Unitarity-safe models of non-minimal inflation in supergravity

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Abstract We show that models of chaotic inflation based on the ϕ^p potential and a linear non-minimal coupling to gravity, $f_{\mathcal{R}} = 1 + c_{\mathcal{R}}\phi$, can be done consistent with data in the context of Supergravity, retaining the perturbative unitarity up to the Planck scale, if we employ logarithmic Kähler potentials with prefactors $-p(1+n)$ or $-p(n+1)-1$, where $-0.035 \lesssim n \lesssim 0.007$ for $p = 2$ or $-0.0145 \lesssim n \lesssim 0.006$ for $p = 4$. Focusing, moreover, on a model employing a gauge non-singlet inflaton, we show that a solution to the μ problem of MSSM and baryogenesis via non-thermal leptogenesis can be also accommodated.

1 Introduction

Inflation established in the presence of a non-minimal coupling between the inflaton ϕ and the Ricci scalar $\mathcal{R}$ is called collectively *non-minimal inflation* (nMI) [1–13]. Between the numerous models, which may be proposed in this context, *universal attractor models* (UAMs) [14] occupy a prominent position since they exhibit an attractor towards an inflationary phase excellently compatible with data [15] for $c_{\mathcal{R}} \gg 1$ and $\phi \leq m_{\text{P}}$ – where m_{P} is the reduced Planck mass. UAMs consider a monomial potential of the type

$$V_{\text{CI}}(\phi) = \lambda^2 \phi^p / 2^{p/2} m_{\text{P}}^{p-4} \quad (1)$$

in conjunction with a strong non-minimal coupling [5, 14]

$$\tilde{f}_{\mathcal{R}}(\phi) = 1 + c_{\mathcal{R}}(\phi/m_{\text{P}})^{q/2} \quad (2)$$

with $p = q$. The emergence of an inflationary plateau in these models can be transparently shown in the *Einstein frame* (EF) where the inflationary potential, $\hat{V}_{\text{attr}}$, takes the form

$$\hat{V}_{\text{attr}} = V_{\text{CI}}/\tilde{f}_{\mathcal{R}}^2 \simeq \lambda^2 m_{\text{P}}^4 / c_{\mathcal{R}}^2, \quad (3)$$

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with the exponent in the denominator being related to the conformal transformation employed [1–5] to move from the *Jordan frame* (JF) to EF.

However, due to the large $c_{\mathcal{R}}$ values needed for the establishment of nMI with $\phi \leq m_{\text{P}}$, the inflationary scale, $\hat{V}_{\text{attr}}^{1/4}$, turns out to be larger than the *Ultraviolet* (UV) cut-off scale [16–18]

$$\Lambda_{\text{UV}}^{\text{attr}} = m_{\text{P}}/c_{\mathcal{R}}^{1/(q/2-1)} \quad \text{with } 2 < q \leq 14/3 \quad (4)$$

of the corresponding effective theory, which thereby breaks down above it. A criticism of these results may be found in Ref. [19], where background-dependent cut-off scales, well larger than $\hat{V}_{\text{attr}}^{1/4}$, are evaluated. This practice is rather questionable, though, preventing the possibility of making perturbative extrapolations of the low-energy theory. Indeed, the low-energy theory expanded around the true low-energy vacuum should break at a scale that is calculable within the low-energy field expansion [20]. Therefore, the presence of $\Lambda_{\text{UV}}^{\text{attr}}$ in Eq. (4) cannot be avoided, and it signals the breakdown of the theory in that field range. Several ways have been proposed to surpass the inconsistency above. E.g., incorporating new degrees of freedom at $\Lambda_{\text{UV}}^{\text{attr}}$ [21], or assuming additional interactions [22], or invoking a large inflaton *vacuum expectation value* (v.e.v) $\langle \phi \rangle$ [23–27], or introducing a sizable kinetic mixing in the inflaton sector which dominates over $\tilde{f}_{\mathcal{R}}$ [28–34].

Here we propose a novel solution – applied only in the context of *Supergravity* (SUGRA) – to the aforementioned problem, by exclusively considering $q = 2$ in Eq. (2) – cf. Ref. [35]. In this case, the canonically normalized inflaton $\hat{\phi}$ is related to the initial field ϕ as $\hat{\phi} \sim c_{\mathcal{R}}\phi$ at the vacuum of the theory, in sharp contrast to what we obtain for $q > 2$ where $\hat{\phi} \simeq \phi$. As a consequence, the small-field series of the various terms of the action expressed in terms of $\hat{\phi}$, does not contain $c_{\mathcal{R}}$ in the numerators, preventing thereby the reduction of Λ_{UV} below m_{P} [7, 18]. The same conclusion may be drawn within the JF since no dangerous inflaton-inflaton-graviton

interaction appears because $\tilde{f}_{\mathcal{R},\phi\phi} = 0$ for $q = 2$ [7]. Note that the importance of a scalar field with a totally or partially linear non-minimal coupling to gravity in unitarizing Higgs inflation within non-SUSY settings is highlighted in Refs. [36, 37].

A permanently linear $\tilde{f}_{\mathcal{R}}$ can be reconciled with an inflationary plateau, similar to that obtained in Eq. (3), in the context of SUGRA, by suitably selecting the employed Kähler potentials. Indeed, this kind of models is realized in SUGRA using logarithmic or semilogarithmic Kähler potentials [8–12] with the prefactor $(-N)$ of the logarithms being related to the exponent of the denominator in Eq. (3). Therefore, by conveniently adjusting N we can achieve, in principle, a flat enough EF potential for any p in Eq. (1) but taking exclusively $q = 2$ in Eq. (2). As we show in the following, this idea works for $p \leq 4$ in Eq. (1) supporting nMI compatible with the present data [15]. For $p = 4$ we also show that the inflaton may be identified with a gauge singlet or non-singlet field. In the latter case, models of non-minimal Higgs inflation are introduced, which may be embedded in a more complete extension of MSSM offering a solution to the μ problem [38] and allowing for an explanation of *baryon asymmetry of the universe* (BAU) [39] via *non-thermal leptogenesis* (nTL) [40, 41]. The resulting models employ one parameter less than those used in Refs. [33, 34] whereas the gauge-symmetry-breaking scale is constrained to values well below the MSSM unification scale contrary to what happens in Refs. [27, 33, 34].

Below, we first – in Sect. 2 – describe the SUGRA set-up of our models and prove that these are unitarity-conserving in Sect. 3. Then, in Sect. 4, we analyze the inflationary dynamics and predictions. In Sect. 5 we concentrate on the case of nMI driven by a Higgs field and propose a possible post-inflationary completion. We conclude in Sect. 6. Unless otherwise stated, we use units where the reduced Planck scale $m_P = 2.4 \times 10^{18}$ GeV is set to be unity.

2 Supergravity framework

In Sect. 2.1 we describe the generic formulation of our models within SUGRA, and then we apply it for a gauge singlet and non-singlet inflaton in Sects. 2.2 and 2.3 respectively.

2.1 General framework

We focus on the part of the EF action within SUGRA related to the complex scalars z^α – denoted by the same superfield symbol – which has the form [8–11]

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(-\frac{1}{2} \widehat{\mathcal{R}} + K_{\alpha\bar{\beta}} \widehat{g}^{\mu\nu} D_\mu z^\alpha D_\nu z^{*\bar{\beta}} - \widehat{V} \right), \quad (5a)$$

where $\widehat{\mathcal{R}}$ is the EF Ricci scalar curvature, D_μ is the gauge covariant derivative, $K_{\alpha\bar{\beta}} = K_{,z^\alpha z^{*\bar{\beta}}}$, and $K^{\alpha\bar{\beta}} K_{\bar{\beta}\gamma} = \delta^\alpha_\gamma$ – throughout subscript of type $,z$ denotes derivation *with respect to* (w.r.t) the field z . Also, $\widehat{V}$ is the EF SUGRA potential which can be found once we select a superpotential W in Eq. (24) and a Kähler potential K via the formula

$$\widehat{V} = e^K \left(K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}}^* W^* - 3|W|^2 \right) + \frac{g^2}{2} \sum_a D_a^2, \quad (5b)$$

where $D_\alpha W = W_{,z^\alpha} + K_{,z^\alpha} W$ is the Kähler covariant derivative and $D_a = z^\alpha (T_a)_\alpha^\beta K_\beta$ are the D term corresponding to a gauge group with generators T_a and (unified) gauge coupling constant g . The remaining terms in the *right-hand side* (r.h.s) of the equation above describes contribution from the F terms. The contribution from the D terms vanishes for a gauge singlet inflaton and can be eliminated during nMI for a gauge non-singlet inflaton, by identifying it with the radial part of a conjugate pair of Higgs superfields – see Sect. 2.3. In both our scenarios, we employ a “stabilizer” field S placed at the origin during nMI. Thanks to this arrangement, the term $3|W|^2$ in $\widehat{V}$ vanishes, avoiding thereby a possible runaway problem, and the derivation of $\widehat{V}$ is facilitated since the non-vanishing terms arise from those proportional to $W_{,S}$ and $W_{,S}^*$ – see Sects. 2.2.2 and 2.3.2 below.

Defining the frame function as

$$-\Omega/N = \exp(-K/N) \Rightarrow K = -N \ln(-\Omega/N), \quad (6)$$

where $N > 0$ is a dimensionless parameter, we can obtain – after a conformal transformation along the lines of Refs. [8–11, 29] – the JF form of $\mathcal{S}$ which is

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\frac{\Omega}{2N} \mathcal{R} + \omega_{\alpha\bar{\beta}} D_\mu z^\alpha D^\mu z^{*\bar{\beta}} - V - \frac{27}{N^3} \Omega \mathcal{A}_\mu \mathcal{A}^\mu \right) \quad \text{with } \omega_{\alpha\bar{\beta}} = \Omega_{\alpha\bar{\beta}} + \frac{3-N}{N} \frac{\Omega_\alpha \Omega_{\bar{\beta}}}{\Omega}. \quad (7a)$$

Here we use the shorthand notation $\Omega_\alpha = \Omega_{,z^\alpha}$, and $\Omega_{\bar{\alpha}} = \Omega_{,z^{*\bar{\alpha}}}$. We also set $V = \widehat{V} \Omega^2 / N^2$ and

$$\mathcal{A}_\mu = -iN \left(\Omega_\alpha D_\mu z^\alpha - \Omega_{\bar{\alpha}} D_\mu z^{*\bar{\alpha}} \right) / 6\Omega. \quad (7b)$$

Although the choice $N = 3$ ensures canonical kinetic terms in Eq. (7a), N may be considered in general as a free parameter with interesting consequences not only on the inflationary observables [7, 25, 26, 29, 32, 42, 43] but also on the consistency of the effective theory, as we show below.

2.2 Gauge-singlet inflaton

Below, in Sect. 2.2.1, we specify the necessary ingredients (super- and Kähler potentials) which allow us to implement our scenario with a gauge-singlet inflaton. Then, in Sect. 2.2.2, we outline the derivation of the inflationary potential.

