Transport coefficients of hot magnetized QCD matter beyond the lowest Landau level approximation

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Received: 2 October 2018 / Accepted: 4 February 2019 / Published online: 13 February 2019

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Abstract In this article, shear viscosity, bulk viscosity, and thermal conductivity of a QCD medium have been studied in the presence of a strong magnetic field. To model the quark–gluon plasma, an extended quasi-particle description of the hot QCD equation of state in the presence of the magnetic field has been adopted. The effects of higher Landau levels on the temperature dependence of viscous coefficients (bulk and shear viscosities) and thermal conductivity have been obtained by considering the $1 \rightarrow 2$ processes in the presence of the strong magnetic field. An effective covariant kinetic theory has been set up in (1+1)-dimensional that includes mean field contributions in terms of quasi-particle dispersions and magnetic field to describe the Landau level dynamics of quarks. The sensitivity of these parameters to the magnitude of the magnetic field has also been explored. Both the magnetic field and mean field contributions have seen to play a significant role in obtaining the temperature behaviour of the transport coefficients of the medium.

1 Introduction

Relativistic heavy-ion collision (RHIC) experiments have reported the presence of strongly coupled matter–Quark–gluon plasma (QGP) as a near-ideal fluid [1–5]. The quantitative estimation of the experimental observables such as the collective flow and transverse momentum spectra of the produced particles from the hydrodynamic simulations involve the dependence upon the transport parameters of the medium. Thus, the transport coefficients are the essential input parameters for the hydrodynamic evolution of the system.

Recent investigations show that intense magnetic field is created in the early stages of the non-central asymmetric collisions [6–9]. This magnetic field affects the thermodynamic and transport properties of the hot dense QCD matter produced in the RHIC. Reference [10] describes the extension of ECHO-QGP [11, 12] to the magnetohydrodynamic regime. The recent major developments regarding the intense magnetic field in heavy-ion collision include the chiral magnetic effect [13–19], chiral vortical effects [20–22] and very recent realization of global Λ -hyperon polarization in non-central RHIC [23, 24]. This sets the motivation to study the transport coefficients in presence of the strong magnetic field. The transport parameters under investigation are the viscous coefficients (shear and bulk) and the thermal conductivity of the hot magnetized QGP.

The dissipative effects are not only significant in the hydrodynamical evolution of QGP, but also in particle production, final-state hadron spectra and other observables derived from them. In the recent works [25, 26], the authors described the properties of matter produced in the energetic heavy-ion collisions with the identified hadrons. Importance of the transport processes in RHIC is well studied [27] and reconfirmed by the recent ALICE results [28–31]. There have been several attempts to evaluate the viscous coefficients in the confined phase using effective models of the hadron gas [32–34]. Furthermore, the significance of viscous effects in the evolution of the Hubble parameter in the QCD era of the early Universe is described in [35–37].

Quantizing quark/antiquark field in the presence of magnetic field gives the Landau levels as energy eigenvalues. The quark/antiquark dynamics is governed by (1 + 1)-dimensional Landau level kinematics whereas gluonic degrees of freedom remain intact in the presence of magnetic field [38, 39]. However, gluonic dynamics can be indirectly affected by the magnetic field through the Debye mass of the system.

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Viscous coefficients can be estimated from Green-Kubo formulation both in the presence and absence of magnetic field [38,40–42]. Lattice results for viscosities to entropy ratio are also well investigated [43–45]. Viscous pressure tensor quantifies the energy-momentum dissipation with the space-time evolution and is characterized by seven viscous coefficients in the presence of magnetic field [46]. The seven viscous coefficients consist of two bulk viscosities (both transverse and longitudinal) and five shear viscosities. The present investigations are focused on the longitudinal component (along the direction of $\vec{B}$) of shear and bulk viscosities since other components of viscosities are negligible in the strong magnetic field. Another key transport coefficient under investigation is the thermal conductivity of the QGP medium. The temperature dependence of thermal conductivity has been studied in the absence of magnetic field in the Ref. [47]. The equations of state (EoS) dependence on the viscous coefficients, electric and thermal conductivities have been studied in Ref. [48]. The first step towards the estimation of transport coefficients from the effective kinetic theory is to include proper collision integral for the processes in the strong field. This can be done within the relaxation time approximation (RTA). Microscopic processes or interactions are the inputs of the transport coefficients and are incorporated through thermal relaxation times. Note that the $1 \rightarrow 2$ processes such as quark–antiquark pair production/annihilation are dominant in the presence of strong magnetic field [49,50].

The prime focus of the present article is to estimate the temperature behaviour of the transport coefficients such as bulk viscosity, shear viscosity and thermal conductivity, incorporating the EoS effects in the presence of the strong magnetic field. Estimation of the transport parameters can be done in two equivalent approaches *viz.*, the hard thermal loop effective theory (HTL) [51–53] and the relativistic semiclassical transport theory [49,54–57]. The present analysis is done with the relativistic transport theory by employing the Chapman–Enskog method. Thermal medium effects are encoded in the quark/antiquark and gluonic degrees of freedom by adopting the effective fugacity quasiparticle model (EQPM) [39,58–60]. The transport coefficients pick up the mean field term (force term) as described in Ref. [61]. The mean field term comes from the local conservations of number current and stress-energy tensor in the covariant effective kinetic theory. In the current analysis, we investigate the mean field corrections in the presence of strong magnetic field and study the temperature behaviour of the transport coefficients. Here, the strong magnetic field restricts the calculations to $(1+1)$ -dimensional (dimensional reduction) covariant effective kinetic theory for quarks and antiquarks.

The manuscript is organized as follows. In Sect. 2, the mathematical formulation for the estimation of transport

coefficients from the effective covariant kinetic theory is discussed along with the quasiparticle description of hot QCD medium in the strong magnetic field. Section 3 deals with the thermal relaxation for the $1 \rightarrow 2$ processes in the strong magnetic field. Predictions of the transport coefficients in the magnetic field are discussed in Sect. 4. Finally, in Sect. 5 the summary and outlook of the are presented.

