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Investigation of zero-modes for a dynamical Dp-brane

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Abstract In this article, we investigate zero-modes for a dynamical (rotating-moving) Dp-brane, coupled to the electromagnetic and tachyonic background fields. This work is done by the boundary state methods, in three cases of bosonic and fermionic boundary states and superstring partition function. By analyzing the obtained zero-modes in either of the cases, interesting results will be obtained. Our findings demonstrate the importance of the zero-mode and its effects on the background fields and the defined internal properties of the described system.

1 Introduction and conclusion

So far many significant aspects of D-branes [1,2], as an essential objects of string/superstring theory, have been discovered. D-branes, interpreted as the classical solutions of the low energy string effective action, could be defined in terms of closed strings. Meanwhile, the boundary state method [3–5], as a powerful technique, can be considered to show the couplings of all closed string states to D-branes. Boundary state method is a beneficial approach in many complicated situations, even when a clear space-time is not accessible. By applying the boundary state method for describing D-branes, different properties and many configurations of these objects have been studied [6–12]. Another description of D-branes, as tachyonic solitons, follows from the boundary string field theory, in which one can codify the information by using the disc partition function of the open string sigma model. In this context, for the case of superstring theory, an effective space-time action can be obtained by the corresponding world-sheet partition function [13–19].

In most articles in which have been studied the issue of the boundary state, the zero-modes have been neglected for simplifying the calculation and hence, the zero-modes effects have been omitted from the system. In this paper, we concentrate specifically on the zero-modes and their effects on the properties of the system under study, such as its dynamics and background fields. This work is done by studying the zero-modes boundary states, for both bosonic and fermionic theory and also for zero-mode superstring partition function, corresponding to a rotating-moving Dp-brane which is coupled to a $U(1)$ gauge potential in the world-volume of the brane (photonic field) and tachyonic background field as open string states.

By investigating the zero-modes in each case, interesting results will be obtained. For example, according to the initial condition and the theory (bosonic or fermionic) which is chosen, the background fields contributions to the zero-modes would be different. In the zero-mode bosonic boundary state, photonic background field would be disappeared. The reason behind that can be explained either by the boundary state method or by the path integral techniques. The former is due to the zero-mode boundary state equation, which is obtained by substituting the zero-mode solution of the closed string equation of motion into the boundary equations. It should be noted that in the case of compact space-time this result could be changed. However, in this paper we assume a non-compact flat space-time and therefore in the bosonic theory, the photonic term would be absent in the zero-mode boundary state. On the other hand, for the fermionic theory, the electromagnetic field would be present and existence of the tachyon field could depend on the choice of the σ's value.

Moreover, we study the explicit form of the fermionic zero-mode boundary state, for both type IIA and type IIB theories, corresponding to the present rotating-moving Dp-brane with tachyonic and photonic boundary action. By evaluating different configurations of the described system, i.e., omitting the background fields and/or rotating-moving structure, our system would overlap some familiar brane structures in the literature. In continue, we have listed some branes configurations and their corresponding zero-mode R-R sector solutions in a classified table. This table shows how the brane properties would affect the R-R zero-mode solution.

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Exploring the zero-mode partition function of the superstring theory, would show the importance of the bosonic zero-mode part of the partition function, and hence, could explain the reason of the absence of photonic field, from the zero-mode superstring partition function, for our described system. Besides, by considering the low limit of the present brane structure, i.e. omitting the dynamical movement, the familiar relation for the zero-mode part of the effective action would be revealed.

The other achievement of this article, is obtaining an elegant interpretation for the potential of this complex system. We show that, by considering the rotating-moving structure and the photonic and tachyonic fields, not only the background field but also the dynamical structure of the system will merge to the potential.

Simultaneous consideration of rotating-moving dynamics, and the electromagnetic and tachyonic background fields, are the main distinctions from conventional literature. This difference, lead our zero-mode equations to a mixed and complex structure, between the pointed parameters, in three cases of bosonic and fermionic zero-mode boundary states and superstring zero-mode partition function, and hence, make the present zero-mode investigation outstanding.

This article is organized as follows. In Sect. 2, The zero-modes corresponding to a rotating-moving Dp-brane with various background fields in the bosonic theory will be constructed. In Sect. 3, The zero-modes in the fermionic theory of such a dynamical system will be extracted. In Sect. 4, The zero-mode partition function in superstring theory associated with a rotating-moving brane will be investigated.

