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Linking infrared and ultraviolet parameters of pion-like states in strongly coupled gauge theories

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Abstract It has been shown previously that in a relativistic constituent-quark model, predictions for the electromagnetic form factor of the π meson match not only experimental data but also, in the limit of large momentum transfers, the asymptotics derived from Quantum Chromodynamics (QCD). This is remarkable since no parameters are introduced to provide for this infrared-ultraviolet link. Here, we follow this approach, going beyond QCD. We obtain numerical relations between the gauge coupling constant, the decay constant and the charge radius of the pion-like meson in general strongly-coupled theories. These relations are compared to published lattice results for $SU(2)$ gauge theory with two fermion flavours, and a good agreement is demonstrated. Further applications of the approach, to be explored elsewhere, include composite Higgs and dark-matter models.

1 Introduction

Obtaining first-principle quantitative predictions concerning strongly coupled bound states remains the main challenge of quantum field theory. The only available direct method, lattice calculations, is complicated and resource consuming in practical implementation, especially when light fermions are involved. Numerous semi-phenomenological approaches have been put forward in order to obtain quantitative description of bound states in Quantum Chromodynamics (QCD), mesons and baryons. In many cases, underlying symmetries of the theory were used as guiding principles in these calculations.

One of these approaches, based on Dirac [1] Instant Form of the Relativistic Hamiltonian Dynamics (RHD; for reviews, see Refs. [2–4]), has been particularly successful in description of electromagnetic properties of light mesons. A distinctive feature of this approach is that the Poincaré invariance is fully exploited and kept unbroken at all steps. The canonical example of the approach's application was the calculation of the π-meson electromagnetic form factor, F_π, as a function of the momentum transfer, Q^2. It has been shown that quantitative results for $F_\pi(Q^2)$ are robust with respect to variations in the most uncertain ingredient of the approach, the phenomenological wave function $\phi(k)$, provided the pion decay constant, f_π, is fixed [5]. Numerical results therefore depend on two phenomenological parameters, one combination of which is fixed through f_π. The remaining combination was fitted (in 1998) from the condition that $F_\pi(Q^2)\big|_{Q^2 \to 0}$ reproduced correctly the experimental data points [6], that is, the pion charge radius [5]. Subsequent measurements [7,8] of $F_\pi(Q^2)$ spanning an order of magnitude larger values of Q^2 agreed with the prediction of the model surprisingly well [9]. In addition, it has been demonstrated that the very same model predicts also the correct QCD asymptotics [10–14] of $F_\pi(Q^2)$ at large Q^2, reproducing both the functional dependence [15] on Q^2 and, for the very same choice of parameters, the numerical coefficient [16]. This is achieved when the constituent-quark mass is switched off [17], independently of the way of this switching.

Altogether, these successes look unusual and may indicate that the model catches some basic dynamical features of the π meson thanks to the full incorporation of the relativistic invariance. This might open the possibility to go beyond QCD and to apply the model to pion-like bound states in other hypothetical strongly coupled theories, e.g. those describing composite Brout–Englert–Higgs scalar [18–20] and/or composite dark-matter particles [21]. At the same time this invites further quantitative tests of the approach which, given the lack of well established strongly coupled gauge theories in Nature, may be performed only by comparison with lattice results or limiting cases. This is the subject of the present work.

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The rest of the paper is organized as follows. In Sect. 2, we give a very brief account of the model and refer to previous works where all details can be found. We discuss in more detail manifestations of the success of the model which motivate the present study. Section 3 presents the method allowing to relate quantitatively parameters of the gauge theory and low-energy meson properties. In Sect. 4, we address a few examples of non-QCD gauge theories for which lattice calculation of the form factor of a pion-like state has been reported, and compare lattice results with those obtained within our approach. We briefly conclude and discuss future applications of our method in Sect. 5.

