Solar system tests in constraining parameters of dyon black holes

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Abstract In the present paper we examine the possibility of constraining dyon black holes based on the available observational data at the scale of the Solar system. For this we consider the classical tests of general relativity, viz., the perihelion precession of the planet Mercury and the deflection of light by the Sun. In connection to mathematical analysis we are considering static and spherically symmetric dyon black hole which carries both the electric and magnetic charge simultaneously, which are encoded it by the parameters λ_0 and β_0 . We constrain these two parameters using the Solar system tests and obtain the permissible range from theoretical analysis based on our model and later on compare them with the available observational data.

1 Introduction

Black holes are believed to be the most promising candidates to address the quantum effects of gravity, as they represent in many ways the possible testing ground for string theory [1–3]. The reason behind this is that, in order to probe gravity beyond general relativity (where classical physics breaks down), one needs a singularity, which can be found in the center of a black hole. Although mathematical formulations have been done for several formal aspects of string theory (quantum gravity), the subject still remains enigmatic as well as remote from other areas of physics as far as measurements or observations are concerned. Sub-field of string theory or string phenomenology otherwise known is well developed that pertains to resolve observational issues of string theory. In the present day scenarios string phenomenology has gained potential to unravel particle physics related as well as cosmological observations in a more deeper sense.

Earlier black hole were considered merely a mathematical entity rather than real existing astrophysical objects of the universe. Recently, especially after the successful detection of gravitational waves from black holes collisions [4,5] and the possibility of looking at an astrophysical black holes [6], black holes have become a hot topic in the current physics research [7–24]. Black holes arise from low energy effective string theories as a result of various compactification schemes. String compactification to four dimensions involves a manifold and generates multiple scalar fields. For example, many black hole solutions have been obtained in the low-energy string theories in which the Kalb-Ramond field, dilaton field and gauge field are incorporated [25–32]. The massless scalar fields coupled with the vector fields, which also naturally occur in the bosonic sector of such four-dimensional effective theories, in addition to gravity and scalar fields.

The Einstein-Maxwell-axion-dilaton system includes all the main features of low-energy string theory, where the two scalar fields couple to the abelian vector fields. The dilaton couples to the square of the Maxwellian field strength F^2 , whereas the axion couples to the topological term $F \wedge F$ and is therefore a pseudoscalar. The presence of these scalar fields affect the basic properties of the black hole. The set of static spherically symmetric solutions of the Einstein-Maxwell-dilaton system, falls in the class of dilaton black holes. They possess either purely a magnetic or electric Maxwell field and have a vanishing axion [26,30]. However, when gravity is coupled to dilaton and gauge field there appears a new axion field and the solutions become dyonic, where the black holes carry electric and magnetic charge simultaneously [31,33].

Now, to be a viable black hole solution, the proposed model need to pass observational tests of astrophysics. One of the possibilities of such observation-based testing black hole model at an astrophysical scale are the solar system tests. The solar system tests are the classical tests of general relativity, which have been analyzed for various gravitational the-

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ories with large non-compactified higher-dimensions [34], by using an analogue of the four-dimensional Schwarzschild metric and have imposed strong constraints on it. This paper basically concerns with the solar system tests, namely the perihelion precession and bending of light for a static spherically symmetric dyonic black hole.

Therefore, the paper however contains a technical result in gravity which treats dyonic black holes. We refer to some references regarding dyonic black holes in string theory and connect the idea with the electromagnetism expecting the possible existence of astrophysical charged black holes. It is to note that electromagnetic black holes, especially the Reissner-Nordström black holes, have been studied widely in literature [35]. However, our paper emphasizes that if such an object do exists, then what range of its parameters are viable to satisfy the observations.