2 Zero-modes in bosonic theory

The action - For exploring the zero-modes for a rotating-moving Dp-brane, the starting point is to describe such a system by means of the sigma-model action in the bosonic theory. The general form of the bulk action is $S = -\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma(\sqrt{-h}h^{\alpha\beta}g_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu)$. For the sake of simplicity, throughout this paper we will use the conformal gauge property and replace the two-dimensional intrinsic metric $h_{\alpha\beta}$ with the two-dimensional flat space-time metric $\eta_{\alpha\beta}$ and use the signature $\eta_{\mu\nu} = diag(-1, 1, \ldots, 1)$ for the metric $g_{\mu\nu}$. Therefore, the bulk action for the closed string in the conformal gauge would be

$$S_{\text{bulk}} = -\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma(\eta_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu), \qquad (1)$$

where Σ is the closed string world-sheet. The world-sheet is parameterized by two parameters (τ, σ) and can be viewed as the map $(\tau, \sigma) \mapsto X^\mu(\tau, \sigma)$. In other word, the corresponding point on the world-sheet with coordinate (τ, σ) would be $\{X^\mu|\mu = 0, 1, \ldots, d-1\}$ on the space-time. Besides, we consider $d^2\sigma \equiv d\sigma d\tau$ and $\partial_\alpha \equiv \frac{\partial}{\partial\sigma^\alpha}$ in which instead of σ and τ one can use variables $\sigma^\alpha|\alpha = 0, 1$ with $\sigma^0 = \tau$ and $\sigma^1 = \sigma$. Note that the world-sheet has the topology of a cylinder which is due to the fact that the fields X^μ satisfy periodic boundary condition for the closed string, i.e. $X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + 2\pi)$ with $0 \le \sigma \le 2\pi$.

In continue, deformations to the original theory are coupled via the boundary terms, as the following boundary action

$$S_{\text{bdry}} = \frac{1}{2\pi\alpha'}\int_{\partial\Sigma} d\sigma(A_\alpha\partial_\sigma X^\alpha + \omega_{\alpha\beta}J_\tau^{\alpha\beta} + iT^2(X^\alpha)), \qquad (2)$$

with the world-sheet boundary $\partial\Sigma$. By assuming the boundary action (2), the following background fields and related terms have been considered: a $U(1)$ gauge field $A_\alpha = -\frac{1}{2}F_{\alpha\beta}X^\beta$ with constant field strength and ∂_σ which is derivative along the boundary. The second term characterizes the rotation and motion of the brane with anti-symmetric angular velocity $\omega_{\alpha\beta}$ and angular momentum density $J_\tau^{\alpha\beta}$. This term is denoted by $\omega_{\alpha\beta}J_\tau^{\alpha\beta} = 2\omega_{\alpha\beta}X^\alpha\partial_\tau X^\beta$, where the component $\omega_{0\bar{\alpha}}|_{\bar{\alpha}\neq 0}$ represents the velocity of the brane along the direction $X^{\bar{\alpha}}$ and $\omega_{\bar{\alpha}\bar{\beta}}$ which interprets its rotation. The final term is a tachyon field with tachyonic profile $T^2 = T_0 + \frac{1}{2}U_{\alpha\beta}X^\alpha X^\beta$, where T_0 and the symmetric matrix $U_{\alpha\beta}$ are considered to be constant. Moreover, we represent $\{X^\alpha|\alpha = 0, 1, \ldots, p\}$ for the world-volume directions of the brane and $\{X^i|i = p+1, \cdots, d-1\}$ for directions perpendicular to it and let $U_{\alpha i} = 0$ for simplicity.

The local symmetries – As a general case, in the sigma model approach some of the gauge symmetries of the string action on a world-sheet with boundaries are modified by turning on the background fields. In the above described actions (Eqs. 1 and 2), the bulk, the background gauge field and the dynamical term actions are diffeomorphism invariant. Besides, the tachyonic boundary action is invariant with respect to a subgroup of the diffeomorphism group. Therefore, Eqs. (1) and (2) are written in world-sheet diffeomorphism invariant way. What about the Weyl symmetry? Except the tachyonic boundary action with broken Weyl-invariance, the other terms, i.e., the bulk, the background gauge field and the rotating-moving term actions are Weyl invariant and have been written in the conformal gauge. However, despite the fact that the tachyonic boundary term does not preserve the Weyl invariance, but still have enough symmetries to impose the conformal gauge [16–19]. Broken Weyl-invariance is often referred to as broken conformal invariance and leads to an off-shell theory.This can be expressed as an example of the background independent string field theory. In this content, one can study properties of the unstable D-branes due to the tachyon condensation (where the unstable D-branes may behave like solitons, corresponding to lower dimensional D-branes) [13–19]. The effect of losing the conformal invariance, due to the presence of tachyon field, would show up not only as corrections to the effective action [24] but also in the boundary states. At the end of this section, we will come back to this point and discuss the effect of this conformal breaking on the boundary states of the described system.

Boundary state - Boundary state equations for the above bosonic action can be achieved by vanishing the variation of the action with respect to $X^\mu(\sigma,\tau)$ as

$$[(\eta_{\alpha\beta}+4\omega_{\alpha\beta})\partial_\tau X^\beta + F_{\alpha\beta}\partial_\sigma X^\beta + iU_{\alpha\beta}X^\beta]|_{\tau=0}\ |B\rangle_{bos} = 0,$$
$$\delta X^i|_{\tau=0}|B\rangle_{bos} = 0, \tag{3}$$

This equation is the key relation for obtaining the zero mode boundary states in both bosonic and fermionic sections.

