Holography of electrically and magnetically charged black branes

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Abstract We construct a new class of black brane solutions in Einstein-Maxwell-dilaton (EMD) theory, which is characterized by two parameters a, b . Based on the obtained solutions, we make detailed analysis on the ground state in zero temperature limit and find that for many cases it exhibits the behavior of vanishing entropy density. We also find that the linear-T resistivity can be realized in a large region of temperature for the specific case of $a^2 = 1/3$, which is the Gubser-Rocha model dual to a ground state with vanishing entropy density. Moreover, for $a = 1$ we analytically construct the black brane which is magnetically charged by virtue of the electric-magnetic (EM) duality. Various transport coefficients are calculated and their temperature dependence are obtained in the high temperature region.

1 Introduction

AdS/CFT correspondence provides a new direction for the study of strongly correlated systems [1–5]. In particular, great progress has been made in modelling and understanding the anomalous scaling behavior of the strange metal phase (see [6] and references therein). Among of them the linear-T resistivity and quadratic-T inverse Hall angle are two prominent properties of the strange metal, which have been widely observed in normal states of high temperature superconductors as well as heavy fermion compounds near a quantum critical point, which is universal in a very wide range of temperature. By holography, the linear-T resistivity has firstly been explored in [7, 8]. Then different scalings between Hall angle and resistivity have also been investigated in holographic framework [6, 9–18]. In particular, both the linear-T resistiv-

ity and quadratic-T inverse Hall angle can be simultaneously reproduced in some special holographic models [19–23].

Currently it is still challenging to achieve the anomalous scales of strange metal over a wide range of temperature in holographic approach. It may be limited by the renormalization group flow which is controlled by the specific bulk geometry subject to Einstein field equations, and the scaling behavior of the near horizon geometry of the background. Therefore, in this direction one usually has two ways to improve the understanding of the transport behavior of the dual system. One way is to consider more general backgrounds within the framework of Einstein's gravity theory. The other way is to introduce additional scaling anomaly which may be characterized by Lifshitz dynamical exponent and hyperscaling violating parameter. In the latter case, the construction of asymptotic hyperscaling-violating and Lifshitz solutions have largely improved the scaling analysis of the exotic behavior in the strange metal [24–33]. In this paper, we will focus on the former case, namely the holographic construction of new backgrounds within the framework of Einstein theory, without the involvement of scale anomaly. In this way the Einstein-Maxwell-Dilaton (EMD) theory provides a nice arena for the study of electric and magnetic transport phenomena in a strongly-coupled system. Previously a particular model constructed in EMD theory is the Gubser-Rocha solution which describes an electrically charged black brane [34]. It is featured by a vanishing entropy density at zero temperature.¹ This model exhibits lots of peculiar properties similar to those of the strange metal, including the linear specific heat [34] and the linear resistivity at low temperature [36]. Also, as a typical model for holographic stud-

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¹ Another important holographic model with vanishing entropy density ground state is presented in [35], in which the black brane is numerically constructed and the near horizon geometry at zero temperature possesses Lifshitz symmetry.

ies, the Gubser-Rocha solution has been extended in various circumstances, see e.g. [37–44].

In this paper, we intend to construct new backgrounds which are applicable for the study of both electric and magnetic transport properties in holographic approach, aiming to provide more comprehensive understanding on the anomalous behavior of strange metals. We first analytically construct a new class of black brane solutions which are electrically charged in Einstein-Maxwell-dilaton (EMD) theory in Sect. 2. In particular, we study the transport behavior in the dual system and find that the linear-T resistivity holds in a large range of temperature for $a^2 = 1/3$. Then by virtue of the electric-magnetic (EM) duality for $a^2 = 1$, we construct a dyonic black brane solution in Sect. 4 and Appendix A. Various transport coefficients are derived, including the resistivity, Hall angle, magnetic resistance and Nernst coefficient. It is expected to provide a useful platform for the study of both electric and magnetic transport behavior in the holographic framework.

2 Electrically charged dilatonic black branes

2.1 Electrically charged dilatonic black branes

We consider Einstein-Maxwell-Dilaton-Axion (EMDA) theory in four dimensional spacetimes with the following action

$$S_{EMD} = \int d^4x \sqrt{-g} \left(R - \frac{Z(\phi)}{4} F_{ab} F^{ab} - \frac{1}{2} (\partial\phi)^2 + V(\phi) - \sum_{I=x,y} (\partial\psi_I)^2 \right). \quad (1)$$

The axionic fields ψ_x, ψ_y are added to break the translation invariant, which is responsible for the finite DC conductivity over a charged black hole background.

The black brane solutions of EMDA theory and their holographic properties have been widely studied in [11, 15, 34, 35, 45–54]. Analytical background can provide a more controllable pattern in studying the holographic characteristics. To obtain an analytical black brane solution, it is crucial to choose an appropriate potential $V(\phi)$. For some specific form of $V(\phi)$, the analytical AdS black brane solutions have been worked out in [15, 34, 52–54]. In this paper, we propose a more general form for the potential and obtain a class of general AdS black brane solutions. The potential $V(\phi)$ and the gauge coupling $Z(\phi)$ we choose here are

$$Z(\phi) = e^{a\phi}, \quad V = V_0(1 + \Phi) + V_1, \quad (2a)$$

$$V_0 = \frac{6(a^2 e^{\frac{\phi}{2a}} + e^{-\frac{a\phi}{2}})^2 - 2a^2 (e^{\frac{\phi}{2a}} - e^{-\frac{a\phi}{2}})^2}{(1 + a^2)^2}, \quad (2b)$$

$$V_1 = \frac{2b}{(1 + a^2)^2} (e^{\frac{1+a^2}{2a}\phi} - 1)^3 (a^2 e^{-\frac{1}{a}\phi} + e^{-\frac{a^2+3}{2a}\phi}), \quad (2c)$$

$$\Phi = -b \left(\frac{e^{\frac{3a^2-1}{2a}\phi} - 1}{3a^2 - 1} + \frac{e^{\frac{a^2-3}{2a}\phi} - 1}{a^2 - 3} - \frac{e^{\frac{a^2-1}{a}\phi} - 1}{a^2 - 1} \right), \quad (2d)$$

where a and b are two free parameters in this EMDA theory. And then, we take the following ansatz

$$ds^2 = \frac{1}{z^2} \left(-f(z) dt^2 + \frac{dz^2}{f(z)} + (1 + \Lambda z)^{\frac{2\beta^2}{1+\beta^2}} (dx^2 + dy^2) \right), \quad (3a)$$

$$A = A_t(z), \quad \phi = \phi(z), \quad \psi_x = kx, \quad \psi_y = ky, \quad (3b)$$

