Gravitational axial perturbations and quasinormal modes of loop quantum black holes

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Abstract Loop quantum gravity (LQG) is a theory that proposes a way to model the behavior of the spacetime in situations where its atomic characteristic arises. Among these situations, the spacetime behavior near the Big Bang or black hole's singularity. The detection of gravitational waves, on the other hand, has opened the way to new perspectives in the investigation of the spacetime structure. In this work, by the use of a WKB method introduced by Schutz and Will (Astrophys J 291:L33, 1985), and after improved by Iyer and Will (Phys Rev D 35:3621, 1987), we study the gravitational wave spectrum emitted by loop quantum black holes, which correspond to a quantized version of the Schwarzschild spacetime by LQG techniques. From the results obtained, loop quantum black holes have been shown stable under axial gravitational perturbations.

1 Introduction

One of most exciting predictions of general relativity is the existence of black holes, objects from which no physical bodies or signals can get loose of their drag due to its strong gravitational field. Going far beyond astrophysics, black holes appear as objects that may help us to clarify one of the most intriguing bone of contention in the current days, the quantum nature of gravity. It is because, in the presence of a black hole's strong gravitational field, quantum features of spacetime must be manifested [3–9].

Loop quantum gravity, on the other hand, is a theory that has given ascent to models that provide a portrait of the quantum features of spacetime unveiled by a black hole. In partic-

ular, in the context of this theory, a black hole metric known as loop quantum black hole (LQBH), or self-dual black hole, has been proposed [10, 11]. This solution corresponds to a quantum corrected Schwarzschild solution and possess the interesting property of self-duality. From this property, the black hole singularity is removed and replaced by another asymptotically flat region, which is an expected effect in a quantum gravity regime. Moreover, LQBHs have been pointed as a possible candidate for dark matter [11, 12] and as the building blocks of a holographic version of loop quantum cosmology [13].

In order to move black holes from a simple mathematical solution of the gravitational equations to objects whose existence in nature is possible, a key point consists in to investigate black hole's stability under perturbations. It is due to the fact that an isolated black hole would never be found in nature. In fact, complex distributions of matter such as accretion disks, galactic nuclei, strong magnetic fields, other stars, etc are always present around black holes, which in turn actively interact with their surroundings. Even if all macroscopic objects and fields in space have been removed, a black hole will interact with the vacuum around it, creating pairs of particles and evaporating due to Hawking phenomena. Besides, in the first moments after its formation, a black hole is in a perturbed state due to gravitational collapse of matter. In this way, a real black hole will be always in a perturbed state.

A black hole's response to a perturbation occurs by emitting gravitational waves whose evolution corresponds, firstly, to a relatively short period of initial outburst of radiation followed by a phase where the black hole get going to vibrating into exponentially decaying oscillations, “quasinormal modes”, whose frequencies and decay times depend only on the intrinsic characteristics of the black hole itself, being indifferent to the details of the collapse. Finally, at a very large

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time, the quasinormal modes are slapped down by power-law or exponential late-time tails.

The issue of black hole stability under perturbations was firstly addressed by Regge and Wheeler [14], and by Zerilli [15], which based on the black hole perturbation theory, have demonstrated the stability of the Schwarzschild metric. The methods used are familiar from quantum mechanics: perturbations caused by an external (e.g., gravitational or electromagnetic) field are taken into account as waves scattering off the respective potential. It is due to the fact that the formalism provided by Regge–Wheeler and Zerilli removes the angular dependence in the perturbation variables by the use of a tensorial generalization of the spherical harmonics which makes possible to translate the solution of the perturbed Einstein's equations in the form of a Schrodinger-like wave equation treatment. Posteriorly, the Regge–Wheeler/Zerilli formalism was extended to the charged [16–18] and rotating [19,20] black hole scenarios. A full description of the black hole perturbation framework can be found in the text by Chandrasekhar [21].