2 Background formalism for dyonic black holes

In the early nineties, Cheng et al. [36] obtained an exact solution of the low-energy string theory representing static spherically symmetric dyonic black hole. They have chosen the four-dimensional effective string action in which gravity is coupled to dilaton and electromagnetic field given by

$$I = \int d^4x \sqrt{-g} [-R + 2(\nabla\phi_0)^2 + e^{-2\phi_0} F^2], \quad (1)$$

We consider here a static and spherically symmetric configuration given by

$$ds^2 = -A(r)c^2 dt^2 + B(r)dr^2 + C(r)(d\theta^2 + \sin^2\theta d\phi^2), \quad (2)$$

where

$$A = \frac{1}{c^2} \left(1 - \frac{2M}{r^2} \sqrt{r^2 + \lambda^2} + \frac{\beta}{r^2} \right), \quad (3)$$

$$B = \frac{r^2}{r^2 + \lambda^2} \left(\frac{1}{1 - \frac{2M}{r^2} \sqrt{r^2 + \lambda^2} + \frac{\beta}{r^2}} \right), \quad (4)$$

$$C = r^2, \quad (5)$$

with $\lambda = (Q_e^2 e^{2\phi_0} - Q_m^2 e^{2\phi_0})/2M$ and $\beta = (Q_e^2 e^{2\phi_0} + Q_m^2 e^{2\phi_0})$.

The electrically (or magnetically) charged dilaton black holes are the special case of the dyonic black holes. However, it is to be noted that unlike the electric or magnetic charged dilaton holes, the Hawking temperature of the dyonic black hole depends on both the electric (Q_e) as well as magnetic (Q_m) charges and vanishes as they tend to extremal values. Except Q_e and Q_m the dyonic black hole solution is characterized by the other two parameters, viz., mass (M) and asymptotic value of the scalar dilaton (ϕ_0).

In the following text we perform the solar system tests on these solutions which are the classical tests of general relativity.

3 Solar system tests for dyonic black holes

The perihelion precession of the Mercury and deflection of light by the Sun are the fundamental tests at the level of the Solar system. These classical tests of general relativity have been used successfully to test the Schwarzschild solutions and some of its generalization. In order to perform the above mentioned tests, we consider the geometry outside of the Sun which is a compact stellar type object comprising of a specific static and spherically symmetric vacuum solutions in the context of dyonic black hole.

3.1 Perihelion precession

The eight major planets of the Solar system are rotating around the Sun in elliptical orbits which are approximately coplanar with each other. The point on the orbit of a planet at which it is closest to the Sun is known as *perihelion*. The perihelion of a particular planet remains fixed in space when the relatively weak interplanetary gravitational interaction is neglected. However, when this interaction is considered the perihelion slowly precesses around the Sun. This perihelion precession of the planets has been treated as one of the most important tests to check the correctness of general relativity.

Now, we begin with the Lagrangian which can be written as

$$\mathcal{L} = -Ac^2\dot{t}^2 + B\dot{r}^2 + C\dot{\theta}^2 + C\sin^2\theta\dot{\phi}^2, \quad (6)$$

where dot over any parameter implies differentiation with respect to the affine parameter τ .

It is known that the gravitational field is isotropic and hence there is conservation of angular momentum. So geodesics of the particles (either massive planets or massless photons) are planar. Without loss of generality, we can choose our coordinates in such a way that this plane is the equatorial plane given by keeping fixed $\theta = \pi/2$.

Therefore, the Lagrangian takes the form

$$\mathcal{L} = -Ac^2\dot{t}^2 + B\dot{r}^2 + C\dot{\phi}^2, \quad (7)$$

with light-like particle photon, $\mathcal{L} = 0$ and for any time-like particle, $\mathcal{L} = 1$.

Considering the generalized coordinates q_i and generalized velocities $\dot{q}_i$, the Euler-Lagrange equations become

$$\frac{d}{ds} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0, \quad (8)$$

given that $Ac^2\dot{i} = E$ and $C\dot{\phi} = L$, where E and L are the energy and momentum of the particle respectively, such that $\dot{i} = E/c^2A$ and $\dot{\phi} = L/C$.

Using the above notations, from Eqs. (7) and (8) we get [25]

$$\left(\frac{dr}{d\phi} \right)^2 + \frac{C}{B} = \frac{\mathcal{L}C^2}{BL^2} + \frac{E^2C^2}{ABL^2c^2}. \quad (9)$$

Again by substituting $r = 1/U$ in the above equation, one can write

$$\left(\frac{dU}{d\phi} \right)^2 + \frac{CU^4}{B} = \frac{\mathcal{L}C^2U^4}{BL^2} + \frac{E^2C^2U^4}{ABL^2c^2}. \quad (10)$$

