$-\sqrt{\frac{2}{3}} c_1 V_{cs}$	$T_{\eta_1}^{Bc} \rightarrow \bar{B}^0 l^- \bar{\nu}$	$\sqrt{3} c_1' V_{cd}$
$T_{\eta_1}^{Bc} \rightarrow \bar{B}_s^0 l^- \bar{\nu}$	$\sqrt{3} c_1' V_{cs}$		

0, $(H_3)^2 = V_{cd}$, $(H_3)^3 = V_{cs}$. Accordingly, we construct the Hamiltonian at the hadron level as follows:

$$\mathcal{H}_{eff} = c_1 (T_{b\bar{c}8})_j^i (H_3)_i \bar{B}^j \bar{\ell} \nu + c_1' (T_{b\bar{c}1})_i^j (H_3)_j \bar{B}^i \bar{\ell} \nu. \quad (11)$$

The decay amplitudes deduced from the Hamiltonian above are listed in Table 2. For completeness, we give the corresponding topological diagrams in Fig. 1d, e. One then obtains the relations between different channels as follows:

$$\begin{aligned} \Gamma(T_{\pi^-}^{Bc} \rightarrow B^- l^- \bar{\nu}) &= 2\Gamma(T_{\pi^0}^{Bc} \rightarrow \bar{B}^0 l^- \bar{\nu}) \\ &= \Gamma(T_{K^0}^{Bc} \rightarrow \bar{B}_s^0 l^- \bar{\nu}) \\ &= 6\Gamma(T_{\eta_8}^{Bc} \rightarrow \bar{B}^0 l^- \bar{\nu}), \\ \Gamma(T_{K^-}^{Bc} \rightarrow B^- l^- \bar{\nu}) &= \Gamma(T_{\bar{K}^0}^{Bc} \rightarrow \bar{B}^0 l^- \bar{\nu}) \\ &= \frac{3}{2}\Gamma(T_{\eta_8}^{Bc} \rightarrow \bar{B}_s^0 l^- \bar{\nu}). \end{aligned}$$

It is necessary to notice that the relations between different channels can be modified with a view to the SU(3) symmetry breaking effects in the charmed or anti-charmed quark decays.

Fig. 2 Topological diagrams for the b -quark non-leptonic decays of tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$. **a–p** Correspond to the decays into a pair of mesons. In **j, l, m, n, p**, the final meson produced by gluons is the

flavor singlet. In **k, o, p**, the initial state is the flavor singlet $T_{b\bar{c}1}$. The diagrams in **c, d, g, h, i** are usually power suppressed as a pair of quark and anti-quark in the initial state can annihilate

4 Non-leptonic $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays

The b quark non-leptonic decays can be divided into four categories:

$$b \rightarrow c\bar{c}d/s, \quad b \rightarrow c\bar{u}d/s, \quad b \rightarrow u\bar{c}d/s, \quad b \rightarrow q_1\bar{q}_2q_3, \quad (12)$$

where $q_{1,2,3}$ represent the light quark. In $\bar{c}$ quark non-leptonic decays, the classifications are given as:

$$\bar{c} \rightarrow \bar{s}d\bar{u}, \quad \bar{c} \rightarrow \bar{u}d\bar{s}/\bar{s}\bar{s}, \quad \bar{c} \rightarrow \bar{d}s\bar{u}, \quad (13)$$

which are Cabibbo allowed, singly Cabibbo suppressed, and doubly Cabibbo suppressed, respectively. In the following, we will study the $T_{b\bar{c}8}/T_{b\bar{c}1}$ non-leptonic decays in order.

4.1 $b \rightarrow c\bar{c}d/s$ transition: two-body decays into mesons

The operators for the $b \rightarrow c\bar{c}d/s$ transition can form a triplet under the SU(3) light quark symmetry, according to that we can write down the effective Hamiltonian of $T_{b\bar{c}8}/T_{b\bar{c}1}$ producing two mesons as follows:

$$\begin{aligned} \mathcal{H}_{eff} &= a_1(T_{b\bar{c}8})_j^i (H_3)^j D_i J/\psi + a'_1(T_{b\bar{c}1})_i^j (H_3)^j D_j J/\psi, \\ \mathcal{H}_{eff} &= a_2(T_{b\bar{c}8})_j^i (H_3)^j D_k M_i^k + a_3(T_{b\bar{c}8})_j^i (H_3)^k D_k M_i^j \\ &\quad + a_4(T_{b\bar{c}8})_j^i (H_3)^k D_i M_k^j \\ &\quad + a'_2(T_{b\bar{c}1})_i^j (H_3)^j D_k M_j^k + a'_3(T_{b\bar{c}1})_i^j (H_3)^k D_k M_j^i \\ &\quad + a'_4(T_{b\bar{c}8})_k^i (H_3)^k D_i M_j^j, \end{aligned} \quad (14)$$

with $(H_3)^2 = V_{cd}^*$ and $(H_3)^3 = V_{cs}^*$. Consequently, the corresponding topological diagrams are given in Fig. 2a–d for the octet tetraquark $T_{b\bar{c}8}$, the diagrams in Fig. 2k, o, m, p represent the processes which correspond with the singlet tetraquark or light singlet meson. In particular, the diagrams in Fig. 2a, b represent $T_{b\bar{c}8}$ decays into D and J/ψ mesons, and the diagrams in Fig. 2c, d denote processes with D and light mesons final states. Expanding the two Hamiltonian above, one obtains the decay amplitudes which are listed in Table 3. In addition, the relations between the different decay widths are given as follows:

Table 3 Tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into anti-charmed meson plus light meson or anti-charmed meson plus J/ψ

Channel	Amplitude	Channel	Amplitude
$T_{\pi}^{Bc} \rightarrow \pi^- D^-$	$(a_3 + a_4) V_{cd}^*$	$T_{\pi}^{Bc} \rightarrow \pi^- D_s^-$	$a_3 V_{cs}^*$
$T_{\pi}^{Bc} \rightarrow K^- D^-$	$a_4 V_{cs}^*$	$T_{\pi^0}^{Bc} \rightarrow \pi^0 D^-$	$\frac{1}{2} (a_2 + 2a_3 + a_4) V_{cd}^*$
$T_{\pi^0}^{Bc} \rightarrow \pi^0 D_s^-$	$a_3 V_{cs}^*$	$T_{\pi^0}^{Bc} \rightarrow \pi^- \bar{D}^0$	$\frac{(a_4 - a_2) V_{cd}^*}{\sqrt{2}}$
$T_{\pi^0}^{Bc} \rightarrow K^0 D_s^-$	$-\frac{a_2 V_{cd}^*}{\sqrt{2}}$	$T_{\pi^0}^{Bc} \rightarrow \bar{K}^0 D^-$	$-\frac{a_4 V_{cs}^*}{\sqrt{2}}$
$T_{\pi^0}^{Bc} \rightarrow K^- \bar{D}^0$	$\frac{a_4 V_{cs}^*}{\sqrt{2}}$	$T_{\pi^0}^{Bc} \rightarrow \eta_8 D^-$	$-\frac{(a_2 + a_4) V_{cd}^*}{2\sqrt{3}}$
$T_{\pi^+}^{Bc} \rightarrow \pi^+ D^-$	$(a_2 + a_3) V_{cd}^*$	$T_{\pi^+}^{Bc} \rightarrow \pi^+ D_s^-$	$a_3 V_{cs}^*$
$T_{\pi^+}^{Bc} \rightarrow \pi^0 \bar{D}^0$	$\frac{(a_2 - a_4) V_{cd}^*}{\sqrt{2}}$	$T_{\pi^+}^{Bc} \rightarrow K^+ D_s^-$	$a_2 V_{cd}^*$
$T_{\pi^+}^{Bc} \rightarrow \bar{K}^0 \bar{D}^0$	$a_4 V_{cs}^*$	$T_{\pi^+}^{Bc} \rightarrow \eta_8 \bar{D}^0$	$\frac{(a_2 + a_4) V_{cd}^*}{\sqrt{6}}$
$T_{K^-}^{Bc} \rightarrow \pi^- D_s^-$	$a_4 V_{cd}^*$	$T_{K^-}^{Bc} \rightarrow K^- D^-$	$a_3 V_{cd}^*$
$T_{K^-}^{Bc} \rightarrow K^- D_s^-$	$(a_3 + a_4) V_{cs}^*$	$T_{K^0}^{Bc} \rightarrow \pi^0 D_s^-$	$-\frac{a_4 V_{cd}^*}{\sqrt{2}}$
$T_{K^0}^{Bc} \rightarrow \bar{K}^0 D^-$	$(a_2 + a_3) V_{cd}^*$	$T_{K^0}^{Bc} \rightarrow \bar{K}^0 D_s^-$	$(a_3 + a_4) V_{cs}^*$
$T_{K^0}^{Bc} \rightarrow K^- \bar{D}^0$	$a_2 V_{cd}^*$	$T_{K^0}^{Bc} \rightarrow \eta_8 D_s^-$	$\frac{(a_4 - 2a_2) V_{cd}^*}{\sqrt{6}}$
$T_{K^0}^{Bc} \rightarrow \pi^0 D^-$	$-\frac{a_2 V_{cs}^*}{\sqrt{2}}$	$T_{K^0}^{Bc} \rightarrow \pi^- \bar{D}^0$	$a_2 V_{cs}^*$
$T_{K^0}^{Bc} \rightarrow K^0 D^-$	$(a_3 + a_4) V_{cd}^*$	$T_{K^0}^{Bc} \rightarrow K^0 D_s^-$	$(a_2 + a_3) V_{cs}^*$
$T_{K^0}^{Bc} \rightarrow \eta_8 D^-$	$\frac{(a_2 - 2a_4) V_{cs}^*}{\sqrt{6}}$	$T_{K^+}^{Bc} \rightarrow \pi^+ D^-$	$a_2 V_{cs}^*$
$T_{K^+}^{Bc} \rightarrow \pi^0 \bar{D}^0$	$\frac{a_2 V_{cs}^*}{\sqrt{2}}$	$T_{K^+}^{Bc} \rightarrow K^+ D^-$	$a_3 V_{cd}^*$
$T_{K^+}^{Bc} \rightarrow K^+ D_s^-$	$(a_2 + a_3) V_{cs}^*$	$T_{K^+}^{Bc} \rightarrow K^0 \bar{D}^0$	$a_4 V_{cd}^*$
$T_{K^+}^{Bc} \rightarrow \eta_8 \bar{D}^0$	$\frac{(a_2 - 2a_4) V_{cs}^*}{\sqrt{6}}$	$T_{\eta_8}^{Bc} \rightarrow \pi^0 D^-$	$-\frac{(a_2 + a_4) V_{cd}^*}{2\sqrt{3}}$
$T_{\eta_8}^{Bc} \rightarrow \pi^- \bar{D}^0$	$\frac{(a_2 + a_4) V_{cd}^*}{\sqrt{6}}$	$T_{\eta_8}^{Bc} \rightarrow K^0 D_s^-$	$\frac{(a_2 - 2a_4) V_{cd}^*}{\sqrt{6}}$
$T_{\eta_8}^{Bc} \rightarrow \bar{K}^0 D^-$	$\frac{(a_4 - 2a_2) V_{cs}^*}{\sqrt{6}}$	$T_{\eta_8}^{Bc} \rightarrow K^- \bar{D}^0$	$\frac{(a_4 - 2a_2) V_{cs}^*}{\sqrt{6}}$
$T_{\eta_8}^{Bc} \rightarrow \eta_8 D^-$	$\frac{1}{6} (a_2 + 6a_3 + a_4) V_{cd}^*$	$T_{\eta_8}^{Bc} \rightarrow \eta_8 D_s^-$	$\frac{1}{3} (2a_2 + 3a_3 + 2a_4) V_{cs}^*$
$T_{\pi^+}^{Bc} \rightarrow \eta_1 \bar{D}^0$	$\sqrt{3} a'_4 (V_{cd})^*$	$T_{\pi^0}^{Bc} \rightarrow \eta_1 D^-$	$-\sqrt{\frac{3}{2}} a'_4 (V_{cd})^*$
$T_{K^+}^{Bc} \rightarrow \eta_1 \bar{D}^0$	$\sqrt{3} a'_4 (V_{cs})^*$	$T_{K^0}^{Bc} \rightarrow \eta_1 D^-$	$\sqrt{3} a'_4 (V_{cs})^*$
$T_{K^0}^{Bc} \rightarrow \eta_1 D_s^-$	$\sqrt{3} a'_4 (V_{cd})^*$	$T_{\eta_8}^{Bc} \rightarrow \eta_1 D^-$	$\frac{a'_4 (V_{cd})^*}{\sqrt{2}}$
$T_{\eta_8}^{Bc} \rightarrow \eta_1 D_s^-$	$-\sqrt{2} a'_4 (V_{cs})^*$	$T_{\eta_1}^{Bc} \rightarrow \pi^0 D^-$	$-\sqrt{\frac{3}{2}} a'_2 (V_{cd})^*$
$T_{\eta_1}^{Bc} \rightarrow \pi^- \bar{D}^0$	$\sqrt{3} a'_2 (V_{cd})^*$	$T_{\eta_1}^{Bc} \rightarrow K^0 D_s^-$	$\sqrt{3} a'_2 (V_{cd})^*$
$T_{\eta_1}^{Bc} \rightarrow \bar{K}^0 D^-$	$\sqrt{3} a'_2 (V_{cs})^*$	$T_{\eta_1}^{Bc} \rightarrow K^- \bar{D}^0$	$\sqrt{3} a'_2 (V_{cs})^*$
$T_{\eta_1}^{Bc} \rightarrow \eta_8 D^-$	$\frac{a'_2 (V_{cd})^*}{\sqrt{2}}$	$T_{\eta_1}^{Bc} \rightarrow \eta_8 D_s^-$	$-\sqrt{2} a'_2 (V_{cs})^*$
$T_{\eta_1}^{Bc} \rightarrow \eta_1 D^-$	$(a'_2 + 3a'_3 + a'_4) (V_{cd})^*$	$T_{\eta_1}^{Bc} \rightarrow \eta_1 D_s^-$	$(a'_2 + 3a'_3 + a'_4) (V_{cs})^*$
$T_{\pi^0}^{Bc} \rightarrow D^- J/\psi$	$-\frac{a_1 V_{cd}^*}{\sqrt{2}}$	$T_{\pi^+}^{Bc} \rightarrow \bar{D}^0 J/\psi$	$a_1 V_{cd}^*$
$T_{K^0}^{Bc} \rightarrow D_s^- J/\psi$	$a_1 V_{cd}^*$	$T_{K^0}^{Bc} \rightarrow D^- J/\psi$	$a_1 V_{cs}^*$
$T_{K^+}^{Bc} \rightarrow \bar{D}^0 J/\psi$	$a_1 V_{cs}^*$	$T_{\eta_8}^{Bc} \rightarrow D^- J/\psi$	$\frac{a_1 V_{cd}^*}{\sqrt{6}}$
$T_{\eta_8}^{Bc} \rightarrow D_s^- J/\psi$	$-\sqrt{\frac{2}{3}} a_1 V_{cs}^*$	$T_{\eta_1}^{Bc} \rightarrow D^- J/\psi$	$\sqrt{3} a'_1 (V_{cd})^*$
$T_{\eta_1}^{Bc} \rightarrow D_s^- J/\psi$	$\sqrt{3} a'_1 (V_{cs})^*$		

