Radiative dynamical mass of planar charged fermion in a constant homogeneous magnetic field

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Abstract The effective Lagrangian and mass operator are calculated for planar charged massive and massless fermions in a constant external homogeneous magnetic field in the one-loop approximation of the 2+1 dimensional quantum electrodynamics (QED₂₊₁). We obtain the renormalizable effective Lagrangian and the fermion mass operator for a charged fermion of mass m and then calculate these quantities for the massless case. The radiative corrections to the mass of charged massless fermion when it occupies the lowest Landau level are found for the cases of the pure QED₂₊₁ as well as the so-called reduced QED₃₊₁ on a 2-brane. The fermion masses were found can be generated dynamically in an external magnetic field in the pure QED₂₊₁ if the charged fermion has small bare mass m_0 and in the reduced QED₃₊₁ on a 2-brane even at $m_0 = 0$. The dynamical mass seems to be likely to be revealed in monolayer graphene in the presence of constant homogeneous magnetic field (normal to the graphene sample).

1 Introduction

Quantum systems of planar charged fermions in external electromagnetic fields are interesting in view of possible applications of the corresponding field-theory models to a number of condensed-matter quantum effects such as, for example, the quantum Hall effect [1] and high-temperature superconductivity [2] as well as in connection with problems of graphene (see, [3–6]). In graphene, the electron dynamics near the Fermi surface can be described by the Dirac equation in 2+1 dimensions for a zero-mass charged fermion [4] though the case of massive charged fermions is also of interest [7]. The field-theory models applied for study are the pure 2+1 dimensional quantum electrodynamics (QED₂₊₁) as well as the so-called reduced QED₃₊₁ on a 2-brane. In the latter model, fermions are confined to a plane, neverthe-

less the electromagnetic interaction between them is three-dimensional [8,9].

The radiative one-loop shift of an electron energy in the ground state in a constant homogeneous magnetic field in QED₂₊₁ was calculated in [10] and the one-loop electron self-energy in the topologically massive QED₂₊₁ at finite temperature and density was obtained in [11]. The effective Lagrangian, the electron mass operator and the density of vacuum electrons (induced by the background field) in an external constant homogeneous magnetic field were derived in the one-loop QED₂₊₁ approximation in [12].

Since the effective fine structure constant in graphene is large, the QED₂₊₁ effects can be significant already in the one-loop approximation. The polarization operator in graphene in a strong constant homogeneous magnetic field perpendicular to the graphene membrane has been obtained in the one-loop approximation of the QED₂₊₁ in [9,13–15]. The effective potential and vacuum current in graphene in a superposition of a constant homogeneous magnetic field and an Aharonov–Bohm vortex was studied in [16]. We also note that the induced vacuum current in the field of a solenoid perpendicular to the graphene sample was investigated in [17], and the vacuum polarization of massive and massless fermions in an Aharonov–Bohm vortex in the one-loop approximation of the QED₂₊₁ was studied in [18]. The one-loop self-energy of a Dirac electron of mass m in a thin medium simulating graphene in the presence of external magnetic field was investigated in the reduced QED₃₊₁ on a 2-brane in [19], in which it was shown that the radiative mass correction in the lowest Landau level does not vanish at the limit $m \rightarrow 0$.

In this work, we calculate the effective Lagrangian and the mass operator of planar charged fermions in the presence of an external constant homogeneous magnetic field in the one-loop approximation of the QED₂₊₁. We also calculate the radiative corrections to the mass of charged fermion when it occupies the lowest Landau level for the cases of the

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pure QED₂₊₁ as well as the so-called reduced QED₃₊₁ on a 2-brane. The fermion masses were found can be generated dynamically in an external magnetic field in the pure QED₂₊₁ if the charged fermion has small bare mass m_0 and in the reduced QED₃₊₁ on a 2-brane even at $m_0 = 0$. The dynamical mass seems to be likely to be revealed in monolayer graphene in the presence of constant homogeneous magnetic field (normal to the graphene sample) though we also see that an one-loop result is not accurate certainly when the coupling constant is large (see, [19]).

