Muon conversion to electron in nuclei within the BLMSSM

Tao Guo^{1,2,a}, Shu-Min Zhao^{3,b}, Xing-Xing Dong^{3,c}, Chun-Gui Duan^{1,d}, Tai-Fu Feng^{3,e}

¹ Department of Physics and Hebei Advanced Thin Films Laboratory, Hebei Normal University, Shijiazhuang 050024, China

² College of Mathematics and Physics, Hebei GEO University, Shijiazhuang 050031, China

³ Department of Physics and Technology, Hebei University, Baoding 071002, China

Received: 9 August 2018 / Accepted: 1 November 2018 / Published online: 13 November 2018

© The Author(s) 2018

Abstract In a supersymmetric extension of the standard model with local gauged baryon and lepton numbers (BLMSSM), there are new sources for lepton flavor violation, because the right-handed neutrinos, new gauginos and Higgs are introduced. We investigate muon conversion to electron in nuclei within the BLMSSM in detail. The numerical results indicate that the $\mu \rightarrow e$ conversion rates in nuclei within the BLMSSM can reach the experimental upper bound, which may be detected in the future experiments.

1 Introduction

The observations of neutrino oscillations [1–3] imply that neutrinos have tiny masses and are mixed [4–7], which have demonstrated that lepton flavor in neutrino sector is not conserved. Nevertheless, in the Standard Model (SM) with three tiny massive neutrinos, the expected rates for the charged lepton flavor violating (LFV) processes are very tiny [8–11]. Thus, Lepton-flavor violation is a window of new physics beyond the SM. Among the various candidates for new physics that produce potentially observable effects in LFV processes, one of the most appealing model is supersymmetric (SUSY) extension of the SM. Here, we can use the neutrino oscillation experimental data to restrain the input parameters in the new models. A neutral Higgs with mass $m_{h^0} = 125.1$ GeV reported by ATLAS [12] and CMS [13, 14] gives a strict constraint on relevant parameter space of the model.

The present sensitivities of the $\mu - e$ conversion rates in different nuclei [15–17] are collected here,

$$\begin{aligned} \text{CR}(\mu \rightarrow e : \text{Au}) &< 7 \times 10^{-13}, \\ \text{CR}(\mu \rightarrow e : \text{Ti}) &< 4.3 \times 10^{-12}, \\ \text{CR}(\mu \rightarrow e : \text{Pb}) &< 4.6 \times 10^{-11}. \end{aligned} \quad (1)$$

These processes have close relation with $l_j \rightarrow l_i \gamma$. In the work [18], the $\mu \rightarrow e$ conversion was studied in $\mu\nu\text{SSM}$. For the models beyond SM, one can violate R parity with the non-conservation of baryon number (B) or lepton number (L) [19–22]. A minimal supersymmetric extension of the SM with local gauged B and L (BLMSSM) was first proposed by the author [23, 24]. The local gauged B is used to explain the matter-antimatter asymmetry in the universe. Right-handed neutrinos in BLMSSM lead to three tiny neutrino masses through the See-Saw mechanism and can account for the neutrino oscillation experiments. So lepton number (L) is expected to be broken spontaneously around TeV scale.

In BLMSSM, the lightest CP-even Higgs mass and the decays $h^0 \rightarrow \gamma\gamma$, $h^0 \rightarrow ZZ(WW)$ were studied in the work [25]. The neutron and lepton electric dipole moments (EDMs) were researched in the CP-violating BLMSSM [26, 27]. In BLMSSM, there are also other works [28–30]. In this work, we analyze the processes on muon conversion to electron in nuclei within the BLMSSM. Compared with MSSM, there are new sources to enlarge the processes via loop contributions. The new scores are produced from: (1) the coupling of new neutralino(lepton neutralino)-slepton-lepton; (2) the right-handed neutrinos mixing with left-handed neutrinos; (3) the sneutrino sector is extended, whose mass squared matrix is 6×6 . In some parameter space of BLMSSM, large corrections to the processes are obtained, and they can easily exceed their experiment upper bounds. Therefore, to enhance the processes on muon conversion to electron in nuclei is possible, and they may be measured in the near future.

After this introduction, we briefly summarize the main ingredients of the BLMSSM, and show the needed mass

^a e-mail: sjzueguotao@163.com

^b e-mail: zhaosm@hbu.edu.cn

^c e-mail: dxx_0304@163.com

^d e-mail: duancg@mail.hebtu.edu.cn

^e e-mail: fengt@hbu.edu.cn

matrices and couplings in Sect. 2. In Sect. 3, the processes $\mu \rightarrow e + q\bar{q}$ are studied in the BLMSSM. The input parameters and numerical analysis are shown in Sect. 4, and our conclusion is given in Sect. 5. Some functions are collected in the Appendix.

2 BLMSSM

BLMSSM is the supersymmetric extension of the SM with local gauged B and L , whose local gauge group is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$ [19,20]. The exotic leptons $\hat{L}_4, \hat{E}_4^c, \hat{N}_4^c, \hat{L}_5^c, \hat{E}_5$ and $\hat{N}_5$ are introduced to cancel L anomaly. As well as, the exotic quarks $\hat{Q}_4, \hat{U}_4^c, \hat{D}_4^c, \hat{Q}_5^c, \hat{U}_5$ and $\hat{D}_5$ are introduced to cancel B anomaly. To break lepton number and baryon number spontaneously, the Higgs superfields $\hat{\Phi}_L, \hat{\phi}_L$ and $\hat{\Phi}_B, \hat{\phi}_B$ are introduced, respectively. The exotic quarks obtain masses from nonzero vacuum expectation values (VEVs) of $\hat{\Phi}_B$ and $\hat{\phi}_B$. While, exotic leptons get masses from VEVs of H_u and H_d . H_u and $\hat{\phi}_L$ give masses to light neutrinos through See-Saw mechanism. In the BLMSSM, the superfields $\hat{X}$ and $\hat{X}'$ are introduced to make the heavy exotic quarks unstable. The above mentioned exotic lepton, quark and Higgs superfields are shown in Table 1.

The superpotential of BLMSSM is [25]

$$\mathcal{W}_{BLMSSM} = \mathcal{W}_{MSSM} + \mathcal{W}_B + \mathcal{W}_L + \mathcal{W}_X,$$

Table 1 Superfields in the BLMSSM

Superfields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_B$	$U(1)_L$
$\hat{L}_4$	1	2	-1/2	0	L_4
$\hat{E}_4^c$	1	1	1	0	$-L_4$
$\hat{N}_4^c$	1	1	0	0	$-L_4$
$\hat{L}_5^c$	1	2	1/2	0	$-3-L_4$
$\hat{E}_5$	1	1	-1	0	$3+L_4$
$\hat{N}_5$	1	1	0	0	$3+L_4$
$\hat{Q}_4$	3	2	1/6	B_4	0
$\hat{U}_4^c$	$\bar{3}$	1	-2/3	$-B_4$	0
$\hat{D}_4^c$	$\bar{3}$	1	1/3	$-B_4$	0
$\hat{Q}_5^c$	$\bar{3}$	2	-1/6	$-1-B_4$	0
$\hat{U}_5$	3	1	2/3	$1+B_4$	0
$\hat{D}_5$	3	1	-1/3	$1+B_4$	0
$\hat{\Phi}_L$	1	1	0	0	-2
$\hat{\phi}_L$	1	1	0	0	2
$\hat{\Phi}_B$	1	1	0	1	0
$\hat{\phi}_B$	1	1	0	-1	0
$\hat{X}$	1	1	0	$2/3+B_4$	0
$\hat{X}'$	1	1	0	$-2/3-B_4$	0