2 Formalism: Transport coefficients at strong magnetic field

The strong magnetic field $\vec{B} = B\hat{z}$ constraints the quarks/antiquarks motion parallel to field with a transverse density of states. The viscous coefficients [38,62] and heavy quark diffusion coefficient [63] have been perturbatively calculated under the regime $\alpha_s |q_f eB| \ll T^2 \ll |q_f eB|$ with the lowest Landau level (LLL) approximation. But the validity of LLL approximation is questionable since higher Landau level contributions are significant at $|eB| = 10m_\pi^2$ in the temperature range above 200 MeV. Here, we are focusing on the more realistic regime $gT \ll \sqrt{|q_f eB|}$ in which higher Landau level (HLL) contributions are significant. In the very recent work [49], Fukushima and Hidaka have been estimated the longitudinal conductivity of magnetized QGP with full Landau level resummation in the regime $gT \ll \sqrt{|q_f eB|}$.

The formalism for the estimation of transport coefficients includes the quasiparticle modeling of the system away from the equilibrium followed by the setting up of the effective kinetic theory for different processes. Quasiparticle models encode the EoS effects, *viz.*, effective fugacity or with effective mass. The later include self-consistent and single parameter quasiparticle models [64–66], NJL and PNJL based quasiparticle models [67–71], effective mass with Polyakov loop [72–75] and recently proposed quasiparticle models based on the Gribov–Zwanziger (GZ) quantization [76–79]. Here, the analysis is done within the effective fugacity quasiparticle model (EQPM) where the medium interactions are encoded through temperature dependent effective quasigluon and quasiquark/antiquark fugacities, z_g and z_q respectively. The extended EQPM describes the QGP medium effects in strong magnetic field [39]. We considered the $(2+1)$ flavor lattice QCD EoS (LEoS) [80,81] and the 3-loop HTLpt EOS [82,83] for the effective description of QGP in the strong magnetic field [39,62].

2.1 Transport coefficients from effective $(1+1)$ -D kinetic theory

In the absence of magnetic field, the particle four flow $\bar{N}^\mu(x)$ can be defined in terms of quasiparticle (dressed) momenta $\vec{p}_k$ within EQPM as [61],

$$\bar{N}^\mu(x) = \sum_{k=1}^N v_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_k} \bar{p}_k^\mu f_k^0(x, \vec{p}_k) + \sum_{k=1}^N \delta\omega v_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_k} \frac{\langle \bar{p}_k^\mu \rangle}{E_k} f_k^0(x, \vec{p}_k), \quad (1)$$

in which v_k is the degeneracy factor of the k^{th} species. Here, we are considering non-zero masses (m_f) for quarks (up, down and strange quarks with masses $m_u = 3$ MeV, $m_d = 5$ MeV and $m_s = 100$ MeV respectively) and hence $E_k = \sqrt{|\vec{p}_k|^2 + m_f^2}$ for quarks/antiquarks and $E_k = |\vec{p}|$ for gluons. The term $\langle \bar{p}^\mu \rangle = \Delta^{\mu\nu} \bar{p}_\nu$ is the irreducible tensor with $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ as the projection operator. The metric has the form $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$. The quasiquark distribution function in local rest frame with the hydrodynamic four-velocity $u^\mu \equiv (1, \mathbf{0})$ is given by,

$$f_{q,g}^0 = \frac{z_{q,g} \exp[-\beta(u^\mu p_\mu)]}{1 \pm z_{q,g} \exp[-\beta(u^\mu p_\mu)]}, \quad (2)$$

with $p^\mu = (E, \vec{p})$. Quasiparticle momenta (dressed momenta) and bare particle four-momenta can be related from the dispersion relations as,

$$\bar{p}^\mu = p^\mu + \delta\omega u^\mu, \quad \delta\omega = T^2 \partial_T \ln(z_{q,g}), \quad (3)$$

which modifies the zeroth component of the four-momenta in the local rest frame. Hence, we have

$$\bar{p}^0 \equiv \omega_k = E_k + \delta\omega. \quad (4)$$

The dispersion relation in Eq. (4) encodes the collective excitation of quasiparton along with the single particle energy. Also, the energy-momentum tensor $\bar{T}^{\mu\nu}$ in terms of dressed momenta takes the following form,

$$\bar{T}^{\mu\nu}(x) = \sum_{k=1}^N v_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_k} \bar{p}_k^\mu \bar{p}_k^\nu f_k^0(x, \vec{p}_k) + \sum_{k=1}^N \delta\omega v_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_k} \frac{\langle \bar{p}_k^\mu \bar{p}_k^\nu \rangle}{E_k} f_k^0(x, \vec{p}_k), \quad (5)$$

where $\langle \bar{p}_k^\mu \bar{p}_k^\nu \rangle = \frac{1}{2}(\Delta^{\mu\alpha} \Delta^{\nu\beta} + \Delta^{\mu\beta} \Delta^{\nu\alpha}) \bar{p}_\alpha \bar{p}_\beta$.

In our case, Eq. (5) should be rewritten for the hot QCD medium in the strong magnetic field $\vec{B} = B\hat{z}$ limit. Thereafter, the transport coefficients could be obtained by realizing the microscopic (transport theory) definition of $\bar{T}^{\mu\nu}$ to the macroscopic decomposition at various order. Recall that the EQPM in the presence of a strong magnetic field is studied by considering the Landau level dynamics in the dispersion relation for quarks whereas gluonic part remain invariant in magnetic field [39, 62]. The quasi-quark/antiquark distribution function in the strong magnetic field background

takes the form as in Eq. (2) with the particle four-momenta $p_\parallel^\mu = (\omega_l, 0, 0, \vec{p}_z)$. The zeroth component of four-momenta becomes,

$$\bar{p}^0 \equiv \omega_l = \sqrt{\vec{p}_z^2 + m_f^2} + 2l |q_f e B| + \delta\omega. \quad (6)$$

where $\sqrt{\vec{p}_z^2 + m_f^2} + 2l |q_f e B| \equiv E_l$ is the Landau level energy eigenvalue in the strong magnetic field.