The most valuable phase in the evolution of black hole perturbations is given by its quasinormal modes which can give us information not only about the black hole stability, but also, as emphasized by Berti [22], ‘how much stable it is’. In other words, quasinormal modes tell us which timescale a black hole radiate away its matter content after formation. By the way, in this context the prefix ‘quasi’ means that the black hole consists in an open system that loses energy due to the emission of gravitational waves. The issue of black hole quasinormal modes is interesting not only by the investigation of black hole stability, but also because gravitational waves have been pointed as a possible experimental way to make contact with black holes and, in this way, with the quantum spacetime characteristics revealed by them. Indeed, recent results from LIGO, which has detected a gravitational wave signal from black holes [23–26], and the last gravitational wave detection from neutron stars [27] have open the floodgates for a large range of possibilities in gravitational physics in the same way that the discovery of infrared radiation by William Herschel the ‘1800s’ opened the possibility of the investigation and application of the electromagnetic spectrum beyond to the visible. It can yet be reinforced by others results from others gravitational antennas such as LISA, VIRGO, and others. In a timely way, the quasinormal fundamental mode constitutes the predominating contribution to such signals in the long period of damping proper oscillations [28]. Moreover, the quasinormal modes’ ringdown phase of the signal is useful to distinguish between black holes and other possible sources [29].

In this work, we shall address the gravitational wave production by LQBHs. Quasinormal frequencies for such black holes have been addressed in the scalar perturbations case in [30,31]. However, in a more realistic scenario, gravitational

perturbations must be included. Investigation on the quasinormal mode spectrum in the context of LQBHs may reveal some advantages front others scenarios under the experimental point of view since the quantum corrections present in this scenario depend on the dimensionless Barbero–Immirzi parameter [32], which as it has been pointed in [33] does not suffers with the problem of mass suppression, as occurs with the parameters of other quantum gravity theories like superstring theory or noncommutative theory. In these theories, the quantum corrections appears as proportional to $(l_{\text{qg}}/M)^m$, where l_{qg} is a quantum gravity motivated dimensionful parameter of the theory, M is the black hole mass, and m is some positive number. In this way, for large black holes, the corrections coming from quantum gravity effects are mass suppressed.

The article is organized as follows: in Sect. 2, we revise the LQBH's theory; in Sect. 3, we calculate the Regge–Wheeler equation for LQBHs; in Sect. 4, we calculate the quasinormal modes of the LQBHs. Section 5 is devoted to conclusions and perspectives. Throughout this paper, we have used $\hbar = c = G = 1$.

2 Loop quantum black holes

Some efforts in order to find out black hole solutions in LQG have been done by several authors [10,34–43]. In this section, we shall analyze a particular solution, called self-dual, that appeared at the primary time from a simplified model of LQG consisting in symmetry reduced models corresponding to homogeneous spacetimes (see [10] and the references therein).

In this way, the LQBH framework which we shall work here is delineated by a quantum gravitationally corrected Schwarzschild metric, written as

$$ds^2 = -G(r)dt^2 + F^{-1}(r)dr^2 + H(r)d\Omega^2 \quad (1)$$

with

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2, \quad (2)$$

where, in the Eq. (1), the metric functions are given by

$$G(r) = \frac{(r - r_+)(r - r_-)(r + r_*)^2}{r^4 + a_0^2},$$

$$F(r) = \frac{(r - r_+)(r - r_-)r^4}{(r + r_*)^2(r^4 + a_0^2)}, \quad (3)$$

and

$$H(r) = r^2 + \frac{a_0^2}{r^2}, \quad (4)$$

where

$$r_+ = 2m; \quad r_- = 2mP^2.$$

In this situation, we have got the presence of two horizons – an event horizon localized at r_+ and a Cauchy horizon localized at r_- .

Furthermore, we have that

$$r_* = \sqrt{r_+ r_-} = 2mP. \quad (5)$$

In the definition above, P is the polymeric function given by

$$P = \frac{\sqrt{1 + \epsilon^2} - 1}{\sqrt{1 + \epsilon^2} + 1}; \quad (6)$$

where $\epsilon = \gamma \delta_b$, being γ the Barbero–Immirzi parameter and δ_b is the polymeric parameter used in the LQG quantization techniques in order to determinate the length of the path along with the connection used to define the holonomies is integrated [10]. Moreover

$$a_0 = \frac{A_{min}}{8\pi}, \quad (7)$$

where A_{min} is the minimal value of area in LQG.