2 Motivation

The model we discuss here [22–26] has been developed for the description of electroweak properties of light strongly interacting two-particle bound states and has been successfully applied to the deuteron [27], π [5,9], ρ [28,29] and K [30] mesons. The model is based on the instant form of the relativistic Hamiltonian dynamics (see e.g. Ref. [3]), supplemented by the so-called modified impulse approximation [24], which is the key ingredient of the approach since it removes certain disadvantages of the instant form. The form factors can be obtained with the use of the Wigner–Eckart theorem for the Poincaré group [31]. Here, we will focus on the bound states similar to the charged π meson, for which all details and explicit formulae are given in Refs. [5,9] (see the Appendix of Ref. [16] for a useful summary). The essential feature of the method, which distinguishes it from many other approaches (see e.g. Ref. [32] for a recent review), is that the explicit Poincaré invariance is kept throughout the calculation.

The model of the electromagnetic structure of the π meson has two principal phenomenological parameters, the constituent-quark mass M and the meson wave-function scale b. The latter is a dimensionful parameter whose definition depends on the particular choice of the wave function; however, it has been shown in Ref. [5] that the dependence on the shape of the wave function diminishes provided the pion decay constant, f_π, is fixed (like other observable quantities, it is expressed through M and b, see Sect. 3). In Ref. [5], the dependence of the form factor on the choice of the wave function was studied. Three different wave-function shapes were considered and it has been shown that the dependence on the shape of the wave function diminishes provided the pion decay constant, f_π, is fixed (like other observable quantities, it is expressed through M and b, see Sect. 3). At large Q^2, the variation in the value of F_π at fixed M and f_π with the change of the wave function does not exceed $\sim 3\%$ for most values of parameters, though it becomes larger at very large values of $M \gg f_\pi$. We estimate the related systematic

uncertainty of the method as $\sim 5\%$ and, following previous studies [16], use the power-law type wave function [33] in the momentum (k) space,

$$\phi(k) \propto \left(4\left(k^2 + M^2\right)\right)^{1/4} k \left(k^2/b^2 + 1\right)^{-3}, \qquad (1)$$

for our numerical examples.

In principle, the model allows for inclusion of two other parameters which have minor impact on numerical results and were never varied; they are related to deviations from point-like constituent quarks and affect the form factor at high momentum transfers, Q^2. One is the coefficient C in the relation between the constituent-quark mass M and its effective radius, C/M; it was always fixed at $C = 0.3$ in previous works and so we do here. The impact of the second parameter, the sum s_q of the anomalous magnetic moments of quarks, is clearly within the overall uncertainty of the method for hypothetical non-QCD theories. For QCD, s_q was determined with the help of Gerasimov sum rules [34] and was found to be $s_q \approx 0.03$; it enters the expressions for the form factor in a sum with quark and antiquark charges (equal to $1 \gg 0.03$) and is therefore expected to have a minor impact on the result. Indeed, we have checked numerically that the effect of its variation within $0 \leq s_q \leq 0.1$ on the form-factor asymptotics is negligible compared to other uncertainties of the model. Similar sum rules justifying a particular value of s_q are unavailable for a general non-QCD theory, and in the numerical calculations presented here we simply put $s_q = 0$.

We turn now to the motivations behind the extension of the π-meson model to a general strongly coupled theory, which we propose and start to study here.

The first motivation is the predictivity of the model. In 1998, the two parameters, M and b, were fixed by fitting two observable quantities, the decay constant, f_π, and the charge radius, $\langle r_\pi^2 \rangle^{1/2}$, of the π meson. This made it possible to calculate the form factor, $F_\pi(Q^2)$, as the function of the momentum transfer. Figure 1 presents the predicted function $F_\pi(Q^2)$ together with experimental data points: data shown in gray were obtained earlier and were used in the fit through $\langle r_\pi^2 \rangle^{1/2}$, while black data points, spanning a further order of magnitude in Q^2, have been obtained after the prediction. The new data have demonstrated an impressive agreement with the calculation ($\chi^2 \approx 4.4$ for 9 degrees of freedom, no free parameter).