Under the above setting, the EMDA theory (1), with the potential (2), has two branches of the asymptotic AdS charged black brane solutions for $\beta = 1/a$ and $\beta = a$, which are

$$\text{Case 1: } \beta = 1/a, \quad (4a)$$

$$f(z) = (1 + \Lambda z)^{\frac{2}{1+a^2}} h(z), \quad (4b)$$

$$h(z) = 1 - \frac{q^2(1 + a^2)}{4\Lambda} z^3 (1 + \Lambda z)^{-\frac{4}{1+a^2}} - k^2 z^2 (1 + \Lambda z)^{\frac{a^2-3}{1+a^2}} - b \left(\frac{(1 + \Lambda z)^{\frac{3a^2-1}{a^2+1}} - 1}{3a^2 - 1} + \frac{(1 + \Lambda z)^{\frac{a^2-3}{a^2+1}} - 1}{a^2 - 3} - \frac{(1 + \Lambda z)^{\frac{2a^2-2}{a^2+1}} - 1}{a^2 - 1} \right), \quad (4c)$$

$$\phi(z) = \frac{2a}{1 + a^2} \ln(\Lambda z + 1), \quad A_t(z) = \mu - \frac{qz}{1 + \Lambda z}, \quad (4d)$$

$$\text{Case 2: } \beta = a, \quad (5a)$$

$$f(z) = (1 + \Lambda z)^{\frac{2a^2}{1+a^2}} h(z), \quad (5b)$$

$$h(z) = 1 + \frac{q^2(1 + a^2)}{4\Lambda} z^3 (1 + \Lambda z)^{\frac{1-3a^2}{1+a^2}} - k^2 z^2 (1 + \Lambda z)^{\frac{1-3a^2}{1+a^2}} - b \left(\frac{(1 + \Lambda z)^{\frac{-3a^2+1}{a^2+1}} - 1}{3a^2 - 1} + \frac{(1 + \Lambda z)^{\frac{-a^2+3}{a^2+1}} - 1}{a^2 - 3} - \frac{(1 + \Lambda z)^{\frac{-2a^2+2}{a^2+1}} - 1}{a^2 - 1} \right), \quad (5c)$$

$$\phi(z) = -\frac{2a}{1 + a^2} \ln(\Lambda z + 1), \quad A_t = \mu - qz. \quad (5d)$$

Some remarks on these solutions are presented in what follows.

- The AdS boundary is located at $z = 0$. μ, q are the chemical potential and charge density of the dual boundary system, respectively.
- The parameter Λ shall be determined in terms of a, b by the horizon condition $h(z_+) = 0$ with z_+ being the position of horizon. Namely, only a, b are free parameters in this model.
- When $b = 0, a^2 = 1/3$, the solution of the case $\beta = 1/a$ reduces to the Gubser-Rocha one [34]. When $b = 0$, the solution of the case $\beta = 1/a$ becomes the well established results in [11, 15, 53, 54]. When $k = q = 0, b \neq 0$, by redefining the parameters, the solution coincides to that in [58].
- To obtain the thermodynamics of the background, one need follow the standard holographic renormalization approach. We would like to recommend article [52, 55, 56], in which the thermodynamics of the above model with $b = 0$ have been well studied.²

We would also like to point out that the above two branches of solutions can be related by a coordinate transformation. That is to say, the second branch of solutions can be obtained from the first one under the following coordinate transformation

$$z \rightarrow \frac{z}{1 - \Lambda z}. \quad (6)$$

Therefore, at this moment, it is enough to consider the first branch with $\beta = 1/a$ only.

Now, in order to make the solutions (4) become a black brane background, one should consider the horizon condition $h(z_+) = 0$, which determines the location of the horizon in terms of Λ and a, b as below

$$1 - \frac{q^2(1+a^2)}{4\Lambda} z_+^3 (1+\Lambda z_+)^{-\frac{4}{1+a^2}} - k^2 z_+^2 (1+\Lambda z_+)^{\frac{a^2-3}{1+a^2}} - b \left(\frac{(1+\Lambda z_+)^{\frac{3a^2-1}{a^2+1}} - 1}{3a^2 - 1} + \frac{(1+\Lambda z_+)^{\frac{a^2-3}{a^2+1}} - 1}{a^2 - 3} - \frac{(1+\Lambda z_+)^{\frac{2a^2-2}{a^2+1}} - 1}{a^2 - 1} \right) = 0. \quad (7)$$

In addition, the derivative of $h(z)$ with respect to z gives

$$h'(z) = -(1+\Lambda z)^{-\frac{4}{1+a^2}} \left(\frac{q^2 z^2 (3a^2 - 1)}{4\Lambda} + \frac{q^2 z^2}{\Lambda(1+\Lambda z)} \right.$$

$$\left. + \frac{b\Lambda^3 z^2}{1+a^2} + \frac{k^2 z (3a^2 - 1)(1+\Lambda z)}{1+a^2} + \frac{k^2 z (3-a^2)}{1+a^2} \right). \quad (8)$$

From now on, we shall set $\Lambda > 0, b \geq 0$. When $k = 0, q = 0$, $h(z)$ monotonically decreases from the boundary $h(z = 0) = 1$ to the horizon, which guarantees $h(z) > 0$ for the neutral black brane background. When $q \neq 0$, we can require $a^2 \geq 1/3$ to ensure $h(z) > 0$.

Then, the temperature and entropy density can be calculated straightforward as

$$T = \frac{(1+\Lambda z_+)^{-\frac{2}{1+a^2}}}{4\pi} \left(\frac{q^2 z_+^2 (3a^2 - 1)}{4\Lambda} + \frac{q^2 z_+^2}{\Lambda(1+\Lambda z_+)} + \frac{b\Lambda^3 z_+^2}{1+a^2} + \frac{k^2 z_+ (3a^2 - 1)(1+\Lambda z_+)}{1+a^2} + \frac{k^2 z_+ (3-a^2)}{1+a^2} \right), \quad (9)$$

$$s = \frac{4\pi}{z_+^2} (1+\Lambda z_+)^{\frac{2}{1+a^2}}. \quad (10)$$

It is obvious that the temperature $T > 0$ when $h(z)$ is monotonic. This is the key point to establish a zero temperature ground state with a zero entropy density, since if $T = 0$ can be attained at a finite z_+ , then, the entropy density above must be finite. On the contrary however, a positive $T > 0$ for all finite z_+ , may decreases and becomes zero as $z_+ \rightarrow 0$.