$$\begin{aligned}
\Gamma(T_{\pi^0}^{Bc} \rightarrow K^- \bar{D}^0) &= \frac{1}{2} \Gamma(T_{\pi^+}^{Bc} \rightarrow K^- D^-) \\
&= \Gamma(T_{\pi^0}^{Bc} \rightarrow \bar{K}^0 D^-) \\
&= \frac{1}{2} \Gamma(T_{\pi^+}^{Bc} \rightarrow \bar{K}^0 \bar{D}^0), \\
\Gamma(T_{\pi^+}^{Bc} \rightarrow \eta_8 \bar{D}^0) &= 2\Gamma(T_{\pi^0}^{Bc} \rightarrow \eta_8 D^-) \\
&= 2\Gamma(T_{\eta_8}^{Bc} \rightarrow \pi^0 D^-) \\
&= \Gamma(T_{\eta_8}^{Bc} \rightarrow \pi^- \bar{D}^0), \\
\Gamma(T_{K^0}^{Bc} \rightarrow \pi^- \bar{D}^0) &= 2\Gamma(T_{K^+}^{Bc} \rightarrow \pi^0 D^-) \\
&= \Gamma(T_{K^+}^{Bc} \rightarrow \pi^+ D^-) \\
&= 2\Gamma(T_{K^+}^{Bc} \rightarrow \pi^0 \bar{D}^0), \\
\Gamma(T_{\bar{K}^0}^{Bc} \rightarrow K^- \bar{D}^0) &= 2\Gamma(T_{\pi^0}^{Bc} \rightarrow K^0 D_s^-) \\
&= \Gamma(T_{\pi^+}^{Bc} \rightarrow K^+ D_s^-), \\
\Gamma(T_{\pi^0}^{Bc} \rightarrow \pi^0 D_s^-) &= \Gamma(T_{\pi^+}^{Bc} \rightarrow \pi^- D_s^-) \\
&= \Gamma(T_{\pi^+}^{Bc} \rightarrow \pi^+ D_s^-), \\
\Gamma(T_{K^+}^{Bc} \rightarrow K^0 \bar{D}^0) &= \Gamma(T_{K^-}^{Bc} \rightarrow \pi^- D_s^-) \\
&= 2\Gamma(T_{\bar{K}^0}^{Bc} \rightarrow \pi^0 D_s^-), \\
\Gamma(T_{K^+}^{Bc} \rightarrow \eta_8 \bar{D}^0) &= \Gamma(T_{K^0}^{Bc} \rightarrow \eta_8 D^-), \\
\Gamma(T_{\eta_8}^{Bc} \rightarrow K^- \bar{D}^0) &= \Gamma(T_{\eta_8}^{Bc} \rightarrow \bar{K}^0 D^-), \\
\Gamma(T_{\pi^+}^{Bc} \rightarrow \pi^- D^-) &= \Gamma(T_{K^0}^{Bc} \rightarrow K^0 D^-), \\
\Gamma(T_{\pi^+}^{Bc} \rightarrow \pi^+ D^-) &= \Gamma(T_{\bar{K}^0}^{Bc} \rightarrow \bar{K}^0 D^-), \\
\Gamma(T_{\pi^+}^{Bc} \rightarrow \pi^0 \bar{D}^0) &= \Gamma(T_{\pi^0}^{Bc} \rightarrow \pi^- \bar{D}^0), \\
\Gamma(T_{\bar{K}^0}^{Bc} \rightarrow \bar{K}^0 D_s^-) &= \Gamma(T_{K^-}^{Bc} \rightarrow K^- D_s^-), \\
\Gamma(T_{K^+}^{Bc} \rightarrow K^+ D^-) &= \Gamma(T_{K^-}^{Bc} \rightarrow K^- D^-), \\
\Gamma(T_{K^+}^{Bc} \rightarrow K^+ D_s^-) &= \Gamma(T_{K^0}^{Bc} \rightarrow K^0 D_s^-), \\
\Gamma(T_{\pi^+}^{Bc} \rightarrow \eta_1 \bar{D}^0) &= 2\Gamma(T_{\pi^0}^{Bc} \rightarrow \eta_1 D^-) \\
&= \Gamma(T_{\bar{K}^0}^{Bc} \rightarrow \eta_1 D_s^-) \\
&= 6\Gamma(T_{\eta_8}^{Bc} \rightarrow \eta_1 D^-), \\
\Gamma(T_{K^+}^{Bc} \rightarrow \eta_1 \bar{D}^0) &= \Gamma(T_{K^0}^{Bc} \rightarrow \eta_1 D^-) \\
&= \frac{3}{2} \Gamma(T_{\eta_8}^{Bc} \rightarrow \eta_1 \bar{D}_s^-), \\
\Gamma(T_{\eta_1}^{Bc} \rightarrow \pi^- \bar{D}^0) &= 2\Gamma(T_{\eta_1}^{Bc} \rightarrow \pi^0 D^-) \\
&= \Gamma(T_{\eta_1}^{Bc} \rightarrow K^0 D_s^-)
\end{aligned}$$

$$\begin{aligned}
&= 6\Gamma(T_{\eta_1}^{Bc} \rightarrow \eta_8 D^-), \\
\Gamma(T_{\eta_1}^{Bc} \rightarrow \bar{K}^0 D^-) &= \Gamma(T_{\eta_1}^{Bc} \rightarrow K^- \bar{D}^0) \\
&= \frac{3}{2} \Gamma(T_{\eta_1}^{Bc} \rightarrow \eta_8 D_s^-), \\
2\Gamma(T_{\pi^0}^{Bc} \rightarrow D^- J/\psi) &= \Gamma(T_{\pi^+}^{Bc} \rightarrow \bar{D}^0 J/\psi) \\
&= \Gamma(T_{\bar{K}^0}^{Bc} \rightarrow D_s^- J/\psi) \\
&= 6\Gamma(T_{\eta_8}^{Bc} \rightarrow D^- J/\psi), \\
\Gamma(T_{K^0}^{Bc} \rightarrow D^- J/\psi) &= \Gamma(T_{K^+}^{Bc} \rightarrow \bar{D}^0 J/\psi) \\
&= \frac{3}{2} \Gamma(T_{\eta_8}^{Bc} \rightarrow D_s^- J/\psi).
\end{aligned}$$

It is necessary to notice that relations for channels involving $\eta(\eta')$ should be further modified due to the mixing effects. Here and in the following, our calculation of decay amplitudes using the SU(3) invariant effective Hamiltonian can be applied to the vector meson, following the replacement $\pi \rightarrow \rho$, $K \rightarrow K^*$, $\eta_8 \rightarrow V_8$, and $\eta_1 \rightarrow V_1$, and taking into account the $\omega - \phi$ mixing effects.

4.2 $b \rightarrow c\bar{u}d/s$ transition: two body decays into mesons

The operators of the $b \rightarrow c\bar{u}d/s$ transition can form an octet according to the SU(3) symmetry, of which nonzero entry $(H_8)_1^2 = V_{ud}^*$ for the $b \rightarrow c\bar{u}d$ transition, and $(H_8)_1^3 = V_{us}^*$ for the $b \rightarrow c\bar{u}s$ transition. As usual, the hadron-level effective Hamiltonian can be constructed as

$$\begin{aligned}
\mathcal{H}_{eff} &= a_6(T_{b\bar{c}8})_j^i (H_8)_k^j M_k^j J/\psi + a_7(T_{b\bar{c}8})_j^i (H_8)_k^j M_k^j J/\psi \\
&\quad + a'_6(T_{b\bar{c}1})_i^j (H_8)_k^j M_k^j J/\psi + a'_7(T_{b\bar{c}8})_j^i (H_8)_i^j M_k^j J/\psi, \\
\mathcal{H}_{eff} &= a_8(T_{b\bar{c}8})_j^i (H_8)_i^j D_k \bar{D}^k + a_9(T_{b\bar{c}8})_j^i (H_8)_i^k D_k \bar{D}^j \\
&\quad + a_{10}(T_{b\bar{c}8})_j^i (H_8)_k^j D_i \bar{D}^k + a'_8(T_{b\bar{c}1})_i^j (H_8)_k^j D_j \bar{D}^k, \\
\mathcal{H}_{eff} &= a_{11}(T_{b\bar{c}8})_j^i (H_8)_i^j M_k^l M_l^k + a_{12}(T_{b\bar{c}8})_j^i (H_8)_i^j M_k^j M_l^k \\
&\quad + a_{13}(T_{b\bar{c}8})_j^i (H_8)_k^j M_i^l M_l^k + a_{14}(T_{b\bar{c}8})_j^i (H_8)_k^l M_i^j M_l^k \\
&\quad + a_{15}(T_{b\bar{c}8})_j^i (H_8)_k^l M_i^k M_l^j + a'_{11}(T_{b\bar{c}1})_i^j (H_8)_j^l M_k^j M_l^k \\
&\quad + a'_{12}(T_{b\bar{c}1})_i^j (H_8)_k^l M_j^j M_l^k + a'_{13}(T_{b\bar{c}8})_j^i (H_8)_i^j M_l^j M_k^k \\
&\quad + a'_{14}(T_{b\bar{c}8})_j^i (H_8)_i^l M_l^j M_k^k + a'_{15}(T_{b\bar{c}8})_j^i (H_8)_i^j M_l^l M_k^k.
\end{aligned} \tag{15}$$

At quark level, the relevant topological diagrams are shown in Fig. 2a–i for the processes without singlet state, in Fig. 2j–p for the processes including the singlet state. Specifically, the topological diagrams of $T_{b\bar{c}8}$ decays into a light meson plus J/ψ are given in Fig. 2b, e, and the Fig. 2a, f, g correspond with the processes of producing D plus $\bar{D}$, the processes that $T_{b\bar{c}8}$ produces two light mesons are represented by topological diagrams in Fig. 2c, d, h, i. One derives the decay

Table 4 Tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into J/ψ plus light meson or charmed meson plus anti-charmed meson or two light mesons

