Scalar perturbations and quasi-normal modes of a nonlinear magnetic-charged black hole surrounded by quintessence

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Received: 7 November 2018 / Accepted: 15 February 2019 / Published online: 27 February 2019

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Abstract We study scalar perturbations and quasi-normal modes of a nonlinear magnetic-charged black hole surrounded by quintessence. The time evolution of scalar perturbations is studied for different parameters associated with the black hole solution. We also study the reflection and transmission coefficients along with absorption cross section for the considered black hole spacetime. It was shown that the real part of the quasi-normal frequency increases with increase in the nonlinear magnetic charge while the module of the imaginary part of the frequency decreases. The analysis of the perturbations with changing quintessential parameter c showed that perturbations with high values of c become unstable.

1 Introduction

The stability of black hole spacetimes is one of the most interesting questions in general relativity and provides us with many answers related to the black hole itself. The study of various types of perturbations such as scalar, electromagnetic and gravitational on a black hole background is an active area of research. Dynamical evolution of any kind of perturbations on a black hole background can be classified into three stages: the initial outburst of the waves, damped oscillation which is also known as quasi-normal modes (QNMs), and the late time power law tail. The first stage completely depends upon the initial perturbing field and it does not give us much information as regards the stability. The second stage is extremely important for black hole stability analysis and it consists of complex frequencies, the real part of which represents the real frequency of the perturbation and the imaginary part represents the damping. These quasi-normal modes also provide information about different black hole parameters such as mass, angular momentum, charge etc.

In pioneering work metric perturbations of the Schwarzschild black hole have been studied by Regge and Wheeler [1] and Zerilli [2]. The author of Ref. [3] analyzed numerically scattering of waves on the Schwarzschild black hole [4]. Later Chandrasekhar presented a monograph about perturbation theory of black holes [5]. The main equation of the perturbation theory is a Schrödinger-like equation and, usually, it can be solved using semiclassical or numerical methods. Many authors performed this type of perturbative investigations of black holes (see, e.g., Refs. [6–8] and the references therein).

The discovery of gravitational waves opened a new window for black hole perturbation physics [9–11]. It has been discussed that the precision of the observation/experiment of gravitational waves allows us to test alternative theories of gravity [12, 13]. Moreover, it was also shown that, regardless of the existence of horizons, waveforms can be formed [14, 15]. On the other hand, it was stated that gravastars cannot be formed in a binary black hole merger process [16]. There is other work related to the study of the ringdown process through perturbation of black holes [17–47].

The solutions of the Einstein equations for black holes have a singularity problem. The appearance of singularities can be considered as a defect of Einstein's general relativity. However, some regular black hole solutions have been obtained by various authors introducing nonlinear electrodynamics into the background gravity [48–54]. Different properties of regular black holes have been studied in the literature [55–60].

Another interesting subject is the vacuum energy or quintessence, the existence of which changes the structure of the spacetime at asymptotic infinity – it will be not flat anymore. Furthermore, there will be a cosmological horizon and behind that the geometry becomes dynamic. Particularly, the effects of a repulsive cosmological constant are widely discussed in [61–83]. The study of quasi-normal modes of

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these black holes surrounded by quintessence is an interesting topic. The effects of the quintessential parameter on quasi-normal frequencies of different black hole spacetimes were studied in [84–92]. Recently, the solution has been obtained of a regular black hole in the presence of quintessence [93]. In this work, we study the stability of this solution and the quasi-normal modes of perturbations. We also study the reflection coefficient, gray-body factor, and absorption coefficient of scalar perturbations.

The paper is organized as follows. In Sect. 2, we review the regular black hole solution surrounded by quintessence. In Sect. 3, we study the basic equations of scalar perturbations and, in Sect. 4, we give the numerical results of evolution of scalar perturbations and quasi-normal modes. Section 5 is devoted to a study of the gray-body factor and absorption coefficient. We summarize our results in Sect. 6.

Throughout the paper we use geometrized units where $G = c = 1$ and a metric with signature $(-, +, +, +)$.