$$\begin{aligned} \mathcal{W}_B &= \lambda_Q \hat{Q}_4 \hat{Q}_5^c \hat{\Phi}_B + \lambda_U \hat{U}_4^c \hat{U}_5 \hat{\phi}_B \\ &\quad + \lambda_D \hat{D}_4^c \hat{D}_5 \hat{\phi}_B + \mu_B \hat{\Phi}_B \hat{\phi}_B \\ &\quad + Y_{u4} \hat{Q}_4 \hat{H}_u \hat{U}_4^c + Y_{d4} \hat{Q}_4 \hat{H}_d \hat{D}_4^c \\ &\quad + Y_{u5} \hat{Q}_5^c \hat{H}_u \hat{U}_5 + Y_{d5} \hat{Q}_5^c \hat{H}_d \hat{D}_5, \\ \mathcal{W}_L &= Y_{e4} \hat{L}_4 \hat{H}_d \hat{E}_4^c + Y_{\nu4} \hat{L}_4 \hat{H}_u \hat{N}_4^c \\ &\quad + Y_{e5} \hat{L}_5^c \hat{H}_u \hat{E}_5 + Y_{\nu5} \hat{L}_5^c \hat{H}_d \hat{N}_5 \\ &\quad + Y_\nu \hat{L} \hat{H}_u \hat{N}^c + \lambda_{N^c} \hat{N}^c \hat{N}^c \hat{\phi}_L + \mu_L \hat{\Phi}_L \hat{\phi}_L, \\ \mathcal{W}_X &= \lambda_1 \hat{Q} \hat{Q}_5^c \hat{X} + \lambda_2 \hat{U}^c \hat{U}_5 \hat{X}' \\ &\quad + \lambda_3 \hat{D}^c \hat{D}_5 \hat{X}' + \mu_X \hat{X} \hat{X}', \end{aligned} \quad (2)$$

where $\mathcal{W}_{MSSM}$ is the superpotential of the MSSM. To save space in the text, the soft breaking terms $\mathcal{L}_{soft}$ [19,25] of the BLMSSM is not shown here.

In this model, we introduce the superfields $\hat{N}^c$ to produce tiny masses of three light neutrinos. The mass matrix of neutrinos in the basis $(\psi_{\nu_L}^I, \psi_{N_R^c}^I)$ is expressed as

$$\begin{aligned} Z_{N_\nu}^\top \begin{pmatrix} 0 & \frac{v_u}{\sqrt{2}} (Y_\nu)^{IJ} \\ \frac{v_u}{\sqrt{2}} (Y_\nu^T)^{IJ} & \frac{\tilde{v}_L}{\sqrt{2}} (\lambda_{N^c})^{IJ} \end{pmatrix} Z_{N_\nu} \\ = \text{diag}(m_{\nu^\alpha}), \alpha = 1 \cdots 6, I, J = 1, 2, 3, \\ \psi_{\nu_L}^I = Z_{N_\nu}^{I\alpha} k_{N_\alpha}^0, \quad \psi_{N_R^c}^I = Z_{N_\nu}^{(I+3)\alpha} k_{N_\alpha}^0, \\ \chi_{N_\alpha}^0 = \begin{pmatrix} k_{N_\alpha}^0 \\ \bar{k}_{N_\alpha}^0 \end{pmatrix}. \end{aligned} \quad (3)$$

Here, $\chi_{N_\alpha}^0$ represent the mass eigenstates of neutrino fields mixed by left-handed and right-handed neutrinos.

The new gaugino λ_L mixes with the superpartners of the $SU(2)_L$ singlets Φ_L and ϕ_L , then they produce three lepton neutralinos

$$\begin{aligned} \mathcal{L}_{\chi_L^0} &= \frac{1}{2} (i\lambda_L, \psi_{\Phi_L}, \psi_{\phi_L}) \\ &\quad \times \begin{pmatrix} 2M_L & 2v_{Lg_L} & -2\tilde{v}_{Lg_L} \\ 2v_{Lg_L} & 0 & -\mu_L \\ -2\tilde{v}_{Lg_L} & -\mu_L & 0 \end{pmatrix} \begin{pmatrix} i\lambda_L \\ \psi_{\Phi_L} \\ \psi_{\phi_L} \end{pmatrix} + h.c. \end{aligned} \quad (4)$$

One can use Z_L to diagonalize the mass matrix in Eq. (4) and obtain three lepton neutralino masses in the end.

From Eq. (2) and the soft breaking terms $\mathcal{L}_{soft}$ [19,25] of the BLMSSM, the mass squared matrix of slepton gets corrections and reads as

$$\begin{pmatrix} (\mathcal{M}_L^2)_{LL} & (\mathcal{M}_L^2)_{LR} \\ (\mathcal{M}_L^2)_{LR}^\dagger & (\mathcal{M}_L^2)_{RR} \end{pmatrix}. \quad (5)$$

$(\mathcal{M}_L^2)_{LL}, (\mathcal{M}_L^2)_{LR}$ and $(\mathcal{M}_L^2)_{RR}$ are shown here

$$\begin{aligned} (\mathcal{M}_L^2)_{LL} &= \frac{(g_1^2 - g_2^2)(v_d^2 - v_u^2)}{8} \delta_{IJ} \\ &\quad + g_L^2 (\tilde{v}_L^2 - v_L^2) \delta_{IJ} + m_{lI}^2 \delta_{IJ} + (m_L^2)_{IJ}, \end{aligned}$$

$$\begin{aligned}
(\mathcal{M}_L^2)_{LR} &= \frac{\mu^* v_u}{\sqrt{2}} (Y_l)_{IJ} - \frac{v_u}{\sqrt{2}} (A'_l)_{IJ} + \frac{v_d}{\sqrt{2}} (A_l)_{IJ}, \\
(\mathcal{M}_L^2)_{RR} &= \frac{g_1^2 (v_u^2 - v_d^2)}{4} \delta_{IJ} - g_L^2 (\bar{v}_L^2 - v_L^2) \delta_{IJ} \\
&\quad + m_{lI}^2 \delta_{IJ} + (m_R^2)_{IJ}.
\end{aligned} \quad (6)$$

The unitary matrix Z_L is used to rotate slepton mass squared matrix to mass eigenstates.

Because of the introduction of right handed neutrinos, in BLMSSM the mass squared matrix of sneutrino is 6×6 . In the base $\tilde{n}^T = (\tilde{\nu}, \tilde{N}^c)$, the concrete forms for the sneutrino mass squared matrix $\mathcal{M}_{\tilde{n}}$ are shown here

$$\begin{aligned}
\mathcal{M}_{\tilde{n}}^2 (\tilde{\nu}_I^* \tilde{\nu}_J) &= \frac{g_1^2 + g_2^2}{8} (v_d^2 - v_u^2) \delta_{IJ} \\
&\quad + g_L^2 (\bar{v}_L^2 - v_L^2) \delta_{IJ} \\
&\quad + \frac{v_u^2}{2} (Y_v^\dagger Y_v)_{IJ} + (m_{\tilde{L}}^2)_{IJ}, \\
\mathcal{M}_{\tilde{n}}^2 (\tilde{N}_I^{c*} \tilde{N}_J^c) &= -g_L^2 (\bar{v}_L^2 - v_L^2) \delta_{IJ} + \frac{v_u^2}{2} (Y_v^\dagger Y_v)_{IJ} \\
&\quad + 2\bar{v}_L^2 (\lambda_{N^c}^\dagger \lambda_{N^c})_{IJ} \\
&\quad + (m_{\tilde{N}^c}^2)_{IJ} + \mu_L \frac{v_L}{\sqrt{2}} (\lambda_{N^c})_{IJ} \\
&\quad - \frac{\bar{v}_L}{\sqrt{2}} (A_{N^c})_{IJ} (\lambda_{N^c})_{IJ}, \\
\mathcal{M}_{\tilde{n}}^2 (\tilde{\nu}_I \tilde{N}_J^c) &= \mu^* \frac{v_d}{\sqrt{2}} (Y_v)_{IJ} - v_u \bar{v}_L (Y_v^\dagger \lambda_{N^c})_{IJ} \\
&\quad + \frac{v_u}{\sqrt{2}} (A_N)_{IJ} (Y_v)_{IJ}.
\end{aligned} \quad (7)$$