Bosonic zero-mode boundary state - As a general strategy for the bosonic theory, the zero-mode coordinate X is conjugate to the total momentum p of the string. In the absence of an external field, the zero-mode boundary-state is the translation-invariant state $|B\rangle^{(0)}_{bos}$ annihilated by p. In the case of having a boundary action $S(X\ldots)$ which depends on X through a space-dependent background field, this zero-mode state is changed by the action of the zero-mode coordinate, to $e^{-S(q)}|B\rangle^{(0)}_{bos}$ [4,5].

Now, let us find the zero-mode bosonic boundary state, first, by applying the zero-mode closed string mode expansion

$$X^\mu(\sigma,\tau) = x^\mu + l^2 p^\mu \tau + \frac{1}{2} il \sum_{m\neq 0}\frac{1}{m}\,(\,\alpha^\mu_m e^{-2im(\tau-\sigma)} + \tilde{\alpha}^\mu_m e^{-2im(\tau+\sigma)}),$$

in Eq. (3). Therefore, in the case of our background fields, boundary state equations are converted to

$$[2\alpha'(\eta_{\alpha\beta}+4\omega_{\alpha\beta})p^\beta + iU_{\alpha\beta}x^\beta]|B\rangle^{(0)}_{bos} = 0, \tag{4}$$
$$(x^i - y^i)|B\rangle^{(0)}_{bos} = 0, \tag{5}$$

Where the set $\{y^i\}$ indicates the position of the brane.

Then, by combining Eqs. (4) and (5) and using the following relation

$$|B\rangle\ = \int_{-\infty}^{\infty}\prod_\alpha dp^\alpha \langle p^\alpha|B\rangle\ \ |p^\alpha\rangle \tag{6}$$

the zero-mode bosonic boundary state is obtained as

$$|B\rangle^{(0)}_{bos} = \int_{-\infty}^{\infty}\prod_\alpha dp^\alpha \exp\left[\alpha'\left(\sum_{\alpha=0}^{p}\left(U^{-1}\mathbf{A}\right)_{\alpha\alpha}(p^\alpha)^2 + \sum_{\alpha,\beta=0,\alpha\neq\beta}^{p}\left(U^{-1}\mathbf{A}+\mathbf{A}^T U^{-1}\right)_{\alpha\beta}p^\alpha p^\beta\right)\right] \times \prod_i \delta(x^i - y^i)|p^i = 0\rangle \otimes \prod_\alpha |p^\alpha\rangle. \tag{7}$$

where $\mathbf{A}_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta}$

The above momentum dependent integral comes from taking into account the first equation of zero modes, i.e, Eq. (4) and a delta function, which is inserted to fix the location of the D-brane (by imposing an extra condition on the position operator) in the transverse direction according to Eq. (5).

It should be noted that for simplifying the calculations, the Kalb-Ramond field $B_{\mu\nu}$ has been omitted from the bulk action. However, even if one considered this field in addition to the gauge potential, then, the results of the zero-mode would not be changed. Because it could be converted into the boundary term by defining $\mathcal{F}_{\alpha\beta} = F_{\alpha\beta} - B_{\alpha\beta}$. Therefore, addition of this term does not have any contribution to the zero-mode bosonic boundary state. Consideration of the bulk 2-form B and the field strength F, would show up only in the oscillating part of the bosonic boundary state.

There is an interesting point that except the photonic field, the other background fields have been contributed in the zero mode boundary state of the bosonic theory. The reason behind that refers to Eq. (4) and the fact that zero mode closed string mode expansion does not contribute to the photonic field. It's physical point of view can be explained as follows: in the case of static brane, the tachyon does not couple to the gauge field because the tachyon is in the U(1) adjoint representation, hence, using the path integral method and considering this decoupling from the gauge field, the zero-mode would just depends on tachyon zero mode [14,15]. in the case of adding a dynmical structure to the same brane, the mentioned decoupling of the gauge field would be exist and the rotating-moving feature of the brane would be appear along with the tachyon field in the zero-mode bosonic boundary state. Thus, inclusion of rotating-moving dynamics of the Dp-brane, which is emerged from the momentum dependent integral, would be create an outstanding mixed structure for the tachyon and this special dynamics of the system. It should be noted that in the case of compact space-time, one could reach to a different result. This is due to the fact that for compact directions, the complete solution of the closed string equation has a $2L^\mu\sigma$ term, proportional to the radius of compactification in the compact direction. For non-compact directions, the closed string winding number L^μ is zero but in the case of having components along the compact directions this term would be remained. Therefore, because of the σ dependency of the photonic term in the bosonic boundary state equation, all of the background fields could be contributed in the zero mode boundary state of the bosonic theory.