In the metric (1), r is only asymptotically the same old radial coordinate since $g_{\theta\theta}$ is not simply r^2 . A more physical radial coordinate is obtained from the shape of the function $H(r)$ within the metric (4)

$$R = \sqrt{r^2 + \frac{a_0^2}{r^2}}, \quad (8)$$

in the sense that it measures the right circumferential distance.

Moreover, the parameter m within the solution is related to the ADM mass M by

$$M = m(1 + P)^2. \quad (9)$$

The Eq. (8) reveals vital aspects of the LQBH's internal structure. From this expression, we have got that, as r decreases from ∞ to 0, R initially decreases from ∞ to $\sqrt{2a_0}$ at $r = \sqrt{a_0}$ so will increase once more to ∞ . The value of R associated with the event horizon is given by

$$R_{EH} = \sqrt{H(r_+)} = \sqrt{(2m)^2 + \left(\frac{a_0}{2m}\right)^2}. \quad (10)$$

An interesting feature of LQBHs scenario is that related to the self-duality of the metric (1). In this case, self-duality means that if one introduces the new coordinates $\tilde{r} = a_0/r$ and $\tilde{t} = tr_*^2/a_0$, the metric preserves its type. The dual radius is given by $r_{dual} = \tilde{r} = \sqrt{a_0}$ and corresponds to the smallest possible surface element. Moreover, since the Eq. (8) may be written as $R = \sqrt{r^2 + \tilde{r}^2}$, it is clear that the solution contains another asymptotically flat Schwarzschild region instead of a singularity within the limit $r \rightarrow 0$. This new region corresponds to a Planck-sized wormhole. Figure 1 shows the Carter–Penrose diagram for the LQBH.

Fig. 1 Carter–Penrose diagram for the LQBH metric. The diagram has two asymptotic regions, one at infinity and the other near the origin, which no observer can reach in a finite time

In addition, the thermodynamical properties of LQBHs have been addressed in the references [11, 44–48]. In particular, within the reference [48], such issue has been addressed by the employment of a tunneling technique which has made possible the introduction of back-reaction effects. By employing the same tunneling method, in the reference [49] it has been investigated the influence of a generalized uncertainty principle on the thermodynamics of LQBHs. Interestingly, from the form of the LQBHs entropy, it has been found out that these objects could be the main constituents of a holographic version of loop quantum cosmology (LQC) [13].

3 Regge–Wheeler formalism for LQBHs

In this section, we shall use a method due to Regge and Wheeler to investigate black hole's axial gravitational perturbations [14]. In this way, we have that, if small perturbations are introduced, the resulting spacetime metric can be written as $g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}$, where $\tilde{g}_{\mu\nu}$ is the background metric and $h_{\mu\nu}$ is the spacetime perturbation. Moreover, the perturbations are much smaller than the background. By placing our attention on the perturbation $h_{\mu\nu}$, we have that, due to spherical symmetry, it can be written in two parts, where one depends on the angular coordinates through the spherical harmonics and the other one depends on the coordinates r e t . In addition, $h_{\mu\nu}$ can be written as a sum of a polar and an axial component.

In this context, we can find out, in particular, that the axial component of the gravitational perturbation of the metric (1),

can be written, under the Regge–Wheeler gauge, as [50]:

$$h_{\mu\nu}^{axial} = \begin{bmatrix} 0 & 0 & 0 & h_0(t, r) \\ 0 & 0 & 0 & h_1(t, r) \\ 0 & 0 & 0 & 0 \\ h_0(t, r) & h_1(t, r) & 0 & 0 \end{bmatrix} \sin \theta \partial_\theta P_l(\cos \theta) e^{im\phi}. \quad (11)$$

In the case of gravitational axial perturbations, we have that

$$\delta R_{\mu\nu} = 0, \quad (12)$$

where

$$\delta R_{\mu\nu} = \delta \Gamma_{\mu\alpha; \nu}^\alpha - \delta \Gamma_{\mu\nu; \alpha}^\alpha. \quad (13)$$