The superfields $\tilde{N}^c$ in BLMSSM lead to the corrections for some couplings existed in MSSM. We give out the corrected couplings such as: W-lepton-neutrino and Z-neutrino-neutrino

$$\begin{aligned}
\mathcal{L}_{WL\nu} &= -\frac{e}{\sqrt{2}s_W} W_\mu^+ \sum_{I=1}^3 \sum_{\alpha=1}^6 Z_{N_\nu}^{I\alpha*} \tilde{\chi}_{N_\alpha}^0 \gamma^\mu P_L e^I, \\
\mathcal{L}_{Z\nu\nu} &= -\frac{e}{2s_W c_W} Z_\mu \sum_{I=1}^3 \sum_{\alpha,\beta=1}^6 Z_{N_\nu}^{I\alpha*} Z_{N_\nu}^{I\beta} \tilde{\chi}_{N_\alpha}^0 \gamma^\mu P_L \tilde{\chi}_{N_\beta}^0,
\end{aligned} \quad (8)$$

where $P_L = \frac{1-\gamma_5}{2}$ and $P_R = \frac{1+\gamma_5}{2}$. We use the abbreviation $s_W = \sin \theta_W$, $c_W = \cos \theta_W$, and θ_W is the Weinberg angle.

Some other adapted couplings are collected here: chargino-lepton-sneutrino, Z-sneutrino-sneutrino and charged Higgs-lepton-neutrino

$$\mathcal{L}_{\chi^\pm L \tilde{\nu}} = -\sum_{I=1}^3 \sum_{i=1}^6 \sum_{j=1}^2 \tilde{\chi}_j^\pm \left(Y_l^I Z_-^{2j*} Z_v^{Ii*} P_R \right.$$

$$\left. + \left[\frac{e}{s_W} Z_+^{1j} Z_v^{Ii*} + Y_v^{Ii} Z_+^{2j} Z_v^{(I+3)i*} \right] P_L \right) e^I \tilde{\nu}^{i*} + h.c. \quad (9)$$

$$\mathcal{L}_{Z\tilde{\nu}\tilde{\nu}} = -\frac{e}{2s_W c_W} Z_\mu \sum_{I=1}^3 \sum_{i,j=1}^6 Z_v^{Ii*} Z_v^{Ij} \tilde{\nu}^{i*} i \left(\overleftrightarrow{\partial}^\mu - \overleftrightarrow{\partial}^\mu \right) \tilde{\nu}^j. \quad (10)$$

$$\begin{aligned}
\mathcal{L}_{H^\pm L \nu} &= \sum_I \sum_{\alpha=1}^6 G^\pm \bar{e}^I \left(Y_l^I \cos \beta Z_{N_\nu}^{I\alpha} P_L \right. \\
&\quad \left. - Y_v^{I\alpha*} \sin \beta Z_{N_\nu}^{(I+3)\alpha} P_R \right) \chi_{N_\alpha}^0 \\
&\quad - \sum_I \sum_{\alpha=1}^6 H^\pm \bar{e}^I \left(Y_l^I \sin \beta Z_{N_\nu}^{I\alpha} P_L \right. \\
&\quad \left. + Y_v^{I\alpha*} \cos \beta Z_{N_\nu}^{(I+3)\alpha} P_R \right) \chi_{N_\alpha}^0 + h.c.
\end{aligned} \quad (11)$$

In BLMSSM, there are new couplings that are deduced from the interactions of gauge and matter multiplets $ig\sqrt{2}T_{ij}^a(\lambda^a \psi_j A_i^* - \bar{\lambda}^a \bar{\psi}_i A_j)$. After calculation, the lepton-slepton-lepton neutralino couplings are obtained

$$\begin{aligned}
\mathcal{L}_{l\chi_L^0 \tilde{L}} &= \sum_{i=1}^6 \sum_{j=1}^3 \sqrt{2} g_L \tilde{\chi}_{L_j}^0 \left(Z_{N_L}^{1j} Z_L^{Ii} P_L \right. \\
&\quad \left. - Z_{N_L}^{1j*} Z_L^{(I+3)i} P_R \right) l^I \tilde{L}_i + h.c.
\end{aligned} \quad (12)$$

3 $\mu \rightarrow e + q\bar{q}$ in the BLMSSM

In this section, the LFV processes $\mu \rightarrow e + q\bar{q}$ are studied in the BLMSSM. Both penguin-type diagrams and box-type diagrams have contributions to the effective Lagrangian. For convenience, the penguin and box diagrams are analyzed in the generic form, which can simplify the work.

3.1 The penguin diagrams

When the external leptons are all on shell, we can generally obtain the γ -penguin contributions in the following form

$$\begin{aligned}
T_{\gamma-p} &= -\bar{e}(p_1) \left[q^2 \gamma_\mu \left(C_1^L P_L + C_1^R P_R \right) \right. \\
&\quad \left. + m_\mu i \sigma_{\mu\nu} q^\nu \left(C_2^L P_L + C_2^R P_R \right) \right] \mu(p) \\
&\quad \times \frac{e^2}{q^2} \bar{q}(p_2) \gamma^\mu q(p_3).
\end{aligned} \quad (13)$$

The relevant Feynman diagrams are shown in Fig. 1. The final Wilson coefficients C_1^L , C_1^R , C_2^L and C_2^R are obtained from the sum of these diagrams' amplitudes.

Fig. 1 The penguin diagrams for $\mu \rightarrow e + q\bar{q}$, with F representing Dirac(Majorana) particles

The contributions from the virtual neutral fermion diagram in the top-left of Fig. 1 are denoted by $C_{\alpha}^{L,R}(n)$, $\alpha = 1, 2$. We give out the deduced results in the following form,

$$C_1^L(n) = \sum_{F=\chi^0, \chi_L^0, \nu} \sum_{S=\tilde{L}, \tilde{L}, H^{\pm}} \frac{1}{6m_W^2} H_R^{SF\bar{e}} H_L^{S^*\mu\bar{F}} I_1(x_F, x_S),$$

$$C_2^L(n) = \sum_{F=\chi^0, \chi_L^0, \nu} \sum_{S=\tilde{L}, \tilde{L}, H^{\pm}} \frac{m_F}{m_{\mu}m_W^2} H_L^{SF\bar{e}} H_L^{S^*\mu\bar{F}} \times [I_2(x_F, x_S) - I_3(x_F, x_S)],$$

$$C_{\alpha}^R(n) = C_{\alpha}^L(n) \Big|_{L \leftrightarrow R}, \quad \alpha = 1, 2. \quad (14)$$

with $x = m^2/m_W^2$ and m representing the mass for the corresponding particle. $H_{L,R}^{SF\bar{e}}$ and $H_{L,R}^{S^*\mu\bar{F}}$ are the corresponding couplings of the left(right)-hand parts in the Lagrangian. The one-loop functions $I_i(x_1, x_2)$, $i = 1 \dots 4$ are collected in Appendix.