2.2 Analysis on the ground state

The ground state with zero entropy is physically acceptable. However, such ground state in holographic model is rare in the present literatures. As we know, the only simple example is the Gubser-Rocha solution [34]. Now, with a more fruitful AdS background (4) at hand we give a detailed analysis, case by case, to find the ground state with a vanishing entropy density. The method has been illustrated in the end of the last subsection, namely, we shall check whether $z_+ \rightarrow \infty$ gives $T \rightarrow 0$ as well as $s \rightarrow 0$.

2.2.1 Neutral black brane background for $q = 0, k = 0$

We first consider the neutral black brane case without the axionic field, $q = 0, k = 0$. The horizon condition (7) and the temperature (9) reduce to

² We are very grateful to Astefanesei for drawing our attention to [58] as well as the correct holographic renormalization approach.

$$b \left(\frac{(1 + \Lambda z_+)^{\frac{3a^2-1}{a^2+1}} - 1}{3a^2 - 1} + \frac{(1 + \Lambda z_+)^{\frac{a^2-3}{a^2+1}} - 1}{a^2 - 3} - \frac{(1 + \Lambda z_+)^{\frac{2a^2-2}{a^2+1}} - 1}{a^2 - 1} \right) = 1, \quad (11)$$

$$T = \frac{b\Lambda^3 z_+^2}{4\pi(1 + a^2)} (1 + \Lambda z_+)^{-\frac{2}{1+a^2}}, \quad (12)$$

Then, we set $\Lambda = \Lambda_0/z_+$. When z_+ is varying, $\Lambda_0 > 0$ is a fixed parameter satisfying

$$b \left(\frac{(1 + \Lambda_0)^{\frac{3a^2-1}{a^2+1}} - 1}{3a^2 - 1} + \frac{(1 + \Lambda_0)^{\frac{a^2-3}{a^2+1}} - 1}{a^2 - 3} - \frac{(1 + \Lambda_0)^{\frac{2a^2-2}{a^2+1}} - 1}{a^2 - 1} \right) = 1, \quad (13)$$

Accordingly, the temperature and the entropy density (10) becomes

$$T = \frac{b\Lambda_0^3}{4\pi z_+(1 + a^2)} (1 + \Lambda_0)^{-\frac{2}{1+a^2}},$$

$$s = \frac{4\pi}{z_+^2} (1 + \Lambda_0)^{\frac{2}{1+a^2}}. \quad (14)$$

When z_+ is varying, we have a simple relation

$$s \propto T^2. \quad (15)$$

Both the s, T tends to zero as $z_+ \rightarrow \infty$. Such neutral background admits a ground state with zero entropy density.

2.2.2 Simple charged black brane background for $b = 0$

Next, we study a simple charged black brane case with $q \neq 0$ but $b = 0$, where the general solution (4) reduces to

$$f = (1 + \Lambda z)^{\frac{2}{1+a^2}} h(z),$$

$$h = 1 - \frac{q^2(1 + a^2)}{4\Lambda} z^3 (1 + \Lambda z)^{-\frac{4}{1+a^2}} - k^2 z^2 (1 + \Lambda z)^{\frac{a^2-3}{1+a^2}}, \quad (16a)$$

$$A_t = \mu - \frac{qz}{1 + \Lambda z}, \quad \phi = \frac{2a}{1 + a^2} \ln(\Lambda z + 1), \quad (16b)$$

The above solutions have been well studied in [15, 52–54]. Especially, when $a^2 = 1/3$, it reduces the famous Gubser-Rocha solution [34]. Here, we try to complete the zero temperature analysis. We will find that not only the Gubser-Rocha solution, i.e., $a^2 = 1/3$, but also the case

of $a^2 > 3, k = 0$, are the vanishing entropy density background at the zero temperature.

We first consider the case without axion fields, namely $k = 0$ where the horizon condition (7) and the temperature (9) reduces to

$$\frac{q^2(1 + a^2)}{4\Lambda} z_+^3 (1 + \Lambda z_+)^{-\frac{4}{1+a^2}} = 1, \quad (17)$$

$$T = \frac{(1 + \Lambda z_+)^{-\frac{2}{1+a^2}}}{4\pi} \left(\frac{q^2 z_+^2 (3a^2 - 1)}{4\Lambda} + \frac{q^2 z_+^2}{\Lambda(1 + \Lambda z_+)} \right). \quad (18)$$

When $a^2 < 1/3, T = 0$ can be achieved at a finite position

$$z_+ = \frac{1}{\mu} \sqrt{\frac{3}{4} \left(\frac{4}{1 - 3a^2} \right)^{\frac{3-a^2}{1+a^2}}}. \quad (19)$$

The corresponding entropy density is

$$s = \frac{\mu^2}{3\pi} \left(\frac{4}{1 - 3a^2} \right)^{\frac{a^2-1}{1+a^2}}, \quad (20)$$

which is of course finite.

However, when $a^2 \geq 1/3$, the story is totally different, because the temperature (18) is always positive. We then check the temperature behavior as $z_+ \rightarrow \infty$.

In the limit of $z_+ \rightarrow \infty$, the relation (17) forces that $\Lambda z_+ + 1 \approx \Lambda z_+ \rightarrow \infty$ and gives

$$\Lambda = \left(\frac{(1 + a^2)q^2}{4} \right)^{\frac{a^2+1}{a^2+5}} z_+^{\frac{3a^2-1}{a^2+5}} \sim z_+^{\frac{3a^2-1}{a^2+5}} \rightarrow \infty. \quad (21)$$

Then the temperature in (18) and the entropy density in (10) approximatively read as

$$T = \frac{3a^2 - 1}{4\pi(1 + a^2)} \left(\frac{(1 + a^2)q^2}{4} \right)^{\frac{2}{a^2+5}} z_+^{\frac{3-a^2}{a^2+5}},$$

$$s = 4\pi \left(\frac{(1 + a^2)q^2}{4} \right)^{\frac{2}{a^2+5}} z_+^{-\frac{2a^2+2}{a^2+5}}. \quad (22)$$

It is clear that for $a^2 \geq 1/3$ the entropy density decreases to zero as $z_+ \rightarrow \infty$. But the zero temperature and zero entropy density can be simultaneously achieved only for $a^2 = 1/3$ or $a^2 > 3$. On the contrary, no zero temperature exists for $3 \geq a^2 > 1/3$. Actually, as $z_+ \rightarrow \infty$, it is a high temperature limit with $T \rightarrow \infty$.