2.2.1 Set-up

This class of models requires the utilization of two gauge singlet chiral superfields, i.e., $z^\alpha = \Phi, S$, with Φ ($\alpha = 1$) and S ($\alpha = 2$) being the inflaton and a “stabilizer” field respectively. More specifically, we adopt the superpotential

$$W_{\text{CI}} = \lambda S \Phi^{p/2}, \quad (8)$$

which can be uniquely determined if we impose two symmetries: (i) an R symmetry under which S and Φ have charges 1 and 0; (ii) a global $U(1)$ symmetry with assigned charges -1 and $2/p$ for S and Φ . To obtain a linear non-minimal coupling of Φ to gravity, though, we have to violate the latter symmetry as regards Φ . Indeed, we propose the following set of Kähler potentials

$$K_1 = -N \ln(1 + c_{\mathcal{R}}(F_{\mathcal{R}} + F_{\mathcal{R}}^*) - F_-/N + F_{1S}), \quad (9a)$$

$$K_2 = -N \ln(1 + c_{\mathcal{R}}(F_{\mathcal{R}} + F_{\mathcal{R}}^*) + F_{1S}) + F_-, \quad (9b)$$

$$K_3 = -N \ln(1 + c_{\mathcal{R}}(F_{\mathcal{R}} + F_{\mathcal{R}}^*) - F_-/N) + F_{2S}, \quad (9c)$$

$$K_4 = -N \ln(1 + c_{\mathcal{R}}(F_{\mathcal{R}} + F_{\mathcal{R}}^*)) + F_- + F_{2S}, \quad (9d)$$

$$K_5 = -N \ln(1 + c_{\mathcal{R}}(F_{\mathcal{R}} + F_{\mathcal{R}}^*)) + F_{3S}. \quad (9e)$$

Recall that $N > 0$. From the involved functions

$$F_{\mathcal{R}} = \Phi/\sqrt{2} \quad \text{and} \quad F_- = -\frac{1}{2}(\Phi - \Phi^*)^2 \quad (10)$$

the first one allows for the introduction of the linear non-minimal coupling of Φ to gravity whereas the second one assures canonical normalization of Φ without any contribution to the non-minimal coupling along the inflationary path – cf. Refs. [14,30,31]. On the other hand, the functions F_{lS} with $l = 1, 2, 3$ offer canonical normalization and safe stabilization of S during and after nMI. Their possible forms are given in Refs. [33,34]. Just for definiteness, we adopt here only their logarithmic form, i.e.,

$$F_{1S} = -\ln(1 + |S|^2/N), \quad (11a)$$

$$F_{2S} = N_S \ln(1 + |S|^2/N_S), \quad (11b)$$

$$F_{3S} = N_S \ln(1 + F_-/N_S + |S|^2/N_S), \quad (11c)$$

with $0 < N_S < 6$. Recall [8–11,44–46] that the simplest term $|S|^2$ leads to instabilities for $K = K_1$ and K_2 and light excitations for $K = K_3 - K_5$. The heaviness of these modes

is required so that the observed curvature perturbation is generated wholly by our inflaton in accordance with the lack of any observational hint [39] for large non-Gaussianity in the cosmic microwave background. Note that all the proposed K 's contain up to quadratic terms of the various fields. Also $F_{\mathcal{R}}$ (and $F_{\mathcal{R}}^*$) is exclusively included in the logarithmic part of the K 's whereas F_- may or may not accompany it in the argument of the logarithm. Note finally that, although quadratic nMI is analyzed in Refs. [6,7,14] too, the present set of K 's is examined for first time.

2.2.2 Inflationary potential

Along the inflationary track determined by the constraints

$$S = \Phi - \Phi^* = 0, \quad \text{or} \quad s = \bar{s} = \theta = 0 \quad (12)$$

if we express Φ and S according to the parametrization

$$\Phi = \phi e^{i\theta}/\sqrt{2} \quad \text{and} \quad S = (s + i\bar{s})/\sqrt{2}, \quad (13)$$

the only surviving term in Eq. (5b) is

$$\widehat{V}_{\text{CI}} = \widehat{V}(\theta = s = \bar{s} = 0) = e^K K^{SS^*} |W_{\text{CI},S}|^2. \quad (14)$$

which, for the K 's in Eqs. (9a)–(9e), reads

$$\widehat{V}_{\text{CI}} = \frac{\lambda^2 \phi^p}{2^{p/2} f_{\mathcal{R}}^N} \cdot \begin{cases} f_{\mathcal{R}} & \text{for } K = K_1, K_2 \\ 1 & \text{for } K = K_3 - K_5, \end{cases} \quad (15)$$

where we define the (inflationary) frame function as

$$f_{\mathcal{R}} = -\frac{\Omega}{N} \Big|_{\text{Eq. (12)}} = 1 + c_{\mathcal{R}} \phi. \quad (16)$$

As expected, $f_{\mathcal{R}}$ coincides with $\tilde{f}_{\mathcal{R}}$ in Eq. (2) for $q = 2$. This form of $f_{\mathcal{R}}$ assures the preservation of unitarity up to $m_{\text{P}} = 1$ as explained in Sect. 1 and verified in Sect. 3. The last factor in Eq. (15) originates from the expression of K^{SS^*} for the various K 's. Indeed, $K_{\alpha\bar{\beta}}$ along the configuration in Eq. (12) takes the form

$$(K_{\alpha\bar{\beta}}) = \text{diag}(\kappa + N c_{\mathcal{R}}^2/2 f_{\mathcal{R}}^2, K_{SS^*}), \quad (17)$$

where

$$K_{SS^*} = \begin{cases} 1/f_{\mathcal{R}} & \text{for } K = K_1, K_2 \\ 1 & \text{for } K = K_3 - K_5 \end{cases} \quad (18)$$

and

$$\kappa = \begin{cases} 1/f_{\mathcal{R}} & \text{for } K = K_1, K_3 \\ 1 & \text{for } K = K_2, K_4, K_5. \end{cases} \quad (19)$$

If we set

$$N = \begin{cases} p(n+1) + 1 & \text{for } K = K_1, K_2 \\ p(n+1) & \text{for } K = K_3 - K_5, \end{cases} \quad (20)$$

we arrive at a universal expression for $\widehat{V}_{\text{CI}}$ which is

$$\widehat{V}_{\text{CI}} = \frac{\lambda^2}{2^{p/2}} \frac{\phi^p}{f_{\mathcal{R}}^{p(1+n)}}. \quad (21)$$

For $n = 0$ and $p = 2$, $\widehat{V}_{\text{CI}}$ reduces to $\widehat{V}_{\text{attr}}$ in Eq. (3) whereas for $p > 2$, $\widehat{V}_{\text{CI}}$ deviates from $\widehat{V}_{\text{attr}}$ although it develops a similar inflationary plateau, for $c_{\mathcal{R}} \gg 1$, since both numerator and denominator are dominated by a term proportional to ϕ^p as in the case of UAMs. The choice $n = 0$ is special since, for integer p , it yields integer N in Eqs. (9a)–(9e), i.e., $N = p + 1$ for $K = K_1$ and K_2 or $N = p$ for $K = K_3 - K_5$. Although integer N 's are more friendly to string theory – and give observationally acceptable results as shown in Sect. 4.2 –, non-integer N 's are also acceptable [25, 26, 29, 32–34, 42, 43] and assist us to cover the whole allowed domain of the observables. More specifically, for $n < 0$, $\widehat{V}_{\text{CI}}$ remains an increasing function of ϕ , whereas for $n > 0$, it develops a local maximum $\widehat{V}_{\text{CI}}(\phi_{\text{max}})$ where

$$\phi_{\text{max}} = \frac{1}{nc_{\mathcal{R}}} \left(1 + \sqrt{\frac{1+n}{1+np}} \right). \quad (22)$$

In a such case we are forced to assume that hilltop [47] nMI occurs with ϕ rolling from the region of the maximum down to smaller values.

Defining the EF canonically normalized fields, denoted by hat, via the relations

$$\frac{d\widehat{\phi}}{d\phi} = \sqrt{K_{\Phi\Phi^*}} = J, \quad \widehat{\theta} = J\theta\phi \quad \text{and} \quad (\widehat{s}, \widehat{\bar{s}}) = \sqrt{K_{SS^*}}(s, \bar{s}) \quad (23)$$

we can verify that the configuration in Eq. (12) is stable w.r.t the excitations of the non-inflaton fields. Taking the limit $c_{\mathcal{R}} \gg 1$ we find the expressions of the masses squared $\widehat{m}_{z^\alpha}^2$ (with $z^\alpha = \theta$ and s) arranged in Table 1,

Table 1 Mass-squared spectrum of the model defined by Eqs. (8) and (10) for $K = K_1 - K_5$ along the path in Eq. (12)

Fields	Eigenstates	Masses squared					
		$K = K_1$	$K = K_2$	$K = K_3$	$K = K_4$	$K = K_5$	
4 Real	$\widehat{\theta}$	$\widehat{m}_\theta^2$	$6(1 - 1/N)\widehat{H}_{\text{Cl}}^2$		$6\widehat{H}_{\text{Cl}}^2$		
Scalars	$\widehat{s}, \widehat{\bar{s}}$	$\widehat{m}_s^2$	$6c_{\mathcal{R}}\phi\widehat{H}_{\text{Cl}}^2/N$		$6\widehat{H}_{\text{Cl}}^2/N_S$		
2 Weyl spinors	$\widehat{\psi}_\pm$	$\widehat{m}_{\psi_\pm}^2$	$3p\left(1 - nc_{\mathcal{R}}\phi^2\right)^2\widehat{H}_{\text{Cl}}^2/Nc_{\mathcal{R}}^2\phi^2$				

which approach rather well the quite lengthy, exact expressions taken into account in our numerical computation. We infer that $\widehat{m}_{z^\alpha}^2 \gg \widehat{H}_{\text{CI}}^2 = \widehat{V}_{\text{CI}}/3$ for $1 < N < 6$ and $K = K_1, K_2$ or for $0 < N_S < 6$ and $K = K_3 - K_5$. Therefore $\widehat{m}_{z^\alpha}^2$ are not only positive but also heavy enough during nMI. In Table 1 we display the masses $\widehat{m}_{\psi_\pm}^2$ of the spinors $\widehat{\psi}_\pm = (\widehat{\psi}_S \pm \widehat{\psi}_\Phi)/\sqrt{2}$ too. We define $\widehat{\psi}_S = \sqrt{K_{SS^*}}\psi_S$ and $\widehat{\psi}_\Phi = \sqrt{K_{\Phi\Phi^*}}\psi_\Phi$ where ψ_Φ and ψ_S are the Weyl spinors associated with S and Φ respectively.

2.3 Gauge non-singlet inflaton

Following the strategy of the previous section, we show below, in Sect. 2.3.1, how we can establish a model of unitarity-conserving nMI driven by a gauge non-singlet inflaton and then, in Sect. 2.3.2, we outline the derivation of the corresponding inflationary potential.

2.3.1 Set-up

In this case, as explained below Eq. (5b), we employ a pair of left-handed chiral superfields, $\bar{\Phi}$ and Φ , oppositely charged under a gauge group, besides the stabilizer, S , which is a gauge-singlet chiral superfield. Here, we take for simplicity the group $U(1)_{B-L}$ where B and L denote the baryon and lepton number respectively. We base our construction on the superpotential [48]

$$W_{\text{HI}} = \lambda S(\bar{\Phi}\Phi - M^2/4), \quad (24)$$

where λ and M are parameters which can be made positive by field redefinitions. W_{HI} is the most general renormalizable superpotential consistent with a continuous R symmetry [48] under which S and W_{HI} are equally charged whereas $\bar{\Phi}\Phi$ is uncharged. To obtain nMI – which is actually promoted to Higgs inflation – with a linear non-minimal coupling to gravity we combine W_{HI} with one of the Kähler potentials in Eqs. (9a)–(9e) where the functions $F_{\mathcal{R}}$ and F_- are now defined as

$$F_{\mathcal{R}} = (\bar{\Phi}\Phi)^{1/2} \quad \text{and} \quad F_- = |\Phi - \bar{\Phi}^*|^2. \quad (25)$$

Note that the proposed K 's respect the symmetries of W_{HI} . As in the case of Sect. 2.2.1, F_- ensures that the kinetic

terms of $\bar{\Phi}$ and Φ do not enter the expression of $f_{\mathcal{R}}$ along the inflationary trough – cf. Refs. [27, 29–32].

Comparing the resulting K 's with the ones used in Refs. [33, 34] we may notice that here F_- is not accompanied by an independent variable c_- and the real function F_+ is here replaced by the combination of the holomorphic function $F_{\mathcal{R}}$ and its anti-holomorphic. Therefore, the present models are more economical since they include one parameter less. On the other hand, the presence of unity in the argument of the logarithms distinguishes clearly the present models from those in Ref. [27] where the absence of unity enforces us to invoke a large inflaton v.e.v.