The diagram in top-right of Fig. 1 represents the virtual charged Fermion diagram and its contribution is

$$C_1^L(c) = \sum_{F=\chi^{\pm}} \sum_{S=\tilde{\nu}} \frac{1}{6m_W^2} H_R^{SF\bar{e}} H_L^{S^*\mu\bar{F}} [I_3(x_F, x_S) - 2I_4(x_F, x_S) - I_1(x_F, x_S)],$$

$$C_2^L(c) = \sum_{F=\chi^{\pm}} \sum_{S=\tilde{\nu}} \frac{m_F}{m_{\mu}m_W^2} H_L^{SF\bar{e}} H_L^{S^*\mu\bar{F}} [I_3(x_F, x_S) - I_4(x_F, x_S) - I_1(x_F, x_S)],$$

$$C_{\alpha}^R(c) = C_{\alpha}^L(c) \Big|_{L \leftrightarrow R}, \quad \alpha = 1, 2. \quad (15)$$

On account of the mixing of three light neutrinos and three heavy neutrinos, the virtual W diagrams in the bot-

tom of Fig. 1 give corrections to the charged LFV process $\mu \rightarrow e + q\bar{q}$. We show the coefficients $C_{\alpha}^{L,R}(W)$ ($\alpha = 1, 2$)

$$C_1^L(W) = \sum_{F=\nu} \frac{-1}{2m_W^2} H_L^{WF\bar{e}} H_L^{W^*\mu\bar{F}} [I_2(x_F, x_W) + I_1(x_F, x_W)],$$

$$C_2^L(W) = \sum_{F=\nu} \frac{1}{m_W^2} H_L^{WF\bar{e}} H_L^{W^*\mu\bar{F}} \left(1 + \frac{m_e}{m_{\mu}}\right) \times [2I_2(x_F, x_W) - \frac{1}{3}I_1(x_F, x_W)],$$

$$C_{\alpha}^R(W) = 0, \quad \alpha = 1, 2. \quad (16)$$

The sum of the total coefficients in Eqs. (14)–(16) are

$$C_{\alpha}^{L,R} = C_{\alpha}^{L,R}(n) + C_{\alpha}^{L,R}(c) + C_{\alpha}^{L,R}(W), \quad \alpha = 1, 2. \quad (17)$$

The contributions from Z -penguin diagrams are depicted by the Fig. 1, similar as γ -penguin diagrams,

$$T_{Z-p} = \frac{e^2}{m_Z^2} \bar{e}(p_1) \gamma_{\mu} (N_L P_L + N_R P_R) \mu(p) \bar{q}(p_2) \gamma^{\mu} \times (H_L^{Ze\bar{e}} P_L + H_R^{Ze\bar{e}} P_R) q(p_3),$$

$$N_{L,R} = N_{L,R}(S) + N_{L,R}(W). \quad (18)$$

The concrete forms of the effective couplings $N_L(S)$, $N_R(S)$, $N_L(W)$ and $N_R(W)$ read as

$$N_L(S) = \frac{1}{2e^2} \sum_{F=\chi^0, \chi^{\pm}, \nu} \sum_{S=\tilde{L}, \tilde{\nu}, H^{\pm}} \times \left[\frac{2m_{F_1}m_{F_2}}{m_W^2} H_R^{SF_2\bar{e}} H_L^{ZF_1\bar{F}_2} H_L^{S^*\mu\bar{F}_1} G_1(x_S, x_{F_2}, x_{F_1}) + H_R^{S_2F\bar{e}} H_R^{ZS_1S_2^*} H_L^{S_1^*\mu\bar{F}} G_2(x_F, x_{S_1}, x_{S_2}) \right]$$

$$\begin{aligned}
& -H_R^{SF_2\bar{e}} H_R^{ZF_1\bar{F}_2} H_L^{S^*\mu\bar{F}_1} G_2(x_S, x_{F_2}, x_{F_1}) \Big] \\
& + \sum_{F=\chi_L^0} \sum_{S=\bar{L}} \left[H_R^{SF_2\bar{e}} H_R^{ZS_1S_2^*} H_L^{S^*\mu\bar{F}} G_2(x_F, x_{S_1}, x_{S_2}) \right], \\
N_R(S) &= N_L(S)|_{L \leftrightarrow R}, \\
N_L(W) &= \frac{c_W}{e s_W} \sum_{F=\nu} H_L^{WF\bar{e}} H_L^{W^*\mu\bar{F}} \\
& \times \left[G_3(x_F, x_W) + 2(x_i + x_j) [I_1(x_F, x_W) - I_2(x_F, x_W)] \right] \\
& + \frac{1}{e^2} \sum_{F_1, F_2=\nu} H_L^{WF_2\bar{e}} H_L^{W^*\mu\bar{F}_1} H_L^{Z^*F_1\bar{F}_2} \\
& \times \left(-\frac{3}{32\pi^2} - G_2(x_W, x_{F_1}, x_{F_2}) \right. \\
& \left. + x_j \left[\frac{1}{3} G_4(x_W, x_{F_1}, x_{F_2}) + G_5(x_W, x_{F_1}, x_{F_2}) \right] \right), \\
N_R(W) &= \frac{c_W}{e s_W} \sum_{F=\nu} H_L^{WF\bar{e}} H_L^{W^*\mu\bar{F}} \\
& \times \left[2\sqrt{x_i x_j} [I_1(x_F, x_W) - I_2(x_F, x_W)] \right] \\
& + \frac{1}{e^2} \sum_{F_1, F_2=\nu} H_L^{WF_2\bar{e}} H_L^{W^*\mu\bar{F}_1} H_L^{Z^*F_1\bar{F}_2} \sqrt{x_i x_j} \\
& \times \left(2G_1(x_W, x_{F_1}, x_{F_2}) - \frac{1}{3} G_4(x_W, x_{F_1}, x_{F_2}) \right. \\
& \left. - 2G_5(x_W, x_{F_1}, x_{F_2}) \right). \quad (19)
\end{aligned}$$

The concrete expressions for the functions G_i ($i = 1, \dots, 7$) are collected in appendix.

3.2 The box-type diagrams

The box-type diagrams drawn in Fig. 2 can be written as

$$T_{\text{box}} = e^2 \sum_{q=u,d} \bar{q} \gamma_\alpha q \bar{e} \gamma^\alpha \left(B_q^L P_L + B_q^R P_R \right) \mu, \quad (20)$$

$$B_q^{L,R} = B_q^{(n)L,R} + B_q^{(c)L,R} + B_q^{(W)L,R} (q = u, d). \quad (21)$$

$B_q(n)$ represent the contributions from the virtual neutral Fermion diagrams in the first line of Fig. 2.