Considering the following transformations $1/B = 1 - f(U)$ and $C = [1/U^2] + [g(U)/U^4]$ the above equation can be written as

$$\left(\frac{dU}{d\phi} \right)^2 + U^2 = U^2 f(U) + f(U)g(U) - g(U) + \frac{\mathcal{L}C^2U^4}{BL^2} + \frac{E^2C^2U^4}{ABL^2c^2} \equiv G(U). \quad (11)$$

Differentiating with respect to U , one gets

$$\frac{d^2U}{d\phi^2} + U = F(U), \quad (12)$$

where

$$F(U) = \frac{1}{2} \frac{dG(U)}{dU}. \quad (13)$$

Since the planetary orbits are nearly circular, so the circular orbit $U = U_0$ can be obtained from the equation

$$F(U_0) = U_0. \quad (14)$$

Now, for the deviation from the circular orbit the perihelion precession is given by

$$\sigma = \frac{1}{2} \left(\frac{dF}{dU} \right)_{U=U_0}. \quad (15)$$

Therefore, for complete rotation, the advancement of perihelion becomes

$$\Delta\phi = \phi - 2\pi \approx 2\pi\sigma. \quad (16)$$

For dyonic black hole the parameters can be written as

$$G(U) = U^2 + (1 + U^2\lambda^2) \left(\frac{1}{L^2} - U^2 \right)$$

$$\times \left(1 - 2MU\sqrt{1 + U^2\lambda^2} + \beta U^2 \right) + \frac{E^2}{L^2} \left(1 + \lambda^2 U^2 \right), \quad (17)$$

$$F(U) = \left[U\sqrt{1 + U^2L^2} (\beta - 2L^2U^2\beta + (1 + E^2 + U^22\beta - L^2(2 + 3U^2\beta)))\lambda^2 + M(1 + U^2\lambda^2)(-1 + U^2 \times (-4\lambda^2 + L^2(3 + 6U^2\lambda^2))) \right] / L^2\sqrt{1 + U^2\lambda^2}. \quad (18)$$

In order to find the circular orbits, we represent the parameters as follows: $\lambda = \lambda_0 M$, $\beta = \beta_0 M^2$ and $U_0 = x_0/M$, where λ_0 , β_0 and x_0 are dimensionless parameters, respectively and find the roots of the equation $F(U_0) = U_0$, which can be written as

$$3x_0^2 - b^2(4x_0^2\lambda_0^2 + 1) = -6x_0^4\lambda_0^2 + \frac{1}{\sqrt{1 + x_0^2\lambda^2}} \left[3x_0^5\beta_0\lambda_0^2 - 2x_0^3 \left\{ -\beta + (-1 + b^2\beta_0)\lambda_0^2 \right\} + x_0(1 + b^2 \left\{ \beta_0 + (1 + E)\lambda_0^2 \right\}) \right], \quad (19)$$

where $b^2 = M^2/L^2$.

To obtain the perihelion precession, one needs to know the value of the parameter L in terms of the orbit parameters. According to Harko et al. [37]

$$\frac{1}{L^2} = \frac{c^2}{GMa(1 - e^2)}, \quad (20)$$

where a is the semi-major axis and e is the eccentricity of the orbit.

For the planet Mercury we have $a = 57.91 \times 10^{11}$ cm, $e = 0.205615$, while the Solar mass $M = M_\odot = 1.989 \times 10^{33}$ gm, $c = 2.998 \times 10^{10}$ cm/s, $G = 6.67 \times 10^{-8}$ cm³/gm s² [38,39]. The Mercury also completes 415.2 revolutions in each century. By the use of these numerical values we first obtain $b^2 = M/a(1 - e^2) = 2.66136 \times 10^8$. Hence putting $E = 1$ we perform a first order series expansion of the square root in Eq. (19) and obtain the standard general relativistic equation $3x_0^2 - x_0 + b^2 = 0$, with the physical solution $x_0^{GR} \approx b^2$. The value of x_0 is obtained by solving the nonlinear algebraic equation Eq. (19) numerically which depends on the parameters λ_0 and β_0 . In general this equation has many roots for different parametric values of λ_0 and β_0 , but we shall concentrate only on the positive values of x_0 which are close to $x_0^{GR} \approx b^2$. Using these numerical values of x_0 we evaluate the perihelion precession angle from Eq. (16). The angle of perihelion precession $\Delta\phi$ for different values of λ_0 and β_0 are shown in Table 1. We note that for some values of parameters the precession is well within

Table 1 Perihilion precession for different parametric values

λ_0	β_0	u_0/b^2	$\Delta\phi$ (arcsec/cen)
0.009	0.1	1.007	43.173
0.01	0.1	1.001	43.657
0.1	-0.1	1.005	42.38
0.1	0.2	0.998	43.672
1	2	1.001	71.60
1	10	0.99	128.887

the observed value of the perihilion precession of the planet Mercury which is $\Delta\phi_{obs} = 43.11 \pm 0.21$ arcsec/cen [40, 41].