From the Eq. (12), we obtain

$$\delta R_{23} = \frac{1}{2} \left[-G^{-1} \frac{\partial h_0}{\partial t} + F \frac{\partial h_1}{\partial r} + \frac{1}{2} F' h_1 + \frac{1}{2} \frac{G'}{G} F h_1 \right] \times \left\{ \cos \theta \frac{\partial P_l(\cos \theta)}{\partial \theta} - \sin \theta \frac{\partial^2 P_l(\cos \theta)}{\partial \theta^2} \right\} = 0, \quad (14)$$

$$\delta R_{13} = \frac{1}{2} \left[G^{-1} \left(\frac{\partial^2 h_0}{\partial t \partial r} - \frac{H'}{H} \frac{\partial h_0}{\partial t} \right) + \left(-\frac{l(l+1)}{H} h_1 - G^{-1} \frac{\partial^2 h_1}{\partial t^2} + \frac{1}{2} F' \frac{H'}{H} h_1 + F \frac{H''}{H} h_1 + \frac{1}{2} \frac{G'}{G} F \frac{H'}{H} h_1 \right) \right] \times \left\{ \sin \theta \frac{\partial_l(\cos \theta)}{\partial \theta} \right\} = 0. \quad (15)$$

In order to handle the equations above, we can observe that values of multipole indices for which $l < s$, are not related to dynamical modes, corresponding to conserved quantities. In this way, we shall consider only the nontrivial radiative multipoles with $l \geq s$. On the other hand, a gravitational perturbation corresponding to $l = 0$ will be related to a black hole mass change, and a perturbation corresponding to $l = 1$ will be related to a displacement as well as to a change on the black hole angular momentum. The most interesting multipole indices values are those that correspond to the cases where the wave can propagate during a time interval large enough to be detected. In this way, only the $l \geq 2$ cases are relevant [51]. Since, for $l \geq 2$, we have $P_{l \geq 2} \neq 0$, we obtain the following radial equations

$$-G^{-1} \frac{\partial h_0}{\partial t} + F \frac{\partial h_1}{\partial r} + \frac{1}{2} F' h_1 + \frac{1}{2} \frac{G'}{G} F h_1 = 0, \quad (16)$$

$$-\frac{\partial^2 h_0}{\partial t \partial r} + \frac{\partial^2 h_1}{\partial t^2} + \frac{H'}{H} \frac{\partial h_0}{\partial t} + \left[\frac{G}{H} l(l+1) - \frac{1}{2} G F' \frac{H'}{H} - \frac{G F H''}{H} - \frac{1}{2} \frac{G' F H'}{H} \right] h_1 = 0. \quad (17)$$

Moreover, from the Eq. (16), we have that

$$\begin{aligned} \frac{\partial h_0}{\partial t} &= (GF)^{1/2} \left\{ (GF)^{1/2} \frac{\partial h_1}{\partial r} + \frac{1}{2} (GF)^{-1/2} G F' h_1 \right. \\ &\quad \left. + \frac{1}{2} (GF)^{-1/2} G' F h_1 \right\} \\ &= (GF)^{1/2} \frac{\partial Q(t, r)}{\partial r} = \frac{\partial Q(t, r)}{\partial x}, \end{aligned} \quad (18)$$

where the function $Q(t, r)$ is defined as

$$Q(t, r) \equiv (GF)^{1/2} h_1(t, r), \quad (19)$$

and the tortoise coordinate x is given by the relation:

$$\frac{\partial r}{\partial x} = (GF)^{1/2}. \quad (20)$$

By integrating the equation above, we obtain

$$\begin{aligned} x &= r - \frac{a_0^2}{rr-r_+} + \frac{a_0^2(r_-+r_+)}{r_-^2 r_+^2} \log(r) \\ &\quad + \frac{(a_0^2+r_-^4)}{r_-^2(r_- - r_+)} \log(r-r_-) + \frac{(a_0^2+r_+^4)}{r_+^2(r_+ - r_-)} \log(r-r_+). \end{aligned} \quad (21)$$