2.3.2 Inflationary potential

Employing the parameterization of S in Eq. (13) and expressing Φ and $\bar{\Phi}$ as follows

$$\Phi = \phi e^{i\theta} \cos \theta_{\Phi} / \sqrt{2} \quad \text{and} \quad \bar{\Phi} = \phi e^{i\bar{\theta}} \sin \theta_{\Phi} / \sqrt{2}, \quad (26)$$

with $0 \leq \theta_{\Phi} \leq \pi/2$, we can determine a D-flat direction from the conditions

$$\bar{s} = s = \theta = \bar{\theta} = 0 \quad \text{and} \quad \theta_{\Phi} = \pi/4. \quad (27)$$

Along this path the only surviving term is again given by Eq. (14) which now reads – cf. Eq. (15)

$$\widehat{V}_{\text{HI}} = \frac{\lambda^2 (\phi^2 - M^2)^2}{16 f_{\mathcal{R}}^N} \cdot \begin{cases} f_{\mathcal{R}} & \text{for } K = K_1, K_2 \\ 1 & \text{for } K = K_3 - K_5, \end{cases} \quad (28)$$

whereas $f_{\mathcal{R}}$ is again given by Eq. (16). If we take into account the definition of n in Eq. (20) we end up with

$$\widehat{V}_{\text{HI}} = \frac{\lambda^2 (\phi^2 - M^2)^2}{16 f_{\mathcal{R}}^{p(1+n)}}, \quad (29)$$

which can be approached rather well by Eq. (21) for $p = 4$, when $M \ll 1$, and λ^2 replaced with $\lambda^2/4$.

To specify $\widehat{\phi}$ in the present case we note that, for all K 's in Eqs. (9a)–(9e) with $F_{\mathcal{R}}$ and F_- given in Eq. (25), $K_{\alpha\bar{\beta}}$ along the configuration in Eq. (27) takes the form

$$(K_{\alpha\bar{\beta}}) = \text{diag} (M_{\pm}, K_{SS^*}), \quad (30)$$

where

$$M_{\pm} = \begin{bmatrix} \kappa + N c_{\mathcal{R}}^2 / 4 f_{\mathcal{R}}^2 & N c_{\mathcal{R}}^2 / 4 f_{\mathcal{R}}^2 \\ N c_{\mathcal{R}}^2 / 4 f_{\mathcal{R}}^2 & \kappa + N c_{\mathcal{R}}^2 / 4 f_{\mathcal{R}}^2 \end{bmatrix} \quad (31)$$

with κ given in Eq. (19). Upon diagonalization we obtain the following eigenvalues

$$\kappa_+ = \kappa + N c_{\mathcal{R}}^2 / 2 f_{\mathcal{R}}^2 \quad \text{and} \quad \kappa_- = \kappa. \quad (32)$$

Inserting Eqs. (26) and (30) in the second term of the r.h.s of Eq. (5a) we can define the EF canonically normalized fields, as follows

$$d\widehat{\phi}/d\phi = J = \sqrt{\kappa_+}, \quad \widehat{\theta}_{\Phi} = \phi \sqrt{\kappa_-} (\theta_{\Phi} - \pi/4), \quad (33a)$$

$$\widehat{\theta}_+ = J \phi \theta_+ / \sqrt{2} \quad \text{and} \quad \widehat{\theta}_- = \sqrt{\kappa_-} \phi \theta_- / \sqrt{2}, \quad (33b)$$

where $\theta_{\pm} = (\bar{\theta} \pm \theta) / \sqrt{2}$ and the normalization of $\widehat{\phi}, \widehat{s}$ and $\widehat{\bar{s}}$ coincides with that found in Eq. (23). Note, in passing, that the spinors $\psi_{\Phi\pm}$ associated with the superfields Φ and $\bar{\Phi}$ are similarly normalized, i.e., $\widehat{\psi}_{\Phi\pm} = \sqrt{\kappa_{\pm}} \psi_{\Phi\pm}$ with $\psi_{\Phi\pm} = (\psi_{\Phi} \pm \psi_{\bar{\Phi}}) / \sqrt{2}$.

To check the stability of inflationary direction in Eq. (27) w.r.t the fluctuations of the non-inflaton fields, we derive the mass-squared spectrum of the various scalars defined in Eqs. (33a) and (33b). Taking the limit $c_{\mathcal{R}} \gg 1$, we find the approximate expressions listed in Table 2 which are rather accurate at the horizon crossing of the pivot scale. As in case of Table 1, we again deduce that $\widehat{m}_{z^{\alpha}}^2 \gg \widehat{H}_{\text{HI}}^2 = \widehat{V}_{\text{HI}}/3$ for $1 < N < 6$ and $K = K_1, K_2$ or for $0 < N_S < 6$ and $K = K_3 - K_5$. In Table 2 we also display the masses M_{BL} of the gauge boson A_{BL} and the corresponding fermions. The non-vanishing of M_{BL} signals the fact that $U(1)_{B-L}$ is broken during nMI and so no cosmic string are produced at

Table 2 Mass-squared spectrum of the model defined by Eqs. (24) and (25) for $K = K_1 - K_5$ along the path in Eq. (27)

Fields	Eigenstates	Masses squared					
			$K = K_1$	$K = K_2$	$K = K_3$	$K = K_4$	$K = K_5$
4 Real Scalars	$\widehat{\theta}_+$	$\widehat{m}_{\theta_+}^2$	$3(1 - 1/N) \widehat{H}_{\text{HI}}^2$		$3 \widehat{H}_{\text{HI}}^2$		
	$\widehat{\theta}_{\Phi}$	$\widehat{m}_{\theta_{\Phi}}^2$	M_{BL}^2	$M_{BL}^2 + 6 \widehat{H}_{\text{HI}}^2$	M_{BL}^2		$M_{BL}^2 + 6(1 + 1/N_S) \widehat{H}_{\text{HI}}^2$
	$\widehat{s}, \widehat{\bar{s}}$	$\widehat{m}_s^2$	$6 \widehat{H}_{\text{HI}}^2 c_{\mathcal{R}} \phi / N$		$6 \widehat{H}_{\text{HI}}^2 / N_S$		
1 Gauge boson	A_{BL}	M_{BL}^2	$g^2 \phi^2 / f_{\mathcal{R}}$	$g^2 \phi^2$	$g^2 \phi^2 / f_{\mathcal{R}}$	$g^2 \phi^2$	
4 Weyl	$\widehat{\psi}_{\pm}$	$\widehat{m}_{\psi_{\pm}}^2$	$3(c_{\mathcal{R}}(N - 5)\phi - 4)^2 \widehat{H}_{\text{HI}}^2 / N c_{\mathcal{R}}^2 \phi^2$		$3(c_{\mathcal{R}}(N - 4)\phi - 4)^2 \widehat{H}_{\text{HI}}^2 / N c_{\mathcal{R}}^2 \phi^2$		
Spinors	$\lambda_{BL}, \widehat{\psi}_{\Phi-}$	M_{BL}^2	$g^2 \phi^2 / f_{\mathcal{R}}$	$g^2 \phi^2$	$g^2 \phi^2 / f_{\mathcal{R}}$	$g^2 \phi^2$	

its end. Finally, the unspecified eigenstates $\hat{\psi}_{\pm}$ are defined as $\hat{\psi}_{\pm} = (\hat{\psi}_{\phi+} \pm \hat{\psi}_S)/\sqrt{2}$ – cf. Table 1.

3 Effective cut-off scale

The motivation of our proposal originates from the fact that $f_{\mathcal{R}}$ in Eq. (16) assures that the perturbative unitarity is retained up to m_P although that the attainment of nMI for $\phi \leq m_P$ requires large $c_{\mathcal{R}}$'s – as expected from the UAMs [5–7] and verified in Sect. 4 below. To show that this achievement is valid, we extract below the UV cut-off scale, Λ_{UV} , expanding the action in the JF – see Sect. 3.1 – and in the EF – see Sect. 3.2. Throughout this section we find it convenient to restore m_P in the formulas. We concentrate also on the versions of our model with gauge singlet inflaton. However, this analysis covers also the case of a gauge non-singlet inflaton for $M \ll m_P$, $p = 4$ and λ^2 replaced by $\lambda^2/4$.

Although the expansions about $\langle\phi\rangle = 0$, presented below, are not valid [19] during nMI, we consider Λ_{UV} extracted this way as the overall cut-off scale of the theory for two reasons: (i) the reheating phase – realized via oscillations about $\langle\phi\rangle$ – is an unavoidable stage of the inflationary dynamics; (ii) the result is within the range of validity of the low-energy theory and so this can be perturbatively extrapolated up to Λ_{UV} .

3.1 Jordan frame computation

Thanks to the special dependence of $f_{\mathcal{R}}$ on ϕ in Eq. (16), there is no interaction between the excitation of ϕ about $\langle\phi\rangle = 0$, $\delta\phi$, and the graviton, $h^{\mu\nu}$ which can jeopardize the validity of perturbative unitarity. To show this, we first expand $g_{\mu\nu}$ about the flat spacetime metric $\eta_{\mu\nu}$ and the inflaton ϕ about its v.e.v.,

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}/m_P \quad \text{and} \quad \phi = 0 + \delta\phi. \quad (34)$$

Retaining only the terms with up to two space-time derivatives of the excitations, the part of the lagrangian corresponding to the two first terms in the r.h.s of Eq. (7a) takes the form [19,25,26]

$$\begin{aligned} \delta\mathcal{L} = & -\frac{\langle f_{\mathcal{R}} \rangle}{8} G_{EH} (h^{\mu\nu}) + \frac{1}{2} \langle f_K \rangle \partial_{\mu} \delta\phi \partial^{\mu} \delta\phi \\ & + \frac{m_P}{2} G_{\mathcal{R}} (h^{\mu\nu}) \left(\langle f_{\mathcal{R},\phi} \rangle \delta\phi + \sum_{\ell=2}^{\infty} \frac{1}{\ell!} \left\langle \frac{d^{\ell} f_{\mathcal{R}}}{d\phi^{\ell}} \right\rangle \delta\phi^{\ell} \right) \\ = & -\frac{1}{8} G_{EH} (\bar{h}^{\mu\nu}) + \frac{1}{2} \partial_{\mu} \bar{\delta\phi} \partial^{\mu} \bar{\delta\phi}, \end{aligned} \quad (35)$$

where the functions G_{EH} and $G_{\mathcal{R}}$ are identical to F_{EH} and $F_{\mathcal{R}}$ defined in Refs. [25,26] – $F_{\mathcal{R}}$ should not to be confused with that given in Eqs. (10) and (25). From Eq. (7a) we compute $f_K = \omega_{\Phi\Phi^*} = 1/\kappa + (N-3)c_{\mathcal{R}}^2/2f_{\mathcal{R}}$. For $\langle\phi\rangle = 0$ we

get $\langle f_K \rangle = 1 + (N-3)c_{\mathcal{R}}^2/2$, $\langle f_{\mathcal{R}} \rangle = 1$ and $\langle f_{\mathcal{R},\phi} \rangle = c_{\mathcal{R}}/m_P$. The JF canonically normalized fields $\bar{h}_{\mu\nu}$ and $\bar{\delta\phi}$, which diagonalise the quadratic part of the form above, are defined as

$$\bar{\delta\phi} \simeq \sqrt{\frac{N}{2}} c_{\mathcal{R}} \delta\phi \quad \text{and} \quad \bar{h}_{\mu\nu} = h_{\mu\nu} + c_{\mathcal{R}} \eta_{\mu\nu} \delta\phi. \quad (36)$$

The UV behavior of the scattering amplitudes is determined by the operators with dimension higher than four which arise from the last term in the second line of Eq. (35). The resulting interactions are proportional to the quantity $\bar{\delta\phi}^{\ell} \square^{\ell} \bar{h}/m_P^{\ell-1}$ with $\square = \partial^{\mu} \partial_{\mu}$ and $\bar{h} = \bar{h}_{\mu}^{\mu}$ – cf. Refs. [19,24–26,35]. Given that $d^{\ell} f_{\mathcal{R}}/d\phi^{\ell}$ vanishes for $\ell > 2$, though, no interaction of that type appears and so the theory does not face any problem with the perturbative unitarity.

