Semileptonic B and B_s decays involving scalar and axial-vector mesons

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Abstract We report our theoretical calculations on the branching fractions for the semileptonic B and B_s decays based on the form factors in the covariant light-front quark model. That is, $B(B_s) \rightarrow (P, V, S, A)\ell\nu_\ell$, where P and V denote the pseudoscalar and vector mesons, respectively, while S denotes the scalar meson with mass above 1 GeV and A the axial-vector meson. The branching fractions for the semileptonic $B \rightarrow P$ and V modes have been measured very well in experiment and our theoretical values are in good agreement with them. The ones for $B \rightarrow S$ and A modes are our theoretical predictions. There is little experimental information on the semileptonic B_s decays although much theoretical effort has been done. In addition, we predict the branching fractions of $B \rightarrow D_0^*(2400)\ell\bar{\nu}_\ell$ and $B_s \rightarrow D_{s0}^{*-}(2317)\ell\bar{\nu}_\ell$ as $(2.31 \pm 0.25) \times 10^{-3}$ and $(3.07 \pm 0.34) \times 10^{-3}$, in order, assuming them as the conventional mesons with quark-antiquark configuration. The high luminosity e^+e^- collider SuperKEKB/Belle-II is running, with the data sample enhanced by a factor of 40 compared to Belle, which will provide huge opportunity for the test of the theoretical predictions and further help understand the inner structure of these scalar and axial-vector mesons, e.g., the glueball content of $f_0(1710)$ and the mixing angles for the axial-vector mesons. These decay channels can also be accessed by the LHCb experiment.

1 Introduction

The CP violation is one of the necessary Sakharov conditions for the emergence of matter-antimatter asymmetry [1–3], which is a key question in the nature. The Cabibbo–Kobayashi–Maskawa (CKM) matrix [4, 5] has been an indispensable skeleton of the Standard Model (SM), which successfully describes the CP violation in the quark sector. In the CKM matrix, the unitarity relation $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$ is fulfilled, and any deviation of this unitarity constraint will be a signal of New Physics. Thus the precise determination of the CKM matrix elements has been a key task and activity in the community of flavor physics [6, 7]. A recent review for the leptonic and semileptonic B decays is compiled in Ref. [8]. In our current work, we will consider $B(B_s) \rightarrow (P, V, S, A)\ell\nu_\ell$ decay,¹ where P and V denote the pseudoscalar and vector mesons, respectively, while S denotes the scalar meson with mass above 1 GeV and A the axial vector meson. In these processes, either $|V_{ub}|$ or $|V_{cb}|$ is involved. And notably, there is a mismatch for the extraction of $|V_{ub}|$ from the inclusive and exclusive decays, which is the so-called $|V_{ub}|$ puzzle [6, 19]. The measurement of these channels can certainly help for rendering more information on the determination of CKM matrix elements, at least as a

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¹ We will consider B^+ as an example, and B^0 decay is the same as B^+ case due to the isospin symmetry. This point can be verified by the branching fractions $\mathcal{B}(B^+ \rightarrow \bar{D}^0\ell\nu_\ell) = \mathcal{B}(B^0 \rightarrow D^-\ell\nu_\ell) = (2.20 \pm 0.10)\%$ [6], and $\mathcal{B}(B^+ \rightarrow \pi^0\ell\nu_\ell) \approx \mathcal{B}(B^0 \rightarrow \pi^-\ell\nu_\ell)/2$, $\mathcal{B}(B^+ \rightarrow \rho^0\ell\nu_\ell) \approx \mathcal{B}(B^0 \rightarrow \rho^-\ell\nu_\ell)/2$ [6] due to the additional factor of $1/\sqrt{2}$ appearing in the quark components in the neutral π and ρ . We also note that these channels could also be treated in the PQCD or light-cone models, e.g., in Refs. [9–18].

supplement to the conventional exclusive decays. Inversely, to calculate the branching fractions, we rely on their concrete values from Particle Data Group (PDG) [6]: $|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$, and $|V_{ub}| = (3.94 \pm 0.36) \times 10^{-3}$ as a combination of the determinations from inclusive and exclusive decays.

