Lowest vector tetraquark states: $Y(4260/4220)$ or $Z_c(4100)$

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Abstract In this article, we take the $Y(4260/4220)$ as the vector tetraquark state with $J^{PC} = 1^{--}$, and construct the $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma_5 C$ type diquark-antidiquark current to study its mass and pole residue with the QCD sum rules in details by taking into account the vacuum condensates up to dimension 10 in a consistent way. The predicted mass $M_Y = 4.24 \pm 0.10$ GeV is in excellent agreement with experimental data and supports assigning the $Y(4260/4220)$ to be the $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma_5 C$ type vector tetraquark state, and disfavors assigning the $Z_c(4100)$ to be the $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma_5 C$ type vector tetraquark state. It is the first time that the QCD sum rules have reproduced the mass of the $Y(4260/4220)$ as a vector tetraquark state.

1 Introduction

In 2005, the BaBar collaboration observed the $Y(4260)$ in the $\pi^+\pi^-J/\psi$ mass spectrum in the initial-state radiation process $e^+e^- \rightarrow \gamma_{ISR}\pi^+\pi^-J/\psi$ [1]. Then the $Y(4260)$ was confirmed by the Belle and CLEO collaborations [2, 3]. There have been several possible assignments for the $Y(4260)$ since its observation, such as the tetraquark state [4–11], hybrid states [12–15], hadro-charmonium state [16], molecular state [17, 18], kinematical effect [19–21], baryonium state [23], etc.

In 2014, the BES collaboration observed a resonance in the $\omega\chi_{c0}$ cross section in the processes $e^+e^- \rightarrow \omega\chi_{c0}/c1/c2$, the measured mass and width are $4230 \pm 8 \pm 6$ MeV and $38 \pm 12 \pm 2$ MeV, respectively [24]. In 2016, the BES collaboration observed the $Y(4220)$ and $Y(4390)$ in the process $e^+e^- \rightarrow \pi^+\pi^-h_c$, the measured masses and widths are $M_{Y(4220)} = 4218.4 \pm 4.0 \pm 0.9$ MeV, $M_{Y(4390)} = 4391.6 \pm 6.3 \pm 1.0$ MeV, $\Gamma_{Y(4220)} = 66.0 \pm 9.0 \pm 0.4$ MeV and $\Gamma_{Y(4390)} = 139.5 \pm 16.1 \pm 0.6$ MeV, respectively [25].

Also in 2016, the BES collaboration observed the $Y(4220)$ and $Y(4320)$ by precisely measuring the cross section of the process $e^+e^- \rightarrow \pi^+\pi^-J/\psi$, the measured masses and widths are $M_{Y(4220)} = 4222.0 \pm 3.1 \pm 1.4$ MeV, $M_{Y(4320)} = 4320.0 \pm 10.4 \pm 7.0$ MeV, $\Gamma_{Y(4220)} = 44.1 \pm 4.3 \pm 2.0$ MeV and $\Gamma_{Y(4320)} = 101.4^{+25.3}_{-19.7} \pm 10.2$ MeV, respectively [26]. The $Y(4260)$ and $Y(4220)$ may be the same particle, while the $Y(4360)$ and $Y(4320)$ may be the same particle according to the analogous masses and widths.

In Ref. [4], L. Maiani et al assign the $Y(4260)$ to be the diquark-antidiquark type tetraquark state with the angular momentum $L = 1$ based on the effective Hamiltonian with the spin-spin and spin-orbit interactions. In the type-II diquark-antidiquark model [5], where the spin-spin interactions between the quarks and antiquarks are neglected, L. Maiani et al interpret the $Y(4008)$, $Y(4260)$, $Y(4290/4220)$ and $Y(4630)$ as the four ground states with $L = 1$. By incorporating the dominant spin-spin, spin-orbit and tensor interactions, A. Ali et al observe that the preferred assignments of the ground state tetraquark states with $L = 1$ are the $Y(4220)$, $Y(4330)$, $Y(4390)$, $Y(4660)$ rather than the $Y(4008)$, $Y(4260)$, $Y(4360)$, $Y(4660)$ [6]. The QCD sum rules can reproduce the experimental values of the masses of the $Y(4360)$ and $Y(4660)$ in the scenario of the tetraquark states [8–11, 27–30].

The diquarks $\varepsilon^{ijk}q_j^T C\Gamma q'_k$ have five structures in Dirac spinor space, where $C\Gamma = C\gamma_5, C, C\gamma_\mu\gamma_5, C\gamma_\mu$ and $C\sigma_{\mu\nu}$ for the scalar, pseudoscalar, vector, axialvector and tensor diquarks, respectively, the i, j, k are color indexes. The attractive interactions of one-gluon exchange favor formation of the diquarks in color antitriplet, flavor antitriplet and spin singlet [31, 32], while the favored configurations are the scalar ($C\gamma_5$) and axialvector ($C\gamma_\mu$) diquark states based on the QCD sum rules [33–37]. We can take the $C\gamma_5$ and $C\gamma_\mu$ diquark states as basic constituents to construct the scalar and axialvector tetraquark states [39–43]. In the non-relativistic quark models, we have to introduce additional P-waves explicitly to study the vector tetraquark states, while in

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the quantum field theory, we can also take other diquark states (C , $C\gamma_\mu\gamma_5$ and $C\sigma_{\mu\nu}$) as basic constituents without introducing the explicit P-waves to study the vector tetraquark states [8–11, 27, 28, 44–46]. However, up to now, the QCD sum rules cannot reproduce the experimental value of the mass of the $Y(4260/4220)$ in the scenario of the tetraquark state [8–11, 27–30]. We often obtain much larger mass than the $M_{Y(4260/4220)}$.