We shall adopt the units where $c = \hbar = 1$.

2 Eigenfunctions, the Green's function for the Dirac equation in a constant magnetic field in 2+1 dimensions. Effective Lagrangian

As far as the charged fermions are supposed to move in a plane the exact solutions and the Green's function for the Dirac equation in a constant homogeneous magnetic field can be found in 2+1 dimensions. The Dirac equation for a fermion in an external electromagnetic field in 2+1 dimensions is written just as in 3+1 dimensions nevertheless the Dirac γ^μ -matrix algebra in 2+1 dimensions is known to be represented in terms of the two-dimensional Pauli matrices σ_j

$$\gamma^0 = \sigma_3, \quad \gamma^1 = i\zeta\sigma_1, \quad \gamma^2 = i\sigma_2, \quad (1)$$

where the parameter $\zeta = \pm 1$ can label two types of fermions in accordance with the signature of the two-dimensional Dirac matrices [20]; it can be applied to characterize two states of the fermion spin (spin "up" and "down") [21].

Then, the Dirac equation for a fermion of mass m and charge $e = -e_0 < 0$ "minimally" interacting with the background electromagnetic field is written in the covariant form as

$$(\gamma_\mu P^\mu - m)\Psi(x) = 0, \quad (2)$$

where $P^\mu = p^\mu - eA^\mu \equiv (P_0, P_1, P_2)$ is the generalized fermion momentum operator (a three-vector) and $p^\mu = (i\partial_t, -i\partial_x, -i\partial_y) \equiv (p_0, p_1, p_2)$. We take the vector potential in the Cartesian coordinates in the Landau gauge $A_0 = 0$, $A_1 = 0$, $A_2 = Bx$, then the magnetic field is defined as $B = \partial_1 A_2 - \partial_2 A_1 \equiv F_{21}$, where $F_{\mu\nu}$ is the electromagnetic field tensor.

The squared Dirac operator is

$$(\gamma P)^2 = P_0^2 - P_1^2 - P_2^2 - e\sigma_3 B. \quad (3)$$

The matrix function E_p , introduced by Ritus [22] for the case of 3+1 dimensions, satisfies the equation

$$(\gamma P)^2 E_p = p^2 E_p, \quad (4)$$

where the eigenvalue p^2 can be any real number and in the magnetic field considered E_p also is an eigenfunction of the operators

$$\begin{aligned} i\partial_t E_p &= p_0 E_p, \quad -i\partial_y E_p = p_2 E_p, \\ (P_1^2 + P_2^2 - e\sigma_3 B) E_p &= 2|eB|k E_p, \quad k = 0, 1, \dots \end{aligned} \quad (5)$$

It is obvious that p_0 , p_2 and $2|eB|n$ label the solutions of Dirac Eq. (2) as well as the E_p eigenfunctions of operators (4) and (5) can also be classified by the eigenvalues $\zeta = \pm 1$ of σ_3 . The eigenfunctions $E_{p\zeta}(t, \mathbf{r})$ are given by

$$E_{p\zeta}(t, \mathbf{r}) = \frac{1}{2^{3/2}\pi} e^{-ip_0 t + ip_2 y} U_n(X) w_\zeta, \quad (6)$$

where the normalized functions $U_n(X)$ are expressed through the Hermite polynomials $H_n(X)$ as

$$\begin{aligned} U_n(X) &= \frac{|eB|^{1/4}}{(2^n n! \pi^{1/2})^{1/2}} e^{-X^2/2} H_n(X), \\ X &= \sqrt{|eB|}(x - p_2/eB), \end{aligned} \quad (7)$$

$n = k + \text{sign}(eB)\zeta/2 - 1/2$, $n = 0, 1, \dots$, and w_ζ are eigenvectors of σ_3 .