Macroscopically, the energy-momentum tensor in the presence of magnetic field $\vec{B} = B\hat{z}$ can be decomposed as [38],

$$\bar{T}^{\mu\nu} = \varepsilon u^\mu u^\nu - P_\perp \Xi^{\mu\nu} + P_\parallel b^\mu b^\nu + \tau^{\mu\nu}, \quad (7)$$

where u^μ is the flow vector and $b^\mu = \epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha} u_\beta / (2B)$ with $B = \sqrt{-B^\mu B_\mu}$. Here, $P_\perp$ and $P_\parallel$ are the transverse and longitudinal components of pressure respectively and holds the relation $P_\perp = P_\parallel - MB$, where the magnetization $M = (\frac{\partial P}{\partial B})_T$. The tensor $\Xi^{\mu\nu} = \Delta^{\mu\nu} + b^\mu b^\nu$, projects out the two-dimensional space orthogonal to both the flow and magnetic field. In the presence of strong magnetic field, the pressure can be defined as,

$$P = P_{\parallel q} + P_g, \quad (8)$$

with $P \equiv \bar{T}^{\mu\nu} b_\mu b_\nu = \bar{T}^{33}$. Here, $P_{\parallel q}$ is the dominant quark and antiquark contribution to the pressure in the strong magnetic field [38, 39] and have the following form,

$$P_\parallel = \sum_{l=0}^{\infty} \frac{|q_f e B|}{\pi^2} N_c \int_0^\infty dp_z \frac{p_z^2}{E_l} \mu_l f_q^0. \quad (9)$$

The integration phase factor in the strong field due to dimensional reduction [84–86] is defined as,

$$\int \frac{d^3 p}{(2\pi)^3} \rightarrow \frac{|q_f e B|}{2\pi} \int_{-\infty}^\infty \frac{dp_z}{2\pi} \mu_l, \quad (10)$$

where $\mu_l = (2 - \delta_{l0})$ is the spin degeneracy factor of the Landau levels. Since gluonic dynamics are not directly affected by the magnetic field, the gluonic contribution P_g retains the same form as in the absence of magnetic field and is well investigated in the work [58]. Note that in the presence of the strong magnetic field quark/antiquark contribution is dominant compared with that of gluons [38, 49, 50]. Also, we can define the quark and antiquark contribution to energy density in the strong field as,

$$\varepsilon_\parallel = \sum_{l=0}^{\infty} \frac{|q_f e B|}{\pi^2} N_c \int_0^\infty dp_z \frac{(\omega_p)^2}{\omega_l} \mu_l f_q^0. \quad (11)$$

Since the quark dynamics is constrained in the $(1+1)$ -dimensional space, both b^μ and u^μ are longitudinal $(1+1)$ -dimensional vector and at the same time b^μ is orthogonal to u^μ . The longitudinal projection operator $\Delta_{\parallel}^{\mu\nu}$ is perpendicular to u^μ and can be constructed from b^μ [87] as,

$$\Delta_{\parallel}^{\mu\nu} \equiv g_{\parallel}^{\mu\nu} - u^\mu u^\nu = -b^\mu b^\nu, \quad (12)$$

where $g_{\parallel}^{\mu\nu} = \text{diag}(1, 0, 0, -1)$. Hence, in the strong magnetic field, the equilibrium energy-momentum tensor from the quark/antiquark part takes the form as follows,

$$T^{\mu\nu} = \varepsilon_{\parallel} u^\mu u^\nu - P_{\parallel} \Delta_{\parallel}^{\mu\nu}. \quad (13)$$

In the strong magnetic field, $T^{\mu\nu}$ can be defined in terms of quasiparticle momenta of quarks and antiquarks as the following,

$$\begin{aligned} T^{\mu\nu}(x) = & \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \mu_l \frac{|q_{fk} eB|}{2\pi} N_c \int_{-\infty}^{\infty} \frac{d\bar{p}_{zk}}{(2\pi)\omega_{lk}} \bar{p}_{\parallel k}^{\mu} \bar{p}_{\parallel k}^{\nu} \\ & \times f_k^0(x, \bar{p}_{zk}) + \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \delta\omega_{lk} \frac{|q_{fk} eB|}{2\pi} N_c \\ & \times \int_{-\infty}^{\infty} \frac{d\bar{p}_{zk}}{(2\pi)\omega_{lk}} \frac{\langle \bar{p}_{\parallel k}^{\mu} \bar{p}_{\parallel k}^{\nu} \rangle}{E_{lk}} \times f_k^0(x, \bar{p}_{zk}), \end{aligned} \quad (14)$$

which give back the expressions as in Eqs. (9) and (11) for the pressure and energy density respectively through the following definitions,

$$\varepsilon_{\parallel} = u^\mu u^\nu T_{\mu\nu}, \quad P_{\parallel} = \Delta_{\parallel}^{\mu\nu} T_{\mu\nu}. \quad (15)$$