The net effects of the relative P-waves between the heavy (anti)quarks and light (anti)quarks in the heavy (anti)diquarks are embodied in the underlined γ_5 in the $C\gamma_5\gamma_5 \otimes \gamma_\mu C$ type and $C\gamma_5 \otimes \gamma_5\gamma_\mu C$ type currents or in the underlined γ^α in the $C\gamma_\alpha\gamma^\alpha \otimes \gamma_\mu C$ type currents [30]. If we introduce the relative P-waves between the heavy (anti)quarks and light (anti)quarks in the heavy (anti)diquarks explicitly, we can obtain the $\overleftrightarrow{\partial}_\mu C\gamma_5 \otimes \gamma_5 C$ type, $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \gamma_5 C$ type, $\overleftrightarrow{\partial}_\mu C\gamma_\alpha \otimes \gamma^\alpha C$ type or $C\gamma_\alpha \otimes \overleftrightarrow{\partial}_\mu \gamma^\alpha C$ type vector currents, for example, $\varepsilon^{ijk}u^{Tj}(x)\overleftrightarrow{\partial}_\mu C\gamma_5 c^k(x)\varepsilon^{imn}\bar{d}^m(x)\gamma_5 C\bar{c}^{Tn}(x)$, $\varepsilon^{ijk}u^{Tj}(x)C\gamma_5 c^k(x)\varepsilon^{imn}\bar{d}^m(x)\overleftrightarrow{\partial}_\mu \gamma_5 C\bar{c}^{Tn}(x)$, where $\overleftrightarrow{\partial}_\mu = \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu$. On the other hand, we can introduce the relative P-waves between diquark and antidiquark explicitly and construct the $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma_5 C$ type and $C\gamma_\alpha \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma^\alpha C$ type currents to interpolate the vector tetraquark states [47], for example, $\varepsilon^{ijk}u^{Tj}(x)C\gamma_5 c^k(x)\overleftrightarrow{\partial}_\mu \varepsilon^{imn}\bar{d}^m(x)\gamma_5 C\bar{c}^{Tn}(x)$.

The masses of the $C\gamma_5\gamma_5 \otimes \gamma_\mu C$ type, $C\gamma_5 \otimes \gamma_5\gamma_\mu C$, $C\gamma_\alpha\gamma^\alpha \otimes \gamma_\mu C$ type, $\overleftrightarrow{\partial}_\mu C\gamma_5 \otimes \gamma_5 C$ type, $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \gamma_5 C$ type, $\overleftrightarrow{\partial}_\mu C\gamma_\alpha \otimes \gamma^\alpha C$ type and $C\gamma_\alpha \otimes \overleftrightarrow{\partial}_\mu \gamma^\alpha C$ type vector tetraquark states maybe differ from the masses of the $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma_5 C$ type and $C\gamma_\alpha \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma^\alpha C$ type vector tetraquark states greatly. In Refs. [8, 9], Zhang and Huang construct the $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma_5 C$ type and $C\gamma_\alpha \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma^\alpha C$ type vector interpolating currents, which have no definite charge conjugation, and study the vector tetraquark states with the QCD sum rules by taking into account the vacuum condensates up to dimension 6 in the operator product expansion, and obtain the masses 4.32 GeV and 4.69 GeV for the $Y(4360)$ and $Y(4660)$ respectively.

In this article, we take the $Y(4260/4220)$ as the vector tetraquark state with the $J^{PC} = 1^{--}$, and construct the $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma_5 C$ type current to study its mass and pole residue with the QCD sum rules in details by taking into account the vacuum condensates up to dimension 10 in a consistent way in the operator product expansion, and use the energy scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_c)^2}$ with the effective c -quark mass $\mathbb{M}_c$ to determine the optimal energy scale of the QCD spectral density [27, 39–43, 48, 49].

Recently, the LHCb collaboration observed evidence for the $\eta_c\pi^-$ resonant state $Z_c(4100)$ with the significance of

more than three standard deviations in a Dalitz plot analysis of the $B^0 \rightarrow \eta_c K^+ \pi^-$ decays, the measured mass and width are $M_{Z_c} = 4096 \pm 20^{+18}_{-22}$ MeV and $\Gamma_{Z_c} = 152 \pm 58^{+60}_{-35}$ MeV respectively [50]. The spin-parity assignments $J^P = 0^+$ and 1^- are both consistent with the experimental data. It is interesting to see which is the lowest vector tetraquark state, the $Y(4260/4220)$ or the $Z_c(4100)$?

The article is arranged as follows: we derive the QCD sum rules for the mass and pole residue of the vector tetraquark state $Y(4260/4220)$ in Sect. 2; in Sect. 3, we present the numerical results and discussions; Sect. 4 is reserved for our conclusion.

2 QCD sum rules for the vector tetraquark state $Y(4260/4220)$

In the following, we write down the two-point correlation function $\Pi_{\mu\nu}(p)$ in the QCD sum rules,