Channel	Amplitude	Channel	Amplitude
$T_{\pi^0}^{Bc} \rightarrow \pi^- J/\psi$	$\frac{(a_6 - a_7) V_{ud}^*}{\sqrt{2}}$	$T_{\pi^0}^{Bc} \rightarrow K^- J/\psi$	$\frac{a_6 V_{us}^*}{\sqrt{2}}$
$T_{\pi^+}^{Bc} \rightarrow \pi^0 J/\psi$	$\frac{(a_7 - a_6) V_{ud}^*}{\sqrt{2}}$	$T_{\pi^+}^{Bc} \rightarrow \bar{K}^0 J/\psi$	$a_6 V_{us}^*$
$T_{\pi^+}^{Bc} \rightarrow \eta_8 J/\psi$	$\frac{(a_6 + a_7) V_{ud}^*}{\sqrt{6}}$	$T_{\bar{K}^0}^{Bc} \rightarrow K^- J/\psi$	$a_7 V_{ud}^*$
$T_{\bar{K}^0}^{Bc} \rightarrow \pi^- J/\psi$	$a_7 V_{us}^*$	$T_{K^+}^{Bc} \rightarrow \pi^0 J/\psi$	$\frac{a_7 V_{us}^*}{\sqrt{2}}$
$T_{K^+}^{Bc} \rightarrow K^0 J/\psi$	$a_6 V_{ud}^*$	$T_{K^+}^{Bc} \rightarrow \eta_8 J/\psi$	$\frac{(a_7 - 2a_6) V_{us}^*}{\sqrt{6}}$
$T_{\eta_8}^{Bc} \rightarrow \pi^- J/\psi$	$\frac{(a_6 + a_7) V_{ud}^*}{\sqrt{6}}$	$T_{\eta_8}^{Bc} \rightarrow K^- J/\psi$	$\frac{(a_6 - 2a_7) V_{us}^*}{\sqrt{6}}$
$T_{\pi^+}^{Bc} \rightarrow \eta_1 J/\psi$	$\sqrt{3} a_7' (V_{ud})^*$	$T_{K^+}^{Bc} \rightarrow \eta_1 J/\psi$	$\sqrt{3} a_7' (V_{us})^*$
$T_{\eta_1}^{Bc} \rightarrow \pi^- J/\psi$	$\sqrt{3} a_6' (V_{ud})^*$	$T_{\eta_1}^{Bc} \rightarrow K^- J/\psi$	$\sqrt{3} a_6' (V_{us})^*$
$T_{\pi^0}^{Bc} \rightarrow D^0 D^-$	$\frac{(a_9 - a_{10}) V_{ud}^*}{\sqrt{2}}$	$T_{\pi^0}^{Bc} \rightarrow D^0 D_s^-$	$\frac{a_9 V_{us}^*}{\sqrt{2}}$
$T_{\pi^+}^{Bc} \rightarrow D^0 \bar{D}^0$	$(a_8 + a_{10}) V_{ud}^*$	$T_{\pi^+}^{Bc} \rightarrow D^+ D^-$	$(a_8 + a_9) V_{ud}^*$
$T_{\pi^+}^{Bc} \rightarrow D^+ D_s^-$	$a_9 V_{us}^*$	$T_{\pi^+}^{Bc} \rightarrow D_s^+ D_s^-$	$a_8 V_{ud}^*$
$T_{\bar{K}^0}^{Bc} \rightarrow D^0 D_s^-$	$a_{10} V_{ud}^*$	$T_{\bar{K}^0}^{Bc} \rightarrow D^0 D^-$	$a_{10} V_{us}^*$
$T_{K^+}^{Bc} \rightarrow D^0 \bar{D}^0$	$(a_8 + a_{10}) V_{us}^*$	$T_{K^+}^{Bc} \rightarrow D^+ D^-$	$a_8 V_{us}^*$
$T_{K^+}^{Bc} \rightarrow D_s^+ D^-$	$a_9 V_{ud}^*$	$T_{K^+}^{Bc} \rightarrow D_s^+ D_s^-$	$(a_8 + a_9) V_{us}^*$
$T_{\eta_8}^{Bc} \rightarrow D^0 D^-$	$\frac{(a_9 + a_{10}) V_{ud}^*}{\sqrt{6}}$	$T_{\eta_8}^{Bc} \rightarrow D^0 D_s^-$	$\frac{(a_9 - 2a_{10}) V_{us}^*}{\sqrt{6}}$
$T_{\eta_1}^{Bc} \rightarrow D^0 D^-$	$\sqrt{3} a_8' (V_{ud})^*$	$T_{\eta_1}^{Bc} \rightarrow D^0 D_s^-$	$\sqrt{3} a_8' (V_{us})^*$
$T_{\pi^-}^{Bc} \rightarrow \pi^- \pi^-$	$2 (a_{14} + a_{15}) V_{ud}^*$	$T_{\pi^-}^{Bc} \rightarrow \pi^- K^-$	$(a_{14} + a_{15}) V_{us}^*$
$T_{\pi^0}^{Bc} \rightarrow \pi^0 \pi^-$	$(a_{14} + a_{15}) V_{ud}^*$	$T_{\pi^0}^{Bc} \rightarrow \pi^0 K^-$	$\frac{1}{2} (a_{12} + 2a_{14} + a_{15}) V_{us}^*$
$T_{\pi^0}^{Bc} \rightarrow \pi^- \bar{K}^0$	$\frac{(a_{12} - a_{15}) V_{us}^*}{\sqrt{2}}$	$T_{\pi^0}^{Bc} \rightarrow \pi^- \eta_8$	$\frac{(a_{12} - a_{13}) V_{ud}^*}{\sqrt{3}}$
$T_{\pi^0}^{Bc} \rightarrow K^0 K^-$	$\frac{(a_{12} - a_{13}) V_{ud}^*}{\sqrt{2}}$	$T_{\pi^0}^{Bc} \rightarrow K^- \eta_8$	$\frac{(a_{15} - a_{12}) V_{us}^*}{2\sqrt{3}}$
$T_{\pi^+}^{Bc} \rightarrow \pi^+ \pi^-$	$(2a_{11} + a_{12} + a_{13} + a_{14}) V_{ud}^*$	$T_{\pi^+}^{Bc} \rightarrow \pi^+ K^-$	$(a_{12} + a_{14}) V_{us}^*$
$T_{\pi^+}^{Bc} \rightarrow \pi^0 \pi^0$	$(2a_{11} + a_{12} + a_{13} - a_{15}) V_{ud}^*$	$T_{\pi^+}^{Bc} \rightarrow \pi^0 \bar{K}^0$	$\frac{(a_{15} - a_{12}) V_{us}^*}{\sqrt{2}}$
$T_{\pi^+}^{Bc} \rightarrow \pi^0 \eta_8$	$\frac{(a_{13} - a_{12}) V_{ud}^*}{\sqrt{3}}$	$T_{\pi^+}^{Bc} \rightarrow K^+ K^-$	$(2a_{11} + a_{13}) V_{ud}^*$
$T_{\pi^+}^{Bc} \rightarrow K^0 \bar{K}^0$	$(2a_{11} + a_{12}) V_{ud}^*$	$T_{\pi^+}^{Bc} \rightarrow \bar{K}^0 \eta_8$	$\frac{(a_{15} - a_{12}) V_{us}^*}{\sqrt{6}}$
$T_{\pi^+}^{Bc} \rightarrow \eta_8 \eta_8$	$\frac{1}{3} (6a_{11} + a_{12} + a_{13} + a_{15}) V_{ud}^*$	$T_{\pi^+}^{Bc} \rightarrow \pi^- K^-$	$(a_{14} + a_{15}) V_{ud}^*$
$T_{K^-}^{Bc} \rightarrow K^- K^-$	$2 (a_{14} + a_{15}) V_{us}^*$	$T_{\bar{K}^0}^{Bc} \rightarrow \pi^0 K^-$	$\frac{(a_{13} - a_{15}) V_{ud}^*}{\sqrt{2}}$
$T_{\bar{K}^0}^{Bc} \rightarrow \pi^- \bar{K}^0$	$(a_{13} + a_{14}) V_{ud}^*$	$T_{\bar{K}^0}^{Bc} \rightarrow \bar{K}^0 K^-$	$(a_{14} + a_{15}) V_{us}^*$
$T_{\bar{K}^0}^{Bc} \rightarrow K^- \eta_8$	$\frac{(a_{15} - a_{13}) V_{ud}^*}{\sqrt{6}}$	$T_{\bar{K}^0}^{Bc} \rightarrow \pi^- K^0$	$(a_{14} + a_{15}) V_{ud}^*$
$T_{\bar{K}^0}^{Bc} \rightarrow \pi^- \eta_8$	$\sqrt{\frac{2}{3}} (a_{13} - a_{15}) V_{us}^*$	$T_{\bar{K}^0}^{Bc} \rightarrow K^0 K^-$	$(a_{13} + a_{14}) V_{us}^*$
$T_{K^+}^{Bc} \rightarrow \pi^+ \pi^-$	$(2a_{11} + a_{13}) V_{us}^*$	$T_{K^+}^{Bc} \rightarrow \pi^0 \pi^0$	$(2a_{11} + a_{13}) V_{us}^*$
$T_{K^+}^{Bc} \rightarrow \pi^0 K^0$	$\frac{(a_{15} - a_{12}) V_{ud}^*}{\sqrt{2}}$	$T_{K^+}^{Bc} \rightarrow \pi^0 \eta_8$	$\frac{(a_{13} - a_{15}) V_{us}^*}{\sqrt{3}}$
$T_{K^+}^{Bc} \rightarrow \pi^- K^+$	$(a_{12} + a_{14}) V_{ud}^*$	$T_{K^+}^{Bc} \rightarrow K^+ K^-$	$(2a_{11} + a_{12} + a_{13} + a_{14}) V_{us}^*$
$T_{K^+}^{Bc} \rightarrow K^0 \bar{K}^0$	$(2a_{11} + a_{12}) V_{us}^*$	$T_{K^+}^{Bc} \rightarrow K^0 \eta_8$	$\frac{(a_{15} - a_{12}) V_{ud}^*}{\sqrt{6}}$
$T_{K^+}^{Bc} \rightarrow \eta_8 \eta_8$	$\frac{1}{3} (6a_{11} + 4a_{12} + a_{13} - 2a_{15}) V_{us}^*$	$T_{\eta_8}^{Bc} \rightarrow \pi^0 K^-$	$\frac{(a_{12} - 2a_{13} + a_{15}) V_{us}^*}{2\sqrt{3}}$
$T_{\eta_8}^{Bc} \rightarrow \pi^- \bar{K}^0$	$\frac{(a_{12} - 2a_{13} + a_{15}) V_{us}^*}{\sqrt{6}}$	$T_{\eta_8}^{Bc} \rightarrow \pi^- \eta_8$	$\frac{1}{3} (a_{12} + a_{13} + 3a_{14} + a_{15}) V_{ud}^*$
$T_{\eta_8}^{Bc} \rightarrow K^0 K^-$	$\frac{(a_{12} + a_{13} - 2a_{15}) V_{ud}^*}{\sqrt{6}}$	$T_{\eta_8}^{Bc} \rightarrow K^- \eta_8$	$\frac{1}{6} (-a_{12} + 2a_{13} + 6a_{14} + 5a_{15}) V_{us}^*$
$T_{\pi^+}^{Bc} \rightarrow \pi^0 \eta_1$	$\sqrt{\frac{3}{2}} (a_{13}' - a_{14}') (V_{ud})^*$	$T_{\pi^+}^{Bc} \rightarrow \bar{K}^0 \eta_1$	$\sqrt{3} a_{14}' (V_{us})^*$

Table 4 continued

Channel	Amplitude	Channel	Amplitude
$T_{\pi^+}^{Bc} \rightarrow \eta_8 \eta_1$	$\frac{(a'_{13}+a'_{14})(V_{ud})^*}{\sqrt{2}}$	$T_{\pi^+}^{Bc} \rightarrow \eta_1 \eta_1$	$2(a'_{13} + a'_{14} + 3a'_{15})(V_{ud})^*$
$T_{\pi^0}^{Bc} \rightarrow \pi^- \eta_1$	$\sqrt{\frac{3}{2}}(a'_{14} - a'_{13})(V_{ud})^*$	$T_{\pi^0}^{Bc} \rightarrow K^- \eta_1$	$\sqrt{\frac{3}{2}}a'_{14}(V_{us})^*$
$T_{K^+}^{Bc} \rightarrow \pi^0 \eta_1$	$\sqrt{\frac{3}{2}}a'_{13}(V_{us})^*$	$T_{K^+}^{Bc} \rightarrow K^0 \eta_1$	$\sqrt{3}a'_{14}(V_{ud})^*$
$T_{K^+}^{Bc} \rightarrow \eta_8 \eta_1$	$\frac{(a'_{13}-2a'_{14})(V_{us})^*}{\sqrt{2}}$	$T_{K^+}^{Bc} \rightarrow \eta_1 \eta_1$	$2(a'_{13} + a'_{14} + 3a'_{15})(V_{us})^*$
$T_{K^0}^{Bc} \rightarrow \pi^- \eta_1$	$\sqrt{3}a'_{13}(V_{us})^*$	$T_{K^0}^{Bc} \rightarrow K^- \eta_1$	$\sqrt{3}a'_{13}(V_{ud})^*$
$T_{\eta_8}^{Bc} \rightarrow \pi^- \eta_1$	$\frac{(a'_{13}+a'_{14})(V_{ud})^*}{\sqrt{2}}$	$T_{\eta_8}^{Bc} \rightarrow K^- \eta_1$	$\frac{(a'_{14}-2a'_{13})(V_{us})^*}{\sqrt{2}}$
$T_{\eta_1}^{Bc} \rightarrow \pi^0 K^-$	$\sqrt{\frac{3}{2}}a'_{11}(V_{us})^*$	$T_{\eta_1}^{Bc} \rightarrow \pi^- \bar{K}^0$	$\sqrt{3}a'_{11}(V_{us})^*$
$T_{\eta_1}^{Bc} \rightarrow \pi^- \eta_8$	$\sqrt{2}a'_{11}(V_{ud})^*$	$T_{\eta_1}^{Bc} \rightarrow \pi^- \eta_1$	$(2a'_{11} + 3a'_{12} + a'_{13} + a'_{14})(V_{ud})^*$
$T_{\eta_1}^{Bc} \rightarrow K^0 K^-$	$\sqrt{3}a'_{11}(V_{ud})^*$	$T_{\eta_1}^{Bc} \rightarrow K^- \eta_8$	$-\frac{a'_{11}(V_{us})^*}{\sqrt{2}}$
$T_{\eta_1}^{Bc} \rightarrow K^- \eta_1$	$(2a'_{11} + 3a'_{12} + a'_{13} + a'_{14})(V_{us})^*$		

amplitudes given in Table 4 respectively. Accordingly, we obtain the relations between different decay widths for J/ψ and a light meson as follows:

$$\begin{aligned}\Gamma(T_{\pi^+}^{Bc} \rightarrow \pi^0 J/\psi) &= \Gamma(T_{\pi^0}^{Bc} \rightarrow \pi^- J/\psi), \\ \Gamma(T_{\pi^+}^{Bc} \rightarrow \bar{K}^0 J/\psi) &= 2\Gamma(T_{\pi^0}^{Bc} \rightarrow K^- J/\psi), \\ \Gamma(T_{K^+}^{Bc} \rightarrow \pi^0 J/\psi) &= \frac{1}{2}\Gamma(T_{K^0}^{Bc} \rightarrow \pi^- J/\psi).\end{aligned}$$