Here, $\bar{p}_{\parallel k}^{\mu} \equiv (\omega_{lk}, 0, 0, p_{zk})$ incorporates the longitudinal components and $\langle \bar{p}_{\parallel k}^{\mu} \bar{p}_{\parallel k}^{\nu} \rangle = \frac{1}{2}(\Delta_{\parallel}^{\mu\alpha} \Delta_{\parallel}^{\nu\beta} + \Delta_{\parallel}^{\mu\beta} \Delta_{\parallel}^{\nu\alpha}) \bar{p}_{\parallel\alpha} \bar{p}_{\parallel\beta}$. For the weak (moderate) magnetic field, one also needs to analyse the transverse dynamics of the hot QCD matter. In these situations, the transverse components of various transport coefficients might play a significant role. These aspects are beyond the scope of the present work and the matter of future extensions of the work. Following the above arguments, four flow N^μ of the quarks and antiquarks in the strong magnetic field has the following form,

$$\begin{aligned} N^\mu(x) = & \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \mu_l \frac{|q_{fk} eB|}{2\pi} N_c \int_{-\infty}^{\infty} \frac{d\bar{p}_{zk}}{(2\pi)\omega_{lk}} \bar{p}_{\parallel k}^{\mu} \\ & \times f_k^0(x, \bar{p}_{zk}) \\ & + \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \delta\omega_{lk} \frac{|q_{fk} eB|}{2\pi} N_c \int_{-\infty}^{\infty} \frac{d\bar{p}_{zk}}{(2\pi)\omega_{lk}} \frac{\langle \bar{p}_{\parallel k}^{\mu} \rangle}{E_{lk}} \\ & \times f_k^0(x, \bar{p}_{zk}), \end{aligned} \quad (16)$$

with $\langle \bar{p}_{\parallel}^{\mu} \rangle = \Delta_{\parallel}^{\mu\nu} \bar{p}_{\parallel\nu}$.

Estimation of the transport coefficients requires the system away from equilibrium. In the current analysis, we are focusing on the dominant quark/antiquark dynamics of the magnetized QGP. Here, we need to set-up the relativistic transport equation, which quantifies the rate of change of quasiquark/antiquark distribution function in terms of collision integral. The thermal relaxation time (τ_{eff}) linearize the collision term ($C(f_q)$) in the following way,

$$\frac{1}{\omega_{lk}} \bar{p}_{\parallel k}^{\mu} \partial_{\mu} f_k^0(x, \bar{p}_{zk}) + F_z \frac{\partial f_k^0}{\partial p_{zk}} = C(f_k) \equiv -\frac{\delta f_k}{\tau_{\text{eff}}}, \quad (17)$$

with $F_z = -\partial_{\mu}(\delta\omega u^\mu u_z)$ is the force term from the conservation of particle density and energy momentum [61]. The local momentum distribution function of quarks can expand as,

$$f_k = f_k^0(p_z) + \delta f_k, \quad \delta f_k = f_k^0(1 \pm f_k^0) \phi_k. \quad (18)$$

Here, ϕ_k defines the deviation of the quasiquark distribution function from its equilibrium. The Eq. (17) gives the effective kinetic theory description of the quasipartons under EQPM in the strong magnetic field. In order to estimate the transport coefficients, we employ the Chapman–Enskog (CE) method. Applying the definition of equilibrium quasiparton momentum distribution function as in Eq. (2), the first term of Eq. (17) gives the number of terms with thermodynamic forces of the transport processes. The second term of Eq. (17) vanishes for a co-moving frame. Finally, we are left with,

$$Q_k X + \langle \bar{p}_{\parallel k}^{\mu} \rangle (\omega_{lk} - h_k) X_{q\mu} - \langle \langle \bar{p}_{\parallel k}^{\mu} \bar{p}_{\parallel k}^{\nu} \rangle \rangle X_{\mu\nu} = -\frac{T\omega_{pk}}{\tau_{\text{eff}}} \phi_k, \quad (19)$$

in which the conformal factor due to the dimensional reduction in the strong field limit is $Q_k = (\bar{p}_{zk}^2 - \omega_{lk}^2 c_s^2)$ where c_s^2 is the speed of sound and h_k is the enthalpy per particle of the system that can be defined from the basic QCD thermodynamics. Here, $\langle \langle P_{\parallel}^{\mu} R_{\parallel}^{\nu} \rangle \rangle = \left\{ \frac{1}{2} \Delta_{\parallel\alpha}^{\mu} \Delta_{\parallel\beta}^{\nu} + \frac{1}{2} \Delta_{\parallel\beta}^{\mu} \Delta_{\parallel\alpha}^{\nu} - \frac{1}{3} \Delta_{\parallel\alpha\beta} \Delta_{\parallel}^{\mu\nu} \right\} P_{\parallel}^{\alpha} R_{\parallel}^{\beta}$. The bulk viscous force, thermal force and shear viscous force are defined respectively as follows,

$$X = \partial_{\mu} u, \quad (20)$$

$$X_q^{\mu} = \left\{ \frac{\nabla^{\mu} T}{T} - \frac{\nabla^{\mu} P}{nh} \right\}, \quad (21)$$

$$X_{\mu\nu} = \langle \langle \partial_{\mu} u_{\nu} \rangle \rangle, \quad (22)$$

where h is the total enthalpy defined as $h = \sum_{k=0}^N h_k$ and n is the total number density of the system. Note that here $\mu = 0, 3$ describes only the longitudinal components in the

strong magnetic field. Also, the deviation function ϕ_k that is the linear combination of these forces can be represented as,

$$\phi_k = A_k X + B_k^\mu X_{q\mu} - C_k^{\mu\nu} X_{\mu\nu}, \quad (23)$$

where the coefficients can be defined from Eq. (19) as,

$$A_k = \frac{Q_k}{\left\{ -\frac{T\omega_{l_k}}{\tau_{\text{eff}}} \right\}}, \quad (24)$$

$$B_k^\mu = \langle \bar{p}_k^\mu \rangle \frac{(\omega_{l_k} - h_k)}{\left\{ -\frac{T\omega_{l_k}}{\tau_{\text{eff}}} \right\}}, \quad (25)$$

$$C_k^{\mu\nu} = \frac{\langle \langle \bar{p}_{\parallel k}^\mu \bar{p}_{\parallel k}^\nu \rangle \rangle}{\left\{ -\frac{T\omega_{l_k}}{\tau_{\text{eff}}} \right\}}. \quad (26)$$

Following this formalism, we can estimate the viscous coefficients and thermal conductivity of the QGP medium in the strong magnetic field.