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T \left\{ J_\mu(x) J_\nu^\dagger(0) \right\} | 0 \rangle, \quad (1)$$

where $J_\mu(x) = J_\mu^+(x)$, $J_\mu^0(x)$ and $J_\mu^-(x)$,

$$\begin{aligned} J_\mu^+(x) &= \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}} u^{Tj}(x) C\gamma_5 c^k(x) \overleftrightarrow{\partial}_\mu \bar{d}^m(x) \gamma_5 C\bar{c}^{Tn}(x), \\ J_\mu^0(x) &= \frac{\varepsilon^{ijk}\varepsilon^{imn}}{2} \left[u^{Tj}(x) C\gamma_5 c^k(x) \overleftrightarrow{\partial}_\mu \bar{u}^m(x) \gamma_5 C\bar{c}^{Tn}(x) \right. \\ &\quad \left. \pm d^{Tj}(x) C\gamma_5 c^k(x) \overleftrightarrow{\partial}_\mu \bar{d}^m(x) \gamma_5 C\bar{c}^{Tn}(x) \right], \\ J_\mu^-(x) &= \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}} d^{Tj}(x) C\gamma_5 c^k(x) \overleftrightarrow{\partial}_\mu \bar{u}^m(x) \gamma_5 C\bar{c}^{Tn}(x), \end{aligned} \quad (2)$$

where the i, j, k, m, n are color indexes. Under charge conjugation transform $\hat{C}$, the currents $J_\mu(x)$ have the property,

$$\hat{C} J_\mu(x) \hat{C}^{-1} = -J_\mu(x). \quad (3)$$

We take the isospin limit by assuming the u and d quarks have degenerate masses, the $J_\mu(x)$ couple to the vector tetraquark states with degenerate masses. In this article, we take $J_\mu(x) = J_\mu^+(x)$.

At the hadronic side, we can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operator $J_\mu(x)$ into the correlation function $\Pi_{\mu\nu}(p)$ to obtain the hadronic representation [51–53]. After isolating the ground state contribution of the vector tetraquark state $Y(4260/4220)$, we get the result,

$$\begin{aligned} \Pi_{\mu\nu}(p) &= \frac{\lambda_Y^2}{M_Y^2 - p^2} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \cdots, \\ &= \Pi(p^2) \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \Pi_0(p^2) \frac{p_\mu p_\nu}{p^2}, \end{aligned} \quad (4)$$

where the pole residue λ_Y is defined by $\langle 0|J_\mu(0)|Y(p)\rangle = \lambda_Y \varepsilon_\mu$, the ε_μ is the polarization vector of the vector tetraquark state $Y(4260/4220)$. The vector and scalar tetraquark states contribute to the components $\Pi(p^2)$ and $\Pi_0(p^2)$, respectively. In this article, we choose the tensor structure $-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}$ for analysis, the scalar tetraquark states have no contaminations.

Now we briefly outline the operator product expansion for the correlation function $\Pi_{\mu\nu}(p)$ in perturbative QCD. We contract the u , d and c quark fields in the correlation function $\Pi_{\mu\nu}(p)$ with Wick theorem, obtain the result:

$$\begin{aligned} \Pi_{\mu\nu}(p) = & -\frac{i\varepsilon^{ijk}\varepsilon^{lmn}\varepsilon^{i'j'k'}\varepsilon^{l'm'n'}}{2} \int d^4x e^{ip\cdot x} \\ & \left\{ \text{Tr} \left[\gamma_5 C^{kk'}(x) \gamma_5 C S^{jj'T}(x) C \right] \partial_\mu \partial_\nu \right. \\ & \text{Tr} \left[\gamma_5 C^{n'n}(-x) \gamma_5 C S^{m'mT}(-x) C \right] \\ & - \partial_\mu \text{Tr} \left[\gamma_5 C^{kk'}(x) \gamma_5 C S^{jj'T}(x) C \right] \partial_\nu \\ & \text{Tr} \left[\gamma_5 C^{n'n}(-x) \gamma_5 C S^{m'mT}(-x) C \right] \\ & - \partial_\nu \text{Tr} \left[\gamma_5 C^{kk'}(x) \gamma_5 C S^{jj'T}(x) C \right] \partial_\mu \\ & \text{Tr} \left[\gamma_5 C^{n'n}(-x) \gamma_5 C S^{m'mT}(-x) C \right] \\ & + \partial_\mu \partial_\nu \text{Tr} \left[\gamma_5 C^{kk'}(x) \gamma_5 C S^{jj'T}(x) C \right] \\ & \left. \text{Tr} \left[\gamma_5 C^{n'n}(-x) \gamma_5 C S^{m'mT}(-x) C \right] \right\}, \end{aligned} \quad (5)$$

where the $S_{ij}(x)$ and $C_{ij}(x)$ are the full u/d and c quark propagators respectively,

$$\begin{aligned} S_{ij}(x) = & \frac{i\delta_{ij}\not{x}}{2\pi^2 x^4} - \frac{\delta_{ij}\langle \bar{q}q \rangle}{12} - \frac{\delta_{ij}x^2 \langle \bar{s}s \sigma G s \rangle}{192} \\ & - \frac{ig_s G_{\alpha\beta}^a t_{ij}^a (\not{x} \sigma^{\alpha\beta} + \sigma^{\alpha\beta} \not{x})}{32\pi^2 x^2} \\ & - \frac{\delta_{ij}x^4 \langle \bar{q}q \rangle \langle g_s^2 GG \rangle}{27648} - \frac{1}{8} \langle \bar{q}_j \sigma^{\mu\nu} q_i \rangle \sigma_{\mu\nu} \\ & + \dots, \end{aligned} \quad (6)$$

$$\begin{aligned} C_{ij}(x) = & \frac{i}{(2\pi)^4} \int d^4k e^{-ik\cdot x} \left\{ \frac{\delta_{ij}}{\not{k} - m_c} \right. \\ & \frac{g_s G_{\alpha\beta}^n t_{ij}^n}{4} \frac{\sigma^{\alpha\beta} (\not{k} + m_c) + (\not{k} + m_c) \sigma^{\alpha\beta}}{(k^2 - m_c^2)^2} \\ & - \frac{g_s^2 (t^a t^b)_{ij} G_{\alpha\beta}^a G_{\mu\nu}^b (f^{\alpha\beta\mu\nu} + f^{\alpha\mu\beta\nu} + f^{\alpha\nu\mu\beta})}{4(k^2 - m_c^2)^5} + \dots \left. \right\}, \\ & f^{\lambda\alpha\beta} = (\not{k} + m_c) \gamma^\lambda (\not{k} + m_c) \gamma^\alpha (\not{k} + m_c) \gamma^\beta (\not{k} + m_c), \\ & f^{\alpha\beta\mu\nu} = (\not{k} + m_c) \gamma^\alpha (\not{k} + m_c) \gamma^\beta (\not{k} + m_c) \gamma^\mu (\not{k} + m_c) \gamma^\nu (\not{k} + m_c), \end{aligned} \quad (7)$$