$$\begin{aligned}
B_q^L(n) &= \sum_{i,j=1}^4 \sum_{k,l=1}^6 \left\{ \frac{1}{8e^2 m_W^2} G_6(x_{\chi_i^0}, x_{\chi_j^0}, x_{\tilde{L}_k}, x_{\tilde{Q}_l}) \right. \\
& \times \left[\left(H_R^{\tilde{L}_k \chi_i^0 \tilde{\mu}} \right)^* H_R^{\tilde{L}_k \chi_j^0 \tilde{e}} H_R^{\tilde{Q}_l \chi_i^0 \tilde{q}} \left(H_R^{\tilde{Q}_l \chi_j^0 \tilde{q}} \right)^* \right. \\
& \left. - \left(H_R^{\tilde{L}_k \chi_i^0 \tilde{\mu}} \right)^* H_R^{\tilde{L}_k \chi_j^0 \tilde{e}} \left(H_R^{\tilde{Q}_l \chi_i^0 \tilde{q}} \right)^* H_R^{\tilde{Q}_l \chi_j^0 \tilde{q}} \right] \\
& \left. - \frac{m_{\chi_i^0} m_{\chi_j^0}}{4e^2 m_W^2} G_7(x_{\chi_i^0}, x_{\chi_j^0}, x_{\tilde{L}_k}, x_{\tilde{Q}_l}) \right\}
\end{aligned}$$

$$\begin{aligned}
& \times \left[\left(H_R^{\tilde{L}_k \chi_i^0 \tilde{\mu}} \right)^* H_R^{\tilde{L}_k \chi_j^0 \tilde{e}} H_L^{\tilde{Q}_l \chi_j^0 \tilde{q}} \left(H_L^{\tilde{Q}_l \chi_i^0 \tilde{q}} \right)^* \right. \\
& \left. - \left(H_R^{\tilde{L}_k \chi_i^0 \tilde{\mu}} \right)^* H_R^{\tilde{L}_k \chi_j^0 \tilde{e}} \left(H_R^{\tilde{Q}_l \chi_i^0 \tilde{q}} \right)^* H_R^{\tilde{Q}_l \chi_j^0 \tilde{q}} \right] \Big\},
\end{aligned}$$

$$B_q^R(n) = B_q^L(n)|_{L \leftrightarrow R}, \quad (q = u, d). \quad (22)$$

The virtual charged Fermion in the middle line of Fig. 2 give contributions denoted by $B_q(c)$.

$$\begin{aligned}
B_d^L(c) &= \sum_{i,j=1}^2 \sum_{k,l=1}^6 \left\{ \frac{1}{8e^2 m_W^2} G_6(x_{\chi_i^\pm}, x_{\chi_j^\pm}, x_{\tilde{\nu}_k}, x_{U_l^+}) \right. \\
& \times \left[\left(H_R^{\tilde{\nu}_k \chi_i^\pm \tilde{\mu}} \right)^* H_R^{\tilde{\nu}_k \chi_j^\pm \tilde{e}} H_R^{U_l^+ \chi_i^\pm \tilde{d}} \left(H_R^{U_l^+ \chi_j^\pm \tilde{d}} \right)^* \right. \\
& \left. - \frac{m_{\chi_i^\pm} m_{\chi_j^\pm}}{4e^2 m_W^2} G_7(x_{\chi_i^\pm}, x_{\chi_j^\pm}, x_{\tilde{\nu}_k}, x_{U_l^+}) \right. \\
& \left. \times \left(H_R^{\tilde{\nu}_k \chi_i^\pm \tilde{\mu}} \right)^* H_R^{\tilde{\nu}_k \chi_j^\pm \tilde{e}} H_L^{U_l^+ \chi_i^\pm \tilde{d}} \left(H_L^{U_l^+ \chi_j^\pm \tilde{d}} \right)^* \right] \Big\}, \\
B_u^L(c) &= \sum_{i,j=1}^2 \sum_{k,l=1}^6 \left\{ \frac{-1}{8e^2 m_W^2} G_6(x_{\chi_i^\pm}, x_{\chi_j^\pm}, x_{\tilde{\nu}_k}, x_{D_l^-}) \right. \\
& \times \left[\left(H_R^{\tilde{\nu}_k \chi_i^\pm \tilde{\mu}} \right)^* H_R^{\tilde{\nu}_k \chi_j^\pm \tilde{e}} \left(H_L^{D_l^- u^c \chi_i^\pm} \right)^* H_L^{D_l^- u^c \chi_j^\pm} \right. \\
& \left. + \frac{m_{\chi_i^\pm} m_{\chi_j^\pm}}{4e^2 m_W^2} G_7(x_{\chi_i^\pm}, x_{\chi_j^\pm}, x_{\tilde{\nu}_k}, x_{D_l^-}) \right. \\
& \left. \times \left(H_R^{\tilde{\nu}_k \chi_i^\pm \tilde{\mu}} \right)^* H_R^{\tilde{\nu}_k \chi_j^\pm \tilde{e}} \left(H_R^{D_l^- u^c \chi_i^\pm} \right)^* H_R^{D_l^- u^c \chi_j^\pm} \right] \Big\},
\end{aligned}$$

$$B_q^R(c) = B_q^L(c)|_{L \leftrightarrow R}. \quad (23)$$

The virtual W produces corrections through the diagrams in the last line of Fig. 2

$$\begin{aligned}
B_d^L(W) &= -\frac{1}{2e^2} \frac{\partial}{\partial x_W} G_2(x_W, x_\nu, x_u) \\
& \times \left(H_L^{W\mu\tilde{\nu}} H_L^{W^*\nu\tilde{e}} H_L^{W^*u\tilde{d}} H_L^{Wd\tilde{u}} \right), \\
B_u^L(W) &= \frac{2}{e^2} \frac{\partial}{\partial x_W} G_2(x_W, x_\nu, x_d) \\
& \times \left(H_L^{W\mu\tilde{\nu}} H_L^{W^*\nu\tilde{e}} H_L^{W^*d\tilde{u}} H_L^{Wud\tilde{d}} \right), \\
B_d^R(W) &= B_u^R(W) = 0. \quad (24)
\end{aligned}$$

3.3 $\mu - e$ conversion rate

Once we know the effective Lagrangian relevant to this process at the quark level, we can calculate the conversion rate

$$\begin{aligned}
CR &= 4\alpha^5 \frac{Z_{eff}^4}{Z} \left| F(q^2) \right|^2 m_\mu^5 \left[\left| Z(A_1^L - A_2^R) \right. \right. \\
& \left. \left. - (2Z + N) \bar{D}_u^L \right. \right. \\
& \left. \left. - (Z + 2N) \bar{D}_d^L \right|^2 \right]
\end{aligned}$$

Fig. 2 The box diagrams for $\mu \rightarrow e + q\bar{q}$, with F representing Dirac(Majorana) particles

$$+ \left| Z \left(A_1^R - A_2^L \right) - (2Z + N) \bar{D}_u^R - (Z + 2N) \bar{D}_d^R \right|^2 \frac{1}{\Gamma_{capt}}, \quad (25)$$

$$\bar{D}_q^L = D_q^L + \frac{Z_L^q + Z_R^q}{2} \frac{F_L}{m_Z^2 s_W^2 c_W^2},$$

$$\bar{D}_q^R = \bar{D}_q^L \Big|_{L \leftrightarrow R}, \quad (q = u, d). \quad (26)$$

with Z and N representing the proton and neutron numbers in a nucleus. Z_{eff} is an effective atomic charge determined in refs [31,32]. $F(q^2)$ is the nuclear form factor and Γ_{capt} denotes the total muon capture rate, while α is the fine structure constant.