The second motivation is the possibility to relate infrared and ultraviolet physics within a single model. In Ref. [15], it has been shown that the model reproduces the *functional form* of the QCD asymptotics [10,11] for $F_\pi(Q^2)$ provided the constituent-quark mass is switched off, $M \to 0$, at $Q^2 \to \infty$. Moreover, numerical calculations of Ref. [16] have demonstrated that the *coefficient* of the asymptotics [12–14] is also reproduced correctly. This is achieved independently of the way M is switched off (see Fig. 2), and
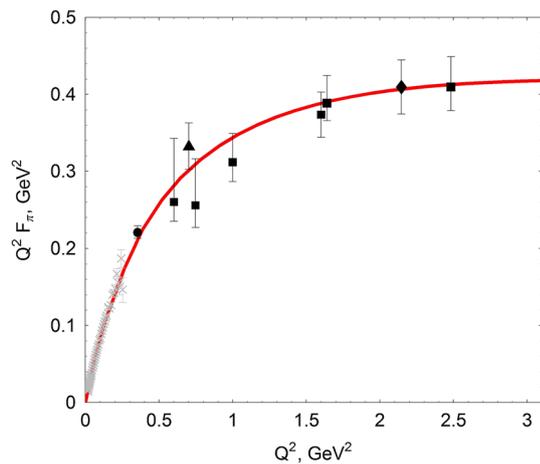
Fig. 1 Motivation I. The π meson form factor: model predictions (full line) versus experimental data. Black data points have been published after the prediction, while gray data points at $Q^2 \to 0$ were used to make the prediction. Symbols correspond to different data sets: Refs. [6] (crosses), [35] (reanalyzed in Ref. [7], circle), [36] (reanalyzed in Ref. [7], triangle), [8] (diamond) and [7] (squares)
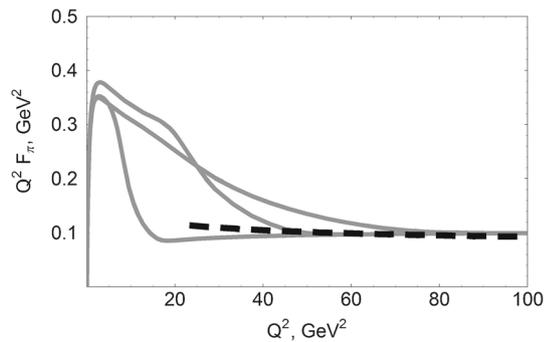

Fig. 2 Motivation II. Once the constituent-quark mass is switched off, $F_\pi(Q^2)$ reaches the asymptotics predicted from QCD [12–14], shown as the thick dashed line. Three thin curves correspond to different ways of switching M off (the explicit expressions of $M(Q^2)$, inspired by Ref. [17], are given in Ref. [16]). The asymptotics does not depend on the details how M is set to zero but does depend on its infrared value

therefore the correct ultraviolet asymptotics of $F_\pi(Q^2)$ is reproduced without any additional parameters (with respect to those two present in the successful infrared model). Reaching the QCD asymptotics is an infrequent, though welcome, feature of infrared models of mesons; to our best knowledge, no other model achieves it without introduction of new tunable parameters. For the QCD case, unusual predictivity of the model allowed for construction of a "parameter-free model of electromagnetic properties of light mesons", successfully connecting corresponding observables of π, ρ and K mesons [28–30].

Meanwhile, methods to relate infrared and ultraviolet physics of strongly coupled gauge theories are in demand for numerous extensions of the Standard Model, notably including models with composite Brout–Englert–Higgs scalars (see

e.g. Refs. [18–20] and references therein) and/or dark-matter particles (in particular, within the asymmetric dark-matter scenario [37,38]; see e.g. Ref. [21] for a recent discussion and collection of references). Recently, the interest in models with new strong dynamics increased considerably because of their possible relation to flavour anomalies observed at the Large Hadron Collider (LHC) [39,40]. In the context of our study, it is interesting to note that in certain scenarios, electromagnetic form factors of composite dark-matter particles determine their cross sections [41] and their knowledge is therefore crucial for tests of the viability of the models and of their experimental and observational predictions.