and $t^n = \frac{\lambda^n}{2}$, the λ^n is the Gell-Mann matrix [53,54]. In Eq. (6), we retain the term $\langle \bar{q}_j \sigma_{\mu\nu} q_i \rangle$ originate from the Fierz rearrangement of the $\langle q_i \bar{q}_j \rangle$ to absorb the gluons emitted from

other quark lines to extract the mixed condensate $\langle \bar{q} g_s \sigma G q \rangle$ [27,39].

It is very difficult (or cumbersome) to carry out the integrals both in the coordinate and momentum spaces directly due to appearance of the partial derives ∂_μ and ∂_ν . We perform integral by parts to exclude the terms proportional to the tensor structure $\frac{p_\mu p_\nu}{p^2}$, which only contributes to the scalar tetraquark states, and simplify the correlation function $\Pi_{\mu\nu}(p)$ greatly,

$$\begin{aligned} \Pi_{\mu\nu}(p) = & 2i\varepsilon^{ijk}\varepsilon^{lmn}\varepsilon^{i'j'k'}\varepsilon^{l'm'n'} \int d^4x e^{ip\cdot x} \\ & \partial_\mu \text{Tr} \left[\gamma_5 C^{kk'}(x) \gamma_5 C S^{jj'T}(x) C \right] \partial_\nu \\ & \text{Tr} \left[\gamma_5 C^{n'n}(-x) \gamma_5 C S^{m'mT}(-x) C \right]. \end{aligned} \quad (8)$$

Then we compute the integrals both in the coordinate and momentum spaces, and obtain the correlation function $\Pi(p^2)$ therefore the spectral density at the level of quark-gluon degrees of freedom.

Once analytical expressions of the QCD spectral density are obtained, we can take the quark-hadron duality below the continuum threshold s_0 and perform Borel transform with respect to the variable $P^2 = -p^2$ to obtain the QCD sum rules:

$$\lambda_Y^2 \exp\left(-\frac{M_Y^2}{T^2}\right) = \int_{4m_c^2}^{s_0} ds \rho(s) \exp\left(-\frac{s}{T^2}\right), \quad (9)$$

where

$$\begin{aligned} \rho(s) = & \rho_0(s) + \rho_3(s) + \rho_4(s) + \rho_5(s) + \rho_6(s) + \rho_7(s) \\ & + \rho_8(s) + \rho_{10}(s), \end{aligned} \quad (10)$$

$$\begin{aligned} \rho_0(s) = & \frac{1}{61440\pi^6} \int dy dz \frac{yz(1-y-z)^4}{1-y} \\ & (s - \bar{m}_c^2)^4 \left[s - \bar{m}_c^2 - 2y(8s - 3\bar{m}_c^2) \right] \\ & - \frac{1}{12288\pi^6} \int dy dz \frac{yz^2(1-y-z)^3}{1-y} \\ & (s - \bar{m}_c^2)^4 (3s - \bar{m}_c^2) \\ & + \frac{1}{3840\pi^6} \int dy dz yz^2 (1-y-z)^3 \\ & (s - \bar{m}_c^2)^3 (18s^2 - 16s\bar{m}_c^2 + 3\bar{m}_c^4) \\ & + \frac{1}{20480\pi^6} \int dy dz yz(1-y-z)^3 \\ & (s - \bar{m}_c^2)^4 \left[s - \bar{m}_c^2 + 4y(8s - 3\bar{m}_c^2) \right], \end{aligned} \quad (11)$$

$$\begin{aligned} \rho_3(s) = & -\frac{m_c \langle \bar{q}q \rangle}{48\pi^4} \int dy dz yz(1-y-z) \\ & (s - \bar{m}_c^2)^2 \left[s - \bar{m}_c^2 + 3y(2s - \bar{m}_c^2) \right], \end{aligned} \quad (12)$$

$$\begin{aligned} \rho_4(s) = & -\frac{m_c^2}{9216\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dy dz \frac{z(1-y-z)^4}{y^2(1-y)} \\ & (s - \bar{m}_c^2) \left[s - \bar{m}_c^2 - 2y(5s - 3\bar{m}_c^2) \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{m_c^2}{9216\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz \frac{z^2(1-y-z)^3}{y^2(1-y)} \\
& \quad (s - \bar{m}_c^2)(9s - 5\bar{m}_c^2) \\
& - \frac{m_c^2}{1152\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz \frac{z^2(1-y-z)^3}{y} \\
& \quad (15s^2 - 20s\bar{m}_c^2 + 6\bar{m}_c^4) \\
& - \frac{m_c^2}{1024\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz \frac{z(1-y-z)^3}{y^2} \\
& \quad (s - \bar{m}_c^2)^2 \\
& + \frac{m_c^2}{3072\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz \left(\frac{z}{y^2} + \frac{y}{z^2} \right) (1-y-z)^3 \\
& \quad (s - \bar{m}_c^2) [s - \bar{m}_c^2 - 2y(5s - 3\bar{m}_c^2)] \\
& + \frac{1}{6144\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz \frac{z(1-y-z)^3}{1-y} \\
& \quad (s - \bar{m}_c^2)^2 [s - \bar{m}_c^2 - 6y(2s - \bar{m}_c^2)] \\
& - \frac{1}{6144\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz \frac{z^2(1-y-z)^2}{1-y} \\
& \quad (s - \bar{m}_c^2)^2 (11s - 5\bar{m}_c^2) \\
& + \frac{1}{256\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz yz^2(1-y-z)^2 \\
& \quad (s - \bar{m}_c^2)(7s^2 - 8s\bar{m}_c^2 + 2\bar{m}_c^4) \\
& - \frac{1}{2048\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz z(1-y-z)^2 \\
& \quad (s - \bar{m}_c^2)^2 [s - \bar{m}_c^2 - 6y(2s - \bar{m}_c^2)], \quad (13)
\end{aligned}$$