Now let us point out to an important note here. As we know, due to the presence of the tachyon field in the boundary state, conformal invariance has been broken. However, we know that D-branes are specified by conformally invariant boundary states which act as sources for all closed string fields. Therefore, in order to have a conformal invariant boundary state, one could deform it under the subset of the full conformal group of the plane, PSL(2,R), which preserves the position and shape of the boundary of the disc. In other words, the loss of conformal invariance introduced by the background tachyon feld is accomodated by a conformal transformation which induces a calculable change in the boundary state. Obtaining the new modified boundary state one could see the fact that the zero mode part of the boundary state does not affected by these transformations. The detailed calculations can be found in [12]. For the sake of simplicity, we do not use the modified form of the boundary state here.

These interesting features caused the present zero-mode bosonic boundary state to be distinguished from the conventional literature.

3 Zero-modes in fermionic theory

The supersymmetric prescription of the bosonic action, in the previous section, is invariant under the global world-sheet supersymmetry. Therefore, one can find the fermionic partners of the bosonic boundary equations by applying the supersymmetry transformations. According to the bosonic boundary state equations (3) we need to insert the following replacements

$$\begin{aligned}
&\partial_\sigma X^\mu(\sigma,\tau) \to (-\psi_0^\mu - i\eta\tilde{\psi}_0^\mu)\\
&\quad+\sum_{k\neq 0}(-i\eta\tilde{\psi}_k^\mu\, e^{-2ik(\tau+\sigma)} - \psi_k^\mu\, e^{-2ik(\tau-\sigma)})\\
&\partial_\tau X^\mu(\sigma,\tau) \to (\psi_0^\mu - i\eta\tilde{\psi}_0^\mu)\\
&\quad+\sum_{k\neq 0}(-i\eta\tilde{\psi}_k^\mu\, e^{-2ik(\tau+\sigma)} + \psi_k^\mu\, e^{-2ik(\tau-\sigma)})
\end{aligned} \tag{8}$$

These two general forms for supersymmetry transformations have been calculated by considering the world-sheet supersymmetry transformations

$$\begin{aligned}
&\partial_+ X^\mu(\sigma,\tau) \to -i\eta\psi_+^\mu(\sigma,\tau),\\
&\quad\partial_- X^\mu(\sigma,\tau) \to \psi_-^\mu(\sigma,\tau),
\end{aligned} \tag{9}$$

$\partial_\pm = (\partial_\tau \pm \partial_\sigma)/2$ and the solution of the equations of motion for the fermions. In addition $\eta = 1$ is due to the GSO projection of the boundary state.

Furthermore, to fulfill applying the supersymmetry transformations of Eq. (3), we need the following replacement for the tachyonic term

$$\begin{aligned}
&X^\beta(\sigma,\tau) \to \sum_{k\neq 0}\frac{1}{2ik}(i\eta\tilde{\psi}_k^\beta\, e^{-2ik(\tau+\sigma)} - \psi_k^\beta\, e^{-2ik(\tau-\sigma)})\\
&+(-i\eta\tilde{\psi}_0^\beta - \psi_0^\beta)\sigma + (\psi_0^\mu - i\eta\tilde{\psi}_0^\mu)
\end{aligned} \tag{10}$$

Note that it is important to separate the zero modes and non-zero modes in Eqs. (8) and (10) to prevent having a wrong answer. The significance of this point will be revealed in the zero-mode fermionic boundary equations.

Considering all these together, the zero-mode fermionic boundary states for the closed string boundary at $\tau = 0$ are as

$$\begin{aligned}
&[(\eta_{\alpha\beta} + 4\omega_{\alpha\beta})(-i\eta\tilde{\psi}_0^\beta + \psi_0^\beta)\\
&\quad+(F_{\alpha\beta} + i\sigma U_{\alpha\beta})(-i\eta\tilde{\psi}_0^\beta - \psi_0^\beta)]|B\rangle^{(0)}_{ferm} = 0\\
&(-i\eta\tilde{\psi}_0^i - \psi_0^i)|B\rangle^{(0)}_{ferm} = 0.
\end{aligned} \tag{11}$$

The two Eq. (11) can be collected into a unified form

$$(d_0^\mu - i\eta S^\mu_{(0)\nu}\tilde{d}_0^\nu)|B\rangle^{(0)}_{R-R} = 0 \tag{12}$$

in which

$$\begin{aligned}
&S_{(0)\mu\nu} = (\Delta_{(0)\alpha\beta}, -\delta_{ij}),\\
&\Delta_{(0)\alpha\beta} = (M_{(0)}^{-1}N_{(0)})_{\alpha\beta},\\
&M_{(0)\alpha\beta} = (\eta_{\alpha\beta} + 4\omega_{\alpha\beta} - F_{\alpha\beta} - i\sigma U_{\alpha\beta}),\\
&N_{(0)\alpha\beta} = (\eta_{\alpha\beta} + 4\omega_{\alpha\beta} + F_{\alpha\beta} + i\sigma U_{\alpha\beta}).
\end{aligned} \tag{13}$$

Clearly the only contribution of the fermionic zero-mode part of the boundary state is that of the R-R sector. It is due to the mode number in the NS-NS sector which runs over half integers.