Now, if we substitute the Eqs. (19) and (20) in (17), we obtain:

$$\begin{aligned} -\frac{d}{dr} \frac{dQ}{dx} + (GF)^{-1/2} \frac{d^2 Q}{dt^2} + \frac{H'}{H} \frac{dQ}{dx} \\ + \left[\frac{G}{H} l(l+1) - \frac{1}{2} G F' \frac{H'}{H} - G F \frac{H''}{H} - \frac{1}{2} G' F \frac{H'}{H} \right] (GF)^{-1/2} Q = 0, \end{aligned} \quad (22)$$

which, by using the definition

$$\Psi(t, x(r)) \equiv \frac{Q(t, r)}{H^{1/2}}, \quad (23)$$

can be written as

$$\begin{aligned} -\frac{d}{dr} \frac{d}{dx} (H^{1/2} \Psi) + \left(\frac{GF}{H} \right)^{-1/2} \frac{d^2 \Psi}{dt^2} + \frac{H'}{H} \frac{d}{dx} (H^{1/2} \Psi) \\ + \left[\frac{G}{H} l(l+1) - \frac{1}{2} G F' \frac{H'}{H} - G F \frac{H''}{H} - \frac{1}{2} G' F \frac{H'}{H} \right] \left(\frac{GF}{H} \right)^{-1/2} \Psi = 0. \end{aligned} \quad (24)$$

The equation above can be rewritten as a Schrodinger-type equation which reads:

$$-\frac{d^2\Psi}{dx^2} + \frac{d^2\Psi}{dt^2} + V_{eff}(r(x))\Psi = 0, \quad (25)$$

where the effective potential is given by

$$V_{eff}(r(x)) = \frac{r^2(r-r_-)(r-r_+)}{(r^4+a_0^2)^4} \left\{ [6r_*^2 - 3(r_+ + r_-) \right.$$

Fig. 2 The effective potential V_{eff} for $l = 2, 3, 4$ as a function of r

Fig. 3 The effective potential V_{eff} for $l = 2, 3, 4$ as a function of x

Table 1 The first LQBH's quasinormal modes for $l = 2$

P	ω_0	ω_1	ω_2
0.1	$0.3987880 - 0.0928877i$	$0.3652110 - 0.2869660i$	$0.3107540 - 0.4942550i$
0.2	$0.4321490 - 0.0941480i$	$0.4057030 - 0.2896530i$	$0.3641310 - 0.4962680i$
0.3	$0.4620600 - 0.0935545i$	$0.4403830 - 0.2866220i$	$0.4064410 - 0.4892180i$
0.4	$0.4872150 - 0.0907627i$	$0.4694450 - 0.2769010i$	$0.4411340 - 0.4709230i$
0.5	$0.5061020 - 0.0855586i$	$0.4920720 - 0.2599850i$	$0.4690900 - 0.4405170i$
0.6	$0.5169450 - 0.0777077i$	$0.5064670 - 0.2352620i$	$0.4886680 - 0.3971150i$
0.7	$0.5177740 - 0.0669562i$	$0.5105030 - 0.2020610i$	$0.4975960 - 0.339798i$
0.8	$0.5066550 - 0.0528541i$	$0.5023270 - 0.1591170i$	$0.4943430 - 0.2667300i$

$$+ l(l+1)(r+r_*)^2]r^8 + 30r^4a_0^2[r(r_+ + r_- - r) - r_*^2] + 2r^4a_0^2l(l+1)(r+r_*)^2 + 3ra_0^4[2r - (r_+ - r_-)] + a_0^4(r+r_*)^2l(l+1) \Big\}. \quad (26)$$

The LQBH potential behavior in relation to the r and x coordinates are, respectively, depicted in the Figs. 2 and 3.

As we can observe, the potential (26) contains quantum gravitational contributions to the classical potential found out by Regge and Wheeler. These contributions depend on the LQG parameters such the polymeric parameter and the minimal area value.

In the next section, we shall analyze how the quantum gravity contributions from LQBH will affect the gravitational wave spectrum emitted by them. In order to do this, we shall analyze its quasinormal modes spectrum with the use of the WKB method.