$$\begin{aligned}
\rho_5(s) &= \frac{m_c \langle \bar{q} g_s \sigma G q \rangle}{64\pi^4} \int dydz yz \\
& \quad (s - \bar{m}_c^2) [s - \bar{m}_c^2 + y(5s - 3\bar{m}_c^2)] \\
& + \frac{m_c \langle \bar{q} g_s \sigma G q \rangle}{128\pi^4} \int dydz y(1-y-z)(1-y) \\
& \quad (s - \bar{m}_c^2)^2 \\
& - \frac{m_c \langle \bar{q} g_s \sigma G q \rangle}{128\pi^4} \int dydz y^2(1-y-z) \\
& \quad (s - \bar{m}_c^2)(9s - 5\bar{m}_c^2) \\
& - \frac{3m_c \langle \bar{q} g_s \sigma G q \rangle}{128\pi^4} \int dydz (y+z)(1-y-z) \\
& \quad (s - \bar{m}_c^2)^2 \\
& + \frac{m_c \langle \bar{q} g_s \sigma G q \rangle}{128\pi^4} \int dydz y(1-y-z) \\
& \quad (s - \bar{m}_c^2) [s - \bar{m}_c^2 - 2y(5s - 3\bar{m}_c^2)], \quad (14)
\end{aligned}$$

$$\rho_6(s) = \frac{m_c^2 \langle \bar{q} q \rangle^2}{12\pi^2} \int dy y(1-y)(s - \tilde{m}_c^2), \quad (15)$$

$$\begin{aligned}
\rho_7(s) &= \frac{m_c^3 \langle \bar{q} q \rangle}{144\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz \left(\frac{z}{y^2} + \frac{y}{z^2} \right) \\
& \quad (1-y-z) [1 + 3y + ys \delta(s - \bar{m}_c^2)] \\
& - \frac{m_c \langle \bar{q} q \rangle}{48\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz \frac{y(1-y-z)}{z}
\end{aligned}$$

$$\begin{aligned}
& [s - \bar{m}_c^2 + y(4s - 3\bar{m}_c^2)] \\
& + \frac{m_c \langle \bar{q} q \rangle}{192\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz z [3(s - \bar{m}_c^2) \\
& - 2y(4s - 3\bar{m}_c^2)] \\
& - \frac{m_c \langle \bar{q} q \rangle}{288\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz y(1-y)(s - \bar{m}_c^2) \\
& + \frac{m_c \langle \bar{q} q \rangle}{288\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dydz y^2(7s - 5\bar{m}_c^2) \\
& - \frac{m_c \langle \bar{q} q \rangle}{288\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dy y(1-y) \\
& [s - \tilde{m}_c^2 + y(4s - 3\tilde{m}_c^2)], \quad (16)
\end{aligned}$$

$$\begin{aligned}
\rho_8(s) &= -\frac{m_c^2 \langle \bar{q} q \rangle \langle \bar{q} g_s \sigma G q \rangle}{24\pi^2} \int dy y(1-y) \\
& \quad [3 + s \delta(s - \tilde{m}_c^2)] \\
& + \frac{m_c^2 \langle \bar{q} q \rangle \langle \bar{q} g_s \sigma G q \rangle}{24\pi^2} \int dy, \quad (17)
\end{aligned}$$

$$\begin{aligned}
\rho_{10}(s) &= \frac{203m_c^2 \langle \bar{q} g_s \sigma G q \rangle^2}{9216\pi^2} \int dy \delta(s - \tilde{m}_c^2) \\
& + \frac{m_c^2 \langle \bar{q} g_s \sigma G q \rangle^2}{32\pi^2} \int dy y(1-y) \\
& \quad \left(1 + \frac{2s}{3T^2} + \frac{s^2}{6T^4} \right) \delta(s - \tilde{m}_c^2) \\
& - \frac{m_c^2 \langle \bar{q} g_s \sigma G q \rangle^2}{48\pi^2} \int dy \left(1 + \frac{s}{2T^2} \right) \delta(s - \tilde{m}_c^2) \\
& - \frac{m_c^4 \langle \bar{q} q \rangle^2}{108T^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dy \frac{1-y}{y^2} \delta(s - \tilde{m}_c^2) \\
& + \frac{m_c^2 \langle \bar{q} q \rangle^2}{36} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dy \frac{1-y}{y} \delta(s - \tilde{m}_c^2) \\
& - \frac{m_c^2 \langle \bar{q} q \rangle^2}{108} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dy \left(1 + \frac{s}{2T^2} \right) \delta(s - \tilde{m}_c^2) \\
& + \frac{m_c^2 \langle \bar{q} q \rangle^2}{36} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int dy y(1-y) \\
& \quad \left(1 + \frac{2s}{3T^2} + \frac{s^2}{6T^4} \right) \delta(s - \tilde{m}_c^2), \quad (18)
\end{aligned}$$

where $\int dydz = \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz$, $y_i = \frac{1+\sqrt{1-4\bar{m}_c^2/s}}{2}$, $y_f = \frac{1-\sqrt{1-4\bar{m}_c^2/s}}{2}$, $z_i = \frac{ym_c^2}{ys-\bar{m}_c^2}$, $\bar{m}_c^2 = \frac{(y+z)m_c^2}{yz}$, $\tilde{m}_c^2 = \frac{m_c^2}{y(1-y)}$, $\int_{y_i}^{y_f} dy \rightarrow \int_0^1 dy$, $\int_{z_i}^{1-y} dz \rightarrow \int_0^{1-y} dz$, when the δ functions $\delta(s - \bar{m}_c^2)$ and $\delta(s - \tilde{m}_c^2)$ appear.

