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Inelastic profiles of protons at 7 and 13 TeV

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Abstract The shapes of the inelastic interaction region of protons with energies $\sqrt{s} = 7$ and 13 GeV are considered. The unitarity condition with different assumptions about the elastic scattering amplitude is used. It is shown that the sign of the imaginary part of the amplitude at large transferred momenta is especially important for the conclusion about this shape at small impact parameters (central collisions).

1 Introduction

Spatial sizes of hadrons are so small (about 1 fm) that direct registration of the interaction region of two colliding protons is inaccessible. From the increase of proton cross sections at high energies one can guess that this region increases as well. It was clearly demonstrated at ISR energies by Amaldi and Schubert [1] and interpreted by Henzi and Valin [2] as an indication that not only the size of the region becomes Larger at high energies, but its interior gets Blacker (less penetrable) and Edgier (steeper). Thus the BEL-regime of this hat-like shape was advocated.

It is crucial that these conclusions have been obtained just from the accurate data about elastic scattering with the help of the unitarity condition. The elastic amplitude was approximated by the square root of the differential cross section with a positive sign. The differential cross section decreases steeply at small transferred momenta such that this region contributes mostly to the elastic cross section. It was recently noticed that the elastic cross section increases with energy faster (see, e.g., the Table 2 in the review paper [3]) than the total cross section from ISR to LHC energies (i.e., from 10 GeV to 13 TeV). These features predetermined numerous speculations [4–22] that the BEL-regime may evolve at higher LHC energies to the toroidal hat with diminishing blackness ("hollow") at the center. The direct experimental

data were used analogously to [1] or their approximations as in [2].

In this paper, the shapes of the proton interaction region are computed at 7 and 13 TeV at different assumptions about the elastic scattering amplitude. It is shown that the choice between the two described possibilities depends on these assumptions. Among them the most important one is about the sign of the imaginary part of the elastic scattering amplitude at large transferred momenta. Unfortunately, no data about it exists now. Therefore, one is tempted to use phenomenological models of the process.

2 The unitarity condition

From the theoretical side, the most reliable information comes from the rigorous statement of the quantum field theory named the unitarity condition. The unitarity of the S-matrix $SS^+ = 1$ relates the amplitude of elastic scattering $f(s, t)$ to the amplitudes of inelastic processes M_n with n particles produced. In the s-channel they are subject to the integral relation (for more details see, e.g., [3,23]) which can be written symbolically as

$$
\text{Im} f(s, t) = I_2(s, t) + g(s, t)
$$
$$
= \int d\Phi_2 f f^* + \sum_n \int d\Phi_n M_n M_n^*. \quad (1)
$$

The variables s and t are the squared energy and transferred momentum of colliding protons in the center of mass system $s = 4E^2 = 4(p^2 + m^2)$, $-t = 2p^2(1 - \cos\theta)$ at the scattering angle θ. The non-linear integral term represents the two-particle intermediate states of the incoming particles. The second term represents the shadowing contribution of inelastic processes to the imaginary part of the elastic scattering amplitude. Following [24] it is called the overlap function. This terminology is ascribed to it because the integral there defines the overlap within the corresponding phase space $d\Phi_n$ between the matrix element M_n of the n-th inelas-

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tic channel and its conjugated counterpart with the collision axis of initial particles deflected by an angle θ in proton elastic scattering. It is positive at $\theta = 0$ but can change sign at $\theta \neq 0$ due to the relative phases of inelastic matrix elements M_n's.

At $t = 0$ it leads to the optical theorem

$$\text{Im} f(s, 0) = \sigma_{tot}/4\sqrt{\pi} \tag{2}$$

and to the general statement that the total cross section is the sum of cross sections of elastic and inelastic processes

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel}, \tag{3}$$

i.e., that the total probability of all processes is equal to one.

It is possible to study the space structure of the interaction region of colliding protons using information about their elastic scattering within the unitarity condition. The whole procedure is simplified because in the space representation one gets an algebraic relation between the elastic and inelastic contributions to the unitarity condition in place of the more complicated non-linear integral term I_2 in Eq. (1).