Next, we consider the case $k \neq 0$. The horizon condition (7) and the temperature (9) with $b = 0$ reduces to

$$1 - \frac{q^2(1 + a^2)}{4\Lambda} z_+^3 (1 + \Lambda z_+)^{-\frac{4}{1+a^2}} - k^2 z_+^2 (1 + \Lambda z_+)^{\frac{a^2-3}{1+a^2}} = 0, \quad (23)$$

$$T = \frac{(1 + \Lambda z_+)^{-\frac{2}{1+a^2}}}{4\pi} \left(\frac{q^2 z_+^2 (3a^2 - 1)}{4\Lambda} + \frac{q^2 z_+^2}{\Lambda(1 + \Lambda z_+)} + \frac{k^2 z_+ (3a^2 - 1)(1 + \Lambda z_+)}{1 + a^2} + \frac{k^2 z_+ (3 - a^2)}{1 + a^2} \right), \quad (24)$$

Then, taking the limit $z_+ \rightarrow \infty$, the horizon condition (23) becomes

$$1 - \frac{q^2(1 + a^2)}{4\Lambda} z_+^3 (\Lambda z_+)^{\frac{-4}{1+a^2}} - k^2 z_+^2 (\Lambda z_+)^{\frac{a^2-3}{1+a^2}} = 0. \quad (25)$$

Immediately, we find that one can not obtain an extremal black brane solution with axionic fields for $a^2 \geq 3$. It is in contrast to the case without axionic fields. Therefore, we conclude that once the axionic fields are taken into account, the simple charge black brane solution with vanishing ground state entropy density can be achieved only for $a^2 = 1/3$.

2.2.3 Special cases for $a^2 = 1, a^2 = 1/3, a^2 = 3$

So far, the discussion is based on the general potential (2). For the special cases $a^2 = 1, a^2 = 1/3, a^2 = 3$, it is convenient to write down the form of Φ in the potential (2) after taking the limit, which are separately given by

$$\Phi(a^2 = 1/3) = -b \left(\frac{\sqrt{3}\phi}{2} - \frac{3e^{-\frac{4\sqrt{3}}{3}\phi} - 3}{8} + \frac{3e^{-\frac{2\sqrt{3}}{3}\phi} - 3}{2} \right), \quad (26a)$$

$$\Phi(a^2 = 1) = -b (\sinh \phi - \phi), \quad (26b)$$

$$\Phi(a^2 = 3) = \frac{-b}{24} \left(3e^{\frac{2\phi}{\sqrt{3}}} \left(e^{\frac{2\phi}{\sqrt{3}}} - 4 \right) + 4\sqrt{3}\phi + 9 \right), \quad (26c)$$

The background solutions of $h(z)$ in (4) also take the form as

$$h\left(a^2 = \frac{1}{3}\right) = 1 - \frac{q^2 z^3}{3\Lambda(1 + \Lambda z)^3} - \frac{k^2 z^2}{(1 + \Lambda z)^2} - b \left(\frac{3}{4} \log(1 + \Lambda z) - \frac{3\Lambda z(3\Lambda z + 2)}{8(\Lambda z + 1)^2} \right), \quad (27a)$$

$$h(a^2 = 1) = 1 - \frac{q^2 z^3}{2\Lambda(1 + \Lambda z)^2} - \frac{k^2 z^2}{(1 + \Lambda z)} - b \left(\frac{\Lambda z(\Lambda z + 2)}{2\Lambda z + 2} - \log(1 + \Lambda z) \right), \quad (27b)$$

$$h(a^2 = 3) = 1 - \frac{q^2 z^3}{\Lambda(1 + \Lambda z)} - k^2 z^2$$

$$- b \left(\frac{1}{8} (\Lambda z(\Lambda z - 2) + 2 \log(1 + \Lambda z)) \right), \quad (27c)$$

However, the special parameters would not change the formula of $h'(z)$ and hence the temperature formula. Namely, we can use the former expressions (8) and (9) directly and fix $a^2 = 1, a^2 = 1/3, a^2 = 3$ respectively.

When we do the zero temperature analysis, the only thing needing to change is the horizon condition (7), which should be replaced by using the above results (27) and require $h(a^2 = 1/3, z_+) = 0, h(a^2 = 1, z_+) = 0, h(a^2 = 3, z_+) = 0$ for $a^2 = 1, a^2 = 1/3, a^2 = 3$ respectively.

It is easy to see that, for the neutral case $q = 0, k = 0$, a zero temperature background with vanishing entropy also exists for these special parameters. While, for $q = 0, k \neq 0$, due to the logarithm divergence as $z_+ \rightarrow \infty$ in (27), the system do not have ground state with vanishing entropy. For the simple charge case $b = 0, k = 0$, special parameters have no influence on the previous discussion. We still have the zero entropy background at zero temperature for $a^2 = 1/3$ or $a^2 > 3$.

3 Linear-T resistivity

In this section, we consider the electric transport behavior of the dual system over the black brane geometry (4). Specifically, we calculate the DC conductivity with the interest in its dependence on the temperature. We find the Gubser-Rocha case exhibits a linear-T resistivity valid in a wide temperature, which coincides to the universal behaviors of the strange metal.

Before the discussion, we should scale the quantities by the charge density q . Namely, in this section z_+, Λ, k, T should be understood as the scaled ones $\sqrt{q}z_+, \Lambda/\sqrt{q}, k/\sqrt{q}, T/\sqrt{q}$. Then, using the standard holographic techniques, one can derive the DC conductivity as

$$\sigma = (1 + \Lambda z_+)^{\frac{2a^2}{1+a^2}} \left(1 + \frac{z_+^2}{2k^2(1 + \Lambda z_+)^2} \right). \quad (28)$$

Usually the relation between σ and T is complicated, since one needs to first solve $z_+(T), \Lambda(T)$ by the relations in Eqs. (7) and (9), and then substitute them into the above expression of the conductivity. It is thus hard to catch a simple universal relation of the resistivity of the temperature. To simplify, one need take some approximation. We firstly consider the 'deep horizon limit' $z_+ \rightarrow \infty$ as appeared in lots of the literature. Recall that when $b \neq 0$, such limit is invalid for the divergence in the horizon conditions, see for instance (27). Thus, we only consider the simple charge black brane case with $b = 0$.