In this article, we carry out the operator product expansion up to the vacuum condensates of dimension-10, and take into account the vacuum condensates which are vacuum expectations of the operators of the orders $\mathcal{O}(\alpha_s^k)$ with $k \leq 1$ consistently. The condensates $\langle g_s^3 GG \rangle$, $\langle \frac{\alpha_s GG}{\pi} \rangle^2$, $\langle \frac{\alpha_s GG}{\pi} \rangle \langle \bar{q} g_s \sigma G q \rangle$ have the dimensions 6, 8, 9, respectively, but they are the vacuum expectations of the operators of the order $\mathcal{O}(\alpha_s^{3/2})$, $\mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha_s^{3/2})$, respectively, and are discarded [27, 39].

We derive Eq. (9) with respect to $\tau = \frac{1}{T^2}$, then eliminate the pole residue λ_Y , and obtain the QCD sum rules for the mass of the vector tetraquark state $Y(4260/4220)$,

$$M_Y^2 = - \frac{\int_{4m_c^2}^{s_0} ds \frac{d}{d\tau} \rho(s) \exp(-\tau s)}{\int_{4m_c^2}^{s_0} ds \rho(s) \exp(-\tau s)}. \quad (19)$$

3 Numerical results and discussions

We take the standard values of the vacuum condensates $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$, $\langle \bar{q}g_s \sigma G q \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$, $\langle \frac{\alpha_s GG}{\pi} \rangle = (0.33 \text{ GeV})^4$ at the energy scale $\mu = 1 \text{ GeV}$ [51–53, 55], and choose the $\overline{MS}$ mass $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$ from the Particle Data Group [56], and set $m_u = m_d = 0$. Moreover, we take into account the energy-scale dependence of the input parameters on the QCD side,

$$\begin{aligned} \langle \bar{q}q \rangle(\mu) &= \langle \bar{q}q \rangle(Q) \left[\frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{12}{25}}, \\ \langle \bar{q}g_s \sigma G q \rangle(\mu) &= \langle \bar{q}g_s \sigma G q \rangle(Q) \left[\frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{2}{25}}, \\ m_c(\mu) &= m_c(m_c) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{25}}, \\ \alpha_s(\mu) &= \frac{1}{b_0 t} \left[1 - \frac{b_1 \log t}{b_0^2 t} + \frac{b_1^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right], \end{aligned} \quad (20)$$

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2}{128\pi^3}$, $\Lambda = 210 \text{ MeV}$, 292 MeV and 332 MeV for the flavors $n_f = 5, 4$ and 3 , respectively [56–58], and evolve all the input parameters to the optimal energy scale μ to extract the mass of the vector tetraquark state $Y(4260/4220)$.

In this article, we search for the ideal Borel parameter T^2 and continuum threshold parameter s_0 to satisfy the following four criteria:

1. Pole dominance at the phenomenological side;
2. Convergence of the operator product expansion;
3. Appearance of the Borel platforms;
4. Satisfying the energy scale formula, using try and error.

In the four-quark system $q\bar{q}'Q\bar{Q}$, the Q -quark serves as a static well potential and combines with the light quark q to form a heavy diquark $\mathcal{D}$ in color antitriplet or combines with the light antiquark $\bar{q}'$ to form a heavy meson-like state or correlation (not a physical meson) in color singlet, while the $\bar{Q}$ -quark serves as another static well potential and

combines with the light antiquark $\bar{q}'$ to form a heavy anti-diquark $\bar{\mathcal{D}}$ in color triplet or combines with the light quark state q to form another heavy meson-like state or correlation (not a physical meson) in color singlet [27, 40–43, 48, 49]. Then the $\mathcal{D}$ and $\bar{\mathcal{D}}$ combine with together to form a compact tetraquark state, the two meson-like states (not two physical mesons) combine together to form a physical molecular state [27, 40–43, 48, 49], the two heavy quarks Q and $\bar{Q}$ stabilize the tetraquark state [7]. The tetraquark states $\mathcal{D}\bar{\mathcal{D}}$ are characterized by the effective heavy quark masses $\mathbb{M}_Q$ and the virtuality $V = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_Q)^2}$. It is natural to take the energy scale $\mu = V = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_Q)^2}$ [27, 40–43, 48, 49]. We cannot obtain energy scale independent QCD sum rules, but we have an energy scale formula to determine the energy scales consistently, which works well even for the hidden-charm pentaquark states [59, 60], the updated value $\mathbb{M}_c = 1.82 \text{ GeV}$ [11].