2 Nonlinear magnetic-charged black hole surrounded by quintessence

A non-rotating magnetic-charged black hole surrounded with quintessence was proposed in [93]. The metric for this black hole solution with mass M and magnetic charge Q is given by

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2 \quad (1)$$

$$f(r) = 1 - \frac{2Mr^2}{r^3 + Q^3} - \frac{c}{r^{3\omega_q+1}}. \quad (2)$$

Here $\omega_q (-1 < \omega_q < -1/3)$ is the quintessential state parameter and c is a positive normalization constant. In the case $c = 0$, we can obtain the nonlinear magnetic-charged black hole in the flat background or the Hayward-like black hole [53].

We can see the behavior of the black hole solution at large and short distances. For large distances, i.e. when $Q \ll r$, the solution corresponds to a weak nonlinear magnetic field. The metric coefficient $f(r)$ becomes

$$f(r) = 1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q+1}}. \quad (3)$$

Therefore, at large distances, the solution behaves as a Schwarzschild black hole surrounded by quintessence [94]. At short distances, i.e. $Q \gg r$, it corresponds to a strong nonlinear magnetic field. The metric coefficient become

$$f(r) = 1 - \frac{\Lambda}{3}r^2 - \frac{c}{r^{3\omega_q+1}}, \quad (4)$$

where $\Lambda = 6M/Q^3 > 0$. Equation (4) behaves like dS geometry, the pressure is negative and thus we can avoid a

singular end-state of the gravitationally collapsed matter. On the other hand, the curvature singularity of the gravitationally collapsed matter should disappear and be replaced by a dS-like geometry core.

3 Massless scalar perturbation

In this section, we shall briefly write the equations of scalar perturbations of a nonlinear magnetic-charged black hole surrounded by quintessence (1). It was pointed out in [8] that, if there is no backreaction on the background, the perturbations of black hole spacetimes can be studied not only by adding the perturbation terms to the spacetime metric, but also by introducing fields into the spacetime metric.

The equation of motion of a massless scalar field Φ in curved spacetime is given by the Klein–Gordon equation,

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi) = 0. \quad (5)$$

Here Φ is a function of the coordinates (t, r, θ, ϕ) . Since the metric is spherically symmetric, the field evolution should be independent of rotations. In order to separate the variables, we write the wave function as

$$\Phi = Y(\theta, \phi)\frac{\psi(r, t)}{r}, \quad (6)$$

where the $Y(\theta, \phi)$ are the spherical harmonics. Now, (5) simplifies to

$$-\frac{\partial^2\psi}{\partial t^2} + \frac{\partial^2\psi}{\partial r_*^2} + (\omega^2 - V(r))\psi(r) = 0, \quad (7)$$

where

$$V(r) = \frac{l(l+1)f(r)}{r^2} + \frac{f(r)f'(r)}{r}. \quad (8)$$

Here l is the spherical harmonic index and r_* is the well-known tortoise coordinate, given by

$$dr_* = \frac{dr}{f(r)}. \quad (9)$$

Here, r_* cannot be evaluated explicitly, because of the nature of the function $f(r)$. When $r \rightarrow r_c$, $r_* \rightarrow \infty$. When $r \rightarrow r_+$, $r_* \rightarrow -\infty$. Here r_+ is the event horizon and r_c is the cosmological horizon.

The effective potential $V(r)$ is plotted in Fig. 1 to see how it changes with the charge Q , the mass M and the spherical harmonic index l . The first panel shows the behavior of the potential for different values of the charge parameter. We can see that the height of the potential increases with the magnetic

Fig. 1 Left panel: behavior of the effective potential $V(r)$ with the charge. Here $l = 3$, $M = 1$ and $c = 0.001$. The solid line, the dotted line and the dashed line correspond to $Q = 0.2$, $Q = 0.6$ and $Q = 1.0$, respectively. Mid panel: Behavior of the effective potential $V(r)$ with mass. Here $l = 3$, $Q = 0.1$ and $c = 0.01$. The solid line, the dotted

line and the dashed line corresponds to $M = 1$, $M = 2$ and $M = 3$ respectively. Right panel: behavior of the effective potential $V(r)$ with the spherical harmonic index. Here $M = 1$, $Q = 0.9$ and $c = 0.01$. The solid line, the dotted line and the dashed line corresponds to $l = 1$, $l = 2$ and $l = 3$, respectively

charge and this represents the suppression of emission modes for higher charge values. The middle panel shows the change in the potential when mass is varied and we can see that the potential decreases for an increase in mass. The right panel shows the increase of the peak of potential when we increase the spherical harmonic index. This again represents the suppression of scalar emission modes for higher values of l .