An important point here is, unlike the bosonic zero-mode boundary state in which photon was omitted, in the fermionic case all the background fields could be participated. This matter depends on the value of the σ. Therefore in the case of $\sigma = 0$ the contribution of tachyonic field would be disappeared and the tachyon would be omitted from the zero-mode boundary state. The final result would affects the fermionic vacuum and hence the spin structure of the boundary state.

To complete our discussion, we should find the explicit form of the fermionic zero-mode boundary state $|B\rangle^{(0)}_{ferm}$ by applying similar considerations as in the bosonic section. According to [4,5], the general strategy is as follows: In the presence of a non vanishing background field, the zero-mode part of the boundary action, will generate some linear combination of n-forms, that span a 2^d-dimensional Hilbert space. This work is done by the action of the zero-mode part on the vacuum. Therefore, the zero-mode part can be classified as

$$e^{-S(\theta_0^\mu,\ldots)}|0;+\rangle = \{\text{polynomial in } \theta_0^\mu\}|0;+\rangle,$$

for the + spin structure, and

$$e^{-S(\pi_0^\mu,\ldots)}|0;-\rangle = \{\text{polynomial in } \pi_0^\mu\}|0;-\rangle,$$

for the − spin structure. In these relations θ_0^μ are the boundary coordinates associated with the zero modes of the + spin structure with the definition $\theta_0^\mu = \psi_0^\mu + i\tilde{\psi}_0^\mu$, and π_0^μ are their corresponding conjugate momenta defined as $\pi_0^\mu = 1/2(\psi_0^\mu - i\tilde{\psi}_0^\mu)$.

The vacuum of one spin structure is the filled Fermi sea of the other, due to the fact that creation operators of one spin structure are the annihilation operators of the other, i.e., $|0;-\rangle = \prod_{\mu=1}^{D}\theta_0^\mu|0;+\rangle$. The states in the zero-mode

Hilbert space can be represented by antisymmetrized products of D-dimensional gamma matrices, this is due to the fact that the n-forms (built out of the θ_0^μ), correspond to the D-dimensional Clifford algebra. Now by considering a duality relation between the above zero-mode forms, and transforming the antisymmetrized products of gamma matrices (by multiplying them by Γ_{11}), the gamma matrix representation of the zero-mode forms can be adopted as

$$e^{-S(\theta_0^\mu,\ldots)}|0;+\rangle \equiv \{\text{sum of } \Gamma^{\mu_1\cdots\mu_n}\}|0;+\rangle,$$
$$e^{-S(\pi_0^\mu,\ldots)}|0;-\rangle \equiv \{\text{sum of } \Gamma^{\mu_1\cdots\mu_n}\}\Gamma_{11}|0;+\rangle \quad (14)$$

Therefore, by unifying these two spin structures provided by the gamma matrix notation, the products of fermionic zero modes can be represented as antisymmetrized products of gamma matrices, along with introducing a notation in which all terms in the expansion with repeated Lorentz indices are to be dropped.

Using above considerations, let us study the explicit form of the fermionic zero-mode boundary state $|B\rangle^{(0)}_{ferm}$ both in type IIA and type IIB theories, for the present case of a rotating-moving Dp-brane with tachyonic and photonic boundary action. Considering the spin fields in the 32-dimensional Majorana representation, i.e, $\mathbf{S}^A$ and $\tilde{\mathbf{S}}^B$, the vacuum for the fermionic zero-modes has the following feature [6,7]

$$|A\rangle|\tilde{B}\rangle = \lim_{z,\bar{z}\to 0} \mathbf{S}^A(z)\tilde{\mathbf{S}}^B(\bar{z})|0\rangle \quad (15)$$

Where $A, B = 1, \ldots, 32$.

Translating above considerations into Hilbert space language, the action of ψ_0^μ and $\tilde{\psi}_0^\mu$ on the R-R vacuum, in a non-chiral basis, can be obtained as

$$\psi_0^\mu|A\rangle|\tilde{B}\rangle = \frac{1}{\sqrt{2}}(\Gamma^\mu)^A_C(1)^B_D|C\rangle|\tilde{D}\rangle$$
$$\tilde{\psi}_0^\mu|A\rangle|\tilde{B}\rangle = \frac{1}{\sqrt{2}}(\Gamma_{11})^A_C(\Gamma^\mu)^B_D|C\rangle|\tilde{D}\rangle \quad (16)$$

Using these definitions, the zero-mode fermionic structure of the present system, i.e, $|B\rangle^{(0)}_{ferm}$, in the language of Hilbert space, can be obtained by considering a solution of Eq. (12) as

$$|B\rangle^{(0)}_{ferm} = \Lambda_{AB}|A\rangle|\tilde{B}\rangle \quad (17)$$

in which Λ satisfies $(\Gamma^\mu)^T\Lambda - i\eta S^\mu\Gamma_{11}\Lambda\Gamma^\nu = 0$, where a chiral representation for the 32×32 Γ-matrices is applied.