Theoretical approaches to calculation of these observables are limited to lattice gauge theories whose interpretation in terms of the underlying continuum renormalizable theory is not always straightforward (see Sect. 4). Another problem for practical applications of the lattice approach is that the numerical methods required to reach reasonable precision in description of light fermion bound states consume lots of computational resources. It is therefore tempting to generalize our method to non-QCD theories, which would open a possibility of fast quantitative description of light bound states (at least of their electromagnetic properties). But how can one verify that the method works and that both successes (prediction of the measured form factor and parameter-free matching with the QCD asymptotics) were not just lucky coincidences? In the absence of a first-principle derivation of our results from a gauge field theory, we can nevertheless test it by comparison with particular lattice results, see Sect. 4.

3 Relating the form factor, the meson decay constant and the gauge coupling constant.

Consider a QCD-like strongly coupled gauge theory allowing for a pion-like state, Π (we keep the notion π for the QCD π meson). "Pion-like" means that Π is a meson (a bound state of a fundamental q_i and antifundamental $\bar{q}_j$ fermions) and is a light pseudo-Goldstone boson of some broken global symmetry acting on q and $\bar{q}$. The bound state arises because of confining interaction determined by the running gauge coupling constant $\alpha(Q^2)$. At one loop, it is expressed by a familiar formula,
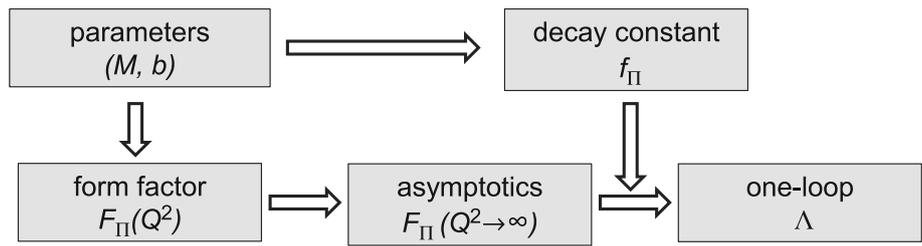
$$\alpha^{1-\text{loop}}(Q^2) = 1 / \left(4\pi b_0 \log \left(Q^2 / \Lambda^2 \right) \right), \tag{2}$$

where

$$b_0 = \frac{1}{48\pi^2} \left(11 C_2^A - 4 \sum_f T_f \right), \tag{3}$$

C_2^A is the Casimir invariant of the adjoint representation of the gauge group and $T_f/2$ is the Dynkin index

Fig. 3 A scheme of relating phenomenological parameters (M, b) to physical parameters (f_Π, Λ); see text

for the representation of fermionic fields (for $SU(N_c)$ gauge theory with N_f flavours of fundamental fermions, $b_0 = \left(11N_c - 4N_f\right)/(48\pi^2)$). This introduces the one-loop dynamical scale Λ which, by the definition (2), is in one-to-one correspondence with $\alpha^{1-\text{loop}}$ calculated at a certain value of Q^2.

To speak about electromagnetic properties of Π, we allow q and $\bar{q}$ to be charged under an extra $U(1)$ gauge group, e.g. the electromagnetic one. Then, the electromagnetic form factor $F_\Pi(Q^2)$ may be defined in the usual way. It is non-zero in two cases: either the sum of charges of q and $\bar{q}$ is nonzero, or it is zero but q and $\bar{q}$ have different masses (corresponding QCD examples are the charged pion and the neutral kaon). In the π-meson model, we kept the masses of constituent u ($\bar{u}$) and d ($\bar{d}$) quarks identical, and therefore we keep them identical for q and $\bar{q}$ here, though the method can be easily generalized to the other case.