1. Shear and bulk viscosity

We can define the pressure tensor from the energy-momentum tensor as in the following way,

$$P^{\mu\nu} = \Delta_{\parallel\sigma}^\mu T^{\sigma\tau} \Delta_{\parallel\tau}^\nu. \quad (27)$$

We can decompose the $P^{\mu\nu}$ in equilibrium and non-equilibrium components of distribution function as follows,

$$P^{\mu\nu} = -P\Delta_{\parallel}^{\mu\nu} + \Pi^{\mu\nu}, \quad (28)$$

where $\Pi^{\mu\nu}$ is the viscous pressure tensor. Following the definition of $T^{\mu\nu}$ as in Eq. (14), $\Pi^{\mu\nu}$ takes the form,

$$\begin{aligned} \Pi^{\mu\nu} = & \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \mu_l \frac{|q_{f_k} e B|}{2\pi} N_c \int_{-\infty}^{\infty} \frac{d\bar{p}_{z_k}}{(2\pi)\omega_{l_k}} \langle \bar{p}_{\parallel k}^\mu \bar{p}_{\parallel k}^\nu \rangle \\ & \times \delta f_k(x, \bar{p}_{z_k}) + \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \delta\omega \mu_l \frac{|q_{f_k} e B|}{2\pi} N_c \\ & \int_{-\infty}^{\infty} \frac{d\bar{p}_{z_k}}{(2\pi)\omega_{l_k}} \frac{\langle \bar{p}_{\parallel k}^\mu \bar{p}_{\parallel k}^\nu \rangle}{E_{l_k}} \times \delta f_k(x, \bar{p}_{z_k}). \end{aligned} \quad (29)$$

In the very strong magnetic field, the pressure tensor has different form as compared to the case without magnetic field. This is due to the $(1+1)$ -dimensional energy eigenvalues of the quarks and antiquarks. Hence, μ and ν can be 0 or 3 in the strong magnetic field, describing the longitudinal components of the viscous pressure tensor. The form of viscous pressure tensor in the strong magnetic field is described in the

recent works by Tuchin [46, 88]. Magnetized plasma is characterized by five shear components. Among the five coefficients, four components are negligible when the strength of the magnetic field is sufficiently higher than the square of the temperature [89]. Here, we are focusing on the non-negligible longitudinal component of shear and bulk viscous coefficients of the hot QGP medium in the strong magnetic field.

Following [48], the longitudinal shear viscous tensor has the following form,

$$\begin{aligned} \bar{\Pi}^{\mu\nu} = & \Pi^{\mu\nu} - \Pi \Delta_{\parallel}^{\mu\nu} \\ = & \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \mu_l \frac{|q_{f_k} e B|}{2\pi} N_c \int_{-\infty}^{\infty} \frac{d\bar{p}_{z_k}}{(2\pi)\omega_{l_k}} \langle \langle \bar{p}_{\parallel k}^\mu \bar{p}_{\parallel k}^\nu \rangle \rangle \\ & \times f_k^0 (1 - f_k^0) \phi_k \\ & + \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \delta\omega \mu_l \frac{|q_{f_k} e B|}{2\pi} N_c \int_{-\infty}^{\infty} \frac{d\bar{p}_{z_k}}{(2\pi)\omega_{l_k}} \frac{\langle \langle \bar{p}_{\parallel k}^\mu \bar{p}_{\parallel k}^\nu \rangle \rangle}{E_{l_k}} \\ & \times f_k^0 (1 - f_k^0) \phi_k. \end{aligned} \quad (30)$$

Also, the bulk viscous part in the longitudinal direction comes out to be,

$$\begin{aligned} \Pi = & \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \mu_l \frac{|q_{f_k} e B|}{2\pi} \frac{N_c}{3} \int_{-\infty}^{\infty} \frac{d\bar{p}_{z_k}}{(2\pi)\omega_{l_k}} \Delta_{\parallel}^{\mu\nu} \\ & \times \bar{p}_{\parallel k}^\mu \bar{p}_{\parallel k}^\nu f_k^0 (1 - f_k^0) \phi_k \\ & + \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \delta\omega \mu_l \frac{|q_{f_k} e B|}{2\pi} \frac{N_c}{3} \int_{-\infty}^{\infty} \frac{d\bar{p}_{z_k}}{(2\pi)\omega_{l_k}} \Delta_{\parallel}^{\mu\nu} \\ & \times \frac{\bar{p}_{\parallel k}^\mu \bar{p}_{\parallel k}^\nu}{E_{l_k}} f_k^0 (1 - f_k^0) \phi_k. \end{aligned} \quad (31)$$

Substituting ϕ_k from Eq. (23) and comparing with the macroscopic definition $\Pi^{\mu\nu} = 2\eta \langle \partial^\mu u^\nu \rangle + \zeta \Delta_{\parallel}^{\mu\nu} \partial \cdot u$, we can obtain the expressions of longitudinal viscosity coefficients in the strong field limit. Note that the longitudinal component of shear viscosity, i.e., in the direction of magnetic field, is defined from $\bar{\Pi}^{33}$ [89]. The longitudinal shear η and bulk viscosity ζ are obtained as,

$$\begin{aligned} \eta = & \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \mu_l \frac{|q_{f_k} e B|}{\pi} \frac{N_c}{9T} \\ & \times \int_{-\infty}^{\infty} \frac{d\bar{p}_{z_k}}{(2\pi)} \frac{|\bar{p}_{z_k}|^4}{\omega_{l_k}^2} \tau_{\text{eff}} f_k^0 (1 - f_k^0) \\ & + \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \delta\omega \mu_l \frac{|q_{f_k} e B|}{\pi} \frac{N_c}{9T} \\ & \times \int_{-\infty}^{\infty} \frac{d\bar{p}_{z_k}}{(2\pi)} \frac{1}{\omega_{l_k}^2} \frac{|\bar{p}_{z_k}|^4}{E_{l_k}} \tau_{\text{eff}} \\ & \times f_k^0 (1 - f_k^0), \end{aligned} \quad (32)$$