Consequently, the following solution can be found

$$\Lambda = C\Gamma^0\Gamma^1\cdots\Gamma^p(\frac{1+i\eta\Gamma_{11}}{1+i\eta})\Upsilon \quad (18)$$

where C is the charge conjugate matrix, and

$$\Upsilon = K\diamond\exp\left\{\frac{1}{2}[(\Delta_{(0)}-1)(\Delta_{(0)}+1)^{-1}]_{\alpha\beta}\Gamma^\alpha\Gamma^\beta\right\}\diamond, \quad (19)$$

in which K is a normalization constant that should be determined according to the boundary action and total boundary state, including the oscillating part of the both bosonic and fermionic sections. Here we do not intend to calculate it. The interested reader can find the detailed calculations in our previous papers [8–11]. Moreover, the symbol $\diamond$ rules that the exponential should be expanded and the indices of the Γ-matrices must be antisymmetrized. As a result, this symbol makes some restrictions. For example, it limits the number of terms for each value of p to a finite number.

Now let's see the above fermionic zero-mode solution how could be changed with some simplifications. Note that the general solution would be the same form as Eq. (17), but due to the initial assumptions, Λ_{AB} would be different. First, consider the Dp-brane as the static object without any background fields. For this structure the fermionic zero-mode solution would be as $\Lambda = C\Gamma^0\Gamma^1\cdots\Gamma^p(\frac{1+i\eta\Gamma_{11}}{1+i\eta})$, in which we have $\Upsilon = 1$. Now by considering the excitations of the open strings attached to the Dp-brane (adding gauge field) Λ would be change according to $\Upsilon = \sqrt{\det(\eta - F)}\diamond\exp\left\{\frac{1}{2}F_{\alpha\beta}\Gamma^\alpha\Gamma^\beta\right\}\diamond$. These two low limits of the present structure are in complete agreement with the literature. It should be noted that according to the initial definition of our gauge filed the prefactor K is the inverse value of [6,7].

In table 1 some branes configurations and their corresponding zero-mode R-R sector solutions have been listed. It should be noted that the T-duality transformation mentioned in the list is performed in the transverse direction i^{th} of the D-brane. And for the next case, at first a rotation in the $(p, p+1)$ plane with angle ϕ and then a T-duality transformation in the $(p+1)^{th}$ direction have been done. The rotated and T-dualized boundary state is the world-sheet realization of a bound state which reproduces the long-distance behavior of the delocalized D-brane classical solutions [6,7].

As it seems, the present system can cover some of those configurations in the low limits (omitting the background fields and/or rotating-moving structure). This outstanding feature is appeared also in the zero-mode partition function in superstring theory.

4 Interpretation of zero-mode partition function in superstring theory

According to the description of the boundary string field theory, a fixed conformal world-sheet action in the bulk (in the

Table 1 some branes configurations and their corresponding zero-mode R-R sector solutions

Brane configuration	R-R zero-mode solution
Static Brane, No external field	$\Lambda = C\Gamma^0\Gamma^1 ... \Gamma^p(\frac{1+i\eta\Gamma_{11}}{1+i\eta})$
T-dualized Brane, No external field	$\Lambda = C\Gamma^0\Gamma^1 \cdots \Gamma^p\Gamma^i(\frac{1+i\eta\Gamma_{11}}{1+i\eta})$
Rotated and T-dualized Brane, No external field	$\Lambda = C\Gamma^0\Gamma^1 \cdots \Gamma^{p-1}(\sin\phi + \cos\phi\Gamma^p\Gamma^{p+1})$
Boosted Brane, No external field	$\Lambda = \frac{1}{\sqrt{1-v^2}}(C[\Gamma^0 + v\Gamma^k]\Gamma^1 \cdots \Gamma^p)(\frac{1+i\eta\Gamma_{11}}{1+i\eta})$
Static Brane, With gauge field	$\Lambda = \sqrt{\det(\eta - F)}C\Gamma^0\Gamma^1 \cdots \Gamma^p(\frac{1+i\eta\Gamma_{11}}{1+i\eta})\diamondsuit e^{\frac{1}{2}F_{\alpha\beta}\Gamma^\alpha\Gamma^\beta}\diamondsuit$
Rotating-Moving Brane, With F and U fields	$\Lambda = \sqrt{\det(\eta - F)}\det\left[\frac{\sqrt{\pi}\Gamma\left(1-\frac{1}{2}(\eta-F)^{-1}U\right)}{\Gamma\left(\frac{1}{2}-\frac{1}{2}(\eta-F)^{-1}U\right)}\right]C\Gamma^0\Gamma^1 \cdots \Gamma^p(\frac{1+i\eta\Gamma_{11}}{1+i\eta})\diamondsuit e^{\frac{1}{2}[\frac{\Delta(0)-1}{\Delta(0)+1}]_{\alpha\beta}\Gamma^\alpha\Gamma^\beta}\diamondsuit$