In this limit $z_+ \rightarrow \infty$, which corresponds to low temperature limit at $a^2 = 1/3$ or high temperature limit for

Fig. 1 The linear-T resistivity behavior in a large temperature range. Here, we have set $k = 10^4$, which ensures $k \gg 1$, $\Lambda \gg 1$ and $\Lambda z_+ \gg 1$

$3 > a^2 > 1/3$, the expression of temperature in (24) can be largely simplified. Together with the relation (25), we have

$$\sigma \sim T^2, \quad 3 > a^2 > 1/3, \quad (29a)$$

$$\sigma \sim 1/T, \quad a^2 = 1/3. \quad (29b)$$

The result indicates that for $1/3 < a^2 < 3$, the resistivity decrease as $1/T^2$ in high temperature limit, which is independent of a . While for $a^2 = 1/3$, this holographic system captures the important property of the strange metal, i.e., linear-T resistivity [8]. However, the behavior of the linear-T resistivity only holds in low temperature limit in [8]. Here we shall address that the linear-T resistivity can also holds for a large temperature range if $\Lambda \gg 1$ as well as $\Lambda z_+ \gg 1$.

We firstly check whether such region exists. When $a^2 = 1/3$, the horizon condition (23) becomes

$$1 - \frac{1}{3\Lambda} z_+^3 (1 + \Lambda z_+)^{-3} - k^2 z_+^2 (1 + \Lambda z_+)^{-2} = 0. \quad (30)$$

To be clear, we rewrite the condition as

$$1 - \frac{1}{3\Lambda^4} y^3 - \frac{k^2}{\Lambda^2} y^2 = 0 \Leftrightarrow 2\Lambda^2 = k^2 y^2 + \sqrt{k^4 y^4 + 4y^3/3}, \quad (31)$$

where $y = (1 + 1/(\Lambda z_+))^{-1}$. We can see that $\Lambda z_+ \gg 1$ leads to $y > 1/2$ and then

$$2\Lambda^2 > k^2/2 + \sqrt{k^4/8 + 1/6} > k^2/2. \quad (32)$$

Thus, when $k \gg 1$ the regime with $\Lambda \gg 1$, $\Lambda z_+ \gg 1$ survives.

Now we turn to explore the relation between the resistivity and the temperature. Using Eqs. (33) and (30), we plot the resistivity as the function of the temperature in Fig. 1. Obviously, the resistivity linearly depends on the temperature. Especially, we can see that the linear-T resistivity survives in a large range of temperature, which shall be further addressed in what follows.

Also, we can obtain an approximate expression of the resistivity. When $a^2 = 1/3$, both the temperature in (24) and the conductivity in (28) reduce to

$$T = \frac{1}{4\pi\sqrt{z_+\Lambda}} \left(2\Lambda + \frac{1}{3\Lambda^3} \right), \quad \sigma = \sqrt{z_+\Lambda} \left(1 + \frac{1}{2\Lambda^2 k^2} \right). \quad (33)$$

When $k \gg 1$ and $\Lambda \gg 1$, $\Lambda z_+ \gg 1$, the above equation can be approximately expressed as

$$T = \frac{1}{4\pi\sqrt{z_+\Lambda}} \left(2\Lambda + \frac{1}{3\Lambda^3} \right) \approx \frac{1}{4\pi\sqrt{z_+k}} \left(2k + \frac{1}{3k^3} \right).$$

Note that in the above equation, we have used the relations, $y \rightarrow 1$ and $\Lambda \rightarrow k$, which can be directly derived from the condition of $\Lambda z_+ \gg 1$. Obviously, T is not sensitive to Λ , but mainly depends on z_+ . The situation for the conductivity is similar. Both T and σ change mainly with z_+ , while Λ can be treated as a constant, i.e. $\Lambda = k$. And then, with the use of the relation $k \gg 1$, the temperature and the conductivity can be further simplified as

$$T \approx \frac{1}{2\pi} \sqrt{\frac{k}{z_+}}, \quad \sigma \approx \sqrt{z_+k}. \quad (34)$$

Immediately, we obtain a linear-T resistivity law

$$\rho \approx \frac{2\pi}{k} T. \quad (35)$$

Next, we shall address that the linear-T resistivity survives in a large range of temperature. To this end, we present two examples as what follows (see Table 1). The exact value of Λ and the temperature T in Table 1 are calculated by the expressions in (30) and (33). The approximate results of the temperature are obtained in terms of the first equation in Eq. (34). Note that we have used the relation $\Lambda \approx k$. From the above table (the second and third rows), we confirm the result that when $k \gg 1$, $\Lambda \gg 1$, and $\Lambda z_+ \gg 1$, Λ is approximately a constant, i.e., $\Lambda \approx k$. In addition, the approximate values of the temperature are consistent with the exact ones, which means that the approximate linear-T resistivity expression, namely Eq. (34), holds very well in the region of $k \gg 1$, $\Lambda \gg 1$, and $\Lambda z_+ \gg 1$. More importantly, from this table, we can see that the temperature T indeed varies in terms of z_+ , crossing a large range.

In summary, the linear-T resistivity is achieved when $\Lambda \approx k \gg 1$, which is a good approximation. It always happens if one considers a system with large parameter $k \gg 1$. In our examples, $T < 160$ is the good regime of linear-T resistivity for $k = 10^4$, while $T < 503$ is, at least, the good regime for $k = 10^5$. Thus, for the parameters satisfying the certain conditions, the linear-T resistivity is achieved for a wide range of temperature.

Table 1 The exact value of Λ and the temperature T obtained by the expressions in (30) and (33), which is presented in the second and third rows, and the approximate results obtained in terms of the first equation in Eq. (34), which is presented in the fourth and fifth rows

	$k = 10^4$	$z_+ = 10^{-3}$	$z_+ = 10^{-2}$	$z_+ = 10^2$		$k = 10^5$	$z_+ = 10^{-2}$	$z_+ = 10^2$
Λ		9000.00	9900.00	9999.99	Λ		99899.99	99999.99
T		477.47	158.36	1.59	T		503.04	5.03
Λ		$\approx 10^4$	$\approx 10^4$	$\approx 10^4$	Λ		$\approx 10^5$	$\approx 10^5$
T		≈ 503.29	≈ 159.16	≈ 1.59	T		≈ 503.29	≈ 5.03

Note added. As this work was being completed, we were informed from Chao Niu that they also find the linear-T resistivity behavior at high temperature region in [59].