and

$$\begin{aligned} \zeta = & \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \mu_l \frac{|q_{fk} eB|}{2\pi} \frac{N_c}{3T} \int_{-\infty}^{\infty} \frac{d\bar{p}_{zk}}{(2\pi)} \frac{1}{\omega_{lk}^2} \left\{ \bar{p}_{zk}^2 - \omega_{lk}^2 c_s^2 \right\}^2 \\ & \times \tau_{\text{eff}} f_k^0 (1 - f_k^0) + \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \delta\omega \mu_l \frac{|q_{fk} eB|}{2\pi} \frac{N_c}{3T} \\ & \times \int_{-\infty}^{\infty} \frac{d\bar{p}_{zk}}{(2\pi)} \frac{1}{\omega_{lk}^2} \left\{ \bar{p}_{zk}^2 - \omega_{lk}^2 c_s^2 \right\}^2 \times \frac{1}{E_{lk}} \tau_{\text{eff}} f_k^0 (1 - f_k^0). \end{aligned} \quad (33)$$

The second term in the Eqs. (32) and (33) gives correction to viscous coefficients due to the quasiparton excitations whereas the first term comes from the usual kinetic theory of bare particles.

2. Thermal conductivity

The heat flow is the difference between the energy flow and enthalpy flow by the particle,

$$I_q^\mu = u_\nu T^{\nu\sigma} \Delta_{\parallel\sigma}^\mu - h N^\sigma \Delta_{\parallel\sigma}^\mu. \quad (34)$$

In terms of the modified/non-equilibrium distribution function Eq. (34) becomes,

$$\begin{aligned} I^\mu = & u_\nu \Delta_{\parallel\sigma}^\mu \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \mu_l \frac{|q_{fk} eB|}{2\pi} N_c \\ & \times \int_{-\infty}^{\infty} \frac{d\bar{p}_{zk}}{(2\pi)\omega_{lk}} \bar{p}_{\parallel k}^\nu \bar{p}_{\parallel k}^\sigma \delta f_k \\ & - h \Delta_{\parallel\sigma}^\mu \left[\sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \mu_l \frac{|q_{fk} eB|}{2\pi} N_c \right. \\ & \times \int_{-\infty}^{\infty} \frac{d\bar{p}_{zk}}{(2\pi)\omega_{lk}} \bar{p}_{\parallel k}^\sigma \delta f_k(x, \bar{p}_{zk}) \\ & + \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \delta\omega \mu_l \frac{|q_{fk} eB|}{2\pi} N_c \\ & \left. \times \int_{-\infty}^{\infty} \frac{d\bar{p}_{zk}}{(2\pi)\omega_{lk}} \frac{\langle \bar{p}_{\parallel k}^\sigma \rangle}{E_{lk}} \delta f_k(x, \bar{p}_{zk}) \right], \end{aligned} \quad (35)$$

in which heat flow retains only non-equilibrium part of the distribution function. After contracting with projection operator and hydrodynamic velocity along with the substitution of δf_k from Eq. (17) and comparing with the macroscopic definition of heat flow, we obtain

$$I^\mu = \lambda T X^\mu. \quad (36)$$

We obtain the thermal conductivity in the strong magnetic field as,

$$\begin{aligned} \lambda = & \left\{ \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \mu_l \frac{|q_{fk} eB|}{2\pi} \frac{N_c}{T^2} \int_{-\infty}^{\infty} \frac{d\bar{p}_{zk}}{(2\pi)} \tau_{\text{eff}} \frac{(\omega_{lk} - h_k)^2}{\omega_{lk}^2} \right. \\ & \times |\bar{p}_{zk}|^2 f_k^0 (1 - f_k^0) \left. \right\} \\ & - \left\{ \sum_{l=0}^{\infty} \sum_{k \in q, \bar{q}} \mu_l \delta\omega \frac{|q_{fk} eB|}{2\pi} \frac{N_c}{T^2} \right. \\ & \times \int_{-\infty}^{\infty} \frac{d\bar{p}_{zk}}{(2\pi)} \tau_{\text{eff}} \frac{h_k (\omega_{lk} - h_k)}{\omega_{lk}^2} \\ & \left. \times \frac{|\bar{p}_{zk}|^2}{E_{lk}} f_k^0 (1 - f_k^0) \right\}. \end{aligned} \quad (37)$$

The second term with $\delta\omega$ in the heat flow comes from the N_μ which encodes the quasiparticle excitation in the thermal conductivity.

3 Thermal relaxation in the strong magnetic field

In the strong magnetic field, the $1 \rightarrow 2$ processes (gluon to quark–antiquark pair) are kinematically possible and are dominant compared to $2 \rightarrow 2$ processes [50]. The thermal relaxation time τ_{eff} can be defined from the collision integral as described in the Eq. (17). For the $1 \rightarrow 2$ processes ($p + p' \rightarrow k$, where primed notation for antiquark), the τ_{eff} in the strong magnetic field can be defined as follows,

$$\begin{aligned} \tau_{\text{eff}}^{-1}(p_z) = & \sum_{l'=0}^{\infty} \int_{-\infty}^{\infty} \frac{dp'_z}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{(2\pi)^2 \delta(k_z - p_z - p'_z)}{2\omega_k 2\omega_{l_p} 2\omega_{l_{p'}}} \\ & \times |M_{p+p' \rightarrow k}|^2 \frac{f_q^0(p'_z)(1 + f_g^0(k))}{(1 - f_q^0(p_z))}, \end{aligned} \quad (38)$$

where M is the matrix element for the process under consideration.