The relations for producing the charmed meson and anti-charmed meson become

$$\Gamma(T_{\pi^0}^{Bc} \rightarrow D^0 D_s^-) = \frac{1}{2}\Gamma(T_{\pi^+}^{Bc} \rightarrow D^+ D_s^-).$$

The relations for producing two light mesons become

$$\begin{aligned}\Gamma(T_{\pi^-}^{Bc} \rightarrow \pi^- \pi^-) &= 2\Gamma(T_{K^-}^{Bc} \rightarrow K^- \pi^-) \\ &= 2\Gamma(T_{K^0}^{Bc} \rightarrow K^0 \pi^-) \\ &= 2\Gamma(T_{\pi^0}^{Bc} \rightarrow \pi^0 \pi^-), \\ \Gamma(T_{\pi^-}^{Bc} \rightarrow \pi^- K^-) &= \frac{1}{2}\Gamma(T_{K^-}^{Bc} \rightarrow K^- K^-) \\ &= \Gamma(T_{K^0}^{Bc} \rightarrow \bar{K}^0 K^-), \\ \Gamma(T_{\pi^0}^{Bc} \rightarrow \pi^- \bar{K}^0) &= 3\Gamma(T_{\pi^+}^{Bc} \rightarrow \eta_8 \bar{K}^0) \\ &= \Gamma(T_{\pi^+}^{Bc} \rightarrow \pi^0 \bar{K}^0), \\ \Gamma(T_{K^+}^{Bc} \rightarrow \pi^0 \pi^0) &= \frac{1}{2}\Gamma(T_{K^+}^{Bc} \rightarrow \pi^+ \pi^-), \\ \Gamma(T_{\pi^+}^{Bc} \rightarrow \pi^0 \eta_8) &= \Gamma(T_{\pi^0}^{Bc} \rightarrow \pi^- \eta_8), \\ \Gamma(T_{\pi^+}^{Bc} \rightarrow \bar{K}^0 \eta_8) &= 2\Gamma(T_{\pi^0}^{Bc} \rightarrow K^- \eta_8),\end{aligned}$$

$$\begin{aligned}\Gamma(T_{\bar{K}^0}^{Bc} \rightarrow \pi^0 K^-) &= 3\Gamma(T_{\bar{K}^0}^{Bc} \rightarrow \eta_8 K^-), \\ \Gamma(T_{K^+}^{Bc} \rightarrow \pi^0 K^0) &= 3\Gamma(T_{K^+}^{Bc} \rightarrow \eta_8 K^0), \\ \Gamma(T_{K^+}^{Bc} \rightarrow \pi^0 \eta_8) &= \frac{1}{2}\Gamma(T_{K^0}^{Bc} \rightarrow \pi^- \eta_8), \\ \Gamma(T_{\eta_1}^{Bc} \rightarrow \pi^- \bar{K}^0) &= 2\Gamma(T_{\eta_1}^{Bc} \rightarrow \pi^0 K^-) \\ &= 6\Gamma(T_{\eta_1}^{Bc} \rightarrow K^- \eta_8), \\ \Gamma(T_{\pi^+}^{Bc} \rightarrow \pi^0 \eta_1) &= \Gamma(T_{\pi^0}^{Bc} \rightarrow \pi^- \eta_1), \\ \Gamma(T_{\pi^+}^{Bc} \rightarrow \eta_8 \eta_1) &= \Gamma(T_{\eta_8}^{Bc} \rightarrow \pi^- \eta_1), \\ \Gamma(T_{K^0}^{Bc} \rightarrow \pi^- \eta_1) &= 2\Gamma(T_{K^+}^{Bc} \rightarrow \pi^0 \eta_1), \\ \Gamma(T_{\eta_1}^{Bc} \rightarrow K^0 K^-) &= \frac{3}{2}\Gamma(T_{\eta_1}^{Bc} \rightarrow \pi^- \eta_8), \\ \Gamma(T_{\pi^+}^{Bc} \rightarrow \bar{K}^0 \eta_1) &= 2\Gamma(T_{\pi^0}^{Bc} \rightarrow K^- \eta_1).\end{aligned}$$

4.3 $b \rightarrow u\bar{c}d/s$ transition: two-body decays into mesons

The operator $(\bar{u}b)(\bar{q}c)$ can form an anti-symmetric $\mathbf{\bar{3}}$ and a symmetric $\mathbf{6}$ representations. In the transition $b \rightarrow u\bar{c}s$, the nonzero components of the anti-symmetric tensor H_3'' and the symmetric tensor H_6 are given respectively as $(H_3'')^{13} = -(H_3'')^{31} = V_{cs}^*$, $(H_6)^{13} = (H_6)^{31} = V_{cs}^*$. In the transition $b \rightarrow u\bar{c}d$, the nonzero components can be obtained by interchanging the subscripts $2 \leftrightarrow 3$, and replacing V_{cs} by V_{cd} . Therefore, the effective Hamiltonian at the hadron level for $T_{b\bar{c}8}/T_{b\bar{c}1}$ producing two mesons is constructed as

$$\begin{aligned}\mathcal{H}_{eff} &= a_{16}(T_{b\bar{c}8})_j^i (H_3'')^{[jk]} D_i D_k + a_{17}(T_{b\bar{c}8})_j^i (H_6)^{\{jk\}} D_i D_k \\ &\quad + a'_{16}(T_{b\bar{c}1})_i^j (H_6)^{\{jk\}} D_j D_k.\end{aligned}\quad (16)$$

Table 5 Tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into two anti-charmed mesons

Channel	Amplitude	Channel	Amplitude
$T_{\pi^-}^{Bc} \rightarrow D^- D^-$	$2(a_{16} + a_{17}) V_{cd}^*$	$T_{\pi^-}^{Bc} \rightarrow D^- D_s^-$	$(a_{16} + a_{17}) V_{cs}^*$
$T_{\pi^0}^{Bc} \rightarrow \bar{D}^0 D^-$	$\sqrt{2} a_{16} V_{cd}^*$	$T_{\pi^0}^{Bc} \rightarrow \bar{D}^0 D_s^-$	$\frac{(a_{16} + a_{17}) V_{cs}^*}{\sqrt{2}}$
$T_{\pi^+}^{Bc} \rightarrow \bar{D}^0 \bar{D}^0$	$2(a_{17} - a_{16}) V_{cd}^*$	$T_{K^-}^{Bc} \rightarrow D^- D_s^-$	$(a_{16} + a_{17}) V_{cd}^*$
$T_{K^-}^{Bc} \rightarrow D_s^- D_s^-$	$2(a_{16} + a_{17}) V_{cs}^*$	$T_{\bar{K}^0}^{Bc} \rightarrow \bar{D}^0 D_s^-$	$(a_{17} - a_{16}) V_{cd}^*$
$T_{K^0}^{Bc} \rightarrow \bar{D}^0 D^-$	$(a_{17} - a_{16}) V_{cs}^*$	$T_{K^+}^{Bc} \rightarrow \bar{D}^0 \bar{D}^0$	$2(a_{17} - a_{16}) V_{cs}^*$
$T_{\eta_8}^{Bc} \rightarrow \bar{D}^0 D^-$	$\sqrt{\frac{2}{3}} a_{17} V_{cd}^*$	$T_{\eta_8}^{Bc} \rightarrow \bar{D}^0 D_s^-$	$\frac{(3a_{16} - a_{17}) V_{cs}^*}{\sqrt{6}}$
$T_{\eta_1}^{Bc} \rightarrow \bar{D}^0 D^-$	$2\sqrt{3} a'_{16} (V_{cd})^*$	$T_{\eta_1}^{Bc} \rightarrow \bar{D}^0 D_s^-$	$2\sqrt{3} a'_{16} (V_{cs})^*$

Also the topological diagrams corresponding with the Hamiltonian above are given in Fig. 2a–d, k, o. One then deduces the decay amplitudes for different channels shown in Table 5, which leads to the relations for decay widths as

$$\begin{aligned}\Gamma(T_{\pi^-}^{Bc} \rightarrow D^- D_s^-) &= \frac{1}{2} \Gamma(T_{K^-}^{Bc} \rightarrow D_s^- D_s^-) \\ &= 2\Gamma(T_{\pi^0}^{Bc} \rightarrow \bar{D}^0 D_s^-), \\ \Gamma(T_{\pi^-}^{Bc} \rightarrow D^- D^-) &= 2\Gamma(T_{K^-}^{Bc} \rightarrow D_s^- D^-), \\ \Gamma(T_{\pi^+}^{Bc} \rightarrow \bar{D}^0 \bar{D}^0) &= 2\Gamma(T_{\bar{K}^0}^{Bc} \rightarrow \bar{D}^0 D_s^-), \\ \Gamma(T_{K^+}^{Bc} \rightarrow \bar{D}^0 \bar{D}^0) &= 2\Gamma(T_{K^0}^{Bc} \rightarrow \bar{D}^0 D^-).\end{aligned}$$

4.4 Charmless $b \rightarrow q_1 \bar{q}_2 q_3$ transition: two body decays into mesons

The charmless tree level operator $(\bar{q}_1 b)(\bar{q}_2 q_3)$ can be decomposed into a triple H_3 , an antisymmetric sextet H_6 and a traceless symmetric H_{15} in upper indices, while the charmless penguin level operator behaves as the triplet H_3 . For the $\Delta S = 0(b \rightarrow d)$ decays, the nonzero components of these irreducible tensors are given as

$$\begin{aligned}(H_3)^2 &= 1, \quad (H_6)_{11}^{12} = -(H_6)_{11}^{21} = (H_6)_{23}^{23} = -(H_6)_{33}^{32} = 1, \\ 2(H_{15})_{11}^{12} &= 2(H_{15})_{11}^{21} = -3(H_{15})_{22}^{22} = -6(H_{15})_{33}^{23} \\ &= -6(H_{15})_{33}^{32} = 6.\end{aligned}\quad (17)$$

For the $\Delta S = 1(b \rightarrow s)$ decays, the nonzero entries in the irreducible tensor H_3, H_6, H_{15} can be obtained from Eq. (17) with the exchange $2 \leftrightarrow 3$. Accordingly, the hadron-level effective Hamiltonian for $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into mesons is constructed as

$$\begin{aligned}\mathcal{H}_{eff} &= b_4(T_{b\bar{c}8})_j^i (H_3)^k D_i M_k^j + b_5(T_{b\bar{c}8})_j^i (H_3)^j D_k M_i^k \\ &+ b_6(T_{b\bar{c}8})_j^i (H_3)^k D_k M_i^j + b_7(T_{b\bar{c}8})_j^i (H_6)^{[jk]} D_l M_i^l \\ &+ b_8(T_{b\bar{c}8})_j^i (H_6)^{[kl]} D_k M_l^j + b_9(T_{b\bar{c}8})_j^i (H_6)^{[jk]} D_l M_i^l \\ &+ b_{10}(T_{b\bar{c}8})_j^i (H_6)^{[jk]} D_k M_l^l + b_{11}(T_{b\bar{c}8})_j^i (H_{15})^{[jk]} D_l M_i^l\end{aligned}$$

$$\begin{aligned}&+ b_{12}(T_{b\bar{c}8})_j^i (H_{15})^{[kl]} D_k M_l^j + b_{13}(T_{b\bar{c}8})_j^i (H_{15})^{[jk]} D_l M_i^l \\ &+ b_{14}(T_{b\bar{c}8})_j^i (H_{15})^{[jk]} D_k M_l^l \\ &+ b'_4(T_{b\bar{c}1})_i^j (H_3)^k D_j M_k^j + b'_5(T_{b\bar{c}1})_i^j (H_3)^k D_k M_j^j \\ &+ b'_6(T_{b\bar{c}8})_j^i (H_3)^j D_i M_k^k + b'_7(T_{b\bar{c}1})_i^j (H_6)^{[kl]} D_k M_l^j \\ &+ b'_8(T_{b\bar{c}8})_j^i (H_6)^{[jk]} D_k M_l^l + b'_9(T_{b\bar{c}1})_i^j (H_{15})^{[kl]} D_k M_l^j \\ &+ b'_{10}(T_{b\bar{c}8})_j^i (H_{15})^{[jk]} D_k M_l^l.\end{aligned}\quad (18)$$

In the two-body decays of the transition, the decay amplitudes are given in Table 6 for the transition $b \rightarrow d$ and Table 7 for the transition $b \rightarrow s$. We obtain no direct relation of these decay widths.