3.2 Deflection of light

Henry Cavendish in 1784 (in an unpublished manuscript) and J.G. Von Soldner in 1804 showed the deviation of light rays near a massive object by using Newtonian gravity. In the early twentieth century Einstein calculated the deflection of light by his general relativity, and this came as twice the Newtonian value. Later on, Arthur Eddington and his collaborators verified this phenomenon by observing the change in position of stars during a total solar eclipse in May 1919. So this deflection of light is also one of the observational verification of general relativity.

To investigate the bending of light in the vicinity of dyon black hole we start with Eq. (7) by assuming $\mathcal{L} = 0$ which now reads

$$\left(\frac{dU}{d\phi}\right)^2 + U^2 = U^2 f(U) + f(U)g(U) - g(U) + \frac{E^2 C^2 U^4}{ABL^2 c^2} \equiv P(U). \quad (21)$$

Differentiating with respect to U , one gets

$$\frac{d^2 U}{d\phi^2} + U = Q(U), \quad (22)$$

where

$$Q(U) = \frac{1}{2} \frac{dP(U)}{dU}. \quad (23)$$

Now, we solve the equation by successive approximation, starting from the straight line (path without gravitating body) as zeroth approximation such that $U = \cos\phi/R_0$, where $\phi = 0$ is the point of nearest approach to the Sun's surface. Ideally, R_0 would be the Solar radius. Substituting this on the right hand side of Eq. (22) for U , we get

$$\frac{d^2 U}{d\phi^2} + U = Q\left(\frac{\cos\phi}{R_0}\right). \quad (24)$$

This gives the general solution for $U = U(\phi)$. The light ray from infinity comes at the asymptotic angle $\phi = -(\pi/2 + \epsilon)$ and goes out to infinity at an asymptotic angle $\phi = \pi/2 + \epsilon$. The angle ϵ is obtained as a solution of the equation $U(\pi/2 + \epsilon) = 0$. The total deflection angle of the light ray is given by $\delta = 2\epsilon$. In dyonic black hole scenario, keeping terms up to $(1/R_0)^6$, above equation takes the following form

$$\begin{aligned} \frac{d^2 U}{d\phi^2} + U = & \frac{21}{8} \lambda_0^4 \left(\frac{\cos\phi}{r_0}\right)^6 - 3\beta_0 \lambda_0^2 \left(\frac{\cos\phi}{r_0}\right)^5 \\ & + \frac{15}{2} \lambda_0^2 \left(\frac{\cos\phi}{r_0}\right)^4 - 2(\beta_0 + \lambda_0^2) \left(\frac{\cos\phi}{r_0}\right)^3 \\ & + 3 \left(\frac{\cos\phi}{r_0}\right)^2 + \frac{c^2}{r_0^2} \lambda_0^2 \left(\frac{\cos\phi}{r_0}\right), \end{aligned} \quad (25)$$

where $L = ER_0/c$, $E = 1$ and representing all the variables in the dimensionless form such that $r_0 = R_0/M$, $\lambda_0 = \lambda/M$ and $\beta_0 = \beta/M^2$.