4 Quasinormal modes from LQBHs

In order to obtain black hole quasinormal modes, several methods have been developed, which date back to the beginning of 70s. The simplest one consists in approximating the black hole potential by a Poschl–Teller one whose analytical solutions are known [52]. Another approximation method consists in the WKB method introduced by Schutz and Will [1]. Such technique is equivalent to find out the poles of the transmission coefficients of a Quantum Mechanics tunneling problem. This treatment was after improved to the third order by Iyer and Will [2], and to the sixth order by Konoplya [28]. Moreover, following Chandrasekhar and Detweiler [53], a shooting treatment in order to match the asymptotic solutions at some intermediate point can be also used. Another important method consists in the direct integration of the perturbation equation in the time domain by the use of light-cone coordinates [54]. In addition, we have yet the continued fraction method developed in 1985 by Leaver [55,56] and later improved by Nollert [57]. For a review of the available

Table 2 The first LQBH's quasinormal modes for $l = 3$

P	ω_0	ω_1	ω_2
0.1	$0.6358020 - 0.0984560i$	$0.6160010 - 0.2989190i$	$0.5814460 - 0.5068190i$
0.2	$0.6773290 - 0.0986630i$	$0.6601870 - 0.2990880i$	$0.6302840 - 0.5061190i$
0.3	$0.7134190 - 0.0970203i$	$0.6988960 - 0.2936260i$	$0.6734060 - 0.4958480i$
0.4	$0.7422610 - 0.0933003i$	$0.7303510 - 0.2819000i$	$0.7092190 - 0.4750270i$
0.5	$0.7618620 - 0.0872996i$	$0.7524960 - 0.2633440i$	$0.7356190 - 0.4427890i$
0.6	$0.7700050 - 0.0788193i$	$0.7630090 - 0.2374000i$	$0.7501440 - 0.3982930i$
0.7	$0.7642560 - 0.0676263i$	$0.7593800 - 0.2034110i$	$0.7502010 - 0.3405640i$
0.8	$0.7422370 - 0.0532427i$	$0.7393210 - 0.1599840i$	$0.7337250 - 0.2674180i$

Table 3 The first LQBH's quasinormal modes for $l = 4$

P	ω_0	ω_1	ω_2
0.1	$0.8558630 - 0.1003000i$	$0.8411430 - 0.3028960i$	$0.8141540 - 0.5102980i$
0.2	$0.9069660 - 0.1001910i$	$0.8940940 - 0.3023160i$	$0.8704570 - 0.5086910i$
0.3	$0.9506180 - 0.0982154i$	$0.9396740 - 0.2960930i$	$0.9194890 - 0.4975590i$
0.4	$0.9846000 - 0.0941816i$	$0.9756130 - 0.2836770i$	$0.9589180 - 0.4760390i$
0.5	$1.0064800 - 0.0879106i$	$0.9993990 - 0.2645530i$	$0.9861180 - 0.4433250i$
0.6	$1.0135200 - 0.0792181i$	$1.0082300 - 0.2381920i$	$0.9981560 - 0.3985940i$
0.7	$1.0027500 - 0.0678756i$	$0.9990480 - 0.2039320i$	$0.9919080 - 0.3408240i$
0.8	$0.9712540 - 0.0533938i$	$0.9690330 - 0.1603310i$	$0.9647010 - 0.2676880i$

techniques, we suggest [58] (see also [28, 59, 60] for other readings about QNMs).