4.5 $\bar{c} \rightarrow \bar{q}_1 q_2 \bar{q}_3$ transition: two body decays into mesons

Under the flavor SU(3) symmetry, the operator $\bar{c} q_1 \bar{q}_2 q_3$ transforms as $\bar{\mathbf{3}} \otimes \mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{3} \oplus \mathbf{6} \oplus \mathbf{15}$. Following the classifications mentioned before, the Cabibbo allowed transition to be $\bar{c} \rightarrow \bar{s} d \bar{u}$, and the nonzero tensor components are given as

$$(H_6)_{31}^2 = -(H_6)_{13}^2 = 1, \quad (H_{15})_{31}^2 = (H_{15})_{13}^2 = 1. \quad (19)$$

In the singly Cabibbo suppressed transition $\bar{c} \rightarrow \bar{u} d \bar{d}$ and $\bar{c} \rightarrow \bar{u} s \bar{s}$, the combination of tensor components are given as

$$\begin{aligned}(H_6)_{31}^3 &= -(H_6)_{13}^3 = (H_6)_{12}^2 = -(H_6)_{21}^2 = \sin(\theta_C), \\ (H_{15})_{31}^3 &= (H_{15})_{13}^3 = -(H_{15})_{12}^2 = -(H_{15})_{21}^2 = \sin(\theta_C).\end{aligned}\quad (20)$$

while for the doubly Cabibbo suppressed transition $\bar{c} \rightarrow \bar{d} s \bar{u}$, we have

$$\begin{aligned}(H_6)_{21}^3 &= -(H_6)_{12}^3 = \sin^2 \theta_C, \\ (H_{15})_{21}^3 &= (H_{15})_{12}^3 = \sin^2 \theta_C.\end{aligned}\quad (21)$$

Therefore, it is convenient to construct the hadron-level effective Hamiltonian for $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into mesons as

Table 6 Tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into anti-charmed and light mesons induced by the charmless $b \rightarrow d$ transition

Channel	Amplitude
$T_{\pi^-}^{Bc} \rightarrow D^- \pi^-$	$b_4 + b_6 + b_9 + b_{10} - 2b_{12} + 3b_{13} + 3b_{14}$
$T_{\pi^0}^{Bc} \rightarrow \bar{D}^0 \pi^-$	$\frac{1}{\sqrt{2}} (b_4 - b_5 + b_7 + b_8 + b_9 + b_{10} + 5b_{11} + 3b_{12} + 3b_{13} - 3b_{14})$
$T_{\pi^0}^{Bc} \rightarrow D^- \pi^0$	$\frac{1}{2} (b_4 + b_5 + 2b_6 - b_7 - b_8 + b_9 + b_{10} - 5b_{11} + b_{12} - 5b_{13} + b_{14})$
$T_{\pi^0}^{Bc} \rightarrow D^- \eta_8$	$\frac{-b_4 - b_5 + b_7 - b_8 + 3b_9 + b_{10} + 5b_{11} + 5b_{12} - 3b_{13} + 5b_{14}}{2\sqrt{3}}$
$T_{\pi^0}^{Bc} \rightarrow D_s^- K^0$	$\frac{-b_5 + b_7 - b_{10} + 5b_{11} + b_{14}}{\sqrt{2}}$
$T_{\pi^+}^{Bc} \rightarrow \bar{D}^0 \pi^0$	$-\frac{b_4 - b_5 + b_7 + b_8 + b_9 + b_{10} - 3b_{11} + 3b_{12} - 5b_{13} - 3b_{14}}{\sqrt{2}}$
$T_{\pi^+}^{Bc} \rightarrow \bar{D}^0 \eta_8$	$\frac{b_4 + b_5 - b_7 + b_8 - 3b_9 - b_{10} + 3b_{11} + 3b_{12} + 3b_{13} + 3b_{14}}{\sqrt{6}}$
$T_{\pi^+}^{Bc} \rightarrow D^- \pi^+$	$b_5 + b_6 - b_7 - b_8 + 3b_{11} + 3b_{12} - 2b_{14}$
$T_{\pi^+}^{Bc} \rightarrow D_s^- K^+$	$b_5 - b_7 + b_{10} + 3b_{11} - b_{14}$
$T_{K^-}^{Bc} \rightarrow D^- K^-$	$b_6 + b_8 + b_{10} - b_{12} + 3b_{14}$
$T_{K^-}^{Bc} \rightarrow D_s^- \pi^-$	$b_4 - b_8 + b_9 - b_{12} + 3b_{13}$
$T_{\bar{K}^0}^{Bc} \rightarrow \bar{D}^0 K^-$	$b_5 + b_7 - b_{10} - b_{11} + 3b_{14}$
$T_{\bar{K}^0}^{Bc} \rightarrow D^- \bar{K}^0$	$b_5 + b_6 + b_7 + b_8 - b_{11} - b_{12} - 2b_{14}$
$T_{\bar{K}^0}^{Bc} \rightarrow D_s^- \pi^0$	$\frac{-b_4 + b_8 - b_9 + b_{12} + 5b_{13}}{\sqrt{2}}$
$T_{\bar{K}^0}^{Bc} \rightarrow D_s^- \eta_8$	$\frac{1}{\sqrt{6}} (b_4 - 2b_5 - 2b_7 - b_8 - 3b_9 - 2b_{10} + 2b_{11} - b_{12} + 3b_{13} + 2b_{14})$
$T_{K^0}^{Bc} \rightarrow D^- K^0$	$b_4 + b_6 - b_9 - b_{10} - 2b_{12} - b_{13} - b_{14}$
$T_{K^+}^{Bc} \rightarrow \bar{D}^0 K^0$	$b_4 + b_8 - b_9 + 3b_{12} - b_{13}$
$T_{K^+}^{Bc} \rightarrow D^- K^+$	$b_6 - b_8 - b_{10} + 3b_{12} - b_{14}$
$T_{\eta_8}^{Bc} \rightarrow \bar{D}^0 \pi^-$	$\frac{b_4 + b_5 + 3b_7 + b_8 + b_9 - b_{10} + 3b_{11} + 3b_{12} + 3b_{13} + 3b_{14}}{\sqrt{6}}$
$T_{\eta_8}^{Bc} \rightarrow D^- \pi^0$	$-\frac{b_4 + b_5 + 3b_7 + b_8 + b_9 - b_{10} + 3b_{11} - 5b_{12} - 5b_{13} - 5b_{14}}{2\sqrt{3}}$
$T_{\eta_8}^{Bc} \rightarrow D^- \eta_8$	$\frac{1}{6} (b_4 + b_5 + 3(2b_6 + b_7 + b_8 - b_9 - b_{10} + b_{11} - b_{12} + b_{13} - b_{14}))$
$T_{\eta_8}^{Bc} \rightarrow D_s^- K^0$	$\frac{-2b_4 + b_5 + 3b_7 + 2b_8 + 2b_9 + b_{10} + 3b_{11} + 2b_{12} + 2b_{13} - b_{14}}{\sqrt{6}}$
$T_{\pi^+}^{Bc} \rightarrow \bar{D}^0 \eta_1$	$\sqrt{3}(b'_6 - b'_8 + 3b'_{10})$
$T_{\pi^0}^{Bc} \rightarrow D^- \eta_1$	$\sqrt{\frac{3}{2}}(-b'_6 + b'_8 + 5b'_{10})$
$T_{\bar{K}^0}^{Bc} \rightarrow D_s^- \eta_1$	$\sqrt{3}(b'_6 + b'_8 - b'_{10})$
$T_{\eta_8}^{Bc} \rightarrow D^- \eta_1$	$\frac{b'_6 + 3(b'_8 + b'_{10})}{\sqrt{2}}$
$T_{\eta_1}^{Bc} \rightarrow \bar{D}^0 \pi^-$	$\sqrt{3}(b'_4 + b'_7 + 3b'_9)$
$T_{\eta_1}^{Bc} \rightarrow D^- \pi^0$	$-\sqrt{\frac{3}{2}}(b'_4 + b'_7 - 5b'_9)$
$T_{\eta_1}^{Bc} \rightarrow D^- \eta_8$	$\frac{b'_4 - 3b'_7 + 3b'_9}{\sqrt{2}}$
$T_{\eta_1}^{Bc} \rightarrow D^- \eta_1$	$b'_4 + 3b'_5 + b'_6$
$T_{\eta_1}^{Bc} \rightarrow D_s^- K^0$	$\sqrt{3}(b'_4 - b'_7 - b'_9)$

$$\begin{aligned}
\mathcal{H}_{eff} = & f_3(T_{b\bar{c}8})_j^i (H_6)_{[ik]}^j \bar{B}^l M_l^k + f_4(T_{b\bar{c}8})_j^i (H_6)_{[ik]}^l \bar{B}^j M_l^k \\
& + f_5(T_{b\bar{c}8})_j^i (H_6)_{[ik]}^l \bar{B}^k M_l^j + f_6(T_{b\bar{c}8})_j^i (H_6)_{[kl]}^j \bar{B}^k M_l^i \\
& + f_7(T_{b\bar{c}8})_j^i (H_{15})_{[ik]}^j \bar{B}^l M_l^k + f_8(T_{b\bar{c}8})_j^i (H_{15})_{[ik]}^l \bar{B}^j M_l^k \\
& + f_9(T_{b\bar{c}8})_j^i (H_{15})_{[ik]}^l \bar{B}^k M_l^j + f_{10}(T_{b\bar{c}8})_j^i (H_{15})_{[kl]}^j \bar{B}^k M_l^i
\end{aligned}$$

$$\begin{aligned}
& + f'_3(T_{b\bar{c}1})_i^i (H_6)_{[jk]}^l \bar{B}^j M_l^k + f'_4(T_{b\bar{c}8})_j^i (H_6)_{[ik]}^j \bar{B}^k M_l^l \\
& + f'_5(T_{b\bar{c}1})_i^i (H_{15})_{[jk]}^l \bar{B}^j M_l^k + f'_6(T_{b\bar{c}8})_j^i (H_{15})_{[ik]}^j \bar{B}^k M_l^l.
\end{aligned} \quad (22)$$

As usual, the corresponding topological diagrams are given in Fig. 3. Expanding the Hamiltonian above to obtain decay amplitudes, and listed in Table 8. One deduces the relations between different decay widths given as

Table 7 Tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into anti-charmed and light mesons induced by the charmless $b \rightarrow s$ transition

Channel	Amplitude	Channel	Amplitude
$T_{\pi^-}^{Bc} \rightarrow D^- K^-$	$b_4 - b_8 + b_9 - b_{12} + 3b_{13}$	$T_{\pi^-}^{Bc} \rightarrow D_s^- \pi^-$	$b_6 + b_8 + b_{10} - b_{12} + 3b_{14}$
$T_{\pi^0}^{Bc} \rightarrow \bar{D}^0 K^-$	$\frac{b_4 + 2b_7 + b_8 + b_9 + 4b_{11} + 3b_{12} + 3b_{13}}{\sqrt{2}}$	$T_{\pi^0}^{Bc} \rightarrow D^- \bar{K}^0$	$\frac{(-b_4 + 2b_7 + b_8 + b_9 + 4b_{11} + b_{12} + b_{13})}{\sqrt{2}}$
$T_{\pi^0}^{Bc} \rightarrow D_s^- \pi^0$	$b_6 + b_{12} + b_{14}$	$T_{\pi^0}^{Bc} \rightarrow D_s^- \eta_8$	$\frac{-2b_7 - b_8 + b_{10} - 4b_{11} + 2b_{12} + 2b_{14}}{\sqrt{3}}$
$T_{\pi^+}^{Bc} \rightarrow \bar{D}^0 \bar{K}^0$	$b_4 + b_8 - b_9 + 3b_{12} - b_{13}$	$T_{\pi^+}^{Bc} \rightarrow D_s^- \pi^+$	$b_6 - b_8 - b_{10} + 3b_{12} - b_{14}$
$T_{K^-}^{Bc} \rightarrow D_s^- K^-$	$b_4 + b_6 + b_9 + b_{10} - 2b_{12} + 3b_{13} + 3b_{14}$	$T_{K^-}^{Bc} \rightarrow D_s^- \bar{K}^0$	$b_4 + b_6 - b_9 - b_{10} - 2b_{12} - b_{13} - b_{14}$
$T_{K^0}^{Bc} \rightarrow \bar{D}^0 \pi^-$	$b_5 + b_7 - b_{10} - b_{11} + 3b_{14}$	$T_{K^0}^{Bc} \rightarrow D^- \pi^0$	$-\frac{b_5 + b_7 + 2b_9 + b_{10} - b_{11} - 4b_{13} - b_{14}}{\sqrt{2}}$
$T_{K^0}^{Bc} \rightarrow D^- \eta_8$	$\frac{-2b_4 + b_5 + b_7 + 2b_8 + b_{10} - b_{11} + 2b_{12} + 6b_{13} - b_{14}}{\sqrt{6}}$	$T_{K^0}^{Bc} \rightarrow D_s^- K^0$	$b_5 + b_6 + b_7 + b_8 - b_{11} - b_{12} - 2b_{14}$
$T_{K^+}^{Bc} \rightarrow \bar{D}^0 \pi^0$	$\frac{(b_5 - b_7 - 2b_9 - b_{10} + 3b_{11} + 4b_{13} + 3b_{14})}{\sqrt{2}}$	$T_{K^+}^{Bc} \rightarrow \bar{D}^0 \eta_8$	$-\frac{2b_4 - b_5 + b_7 + 2b_8 + b_{10} - 3b_{11} + 6b_{12} - 6b_{13} - 3b_{14}}{\sqrt{6}}$
$T_{K^+}^{Bc} \rightarrow D^- \pi^+$	$b_5 - b_7 + b_{10} + 3b_{11} - b_{14}$	$T_{K^+}^{Bc} \rightarrow D_s^- K^+$	$b_5 + b_6 - b_7 - b_8 + 3b_{11} + 3b_{12} - 2b_{14}$
$T_{\eta_8}^{Bc} \rightarrow \bar{D}^0 K^-$	$\frac{b_4 - 2b_5 + b_8 + b_9 + 2b_{10} + 6b_{11} + 3b_{12} + 3b_{13} - 6b_{14}}{\sqrt{6}}$	$T_{\eta_8}^{Bc} \rightarrow D^- \bar{K}^0$	$\frac{b_4 - 2b_5 - b_8 - b_9 - 2b_{10} + 6b_{11} - b_{12} - b_{13} + 2b_{14}}{\sqrt{6}}$
$T_{\eta_8}^{Bc} \rightarrow D_s^- \pi^0$	$\frac{-b_8 + 2b_9 + b_{10} + 2b_{12} - 4b_{13} + 2b_{14}}{\sqrt{3}}$	$T_{\eta_8}^{Bc} \rightarrow D_s^- \eta_8$	$\frac{2b_4}{3} + \frac{2b_5}{3} + b_6 - 2b_{11} - b_{12} - 2b_{13} - b_{14}$
$T_{\pi_0}^{Bc} \rightarrow D_s^- \eta_1$	$\sqrt{6}(b'_8 + 2b'_{10})$	$T_{K^+}^{Bc} \rightarrow \bar{D}^0 \eta_1$	$\sqrt{3}(b'_6 - b'_8 + 3b'_{10})$
$T_{K^0}^{Bc} \rightarrow D^- \eta_1$	$\sqrt{3}(b'_6 + b'_8 - b'_{10})$	$T_{\eta_8}^{Bc} \rightarrow D_s^- \eta_1$	$\sqrt{2}(3b'_{10} - b'_6)$
$T_{\eta_1}^{Bc} \rightarrow \bar{D}^0 K^-$	$\sqrt{3}(b'_4 + b'_7 + 3b'_9)$	$T_{\eta_1}^{Bc} \rightarrow D^- \bar{K}^0$	$\sqrt{3}(b'_4 - b'_7 - b'_9)$
$T_{\eta_1}^{Bc} \rightarrow D_s^- \pi^0$	$-\sqrt{6}(b'_7 - 2b'_9)$	$T_{\eta_1}^{Bc} \rightarrow D_s^- \eta_8$	$-\sqrt{2}(b'_4 - 3b'_9)$
$T_{\eta_1}^{Bc} \rightarrow D_s^- \eta_1$	$b'_4 + 3b'_5 + b'_6$