In Refs. [27, 39–43], we study the hidden-charm or hidden-bottom tetraquark states, the heavy diquarks and heavy antidiquarks are in relative S-wave, if there exist relative P-waves, the P-waves lie in between the heavy (anti)quark and light (anti)quark in the heavy (anti)diquark. In the present work, we study the vector tetraquark state which has a relative P-wave between the charmed diquark and charmed antidiquark. If a relative P-wave costs about 0.5 GeV , then the energy scale formula is modified to be

$$\mu = \sqrt{M_Y^2 - (2\mathbb{M}_c + 0.5 \text{ GeV})^2} = \sqrt{M_Y^2 - (4.1 \text{ GeV})^2}. \quad (21)$$

In calculations, we observe that if we take the continuum threshold parameter $\sqrt{s_0} = 4.8 \pm 0.1 \text{ GeV}$, Borel parameter $T^2 = (2.2 - 2.8) \text{ GeV}^2$, energy scale $\mu = 1.1 \text{ GeV}$, the pole contribution of the ground state vector tetraquark state $Y(4260/4220)$ is about $(49 - 81)\%$, the predicted mass is about $M_Y = 4.24 \text{ GeV}$, the modified energy scale formula is well satisfied.

In Fig. 1, we plot the pole contribution with variation of the Borel parameter, from the figure, we can see that the pole contribution decreases monotonously with increase of the Borel parameter, the pole contribution reaches about 50% at the point $T^2 = 2.8 \text{ GeV}^2$ and $\sqrt{s_0} = 4.7 \text{ GeV}$, we can obtain the upper bound $T_{max}^2 = 2.8 \text{ GeV}^2$. In Fig. 2, we plot the contributions of the vacuum condensates of dimension n in the operator product expansion, which are defined by

$$D(n) = \frac{\int_{4m_c^2}^{s_0} ds \rho_n(s) \exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{s_0} ds \rho(s) \exp\left(-\frac{s}{T^2}\right)}. \quad (22)$$

From the figure, we can see that the contributions of the vacuum condensates of dimensions 3, 5, 6 and 8 are very large, and change quickly with variation of the Borel param-

Fig. 1 The pole contributions with variation of the Borel parameter T^2 , where the A , B and C denote the threshold parameters $\sqrt{s_0} = 4.7$ GeV, 4.8 GeV and 4.9 GeV, respectively

Fig. 2 The contributions of the vacuum condensates of dimension n with variation of the Borel parameter T^2 for the threshold parameter $\sqrt{s_0} = 4.8$ GeV

eter T^2 at the region $T^2 < 2.2 \text{ GeV}^2$, the operator product expansion is not convergent, we can obtain the lower bound $T_{min}^2 = 2.2 \text{ GeV}^2$. At the region $T^2 \geq 2.2 \text{ GeV}^2$, the contribution of the vacuum condensate of dimension $n = 3$ is large, but the contributions of the vacuum condensates of dimensions 3, 5, 6, 8 have the hierarchy $D(3) \gg |D(5)| \sim D(6) \gg |D(8)|$, the contributions of the vacuum condensates of the dimensions 4, 7, 10 are tiny, the operator product expansion is convergent. The Borel window is $T^2 = (2.2 - 2.8) \text{ GeV}^2$, where operator product expansion is well convergent.

We take into account all uncertainties of the input parameters, and obtain the values of the mass and pole residue of the vector tetraquark state $Y(4260/4220)$, which are shown explicitly in Figs. 3, 4,

$$\begin{aligned} M_Y &= 4.24 \pm 0.10 \text{ GeV}, \\ \lambda_Y &= (2.31 \pm 0.45) \times 10^{-2} \text{ GeV}^6. \end{aligned} \quad (23)$$

Fig. 3 The mass of the $Y(4260/4220)$ as vector tetraquark state with variation of the Borel parameter T^2

Fig. 4 The pole residue of the $Y(4260/4220)$ as vector tetraquark state with variation of the Borel parameter T^2

From Figs. 3, 4, we can see that there appear platforms in the Borel window. Now the four criteria of the QCD sum rules are all satisfied, and we expect to make reliable predictions.

The predicted mass $M_Y = 4.24 \pm 0.10 \text{ GeV}$ is in excellent agreement with the experimental value $M_{Y(4220)} = 4222.0 \pm 3.1 \pm 1.4 \text{ MeV}$ from the BESIII collaboration [26], or the experimental value $M_{Y(4260)} = 4230.0 \pm 8.0 \text{ MeV}$ from Particle Data Group [56], which supports assigning the $Y(4260/4220)$ to be the $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma_5 C$ type vector tetraquark state. The average value of the width of the $Y(4260)$ is $55 \pm 19 \text{ MeV}$, the relative P-wave between the diquark and antidiquark disfavors rearrangement of the quarks to form meson pairs, which can account for the small width.

From Fig. 3, we can see that the mass $M_{Z_c(4100)}$ lies below the lower bound of the predicted mass of the $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma_5 C$ type vector tetraquark state $c\bar{c}q\bar{q}$, which disfavors assigning the $Z_c(4100)$ to be the $C\gamma_5 \otimes \overleftrightarrow{\partial}_\mu \otimes \gamma_5 C$ type vector tetraquark state $c\bar{c}q\bar{q}$.