To define the geometry of the collision we must express all characteristics presented by the angle θ and the transferred momentum t in terms of the transverse distance between the trajectories of the centers of the colliding protons called the impact parameter, b. This is easily carried out using the Fourier-Bessel transform of the amplitude f which retranslates the momentum data to the corresponding transverse space features and is written as

$$i\Gamma(s, b) = \frac{1}{2\sqrt{\pi}} \int_0^\infty d|t| f(s, t) J_0(b\sqrt{|t|}). \tag{4}$$

The unitarity condition in the b-representation reduces to the algebraic expression

$$G(s, b) = 2\text{Re}\Gamma(s, b) - |\Gamma(s, b)|^2. \tag{5}$$

The left-hand side (the overlap function in the b-representation) describes the transverse impact-parameter profile of inelastic collisions of protons. It is just the Fourier-Bessel transform of the overlap function g. It satisfies the inequalities $0 \leq G(s, b) \leq 1$ and determines how absorptive the interaction region is, depending on the impact parameter (with $G = 1$ for full absorption and $G = 0$ for complete transparency). The profile of elastic processes is determined by the subtrahend in Eq. (5). If $G(s, b)$ is integrated over all impact parameters, it leads to the cross section for inelastic processes. The terms on the right-hand side would produce the total cross section and the elastic cross section, correspondingly, as should be the case according to Eq. (3). The overlap function is often discussed in relation

with the opacity (or the eikonal phase) $\Omega(s, b)$ such that $G(s, b) = 1 - \exp(-\Omega(s, b))$. Thus, full absorption corresponds to $\Omega = \infty$ and complete transparency to $\Omega = 0$.

3 Shapes of the interaction region

To calculate the contribution of different t-regions to the unitarity condition (5) one must know, in principle, the behavior of the real and imaginary parts of the elastic scattering amplitude f (or its modulus and phase) in the whole interval of the transferred momenta. This is inaccessible now and one has to rely on experimental data and some assumptions.

In experiment, the modulus $|f(s, t)|$ is given by the shape of the differential cross section $d\sigma/dt$

$$\frac{d\sigma}{dt} = |f(s, t)|^2 \equiv (\text{Re} f(s, t))^2 + (\text{Im} f(s, t))^2. \tag{6}$$

The phase of the amplitude is much less known. At a different level of confidence it can be estimated at some values of the transferred momentum. The ratio $\rho_0(s)$ of the real to imaginary parts at near forward scattering is extracted from the interference of the nuclear and Coulomb amplitudes at various energies s:

$$\rho_0(s) = \frac{\text{Re} f(s, 0)}{\text{Im} f(s, 0)}. \tag{7}$$

From the optical theorem one gets $\text{Im} f(s, 0) > 0$. Some statements can be gained from the dispersion relations about the energy behavior of ρ_0. At LHC energies it must be positive and rather small (about 0.1–0.15) [25,26]. It is confirmed by experiment. Thus $\text{Re} f(s, 0) > 0$ also.

According to rather well founded theoretical predictions [27,28] this ratio (i.e. the real part) must become zero (or the phase equals $\pi/2$) at comparatively small transferred momenta inside the diffraction cone. The fits of $d\sigma/dt \propto \exp(Bt)$ at rather small transferred momenta show approximately exponential decrease with the slope B. It looks as if the real part is small, falls down steeper than the imaginary part and changes sign inside the diffraction cone. Namely this region contributes mostly to the elastic cross section because at larger transferred momenta $d\sigma/dt$ becomes many orders of magnitude smaller.