4 Dyonic dilatonic black brane and its Transports

The lack of exact dyonic solution in gravity theory prevents us from investigating the magnetic transport behavior of the dual system in an analytical manner. Fortunately, in EMDA theory (1), we are able to find such an analytical dyonic solution for the special case of $a^2 = 1$ by virtue of the electromagnetic self-duality. Here we just list the dyonic solutions as below. The detailed derivation can be found in Appendix A. Moreover, we point out that an AdS dyonic solution with $b = 0$ as well as $k = 0$ has previously been reported in [57].³ In [52], by detailed analysis on the boundary condition it is argued that a dyonic solution may only exist at $a = 1$ ($\xi = 1$ in their paper). Here, interestingly enough, we provide an interpretation for this fact from a different angle of view, namely the electromagnetic self-duality.

For $a = 1$, the EMDA theory with the potential (2) exists the dyonic black brane solution as the following form

$$ds^2 = \frac{1}{z^2} \left(-f(z)dt^2 + \frac{dz^2}{f(z)} + (1 + \Lambda z)(dx^2 + dy^2) \right), \quad (36a)$$

$$\phi = \ln(\Lambda z + 1), \quad (36b)$$

$$A = \left(\mu - \frac{qz}{1 + \Lambda z} \right) dt + Bx dy, \quad (36c)$$

$$f = (1 + \Lambda z)h(z), \quad (36d)$$

$$h = 1 - \frac{q^2}{2\Lambda} z^3 (1 + \Lambda z)^{-2} + \frac{B^2}{2\Lambda} z^3 (1 + \Lambda z)^{-1} - k^2 z^2 (1 + \Lambda z)^{-1} - b \left(\frac{\Lambda z (\Lambda z + 2)}{2(1 + \Lambda z)} - \log(1 + \Lambda z) \right), \quad (36e)$$

where B is a constant magnetic field. The horizon condition $h(z_+) = 0$ gives rise to

$$1 - \frac{q^2}{2\Lambda} z_+^3 (1 + \Lambda z_+)^{-2} + \frac{B^2}{2\Lambda} z_+^3 (1 + \Lambda z_+)^{-1} - k^2 z_+^2 (1 + \Lambda z_+)^{-1} - b \left(\frac{\Lambda z_+ (\Lambda z_+ + 2)}{2(1 + \Lambda z_+)} - \log(1 + \Lambda z_+) \right) = 0, \quad (37)$$

The temperature of this black brane is

$$T = \frac{(1 + \Lambda z_+)^{-1}}{4\pi} \left(\frac{q^2 z_+^2}{2\Lambda} + \frac{q^2 z_+^2}{\Lambda(1 + \Lambda z_+)} + k^2 z_+ (1 + \Lambda z_+) + k^2 z_+ + \frac{b\Lambda^3 z_+^2}{2} - \frac{B^2 z_+^2}{2\Lambda} - \frac{B^2 z_+^2}{\Lambda} (1 + \Lambda z_+) \right), \quad (38)$$

Now, we turn to study the DC transports. Employing the standard techniques developed in [18, 37, 64], we obtain the thermoelectric conductivities over the dyonic black brane geometry (36) as

$$\sigma_{xx} = \frac{H(q^2 + HZ + B^2 Z^2)}{B^2 q^2 + (B^2 Z + H)^2}, \quad \sigma_{xy} = \frac{Bq(q^2 + 2HZ + B^2 Z^2)}{B^2 q^2 + (B^2 Z + H)^2}, \quad (39a)$$

$$\alpha_{xx} = \frac{Hsq}{B^2 q^2 + (B^2 Z + H)^2}, \quad \alpha_{xy} = \frac{Bs(q^2 + HZ + B^2 Z^2)}{B^2 q^2 + (B^2 Z + H)^2}, \quad (39b)$$

$$\bar{\kappa}_{xx} = \frac{s^2 T (B^2 Z + H)}{B^2 q^2 + (B^2 Z + H)^2}, \quad \bar{\kappa}_{xy} = \frac{Bqs^2 T}{B^2 q^2 + (B^2 Z + H)^2}, \quad (39c)$$

where $Z \equiv Z(\phi)|_{z_+}$, $H \equiv H(\phi)|_{z_+}$ and q are, respectively

$$Z = (1 + \Lambda z_+), \quad H = \frac{2k^2(1 + \Lambda z_+)}{z_+^2}, \quad q = \frac{(1 + \Lambda z_+)\mu}{z_+}. \quad (40)$$

³ We are very grateful to Gouteráux for drawing our attention to the work in [52, 57].

And then, we give the charge transport coefficients, the DC resistivity ρ_{dc} and the thermopower S as

$$\rho_{dc} = \frac{1}{\sigma_{xx}(B=0)} = \frac{2k^2}{(1+\Lambda z_+)(2k^2+\mu^2)}, \quad (41a)$$

$$S = \frac{\alpha_{xx}(B=0)}{\sigma_{xx}(B=0)} = \frac{4\pi\mu}{z_+(2k^2+\mu^2)}. \quad (41b)$$

Also the magnetic transport coefficients, including the Hall angle $\tan\theta_H$, the Hall Lorentz ratio L_H , the magnetic resistance ρ_B and Nernst coefficient ν are calculated explicitly as

$$\tan\theta_H = \frac{\sigma_{xy}}{\sigma_{xx}} = \frac{B\mu z_+(B^2 z_+^2 + 4k^2 + \mu^2)}{2k^2(B^2 z_+^2 + 2k^2 + \mu^2)}, \quad (42a)$$

$$L_H = \frac{\bar{\kappa}_{xy}}{T\sigma_{xy}} = \frac{16\pi^2}{z_+^2(B^2 z_+^2 + 4k^2 + \mu^2)}, \quad (42b)$$

$$\rho_B = \frac{\rho_{xx} - \rho_{xx}(B=0)}{\rho_{xx}(B=0)} = \frac{2B^2 k^2 z_+^2}{\mu^2(B^2 z_+^2 + 4k^2) + 4k^4 + \mu^4}, \quad (42c)$$

$$\begin{aligned} \nu &= \frac{1}{B} \left(\frac{\alpha_{xy}}{\sigma_{xx}} - S \tan\theta_H \right) \\ &= \frac{4\pi(B^2 z_+^2 + 2k^2)}{(2k^2 + \mu^2)(B^2 z_+^2 + 2k^2 + \mu^2)}. \end{aligned} \quad (42d)$$