The approach described in detail in Refs. [5,9] (see the Appendix of Ref. [16] for a collection of all necessary formulae) allows one to calculate $F_\Pi(Q^2)$, starting from two phenomenological parameters, M and b, discussed above. It includes switching off the constituent-quark mass M smoothly at a certain energy scale, much lower than the values of Q^2 at which one expects the high-energy asymptotics to settle down. An explicit expression for the meson decay constant f_Π relates it to the parameters of the model, M and b,

$$f_\Pi = \frac{M\sqrt{3}}{\pi} \int \frac{k^2 \, dk}{(k^2 + M^2)^{3/4}} \, u(k). \tag{4}$$

On the other hand, at large Q^2, the gauge-theory asymptotics [12–14] is

$$Q^2 F_\Pi(Q^2)\Big|_{Q^2\to\infty} \to 8\pi f_\Pi^2 \alpha^{1-\text{loop}}(Q^2). \tag{5}$$

The right-hand side of Eq. (5) depends on two quantities, f_Π and Λ. Since f_Π is determined from M and b by means of Eq. (4), calculation of $F_\Pi(Q^2)$ at large Q^2 allows to determine one-loop Λ of the underlying gauge theory. In this way, we relate two phenomenological parameters, M and b, to two physical parameters, f_Π and Λ. Our method thus makes it possible to calculate $F_\Pi(Q^2)$ in the infrared region, $Q^2 < \Lambda^2$, starting from Λ and f_Π. The logic of the method is illustrated schematically in Fig. 3.
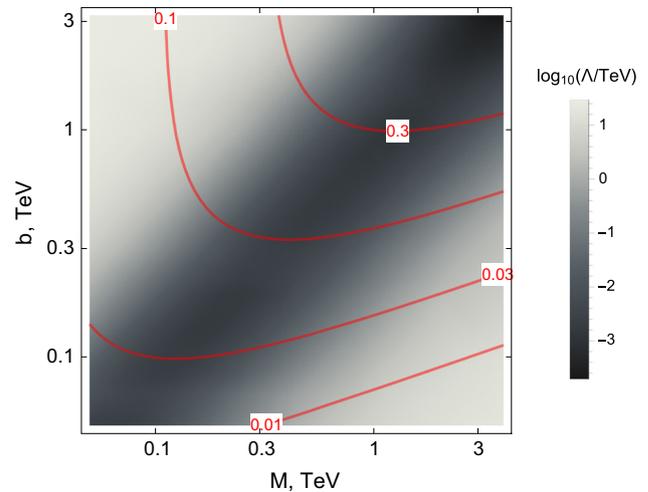

Fig. 4 Relation between (M, b) and (Λ, f_Π) for an $SU(2)$ gauge theory with $N_f = 2$ fundamental fermions. Different levels of shading correspond to different values of Λ, see the scale to the right. Full thick red lines are contours of equal F_Π corresponding to the values written on the contours (in TeV)

We turn now to a numerical realization of the method. We illustrate it for an $SU(2)$ gauge theory with $N_f = 2$ fundamental fermions. Our approach does not allow to trace explicitly the influence of the current quark masses; together with the choice of the gauge group and the matter content, it determines the value of f_Π which is treated as independent parameter of the model (different values of f_Π at a fixed value of Λ correspond to different gauge groups and/or different current quark masses). We choose opposite charges of q_1 ($+1/2$) and q_2 ($-1/2$), so that $\Pi = q_1 \bar{q}_2$ has the unit charge. As we have already pointed out, we assume equal masses of q_1 and q_2 in this example.