Fig. 3 Topological diagrams for the $\bar{c}$ -quark non-leptonic decays of tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$. The two-body processes are given in **a–f**. The $\bar{c}$ -quark decays have the similar structures with the b -quark decays. The **a–l** contribute to the process of $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into B plus a light meson

$$\Gamma(T_{\pi^-}^{Bc} \rightarrow B^- \pi^-) = \Gamma(T_{K^-}^{Bc} \rightarrow B^- K^-),$$

$$\Gamma(T_{\pi^0}^{Bc} \rightarrow B^- \pi^0) = \Gamma(T_{\eta_8}^{Bc} \rightarrow B^- \eta_8),$$

$$\Gamma(T_{\pi^+}^{Bc} \rightarrow B^- \pi^+) = \Gamma(T_{K^+}^{Bc} \rightarrow B^- K^+),$$

$$\Gamma(T_{\pi^+}^{Bc} \rightarrow \bar{B}_s^0 \pi^0) = \Gamma(T_{\pi^0}^{Bc} \rightarrow \bar{B}_s^0 \pi^-),$$

$$\Gamma(T_{K^0}^{Bc} \rightarrow B^- K^0) = \Gamma(T_{K^0}^{Bc} \rightarrow B^- \bar{K}^0),$$

$$\Gamma(T_{K^0}^{Bc} \rightarrow \bar{B}_s^0 \pi^-) = \Gamma(T_{K^0}^{Bc} \rightarrow \bar{B}^0 K^-),$$

Table 8 Tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$ decays into a B meson and light meson

Channel	Amplitude	Channel	Amplitude
$T_{\pi^0}^{Bc} \rightarrow B^- K^0$	$-\frac{f_4+f_6+f_8-f_{10}}{\sqrt{2}}$	$T_{\pi^0}^{Bc} \rightarrow \bar{B}_s^0 \pi^-$	$-\frac{f_5+f_6-f_9+f_{10}}{\sqrt{2}}$
$T_{\pi^+}^{Bc} \rightarrow B^- K^+$	$-f_3 - f_6 + f_7 + f_{10}$	$T_{\pi^+}^{Bc} \rightarrow \bar{B}^0 K^0$	$-f_3 - f_4 + f_7 + f_8$
$T_{\pi^+}^{Bc} \rightarrow \bar{B}_s^0 \pi^0$	$\frac{f_5+f_6-f_9+f_{10}}{\sqrt{2}}$	$T_{\pi^+}^{Bc} \rightarrow \bar{B}_s^0 \eta_8$	$\frac{(2f_3-f_5+f_6-2f_7+f_9+f_{10})}{\sqrt{6}}$
$T_{K^-}^{Bc} \rightarrow B^- \pi^-$	$f_4 + f_5 + f_8 + f_9$	$T_{K^0}^{Bc} \rightarrow B^- \pi^0$	$\frac{f_3-f_5+f_7-f_9}{\sqrt{2}}$
$T_{K^0}^{Bc} \rightarrow B^- \eta_8$	$\frac{f_3+f_5+2f_6+f_7+f_9-2f_{10}}{\sqrt{6}}$	$T_{K^0}^{Bc} \rightarrow \bar{B}^0 \pi^-$	$f_3 + f_4 + f_7 + f_8$
$T_{K^0}^{Bc} \rightarrow \bar{B}_s^0 K^-$	$f_3 + f_6 + f_7 + f_{10}$	$T_{K^+}^{Bc} \rightarrow \bar{B}_s^0 K^0$	$-f_4 - f_5 + f_8 + f_9$
$T_{\eta_8}^{Bc} \rightarrow B^- K^0$	$-\frac{f_4+2f_5+f_6-f_8+2f_9-f_{10}}{\sqrt{6}}$	$T_{\eta_8}^{Bc} \rightarrow \bar{B}_s^0 \pi^-$	$-\frac{2f_4-f_5+f_6-2f_8+f_9+f_{10}}{\sqrt{6}}$
$T_{\pi^+}^{Bc} \rightarrow \bar{B}_s^0 \eta_1$	$\sqrt{3}(f'_6 - f'_4)$	$T_{K^0}^{Bc} \rightarrow B^- \eta_1$	$\sqrt{3}(f'_4 + f'_6)$
$T_{\eta_1}^{Bc} \rightarrow B^- K^0$	$\sqrt{3}(f'_5 - f'_3)$	$T_{\eta_1}^{Bc} \rightarrow \bar{B}^0 \pi^-$	$\sqrt{3}(f'_3 + f'_5)$
$T_{\pi^-}^{Bc} \rightarrow B^- \pi^-$	$(f_4 + f_5 + f_8 + f_9)(-sC)$	$T_{\pi^0}^{Bc} \rightarrow B^- \pi^0$	$\frac{(f_3-f_4-f_5+f_6+f_7+f_8-f_9-f_{10})sC}{2}$
$T_{\pi^0}^{Bc} \rightarrow B^- \eta_8$	$\frac{(f_3+3f_4+f_5-f_6+f_7-3f_8+f_9+f_{10})sC}{2\sqrt{3}}$	$T_{\pi^0}^{Bc} \rightarrow \bar{B}^0 \pi^-$	$\frac{(f_3+f_4+f_5+f_6+f_7+f_8-f_9+f_{10})sC}{\sqrt{2}}$
$T_{\pi^0}^{Bc} \rightarrow \bar{B}_s^0 K^-$	$\frac{(f_3-f_5+f_7+f_9)sC}{\sqrt{2}}$	$T_{\pi^+}^{Bc} \rightarrow B^- \pi^+$	$(f_3 + f_6 - f_7 - f_{10})sC$
$T_{\pi^+}^{Bc} \rightarrow \bar{B}^0 \pi^0$	$-\frac{(f_3+f_4+f_5+f_6-f_7-f_8-f_9+f_{10})sC}{\sqrt{2}}$	$T_{\pi^+}^{Bc} \rightarrow \bar{B}^0 \eta_8$	$\frac{(f_3+3f_4+f_5-f_6-f_7-3f_8-f_9-f_{10})sC}{\sqrt{6}}$
$T_{\pi^+}^{Bc} \rightarrow \bar{B}_s^0 \bar{K}^0$	$(f_3 - f_5 - f_7 + f_9)sC$	$T_{K^-}^{Bc} \rightarrow B^- K^-$	$(f_4 + f_5 + f_8 + f_9)sC$
$T_{K^0}^{Bc} \rightarrow B^- \bar{K}^0$	$(f_5 + f_6 + f_9 - f_{10})sC$	$T_{K^0}^{Bc} \rightarrow \bar{B}^0 K^-$	$(f_4 - f_6 + f_8 - f_{10})sC$
$T_{K^0}^{Bc} \rightarrow B^- K^0$	$(f_5 + f_6 + f_9 - f_{10})(-sC)$	$T_{K^0}^{Bc} \rightarrow \bar{B}_s^0 \pi^-$	$(f_4 - f_6 + f_8 - f_{10})(-sC)$
$T_{K^+}^{Bc} \rightarrow B^- K^+$	$(f_3 + f_6 - f_7 - f_{10})(-sC)$	$T_{K^+}^{Bc} \rightarrow \bar{B}^0 K^0$	$(f_3 - f_5 - f_7 + f_9)(-sC)$
$T_{K^+}^{Bc} \rightarrow \bar{B}_s^0 \pi^0$	$\frac{(-f_4+f_6+f_8+f_{10})sC}{\sqrt{2}}$	$T_{K^+}^{Bc} \rightarrow \bar{B}_s^0 \eta_8$	$\frac{(2f_3+3f_4+2f_5+f_6-2f_7-3f_8-2f_9+f_{10})sC}{\sqrt{6}}$
$T_{\eta_8}^{Bc} \rightarrow B^- \pi^0$	$-\frac{(3f_3+f_4-f_5+f_6+3f_7-f_8-f_9-f_{10})sC}{2\sqrt{3}}$	$T_{\eta_8}^{Bc} \rightarrow B^- \eta_8$	$-\frac{(f_3-f_4-f_5+f_6+f_7+f_8-f_9-f_{10})sC}{2}$
$T_{\eta_8}^{Bc} \rightarrow \bar{B}^0 \pi^-$	$-\frac{(3f_3+f_4-f_5+f_6+3f_7+f_8+f_9+f_{10})sC}{\sqrt{6}}$	$T_{\eta_8}^{Bc} \rightarrow \bar{B}_s^0 K^-$	$-\frac{(3f_3+2f_4+f_5+2f_6+3f_7+2f_8-f_9+2f_{10})sC}{\sqrt{6}}$
$T_{\pi^+}^{Bc} \rightarrow \bar{B}^0 \eta_1$	$\sqrt{3}(f'_4 - f'_6)sC$	$T_{\pi^0}^{Bc} \rightarrow B^- \eta_1$	$\sqrt{\frac{3}{2}}(f'_4 + f'_6)sC$
$T_{K^+}^{Bc} \rightarrow \bar{B}_s^0 \eta_1$	$\sqrt{3}(f'_6 - f'_4)sC$	$T_{\eta_8}^{Bc} \rightarrow B^- \eta_1$	$-\frac{3(f'_4+f'_6)sC}{\sqrt{2}}$
$T_{\eta_1}^{Bc} \rightarrow B^- \pi^0$	$-\sqrt{\frac{3}{2}}(f'_3 - f'_5)sC$	$T_{\eta_1}^{Bc} \rightarrow B^- \eta_8$	$\frac{3(f'_3-f'_5)sC}{\sqrt{2}}$
$T_{\eta_1}^{Bc} \rightarrow \bar{B}^0 \pi^-$	$-\sqrt{3}(f'_3 + f'_5)sC$	$T_{\eta_1}^{Bc} \rightarrow \bar{B}_s^0 K^-$	$\sqrt{3}(f'_3 + f'_5)sC$
$T_{\pi^-}^{Bc} \rightarrow B^- K^-$	$(f_4 + f_5 + f_8 + f_9)sC^2$	$T_{\pi^0}^{Bc} \rightarrow B^- \bar{K}^0$	$-\frac{(f_4+f_5-f_8+f_9)sC^2}{\sqrt{2}}$
$T_{\pi^0}^{Bc} \rightarrow \bar{B}^0 K^-$	$-\frac{(f_4+f_5+f_8-f_9)sC^2}{\sqrt{2}}$	$T_{\pi^+}^{Bc} \rightarrow \bar{B}^0 \bar{K}^0$	$(f_4 + f_5 - f_8 - f_9)(-sC^2)$
$T_{K^0}^{Bc} \rightarrow B^- \pi^0$	$\frac{(f_3+f_6+f_7-f_{10})sC^2}{\sqrt{2}}$	$T_{K^0}^{Bc} \rightarrow B^- \eta_8$	$\frac{(f_3-2f_5-f_6+f_7-2f_9+f_{10})sC^2}{\sqrt{6}}$
$T_{K^0}^{Bc} \rightarrow \bar{B}^0 \pi^-$	$(f_3 + f_6 + f_7 + f_{10})sC^2$	$T_{K^0}^{Bc} \rightarrow \bar{B}_s^0 K^-$	$(f_3 + f_4 + f_7 + f_8)sC^2$
$T_{K^+}^{Bc} \rightarrow B^- \pi^+$	$(f_3 + f_6 - f_7 - f_{10})(-sC^2)$	$T_{K^+}^{Bc} \rightarrow \bar{B}^0 \pi^0$	$\frac{1}{\sqrt{2}}(f_3 + f_6 - f_7 + f_{10})sC^2$
$T_{K^+}^{Bc} \rightarrow \bar{B}^0 \eta_8$	$-\frac{(f_3-2f_5-f_6-f_7+2f_9-f_{10})sC^2}{\sqrt{6}}$	$T_{K^+}^{Bc} \rightarrow \bar{B}_s^0 \bar{K}^0$	$(f_3 + f_4 - f_7 - f_8)(-sC^2)$
$T_{\eta_8}^{Bc} \rightarrow B^- \bar{K}^0$	$-\frac{(f_4-f_5-2f_6-f_8-f_9+2f_{10})sC^2}{\sqrt{6}}$	$T_{\eta_8}^{Bc} \rightarrow \bar{B}^0 K^-$	$\frac{(f_4-f_5-2f_6+f_8+f_9-2f_{10})sC^2}{\sqrt{6}}$
$T_{K^+}^{Bc} \rightarrow \bar{B}^0 \eta_1$	$-\sqrt{3}(f'_4 - f'_6)sC^2$	$T_{\eta_1}^{Bc} \rightarrow B^- \bar{K}^0$	$-\sqrt{3}(f'_3 - f'_5)sC^2$
$T_{\eta_1}^{Bc} \rightarrow \bar{B}^0 K^-$	$\sqrt{3}(f'_3 + f'_5)sC^2$		