The eigenfunctions of the Dirac equation Hamiltonian

$$H_D = \sigma_1 P_2 - \sigma_2 P_1 + \sigma_3 m \quad (8)$$

are

$$\begin{aligned} \Psi(t, \mathbf{r}) &= \frac{1}{\sqrt{2E_n}} \begin{pmatrix} \sqrt{E_n + \zeta m} U_n(X) \\ -\text{sign}(eB) \sqrt{E_n - \zeta m} U_{n-1}(X) \end{pmatrix} \\ &\times \exp(-iE_n t + ip_2 y), \end{aligned} \quad (9)$$

where

$$E_n = \sqrt{m^2 + 2n|eB|} \quad (10)$$

is the energy eigenvalues (the Landau levels). All the energy levels except the lowest level ($n = 0$) with $\zeta = 1$ for $eB > 0$ and $\zeta = -1$ for $eB < 0$ are doubly degenerate on spin $\zeta = \pm 1$. This means that the eigenvalues of the fermion energy except the lowest level are actually spin-independent in the configuration under investigation. For definiteness, we consider the case where $eB < 0$. It should be also emphasized that $\Psi(t, \mathbf{r})$ are not eigenfunctions of σ_3 .

The electron propagator in an external constant homogeneous magnetic field in 3+1 dimensions was found for the first time by Schwinger [23]. In the 2+1 dimensions it was obtained in [24] in the momentum representation (see, also, [15]) and in [12] in the coordinate (proper-time) representation in the form

$$\begin{aligned}
& S^c(t, t', \mathbf{r}, \mathbf{r}', B) \\
&= -\frac{i^{1/2}}{8\pi^{3/2}} \int_0^\infty \frac{dz}{s^{1/2} \sin z} \exp \left[-i(m^2 - i\epsilon) - i \frac{(t - t')^2}{4s} \right. \\
&\quad \left. + iz \frac{(x - x')^2 + (y - y')^2}{4s \tan z} + ieB \frac{(x - x')(y - y')}{2} \right] \\
&\quad \times \left[\left(\frac{\gamma^0(t - t')}{2s} + m \right) \exp(i\zeta \sigma_3 z) \right. \\
&\quad \left. - z \frac{(\gamma^1(x - x') + \gamma^2(y - y'))}{2s \sin z} \right], \quad (11)
\end{aligned}$$

where $z = |eB|s$ and s is the “proper time”.

The effective action $V(B) = \int d^3x L_{eff}(B)$, where $L_{eff}(B)$ is the effective Lagrangian and $x^\mu = x^0 = t, x^1 = x, x^2 = y$, is determined the causal Green's function S^c as follows

$$V(B) = i \text{Tr} \log S^c = i \int d^3x \text{tr}(x | \log S^c | x), \quad (12)$$

and from Eq. (11) we derive

$$L_{eff}(B) = \frac{(-i)^{1/2}}{8\pi^{3/2}} \int_0^\infty \frac{ds}{s^{3/2}} e^{-im^2s} \left(|eB| \cot |eB|s - \frac{1}{s} \right) \quad (13)$$

Rotating the integration contour by $-\pi/2$, we have

$$L_{eff}(B) = -\frac{|eB|^{3/2}}{8\pi^{3/2}} \int_0^\infty \frac{dz}{z^{3/2}} e^{-m^2z/|eB|} \left(\coth z - \frac{1}{z} \right) \quad (14)$$

and integrating (14), one obtains (see, [12])

$$L_{eff}(B) = -\frac{(eB)^2}{24m\pi} \left(1 - \frac{(eB)^2}{20m^4} \right), \quad |eB| \ll m^2 \quad (15)$$

and

$$L_{eff}(B) \approx \frac{(eB)^2}{12m\pi^{3/2}}, \quad |eB| \gg m^2. \quad (16)$$

In Eq. (15) the first term can be interpreted as the magnetic energy of the vacuum [25]. In the QED₃₊₁ the nonrenormalized effective Lagrangian involves a similar B^2 term with a logarithmic factor whose argument depends on the cutoff parameter. This term is then combined with the B^2 term in the initial Lagrangian $L_0(B^2)$, which implies fermion-charge and external magnetic-field renormalizations.