In Refs. [61, 62], we study the $C\gamma_\mu \otimes \gamma^\mu C$ -type, $C\gamma_\mu \gamma_5 \otimes \gamma_5 \gamma^\mu C$ -type, $C\gamma_5 \otimes \gamma_5 C$ -type, $C \otimes C$ -type $c\bar{s}\bar{s}$ scalar

tetraquark states with the QCD sum rules in a systematic way, and obtain the predictions $M_{C\gamma_\mu \otimes \gamma^\mu C} = 3.92^{+0.19}_{-0.18}$ GeV and $M_{C\gamma_5 \otimes \gamma_5 C} = 3.89 \pm 0.05$ GeV, which support assigning the $X(3915)$ to be the $C\gamma_\mu \otimes \gamma^\mu C$ -type or $C\gamma_5 \otimes \gamma_5 C$ -type $c\bar{s}\bar{c}\bar{s}$ scalar tetraquark state. In fact, the $SU(3)$ breaking effects of the masses of the $c\bar{s}\bar{c}\bar{s}$ and $cq\bar{c}\bar{q}$ tetraquark states from the QCD sum rules are rather small, if the scalar tetraquark state $cq\bar{c}\bar{q}$ has the mass $M_{C\gamma_\mu \otimes \gamma^\mu C} = 3.92^{+0.19}_{-0.18}$ GeV, which is compatible with the LHCb data $M_{Z_c} = 4096 \pm 20^{+18}_{-22}$ MeV and $\Gamma_{Z_c} = 152 \pm 58^{+60}_{-35}$ MeV considering the uncertainties [50], and favors assigning the $Z_c(4100)$ to be the $C\gamma_\mu \otimes \gamma^\mu C$ -type scalar tetraquark state.

In Ref. [30], we choose the $C\otimes\gamma_\mu C$ type and $C\gamma_5\otimes\gamma_5\gamma_\mu C$ type vector currents to study the vector tetraquark states, the net effects of the relative P-waves are embodied in the underlined γ_5 in the $C\gamma_5\gamma_5\otimes\gamma_\mu C$ type and $C\gamma_5\otimes\gamma_5\gamma_\mu C$ type currents or in the underlined γ^α in the $C\gamma_\alpha\gamma^\alpha\otimes\gamma_\mu C$ type currents, and obtain the masses $M_{C\otimes\gamma_\mu C} = 4.59 \pm 0.08$ GeV and $M_{C\gamma_5\otimes\gamma_5\gamma_\mu C} = 4.34 \pm 0.08$ GeV. The $C\otimes\gamma_\mu C$ type tetraquark states have larger masses than the corresponding $C\gamma_5\otimes\gamma_5\gamma_\mu C$ type tetraquark states, as $C\otimes\gamma_\mu C = [C\gamma_5\gamma_5\otimes\gamma_\mu C] \oplus [C\gamma_\alpha\gamma^\alpha\otimes\gamma_\mu C]$ and $C\gamma_5\otimes\gamma_5\gamma_\mu C = C\gamma_5\otimes\gamma_5\gamma_\mu C$, the $C\gamma_\mu$ diquark states have slightly larger masses than the corresponding $C\gamma_5$ diquark states from the QCD sum rules [33–35]. The vector tetraquark masses $M_{C\otimes\gamma_\mu C}$ and $M_{C\gamma_5\otimes\gamma_5\gamma_\mu C}$ differ from the vector tetraquark mass $M_{C\gamma_5\otimes\vec{\partial}_\mu\otimes\gamma_5 C}$ greatly. For the conventional ground state $c\bar{q}$ mesons, the energy gaps between the S-wave and P-wave states are about 0.5 GeV, if the relative P-waves between the q -quark and c -quark in the diquark states cq cost about 0.5 GeV [56], the masses of the $\vec{\partial}_\mu C\gamma_5\otimes\gamma_5 C$ type, $C\gamma_5\otimes\vec{\partial}_\mu\gamma_5 C$ type, $\vec{\partial}_\mu C\gamma_\alpha\otimes\gamma^\alpha C$ type and $C\gamma_\alpha\otimes\vec{\partial}_\mu\gamma^\alpha C$ vector tetraquark states are estimated to be 4.4 GeV according the $C\gamma_\mu\otimes\gamma^\mu C$ -type and $C\gamma_5\otimes\gamma_5 C$ -type scalar tetraquark masses [61,62], which differs from the present prediction $M_{C\gamma_5\otimes\vec{\partial}_\mu\otimes\gamma_5 C} = 4.24 \pm 0.10$ GeV greatly. Before draw a definite conclusion, we should study the masses of the $\vec{\partial}_\mu C\gamma_5\otimes\gamma_5 C$ type, $C\gamma_5\otimes\vec{\partial}_\mu\gamma_5 C$ type, $\vec{\partial}_\mu C\gamma_\alpha\otimes\gamma^\alpha C$ type and $C\gamma_\alpha\otimes\vec{\partial}_\mu\gamma^\alpha C$ vector tetraquark states with the QCD sum rules directly, this is our next work.

4 Conclusion

In this article, we take the $Y(4260/4220)$ as the vector tetraquark state with $J^{PC} = 1^{--}$, and construct the $C\gamma_5\otimes\vec{\partial}_\mu\otimes\gamma_5 C$ type current to study its mass and pole residue with the QCD sum rules in details by taking into account the vacuum condensates up to dimension 10 in a consistent way in the operator product expansion,

and use the modified energy scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2M_c + 0.5\text{GeV})^2}$ with the effective c -quark mass M_c to determine the optimal energy scale of the QCD spectral density. The predicted mass $M_Y = 4.24 \pm 0.10$ GeV is in excellent agreement with the experimental value $M_{Y(4220)} = 4222.0 \pm 3.1 \pm 1.4$ MeV from the BESIII collaboration or the experimental value $M_{Y(4260)} = 4230.0 \pm 8.0$ MeV from Particle Data Group, and supports assigning the $Y(4260/4220)$ to be the $C\gamma_5\otimes\vec{\partial}_\mu\otimes\gamma_5 C$ type vector tetraquark state, and disfavors assigning the $Z_c(4100)$ to be the $C\gamma_5\otimes\vec{\partial}_\mu\otimes\gamma_5 C$ type vector tetraquark state. It is the first time that the QCD sum rules have reproduced the mass of the $Y(4260/4220)$ as a vector tetraquark state.

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