Figure 4 illustrates the calculation outlined above: f_Π and Λ versus M and b. All these quantities are dimensionful; keeping in mind future applications to composite models, see also Sec. 4, for which $f_\Pi = v = 246$ GeV, we change M and b between 0.05 and 10 TeV and determine Λ from asymptotics above $Q^2 = 900$ TeV^2. It is interesting to note that the inverse relation is not single-valued: as one can see from Fig. 4, there are two pairs of (M, b) corresponding to the same values of (f_Π, Λ) and the branches $M < b$ and $M > b$.
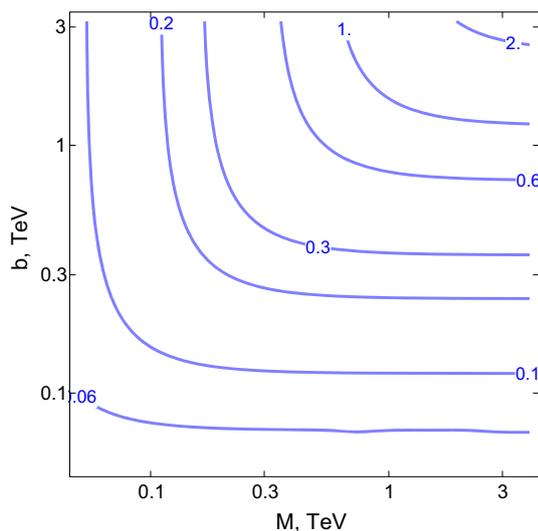
Fig. 5 Inverse charge radius of Π versus M and b in an $SU(2)$ gauge theory with $N_f = 2$ fundamental fermions. Full thick blue lines are contours of equal $1/r_\Pi$ corresponding to the values written on the contours (in TeV)

Fig. 6 Inverse charge radius of Π versus f_Π in an $SU(2)$ gauge theory with $N_f = 2$ fundamental fermions. Different colors correspond to different values of Λ, see the color code to the right. Full lines correspond to the branch $M < b$ ("strong coupling"), dashed lines correspond to the branch $M > b$ ("heavy quarks")

To characterise the behaviour of $F_\Pi(Q^2)$ at $Q^2 \to 0$, it is convenient to determine the Π charge radius, $r_\Pi \equiv \langle r_\Pi^2 \rangle^{1/2}$,

$$r_\Pi^2 = -6 \left. \frac{dF_\Pi(Q^2)}{dQ^2} \right|_{Q^2 \to 0}.$$

Note that it is the charge radius which determines the cross section important for the search of composite dark matter states [41,42], though they may be more complicated than Π. While our approach generates the full $F_\Pi(Q^2)$ function, we will concentrate on r_Π for the moment.

Qualitatively, the behaviour of r_Π at the two branches, see Fig. 5, is easily understood. For $M < b$, the theory is in its strong-coupling regime, the "quarks" are light compared to Λ and the meson wave-function size in the momentum space, b, depends mostly on Λ (the strong interaction) while the size of the meson r_Π gets contribution both from M and b. The $M > b$ case corresponds to (relatively) heavy, almost point-like "quarks", and the size of the meson is fully determined by the interaction.

Figure 6 presents the dependence of r_Π on f_Π for different values of Λ. The leading dependence is $1/r_\Pi \propto f_\Pi$ with the coefficient depending on Λ. For the "strong-coupling" branch and for a fixed f_Π, varying Λ gives only small (but measurable) corrections.[1]

4 Comparison to lattice results

In this section, we use published lattice results on $F_\Pi(Q^2)$ in $SU(2)$ gauge theory with $N_f = 2$. This result was presented in Ref. [42], while some required information about the lattice calculation was given in Ref. [43], based on the same numerical simulations.

Table I of Ref. [42] gives the values of $F_\Pi(\hat{Q}^2)$ calculated for three versions of the lattice calculations with different parameters of the lattice Lagrangian. Here, $\hat{Q}^2 \equiv (Qa)^2$, where a is the lattice spacing, fixed from the condition $f_\Pi = 246$ GeV (the latter choice is of course arbitrary and is motivated by studies of theories with a composite Brout–Englert–Higgs scalar),

$$f_\Pi = \frac{1}{a} Z_a f_\Pi^{\text{lat}},$$

Z_a is the renormalization constant, whose value used in Refs. [42,43] is

$$Z_a = 1 - k \left(\frac{g_0}{4\pi} \right)^2 \frac{N_c^2 - 1}{2N_c}.$$

In this expression, g_0 is the bare gauge coupling constant (related to the lattice coupling constant, β, by $\beta = 2N_c/g_0^2$), $N_c = 2$ is the "number of colours" in the gauge group $SU(N_c)$ and $k \approx 15.7$ is a numerical coefficient determined in Ref. [44]. Values of f_Π^{lat} for all lattice calculations used are given in Table II of Ref. [43].