The integral contribution of the real part to the unitarity condition should be small (see estimates in [29] and Fig. 2 below) and is neglected in what follows. Then the unitarity condition is written as

$$G(s, b) = \zeta(s, b)(2 - \zeta(s, b))$$
$$= \text{Re}\Gamma(s, b)(2 - \text{Re}\Gamma(s, b)). \tag{8}$$

Thus, according to Eq. (4), the shape of $G(s, b)$ is determined by the integral contribution over all transferred momenta
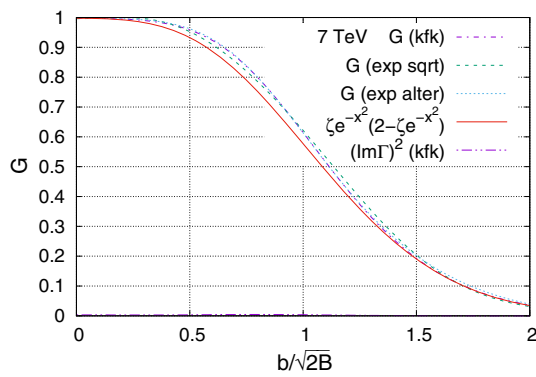
Fig. 1 The transverse impact-parameter profile (G) of inelastic collisions of protons at 7 TeV in different assumptions: kfk-model (dash-dotted line), $\mathrm{Im}\,f(s,t) = \sqrt{d\sigma/dt}$ calculated from experimental data (dash line), $\mathrm{Im}\,f(s,t) = \sqrt{d\sigma/dt}$ inside the diffraction cone and $\mathrm{Im}\,f(s,t) = -\sqrt{d\sigma/dt}$ outside cone also calculated from experimental data (dotted line), shape $\zeta \exp(-x^2)(2-\zeta \exp(-x^2))$ with $\zeta = 0.95058$ (solid line). Also it is shown square of imaginary part of Γ for kfk-model (dash-double dot line, very small)

from the imaginary part of the elastic amplitude. The sign of $\mathrm{Im}\,f$ can not be determined from $d\sigma/dt$. The peculiar t-dependence of the differential cross section can help us in choosing different possible alternatives. Its steep exponential decrease with t turns out to a minimum, slight increase and further exponential (with a smaller slope!) fall off at much lower values of $d\sigma/dt$. It is speculated (and supported by the phenomenological model [30]) that the minimum results from the imaginary part becoming zero near its position and changing sign to negative at larger $|t|$. The phase is equal $3\pi/2$ if the real part is still negative there.

Thus we compare two assumptions about $\mathrm{Im}\,f(s,t)$: 1) it is given either by $+\sqrt{d\sigma/dt}$ at all t or 2) is positive inside the cone and negative $-\sqrt{d\sigma/dt}$ outside it (at $|t| > |t_0|$, where t_0 is the minimum position). The shapes of the interaction region have been computed for these two assumptions with spline interpolation of experimental data for $d\sigma/dt$ used[1]. They are shown in Figs. 1 and 2 for 7 and 13 TeV.

It is clearly seen (Fig. 3) that the assumption about the everywhere positive imaginary part leads to the dip for central collisions at $b = 0$ (especially noticed at 13 TeV), i.e. to the toroid. If the imaginary part becomes negative at large transferred momenta, no dip appears and the BEL-shape is recovered.

The assumption of the positive imaginary part at all t has just lead to the earlier speculation about the toroidal shape of the interaction region (see the review paper [16]). In particular, this conclusion was supported if the exponential form of the imaginary part in the diffraction cone with experimental