3.2 Einstein frame computation

Alternatively, Λ_{UV} can be determined in EF, analyzing the small-field behavior of our models following Ref. [18]. We focus first on the second term in the r.h.s of (5a) for $\mu = \nu = 0$ and $K_{\Phi\Phi^*} = J^2$ given in Eq. (23). Expanding it about $\langle\phi\rangle = 0$, we arrive at

$$\begin{aligned} J^2 \dot{\phi}^2 = & \left(1 + \frac{1}{2} N c_{\mathcal{R}}^2 - N c_{\mathcal{R}}^3 \frac{\delta\phi}{m_P} + \frac{3}{2} N c_{\mathcal{R}}^4 \frac{\delta\phi^2}{m_P^2} \right. \\ & \left. - 2 N c_{\mathcal{R}}^5 \frac{\delta\phi^3}{m_P^3} + \dots \right) \dot{\delta\phi}^2, \end{aligned} \quad (37)$$

where $\delta\phi$ is defined in Eq. (34) and we set $\dot{\phi} = \dot{\delta\phi}$. If the EF (canonically normalized) inflaton, $\widehat{\delta\phi}$, was identical to $\delta\phi$ at the vacuum of the theory – as happens for the UAMs [16–18] with $q > 2$ – Λ_{UV} would have been equal to $m_P/c_{\mathcal{R}}$ since this is the lowest of the denominators above for $c_{\mathcal{R}} \gg 1$. However, here we have $\widehat{\delta\phi} = \langle J \rangle \delta\phi \gg \delta\phi$ since

$$\langle J \rangle = \sqrt{1 + N c_{\mathcal{R}}^2/2} \simeq c_{\mathcal{R}} \sqrt{N/2}. \quad (38)$$

Therefore, expressing Eq. (37) in terms of $\widehat{\delta\phi}$ and $\dot{\widehat{\delta\phi}} = \langle J \rangle \dot{\delta\phi}$, we see that the $c_{\mathcal{R}}$'s are cancelled out and we end up with the result

$$J^2 \dot{\phi}^2 = \left(1 - 2 \frac{\sqrt{2}}{\sqrt{N}} \frac{\widehat{\delta\phi}}{m_P} + \frac{6}{N} \frac{\widehat{\delta\phi}^2}{m_P^2} - 8 \frac{\sqrt{2}}{N^{\frac{3}{2}}} \frac{\widehat{\delta\phi}^3}{m_P^3} + \dots \right) \dot{\widehat{\delta\phi}}^2 \quad (39a)$$

where we neglect terms suppressed by powers of $c_{\mathcal{R}}$ in the denominator. This expression is valid for both cases in Eq. (17) and agrees with that in Ref. [7] for $N = 3(1+n)$. Expanding similarly $\widehat{V}_{CI}$, see Eq. (21), in terms of $\widehat{\delta\phi}$ we

have

$$\widehat{V}_{\text{CI}} = \lambda^2 m_{\text{P}}^4 \left(\frac{\widehat{\delta\phi}}{\sqrt{N} c_{\mathcal{R}}} \right)^p \left(1 - \sqrt{\frac{2}{N}} (1+n) p \frac{\widehat{\delta\phi}}{m_{\text{P}}} + \frac{p}{N} (1 + p(1+n))(1+n) \frac{\widehat{\delta\phi}^2}{m_{\text{P}}^2} + \dots \right). \quad (39b)$$

Since no numerator proportional to $c_{\mathcal{R}}$ arises in Eqs. (39a) and (39b), we conclude that $\Lambda_{\text{UV}} = m_{\text{P}}$. As a consequence, our inflationary model is not sensitive to the UV completion of the theory, provided that $\widehat{V}_{\text{CI}}^{1/4}(\phi) \ll m_{\text{P}}$ with $\phi \leq m_{\text{P}}$. These prerequisites are readily fulfilled as we see in Sect. 4.2.

4 Inflation analysis

In Sects. 4.1 and 4.2 below we examine semi-analytically and numerically respectively, if $\widehat{V}_{\text{CI}}$ in Eq. (21) may be consistent with a number of observational constraints. The analysis can be easily adapted to the case of $\widehat{V}_{\text{HI}}$ in Eq. (29) performing the replacements mentioned in Sect. 3.

4.1 Semi-analytic results

The period of slow-roll nMI is determined in the EF by the condition – see, e.g., Refs. [49, 50]:

$$\max\{\widehat{\epsilon}(\phi), |\widehat{\eta}(\phi)|\} \leq 1, \quad (40a)$$

where the slow-roll parameters $\widehat{\epsilon}$ and $\widehat{\eta}$ read

$$\widehat{\epsilon} = \left(\widehat{V}_{\text{CI},\widehat{\phi}} / \sqrt{2\widehat{V}_{\text{CI}}} \right)^2 \quad \text{and} \quad \widehat{\eta} = \widehat{V}_{\text{CI},\widehat{\phi}\widehat{\phi}} / \widehat{V}_{\text{CI}} \quad (40b)$$

and can be derived employing $J \simeq \sqrt{N/2\phi^2}$ in Eq. (23) without express explicitly $\widehat{V}_{\text{CI}}$ in terms of $\widehat{\phi}$. Since J for $K = K_1, K_3$ deviates slightly from that for $K = K_2, K_4, K_5$ – see Eq. (19) – we have a discrimination as regards the expressions of $\widehat{\epsilon}$ and $\widehat{\eta}$ in these two cases. Indeed, our results are

$$\widehat{\epsilon} \simeq p^2 \frac{f_n^2}{c_{\mathcal{R}}^2 \phi^2} \begin{cases} 1/N & \text{for } K = K_1, K_3, \\ 1/(N + 2\phi^2) & \text{for } K = K_2, K_4, K_5, \end{cases} \quad (41)$$

where $f_n = 1 - nc_{\mathcal{R}}\phi$. Similarly, we obtain

$$\frac{\widehat{\eta}}{2p} \simeq \begin{cases} (pf_n^2 - f_{\mathcal{R}})/Nc_{\mathcal{R}}^2\phi^2 & \text{for } K = K_1, K_3, \\ (Nc_{\mathcal{R}}\phi f_n + (N + 2\phi^2)(c_{\mathcal{R}}\phi(f_n - 1) + pf_n^2)) / c_{\mathcal{R}}^2\phi^2(N + 2\phi^2) & \text{for } K = K_2, K_4, K_5. \end{cases} \quad (42)$$

For any of the K 's above we can numerically verify that Eq. (40a) is saturated for $\phi = \phi_{\text{f}}$, which is found from the condition

$$\widehat{\epsilon}(\phi_{\text{f}}) \simeq 1 \Rightarrow \phi_{\text{f}} \simeq \frac{p}{c_{\mathcal{R}}} \frac{\sqrt{N} - pn}{N - n^2 p^2}. \quad (43)$$

Apart from irrelevant constant prefactors, the formulas above for $n = 0$ and $K = K_1, K_3$ reduce to the ones obtained for quadratic nMI – cf. Refs. [6, 13].

The number of e-foldings $\widehat{N}_{\star}$ that the pivot scale $k_{\star} = 0.05/\text{Mpc}$ experiences during nMI and the amplitude A_s of the power spectrum of the curvature perturbations generated by ϕ can be computed using the standard formulae

$$(a) \quad \widehat{N}_{\star} = \int_{\widehat{\phi}_{\text{f}}}^{\widehat{\phi}_{\star}} d\widehat{\phi} \frac{\widehat{V}_{\text{CI}}}{\widehat{V}_{\text{CI},\widehat{\phi}}} \quad \text{and} \quad (b) \quad A_s = \frac{1}{12\pi^2} \frac{\widehat{V}_{\text{CI}}^3(\widehat{\phi}_{\star})}{\widehat{V}_{\text{CI},\widehat{\phi}}^2(\widehat{\phi}_{\star})}, \quad (44)$$

where $\phi_{\star} [\widehat{\phi}_{\star}]$ is the value of ϕ [$\widehat{\phi}$] when $k_{\star}$ crosses the inflationary horizon. Taking into account $\phi_{\star} \gg \phi_{\text{f}}$, we can derive $\widehat{N}_{\star}$. We single out the following cases:

- For $n = 0$ and any K in Eqs. (9a)–(9e), we obtain

$$\widehat{N}_{\star} = \frac{Nc_{\mathcal{R}}}{2p} \phi_{\star} \Rightarrow \phi_{\star} \simeq \frac{2p\widehat{N}_{\star}}{Nc_{\mathcal{R}}}. \quad (45a)$$

Note that for $n \neq 0$ the formulas below for $\widehat{N}_{\star}$ cannot be reduced to the previous one.

- For $n \neq 0$ and $K = K_1, K_3$, we obtain

$$\widehat{N}_{\star} = -\frac{N \ln f_{n\star}}{2n(1+n)p} \Rightarrow \phi_{\star} \simeq \frac{1 - e_n}{nc_{\mathcal{R}}}, \quad (45b)$$

where $e_n = e^{-2pn(n+1)\widehat{N}_{\star}/N}$ and $f_{n\star} = f_n(\phi_{\star})$.

- For $n < 0$ and $K = K_2, K_4, K_5$, we obtain

$$\widehat{N}_{\star} = -\frac{N \ln f_{n\star}}{2n(1+n)p} - \frac{\phi_{\star}^2}{2np} \Rightarrow \phi_{\star} \simeq \sqrt{\frac{N}{2(n+1)}} W_0(y), \quad (45c)$$

where W_k is the Lambert or product logarithmic function [51] with $k = 0$ and

$$y = 2(1+n)e_n^2/Nn^2c_{\mathcal{R}}^2. \quad (45d)$$

- For $n > 0$ and $K = K_2, K_4, K_5$, we obtain

$$\hat{N}_\star = -\frac{2+n^2c_{\mathcal{R}}^2N}{2nc_{\mathcal{R}}^2n^3(1+n)p} \ln f_{n\star} - \frac{3+f_{n\star}}{n^2pc_{\mathcal{R}}} \phi_\star. \quad (45e)$$

Here we are not able to solve the equation above w.r.t $\phi_\star$. As a consequence, it is not doable to find an analytical expression for $\phi_\star$ and the inflationary observables – see below. Therefore, in this portion of parameter space, our last resort is the numerical computation, whose the results are presented in Sect. 4.2.

In all cases above, there is a lower bound on $c_{\mathcal{R}}$, above which $\phi_\star \leq 1$, as in the original UAMs [14]. E.g., for $n = 0$

$$\phi_\star \leq 1 \Rightarrow c_{\mathcal{R}} \geq 2p\hat{N}_\star/N \gg 1, \quad (46)$$

with $\hat{N}_\star \simeq (52 - 59)$ for $p = 2 - 4$. As shown in Sect. 3, these large $c_{\mathcal{R}}$'s do not disturb the validity of perturbative unitarity up to m_p . On the other hand, the EF field, $\hat{\phi}$ may be transplanckian, since integrating the first equation in Eq. (23) with J given below Eq. (40b), we find

$$\hat{\phi} \simeq \hat{\phi}_c + \sqrt{N/2} \ln \phi, \quad (47)$$

with $\hat{\phi}_c$ being a constant of integration. If we set, e.g., $\hat{\phi}_c = 0$ and $\phi \simeq (0.001 - 1)$, then $|\hat{\phi}| > 1$. Despite this fact, our proposal is stabilized against corrections from higher order terms in W_{Cl} and/or K 's in Eqs. (8) and (9a)–(9e), since these terms are exclusively expressed as functions of the initial field Φ and remain harmless for $\phi = \sqrt{2}|\Phi| \leq 1$ – cf. Ref. [14].

From Eq. (44b) we can derive a relation between λ and $c_{\mathcal{R}}^{p/2}$ for fixed n , in sharp contrast to UAMs where the same condition implies a relation between λ and $c_{\mathcal{R}}$ [6, 12, 13]. Given that $c_{\mathcal{R}}$ assumes large values, we expect that λ increases with p and rapidly (for $p \geq 5$ as we find numerically) violates the perturbative bound $\lambda \leq 2\sqrt{\pi} \simeq 3.5$. In particular, our results can be cast as following:

- For $n = 0$ and any K in Eqs. (9a)–(9e), we obtain

$$\lambda = 2^{\frac{1}{2}-\frac{p}{4}} \pi \sqrt{3N A_s} c_{\mathcal{R}}^{p/2} / \hat{N}_\star. \quad (48a)$$

- For $n \neq 0$ and $K = K_1, K_3$, we obtain

$$\lambda = 2^{\frac{3}{2}+\frac{p}{4}} p \pi \sqrt{3A_s} (1 - n f_{\mathcal{R}\star}) f_{\mathcal{R}\star}^{\frac{np}{2}-1} c_{\mathcal{R}}^{p/2} / N^{1/2}, \quad (48b)$$

where $f_{\mathcal{R}\star} = f_{\mathcal{R}}(\phi_\star)$. Taking into account that $n \ll 1$ and $f_{\mathcal{R}\star}$ is almost proportional to $c_{\mathcal{R}}$ for large $c_{\mathcal{R}}$, we can easily convince ourselves that the output above implies that $\lambda/c_{\mathcal{R}}^{p/2}$ remains constant for fixed n .