For massless case it is convenient to write the one-loop correction L_1 to the Lagrangian density $L_0 = -B^2/4\pi$ for the magnetic field as

$$L_1 = \frac{|eB|}{2\pi} \sum_{n=1}^\infty |E'_n|, \quad (17)$$

where the factor $|eB|/2\pi$ takes into account quantum degeneracy of Landau levels per unit surface area, $E'_n = \sqrt{2|eB|n}$

and the zero modes are to be omitted (see, [26,27]). A massless fermion does not have a spin degree of freedom in 2+1 dimensions [28] but the Dirac equation for charged massless fermions in an external magnetic field in 2+1 dimensions keeps the spin parameter. Therefore, all the energy levels except the lowest level $n = 0$ are also doubly degenerate. This sum is divergent. Using for $|E'_n|$ the Fock-Schwinger proper-time representation in the form [29]

$$\sqrt{2|eB|n} = -(\pi)^{-1/2} \int_0^\infty \frac{ds}{s^{1/2}} \frac{d}{ds} e^{-2|eB|ns} \quad (18)$$

and applying for (17) the zeta-function regularization [26,27,30], after some calculations one can bring Eq. (17) formally to the form

$$\begin{aligned}
L_1 &= -\frac{(eB)^{3/2}}{2\sqrt{2}\pi^{3/2}} \int_0^\infty \frac{x^{-3/2}}{e^x - 1} dx \\
&\equiv -\frac{(eB)^{3/2}}{2\sqrt{2}\pi^{3/2}} \Gamma(-1/2) \zeta(-1/2), \quad (19)
\end{aligned}$$

where $\Gamma(z)$ and $\zeta(z)$ are respectively the gamma and ζ -functions. Then using the Riemann identity [31]

$$\zeta(z) = \Gamma(1-z) 2^z \pi^{z-1} \cos[\pi(1-z)/2] \zeta(1-z), \quad (20)$$

we obtain the renormalizable effective Lagrangian for charged massless fermions in the final form :

$$L_1 = -\frac{(eB)^{3/2}}{4\sqrt{2}\pi^2} \zeta(3/2). \quad (21)$$

This formula coincides with the corresponding result obtained in [26,27].

3 Mass operator of a charged fermion in a constant homogeneous magnetic field

In the coordinate representation, the mass operator of a charged massive fermion in the considered external field in 2+1 dimensions is given by

$$\begin{aligned}
M(x, x') &= ie^2 \gamma^\mu S^c(x, x') \gamma^\nu S_{\mu\nu}(x - x') \\
&\quad - ie^2 \gamma^\mu S_{\mu\nu}(x - x') \text{tr}(\gamma^\nu S^c(x, x')), \quad (22)
\end{aligned}$$

where $S_{\mu\nu}(x - x')$ is the photon propagation function. We note that in the QED₃₊₁, the second term in the right-hand side of Eq. (22) is absent in the constant homogeneous electromagnetic field because the induced vacuum electric current vanishes. In the pure QED₂₊₁, the Chern–Simons field dynamically generated in the effective Heisenberg–Euler Lagrangian by the background magnetic field contributes in this second term in the model under consideration. The second term in the mass operator of a charged fermion in a constant homogeneous magnetic field $M(x, x')$ violates the spatial and temporal invariance and therefore cannot be observed, even in principle. We shall not consider this term.

The effect due to the Chern–Simons field can possibly be observed in a condensed-matter model with a finite density of fermions as the density of charged fermions induced by the background magnetic field.