space of two dimensional world-sheet theories on the disk) with arbitrary boundary deformations, construct a configuration space for the boundary string field theory. In this context, propagating a closed string from the boundary of the disk can be explained as the disk partition function in the closed string theory. It has been demonstrated that the vacuum amplitude of the boundary state can be described as the partition function

$$Z^{Disk}_{S_{bdry}} = \langle vacuum|B; S_{bdry}\rangle. \tag{20}$$

Now by inserting both the bosonic and fermionic zero-mode boundary states obtained in the previous sections, integrating on the momenta and projecting that onto the bra-vacuum, the zero-mode part of the partition function in superstring theory would be appeared as

$$Z^{(0)}_{Super} = \left(-\frac{\pi}{\alpha'}\right)^{(p+1)/2} \frac{1}{\sqrt{\det(H+D)}}, \tag{21}$$

where $D_{\alpha\beta} = (U^{-1}\mathbf{A})_{\alpha\alpha}\delta_{\alpha\beta}$ and

$$H_{\alpha\beta} = \begin{cases} (U^{-1}\mathbf{A} + \mathbf{A}^T U^{-1})_{\alpha\beta} & , \quad \alpha \neq \beta, \\ 0 & , \quad \alpha = \beta, \end{cases}. \tag{22}$$

Note that the NS-NS and R-R sectors of the fermionic boundary states have contribution in the partition function but not in the zero-mode part, hence, they would not appear here. Moreover, the explicit form of the fermionic zero-mode state $|B\rangle^{(0)}_{ferm}$ and its contribution to the spin structure have been projected out, according to the above description, and therefore omitted from the zero-mode partition function in superstring theory. Thus, the significant part of the zero-mode partition function is that of the bosonic one.

As it seems, the photonic field has been disappeared. The reason behind this can be explained in two different ways: First, this is due to the fact that the tachyon is in the $U(1)$ adjoint representation, hence, it could not couple to the gauge field. Second, it can be explained by applying the boundary state method, in which by making use of the bosonic zero-mode boundary state (7), the photonic term would not be present any more.

A complex mixture of tachyonic background field and the rotating-moving dynamics of the system, distinguish this equation. In order to see this issue in detail, let us calculate the explicit form of the symmetric matrix $\Phi = H + D$ which is the heart of our zero-mode part of the superstring partition function. For this purpose, consider a $D2$-brane with the linear velocity $v_{\bar{\alpha}}$ where α could get the values of 1, 2, and the angular velocity $(\omega_{12}) = \overline{\omega}$. Then the matrix elements of Φ would be

$$\begin{aligned}
\Phi_{00} &= (U^{-1})_{00} - 4v_1(U^{-1})_{01} - 4v_2(U^{-1})_{02}\ , \\
\Phi_{01} &= 2(U^{-1})_{01} - 4v_1\left((U^{-1})_{00} + (U^{-1})_{11}\right) \\
&\quad -4\overline{\omega}(U^{-1})_{02} - 4v_2(U^{-1})_{21}\ , \\
\Phi_{02} &= 2(U^{-1})_{02} - 4v_2\left((U^{-1})_{00} + (U^{-1})_{22}\right) \\
&\quad +4\overline{\omega}(U^{-1})_{01} - 4v_1(U^{-1})_{12}\ , \\
\Phi_{11} &= (U^{-1})_{11} - 4v_1(U^{-1})_{10} - 4\overline{\omega}(U^{-1})_{12}\ , \\
\Phi_{12} &= 2(U^{-1})_{12} + 4\overline{\omega}\left((U^{-1})_{11} - (U^{-1})_{22}\right) \\
&\quad -4v_2(U^{-1})_{10} - 4v_1(U^{-1})_{02}\ , \\
\Phi_{22} &= (U^{-1})_{22} - 4v_2(U^{-1})_{20} + 4\overline{\omega}(U^{-1})_{21}.
\end{aligned} \tag{23}$$

Clearly, a complicated structure between the elements of the tachyon background field, rotation and motion of the brane can be seen from the above relations which distinguish our work from the others where U_is do not mix with any other couplings.

In continue let's introduce a new interpretation for the zero-modes of the present superstring partition function, by concentrating on the effective action. In order to reach this concept first, consider the familiar tachyon effective action with photonic term in superstring theory [13,25–27]

$$\begin{aligned}
L = -V(T)\sqrt{-\det(G_{\mu\nu} + F_{\mu\nu})}K((G \\
+F)^{\mu\nu}\partial_\mu T\partial_\nu T).
\end{aligned} \tag{24}$$

Then look at the effective action associated with our rotating-moving Dp-brane with background fields extracted from the boundary string field theory [24]

$$L' = -V(T)\frac{T_p}{g_s}\frac{\left(-\frac{\pi}{\alpha'}\right)^{(p+1)/2}}{\sqrt{\det(H+D)}}\sqrt{-\det(\eta+F)}K'((\eta+F)^{\mu\nu}\partial_\mu T\partial_\nu T) \quad (25)$$

Where the tachyonic potential term $V(T)$ in superstring field theory action is proportional to $e^{-T^2/4}$ due to the tachyon profile and K and K' are two general functionals according to the considered system.