4 Numerical results

4.1 Evolution of scalar perturbations

In order to study the scalar perturbations of a nonlinear magnetic-charged black hole surrounded by quintessence, we rewrite the wave equation for the propagation of scalar perturbations (7) in null coordinates defined by

$$du = dt - dr_*, \quad dv = dt + dr_*. \quad (10)$$

The wave equation becomes

$$\left(4 \frac{\partial^2}{\partial u \partial v} + V(u, v)\right) \psi(u, v) = 0. \quad (11)$$

We solve Eq. (11) numerically with a simple initial condition, i.e., a Gaussian pulse profile centered around v_c with width σ ,

$$\psi(u = u_0, v) = \exp \left[-\frac{(v - v_c)^2}{2\sigma^2} \right]. \quad (12)$$

The integration is done on a triangular grid. This method is extensively reviewed in [43, 95, 96]. We follow [96] for discretization of the wave function. During the integration of Eq. (11), we extract the values of the field ψ at constant r_* and allow the field to evolve to larger values of t .

We plot the time evolution of scalar perturbations in Fig. 2 for different parameters. The left panel shows that scalar perturbations with larger l live longer. In the middle panel, we plot ψ for different values of c and we see that the perturbations start to become unstable for higher values of c . In the right panel of Fig. 2, we can see the behavior of the perturbations with respect to the charge Q .

4.2 Quasi-normal modes

One of our main goals in this paper is to study the quasi-normal modes (QNMs) and the stability of the perturbations in a nonlinear magnetic-charged black hole spacetime surrounded by quintessence. We shall focus on massless scalar perturbations here.

QNMs for a perturbed black hole spacetime are the solutions to the wave equation given in (7). In order to obtain these solutions, one has to impose proper boundary conditions. We impose the requirement that the wave at the horizon be purely incoming and that the wave at spatial infinity be purely outgoing:

$$\begin{aligned} \psi(r) &\sim e^{-i\omega r_*} \quad \text{as } r_* \rightarrow -\infty (r \rightarrow r_h), \\ \psi(r) &\sim e^{i\omega r_*} \quad \text{as } r_* \rightarrow +\infty (r \rightarrow r_c), \end{aligned} \quad (13)$$

where r_h and r_c are event horizon and cosmological horizon, respectively.

It is impossible to analytically solve the time-independent, second-order differential equation (7) with the potential (8) for a nonlinear magnetic-charged black hole surrounded by quintessence. Therefore, we use the sixth-order WKB method for numerical calculations which is given by the relation

$$\frac{i(\omega^2 - V_0)}{\sqrt{-2V_0''}} + \sum_{j=2}^6 \Lambda_j = n + \frac{1}{2}, \quad (14)$$

Fig. 2 Left panel: time evolution behavior of scalar perturbations for $l = 0$, $l = 1$ and $l = 2$. Mid panel: Time evolution behavior of scalar perturbations for $c = 0.001$, $c = 0.0001$, $c = 0.005$. Right panel: time evolution behavior of scalar perturbations for $Q = 0.2$, $Q = 0.6$, $Q = 0.9$

Table 1 Fundamental quasi-normal frequencies ($n = 0$) for scalar perturbations of a nonlinear magnetic-charged black hole surrounded by quintessence. Here $\omega_q = -2/3$, $l = 2$

Q	c	$\text{Re}(\omega)$	$\text{Im}(\omega)$
0.1	0.0	0.48366	-0.096758
0.1	0.0001	0.483432	-0.096701
0.1	0.001	0.48137	-0.096191
0.5	0.0	0.485999	-0.095791
0.5	0.0001	0.485769	-0.095736
0.5	0.001	0.483698	-0.095235
0.9	0.0	0.498703	-0.089525
0.9	0.0001	0.498468	-0.089477
0.9	0.001	0.496348	-0.089045

Table 2 Fundamental quasi-normal frequencies ($n = 0$) for scalar perturbations of a nonlinear magnetic-charged black hole surrounded by quintessence. Here $\omega_q = -2/3$, $Q = 0.6$

l	c	$\text{Re}(\omega)$	$\text{Im}(\omega)$
1	0.0	0.295528	-0.096078
1	0.0001	0.295377	-0.096024
1	0.001	0.294019	-0.095544
2	0.0	0.487769	-0.095027
2	0.0001	0.487538	-0.094973
2	0.001	0.485461	-0.094480
3	0.0	0.681045	-0.094792
3	0.0001	0.68073	-0.094737
3	0.001	0.677896	-0.094240

where a prime denotes the derivative with respect to the tortoise coordinate r_* and V_0 stands for the value of effective potential at its local maxima. j denotes the order of the WKB approximation and Λ_j is a correction term corresponding to the j th order. Expressions for Λ_j can be found in [97,98].