We shall use the usual photon propagator in 2+1 dimensions (in the Feynman gauge)

$$\frac{1}{k^2 - i\epsilon} = i \int_0^\infty du e^{-i(k^2 - i\epsilon)u}, \quad (23)$$

as well as the “effective” internal photon propagator for the reduced QED₃₊₁ on a 2-brane in which the photon is allowed to also propagate in the “bulk” [8]

$$\frac{1}{\sqrt{k^2 - i\epsilon}} = \sqrt{\frac{i}{\pi}} \int_0^\infty du \frac{e^{-i(k^2 - i\epsilon)u}}{\sqrt{u}}, \quad (24)$$

In the coordinate (proper-time) representation the photon propagator in 2+1 dimensions reads as

$$S_{\mu\nu}(t - t', \mathbf{r} - \mathbf{r}') = g_{\mu\nu} \frac{i^{1/2}}{8\pi^{3/2}} \int_0^\infty \frac{du}{u^{3/2}} \times \exp \left[-\epsilon u + i \frac{(t - t')^2 - (x - x')^2 - (y - y')^2}{4u} \right]. \quad (25)$$

The internal photon propagator for the reduced QED₃₊₁ on a 2-brane is given by Eq. (25) in which $i^{1/2}/8\pi^{3/2}$ and $u^{-3/2}$ are replaced by $i/8\pi^{1/2}$ and u^{-2} , respectively.

We determine the mass operator in the E_p representation as

$$M(p, p', B) = \int d^3z \int d^3z' \bar{E}_p(z) M(z, z', B) E'_p(z'), \quad (26)$$

where $\bar{E}_p = \gamma^0 E_p^\dagger \gamma^0$ and $E_p^\dagger$ is the Hermitian conjugate matrix function. The mass operator is diagonal in the E_p representation

$$M(p, p', B) = \delta^3(p - p') M(p, B). \quad (27)$$

We now calculate the mass operator in the special reference frame with using the fermion propagator (11), the photon propagator (25) and E_p -matrix function (6). It is convenient to introduce variables $t_- = t - t'$, $x_- = x - x'$, $y_- = y - y'$, $t_+ = (t + t')/2$, $x_+ = (x + x')/2$, $y_+ = (y + y')/2$. It is seen that the t_+ and y_+ integrations give $2\pi\delta(p_0 - p'_0)$ and $2\pi\delta(p_2 - p'_2)$. We then use the arising δ -functions, integrate with respect to t_- and obtain in the terms independent of t_- the factor

$$2\sqrt{\frac{\pi su}{i(s+u)}} \exp \left(\frac{ip_0^2 su}{s+u} \right).$$

The first-order terms in t_- then contain an extra factor $2p_0 su/(s+u)$.

We integrate over x_+ with applying formula

$$\int_{-\infty}^\infty \sqrt{eB} \exp(-iy_- x) U_n(\eta) U_k(\eta') dx = \exp[i(n-k)v] I_{nk}(z), \quad (28)$$

where $U_l(\eta)$ is given by Eq. (7), $\eta = \sqrt{eB}(x + x_-)$, $\eta' = \sqrt{eB}(x - x_-)$, $v = \arctan(y_-/x_-)$, and $I_{nk}(z)$ is the Laguerre function in $z = |eB|(x_-^2 + y_-^2)/2$, related to the Laguerre polynomial

$$L_s^l(z) = e^z z^{-l} \frac{d^s}{dz^s} (z^{s+l} e^{-z})$$

as

$$I_{nk}(z) = \frac{1}{\sqrt{n!k!}} e^{-z/2} z^{(n-k)/2} L_k^{n-k}(z). \quad (29)$$

The integration over x_- and y_- is performed in polar coordinates with using the formulas

$$\int_0^{2\pi} d\phi e^{i(n-k)\phi} = 2\pi \delta_{nk}, \quad \phi = \arctan(y_-/x_-),$$

$$\int_0^\infty dz e^{-az} L_n^0(z) = \frac{(a-1)^n}{a^{n+1}}, \quad \text{Re}(a) > 0. \quad (30)$$

After these integrations the mass operator (27) at $su/(s+u) \rightarrow 0$ has the ultraviolet divergency of the vacuum origin and needs in the renormalization. The renormalized mass operator in an external electromagnetic field can be represented in the form

$$M_r(p, B) = M(p, B) - M^0 + M^0 - (M^0)_{\gamma p=m} - \left(\frac{\partial M^0}{\partial \gamma p} \right)_{\gamma p=m} (\gamma p - m) \equiv M_c(p, B) + M_r^0, \quad (31)$$

where $M^0 = M(p, B=0)$, the expression $M_c(p, B) = M(p, B) - M^0$ does not contain the divergences and M_r^0 is the regularized mass operator of fermion in the vacuum, which vanishes on the mass shell.