4 Numerical results

In this section, we discuss the numerical results, and consider the experimental constraints from the lightest neutral CP-even Higgs mass $m_{h0} \simeq 125.1$ GeV [12–14] and the neutrino experiment data. In this model, the LFV processes $l_j \rightarrow l_i \gamma$ and $l_j \rightarrow 3l_i$ are studied in our previous work [30], and their constraints are also taken into account. In this work, we use the parameters [28,29]

$$L_4 = \frac{3}{2}, \quad \mu = 0.5 \text{ GeV}, \quad (A_l)_{ii} = -2000 \text{ GeV},$$

$$(A'_l)_{ii} = A'_L = 300 \text{ GeV},$$

$$(A_{N^c})_{ii} = (A_N)_{ii} = -500 \text{ GeV},$$

$$(A_u)_{ii} = (A_d)_{ii} = (A'_d)_{ii} = (m_{\tilde{N}^c})_{ii} = 1000 \text{ GeV},$$

$$(A'_u)_{ii} = 800 \text{ GeV},$$

$$(m_{\tilde{Q}})_{ii} = (m_{\tilde{U}})_{ii} = (m_{\tilde{D}})_{ii} = 2 \times 10^3 \text{ GeV}, \quad \lambda_{N^c} = 1. \quad (27)$$

The Yukawa couplings of neutrinos $(Y_\nu)^{IJ}$, $(I, J = 1, 2, 3)$ are at the order of $10^{-8} - 10^{-6}$, whose effects to this processes are tiny and can be ignored safely. To simplify the numerical discussion, we use the following relations

$$(A_l)_{ii} = AL, \quad (A_{N^c})_{ii} = (A_N)_{ii} = AN,$$

$$(A'_l)_{ii} = A'_L,$$

$$(m_{\tilde{N}^c}^2)_{ii} = M_{sn}^2, \quad (m_{\tilde{L}}^2)_{ii} = (m_{\tilde{R}}^2)_{ii} = s_m^2,$$

$$\text{for } i = 1, 2, 3.$$

$$(m_{\tilde{L}}^2)_{ij} = (m_{\tilde{R}}^2)_{ij} = M_{Lf},$$

$$\text{for } i, j = 1, 2, 3 \text{ and } i \neq j. \quad (28)$$

If we do not specially declare, the non-diagonal elements of the used parameters should be zero.

Fig. 3 With $m_2 = 1000 \text{ GeV}$, $M_{Lf} = 10^4 \text{ GeV}^2$, $S_m^2 = 13 \text{ TeV}^2$, $\tan\beta = 5.0$, $\tan\beta_L = 2.0$, and $g_L = \frac{1}{6}$, $\mu \rightarrow e$ conversion rate in nuclei Au versus m_1 with $V_{Lt} = 3000 \text{ GeV}$ (solid line) and 6000 GeV (dotted line)

4.1 $\mu \rightarrow e$ conversion rate in nuclei Au

The experimental upper bound for the $\mu \rightarrow e$ conversion rate in nuclei Au is around 7.0×10^{-13} . The parameters $m_2 = 1000 \text{ GeV}$, $M_{Lf} = 10^4 \text{ GeV}^2$ are supposed in the calculation of this process. The parameter m_1 is related to the mass matrix of the neutralino, which means the contributions from neutralino-slepton can be influenced by the parameter m_1 . For $S_m^2 = 13 \text{ TeV}^2$, $\tan\beta = 5.0$, $\tan\beta_L = 2.0$, and $g_L = \frac{1}{6}$, we plot the results versus m_1 with $V_{Lt} = 3000 \text{ GeV}$ and 6000 GeV in Fig. 3. We can see that the results decrease quickly with the increase of m_1 . As $V_{Lt} = 6000 \text{ GeV}$, the results are slightly smaller than the corresponding results with $V_{Lt} = 3000 \text{ GeV}$. This implies that m_1 is a sensitive parameter and has a strong effect on muon conversion to electron in nuclei. Compared with m_1 , the effect from V_{Lt} is very small.

$\tan\beta$ is related to v_u and v_d , and appears in almost all mass matrices of particles contributing to the $\mu \rightarrow e$ processes. With $m_1 = 500 \text{ GeV}$, $V_{Lt} = 3000 \text{ GeV}$, $\tan\beta_L = 2.0$, and $g_L = \frac{1}{6}$, Fig. 4 shows the variation of the $\mu \rightarrow e$ conversion rate in nuclei Au with the parameter $\tan\beta$ and S_m^2 . It indicates that the results change significantly with $\tan\beta$. When $\tan\beta$ is in the region $(0 \sim 6)$, the results decrease significantly, but in the range of $\tan\beta > 6$, we find that the results increase sharply. Only when the value of $\tan\beta$ is about 6, the results of $\mu \rightarrow e$ conversion rate in nuclei Au are close and not higher than the experimental upper bound.

The parameters g_L , $\tan\beta_L$ and V_{Lt} all present in the mass squared matrices of sleptons, sneutrinos and lepton neutralinos. Therefore, these three parameters affect the results through slepton-neutrino, sneutrinos-chargino and slepton-lepton neutralino contributions.

As $m_1 = 1000 \text{ GeV}$, $\tan\beta = 5.5$, $\tan\beta_L = 2$, $S_m^2 = 16 \text{ TeV}^2$, g_L versus V_{Lt} are scanned in Fig. 5. We find that the allowed scope of V_{Lt} shrinks and the value of V_{Lt} decreases

Fig. 4 With $m_1 = 500 \text{ GeV}$, $m_2 = 1000 \text{ GeV}$, $M_{Lf} = 10^4 \text{ GeV}^2$, $V_{Lt} = 3000 \text{ GeV}$, $\tan\beta_L = 2.0$, and $g_L = \frac{1}{6}$, the contribution to $\mu \rightarrow e$ conversion rate in nuclei Au versus $\tan\beta$ with $S_m^2 = 12 \text{ TeV}^2$ (solid line) and 16 TeV^2 (dotted line)

Fig. 5 For $\mu \rightarrow e$ conversion rate in nuclei Au, the allowed parameter space in the plane of g_L versus V_{Lt} with $m_1 = 1000 \text{ GeV}$, $m_2 = 1000 \text{ GeV}$, $\tan\beta = 5.5$, $S_m^2 = 16 \text{ TeV}^2$, $M_{Lf} = 10^4 \text{ GeV}^2$ and $\tan\beta_L = 2$

with the enlarging g_L . Therefore, the value of g_L should not be too large. Generally, we take $0.05 \leq g_L \leq 0.3$ and $V_{Lt} \sim 3 \text{ TeV}$ in our numerical calculations.

As the parameters $m_1 = 1000 \text{ GeV}$, $\tan\beta = 6.0$, $S_m^2 = 12 \text{ TeV}^2$ and $V_{Lt} = 3000 \text{ GeV}$, we plot the allowed results with $\tan\beta_L$ versus g_L in Fig. 6. When $g_L < 0.43$, the parameter $\tan\beta_L$ can vary in the region of $(0 \sim 4)$. It implies that g_L is a sensitive parameter to the numerical results and the value of g_L should not be larger than 0.43.

4.2 $\mu \rightarrow e$ conversion rate in nuclei Ti

In a similar way, the $\mu \rightarrow e$ conversion rate in nuclei Ti is numerically studied and its experimental upper bound is around 4.3×10^{-12} . In this subsection, we use the parameters $\tan\beta = 2.0$, $\tan\beta_L = 2.0$, $g_L = \frac{1}{6}$ and $V_{Lt} = 3000 \text{ GeV}$. The parameter m_2 presents in the mass matrixes of neutralino and chargino. This parameter affects the numerical results through the neutralino-slepton and chargino-sneutrino con-

Fig. 6 For $\mu \rightarrow e$ conversion rate in nuclei Au, the allowed parameter space in the plane of $\tan \beta_L$ versus g_L with $m_1 = 1000$ GeV, $m_2 = 1000$ GeV, $\tan \beta = 6$, $S_m^2 = 12 \text{ TeV}^2$, $M_{Lf} = 10^4 \text{ GeV}^2$ and $V_{Li} = 3000$ GeV

Fig. 7 With $\tan \beta = 2.0$, $S_m^2 = 10 \text{ TeV}^2$, $M_{Lf} = 10^4 \text{ GeV}^2$, $\tan \beta_L = 2.0$, $g_L = \frac{1}{6}$ and $V_{Li} = 3000$ GeV, $\mu \rightarrow e$ conversion rate in nuclei Ti versus m_2 is plotted for $m_1 = 2000$ GeV (solid line) and 1000 GeV (dotted line)

tributions. With $S_m^2 = 10 \text{ TeV}^2$, $M_{Lf} = 10^4 \text{ GeV}^2$, we plot the results versus m_2 with $m_1 = 1000$ GeV and 2000 GeV by the solid and dotted lines in Fig. 7. We can see that the results decrease with the increase of m_2 . The results of dotted line are slightly larger than solid line and all the results are in the region ($1.4 \times 10^{-12} \sim 6.4 \times 10^{-12}$). This implies that m_2 should have impact on the results to some extent.