- For $n < 0$ and $K = K_2, K_4, K_5$, we obtain

$$\lambda = 2^{\frac{3}{2}+\frac{p}{4}} p \pi \sqrt{3A_s} \frac{(1 - n f_{\mathcal{R}\star}) f_{\mathcal{R}\star}^{\frac{np}{2}-1} c_{\mathcal{R}}^{1+\frac{p}{2}}}{(2f_{\mathcal{R}\star}^2 + Nc_{\mathcal{R}}^2)^{1/2}}, \quad (48c)$$

from which we can again verify that the approximate proportionality of λ on $c_{\mathcal{R}}^{p/2}$ holds.

The remaining inflationary observables – i.e., the (scalar) spectral index n_s , its running a_s , and the scalar-to-tensor ratio r – are found from the relations [49, 50]

$$n_s = 1 - 6\hat{\epsilon}_\star + 2\hat{\eta}_\star, \quad r = 16\hat{\epsilon}_\star, \quad (49a)$$

$$a_s = 2 \left(4\hat{\eta}_\star^2 - (n_s - 1)^2 \right) / 3 - 2\hat{\xi}_\star, \quad (49b)$$

where the variables with subscript $\star$ are evaluated at $\phi = \phi_\star$ and $\hat{\xi} = \hat{V}_{\text{Cl},\hat{\phi}} \hat{V}_{\text{Cl},\hat{\phi}\hat{\phi}} / \hat{V}_{\text{Cl}}^2$. Inserting $\phi_\star$ from Eqs. (45a) (45b) and (45c) into Eq. (40b) and then into equations above we can obtain some analytical estimates. Namely:

- For $K = K_1$ and K_3 we end up with a unified result

$$\begin{aligned} n_s &\simeq 1 - \frac{4p}{N f_{\mathcal{R}\star}} - \frac{2p^2}{N} \left(n - \frac{1}{f_{\mathcal{R}\star}} \right)^2 \\ &\simeq 1 - \frac{2}{\hat{N}_\star} + \frac{np}{\hat{N}_\star} - \frac{2np}{N}; \end{aligned} \quad (50a)$$

$$r \simeq \frac{16p^2}{N} \left(n - \frac{1}{f_{\mathcal{R}\star}} \right)^2 \simeq \frac{4N}{\hat{N}_\star^2} - \frac{8np}{\hat{N}_\star}; \quad (50b)$$

$$\begin{aligned} a_s &\simeq 8p^2(n f_{\mathcal{R}\star} - 1) \frac{1 + (1+n)p + f_{\mathcal{R}\star}(1 - np)}{N^2 f_{\mathcal{R}\star}^3} \\ &\simeq -\frac{2}{\hat{N}_\star^2} + n \frac{1+p}{\hat{N}_\star^2}. \end{aligned} \quad (50c)$$

For $n = 0$ the above results are also valid for $K = K_2, K_4$ and K_5 and yield observables identical with those obtained within UAMs [6, 8–12].

- For $n < 0$ and $K = K_2, K_4$ and K_5 we arrive at the following results

$$\begin{aligned} n_s &\simeq 1 + \frac{2pc_{\mathcal{R}}^2}{f_{\mathcal{R}\star}^2(2f_{\mathcal{R}\star}^2 + Nc_{\mathcal{R}}^2)^2} \left(4f_{\mathcal{R}\star}^3(n f_{\mathcal{R}\star} - 2) \right. \\ &\quad \left. - 2Nc_{\mathcal{R}}^2 f_{\mathcal{R}\star} - (n f_{\mathcal{R}\star} - 1)^2(2f_{\mathcal{R}\star}^2 + Nc_{\mathcal{R}}^2)p \right); \end{aligned} \quad (51a)$$

$$r \simeq \frac{16p^2}{N + 2f_{\mathcal{R}\star}^2/c_{\mathcal{R}}^2} \left(n - \frac{1}{f_{\mathcal{R}\star}} \right)^2 \quad (51b)$$

with negligibly small a_s , as we find out numerically. Contrary to our previous results, here a $c_{\mathcal{R}}$ dependence arises which complicates somehow the investigation of these models – see Sect. 4.2.

Fig. 2 The inflationary potential $\widehat{V}_{\text{CI}}$ as a function of ϕ for $K = K_1$, $\phi_* = 1$, $n = 0$ and $p = 2$ (light gray line) or $p = 4$ (gray line). The values corresponding to ϕ_* , ϕ_f are also indicated

Fixing, in addition, $n = 0$ and $K = K_1$, $\phi_* = 1$ – i.e. confining the corresponding $c_{\mathcal{R}}$ and λ values to their lowest possible values enforcing Eq. (48a) – we illustrate in Fig. 2 the structure of $\widehat{V}_{\text{CI}}$ as a function of ϕ for $p = 2$ (light gray line) or $p = 4$ (gray line). More specifically, we find $\lambda = 1.173 \cdot 10^{-3}$ or 0.257 and $c_{\mathcal{R}} = 75$ or 99 for $p = 2$ or $p = 4$ respectively. We see that in both cases $\widehat{V}_{\text{CI}}$ develops a plateau with magnitude 10^{-10} which is similar to that obtained in Starobinsky inflation [24, 44–46] but one order of lower than that obtained from the models analyzed in Refs. [32–34] where r is a little more enhanced. Contrary to Starobinsky inflation, though, $\widehat{V}_{\text{CI}}$ is well defined at the origin as happens within the original UAMs and those in Refs. [32–34]. Obviously, the requirement – mentioned in Sect. 3 – $\widehat{V}_{\text{CI}}^{1/4} \ll 1$ dictated from the validity of the effective theory is readily fulfilled.

Practically the same observables for $n = 0$ are shown in the right plot of Fig. 1. In that plot, though, we see that the models' predictions are confined to $n_s \lesssim 0.974$, n may deviate more appreciably from zero (mainly for $p = 2$) and the maximal possible r is somewhat larger. Moreover, these predictions depend harder on $c_{\mathcal{R}}$ for $|n| > 0.01$, as expected from Eqs. (51a) and (51b). Therefore, in that regime, we could say that these models are less predictive than those based on $K = K_1$ and K_3 . Our results below are presented for $c_{\mathcal{R}}$ such that $\phi_* \simeq 1$. Namely, for $p = 2$, we find

$$0.7 \gtrsim n/0.01 \gtrsim -3.5, \quad 9.57 \lesssim n_s/0.1 \lesssim 9.68, \quad (56a)$$

$$1.3 \lesssim r/10^{-3} \lesssim 20 \quad \text{and} \quad 0.98 \lesssim 10^5 \lambda/c_{\mathcal{R}} \lesssim 2.8, \quad (56b)$$

whereas for $p = 4$ we have

$$0.6 \gtrsim n/0.01 \gtrsim -1.45, \quad 9.57 \lesssim n_s/0.1 \lesssim 9.72, \quad (57a)$$

$$2 \lesssim r/10^{-3} \lesssim 14 \quad \text{and} \quad 1.8 \lesssim 10^5 \lambda/c_{\mathcal{R}}^2 \lesssim 3.6. \quad (57b)$$

The latter results deviate more drastically from those in Refs. [27, 33, 34]. From the data of both plots of Fig. 1, we remark

Fig. 3 Allowed values of $c_{\mathcal{R}}$ versus $\sqrt{\lambda}$ for $p = 4$ and various n 's and K 's. The conventions adopted for the various lines are also shown

that $r \gtrsim 0.0013$. These r values are testable by the forthcoming experiments [53–56], which are expected to measure r with an accuracy of 10^{-3} . The tuning, finally, required for the attainment of hilltop nMI for $n > 0$ is very low, since $\phi_{\text{max}} \gg \phi_*$.

Although $\lambda/c_{\mathcal{R}}^{p/2}$ is constant for fixed n , the amplitudes of λ and $c_{\mathcal{R}}$ can be bounded. This fact is illustrated in Fig. 3 where we display the allowed values $c_{\mathcal{R}}$ versus $\sqrt{\lambda}$ for $p = 4$ and $K = K_1$ (gray lines) or $K = K_5$ (light gray lines). We take $n = 0$ (solid lines) and $n = -0.004$ (dashed lines). As anticipated in Eq. (46) for any n there is a lower bound on $c_{\mathcal{R}}$, above which $\phi_* \leq 1$ stabilizing thereby the results against corrections from higher order terms – e.g., $(\bar{\Phi}\Phi)^l$ with $l > 1$ in Eq. (24). The perturbative bound $\lambda = 3.5$ limits the various lines at the other end. We observe that the ranges of the allowed lines are much more limited compared to other models – cf. Refs. [12, 30, 31] – and displaced to higher λ values as seen, also, by Eqs. (48a) and (48b). We find that for $p = 5$, λ corresponding to lowest possible $c_{\mathcal{R}}$ violates the perturbative bound and so, our proposal can not be applied for $p > 4$.

5 A post-inflationary completion

In a couple of recent papers [27, 33, 34] we attempt to connect the high-scale inflationary scenario based on $W = W_{\text{HI}}$ in Eq. (24) with the low energy physics, taking into account constraints from the observed BAU, neutrino data and MSSM phenomenology. It would be, therefore, interesting to check if this scheme can be applied also in the case of our present set-up where W_{HI} in Eq. (24) cooperates with the K 's in Eqs. (9a) – (9e) where $F_{\mathcal{R}}$ and F_- given in Eq. (25). The necessary extra ingredients for such a scenario are described in Sect. 5.1. Next, we show how we can correlate nMI with

the generation of the μ term of MSSM – see Sect. 5.2 – and the generation of BAU via nTL – see Sect. 5.3. Hereafter, we restore units, i.e., we take $m_P = 2.433 \cdot 10^{18}$ GeV.

5.1 Relevant set-up

Following the post-inflationary setting of Refs. [33,34] we consider a $B - L$ extension of MSSM with the field content charged under $B - L$ and R as displayed in Table 1 therein. The superpotential of the model contains W_{HI} in Eq. (24), the superpotential of MSSM with $\mu = 0$ and the following two terms

$$W_\mu = \lambda_\mu S H_u H_d, \quad (58a)$$

$$W_{\text{RHN}} = \lambda_{iN^c} \tilde{\Phi} N_i^{c2} + h_{Nij} N_i^c L_j H_u. \quad (58b)$$

From the terms above, the first one inspired by Ref. [38] helps to justify the existence of the μ term within MSSM, whereas the second one allows for the implementation of (type I) see-saw mechanism (providing masses to light neutrinos) and supports a robust baryogenesis scenario through nTL. Let us note that L_i denotes the i -th generation $SU(2)_L$ doublet left-handed lepton superfields, and H_u [H_d] is the $SU(2)_L$ doublet Higgs superfield which couples to the up [down] quark superfields. Also, we assume that the superfields N_j^c have been rotated in the family space so that the coupling constants λ_i are real and positive. This is the so-called [24, 33,34] N_i^c basis, where the N_i^c masses, M_{iN^c} , are diagonal, real and positive.

We assume that the extra scalar fields $X^\beta = H_u, H_d, \tilde{N}_i^c$ have identical kinetic terms as the stabilizer field S expressed by the functions F_{lS} with $l = 1, 2, 3$ in Eqs. (11a) – (11c) – see Refs. [33,34]. Therefore, N_S may be renamed N_X henceforth. The inflationary trajectory in Eq. (12) has to be supplemented by the conditions

$$H_u = H_d = \tilde{N}_i^c = 0, \quad (59)$$

and the stability of this path has to be checked, parameterizing the complex fields above as we do for S in Eq. (13). The relevant masses squared are listed in Table 3 for $K = K_1 - K_5$, with hatted fields being defined as $\hat{s}$ and $\hat{\bar{s}}$ in Eq. (23).