As a result of this renormalization, we obtain the renormalized mass operator with the photon propagator (25) in the form

$$M_c(p, B) = -e^2 \frac{i^{3/2}}{8\pi^{3/2}} \int_0^\infty ds \int_0^\infty du \left[\frac{|eB|}{\Delta(s+u)^{1/2} \sin(|eB|s)} \exp[-ism^2 + ip_0^2 su/(s+u) - 2in\phi] \times [(3MI - \gamma^0 P_0) e^{-i\phi} - \gamma^2 P_2] - \frac{\exp[-ism^2 + ip_0^2 su/(s+u)]}{(s+u)^{3/2}} \left[3mI - \sigma_3 \frac{p_0 u}{s+u} \right] \right], \quad (32)$$

where $\Delta = \sqrt{(eBu)^2 + (1 + eBu \cot(eBs))^2}$, $\phi = \arctan [eBu/(1 + eBu \cot(eBs))]$, I is the unit two-column matrix, and

$$\begin{aligned} Mi &= \frac{p_0 u}{s + u} \sin(|eB|s) + m \cos(|eB|s), \\ P_0 &= \frac{p_0 u}{s + u} \cos(|eB|s) + im \sin(|eB|s), \\ P_2 &= \sqrt{2|eB|n} \frac{eBu}{\Delta \sin(|eB|s)}. \end{aligned} \quad (33)$$

Putting $s' = |eB|s$, $u' = |eB|u$, we obtain

$$\begin{aligned} M_c(p, B) &= -e^2 \frac{i^{3/2}}{8\pi^{3/2} \sqrt{|eB|}} \int_0^\infty ds' \\ &\int_0^\infty du' e^{[-is'm^2/|eB| + ip_0^2 s' u' / |eB|(s' + u')]} \\ &\times \left[((3MI - \sigma_3 P_0) \exp[-i(2n + 1)\phi] \right. \\ &\quad \left. - i\sigma_2 \frac{\sqrt{2|eB|} nu' \exp[-i2n\phi]}{\Delta'} \right) \frac{1}{\Delta' \sqrt{s' + u'}} - \\ &\quad \left. - \left[3mI - \sigma_3 \frac{p_0 u'}{s' + u'} \right] \frac{1}{(s' + u')^{3/2}} \right], \end{aligned} \quad (34)$$

where $\Delta' = \sqrt{u'^2 + 2u' \sin s' \cos s' + \sin^2 s'}$, $\phi = \arctan [u'/(1 + u' \cot(s'))]$.

It is worth while noting that the fermion propagator (i.e. the fermion Green's function) in a constant homogeneous magnetic field in 2+1 dimensions (11) is derived with taking account of all Landau levels.

Now, let us perform a known change of variables (see [32], p. 44, Eq. (3.12) and, also, [19]) $s' = sv$, $u' = s(1-v)$, and introduce a variable $z = isv^2$, which, in fact, is equivalent to the turning of integration path over s on $\pi/2$ (a Wick rotation). We shall calculate the mass correction for the (on mass-shell) level $p_0 = m$, $n = 0$. As a result of all these transformations, the mass operator can be written as

$$\begin{aligned} M_c(B) &= \frac{e^2 m}{8\pi^{3/2}} \int_0^\infty dz \sqrt{z} e^{-(z/l)} \int_0^1 \frac{dv}{v} \\ &\times \left[\begin{pmatrix} 4 - 2v + 2v \exp[-2z/v] & 0 \\ 0 & 8 - 4v + 2v \exp[-2z/v] \end{pmatrix} \right. \\ &\quad \left. \times \frac{1}{2z(1-v) + v^2(1 - \exp[-2z/v])} - \begin{pmatrix} 2 + v & 0 \\ 0 & 4 - v \end{pmatrix} \frac{1}{z} \right], \end{aligned} \quad (35)$$

where $l = |eB|/m^2$.