In Tables 1 and 2, we list the fundamental quasi-normal frequencies $n = 0$ of scalar perturbations of a nonlinear magnetic-charged black hole surrounded by quintessence.

Table 1 shows the variation with respect to the charge for different values of c . The real part of the frequency decreases with increasing c . With increasing charge the real part of the frequency increases but the magnitude of the imaginary part of the frequency decreases. In Table 2, we list the fundamental quasi-normal frequencies with respect to smaller spherical harmonic index l for different values of c . We can see that the real part of the frequency decreases and the magnitude of the imaginary part of the frequency also decreases with increase in c for the same l . Here we have to note that the WKB method has low accuracy in the small l regime.

In Fig. 3 we plot the quasi-normal frequencies of scalar perturbations with respect to the charge parameter Q for $c = 0$, $c = 0.0001$ and $c = 0.001$. The left panel shows the real part of the frequency and the right panel shows the imaginary part of the frequency. Here we consider $M = 1$ and $l = 2$ and $\omega_q = -2/3$.

In Fig. 4, we plot the quasi-normal frequencies with respect to spherical harmonic index l for $c = 0$, $c = 0.0001$ and $c = 0.001$. The left panel shows the real frequencies and the right panel shows the imaginary frequencies. Here we consider $M = 1$, $Q = 0.6$ and $\omega_q = -2/3$.

In Fig. 5, we plot the quasi-normal frequencies with respect to the charge parameter Q for $n = 0$, $n = 1$. The left panel shows the real frequencies and the right panel shows the imaginary frequencies. Here we consider $M = 1$, $c = 0.001$, $\omega_q = -2/3$ and $l = 2$.

In Fig. 6, we plot the quasi-normal frequencies with respect to spherical harmonic index for different values of quintessential parameter ω_q . The left panel shows the increase of real part of the frequency with l and we see that the change with respect to ω_q is very small. The right panel shows the magnitude of imaginary part of quasi-normal frequencies which decreases with increasing l .

In Fig. 7, we plot the quasi-normal frequencies with respect to quintessential parameter ω_q . The left panel shows the increase of real part of the quasi-normal frequency with the increase in quintessential parameter ω_q and the right panel shows the decrease of imaginary part of quasi-normal with the increase in quintessential parameter.

Fig. 3 The left panel shows the dependence of real part of quasi-normal frequencies on Q for $c = 0$, $c = 0.0001$ and $c = 0.001$. The right panel shows the dependence of imaginary part of quasi-normal frequencies on Q for $c = 0$, $c = 0.0001$ and $c = 0.001$

Fig. 4 The left panel shows the dependence of real part of quasi-normal frequencies on l for $c = 0$, $c = 0.0001$ and $c = 0.001$. The right panel shows the dependence of imaginary part of quasi-normal frequencies on l for $c = 0$, $c = 0.0001$ and $c = 0.001$

Fig. 5 The left panel shows the dependence of real part of quasi-normal frequencies on Q for $n = 0$ and $n = 1$. The right panel shows the dependence of imaginary part of quasi-normal frequencies on Q for $n = 0$ and $n = 1$

Fig. 6 The left panel shows the dependence of real part of quasi-normal frequencies on l for $\omega_q = -5/6$, $\omega_q = -2/3$ and $\omega_q = -1/2$. The right panel shows the dependence of imaginary part of quasi-normal

frequencies on l for $\omega_q = -5/6$, $\omega_q = -2/3$ and $\omega_q = -1/2$. Here $M = 1$, $n = 0$, $Q = 0.8$ and $c = 0.0001$

Fig. 7 The left panel shows the dependence of real part of quasi-normal frequencies on ω_q . The right panel shows the dependence of imaginary part of quasi-normal frequencies on ω_q . Here $M = 1$, $l = 10$, $n = 0$, $Q = 0.8$ and $c = 0.0001$

5 Gray-body factors and absorption coefficient

5.1 Nature of gray-body factors

In this subsection, we shall discuss the reflection coefficients $R(\omega)$ and transmission coefficients $T(\omega)$ for different parameter spaces for scalar perturbations around a nonlinear magnetically charged black hole surrounded by quintessence. Our analysis in this section will be analytic and we shall use the third-order WKB method found in [42, 99–101] for our computations.