In this section, we shall use a WKB method due to Schutz and Will [1], and further improved by Iyer and Will [2] in order to obtain the LQBH's quasinormal modes in the third order approximation. Following this method, if one supposes that $\Psi(t, x(r))$ has a harmonic asymptotic behavior in t , $\Psi \sim e^{-i\omega(t \pm x)}$, and $V_{eff}(r(x)) \rightarrow 0$ as $x \rightarrow \pm\infty$, we obtain the following wave equation:

$$\frac{d^2\Psi}{dx^2} + \omega_n^2\Psi - V_{eff}(r(x))\Psi = 0. \quad (27)$$

In the equation above, the frequencies ω_n are determined at third order approximation by the following relation:

$$\omega_n^2 = [V_0 + \Delta] - i\left(n + \frac{1}{2}\right)(-2V_0'')^{1/2}(1 + \Omega), \quad (28)$$

where

$$\Delta = \frac{1}{8} \left[\frac{V_0^{(4)}}{V_0''} \right] \left(\frac{1}{4} + \alpha^2 \right) - \frac{1}{288} \left(\frac{V_0'''}{V_0''} \right)^2 (7 + 60\alpha^2), \quad (29)$$

$$\Omega = -\frac{1}{2V_0''} \left[\frac{5}{6912} \left(\frac{V_0'''}{V_0''} \right)^4 (77 + 188\alpha^2) - \frac{1}{384} \left(\frac{(V_0''')^2(V_0^{(4)})}{(V_0'')^3} \right) (51 + 100\alpha^2) \right]$$

Fig. 4 Behavior of the quasinormal modes, **a** real part and **b** imaginary part of ω for $l = 2$ and $P = 0.1, 0.3, 0.6$

Fig. 5 Behavior of the quasinormal modes, **a** real part and **b** imaginary part of ω for $l = 3$ and $P = 0.1, 0.3, 0.6$

Fig. 6 Behavior of the quasinormal modes, **a** real part and **b** imaginary part of ω for $l = 4$ and $P = 0.1, 0.3, 0.6$

$$\begin{aligned}
 & + \frac{1}{2304} \left(\frac{V_0^{(4)}}{V_0''} \right)^2 (65 + 68\alpha^2) \\
 & + \frac{1}{288} \left(\frac{V_0''' V_0^{(5)}}{(V_0'')^2} \right) (19 + 28\alpha^2) \\
 & - \frac{1}{288} \left(\frac{V_0^{(6)}}{V_0''} \right) (5 + 4\alpha^2) \Big]. \quad (30)
 \end{aligned}$$

In the relations above, $\alpha = n + \frac{1}{2}$ and $V_0^{(n)}$ denotes the n -order derivative of the potential on the maximum x_0 of the potential.

By the use of the potential (26) in the relations (27), (28), (29), and (30), we can find out the quasinormal frequencies for a LQBH, which have been depicted in the Tables 1, 2 and 3. One may find the Schwarzschild case in Ref. [61].

Fig. 7 Imaginary $\times$ Real part of ω for **a** $l = 2$, **b** $l = 3$, **c** $l = 4$ and $P = 0.1, P = 0.3, P = 0.6$

The presented results imply in a larger oscillation, as well as a slower damping as the polymeric parameter increases. In this way, the LQBHs decay slower as the quantum gravity contribution becomes more significant. Moreover, as we can see, LQBHs are stable under axial perturbations due to the negative sign of the imaginary part of frequencies for several values of the polymeric parameter.

In order to visualize, in a better way, the effects of the quantum gravity corrections present in the LQBH scenario on the black hole quasinormal spectrum, we have depicted in the Figs. 4, 5 and 6 the behavior of the LQBH quasinormal modes, as a function of n , for different values of the polymeric function P . In this way, we have (a) real part and (b) imaginary part of ω_n for $l = 2, 3, 4$, by consideration of the following values of the polymeric function: $P = 0.1$, $P = 0.3$ and $P = 0.6$. For the sake of comparison it was also depicted the behavior of the quasinormal spectrum of a Schwarzschild black hole.

Moreover, it is also interesting to plot the frequencies in the complex plane. In this way, in Fig. 7 this is done by considering three families of multipoles $l = 2, 3, 4$. Looking at the right side of the figure, we conclude that the frequency curves are moving clockwise as P grows, which consists in the inverse effect we have in the case of scalar perturbations [30,31]. Such twisting effect becomes more apparent for larger values of the angular quantum number. We also present the behavior for the Schwarzschild black hole.

From the results found out, LQBHs quasinormal modes depend directly from the polymeric parameter, where a larger oscillation, as well as a slower damping occurs as the value of the polymeric parameter increases. As a result, the LQBHs decay slower when the quantum gravity contribution becomes more important.