In order to determine the mass correction of a charged fermion δm in the lowest Landau level $n = 0$, $\zeta = -1$, we must calculate the matrix element of the self-energy operator (35) between two states with $n = 0$, $\zeta = -1$, defined by Eq. (9). After simple calculations, we obtain

$$\begin{aligned} \delta m &= \frac{e^2 m}{8\pi^{3/2}} \int_0^\infty dz \sqrt{z} e^{-(z/l)} \int_0^1 \frac{dv}{v} \\ &\left[\frac{8 - 4v + 2v \exp[-2z/v]}{2z(1-v) + v^2(1 - \exp[-2z/v])} - \frac{4-v}{z} \right]. \end{aligned} \quad (36)$$

The mass correction of a charged fermion δm_r in the level $n = 0$, $\zeta = -1$ for the reduced QED₃₊₁ on a 2-brane is given by

$$\begin{aligned} \delta m_r &= \frac{e^2 m}{8\pi} \int_0^\infty dz e^{-(z/l)} \int_0^1 \frac{dv}{\sqrt{1-v}} \\ &\left[\frac{8 - 4v + 2v \exp[-2z/v]}{2z(1-v) + v^2(1 - \exp[-2z/v])} - \frac{4-v}{z} \right]. \end{aligned} \quad (37)$$

It is well to note that Eqs. (35), (36) and (37) are taken into account the contributions of the virtual electrons occupying all Landau levels, but also when obtaining Eq. (35) from Eq. (34) we took account of that the needed diagonal matrix element of operator (34) must be calculated for the lowest state with $n = 0$, $\zeta = -1$.

It is useful the integration over z in the limits $[0, \infty)$ to divide into two regions $[0, z_0]$ plus $[z_0, \infty)$ [33] (see, also, [19]) with $z_0 \sim 1$. Then, when integrating into region $[z_0, \infty)$ the exponentials can be neglected and integral between the limits 0 and z_0 is small compared with that in the limits $[z_0, \infty)$ and it can be neglected. Having made the integrations over v we obtained

$$\delta m = \frac{e^2 m}{8\pi^{3/2}} \int_{z_0}^\infty dz e^{-(z/l)} \left(\frac{4}{\sqrt{z}} + \frac{2 \ln(2z)}{\sqrt{z}} \right) \quad (38)$$

and

$$\delta m_r = \frac{e^2 m}{8\pi} \int_{z_0}^\infty dz e^{-(z/l)} \left(\frac{2\sqrt{2}\pi}{\sqrt{z}} - \frac{20}{3z} \right). \quad (39)$$

It should be emphasized that though the first term in integrand of (39) has the form $2\sqrt{2}\pi/\sqrt{z}$ at $z \gg 1$ the exact calculation of corresponding integral over v at $z = 1$ gives $2\sqrt{2}\pi$.

Finally, integrating over z we find the mass corrections in the form

$$\begin{aligned} \delta m &= \frac{e^2}{4\pi} \sqrt{|eB|} \left[\left(\ln \left(\frac{|eB|}{m^2} \right) + 2 + \ln 2 \right) \operatorname{erfc} \left(\frac{1}{\sqrt{l}} \right) \right. \\ &\quad \left. - 2 \ln 2 - \mathcal{C} \right] \end{aligned} \quad (40)$$

and

$$\delta m_r = \frac{e^2}{4\pi} \sqrt{|eB|} \left[\sqrt{2}\pi^{3/2} \operatorname{erfc} \left(\frac{1}{\sqrt{l}} \right) - \frac{10}{3\sqrt{l}} \Gamma \left(0, \frac{1}{l} \right) \right], \quad (41)$$

where (see, for example, [35])

$$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2} dt, \quad \Gamma(a, z) = \int_z^\infty e^{-t} t^{a-1} dt \quad (42)$$

and we put $z_0 = 1$. In the limit $l \rightarrow \infty$

$$\operatorname{erfc}\left(\frac{1}{\sqrt{l}}\right) \approx 1 - \frac{2}{\sqrt{\pi l}}, \quad \Gamma\left(0, \frac{1}{l}\right) \approx \ln l - \mathcal{C}. \quad (43)$$