In order to simplify the expression of temperature, we will only consider the case $b=0$ in the follows, in which we can take a large z_+ limit. The horizon condition (37) and the temperature (38) reduce to

$$1 - \frac{q^2}{2\Lambda} z_+^3 (1 + \Lambda z_+)^{-2} + \frac{B^2}{2\Lambda} z_+^3 (1 + \Lambda z_+)^{-1} - k^2 z_+^2 (1 + \Lambda z_+)^{-1} = 0, \quad (43)$$

$$4\pi T = (1 + \Lambda z_+) \times \left(\frac{3}{z_+} - \frac{\Lambda}{1 + \Lambda z_+} - \frac{q^2 z_+^3}{2(1 + \Lambda z_+)^3} - \frac{k^2 z_+}{1 + \Lambda z_+} \right). \quad (44)$$

We are particularly interested in the transport behavior of this system in the high temperature limit, in which $\Lambda z_+ \gg 1$ such that $1 + \Lambda z_+ \approx \Lambda z_+$. In this case, Eqs. (43) and (44) reduce to

$$1 - \frac{q^2}{2\Lambda^3} z_+ - \frac{k^2}{\Lambda} z_+ + \frac{B^2}{2\Lambda^2} z_+^2 = 0, \quad (45a)$$

$$4\pi T = \Lambda \left(2 - \frac{q^2}{2\Lambda^3} z_+ - \frac{k^2}{\Lambda} z_+ \right). \quad (45b)$$

Next we consider the situation that the magnetic field B is small, then z_+ and Λ can be solved as

$$\Lambda = 4\pi T + \frac{2048\pi^5 T^5}{(32\pi^2 k^2 T^2 + q^2)^2} B^2 + \mathcal{O}(B^4), \quad (46a)$$

$$\begin{aligned} z_+ &= \frac{128\pi^3 T^3}{32\pi^2 k^2 T^2 + q^2} \\ &+ \frac{262144\pi^7 T^7 (16\pi^2 k^2 T^2 + q^2)}{(32\pi^2 k^2 T^2 + q^2)^4} B^2 + \mathcal{O}(B^4). \end{aligned} \quad (46b)$$

In the above equations, we have expressed z_+ and Λ up to the second order of B . As a consequence, we give the charge and magnetic transport coefficients up to the first order as

$$\rho_{dc} = \frac{k^2}{16\pi^2 T^2}, \quad S = \frac{q}{8\pi T^2}, \quad (47a)$$

$$\tan\theta_H = B \frac{16\pi^2 q T^2 (64\pi^2 k^2 T^2 + q^2)}{k^2 (32\pi^2 k^2 T^2 + q^2)^2} + \mathcal{O}(B^3), \quad (47b)$$

$$L_H = \frac{16\pi^2 k^4}{64\pi^2 k^2 T^2 + q^2} + \frac{q^2}{64\pi^2 T^4} + \mathcal{O}(B^2), \quad (47c)$$

$$\begin{aligned} \rho_B &= -B^2 \frac{512(\pi^4 T^4 (192\pi^2 k^2 q^2 T^2 + 1024(3\pi^4 k^4 T^4 - 16\pi^6 k^2 T^6) + 3q^4))}{(32\pi^2 k^2 T^2 + q^2)^4} \\ &+ \mathcal{O}(B^4), \end{aligned} \quad (47d)$$

$$\nu = \frac{2048\pi^5 k^2 T^4}{(32\pi^2 k^2 T^2 + q^2)^2} + \mathcal{O}(B^2). \quad (47e)$$

Fig. 2 The transports as the function of temperature T in the high temperature region. Here we have set $k = 1$, $q = 10$ and $B = 1$

The characteristics of the transport behavior in the high temperature region are summarized as what follows.

- Both DC resistivity ρ_{dc} and thermopower S decrease with $1/T^2$ at high temperature, which implies the thermal transport is dominant over the electric and electrothermal transport.
- With the increase of temperature, the Hall angle $1/\tan \theta_H$ and Hall Lorentz ratio decrease (see the plots in Fig. 2), while the magneticresistance ρ_B increases (see the left below plot in Fig. 2). The Nernst coefficient ν becomes a constant in the limit of high temperature (see the right below plot in Fig. 2).

5 Conclusions and discussions

In this paper we have constructed a new class of charged black brane solutions in EMDA theory, which is characterized by two free parameters a , b , which could be viewed as the extension of various charged solutions with $b = 0$ in literature [15, 34, 52–54].

For different a , b , the background exhibits distinct behavior in zero temperature limit. In the neutral background $q = 0$, the zero temperature ground state with zero entropy density always exists for any a , b , while in the simple charged case $q \neq 0$, $b = 0$, it depends on a . For $a^2 < 1/3$, the zero temperature can be achieved at finite horizon position. But the entropy density is finite as well at the zero temperature. It is interesting to notice that a ground state with vanishing entropy density is allowed for $a^2 = 1/3$, which has previously been obtained in Gubser-Rocha model, and for $a^2 > 3$.

While for $3 > a^2 > 1/3$, the zero temperature can not be achieved and the deep horizon $z_+ \gg 1$ corresponds to a high temperature limit. When the translation invariance is broken by adding axion fields, by contrast, the vanishing entropy density ground state can be achieved only for $a^2 = 1/3$. For this special case we have demonstrated that the dual system is characterized by a linear- T resistivity in a large range of temperature, reminiscent of the key feature of the strange metal. We have also obtained dyonic black brane by virtue of the EM duality for $a = 1$. The transport coefficients have been calculated and their temperature dependence have been analyzed in high temperature region. We expect the EM duality as a valuable strategy may be applicable to more general gravity theories such that more analytic solutions of dyonic black brane could be constructed, which should be helpful for us to investigate the magnetic transport behavior of the dual system by holography.

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Appendix A: Electrically, magnetically charged dilatonic black branes and EM duality

The well-known dyonic black brane solution is given in Einstein-Maxwell theory, which enjoys S-duality. While, usually it is hard to obtain new analytical solutions for dyonic black brane. In this section, we shall study the electrically, magnetically charged black brane solutions in EMD theory (1) with the help of EM duality.