Black holes are believed to be thermal systems with an associated temperature and entropy. Therefore black holes radiate and the radiation is known as Hawking radiation [102, 103]. Hawking showed that, at the event horizon, the emission rate of a black hole in a mode with frequency ω is given by

$$\Gamma(\omega) = \frac{1}{e^{\beta\omega} \pm 1} \frac{d^3k}{(2\pi)^3}. \quad (15)$$

Here, β is the inverse of Hawking temperature and the plus (minus) sign corresponds to fermions (bosons). But the geometry outside the event horizon might have an important effect on the emission rate measured by an observer located far away. In other words, the geometry outside the horizon will act as a potential barrier for the Hawking radiation emitted from the black hole. A part of the radiation will be tunneled through the potential barrier and reach the distant observer and the other part will be reflected back towards the black hole. The radiation recorded by the distant observer will no longer appear as a blackbody. Mathematically, we can write emission rate measured by an observer at infinity for a frequency mode ω as

$$\Gamma(\omega) = \frac{\gamma(\omega)}{e^{\beta\omega} \pm 1} \frac{d^3k}{(2\pi)^3}, \quad (16)$$

where $\gamma(\omega)$ is called the gray-body factor. This quantity gets its name from the fact that it modifies the emitted spectrum of a black hole to a gray-body. The gray-body factor is naturally defined as

$$\gamma(\omega) = |T(\omega)|^2. \quad (17)$$

Now, the reflected and the transmitted waves can be represented as follows:

$$\begin{aligned} \psi(r_*) &= T(\omega)e^{-i\omega r_*}, \quad r_* \rightarrow -\infty (r \rightarrow r_h) \\ \psi(r_*) &= e^{-i\omega r_*} + R(\omega)e^{-i\omega r_*}, \quad r_* \rightarrow +\infty (r \rightarrow r_c), \end{aligned} \quad (18)$$

where $R(\omega)$ and $T(\omega)$ are the reflection and transmission coefficient, respectively, and they are related by

$$|R(\omega)|^2 + |T(\omega)|^2 = 1. \quad (19)$$

Now let us discuss the WKB method developed in [42, 99–101]. If r_0 is the value of r where the potential $V(r)$ is the maximum, then depending on the relation between ω and $V(r_0)$, there are three cases to consider:

- $\omega^2 \ll V(r_0)$. Here the transmission coefficient is close to zero and the reflection coefficient is almost equal to one.
- $\omega^2 \gg V(r_0)$. Here the transmission coefficient is close to one and the reflection coefficient is almost equal to zero.
- $\omega^2 \sim V(r_0)$. We shall consider this case because the WKB approximation has high value of accuracy for $\omega^2 \sim V(r_0)$.

In this approximation, the reflection coefficient is given by

$$R(\omega) = (1 + e^{-2\pi i\alpha})^{-1/2}, \quad (20)$$

where α is given by

$$\alpha = \frac{i(\omega^2 - V_0)}{\sqrt{-2V_0''}} - \Lambda_2 - \Lambda_3. \quad (21)$$