In this way, we know f_Π and $F_\Pi(Q^2)$, but we also need to know the gauge coupling constant in the large-Q^2 limit, or equivalently one-loop Λ, to perform a parameter-free test of

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[1] For the QCD π meson, described by this branch, it was this correction which allowed to trace the correct Λ-dependent numerical coefficient in the asymptotics, see Ref. [16].
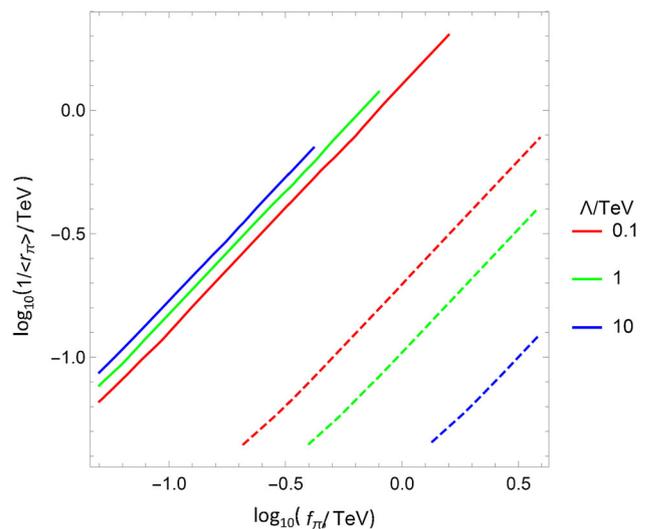
Table 1 Parameters of the lattice calculations (see text)

Case	(β, m_0)	g_0	Z_a	f_Π^{lat}	$1/a$, GeV	Λ_{lat}, GeV	Λ, GeV
I	(2.2, −0.72)	1.348	0.864	0.0664 (5)	4286	9.04	298
II	(2.2, −0.75)	1.348	0.864	0.0457 (7)	6227	13.14	423
III	(2.0, −0.947)	1.414	0.851	0.0754 (8)	3834	15.03	497

the results obtained in our approach. It is, generally, a non-trivial task to relate Λ and g_0, because lattice and continuum models use different ways of renormalization (see e.g. Refs. [45,46] for reviews). To extract the physical value of α or Λ from a lattice calculation, a certain observable (related, for instance, to the force of interaction between fermions) is usually calculated. These calculations have not been performed for configurations used in Ref. [42].

Fortunately, for our purposes, it is sufficient to follow a different approach. Indeed, what we need in Eq. (5) is, by definition, the one-loop coupling constant which enters the asymptotics at large Q^2. In the asymptotical region, one can expand coupling constants, determined in different schemes, in powers of each other (see e.g. Ref. [47]). One obtains an expression for the lattice strong-coupling scale Λ_{lat},

$$\Lambda_{\text{lat}} = \frac{1}{a} \left(b_0 g_0^2 \right)^{-\frac{b_1}{2b_0^2}} \exp\left(-\frac{1}{2b_0 g_0^2} \right) \times c,$$

where the correction

$$c = \exp\left[-\int_0^{g_0} \left(\frac{1}{\beta(t)} + \frac{1}{b_0 t^3} - \frac{b_1}{b_0^2 t} \right) dt \right] \simeq 1 + \mathcal{O}(g_0^2)$$

and the two-loop beta function is

$$\beta(t) = -t^3 \left(b_0 + b_1 t \right);$$

the coefficient b_0 is determined in Eq. (3) and, for an $SU(N_c)$ gauge theory with N_f flavours of fermions in the fundamental representation,

$$b_1 = \frac{1}{(4\pi)^2} \left(\frac{34}{3} N_c^2 - \frac{10}{3} N_c N_f - \frac{N_c^2 - 1}{N_c} N_f \right).$$

The relation between the continuum Λ (determined, e.g., in the $\overline{\text{MS}}$ renormalization scheme) and Λ_{lat} reads as

$$\Lambda = \Lambda_{\text{lat}} \exp\left(-\frac{l_0}{2b_0} \right),$$

where

$$l_0 = \frac{1}{8N_c} + k_c N_c + k_f N_f,$$

the coefficient $k_c = -0.16995599$ was determined in Ref. [48] and, for the Wilson fermions used in the lattice calculation we discuss, $k_f = 0.0066959993$, see Ref. [49].