However, from the recent gravitational wave observations [23–27], we have that, for macroscopic black holes, with mass of the order of the gravitational wave spectrum must agree with that given by classical General Relativity theory. In this way, for such black holes, one should not expect that quantum corrections would result in significant changes to the quasinormal radiation spectrum.

In order to face up such observational facts with our results, we have multiplied the quasinormal modes by the factor $2\pi(5.142 \text{ KHz}) M_\odot/M_{BH}$, in order to include the information about the black hole mass [28]. The results have been plotted in the Fig. 8, for the case where $l = 2$, and $n = 0$, for black hole masses in the same range of that investigated in the recent observations [23–27]. From such results, since the polymeric function does not suffers with the mass suppression problem, we have that the recent gravitational wave observations impose a limitation to our model, where it remains valid only for $P \leq 0.1$. Such result could be used in order to impose a limitation in the value of the Barbero–Immirzi parameter, in the context of LQG.

Fig. 8 Imaginary and Real part of ω for $l = 2$, $n = 0$, and $P = 0.1$, $P = 0.4$, $P = 0.8$

5 Conclusions and perspectives

Black holes offer a scenario where quantum fluctuations of spacetime may appear and can be a good laboratory in order to verify the validity of predictions coming from different approaches that have arisen as candidates for a theory of quantum gravity like superstring theory and LQG. In particular, LQBHs provide us with a way to investigate quantum gravity corrections from LQG.

Due to its thermodynamical properties, the importance of LQBHs has been shown to go beyond the simple verification of LQG predictions to the gravitational field produced by a black hole, but it extends to the discussion of other relevant themes like the problem of dark matter and the problem of the initial state of the cosmos. It is because, LQBHs in a different way from classical Schwarzschild black holes must get thermodynamical equilibrium with radiation due to the anomalous behavior of its temperature near the Planck scale [11]. In this way, LQBHs have the necessary thermodynamical stability in order to be conceived as possible candidates to dark matter [11, 12]. Moreover, from the form of the entropy-area relation associated with LQBHs, they have been pointed as the building blocks of a holographic description of loop quantum cosmology [13].

Gravitational waves, on the other hand have opened the doors to a new world in physics and may establish a bridge between the quantum gravity world revealed by a black hole and experimental investigations in gravity. In this way, motivated by the new perspectives opened by the detection of gravitational waves, we have investigated the stability of

LQBHs under axial gravitational perturbations and its quasinormal spectrum.

In order to investigate how LQG corrections to a black hole scenario can influence in the gravitational wave emission, we have obtained the LQBH's quasinormal frequencies by the use of the WKB approach. The Regge–Wheeler potential corresponding to the gravitational perturbations around the LQBH was obtained and the related Regge–Wheeler equation has been solved. After, the quasinormal frequencies have been obtained by the use of 3th-order WKB approximation, demonstrating that such black holes are stable under axial perturbations.

For the aforementioned importance of LQBHs as candidates to dark matter and to construct a holographic version of LQC, it is crucial that they should be stable under gravitational perturbations. Such stability has been met in the results found out in the present paper. Additional contributions can be obtained, beyond the 3th order approximation, by the use of the methods developed by Konoplya [61] in a forthcoming analysis.

From the results found out, LQBHs quasinormal modes depend directly from the polymeric parameter. However, from the recent gravitational wave observations [23–27], for macroscopic black holes, the gravitational wave spectrum must agree with that given by classical General Relativity theory. In this way, for such black holes, one should not expect that quantum corrections would result in significant changes to the quasinormal radiation spectrum.

By analyzing our results for different black holes masses, we have found out a limitation to our model where it remains valid only for $P \leq 0.1$. Others black hole models in the context of LQG can be investigated in future works. In fact some efforts have been done in order to obtain a consistent black hole solution from LQG [10, 34–43]. (For a review on this issue, see [62]. For some more recent works on this issue, see [63, 64]). The analysis of the gravitational wave spectrum from such solutions can be used as an additional criteria in order to verify, or falsify, some of such proposals.

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