Within the LLL approximation the momentum dependent thermal relaxation time takes the following form in the regime $p_z' \sim 0$, as [62,90],

$$(\tau_{\text{eff}}^{-1})_{l=0} = \frac{2\alpha_{\text{eff}} C_F m_f^2}{\omega_q (1 - f_q^0)} \frac{z_q}{(z_q + 1)} (1 + f_g^0(E_{p_z})) \ln(T/m), \quad (39)$$

where C_F is the Casimir factor of the processes and α_{eff} is the effective coupling constant defined from the Debye screening mass [62].

The Impact of the higher Landau levels on the matrix element and distribution function for the $1 \rightarrow 2$ processes is explored in the very recent work [49]. Including these HLL effects, the τ_{eff} of the $1 \rightarrow 2$ processes has the following form,

$$\tau_{\text{eff}}^{-1}(p_z) = \frac{1}{4\omega_{l_q}} \frac{1}{(1 - f_q^0(p_z))} \sum_{l' \geq l} \int_{-\infty}^{\infty} \frac{dp'_z}{2\pi} \frac{1}{2\omega_{l'_q}} X(l, l', \xi) \times f_q^0(p'_z)(1 + f_g^0(p'_z + p_z)), \quad (40)$$

where ξ is defined as,

$$\xi = \frac{(\omega_{l_q} + \omega_{l'_q})^2 - (p_z + p'_z)^2}{2 |q_f eB|}, \quad (41)$$

and $X(l, l', \xi)$ takes the form as follows,

$$X(l, l', \xi) = 4\pi\alpha_{\text{eff}} N_c C_F \frac{l!}{l'!} e^{-\xi} \xi^{l'-l} \left[\left(4m_f^2 - 4 |q_f eB| (l + l' - \xi) \frac{1}{\xi} (l + l') \right) F(l, l', \xi) + 16 |q_f eB| l' (l + l') \frac{1}{\xi} L_l^{(l'-l)}(\xi) L_{l-1}^{(l'-l)}(\xi) \right], \quad (42)$$

with $F(l, l', \xi) = [L_l^{(l'-l)}(\xi)]^2 + \frac{l'}{l} [L_{l-1}^{(l'-l)}(\xi)]^2$ for $l > 0$ and $F(l, l', \xi) = 1$ for the lowest Landau level. Here, α_{eff} is the effective coupling constant and is defined from the Debye screening masses of the QGP [48, 91–94].

The effective thermal relaxation time controls the behaviour of transport coefficients critically. Note that in the limit $T^2 \ll |q_f eB|$, LLL approximation is valid so that $X(l = 0, l' = 0, \xi) \approx 16\pi(\alpha_{\text{eff}})m_f^2 N_c C_F$, where $e^{-\xi} \approx 1$ in this regime. Hence, the τ_{eff} as defined in the Eq. (40) can be reduced to the LLL result as defined in Eq. (39) in the limit $T^2 \ll |q_f eB|$. Following the parton distribution function within the EQPM framework, the thermal average of τ_{eff} can be defined as,

$$\langle \tau_{\text{eff}} \rangle = \frac{\sum_{l=0}^{\infty} \int_{-\infty}^{\infty} dp_z \tau_{\text{eff}} f_q^0}{\sum_{l=0}^{\infty} \int_{-\infty}^{\infty} dp_z f_q^0}. \quad (43)$$

Notably, the thermal average is taken merely to explore the temperature behaviour of $\langle \tau_{\text{eff}} \rangle$ with the inclusion of the effects of HLLs and analysed in the next section. While computing the transport coefficients the momentum dependence of the τ_{eff} has been employed.

4 Results and discussions

Let us initiate the discussion with the temperature behaviour of thermal relaxation time of the quarks (up, down and strange quarks with masses $m_u = 3$ MeV, $m_d = 5$ MeV and $m_s = 100$ MeV respectively) for the dominant $1 \rightarrow 2$

Fig. 1 The temperature dependence of thermal relaxation time for quarks at $|eB| = 10m_\pi^2$

Fig. 2 The effects of HLLs on the temperature behaviour of ζ/s at $|eB| = 10m_\pi^2$. Behaviour of ζ/s is comparing with the result at $\mathbf{B} = 0$ of Mitra et al. [61]

processes in the presence of the strong magnetic field. The τ_{eff} has been plotted as a function of $\frac{T}{T_c}$ for $|eB| = 10m_\pi^2$ considering up to 50 LLs in the Fig. 1. The relaxation time exhibits the decreasing trend with increasing temperature. In the limit, $T^2 \ll |q_f eB|$, τ_{eff} defined in Eq. (40) reduced to the LLL result as described in [62]. To encode the EoS effects in the τ_{eff} , the quasiparticle parton distribution functions are introduced along with the effective coupling constant.

Following the Eq. (33), the temperature dependence of bulk viscosity depends on the term $\frac{1}{\omega_p^2} (p_{zk}^2 - \omega_p^2 c_s^2)^2$ and τ_{eff} , where c_s^2 can be obtained from the QCD thermodynamics. The ratio of longitudinal bulk viscosity to entropy density for the $1 \rightarrow 2$ processes at $|eB| = 10m_\pi^2$ has been plotted as a function of T/T_c in the Fig. 2. The temperature dependence of the ζ/s in the strong magnetic field indicates its rising behaviour near T_c . The behaviour of longitudinal shear viscosity for the $1 \rightarrow 2$ processes with T/T_c at $|eB| = 10m_\pi^2$ is shown in Fig. 3. Since the driving force for the longitudinal shear viscosity is in the direction of the magnetic field, the Lorentz force does not interfere in the calculation. Quantitatively, η/s with the HLL contributions remains within the same range of the lattice data [43] and NJL model

Fig. 3 The effects of HLLs on the temperature behaviour of η/s at $|eB| = 10m_\pi^2$. Lattice data [43] and result of Marty et al. [47] for η/s are in the absence of magnetic field

result in [47] at $B = 0$. This observation is in line with the result that longitudinal conductivity with HLLs contributions remains within the range of the lattice result at zero magnetic field [49]. For the numerical estimation of ζ/s and η/s , we truncate the Landau level sum at $l_{\max} = 50$. We observe that the HLL contributions are significant in the estimation of the viscous coefficients whereas the LLL approximation has an enhancement as m_f tends to zero. Our observations on the effects of HLLs to the transport coefficients are qualitatively consistent with the results of the recent work of Fukushima and Hidaka [49].