M_{Lf} are the non-diagonal elements of $m_{\tilde{L}}^2$ and $m_{\tilde{R}}^2$ in the slepton mass matrix. For $m_1 = 1000$ GeV and $S_m^2 = 12 \text{ TeV}^2$, we study the $\mu \rightarrow e$ conversion rate in nuclei Ti versus M_{Lf} with $m_2 = 1000$ GeV (solid line) and $m_2 = 2000$ GeV (dotted line) in Fig. 8. As $M_{Lf} = 0$, the conversion ratio for $\mu \rightarrow e$ is almost zero, but the results increase sharply with $M_{Lf} > 0$. We deduce that non-zero M_{Lf} is a sensitive parameter and has a strong effect on muon conversion to electron in nuclei.

Fig. 8 With $m_1 = 1000$ GeV, $\tan \beta = 2.0$, $S_m^2 = 12 \text{ TeV}^2$, $\tan \beta_L = 2.0$, $g_L = \frac{1}{6}$ and $V_{Li} = 3000$ GeV, $\mu \rightarrow e$ conversion rate in nuclei Ti versus M_{Lf} with $m_2 = 1000$ GeV (solid line) and 2000 GeV (dotted line)

Fig. 9 With $m_1 = 1000$ GeV, $m_2 = 1000$ GeV, $M_{Lf} = 10^4 \text{ GeV}^2$, $\tan \beta_L = 2.0$, $g_L = \frac{1}{6}$ and $V_{Li} = 3000$ GeV, the contribution to $\mu \rightarrow e$ conversion rate in nuclei Pb versus S_m with $\tan \beta = 2$ (solid line) and 3 (dotted line)

4.3 $\mu \rightarrow e$ conversion rate in nuclei Pb

The experimental upper bound of $\mu \rightarrow e$ conversion rate in nuclei Pb is around 4.6×10^{-11} . In this subsection, we use the parameters $m_2 = 1000$ GeV, $\tan \beta_L = 2.0$, $g_L = \frac{1}{6}$ and $M_{Lf} = 10^4 \text{ GeV}^2$. S_m are the diagonal elements of $m_{\tilde{L}}^2$ and $m_{\tilde{R}}^2$ in the slepton mass matrix, which can affect slepton-neutralino and slepton-lepton neutralino contributions in the $\mu \rightarrow e$ process.

With $m_1 = 1000$ GeV, $V_{Li} = 3000$ GeV, we plot the conversion ratio for $\mu \rightarrow e$ in nuclei Pb versus S_m with $\tan \beta = 2.0$ (solid line) and $\tan \beta = 3.0$ (dotted line) in Fig. 9. These two lines decrease quickly with S_m enlarging from 1400 to 3000 GeV, which indicates that S_m is a very sensitive parameter to the numerical results. When $S_m > 3000$ GeV, the results decrease slowly and the conversion ratios are around ($10^{-12} - 10^{-13}$).

We focus on V_{Li} which is a special parameter in BLMSSM, and with $m_1 = 500$ GeV, $\tan \beta = 13$, we plot

Fig. 10 With $m_1 = 500$ GeV, $m_2 = 1000$ GeV, $\tan\beta = 13$, $M_{L_f} = 10^4$ GeV², $\tan\beta_L = 2.0$, and $g_L = \frac{1}{6}$, $\mu \rightarrow e$ conversion rate in nuclei Pb versus V_{L_t} with $S_m^2 = 5$ TeV² (solid line) and $S_m^2 = 6$ TeV² (dotted line)

the conversion ratio for $\mu \rightarrow e$ in nuclei Pb versus V_{L_t} with $S_m^2 = 5$ TeV² (solid line) and $S_m^2 = 6$ TeV² (dotted line) in Fig. 10. Overall, the results of dotted line are about $0.5 \times 10^{-11} \sim 1.2 \times 10^{-11}$ larger than the solid line. In the range of $V_{L_t} = (0 \sim 10000$ GeV), the two lines decrease quickly with the enlarging V_{L_t} . We can see S_m and V_{L_t} are sensitive parameters to the numerical results.

5 Discussion and conclusion

In the framework of the BLMSSM model, we study the LFV processes $\mu \rightarrow e + q\bar{q}$. In the processes, we consider some new parameters and contributions, such as the newly introduced parameters g_L , $\tan\beta_L$ and V_{L_t} . Combined with the numerical results discussed in the Sect. 4, different parameters have different effects on the processes. The parameter g_L presents in the mass squared matrices of sleptons, sneutrinos and lepton neutralinos. Numerical analysis shows that g_L has obvious influence on the results, the value of g_L should not be too large. As sensitive parameters, S_m and M_{L_f} are respectively diagonal and non-diagonal elements of matrixes for $m_{\tilde{L}}$ and $m_{\tilde{R}}$. Both S_m and M_{L_f} have significant impacts on the results. $\tan\beta$ is related to v_u and v_d , and appears in almost all mass matrices of particles contributing to the $\mu \rightarrow e$ processes. The value of $\tan\beta$ is critical to these processes. With the improvement of experimental accuracy, we believe that there will be some discoveries for μ to e conversion in the near future.

Acknowledgements Supported by National Natural Science Foundation of China (No. 11535002 and No.11575052), Physics laboratory center of Hebei GEO University(Hebei experimental teaching demonstration center).

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.

Funded by SCOAP³.

6 Appendix

In this section, we give out the corresponding one loop functions. $G_2(x_1, x_2, x_3)$ and $G_3(x_1, x_2, x_3)$ have infinite term, and to obtain finite results we use $\overline{MS}$ subtraction and $\overline{DR}$ scheme.