By comparing the effective action (24), (static brane with tachyonic and photonic background fields) and the generalized form of our Lagrangian in (25), (rotating-moving brane with the same fields), the potential term of the present dynamical Dp-brane with photonic and tachyonic background fields can be obtained as

$$V_{system} = \frac{T_p}{g_s}V(T)Z^{(0)}_{Super}, \quad (26)$$

An interesting point here is that the dynamics of this Dp-brane (rotating-moving movement) is just inserted into the system's potential term and not in any other part of the effective action. This matter can be explained either by the boundary state equations or the path integral calculations. Therefore, unlike the the former case in which the only argument of the potential is the zero-mode of the tachyon field, the potential of this dynamical Dp-brane is proportional to the zero-mode part of the superstring partition function, including a mixture of tachyon and rotating-moving dynamics. Hence, in addition to the tachyonic field, the dynamical structure of this system also is merged to the potential.

Now let us investigate the zero-mode partition function of the described system for the static case, i.e, in the absence of ω-term. This can be done by setting $\omega = 0$ in the above relations. But it should be noted that we calculated these equations in the momentum space, then, let us return back and calculate the zero-mode partition function in the x-space as

$$Z^{(0)} = (-4\pi\alpha')^{\frac{p+1}{2}}\frac{1}{\sqrt{\det U}}, \quad (27)$$

This is the familiar relation for the zero-mode part of the effective action in [14,15,28,29].

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Author's comment: The data supporting the findings of this study are available within the article.]

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References

1. J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, 1998)
2. C. Johnson , D-Branes (Cambridge University Press, 2003)
3. M. Frau, I. Pesando, S. Sciuto, A. Lerda, R. Russo, Phys. Lett. B **400**, 52 (1997)
4. C.G. Callan, I.R. Klebanov, Nucl. Phys. B **465**, 473 (1996)
5. C.G. Callan, C. Lovelace, C.R. Nappi, S.A. Yost, Nucl. Phys. B **308**, 221 (1988)
6. P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda, R. Russo, Nucl. Phys. B **507**, 259 (1997)
7. P. Di Vecchia and A. Liccardo, arXiv:hep-th/9912275
8. H. Arfaei, D. Kamani, Phys. Lett. B **452**, 54 (1999)
9. F. Safarzadeh-Maleki, D. Kamani, Phys. Rev. D **90**, 107902 (2014)
10. F. Safarzadeh-Maleki, D. Kamani, Zh Eksp, Theor. Phys. **146**, 769 (2014)
11. Z. Rezaei, Phys. Rev. D **85**, 086011 (2012)
12. F. Safarzadeh-Maleki, D. Kamani, Phys. Rev. D **89**, 026006 (2014)
13. D. Kutasov, M. Marino, G. Moore, JHEP **0010**, 045 (2000)
14. P. Kraus, F. Larsen, Phys. Rev. D **63**, 106004 (2001)
15. T. Takayanagi, S. Terashima, T. Uesugi, JHEP **0103**, 019 (2001)
16. G. Arutyunov, A. Pankiewicz, B. Stefanski Jr., JHEP **0106**, 049 (2001)
17. S.P. de Alwis, Phys. Lett. B **505**, 215 (2001)
18. K. Okuyama, Phys. Lett. B **499**, 305 (2001)
19. T. Lee, K.S. Viswanathan, Y. Yang, J. Korean Phys. Soc. **42**, 34 (2003)
20. Jorge Alfaro, Phys. Rev. Lett. **94**, 221302 (2005)
21. Nick E. Mavromatos, Int. J. Mod. Phys. A **25**, 5409–5485 (2010)
22. Jun-Jie Wei, Bin-Bin Zhang, Lang Shao, Xue-Feng Wu, Peter Mszros, arXiv:1612.09425
23. Guo-Ping Li, Jin Pu, Qing-Quan Jiang, Xiao-Tao Zu, https://doi.org/10.1140/epjc/s10052-017-5220-z, arXiv:1703.06436
24. F. Safarzadeh-Maleki, EPL **115**(1), 11001 (2016)
25. K. Kamimura, J. Simon, Nucl. Phys. B **585**, 219 (2000)
26. E.A. Bergshoeff, M. de Roo, T.C. de Wit, E. Eyras, S. Panda, JHEP **0005**, 009 (2000)
27. N.D. Lambert, I. Sachs, Phys. Rev. D **67**, 026005 (2003)
28. T. Lee, Phys. Rev. D **64**, 106004 (2001)
29. T. Lee, Phys. Lett. B **520**, 385–390 (2001)