Also we set

$$\hat{h}_\pm = (\hat{h}_u \pm \hat{h}_d)/\sqrt{2} \quad \text{and} \quad \hat{\bar{h}}_\pm = (\hat{\bar{h}}_u \pm \hat{\bar{h}}_d)/\sqrt{2}. \quad (60)$$

In Table 3 we see that $\hat{m}_{i\bar{\nu}^c}^2 \gtrsim \hat{H}_{\text{HI}}^2$ and $\hat{m}_{h+}^2 \gtrsim \hat{H}_{\text{HI}}^2$ for every ϕ , whereas imposing $\hat{m}_{h-}^2 \gtrsim \hat{H}_{\text{HI}}^2$ dictates

$$\lambda_\mu \lesssim \frac{\lambda\phi^2}{4} \cdot \begin{cases} (2/3 + c_{\mathcal{R}}\phi/N)/f_{\mathcal{R}} & \text{for } K = K_1, K_2, \\ (2/3 + 1/N_X) & \text{for } K = K_3 - K_5. \end{cases} \quad (61)$$

Both bounds above depend on ϕ and assume their lowest values for $\phi \simeq \phi_f$. Taking, e.g., $n = 0$ and $N_X = 2$ the equations above imply

$$\lambda_\mu \lesssim \begin{cases} 1.5 \cdot 10^{-5} & \text{for } K = K_1, K_2, \\ 5.3 \cdot 10^{-5} & \text{for } K = K_3 - K_5. \end{cases} \quad (62)$$

Since ϕ_f is inverse proportional to $c_{\mathcal{R}}$ and λ is proportional to $c_{\mathcal{R}}^2$ according to Eqs. (43) and (48a) respectively, the bounds above are practically independent of the variation of λ and ϕ_f . Similar bounds are obtained in Refs. [24,27,33,34] and should not be characterized as unnatural, given that the Yukawa coupling constant which provides masses to the up-type quarks, is of the same order of magnitude at a high scale – cf. Ref. [57].

5.2 Solution to the μ problem of MSSM

Supplementing, in Sect. 5.2.2, with soft SUSY breaking terms the SUSY limit of the SUGRA potential – found in Sect. 5.2.1 – we can show that our model assists us to understand the origin of μ term of MSSM, consistently with the low-energy phenomenology – see Sect. 5.2.3.

5.2.1 SUSY potential

The presence of unity in the arguments of the logarithms of K 's in Eqs. (9a)–(9e) with $F_{\mathcal{R}}$ and F_- defined in Eq. (25) allows us to expand them for $m_P \rightarrow \infty$ up to quadratic terms obtaining $\tilde{K}$. Focusing on the $S - \tilde{\Phi} - \Phi$ system we obtain

Table 3 Mass-squared spectrum of the non-inflaton sector for various K 's along the path in Eqs. (27) and (59)

Fields	Eigenstates	Masses squared		
			$K = K_1, K_2$	$K = K_3, K_4, K_5$
10 Real	$\hat{h}_\pm, \hat{\bar{h}}_\pm$	$\hat{m}_{h\pm}^2$	$3\hat{H}_{\text{HI}}^2 (1 + c_{\mathcal{R}}\phi/N \pm 4\lambda_\mu f_{\mathcal{R}}/\lambda\phi^2)$	$3\hat{H}_{\text{HI}}^2 (1 + 1/N_X \pm 4\lambda_\mu/\lambda\phi^2)$
Scalars	$\hat{\bar{\nu}}_i^c, \hat{\bar{\nu}}_i$	$\hat{m}_{i\bar{\nu}^c}^2$	$3\hat{H}_{\text{HI}}^2 c_{\mathcal{R}} (\phi/N + 16\lambda_{iN^c}^2/\lambda^2\phi^2)$	$3\hat{H}_{\text{HI}}^2 (1 + 1/N_X + 16\lambda_{iN^c}^2/\lambda^2\phi^2)$
3 Weyl spinors	$\hat{N}_i^c$	$\hat{m}_{iN^c}^2$	$48\hat{H}_{\text{HI}}^2 \lambda_{iN^c}^2/\lambda^2\phi^2$	

$$\tilde{K} = -N m_{\text{PC}} \mathcal{R} (F_{\mathcal{R}} + F_{\mathcal{R}}^*) + \frac{1}{2} c_{\mathcal{R}}^2 N (F_{\mathcal{R}} + F_{\mathcal{R}}^*)^2 + F_- + |S|^2 \quad (63)$$

from which we can then compute

$$(\tilde{K}_{\alpha\bar{\beta}}) = \text{diag}(\tilde{M}_{\pm}, 1) \quad (64a)$$

where the matrix $\tilde{M}_{\pm}$ reads

$$\tilde{M}_{\pm} = \begin{pmatrix} 1 + N c_{\mathcal{R}}^2 |\bar{\Phi}|^2/4 |F_{\mathcal{R}}|^2 & N c_{\mathcal{R}}^2 \bar{\Phi} \Phi^*/4 |F_{\mathcal{R}}|^2 \\ N c_{\mathcal{R}}^2 \bar{\Phi}^* \Phi/4 |F_{\mathcal{R}}|^2 & 1 + N c_{\mathcal{R}}^2 |\Phi|^2/4 |F_{\mathcal{R}}|^2 \end{pmatrix}. \quad (64b)$$

After calculating $\tilde{K}^{-1}$ and substituting it into the SUSY limit

$$V_{\text{SUSY}} = \tilde{K}^{\alpha\bar{\beta}} W_{\text{HI}\alpha} W_{\text{HI}\bar{\beta}}^* + \frac{g^2}{2} \sum_a D_a D_a, \quad (65)$$

of $\hat{V}_{\text{HI}}$ in Eq. (5b), we find

$$V_{\text{SUSY}} = \lambda^2 \left| \bar{\Phi} \Phi - \frac{1}{4} M^2 \right|^2 + \frac{\lambda^2 |S|^2}{\det \tilde{M}_{\pm}} (|\Phi|^2 + |\bar{\Phi}|^2) + \frac{g^2}{2} (|\Phi|^2 - |\bar{\Phi}|^2)^2, \quad (66)$$

where the non-diagonal contributions in the F terms proportional to $W_{\text{HI},\Phi} W_{\text{HI},\bar{\Phi}}^*$ and $W_{\text{HI},\bar{\Phi}} W_{\text{HI},\Phi}^*$ are cancelled out and

$$\det \tilde{M}_{\pm} = 1 + c_{\mathcal{R}}^2 N (|\Phi|^2 + |\bar{\Phi}|^2) / 4 |F_{\mathcal{R}}|^2. \quad (67)$$

From Eq. (66), we find that the SUSY vacuum lies along the D-flat direction $|\bar{\Phi}| = |\Phi|$ with

$$\langle S \rangle = 0 \quad \text{and} \quad |\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| = M/2. \quad (68)$$

As a consequence, $\langle \Phi \rangle$ and $\langle \bar{\Phi} \rangle$ break spontaneously $U(1)_{B-L}$ down to $\mathbb{Z}_2^{B-L}$. Since $U(1)_{B-L}$ is already broken during nMI, no cosmic string are formed. Given, finally, that X^β participate in $\tilde{K}$, see Eq. (63), with terms similar to the one we have for S , we can easily verify that their v.e.vs lie along the direction in Eq. (59).

5.2.2 Generation of the μ term of MSSM

The contributions from the TeV scale soft SUSY breaking terms, although negligible during nMI may shift slightly $\langle S \rangle$ from zero in Eq. (68). The relevant potential terms are

$$V_{\text{soft}} = \left(\lambda A_\lambda S \bar{\Phi} \Phi - a_S S \lambda M^2/4 + \text{h.c.} \right) + m_\alpha^2 |X^\alpha|^2, \quad (69)$$

where m_α , A_λ and a_S are soft SUSY breaking mass parameters. Considering V_{soft} together with V_{SUSY} from Eq. (66) we end up with the total low energy potential

$$V_{\text{tot}} = V_{\text{SUSY}} + V_{\text{soft}}, \quad (70)$$

which, replacing Φ and $\bar{\Phi}$ by their SUSY v.e.vs from Eq. (68), can be rephrased as

$$\langle V_{\text{tot}}(S) \rangle = \lambda^2 M^2 S^2 / (N c_{\mathcal{R}}^2 + 2) - \lambda a_{3/2} m_{3/2} M^2 S, \quad (71a)$$

if we neglect $m_\alpha \ll M$ and set

$$|A_\lambda| + |a_S| = 2 a_{3/2} m_{3/2}. \quad (71b)$$

Here $m_{3/2}$ is the gravitino ($\tilde{G}$) mass and $a_{3/2} > 0$ a parameter of order unity which parameterizes our ignorance for the dependence of $|A_\lambda|$ and $|a_S|$ on $m_{3/2}$ – note that the phases of A_λ and a_S have been chosen so that V_{tot} is minimized and S has been rotated in the real axis by an appropriate R -transformation. The extremum condition for $\langle V_{\text{tot}}(S) \rangle$ in Eq. (71a) w.r.t S leads to a non-vanishing $\langle S \rangle$ as follows

$$\frac{d}{dS} \langle V_{\text{tot}}(S) \rangle = 0 \Rightarrow \langle S \rangle \simeq a_{3/2} m_{3/2} (N c_{\mathcal{R}}^2 + 2) / 2\lambda. \quad (72a)$$

The generated μ term from W_μ in Eq. (58a) is

$$\mu = \lambda_\mu \langle S \rangle \simeq \lambda_\mu a_{3/2} m_{3/2} (N c_{\mathcal{R}}^2 + 2) / 2\lambda, \quad (72b)$$

which, although similar, is clearly distinguishable from the results obtained in Refs. [24,27,33,34]. The resulting μ above depends only on n and λ_μ since $\lambda/c_{\mathcal{R}}^2$ is fixed for frozen n by virtue of Eqs. (48a)–(48c). As a consequence, we may verify that any $|\mu|$ value is accessible for the λ_μ 's allowed by Eq. (62) without any ugly hierarchy between $m_{3/2}$ and μ .

5.2.3 Link to the MSSM phenomenology

The subgroup, $\mathbb{Z}_2^R$ of $U(1)_R$ – which remains unbroken after the consideration of the SUSY breaking effects in Eq. (69) – combined with the $\mathbb{Z}_2^f$ fermion parity yields the well-known R -parity. This symmetry guarantees the stability of the *lightest SUSY particle* (LSP), providing thereby a well-motivated *cold dark matter* (CDM) candidate.

The candidacy of LSP may be successful, if it generates the correct CDM abundance [39] within a concrete low-energy framework. In the case under consideration [33,34] this could

Table 4 The required λ_μ values rendering our models for $n = 0$ compatible with the best-fit points of the CMSSM as found in Ref. [58] with the assumptions in Eq. (74)

CMSSM region	Parameters				λ_μ (10^{-6})			
	$ A_0 $ (TeV)	m_0 (TeV)	$ \mu $ (TeV)	$a_{3/2}$	$K = K_1$	$K = K_2$	$K = K_3$	$K = K_4, K_5$
A/H funnel	9.9244	9.136	1.409	1.086	2.946	2.867	3.28	3.195
$\tilde{\tau}_1 - \chi$ coannihilation	1.2271	1.476	2.62	0.831	<i>44.3</i>	<i>43.13</i>	49.35	48.06
$\tilde{t}_1 - \chi$ coannihilation	9.965	4.269	4.073	2.33	8.58	8.27	9.461	9.214
$\tilde{\chi}_1^\pm - \chi$ coannihilation	9.2061	9.000	0.983	1.023	2.215	2.156	2.46	2.4

be the *Constrained MSSM* (CMSSM), which employs the following free parameters

$$\text{sign}\mu, \tan\beta = \langle H_u \rangle / \langle H_d \rangle, M_{1/2}, m_0 \text{ and } A_0, \quad (73)$$

where $\text{sign}\mu$ is the sign of μ , and the three last mass parameters denote the common gaugino mass, scalar mass and trilinear coupling constant, respectively, defined (normally) at M_{GUT} . The parameter $|\mu|$ is not free, since it is computed at low scale by enforcing the conditions for the electroweak symmetry breaking. The values of the parameters in Eq. (73) can be tightly restricted imposing a number of cosmo-phenomenological constraints from which the consistency of LSP relic density with observations plays a central role. Some updated results are recently presented in Ref. [58], where we can also find the best-fit values of $|A_0|$, m_0 and $|\mu|$ listed in the second, third and fourth leftmost columns of Table 4. We see that there are four allowed regions characterized by the specific mechanism for suppressing the relic density of the LSP which is the lightest neutralino (χ) – note that $\tilde{\tau}_1$, $\tilde{t}_1$ and $\tilde{\chi}_1^\pm$ stand for the lightest stau, stop and chargino eigenstate.