$$I_1(x_1, x_2) = \frac{1}{96\pi^2} \left[\frac{11 + 6 \ln x_2}{(x_2 - x_1)} - \frac{15x_2 + 18x_2 \ln x_2}{(x_2 - x_1)^2} + \frac{6x_2^2 + 18x_2^2 \ln x_2}{(x_2 - x_1)^3} + \frac{6x_1^3 \ln x_1 - 6x_2^3 \ln x_2}{(x_2 - x_1)^4} \right]. \quad (29)$$

$$I_2(x_1, x_2) = \frac{1}{32\pi^2} \left[\frac{3 + 2 \ln x_2}{(x_2 - x_1)} - \frac{2x_2 + 4x_2 \ln x_2}{(x_2 - x_1)^2} - \frac{2x_1^2 \ln x_1}{(x_2 - x_1)^3} + \frac{2x_2^2 \ln x_2}{(x_2 - x_1)^3} \right], \quad (30)$$

$$I_3(x_1, x_2) = \frac{1}{16\pi^2} \left[\frac{1 + \ln x_2}{(x_2 - x_1)} + \frac{x_1 \ln x_1 - x_2 \ln x_2}{(x_2 - x_1)^2} \right]. \quad (31)$$

$$I_4(x_1, x_2) = \frac{1}{16\pi^2} \left[-\frac{1 + \ln x_1}{(x_2 - x_1)} - \frac{x_1 \ln x_1 - x_2 \ln x_2}{(x_2 - x_1)^2} \right]. \quad (32)$$

$$G_1(x_1, x_2, x_3) = \frac{1}{16\pi^2} \left[\frac{x_1 \ln x_1}{(x_1 - x_2)(x_1 - x_3)} + \frac{x_2 \ln x_2}{(x_2 - x_1)(x_2 - x_3)} + \frac{x_3 \ln x_3}{(x_3 - x_1)(x_3 - x_2)} \right], \quad (33)$$

$$G_2(x_1, x_2, x_3) = \frac{1}{16\pi^2} \left[-(\Delta + 1 + \ln x_\mu) + \frac{x_1^2 \ln x_1}{(x_1 - x_2)(x_1 - x_3)} + \frac{x_2^2 \ln x_2}{(x_2 - x_1)(x_2 - x_3)} + \frac{x_3^2 \ln x_3}{(x_3 - x_1)(x_3 - x_2)} \right]. \quad (34)$$

$$G_3(x_1, x_2) = \frac{-1}{16\pi^2} \left((\Delta + \ln x_\mu + 1) + \frac{x_2^2 \ln x_2 - x_1^2 \ln x_1}{(x_2 - x_1)^2} + \frac{x_2 + 2x_2 \ln x_2}{x_1 - x_2} - \frac{1}{2} \right),$$

$$G_4(x_1, x_2, x_3) = \frac{1}{32\pi^2} \left(\frac{2x_1^3 [3x_1(x_1 - x_2 - x_3) + x_2^2 + x_2x_3 + x_3^2] \ln x_1}{(x_1 - x_2)^3(x_1 - x_3)^3} - \frac{2(3x_1^2 - 3x_1x_2 + x_2^2)x_2 \ln x_2}{(x_1 - x_2)^3(x_2 - x_3)} + \frac{2(3x_1^2 - 3x_1x_3 + x_3^2)x_3 \ln x_3}{(x_1 - x_3)^3(x_2 - x_3)} \right)$$

$$\begin{aligned}
& -\frac{x_1[5x_1^2 - 3x_1(x_2 + x_3) + x_2x_3]}{(x_1 - x_2)^2(x_1 - x_3)^2}), \\
G_5(x_1, x_2, x_3) = & \frac{1}{16\pi^2} \left(\frac{x_1^2(2x_1 - x_2 - x_3) \ln x_1}{(x_1 - x_2)^2(x_1 - x_3)^2} \right. \\
& + \frac{x_2(x_2 - 2x_1) \ln x_2}{(x_1 - x_2)^2(x_2 - x_3)} \\
& \left. - \frac{x_1}{(x_1 - x_2)(x_1 - x_3)} + \frac{x_3(2x_1 - x_3) \ln x_3}{(x_1 - x_3)^2(x_2 - x_3)} \right). \quad (35)
\end{aligned}$$

$$\begin{aligned}
G_6(x_1, x_2, x_3, x_4) = & \frac{1}{16\pi^2} \left[\frac{x_1^2 \ln x_1}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)} \right. \\
& + \frac{x_2^2 \ln x_2}{(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)} \\
& + \frac{x_3^2 \ln x_3}{(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)} \\
& \left. + \frac{x_4^2 \ln x_4}{(x_4 - x_1)(x_4 - x_2)(x_4 - x_3)} \right], \\
G_7(x_1, x_2, x_3, x_4) = & \frac{1}{16\pi^2} \left[\frac{x_1 \ln x_1}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)} \right. \\
& + \frac{x_2 \ln x_2}{(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)} \\
& + \frac{x_3 \ln x_3}{(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)} \\
& \left. + \frac{x_4 \ln x_4}{(x_4 - x_1)(x_4 - x_2)(x_4 - x_3)} \right]. \quad (36)
\end{aligned}$$

References

1. K. Abe et al., (T2K Collab), Phys. Rev. Lett. **107**, 041801 (2011)
2. J. Ahn et al., (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012)
3. F. An et al., (DAYA-BAY Collab), Phys. Rev. Lett. **108**, 171803 (2012)
4. E. Ma, A. Natale, O. Popov, Phys. Lett. B **764**, 114–116 (2015)
5. I. Girardi, S.T. Petcov, A.V. Titov, Nucl. Phys. B **894**, 733–768 (2015)
6. P. Ghosh, S. Roy, JHEP **0904**, 069 (2009)
7. P. Ghosh, P. Dey, B. Mukhopadhyaya, S. Roy, JHEP **1005**, 087 (2010)
8. S.T. Petcov, Sov. J. Nucl. Phys. **25**, 340 (1977)
9. G. Mann, T. Riemann, Ann. Phys. **40**, 334 (1984)
10. J. I. Illana, M. Jack, T. Riemann, [arXiv:hep-ph/0001273](https://arxiv.org/abs/hep-ph/0001273)
11. J.I. Illana, T. Riemann, Phys. Rev. D **63**, 053004 (2001)
12. ATLAS Collaboration, Phys. Lett. B **716**, 1 (2012)
13. CMS Collaboration, Phys. Lett. B **716**, 30 (2012)
14. CMS Collaboration, JHEP **06**, 081 (2013)
15. P. Paradisi, JHEP **10**, 006 (2005)
16. J. Gierbach, S. Mertens, U. Nierste, S. Wiesenfeldt, JHEP **05**, 026 (2010)
17. J. Rosiek, P.H. Chankowski, A. Dedes, S. Jager, P. Tanedo, Comput. Phys. Commun. **181**, 2180 (2010)
18. H.B. Zhang, T.F. Feng, G.H. Luo, Z.F. Ge, S.M. Zhao, JHEP **07**, 069 (2013)
19. P.F. Perez, Phys. Lett. B **711**, 353 (2012)
20. J.M. Arnold, P.F. Perez, B. Fornal, S. Spinner, Phys. Rev. D **85**, 115024 (2012)
21. R. Barbieri, A. Masiero, Nucl. Phys. B **267**, 679 (1986)
22. S. Dimopoulos, L.J. Hall, Phys. Lett. B **207**, 210 (1987)
23. P.F. Perez, M.B. Wise, JHEP **1108**, 068 (2011)
24. P.F. Perez, M.B. Wise, Phys. Rev. D **82**, 011901 (2010)
25. T.F. Feng, S.M. Zhao, H.B. Zhang et al., Nucl. Phys. B **871**, 223 (2013)
26. S.M. Zhao, T.F. Feng, B. Yan et al., JHEP **1310**, 020 (2013)
27. S.M. Zhao, T.F. Feng, X.J. Zhan, H.B. Zhang, B. Yan, JHEP **1507**, 124 (2015)
28. B. Chen, S.M. Zhao, B. Yan, H.B. Zhang, T.F. Feng, Commun. Theor. Phys. **61**, 619–623 (2014)
29. S.M. Zhao, T.F. Feng, H.B. Zhang et al., JHEP **1411**, 119 (2014)
30. S.M. Zhao, T.F. Feng, H.B. Zhang et al., Phys. Rev. D **92**, 115016 (2015)
31. J.C. Sen, Phys. Rev. **113**, 679 (1959)
32. H.C. Chiang, E. Oset, T.S. Kosmas, A. Faessler, J.D. Vergados, Nucl. Phys. A **559**, 526 (1993)