Table 9 Cabibbo allowed $T_{b\bar{c}8}/T_{b\bar{c}1}$ $\bar{c}$ -quark decays

$T_{\pi^0}^{Bc} \rightarrow B^- K^0$	$T_{\pi^0}^{Bc} \rightarrow \bar{B}_s^0 \pi^-$		
$T_{\pi^+}^{Bc} \rightarrow B^- K^+$	$T_{\pi^+}^{Bc} \rightarrow \bar{B}^0 K^0$	$T_{\pi^+}^{Bc} \rightarrow \bar{B}_s^0 \eta_1$	
$T_{K^-}^{Bc} \rightarrow B^- l^- \bar{\nu}$	$T_{K^-}^{Bc} \rightarrow B^- \pi^-$		
$T_{\bar{K}^0}^{Bc} \rightarrow \bar{B}^0 l^- \bar{\nu}$	$T_{\bar{K}^0}^{Bc} \rightarrow \bar{B}^0 \pi^-$	$T_{\bar{K}^0}^{Bc} \rightarrow \bar{B}_s^0 K^-$	$T_{\bar{K}^0}^{Bc} \rightarrow B^- \eta_1$
$T_{K^+}^{Bc} \rightarrow \bar{B}_s^0 K^0$			
$T_{\eta_8}^{Bc} \rightarrow \bar{B}_s^0 l^- \bar{\nu}$	$T_{\eta_8}^{Bc} \rightarrow B^- K^0$	$T_{\eta_8}^{Bc} \rightarrow \bar{B}_s^0 \pi^-$	
$T_{\eta_1}^{Bc} \rightarrow \bar{B}_s^0 l^- \bar{\nu}$	$T_{\eta_1}^{Bc} \rightarrow B^- K^0$	$T_{\eta_1}^{Bc} \rightarrow \bar{B}_s^0 \pi^-$	

Table 10 Golden channels for $T_{b\bar{c}8}/T_{b\bar{c}1}$ b -quark decays

$T_{\pi^-}^{Bc} \rightarrow \pi^- l^- \bar{\nu}$	$T_{\pi^-}^{Bc} \rightarrow \pi^- D_s^-$	$T_{\pi^-}^{Bc} \rightarrow K^- D^-$	$T_{\pi^-}^{Bc} \rightarrow \pi^- \pi^-$	$T_{\pi^-}^{Bc} \rightarrow D^- D_s^-$
$T_{\pi^0}^{Bc} \rightarrow \pi^0 l^- \bar{\nu}$	$T_{\pi^0}^{Bc} \rightarrow \pi^- J/\psi$	$T_{\pi^0}^{Bc} \rightarrow \bar{K}^0 D^-$	$T_{\pi^0}^{Bc} \rightarrow K^- \bar{D}^0$	$T_{\pi^0}^{Bc} \rightarrow D^0 D^-$
$T_{\pi^0}^{Bc} \rightarrow K^0 K^-$	$T_{\pi^0}^{Bc} \rightarrow \bar{D}^0 D_s^-$	$T_{\pi^0}^{Bc} \rightarrow \pi^- \eta_1$		
$T_{\pi^+}^{Bc} \rightarrow \pi^+ l^- \bar{\nu}$	$T_{\pi^+}^{Bc} \rightarrow \pi^+ D_s^-$	$T_{\pi^+}^{Bc} \rightarrow \bar{K}^0 \bar{D}^0$	$T_{\pi^+}^{Bc} \rightarrow D^0 \bar{D}^0$	$T_{\pi^+}^{Bc} \rightarrow D^+ D^-$
$T_{\pi^+}^{Bc} \rightarrow D_s^+ D_s^-$	$T_{\pi^+}^{Bc} \rightarrow \pi^+ \pi^-$	$T_{\pi^+}^{Bc} \rightarrow K^+ K^-$	$T_{\pi^+}^{Bc} \rightarrow K^0 \bar{K}^0$	$T_{\pi^+}^{Bc} \rightarrow \eta_1 J/\psi$
$T_{\pi^+}^{Bc} \rightarrow \pi^0 \eta_1$	$T_{\pi^+}^{Bc} \rightarrow \eta_8 \eta_1$	$T_{\pi^+}^{Bc} \rightarrow \eta_1 \eta_1$		
$T_{K^-}^{Bc} \rightarrow K^- l^- \bar{\nu}$	$T_{K^-}^{Bc} \rightarrow K^- D_s^-$	$T_{K^-}^{Bc} \rightarrow \pi^- K^-$	$T_{K^-}^{Bc} \rightarrow D_s^- D_s^-$	
$T_{\bar{K}^0}^{Bc} \rightarrow \bar{K}^0 l^- \bar{\nu}$	$T_{\bar{K}^0}^{Bc} \rightarrow K^- J/\psi$	$T_{\bar{K}^0}^{Bc} \rightarrow \bar{K}^0 D_s^-$	$T_{\bar{K}^0}^{Bc} \rightarrow D^0 D_s^-$	$T_{\bar{K}^0}^{Bc} \rightarrow \pi^- \bar{K}^0$
$T_{\bar{K}^0}^{Bc} \rightarrow K^- \eta_1$				
$T_{K^0}^{Bc} \rightarrow K^0 l^- \bar{\nu}$	$T_{K^0}^{Bc} \rightarrow D^- J/\psi$	$T_{K^0}^{Bc} \rightarrow \pi^- \bar{D}^0$	$T_{K^0}^{Bc} \rightarrow K^0 D_s^-$	$T_{K^0}^{Bc} \rightarrow \pi^- K^0$
$T_{K^0}^{Bc} \rightarrow \bar{D}^0 D^-$	$T_{K^0}^{Bc} \rightarrow \eta_1 D^-$			
$T_{K^+}^{Bc} \rightarrow K^+ l^- \bar{\nu}$	$T_{K^+}^{Bc} \rightarrow \bar{D}^0 J/\psi$	$T_{K^+}^{Bc} \rightarrow K^0 J/\psi$	$T_{K^+}^{Bc} \rightarrow \pi^+ D^-$	$T_{K^+}^{Bc} \rightarrow K^+ D_s^-$
$T_{K^+}^{Bc} \rightarrow D_s^+ D^-$	$T_{K^+}^{Bc} \rightarrow \pi^- K^+$	$T_{K^+}^{Bc} \rightarrow \bar{D}^0 \bar{D}^0$	$T_{K^+}^{Bc} \rightarrow \eta_1 \bar{D}^0$	$T_{K^+}^{Bc} \rightarrow K^0 \eta_1$
$T_{\eta_8}^{Bc} \rightarrow \eta_8 l^- \bar{\nu}$	$T_{\eta_8}^{Bc} \rightarrow D_s^- J/\psi$	$T_{\eta_8}^{Bc} \rightarrow \pi^- J/\psi$	$T_{\eta_8}^{Bc} \rightarrow \bar{K}^0 D^-$	$T_{\eta_8}^{Bc} \rightarrow K^- \bar{D}^0$
$T_{\eta_8}^{Bc} \rightarrow D^0 D^-$	$T_{\eta_8}^{Bc} \rightarrow K^0 K^-$	$T_{\eta_8}^{Bc} \rightarrow \bar{D}^0 D_s^-$	$T_{\eta_8}^{Bc} \rightarrow \eta_1 D_s^-$	$T_{\eta_8}^{Bc} \rightarrow \pi^- \eta_1$
$T_{\eta_1}^{Bc} \rightarrow \eta_1 l^- \bar{\nu}$	$T_{\eta_1}^{Bc} \rightarrow J/\psi l^- \bar{\nu}$	$T_{\eta_1}^{Bc} \rightarrow \bar{K}^0 D^-$	$T_{\eta_1}^{Bc} \rightarrow K^- \bar{D}^0$	$T_{\eta_1}^{Bc} \rightarrow \eta_8 D_s^-$
$T_{\eta_1}^{Bc} \rightarrow \eta_1 D_s^-$	$T_{\eta_1}^{Bc} \rightarrow D_s^- J/\psi$	$T_{\eta_1}^{Bc} \rightarrow \pi^- J/\psi$	$T_{\eta_1}^{Bc} \rightarrow D^0 D^-$	$T_{\eta_1}^{Bc} \rightarrow \pi^- \eta_8$
$T_{\eta_1}^{Bc} \rightarrow K^0 K^-$	$T_{\eta_1}^{Bc} \rightarrow \bar{D}^0 D_s^-$	$T_{\eta_1}^{Bc} \rightarrow \pi^- \eta_1$		

$$\begin{aligned} \Gamma(T_{K^+}^{Bc} \rightarrow \bar{B}^0 K^0) &= \Gamma(T_{\pi^+}^{Bc} \rightarrow \bar{B}_s^0 \bar{K}^0), \\ \Gamma(T_{\pi^+}^{Bc} \rightarrow \bar{B}^0 \eta_1) &= \Gamma(T_{K^+}^{Bc} \rightarrow \bar{B}_s^0 \eta_1), \\ \Gamma(T_{\eta_8}^{Bc} \rightarrow B^- \eta_1) &= 3\Gamma(T_{\pi^0}^{Bc} \rightarrow B^- \eta_1), \\ \Gamma(T_{\eta_1}^{Bc} \rightarrow B^- \eta_8) &= 3\Gamma(T_{\eta_1}^{Bc} \rightarrow B^- \pi^0), \\ \Gamma(T_{\eta_1}^{Bc} \rightarrow \bar{B}^0 \pi^-) &= \Gamma(T_{\eta_1}^{Bc} \rightarrow \bar{B}_s^0 K^-). \end{aligned}$$

5 Golden decay channels

In this section, we will discuss the golden channels to reconstruct the $T_{b\bar{c}8}/T_{b\bar{c}1}$ and give an estimate of the decay branching fractions. In our analysis given in the previous sections,

the final meson can be replaced by its corresponding counterpart with the same quark constituent but different quantum numbers. For instance, one may replace $\bar{K}^0$ by $\bar{K}^{*0}$.

Golden decay channels must satisfy the following criteria:

- *Large branching fractions* For charm quark decays, one should use the Cabibbo allowed decay modes, while for bottom quark, the quark level transition $b \rightarrow c\bar{u}d$ or $b \rightarrow c\bar{s}s$ gives the largest branching fractions.
- *Large detection efficiency* At hadron colliders like LHC, charged particle has better chance to be detected than neutral state. So we will remove the channels with $\pi^0, \eta, \phi, \rho^\pm (\rightarrow \pi^\pm \pi^0), K^{*\pm} (\rightarrow K^\pm \pi^0)$ and ω in final states, but keep the modes with $\pi^\pm, K^0 (\rightarrow \pi^+ \pi^-), \rho^0 (\rightarrow \pi^+ \pi^-)$.

The two-body decay modes that can be used to reconstruct the $T_{b\bar{c}8}/T_{b\bar{c}1}$ are collected in Tables 9 and 10. Some comments are given in order.

From the data on the charmed meson decays, we can infer that the typical branching fraction of these Cabibbo allowed decay channels for charm quark decay in Table 9 is at a few percent level. On the experimental side, to construct the bottom meson in final states, another factor, at the order 10^{-3} or even smaller, due to the weak decay of bottom meson is needed. So the branching fraction for the decay chains to reconstruct the $T_{b\bar{c}8}/T_{b\bar{c}1}$ might reach the order 10^{-5} , or smaller.

If the b quark in $T_{b\bar{c}8}/T_{b\bar{c}1}$ decay first, from the data on the bottom mesons decays, one may deduce that the typical branching fraction is at the order 10^{-3} . The final states which include the J/ψ or D meson such as $T_{\pi^-}^{Bc} \rightarrow K^- D^-$ would introduce a factor 10^{-3} to reconstruct in the experiment. So the branching fraction of these channels may also reach the order 10^{-5} .

The channels with two light mesons such as $T_{\pi^-}^{Bc} \rightarrow \pi^- \pi^-$ require the annihilation of the two heavy quarks. But since the CKM matrix element V_{cb} , these channels might have sizable decay branching fractions.

6 Conclusions

Tetraquarks with the quark content $[bq_i][\bar{c}\bar{q}_j]$ are of great research interest but have not been discovered yet. In this paper, we have systematically studied the weak decays of the doubly-heavy tetraquark $T_{b\bar{c}8}/T_{b\bar{c}1}$ under the SU(3) flavor symmetry, which include the semileptonic and nonleptonic b and $\bar{c}$ quark decays. The $\bar{c}$ quark decays are dominant, of which the typical branching fraction is at a few percents level.

Using the building blocks in SU(3), we construct the effective Hamiltonian at the hadron level for their weak decays. The nonperturbative effects are parametrized into a few quantities ($a_i, b_j, \dots$). Therefore, one can easily derive the decay amplitudes, based on which relations between different channels can be obtained. Finally, we give a list of the golden channels which is useful to search for the $T_{b\bar{c}8}/T_{b\bar{c}1}$ state tetraquark in future experiments.