Here $\mathcal{C} = 0.57721$ is the Euler constant [36].

At the limit $m \rightarrow 0$, we obtain

$$\delta m = \frac{e^2}{4\pi} \sqrt{|eB|} \left[\ln\left(\frac{|eB|}{m_0^2}\right) + 2 - \ln 2 - \mathcal{C} \right], \quad m_0 \rightarrow 0 \quad (44)$$

and

$$\delta m_r = \frac{e^2}{2\sqrt{2}} \sqrt{\pi |eB|}. \quad (45)$$

Equation (45) coincides with Eq. (63) obtained in [19].

In order to estimate the magnitude of the magnetic field strength which can be of interest for application to graphene, we recover the units c and $\hbar$ and rewrite Eq. (45) as follows

$$\delta m_r c^2 = \frac{\alpha \sqrt{\pi}}{2\sqrt{2}} \sqrt{|eB| \hbar c}, \quad \alpha = \frac{e^2}{\hbar c}. \quad (46)$$

The numerical estimation is obtained in the form

$$\delta m_r c^2 \approx 0.5 \cdot 10^{-4} \alpha \sqrt{|B|} eV. \quad (47)$$

If the constant α is considered as “effective fine structure constant” for graphene then $\alpha \sim 1$, and as it follows from the Eq. (47) the mass correction $\delta m_r c^2 \approx 1eV$ in the magnetic field of the strength $B = 4T (4 \cdot 10^4 G)$. Of course, there is a question about the validity of one-loop result when the corresponding constant of expansion can be of order 1 but the study of higher orders lie beyond the scope of given work.

It should be noted that Eqs. (44) and (45) describe the main terms of radiative mass corrections in massive QED_{2+1} and reduced QED_{3+1} on a 2-brane, respectively, in the limits $|eB| \gg m^2$. We see these corrections differ, in principle, from the mass correction in standard massive QED_{3+1} in the limits $|eB| \gg m^2$ (see, [33, 37]), in which the leading term is determined by

$$\delta m_{3+1} = \frac{e^2}{4\pi} m \ln^2 \left(\frac{|eB|}{m^2} \right). \quad (48)$$

Equations (44) and (45) show that, in the presence of external constant homogeneous magnetic field, the fermion mass can be generated dynamically in QED_{2+1} at small bare mass m_0 and in reduced QED_{3+1} on a 2-brane at $m_0 = 0$. Thus, the latter model, in a constant homogeneous magnetic field, cannot stay massless in the one-loop approximation [19].

4 Resume

In this work we calculated the effective Lagrangian and the mass operator of planar charged fermions in an external constant homogeneous magnetic field in the one-loop approximation of the QED_{2+1} . We also calculated the mass corrections of charged fermion in the lowest Landau level. The fermion mass can be generated dynamically in a constant homogeneous magnetic field in the pure QED_{2+1} if the charged fermion has small bare mass m_0 and in the reduced QED_{3+1} on a 2-brane even at $m_0 = 0$. We believe the mass generation to be due to renormalization and to closely resemble to the dimensional transmutation phenomenon which occurs in massless relativistic field theories [38]. In our case the cutoff parameter, in fact, depending upon eB transmutes in an arbitrary mass. The dynamical mass seems to be likely to be revealed in monolayer graphene in the presence of constant homogeneous magnetic field (normal to the graphene sample).

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This is a theoretical study and no experimental data has been listed.]

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