The inputs from Ref. [58] can be deployed within our setting, if we identify, e.g.,

$$m_0 = m_{3/2} \quad \text{and} \quad |A_0| = |A_\lambda| = |a_S|. \quad (74)$$

Fixing also $n = 0$, we first derive $a_{3/2}$ from Eq. (71b) – see fifth column of Table 4 – and then the λ_μ values which yield the phenomenologically desired $|\mu|$ – ignoring renormalization group effects. The outputs w.r.t λ_μ of our computation are listed in the four rightmost columns of Table 4 for $K = K_1 - K_5$. From these we infer that the required λ_μ values vary slightly depending on N and $c_{\mathcal{R}}$ required by each K and, besides the ones written in italics, are comfortably compatible with Eq. (62). Therefore, the whole inflationary scenario can be successfully combined with all the allowed regions CMSSM besides the $\tilde{\tau}_1 - \chi$ coannihilation region for $K = K_1$ and K_2 . On the other hand, the $m_{3/2}$'s used in all the CMSSM regions can be consistent with the gravitino limit on reheat temperature T_{rh} , under the assumption of the

unstable $\tilde{G}$, for the T_{rh} values necessitated for satisfactory leptogenesis – see Sect. 5.3.2.

5.3 Non-thermal leptogenesis and neutrino masses

Besides the generation of μ term, our post-inflationary setting offers a graceful exit from the inflationary phase (see Sect. 5.3.1) and explains the observed BAU (see Sect. 5.3.2) consistently with the $\tilde{G}$ constraint and the low energy neutrino data. Our results are summarized in Sect. 5.3.3.

5.3.1 Inflaton mass and decay

When nMI is over, the inflaton continues to roll down towards the SUSY vacuum, Eq. (68). Soon after, it settles into a phase of damped oscillations around the minimum of $\hat{V}_{\text{HI}}$. The (canonically normalized) inflaton,

$$\widehat{\delta\phi} = \langle J \rangle \delta\phi \quad \text{with } \delta\phi = \phi - M, \quad (75)$$

where $\langle J \rangle$ is estimated from Eq. (38), acquires mass given by

$$\widehat{m}_{\delta\phi} = \left\langle \widehat{V}_{\text{HI}, \widehat{\delta\phi}} \right\rangle^{1/2} = \frac{\lambda M}{\sqrt{2} \langle J \rangle} \simeq \frac{\lambda M}{\sqrt{N} c_{\mathcal{R}}}. \quad (76)$$

As we see, $\widehat{m}_{\delta\phi}$ depends crucially on M which is bounded from above by the requirement $\langle f_{\mathcal{R}} \rangle = 1$ ensuring the establishment of the conventional Einstein gravity at the vacuum. This bound is translated to an upper bound on the mass $\langle M_{BL} \rangle$ that the $B-L$ gauge boson acquires for $\phi = \langle \phi \rangle$ – see Table 2. Namely we obtain $\langle M_{BL} \rangle \leq 10^{14}$ GeV, which is lower than the value $M_{\text{GUT}} \simeq 2 \cdot 10^{16}$ GeV dictated by the unification of the MSSM gauge coupling constants – cf. Refs. [27, 33, 34]. However, since $U(1)_{B-L}$ gauge symmetry does not disturb this unification, we can treat $\langle M_{BL} \rangle = gM$ as a free parameter with $g \simeq 0.5 - 0.7$ being the value of the GUT gauge coupling at the scale $\langle M_{BL} \rangle$.

During the phase of its oscillations at the SUSY vacuum, $\widehat{\delta\phi}$ decays perturbatively reheating the Universe at a reheat temperature given by [59]

$$T_{\text{rh}} = (40/\pi^2 g_{\text{rh}})^{1/4} \widehat{\Gamma}_{\delta\phi}^{1/2} m_{\text{P}}^{1/2}, \quad (77a)$$

where the unusual – cf. Refs. [27, 33, 34] – prefactor is consistent with $w_{\text{rh}} \simeq 0.33$ [59] and we set $g_{\text{rh}} = 228.75$ as in Eq. (52a). The total decay width of $\widehat{\delta\phi}$ is found to be

$$\widehat{\Gamma}_{\delta\phi} = \widehat{\Gamma}_{\delta\phi \rightarrow N_i^c} + \widehat{\Gamma}_{\delta\phi \rightarrow H} + \widehat{\Gamma}_{\delta\phi \rightarrow XYZ}, \quad (77b)$$

where the individual decay widths are

$$\widehat{\Gamma}_{\delta\phi \rightarrow N_i^c} = \frac{g_{iN^c}^2}{16\pi} \widehat{m}_{\delta\phi} \left(1 - \frac{4M_{iN^c}^2}{\widehat{m}_{\delta\phi}^2}\right)^{3/2}; \quad (78a)$$

$$\widehat{\Gamma}_{\delta\phi \rightarrow H} = \frac{2}{8\pi} g_H^2 \widehat{m}_{\delta\phi}; \quad (78b)$$

$$\widehat{\Gamma}_{\delta\phi \rightarrow XYZ} = g_y^2 \frac{7}{256\pi^3} \frac{\widehat{m}_{\delta\phi}^3}{m_{\text{P}}^2}, \quad (78c)$$

and the relevant coupling constants are defined as follows

$$g_{iN^c} = \frac{\lambda_{iN^c}}{\langle J \rangle} \left(1 - N c_{\mathcal{R}} \frac{M}{m_{\text{P}}}\right), \quad (79a)$$

$$g_H \simeq \lambda_{\mu}/\sqrt{2} \quad \text{and} \quad g_y = y_3 N \langle J \rangle c_{\mathcal{R}}/2. \quad (79b)$$

The decay widths above arise from the lagrangian terms

$$\begin{aligned} \mathcal{L}_{\widehat{\delta\phi} \rightarrow N_i^c N_i^c} &= -\frac{1}{2} e^{K/2m_{\text{P}}^2} W_{\text{RHN}, N_i^c N_i^c} N_i^c N_i^c + \text{h.c.} \\ &= g_{iN^c} \widehat{\delta\phi} (N_i^c N_i^c + \text{h.c.}) + \dots; \end{aligned} \quad (80a)$$

$$\begin{aligned} \mathcal{L}_{\widehat{\delta\phi} \rightarrow H_u H_d} &= -e^{K/m_{\text{P}}^2} K^{SS*} |W_{\mu,S}|^2 \\ &= -g_H \widehat{m}_{\delta\phi} \widehat{\delta\phi} (H_u^* H_d^* + \text{h.c.}) + \dots; \end{aligned} \quad (80b)$$

$$\begin{aligned} \mathcal{L}_{\widehat{\delta\phi} \rightarrow X\psi_Y\psi_Z} &= -\frac{1}{2} e^{K/2m_{\text{P}}^2} (W_{y,YZ} \psi_Y \psi_Z) + \text{h.c.} \\ &= -g_y \frac{\widehat{\delta\phi}}{m_{\text{P}}} (X\psi_Y\psi_Z) + \text{h.c.}, \end{aligned} \quad (80c)$$

describing respectively $\widehat{\delta\phi}$ decay into a pair of N_i^c with masses $M_{jN^c} = \lambda_{jN^c} M < \widehat{m}_{\delta\phi}/2$, H_u and H_d and three MSSM (s)-particles X, Y, Z involved in a typical trilinear superpotential term $W_y = yXYZ$. Here, ψ_X, ψ_Y and ψ_Z are the chiral fermions associated with the superfields X, Y and Z whose scalar components are denoted with the superfield symbols and $y = y_3 \simeq (0.4 - 0.6)$ is a Yukawa coupling constant of the third generation.

5.3.2 Lepton-number and gravitino abundances

For $T_{\text{rh}} < M_{iN^c}$, the out-of-equilibrium decay of N_i^c generates a lepton-number asymmetry (per N_i^c decay), ε_i estimated from Ref. [60]. The resulting lepton-number asymmetry is partially converted through sphaleron effects into a yield of the observed BAU

$$Y_B = -0.35 \frac{3}{2} \frac{T_{\text{rh}}}{\widehat{m}_{\delta\phi}} \sum_i \frac{\widehat{\Gamma}_{\delta\phi \rightarrow N_i^c}}{\widehat{\Gamma}_{\delta\phi}} \varepsilon_i, \quad (81)$$

where $\langle H_u \rangle \simeq 174$ GeV, for large $\tan \beta$ and m_{D} is the Dirac mass matrix of neutrinos, ν_i . The ratio $(3/2)$ is again [59] consistent with $w_{\text{rh}} = 0.33$. The expression above has to reproduce the observational result [39]

$$Y_B = (8.64_{-0.16}^{+0.15}) \cdot 10^{-11}. \quad (82)$$

The validity of Eq. (81) requires that the $\widehat{\delta\phi}$ decay into a pair of N_i^c 's is kinematically allowed for at least one species of the N_i^c 's and also that there is no erasure of the produced Y_L due to N_1^c mediated inverse decays and $\Delta L = 1$ scatterings. These prerequisites are ensured if we impose

$$(a) \quad \widehat{m}_{\delta\phi} \geq 2M_{1N^c} \quad \text{and} \quad (b) \quad M_{1N^c} \gtrsim 10T_{\text{rh}}. \quad (83)$$

The quantity ε_i can be expressed in terms of the Dirac masses of $\nu_i, m_{i\text{D}}$, arising from the third term of Eq. (24). Employing the (type I) seesaw formula we can then obtain the light-neutrino mass matrix m_{ν} in terms of $m_{i\text{D}}$ and M_{iN^c} . As a consequence, nTL can be nicely linked to low energy neutrino data. We take as inputs the recently updated best-fit values [61] – cf. Refs. [33, 34] – on the neutrino mass-squared differences, $\Delta m_{21}^2 = 7.56 \cdot 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = 2.55 \cdot 10^{-3} \text{ eV}^2$ [$\Delta m_{31}^2 = 2.49 \cdot 10^{-3} \text{ eV}^2$], on the mixing angles, $\sin^2 \theta_{12} = 0.321$, $\sin^2 \theta_{13} = 0.02155$ [$\sin^2 \theta_{13} = 0.0214$] and $\sin^2 \theta_{23} = 0.43$ [$\sin^2 \theta_{23} = 0.596$] and the CP-violating Dirac phase $\delta = 1.4\pi$ [$\delta = 1.44\pi$] for *normal* [*inverted*] *ordered* (NO [IO]) *neutrino masses*, $m_{i\nu}$'s. Furthermore, the sum of $m_{i\nu}$'s is bounded from above at 95% c.l. by the data [39]

$$\sum_i m_{i\nu} \leq 0.23 \text{ eV}. \quad (84)$$

The required T_{rh} in Eq. (81) must be compatible with constraints on the $\widetilde{G}$ abundance, $Y_{3/2}$, at the onset of *nucleosynthesis* (BBN), which is estimated to be [62–67]

$$Y_{3/2} \simeq 1.9 \cdot 10^{-22} T_{\text{rh}}/\text{GeV}, \quad (85)$$

where we take into account only thermal production of $\widetilde{G}$, and assume that $\widetilde{G}$ is much heavier than the MSSM gauginos. Non-thermal contributions to $Y_{3/2}$ [68] are also possible but strongly dependent on the mechanism of soft SUSY breaking. No precise computation of this contribution exists within nMI adopting the simplest Polonyi model of SUSY breaking [69, 70]. It is notable, though, that the non-thermal contribution to $Y_{3/2}$ in models with stabilizer field, as in our case, is significantly suppressed compared to the thermal one.