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[1] We have used so called Akima spline [31] between experimental points and $\exp(-B|t|)$ with $B = 19.89$ at 7 TeV and $B = 20.36$ at 13 TeV from $t = 0$ to the first experimental point
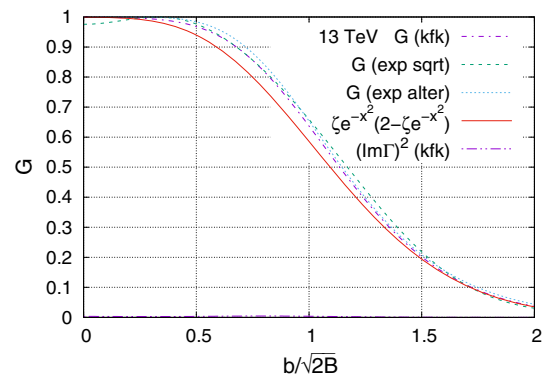
Fig. 2 The same as Fig. 1 but at 13 TeV and with $\zeta = 0.96906$
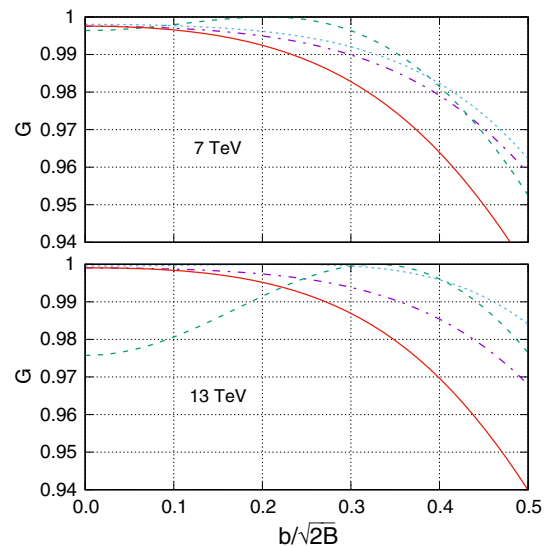
Fig. 3 Area near $b = 0$ of Figs. 1 and 2 in more details

values for its slope B was extended to all transferred momenta [12,13] in place of using its experimental form at large $|t|$. Then the positive exponential tail of the elastic amplitude with rather large slope $B/2$ provides the slight dip at $b = 0$. Albeit it is much smaller than for our first variant due to the lower tail and can not be resolved at the scale of Figs. 1 and 2 where it is shown as $\zeta \exp(-x^2)(2 - \zeta \exp(-x^2))$. For that case, the shrinkage of the diffraction cone at high energies (the energy increase of the slope B) is directly related to the energy increase of the ratio of elastic to total cross section ($B \approx \sigma_{el}/16\pi \sigma_{tot}^2$).

Another shape of the interaction region of the BEL-type follows from the so called kfk-model [30]. This phenomenological model uses some QCD inspired ideas and predicts the real and imaginary parts of the amplitude with parameters derived from precise fits of measured cross sections. The imaginary part possesses zero near the minimum of the differential cross section (which is filled by a small real part) and is negative at large transferred momenta. It reminds our

second variant. Therefore $G(s, b)$ has no dip at the center $b = 0$. It is also demonstrated in Figs. 1 and 2. Some analysis of the kfk-model showing its "anatomy" was done in [29]. It is possible to verify our assumption about smallness of the real part of the amplitude for this model. The real part of the amplitude has been computed at 7 and 13 TeV. Its contribution to the shape of $G(s, b)$ in Eq. (5) happens to be extremely small (within the limits of experimental accuracy) and can be neglected (see Fig. 2).

Quite special feature of $G(s, b)$ at ISR energies was noticed in [1] where genuine experimental data were used. At its tail of large impact parameters from 2 fm to 2.5 fm a slight bump was observed. No bump was obtained in [2] where some interpolation of the data was used. Our results in Figs. 1 and 2 do not show any indication on such a bump. The corresponding values of $b = 2\sqrt{2B} \approx 2.5$ fm are similar to those in [1].

4 Conclusion

According to Eqs. (4, 5), the spatial shape of the proton interaction region is determined by the integrals of the elastic scattering amplitude over all transferred momenta. The knowledge of its modulus obtainable from measurable differential cross sections is not enough to compute them. The prescription $\text{Im} f \approx |f|$ leads to the toroidal shape at the highest LHC energies, while the negative values of $\text{Im} f$ at large $|t|$ can recover the BEL-regime.

Thus, the problem of the spatial shape of the proton interaction region can not be solved rigorously unless the behavior (and the sign of the imaginary part!) of the elastic scattering amplitude is known. Unfortunately, there seem to be no ways to get precise information about it now. Therefore, one has to rely on "reasonable" speculations and phenomenological models confronted to a wide spectrum of experimental data.

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