In the third-order WKB approximation Λ_2 and Λ_3 are given by

$$\begin{aligned} \Lambda_2 &= \frac{1}{\sqrt{-2V^{(2)}(r_0)}} \left[\frac{1}{8} \left(\frac{V^{(4)}(r_0)}{V^{(2)}(r_0)} \right) \left(b^2 + \frac{1}{4} \right) \right. \\ &\quad \left. - \frac{1}{288} \left(\frac{V^{(3)}(r_0)}{V^{(2)}(r_0)} \right)^2 (7 + 60b^2) \right], \\ \Lambda_3 &= \frac{n + \frac{1}{2}}{-2V^{(2)}(r_0)} \left[\frac{5}{6912} \left(\frac{V^{(3)}(r_0)}{V^{(2)}(r_0)} \right)^4 (188b^2 + 77) \right. \\ &\quad - \frac{1}{384} \left(\frac{(V^{(3)})^2(r_0)V^{(4)}(r_0)}{(V^{(2)})^3(r_0)} \right) (51 + 100b^2) \\ &\quad + \frac{1}{2304} \left(\frac{V^{(4)}(r_0)}{V^{(2)}(r_0)} \right)^2 (68b^2 + 67) \\ &\quad - \frac{1}{288} \left(\frac{V^{(6)}(r_0)}{V^{(2)}(r_0)} \right) (4b^2 + 5) \\ &\quad \left. + \frac{1}{288} \left(\frac{V^{(3)}(r_0)V^{(5)}(r_0)}{(V^{(2)})^2(r_0)} \right) (28b^2 + 19) \right]. \end{aligned} \quad (22)$$

In these expressions $b = n + \frac{1}{2}$, $V^n(r_0) = d^n V/dr_*^n$ at $r = r_0$.

In the left and middle panel of Figs. 8, 9, 10 and 12, we have plotted the dependence of reflection and absorption coefficient on the frequency of scalar perturbations, respectively. Figure 8 shows the variation for different spherical harmonic indices l , Fig. 9 shows the variation for different charges Q , Fig. 10 shows the variation for different quintessential parameter values c and Fig. 12 shows the variation for different values of quintessential parameter ω_q .

When the spherical harmonic index is varied, the transmission coefficient $|T(\omega)|^2$ becomes smaller and hence the reflection coefficient $|R(\omega)|^2$ becomes larger as seen in the

Fig. 8 The left panel shows the dependence of $|R(\omega)|^2$ on ω . The middle panel shows the dependence of $|T(\omega)|^2$ or the gray-body factor $\gamma(\omega)$ on ω . Here $M = 1$, $Q = 0.2$, $c = 0.0001$ and $\omega_q = -2/3$. The

right panel shows the dependence of total absorption cross section σ on ω for $Q = 0.4$, $Q = 0.6$ and $Q = 0.8$

Fig. 9 The left panel shows the dependence of $|R(\omega)|^2$ on ω . The middle panel shows the dependence of $|T(\omega)|^2$ or the gray-body factor $\gamma(\omega)$ on ω . The right panel shows the dependence of partial absorption cross section σ_l on ω . Here $M = 1$, $l = 3$, $c = 0.0001$ and $\omega_q = -2/3$

Fig. 10 The left panel shows the dependence of $|R(\omega)|^2$ on ω . The middle panel shows the dependence of $|T(\omega)|^2$ or the gray-body factor $\gamma(\omega)$ on ω . The right panel shows the dependence of partial absorption cross section σ_l on ω . Here $M = 1$, $Q = 0.2$, $l = 3$ and $\omega_q = -2/3$

Fig. 11 Plot shows the dependence of gray-body factor on ω for different values of ω_q . The difference is indistinguishable here

middle and left panel of Fig. 8. The transmission coefficient $|T(\omega)|^2$ decreases with the increase in magnetic charge and hence the reflection coefficient $|R(\omega)|^2$ increases as can be seen in the middle and left panel of Fig. 9. Similarly, $|T(\omega)|^2$ increases with increase in quintessential parameter c and hence $|R(\omega)|^2$ decreases as seen in the middle and left panel of Fig. 10. The change of transmission and reflection coefficient is very small when we vary the quintessential parameter ω_q as can be seen from Fig. 11. In order to see the variation, we plot $|R(\omega)|^2$, $|T(\omega)|^2$ with higher resolution. We can see that transmission coefficient $|T(\omega)|^2$ decreases for increase in quintessential parameter ω_q and hence reflection coefficient $|R(\omega)|^2$ increases.