Table 5 Parameters yielding the correct BAU for $K = K_1$ or K_2 , $\langle M_{BL} \rangle = 10^{12}$ GeV, $n = 0$, $\lambda_\mu = 10^{-6}$, $y_3 = 0.5$ and various neutrino mass schemes

Cases	A	B	C	D	E	F	G
Parameters	Normal Hierarchy		Almost Degeneracy			Inverted Hierarchy	
Low scale parameters (masses in eV)							
$m_{1\nu}/0.1$	0.01	0.1	0.5	0.6	0.7	0.51	0.5
$m_{2\nu}/0.1$	0.09	0.13	0.51	0.61	0.71	0.52	0.51
$m_{3\nu}/0.1$	0.5	0.51	0.71	0.78	0.5	0.1	0.05
$\sum_i m_{i\nu}/0.1$	0.6	0.74	1.7	2	1.9	1.1	1
φ_1	$-\pi/9$	$-2\pi/3$	π	$\pi/2$	$\pi/2$	$3\pi/5$	$2\pi/3$
$-\varphi_2$	$\pi/2$	0	$-\pi/2$	$2\pi/3$	$2\pi/3$	$\pi/2$	$\pi/2$
Leptogenesis-scale mass parameters in GeV							
$m_{1D}/0.1$	0.38	0.69	3.23	2.25	0.94	50	1.9
m_{2D}	1	5	0.2	0.15	0.3	0.097	0.2
m_{3D}	1	10	5	10	9	6	5
$M_{1N^c}/10^8$	5.4	8.4	9.9	4.7	2.2	7.9	12.6
$M_{2N^c}/10^9$	13	755	2.5	0.96	1.57	219	3.5
$M_{3N^c}/10^{11}$	3.2	18.8	1.6	6.1	5	12.7	12.1
Decay channels of $\widehat{\delta\phi}$							
$\widehat{\delta\phi} \rightarrow$	N_1^c	N_1^c	N_1^c	$N_{1,2}^c$	$N_{1,2}^c$	N_1^c	N_1^c
Resulting B -yield							
$Y_B/10^{-11}$	8.7	8.6	8.6	8.6	8.5	8.7	8.6
Resulting T_{rh} (in GeV)							
$T_{\text{rh}}/10^7$	2.8	3	3	3.1	2.7	2.9	3.3

On the other hand, $Y_{3/2}$ is bounded from above in order to avoid spoiling the success of the BBN. For the typical case where $\tilde{G}$ decays with a tiny hadronic branching ratio, we obtain [64–67] an upper bound on T_{rh} , i.e.,

$$T_{\text{rh}} \lesssim 5.3 \cdot \begin{cases} 10^7 \text{ GeV} \\ 10^8 \text{ GeV} \\ 10^9 \text{ GeV} \end{cases} \quad \text{for } m_{3/2} \simeq \begin{cases} 0.69 \text{ TeV}, \\ 10.6 \text{ TeV}, \\ 13.5 \text{ TeV}. \end{cases} \quad (86)$$

The bounds above can be somehow relaxed in the case of a stable $\tilde{G}$.

5.3.3 Results

Confronting with observations Y_B and T_{rh} which depend on $\widehat{m}_{\delta\phi}$, M_{iN^c} and m_{iD} 's – see Eqs. (81) and (86) – we can further constrain the parameter space of the our models. In our investigation we follow the bottom-up approach detailed in Refs. [33, 34], according to which we find the M_{iN^c} 's by using as inputs the m_{iD} 's, a reference mass of the ν_i 's – $m_{1\nu}$ for NO $m_{i\nu}$'s, or $m_{3\nu}$ for IO $m_{i\nu}$'s –, the two Majorana phases φ_1 and φ_2 of the PMNS matrix, and the best-fit values for the low energy parameters of neutrino physics mentioned in Sect. 5.3.2. In our numerical code, we also estimate [33, 34] the renormalization-group evolved values of the latter parameters at the scale of nTL, $\Lambda_L = \widehat{m}_{\delta\phi}$, by considering the MSSM with $\tan \beta \simeq 50$ as an effective theory between

Λ_L and the soft SUSY breaking scale, $M_{\text{SUSY}} = 1.5$ TeV. We evaluate the M_{iN^c} 's at Λ_L , and we neglect any possible running of the m_{iD} 's and M_{iN^c} 's. The so obtained M_{iN^c} 's clearly correspond to the scale Λ_L .

Some representative values of the parameters which yield Y_B and T_{rh} compatible with Eqs. (82) and (86), respectively, are arranged in Table 5. We take $n = 0$ – to avoid any tuning as regards the inflationary inputs –, $\lambda_\mu = 10^{-6}$ in accordance with Eq. (62), and $\langle M_{BL} \rangle = 10^{12}$ GeV. Note that we consider $\langle M_{BL} \rangle$ as a free parameter since the unification value – imposed in Refs. [27, 33, 34] – is not reconciled with the reappearance of Einstein gravity at low energies, i.e., $\langle f_{\mathcal{R}} \rangle = 1$. Setting $g = 0.7$ in the formula giving M_{BL} in Table 2, we obtain $M = 1.43 \cdot 10^{12}$ GeV resulting via Eq. (76) to $2.8 \leq \widehat{m}_{\delta\phi}/10^9 \text{ GeV} \leq 4.1$ – the variation is due to the choice of K . Although this amount of uncertainty does not cause any essential alteration of the final outputs, we mention just for definiteness that we take throughout $K = K_1$ corresponding to $\widehat{m}_{\delta\phi} = 3.3 \cdot 10^{10}$ GeV. We consider NO (cases A and B), almost degenerate (cases C, D and E) and IO (cases F and G) $m_{i\nu}$'s. In all cases, the current limit in Eq. (84) is safely met. This is more restrictive than the 90% c.l. upper bound arising from the effective electron neutrino mass m_β in β -decay [71–73] by various experiments. Indeed, the current upper bounds on m_β are comfortably satisfied by the values found in our set-up

Fig. 4 Contours, yielding the central Y_B in Eq. (5.8) consistently with the inflationary requirements, in the **a** $\langle M_{BL} \rangle - m_{1D}$ and **b** $\hat{m}_{\delta\phi} - M_{1N^c}$ plane. We take $K = K_1$, $n = 0$, $\lambda_\mu = 10^{-6}$ and the values of $m_{i\nu}$,

m_{2D} , m_{3D} , ϕ_1 and ϕ_2 which correspond to the cases A (solid line), C (dashed line) and E (dot-dashed line) of Table 5

$$0.002 \leq m_\beta/\text{eV} \leq 0.036, \quad (87)$$

where the lower and upper bound corresponds to case A and C respectively.

The gauge symmetry considered here does not predict any particular Yukawa unification pattern and so, the m_{iD} 's are free parameters. This fact allows us to consider m_{iD} 's which are not hierarchical depending on the generation. Also, it facilitates the fulfilment of Eq. (83b) since m_{1D} affects heavily M_{1N^c} . Care is also taken so that the perturbativity of λ_{iN^c} – defined below Eq. (80c) – holds, i.e., $\lambda_{iN^c}^2/4\pi \leq 1$. The inflaton $\hat{\delta\phi}$ decays mostly into N_1^c 's – see cases A – E. In all cases $\hat{\Gamma}_{\delta\phi \rightarrow N_1^c} < \hat{\Gamma}_{\delta\phi \rightarrow H}$ and so the ratios $\hat{\Gamma}_{\delta\phi \rightarrow N_1^c}/\hat{\Gamma}_{\delta\phi}$ introduce a considerable reduction in the derivation of Y_B . Namely, we obtain

$$0.07 \lesssim \hat{\Gamma}_{\delta\phi \rightarrow N_1^c}/\hat{\Gamma}_{\delta\phi} \lesssim 0.35 \quad (88)$$

where the lower [upper] bound comes out in case E [G]. In Table 5 we also display the values of T_{rh} , the majority of which are close to $3 \cdot 10^7$ GeV, and consistent with Eq. (86) for $m_{3/2} \gtrsim 1$ TeV. These values are in nice agreement with the ones needed for the solution of the μ problem of MSSM – see, e.g., Table 4. Thanks to our non-thermal set-up, successful leptogenesis can be accommodated with T_{rh} 's lower than those necessitated in the thermal regime – cf. Ref. [74].

In order to investigate the robustness of the conclusions inferred from Table 5, we examine also how the central value of Y_B in Eq. (82) can be achieved by varying $\langle M_{BL} \rangle$, or $\hat{m}_{\delta\phi}$, and adjusting conveniently m_{1D} or M_{1N^c} – see Fig. 4a and b respectively. We fix again $n = 0$ and $\lambda_\mu = 10^{-6}$. Since the range of Y_B in Eq. (82) is very narrow, the 95% c.l. width of these contours is negligible. The convention adopted for

the various lines is also depicted. In particular, we use solid, dashed and dot-dashed line when the remaining inputs – i.e. $m_{i\nu}$, m_{2D} , m_{3D} , ϕ_1 , and ϕ_2 – correspond to the cases A, C and E of Table 5, respectively. At the lower limit of these lines nTL becomes inefficient (due to low T_{rh}) failing to reach the value in Eq. (82). At the other end, these lines terminate at the values of m_{1D} beyond which Eq. (83b) is violated and, therefore, washout effects start becoming significant. Along the depicted contours, the resulting M_{2N^c} and M_{3N^c} remain close to their values presented in the corresponding cases of Table 4. As regards the other quantities, in all we obtain

$$0.04 \lesssim T_{\text{rh}}/10^8 \text{ GeV} \lesssim 13, \quad (89a)$$

$$0.03 \lesssim \hat{m}_{\delta\phi}/10^{10} \text{ GeV} \lesssim 4.64, \quad (89b)$$

with the lower and upper bound obtained in the limits of the solid line which represent the most ample region of parameters satisfying the imposed requirements. These values are much lower than those obtained in Ref. [27] and a little lower than those found in Refs. [33, 34], mainly due to lower $\langle M_{BL} \rangle$'s employed here.

As a bottom line, nTL is a realistic possibility within our setting. It can be comfortably reconciled with the $\tilde{G}$ constraint even for $m_{3/2} \sim 1$ TeV as deduced from Eqs. (89b) and (86) adopting a sufficiently low $\langle M_{BL} \rangle$.

6 Conclusions

Motivated by the fact that a strong linear non-minimal coupling of the inflaton to gravity does not cause any problem with the validity of the effective theory up to the Planck scale, we explored the possibility to attain observationally viable

nMI (i.e. non-minimal inflation) in the context of standard SUGRA by strictly employing this coupling. We showed that nMI is easily achieved, for $p \leq 4$ in the superpotential of Eq. (8), by conveniently adjusting the prefactor ($-N$) of the logarithmic part of the relevant Kähler potentials K given in Eqs. (9a)–(9e), where the relevant functions $F_{\mathcal{R}}$ and F_- are shown in Eq. (10) for a gauge-singlet inflaton. For appropriately selected integer N 's – i.e., setting $n = 0$ in Eq. (20) –, our models retain the predictive power of well-known universal attractor models – which employ a non-minimal coupling functionally related to the potential – and yield similar results. Allowing for non-integer N values, this predictability is lost since the observables depend on the adopted K and n in Eq. (20) and may yield any n_s in its allowed region and $0.0013 \leq r \leq 0.02$.

This scheme works also for a gauge non-singlet inflaton employing the superpotential shown in Eq. (24) and the functions $F_{\mathcal{R}}$ and F_- in Eq. (25). Embedding these models within a $B - L$ extension of MSSM, we showed that a μ term is easily generated and the baryon asymmetry in the Universe is naturally explained via non-thermal leptogenesis. The $B - L$ breaking scale $\langle M_{BL} \rangle$, though, has to take values lower than the MSSM unification scale and so, the present scheme is similarly predictive with that of Refs. [33, 34] which employs one more parameter in the Kähler potentials but allows for $\langle M_{BL} \rangle$'s fixed by the gauge coupling unification within MSSM. Our scenario can be comfortably tolerated with almost all the allowed regions of the CMSSM with gravitino as low as 1 TeV. Moreover, leptogenesis is realized through the out-of equilibrium decay of the inflaton to the right-handed neutrinos N_1^c and/or N_2^c with masses lower than $2.32 \cdot 10^{10}$ GeV, and a reheat temperature $T_{\text{rh}} \leq 10^9$ GeV taking $\langle M_{BL} \rangle \leq 10^{13}$ GeV.

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