The present analysis is done by employing the effective covariant kinetic theory using the Chapman–Enskog method including the effects of HLLs. The mean field force term which emerges from the effective theory indeed appears as the mean field corrections to the transport coefficients. The second term in the Eqs. (32) and (33) describes the mean field contribution to the longitudinal shear viscosity and bulk viscosity in the presence of magnetic field, respectively. The mean field term consists of the term $\delta\omega$ which is the temperature gradient of the effective fugacity $z_{g/q}$. The temperature behaviours of the viscous coefficients (bulk and shear viscosities) in the magnetic field with and without the mean field corrections are shown in Fig. 4 (left panel). At higher temperature, the effects are negligible since the effective fugacity

Fig. 5 Thermal conductivity as a function of T/T_c at $|eB| = 10m_\pi^2$. Behaviour of λ/T^2 is comparing with the result at $\mathbf{B} = 0$ of Marty et al. [47]

behaves as a slowly varying function of temperature there. Hence, the mean field corrections due to the quasiparticle excitations are significant at temperature region closer to T_c . The magnetic field dependence of the bulk viscosity and shear viscosity have been plotted in the Fig. 4 (right panel). In the strong magnetic field limit, the viscous coefficients could be computed within LLL approximation. The inclusion of HLLs reflects the non-trivial (non-monotonic) magnetic field dependence of the transport coefficients. Similar non-monotonic structure in the magnetic field dependence of longitudinal conductivity with HLLs is described in [49].

Mean field corrections to the thermal conductivity is explicitly shown in Eq. (37) in which thermal relaxation incorporates the microscopic interactions. We depicted the temperature behaviour of λ/T^2 in Fig. 5. The HLL effects of the transport coefficients are entering through the thermal relaxation time and the quasiparticle distribution function. These effects are significant in the estimation of transport coefficients in the presence of a magnetic field. The temperature behaviour of the dimensionless quantity λ/T^2 in the absence of the magnetic field is well investigated [47,48] and is in the order of $100 - 25$ within the temperature range $(1 - 4) \frac{T}{T_c}$, which is quantitatively consistent with our result.

The viscous coefficients of the strongly interacting matter could be employed to obtain the viscous corrections to the

Fig. 4 Temperature dependence of ζ/s and η/s with and without mean field correction at $|eB| = 10m_\pi^2$ for $1 \rightarrow 2$ processes (left panel). Magnetic field dependence of ζ/s and η/s (right panel)

experimental observables (hadron spectra, dilepton spectra etc.) in the RHIC. The dissipative effects and EoS dependence of the confined phase have been estimated within lattice QCD [25, 26]. These aspects along with the estimation of electric conductivity [95] within our model while including the HLLs is beyond the scope of the present analysis and is a matter of future investigations.

5 Conclusion and outlook

In conclusion, we have computed the temperature behaviour of the transport parameters such as longitudinal viscous coefficients (shear and bulk viscosities) and thermal conductivity for the $1 \rightarrow 2$ processes in the strong magnetic field background while including the effects of HLLs. Thermal relaxation time is computed in magnetized QGP incorporating the HLL contributions. Setting up an effective covariant kinetic theory within EQPM in the magnetic field induces mean field contributions to the transport coefficients. We employed the Chapman–Enskog method in the effective kinetic theory for the computation of transport coefficients. The transport coefficients that have been estimated are influenced by the thermal medium and magnetic field. Hot QCD effects are incorporated through the quasiparton degrees of freedom along with effective coupling and the medium effects are found to be negligible at very high temperature. We focused on the weakly coupled regime of the perturbative QCD within the limit $gT \ll \sqrt{|q_f e B|}$ in which HLL contributions are significant. Notably, the inclusion of HLL contributions are essential to explain the transport processes at the high temperature regimes. Furthermore, effects of the mean field term are seen to be quite significant as far as the temperature behavior of the above mentioned transport coefficients is concerned (for the temperatures which are not very far away from T_c).

An immediate future extension of the work is to investigate the aspects of non-linear electromagnetic responses of the hot QGP with the mean field contribution along with the effective description of magnetohydrodynamic waves in the QGP medium. In addition, the estimation of all transport coefficients from covariant kinetic theory within the effective fugacity quasiparticle model using more realistic collision integral, for example, BGK (Bhatnagar, Gross and Krook) collision term, in the strong magnetic field would be another direction to work.

Acknowledgements VC would like to acknowledge Science and Engineering Research Board (SERB), Govt. of India for the Early Career Research Award (ECRA/2016/000683) and Department of Science and Technology (DST), Govt. of India for INSPIRE-Faculty Fellowship (IFA-13/PH-55). SG would like to acknowledge the Indian Institute of Technology Gandhinagar for the postdoctoral fellowship. SM would like to acknowledge SERB-INDO US forum to conduct the Postdoc-

toral research in USA. We record our sincere gratitude to the people of India for their generous support for the research in basic sciences.

Data Availability Statement This manuscript has no associated data or the data will not be deposited [Authors' comment: The plots are the graphical depiction of various mathematical expressions mentioned in the manuscript. There are no other data involved.]

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