We solve this equation considering the first approximation and substitute $\phi = \pi/2 + \epsilon$, $U = 0$ and use the relations $\cos(\pi/2 + \epsilon) = -\sin\epsilon$, $\cos(\pi + 2\epsilon) = -\cos 2\epsilon$, $\sin\epsilon \approx \epsilon$ and $\cos 2\epsilon \approx 1$. Eventually, for ϵ we get

$$\begin{aligned} \epsilon = & \left[\left(40R_0^3(16R_0 - 3\pi\beta_0) \right. \right. \\ & + 10R_0(4R_0(32 + (-3 + 2c^2M^2)\pi R_0) \\ & + 15\pi\beta_0)\lambda_0^2 + 384\lambda_0^4) \Big] / \left[\left(5R_0(64C_1R_0^4 - 7\beta_0\lambda_0^2) \right. \right. \\ & \left. \left. + 8R_0^2(\beta_0 + \lambda_0^2 - 2c^2M^2\lambda_0^2) \right) \right]. \end{aligned} \quad (26)$$

Here C_1 is the arbitrary constant. For the Sun, by taking $R_0 = R_\odot = 6.955 \times 10^{10}$ cm, where $R_\odot$ is the radius of the Sun, one can obtain the value of r_0 as $r_0 = 4.71194 \times 10^5$.

We evaluate the angle of light deflection $\delta\phi = 2\epsilon$ for different values of parameter λ_0 and β_0 in Table 2. It is to be noted that for various parameters the theoretical values of angle of deflections are in agreement with the observational value obtained from long baseline radio interferometry [42, 43], which gives $\delta\phi_{LD} = \delta\phi_{LD}^{(GR)}(1 + \Delta_{LD})$, with $\Delta_{LD} \leq 0.0002 \pm 0.0008$, where $\delta\phi_{LD}^{(GR)} = 1.7510$ arcsec.

Table 2 Angle of deflection for different parametric values

λ_0	β_0	C_1	$\delta\phi$
0.009	0.1	0.48	1.78
0.01	0.1	0.59	1.753
0.013	0.2	1	1.79
0.011	0.1	1	1.70
0.1	-0.1	60.1	1.752
0.1	0.2	60.1	1.75
1	2	5938	1.75

4 Discussion and conclusion

The classical tests of general relativity provide a very powerful tool for constraining the allowed parameter space of various astrophysical solutions, such as the black hole and brane world solutions. It provides a deeper insight into the physical nature and properties of the corresponding astrophysical object and space-time metric. In the present work, we have analyzed the dyonic black hole, which carries both electric and magnetic charge simultaneously. The black holes have been considered so far as hypothetical as well as intrigued objects which have sparked interest due to recent detection of the gravitational waves formed by the merger of black holes.

In this context, the classical tests of general relativity, namely the perihelion precession and deflection of light are considered for the specific static and spherically symmetric solutions of dyonic black hole. This black hole has two free parameters, λ_0 and β_0 , which quantify the electric and magnetic charge in it. We have constrained the parameters by the perihelion precession of the planet Mercury and found $0.01 \geq \lambda_0 \geq 0.009$ and $\beta_0 = 0.1$ which agree satisfactorily to the observational result. However, from the study of the deflection of light by the Sun it is noticed that the admissible range of the parameters can be extended as $0.1 \geq \lambda_0 \geq 0.009$ and $0.02 \leq \beta_0 \leq 0.1$ (vide Table 2).

At this juncture we would like to emphasize that a crucial issue of the present study consists of the perihelion precession. Indeed, the theoretically predicted values of Table 1 can be compared to the most recent experimental determinations of the anomalous perihelion precessions for all the inner planets along with the Saturn by the team producing the planetary ephemerides. It must be stressed here that, actually, there are no non-zero anomalous perihelion precessions at a statistically significant level since all are statistically compatible with zero. Indeed, the best values are smaller than the errors released. Thus, such values can be used to infer upper bounds on the parameters of the perihelion precessions as shown in Table 1 by comparing them with such recent experimentally determined values. It has been done in a number of papers in the literature for various exotic models of gravity, e.g., the work by Iorio et al. [44] which can be retrieved

from Table 1 of Iorio [45]. These have been determined by Pitjev and Pitjeva [46, 47] with the EPM ephemerides and by Fienga et al. [48] with the INPOP ephemerides. In this way, it can be discovered from Table 3 that how tremendously outdated is the uncertainty of 0.21 arcseconds per century as shown in Table 1. Indeed, the latest uncertainties are much more smaller and Table 3 further gives stringent bound on the model parameters.

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Table 3 Perihelion precession for different parametric values with higher precision

λ_0	β_0	u_0/b^2	$\Delta\phi$ (arcsec/cen)
Venus	0.009	0.1	8.736
	0.01	0.1	8.630
Mars	0.01	0.1	1.353
Saturn	0.009	0.099	0.0137
	0.01	0.1	0.0138

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