5.2 Absorption cross section

In this subsection we shall discuss the partial absorption cross section in the context of scalar perturbations for a nonlinear magnetically charged black hole surrounded by quintessence. Partial absorption cross section and total absorption cross section are defined as

$$\sigma_l = \frac{\pi(2l+1)}{\omega^2} |T(\omega)|^2, \quad (23)$$

$$\sigma = \sum_l \frac{\pi(2l+1)}{\omega^2} |T(\omega)|^2. \quad (24)$$

We plot the variation of partial absorption cross section σ_l with respect to the frequency of scalar perturbations in the right panels of Figs. 9, 10 and 12. Figure 9 shows the variation for different charges Q and Fig. 10 shows the variation for different quintessential parameter values c . Figure 12 shows the variation of partial absorption with respect to the quintessential parameter ω_q . In these cases, the variation is extremely small. In the right panel of Fig. 8, we have plotted the total absorption cross section for different charges and for convenience we have summed over $l = 1$ to $l = 10$ modes to determine σ . Total absorption cross section decreases for increasing value of charge parameter Q . As the transmission coefficient attains the value of one at some critical value of ω , the total absorption cross section falls off as $1/\omega^2$ regardless of the black hole parameters. Therefore we observe the fall-off region in this particular plot.

Fig. 12 The left panel shows the dependence of $|R(\omega)|^2$ on ω . The middle panel shows the dependence of $|T(\omega)|^2$ or the gray-body factor $\gamma(\omega)$ on ω . The right panel shows the dependence of partial absorption cross section σ_l on ω . Here $M = 1$, $Q = 0.6$, $l = 2$ and $c = 0.0001$

6 Summary and conclusion

In this paper, we have focused on scalar perturbations of a nonlinear magnetic-charged black hole surrounded by quintessence. First, we numerically calculated the time evolution of scalar perturbations around the considered black hole spacetime and we found that perturbations with higher spherical harmonic index l live longer. We also saw the behavior of perturbations with changing cosmological constant parameter c and we note that perturbations with the higher value of c becomes unstable. The behavior of the perturbation for different values of the charge parameter is also studied and reported.

We have used the sixth-order WKB method to calculate the quasi-normal frequencies of scalar perturbations for a nonlinear magnetic-charged black hole surrounded by quintessence. We studied the dependence of quasi-normal frequencies on charge Q , spherical harmonic index l and cosmological constant parameter c . We see that the real part of quasi-normal frequency increases with increase in charge Q for both fundamental ($n = 0$) and first overtone mode ($n = 1$) but the magnitude of the imaginary part of the frequency decreases. The magnitude of the imaginary part of the frequency decreases with increase in charge for different values of c . The real part of the quasi-normal frequencies increase monotonically with respect to l but the variation with respect to c is extremely small and the lines overlap on the plot. The magnitude of the imaginary frequencies decreases with an increase in l and c .

We have further studied the gray-body factors $\gamma(\omega)$ and the partial absorption cross section σ_l of scalar perturbations for a nonlinear magnetic-charged black hole surrounded by quintessence. We studied the dependence of these quantities for different parameters of the black hole spacetime. The transmission coefficient $|T(\omega)|^2$, or the gray-body factor $\gamma(\omega)$, becomes smaller and hence reflection coefficient $|R(\omega)|^2$ becomes larger with increase in l . The transmission coefficient $|T(\omega)|^2$ decreases with the increase in charge and hence the reflection coefficient $|R(\omega)|^2$ increases. Similarly, $|T(\omega)|^2$ increases with increase in quintessential parameter c

and hence $|R(\omega)|^2$ decreases but the change is too small. We also investigated the effect of changing quintessential parameter ω_q on transmission and reflection coefficient and partial absorption cross section and we found that the effect of ω_q on these quantities are very small.

For future directions of research, it would be interesting to study the electromagnetic and gravitational perturbations for the nonlinear magnetic-charged black hole surrounded by quintessence. We hope to understand the behavior of these perturbations with respect to the cosmological constant parameter c .

Acknowledgements The authors thank Dmitry Ayzenberg and Askar Abdikamalov for helpful discussion. This work was supported by the National Natural Science Foundation of China (Grant no. U1531117) and Fudan University (Grant no. IDH1512060). H.C. also acknowledges support from the China Scholarship Council (CSC), Grant no. 2017GXZ019020. The research is supported in part by Grant no. VA-FA-F-2-008 and no. YFA-Ftech-2018-8 of the Uzbekistan Ministry for Innovation Development, by the Abdus Salam International Centre for Theoretical Physics through Grant no. OEA-NT-01 and by Erasmus+ exchange grant between Silesian University in Opava and National University of Uzbekistan. A.A. thanks the Nazarbayev University for the hospitality.

Data Availability Statement This manuscript has no associated data or the data will not be deposited [Authors' comment: This is a theoretical work and all results can be easily obtained from the equations presented in the manuscript.]

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