Acknowledgements We thank Prof. Wei Wang for useful discussions. This work is supported in part by National Natural Science Foundation of China under Grant Nos. 11575110, 11655002, 11675091, 11735010, and 11835015, by Natural Science Foundation of Shanghai under Grant No. 15DZ22272100, by Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This is a theoretical study and no experimental data has been listed.]

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

References

1. S.K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003). <https://doi.org/10.1103/PhysRevLett.91.262001>. arXiv:hep-ex/0309032
2. M. Tanabashi et al. [Particle Data Group], Phys. Rev. D **98**(3), 030001 (2018). <https://doi.org/10.1103/PhysRevD.98.030001>
3. H.X. Chen, W. Chen, X. Liu, S.L. Zhu, Phys. Rept. **639**, 1 (2016). <https://doi.org/10.1016/j.physrep.2016.05.004>. arXiv:1601.02092 [hep-ph]
4. F.K. Guo, C. Hanhart, U.G. Meißner, Q. Wang, Q. Zhao, B.S. Zou, Rev. Mod. Phys. **90**(1), 015004 (2018). <https://doi.org/10.1103/RevModPhys.90.015004>. arXiv:1705.00141 [hep-ph]
5. F.K. Guo, C. Hidalgo-Duque, J. Nieves, M.P. Valderrama, Phys. Rev. D **88**, 054007 (2013). <https://doi.org/10.1103/PhysRevD.88.054007>. arXiv:1303.6608 [hep-ph]
6. M. Cleven, Q. Wang, F.K. Guo, C. Hanhart, U.G. Meißner, Q. Zhao, Phys. Rev. D **87**(7), 074006 (2013). <https://doi.org/10.1103/PhysRevD.87.074006>. arXiv:1301.6461 [hep-ph]
7. F.K. Guo, U.G. Meißner, W. Wang, Commun. Theor. Phys. **61**, 354 (2014). <https://doi.org/10.1088/0253-6102/61/3/14>. arXiv:1308.0193 [hep-ph]
8. X.H. Liu, G. Li, Eur. Phys. J. C **76**(8), 455 (2016). <https://doi.org/10.1140/epjc/s10052-016-4308-1>. arXiv:1603.00708 [hep-ph]
9. G. Li, X.H. Liu, Z. Zhou, Phys. Rev. D **90**(5), 054006 (2014). <https://doi.org/10.1103/PhysRevD.90.054006>. arXiv:1409.0754 [hep-ph]
10. G. Li, Eur. Phys. J. C **73**(11), 2621 (2013). <https://doi.org/10.1140/epjc/s10052-013-2621-5>. arXiv:1304.4458 [hep-ph]
11. F.K. Guo, U.G. Meißner, W. Wang, Z. Yang, Eur. Phys. J. C **74**(9), 3063 (2014). <https://doi.org/10.1140/epjc/s10052-014-3063-4>. arXiv:1402.6236 [hep-ph]
12. F.K. Guo, U.G. Meißner, W. Wang, Z. Yang, JHEP **1405**, 138 (2014). [https://doi.org/10.1007/JHEP05\(2014\)138](https://doi.org/10.1007/JHEP05(2014)138). arXiv:1403.4032 [hep-ph]
13. F.K. Guo, C. Hanhart, Q. Wang, Q. Zhao, Phys. Rev. D **91**(5), 051504 (2015). <https://doi.org/10.1103/PhysRevD.91.051504>. arXiv:1411.5584 [hep-ph]
14. Y.H. Chen, M. Cleven, J.T. Daub, F.K. Guo, C. Hanhart, B. Kubis, U.G. Meißner, B.S. Zou, Phys. Rev. D **95**(3), 034022 (2017). <https://doi.org/10.1103/PhysRevD.95.034022>. arXiv:1611.00913 [hep-ph]
15. Q. Wang, M. Cleven, F.K. Guo, C. Hanhart, U.G. Meißner, X.G. Wu, Q. Zhao, Phys. Rev. D **89**(3), 034001 (2014). <https://doi.org/10.1103/PhysRevD.89.034001>. arXiv:1309.4303 [hep-ph]
16. G. Li, F. Shao, C.W. Zhao, Q. Zhao, Phys. Rev. D **87**(3), 034020 (2013). arXiv:1212.3784 [hep-ph]
17. G. Li, W. Wang, Phys. Lett. B **733**, 100 (2014). arXiv:1402.6463 [hep-ph]
18. M. Albaladejo, F.K. Guo, C. Hanhart, U.G. Meißner, J. Nieves, A. Nogga, Z. Yang, Chin. Phys. C **41**(12), 121001 (2017). <https://doi.org/10.1088/1674-1137/41/12/121001>. arXiv:1709.09101 [hep-ph]
19. X.-H. Liu, G. Li, Phys. Rev. D **88**, 014013 (2013). arXiv:1306.1384 [hep-ph]

20. F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, Phys. Lett. B **725**, 127 (2013). [arXiv:1306.3096](#) [hep-ph]
21. M.B. Voloshin, Phys. Rev. D **87**(7), 074011 (2013). [arXiv:1301.5068](#) [hep-ph]
22. M.B. Voloshin, Phys. Rev. D **84**, 031502 (2011). [arXiv:1105.5829](#) [hep-ph]
23. D.-Y. Chen, X. Liu, Phys. Rev. D **84**, 094003 (2011). [arXiv:1106.3798](#) [hep-ph]
24. G. Li, X.-H. Liu, Phys. Rev. D **88**, 094008 (2013). [arXiv:1307.2622](#) [hep-ph]
25. D.-Y. Chen, X. Liu, T. Matsuki, Phys. Rev. D **84**, 074032 (2011). [arXiv:1108.4458](#) [hep-ph]
26. D.-Y. Chen, X. Liu, T. Matsuki. [arXiv:1208.2411](#) [hep-ph]
27. A.E. Bondar, A. Garmash, A.I. Milstein, R. Mizuk, M.B. Voloshin, Phys. Rev. D **84**, 054010 (2011). [arXiv:1105.4473](#) [hep-ph]
28. G. Li, Z. Zhou, Phys. Rev. D **91**(3), 034020 (2015). [arXiv:1502.02936](#) [hep-ph]
29. D.-Y. Chen, X. Liu, T. Matsuki, Phys. Rev. D **88**, 014034 (2013). [arXiv:1306.2080](#) [hep-ph]
30. Y.J. Shi, W. Wang, Phys. Rev. D **92**(7), 074038 (2015). [https://doi.org/10.1103/PhysRevD.92.074038](#). [arXiv:1507.07692](#) [hep-ph]
31. W. Wang, Q. Zhao, Phys. Lett. B **755**, 261 (2016). [https://doi.org/10.1016/j.physletb.2016.02.012](#). [arXiv:1512.03123](#) [hep-ph]
32. Q. Wu, G. Li, F. Shao, R. Wang, Phys. Rev. D **94**(1), 014015 (2016)
33. W. Wang, R.L. Zhu, Phys. Rev. D **96**(1), 014024 (2017). [https://doi.org/10.1103/PhysRevD.96.014024](#). [arXiv:1704.00179](#) [hep-ph]
34. H. Sundu, B. Barsbay, S.S. Agaev, K. Azizi, Eur. Phys. J. A **54**(7), 124 (2018). [https://doi.org/10.1140/epja/i2018-12552-0](#). [arXiv:1804.04525](#) [hep-ph]
35. V.M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. **117**(2), 022003 (2016). [https://doi.org/10.1103/PhysRevLett.117.022003](#). [arXiv:1602.07588](#) [hep-ex]
36. R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. **117**(15), 152003 (2016). [https://doi.org/10.1103/PhysRevLett.117.152003](#). [https://doi.org/10.1103/PhysRevLett.117.152003](#). [arXiv:1608.00435](#) [hep-ex] [**Addendum: (Phys. Rev. Lett. 118(10), 109904 (2017))**]
37. A.M. Sirunyan et al. [CMS Collaboration], Phys. Rev. Lett. **120**(20), 202005 (2018). [https://doi.org/10.1103/PhysRevLett.120.202005](#). [arXiv:1712.06144](#) [hep-ex]
38. T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. **120**(20), 202006 (2018). [https://doi.org/10.1103/PhysRevLett.120.202006](#). [arXiv:1712.09620](#) [hep-ex]
39. M. Aaboud et al. [ATLAS Collaboration], Phys. Rev. Lett. **120**(20), 202007 (2018). [https://doi.org/10.1103/PhysRevLett.120.202007](#). [arXiv:1802.01840](#) [hep-ex]
40. M. Gell-Mann, Phys. Lett. **8**, 214 (1964). [https://doi.org/10.1016/S0031-9163\(64\)92001-3](#)
41. S.L. Zhu, Int. J. Mod. Phys. E **17**, 283 (2008). [https://doi.org/10.1142/S0218301308009446](#). [arXiv:hep-ph/0703225](#)
42. M. Nielsen, F.S. Navarra, S.H. Lee, Phys. Rept. **497**, 41 (2010). [https://doi.org/10.1016/j.physrep.2010.07.005](#). [arXiv:0911.1958](#) [hep-ph]
43. N. Brambilla et al., Eur. Phys. J. C **71**, 1534 (2011). [https://doi.org/10.1140/epjc/s10052-010-1534-9](#). [arXiv:1010.5827](#) [hep-ph]
44. M. Nielsen, F.S. Navarra, Mod. Phys. Lett. A **29**, 1430005 (2014). [https://doi.org/10.1142/S0217732314300055](#). [arXiv:1401.2913](#) [hep-ph]
45. R.L. Jaffe, F. Wilczek, Phys. Rev. Lett. **91**, 232003 (2003). [https://doi.org/10.1103/PhysRevLett.91.232003](#). [arXiv:hep-ph/0307341](#)
46. R.L. Jaffe, Phys. Rept. **409**, 1 (2005). [https://doi.org/10.1016/j.physrep.2004.11.005](#). [arXiv:hep-ph/0409065](#)
47. M.J. Savage, M.B. Wise, Phys. Rev. D **39**, 3346 (1989). [https://doi.org/10.1103/PhysRevD.39.3346](#), [https://doi.org/10.1103/PhysRevD.40.3127](#) [**Erratum: Phys. Rev. D 40, 3127 (1989)**]
48. M. Gronau, O.F. Hernandez, D. London, J.L. Rosner, Phys. Rev. D **52**, 6356 (1995). [https://doi.org/10.1103/PhysRevD.52.6356](#). [arXiv:hep-ph/9504326](#)
49. X.G. He, Eur. Phys. J. C **9**, 443 (1999). [https://doi.org/10.1007/s100529900064](#). [arXiv:hep-ph/9810397](#)
50. X.G. He, Y.K. Hsiao, J.Q. Shi, Y.L. Wu, Y.F. Zhou, Phys. Rev. D **64**, 034002 (2001). [https://doi.org/10.1103/PhysRevD.64.034002](#). [arXiv:hep-ph/0011337](#)
51. C.W. Chiang, M. Gronau, J.L. Rosner, D.A. Suprun, Phys. Rev. D **70**, 034020 (2004). [https://doi.org/10.1103/PhysRevD.70.034020](#). [arXiv:hep-ph/0404073](#)
52. Y. Li, C.D. Lu, W. Wang, Phys. Rev. D **77**, 054001 (2008). [https://doi.org/10.1103/PhysRevD.77.054001](#). [arXiv:0711.0497](#) [hep-ph]
53. W. Wang, C.D. Lu, Phys. Rev. D **82**, 034016 (2010). [https://doi.org/10.1103/PhysRevD.82.034016](#). [arXiv:0910.0613](#) [hep-ph]
54. H.Y. Cheng, S. Oh, JHEP **1109**, 024 (2011). [https://doi.org/10.1007/JHEP09\(2011\)024](#). [arXiv:1104.4144](#) [hep-ph]
55. Y.K. Hsiao, C.F. Chang, X.G. He, Phys. Rev. D **93**(11), 114002 (2016). [https://doi.org/10.1103/PhysRevD.93.114002](#). [arXiv:1512.09223](#) [hep-ph]
56. C.D. Lu, W. Wang, F.S. Yu, Phys. Rev. D **93**(5), 056008 (2016). [https://doi.org/10.1103/PhysRevD.93.056008](#). [arXiv:1601.04241](#) [hep-ph]
57. X.G. He, W. Wang, R.L. Zhu, J. Phys. G **44**(1), 014003 (2017). [https://doi.org/10.1088/0954-3899/44/1/014003](#), [https://doi.org/10.1088/0022-3727/44/27/274003](#). [arXiv:1606.00097](#) [hep-ph]
58. W. Wang, F.S. Yu, Z.X. Zhao, Eur. Phys. J. C **77**(11), 781 (2017). [https://doi.org/10.1140/epjc/s10052-017-5360-1](#). [arXiv:1707.02834](#) [hep-ph]
59. W. Wang, Z.P. Xing, J. Xu, Eur. Phys. J. C **77**(11), 800 (2017). [https://doi.org/10.1140/epjc/s10052-017-5363-y](#). [arXiv:1707.06570](#) [hep-ph]
60. Y.J. Shi, W. Wang, Y. Xing, J. Xu, Eur. Phys. J. C **78**(1), 56 (2018). [https://doi.org/10.1140/epjc/s10052-018-5532-7](#). [arXiv:1712.03830](#) [hep-ph]
61. W. Wang, J. Xu, Phys. Rev. D **97**(9), 093007 (2018). [https://doi.org/10.1103/PhysRevD.97.093007](#). [arXiv:1803.01476](#) [hep-ph]
62. X.G. He, W. Wang, Chin. Phys. C **42**, 103108 (2018). [https://doi.org/10.1088/1674-1137/42/10/103108](#). [arXiv:1803.04227](#) [hep-ph]