The precision of this method is not very high. The main source of errors is in the use of perturbative expressions for Λ

Table 2 Comparison of r_Π obtained in our calculation with lattice results

Case	r_Π, TeV^{-1}		
	Lattice	$M < b$	$M > b$
I	1.6	1.9	1.0
II	1.2	1.9	1.0
III	1.4	1.8	1.0

and Z_a (the latter affects Λ_{lat} through the value of a). Particular values of the coupling constants used for the simulations we address correspond to $g_0 \sim 1.4$, so that the precision is limited by the loop factor $\sim g_0^2/(4\pi) \sim 16\%$. Additional uncertainties appear in the lattice calculation of f_Π^{lat} and of other quantities, so we use a conservative estimate of $\pm 20\%$ precision in Λ (remember that f_Π is fixed and all uncertainties in f_Π^{lat} are translated into those of a).

Table 1 lists three different sets of lattice data we use, determined by β and by the bare fermion mass m_0, together with useful values of corresponding parameters. Table 2 gives, for the three cases, results of the lattice calculations of $F_\Pi(Q^2)$ (data points) together with calculations by our method, Sect. 3, for values of $f_\Pi = 246$ GeV and Λ given in Table 1. We see that, within the precision behind these numbers, the agreement is reasonable, which represents a highly nontrivial test of our method.

5 Conclusions and outlook

In this work, we started from a method to calculate the electromagnetic form factor of the π meson, $F_\pi(Q^2)$, which has been shown previously (i) to predict experimental values of $F_\pi(Q^2)$, measured later, without tuning of parameters, and (ii) to obtain correct QCD asymptotics at $Q^2 \to \infty$, again with no additional parameters introduced or tuned. This allowed us to link infrared and ultraviolet regimes in a non-trivial way. Then we pretended that this method is general and applied it to non-QCD gauge theories, concentrating on a pion-like state Π. We presented a method to obtain general numerical relations between intrinsic parameters of the theory, f_Π and Λ, and phenomenological parameters of the model, which allows one to calculate the form factor, $F_\Pi(Q^2)$, starting from known values of these physical parameters, without fitting anything to experimental data. We

then tested the outcome of this method versus known lattice results for $F_\Pi(Q^2)$ for a $SU(2)$ gauge theory with $N_f = 2$ flavours of fundamental fermions and obtained a reasonable agreement in accordance with the precision, limited mostly by uncertainties in determination of Λ corresponding to the lattice calculations.

Together with the success in the description of the real QCD π meson, this result supports the proposal that our method, based on Relativistic Hamiltonian Dynamics and presumably aimed at low energies only, may nevertheless be used for calculation of electromagnetic properties of bound states in strongly coupled gauge theories defined in their ultraviolet limit. The physical reason behind this is probably related to the relativistic invariance carefully preserved throughout our calculations, in contrast with some other approaches.

At least in its present form, the approach is not universal since (i) it addresses a particular problem, calculation of the electromagnetic form factor, and (ii) it uses, as the input parameters, the gauge coupling constant (through Λ) and the decay constant of the pion-like state, f_Π. The latter, in principle, should be expressed through parameters of the Lagrangian, the coupling constant and fermion masses, but this is beyond the capabilities of our method. Still, it is an unusual success and opens the possibility of physical applications, the most straightforward one dealing with composite-Higgs models with composite dark-matter particles. In this case, f_Π is fixed from the electroweak symmetry breaking and is therefore known. The dark-matter particle is another, different from Π, meson, but its phenomenologically interesting cross section is determined by its electromagnetic form factor [41], which can be calculated by the method suggested here. This interesting approach will be followed elsewhere.

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