1. Pure magnetic solution

We first construct the purely magnetic solution. For this purpose, we introduce EM duality of the EMD theory (1). We define the dual field strength G_{ab} by the Hodge star operation

$$G_{ab} := \frac{Z(\phi)}{2} \epsilon_{abcd} F^{cd}, \quad (\text{A1})$$

where $G = dH$ with H_a being the dual gauge field and ϵ_{abcd} is the completely antisymmetric Levi-Civita tensor. And then, we can write down the dual one of EMD theory as

$$\hat{S}_{EMD} = \int d^4x \sqrt{-g} \left(R - \frac{1}{4Z(\phi)} G_{ab} G^{ab} - \frac{1}{2} (\partial\phi)^2 + V(\phi) - \sum_{I=x,y} (\partial\psi_I)^2 \right). \quad (\text{A2})$$

From Eqs. (A1) and (A2), it is easy to find that under the EM duality, the gauge coupling transforms as $Z(\phi) \rightarrow 1/Z(\phi)$, which also implies a weak-strong coupling duality. Especially, there is a correspondence between the electric field F_{tx} of the original theory and the magnetic field G_{yz} of its dual one. Therefore, by EM duality we could quickly obtain a purely magnetic solution of the dual EMD theory (A2). We demonstrate it as follows.

Given an electrically charged solution for the action in (1) which has a gauge coupling with an exponential function $Z(\phi) = e^{a\phi}$, one can obtain a purely magnetic solution for the dual action in (A2) with gauge coupling $1/Z(\phi) = e^{-a\phi}$ by virtue of the EM-duality, namely replace $q \rightarrow B$ in (5). Next, we change $a \rightarrow -a$ in the magnetic solution. The final result is a solution for the action with gauge coupling $Z(\phi)$ and potential $\tilde{V}(\phi) = V(a \rightarrow -a, \phi)$, which is

$$\hat{S}_{EMD} = \int d^4x \sqrt{-g} \left(R - \frac{e^{a\phi}}{4} F^2 - \frac{1}{2} (\partial\phi)^2 + \tilde{V}(\phi) - \sum_{I=x,y} (\partial\psi_I)^2 \right). \quad (\text{A3})$$

One can obtain the following purely magnetic solution

$$ds^2 = \frac{1}{z^2} \left(-\tilde{f}(z) dt^2 + \frac{dz^2}{\tilde{f}(z)} + (1 + \Lambda z) \frac{2a^2}{1+a^2} (dx^2 + dy^2) \right), \quad (\text{A4a})$$

$$\tilde{f}(z) = (1 + \Lambda z) \frac{2a^2}{1+a^2} h(z), \quad (\text{A4b})$$

$$h(z) = 1 + \frac{B^2(1+a^2)}{4\Lambda} z^3 (1 + \Lambda z) \frac{1-3a^2}{1+a^2} - k^2 z^2 (1 + \Lambda z) \frac{1-3a^2}{1+a^2} - b \left(\frac{(1 + \Lambda z) \frac{-3a^2+1}{a^2+1} - 1}{3a^2 - 1} + \frac{(1 + \Lambda z) \frac{-a^2+3}{a^2+1} - 1}{a^2 - 3} - \frac{(1 + \Lambda z) \frac{-2a^2+2}{a^2+1} - 1}{a^2 - 1} \right), \quad (\text{A4c})$$

$$\phi = \frac{2a}{1+a^2} \ln(\Lambda z + 1), \quad \tilde{A} = Bx dy, \quad (\text{A4d})$$

Note that this magnetic solution has also been found in [52].

2. EM S-duality and dyonic black brane solution

In this subsection, we construct the dyonic black brane solution from the EMDA theory (1) for $a^2 = 1$, in which the theory (1) is S-duality, namely the charge and its dual magnetic solutions are both valid for the same action.

For this case, the potential becomes

$$V = V_0(1 + \Phi) + V_1, \quad (\text{A5a})$$

$$V_0 = \frac{3(e^{\frac{1}{2}\phi} + e^{-\frac{1}{2}\phi})^2 - (e^{\frac{1}{2}\phi} - e^{-\frac{1}{2}\phi})^2}{2}, \quad (\text{A5b})$$

$$V_1 = \frac{b}{2} (e^\phi - 1)^3 (e^{-\phi} + e^{-2\phi}), \quad (\text{A5c})$$

$$\Phi = -b (\sinh \phi - \phi), \quad (\text{A5d})$$

the charged black brane solution (4) becomes

$$ds^2 = \frac{1}{z^2} \left(-f(z) dt^2 + \frac{dz^2}{f(z)} + (1 + \Lambda z) (dx^2 + dy^2) \right), \quad (\text{A6a})$$

$$f = (1 + \Lambda z) h(z), \quad (\text{A6b})$$

$$h(z) = 1 - \frac{q^2 z^3}{2\Lambda(1 + \Lambda z)^2} - \frac{k^2 z^2}{(1 + \Lambda z)} - b \left(\frac{\Lambda z(\Lambda z + 2)}{2\Lambda z + 2} - \log(1 + \Lambda z) \right), \quad (\text{A6c})$$

$$A = A_t(z) dt = \left(\mu - \frac{qz}{1 + \Lambda z} \right) dt, \quad \phi = \ln(\Lambda z + 1). \quad (\text{A6d})$$

Since for $a^2 = 1$, the theory is S-duality, we also has the magnetic black brane solution

$$\tilde{f} = (1 + \Lambda z) h(z), \quad (\text{A7a})$$

$$h(z) = 1 + \frac{B^2 z^3}{2\Lambda(1+\Lambda z)} - \frac{k^2 z^2}{(1+\Lambda z)} - b \left(\frac{\Lambda z(\Lambda z + 2)}{2\Lambda z + 2} - \log(1 + \Lambda z) \right), \quad (\text{A7b})$$

$$\tilde{A} = Bx dy, \quad \phi = \ln(\Lambda z + 1). \quad (\text{A7c})$$

Combining the charge black brane solution and the magnetic one, one can easily construct the dyonic black brane solution from the EMDA theory (1), which is

$$f = (1 + \Lambda z) \left(1 - \frac{q^2}{2\Lambda} z^3 (1 + \Lambda z)^{-2} + \frac{B^2}{2\Lambda} z^3 (1 + \Lambda z)^{-1} - b \left(\frac{\Lambda z(\Lambda z + 2)}{2\Lambda z + 2} - \log(1 + \Lambda z) \right) \right), \quad (\text{A8a})$$

$$A = \left(\mu - \frac{qz}{1 + \Lambda z} \right) dt + Bx dy, \quad \phi = \ln(\Lambda z + 1). \quad (\text{A8b})$$

The line element is also (A6a).

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