For the axial-vector mesons which contain the superposition of quark contents $s\bar{s}$ and $q\bar{q} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$, $f_1(1285)$ and $f_1(1420)$, $h_1(1170)$ and $h_1(1380)$, and K_{1A} and K_{1B} do mix. The mixing angles are not fully fixed yet, see e.g., Refs. [20,21]. The structure of the scalar meson is more obscure, see the review [22]. For example, f_0 states ($f_0(1370)$, $f_0(1500)$, $f_0(1710)$) are interpreted as the mixed states of $q\bar{q}$, $s\bar{s}$ and glueball (G), but which one consists mainly of G is not fully determined.² The observables depend on, or even are sensitive to these mixing angles. The three-body semileptonic decay is an ideal place to study the weak hadronic transition form factor as well as the underlying structure of such mesons due to the absence of the final-state interactions (FSIs) between hadrons.³ As such, the theoretical prediction of the relevant semileptonic decay channels becomes crucial for the future experimental measurement.

From the experimental point of view, Belle has accumulated huge data samples, that can be exploited to measure the branching fractions and hadronic transition form factors for the various semileptonic decay channels, in order to test or constrain the various theoretical models. There are $(772 \pm 11) \times 10^6$ $B\bar{B}$ [30] and $(6.53 \pm 0.66) \times 10^6$ $B_s\bar{B}_s$ pairs [31] collected at $\Upsilon(4S)$ and $\Upsilon(5S)$ resonances, respectively, by the Belle detector at the KEKB asymmetric energy electron-positron collider. The statistics will be enhanced by a factor of 40 for Belle-II, and by the mid of next decade, 50 times more data is expected comparing to the Belle experiment. Our predicted branching fractions are typically in the order of 10^{-5} so that they can be, in principle, easily accessed by the Belle/Belle-II and LHCb experiments.

2 Theoretical framework

The transition form factor is a probe to the inner structure of the hadron. Among the various theoretical tools for the form factor, we will concentrate on the application of the light-front quark model (LFQM) [32,33] and its covariant extension (CLFQM) [34], see also [35–37]. A distinct feature of the light-front frame is that the diagrams involving quarks created out of or annihilating into the vacuum can be eliminated, i.e., only the valence quarks are considered in

the meson or baryon [38]. This leads to a relativistic quark model which retains the $q\bar{q}$ structure for a meson. The relevant form factors can be extracted by choosing the plus component of the matrix elements in the LFQM. In fact, there is spurious contribution proportional to the lightlike four-vector $\omega = (2, 0, 0_\perp)$ in transforming the covariant Feynmann integral into the light-front form, which makes the theory non-covariant. The covariance requires inclusion of the zero-mode effect which eliminates the undesired ω dependence. Such development is elaborated in Ref. [34]. In this way, all the form factors that are necessary to represent the Lorentz structure of a hadronic matrix element can be calculated on the same footing, which is not possible in the standard LFQM. In the framework of CLFQM, the vertex function of a meson (bound state) coupling to its constituent quarks consists of the momentum part and also the spin part, where the former describes the momentum distribution of the constituent quarks, and the latter is constructed from the light-front helicity state involving the Melosh transformation. In the vertex wave function, there is a free parameter β , that will be fixed by the decay constant of the meson. The fermion line is just represented by the relativistic propagator. The electroweak vertex is given by the Standard Model. Following the line of Ref. [34], Cheng, Chua and Hwang have systematically studied the decay constants and form factors for the S - and P -wave mesons in 2003 [39], while an update was done in Ref. [40] in two points: (i) The experimental information of the branching fractions wherever available or the lattice results for the decay constants was used to constrain the parameters β in the wave functions; (ii) the extension to the counterpart with s quark (D_s and B_s) has also been considered.

All the form factors for $B(B_s) \rightarrow (P, V, A, S)$ transitions considered by us have been calculated in Refs. [39,40], and thus we omit to repeat these calculations. However, we note that to make a direct comparison between theory and experiment, we should further provide the branching fractions which are the true observables in experiment and can be directly accessed to test our theoretical predictions. This constitutes one of our main results. Similar studies have been done for the case of charmed meson, D and D_s decay [41], where the formalism corresponding to the differential decay rates and branching fractions are explicitly given. Those expressions are certainly applicable to the B and B_s decay with only some replacements of the relevant masses. While Ref. [41] has aroused great interest of BES colleagues and some of our results have been confirmed, a natural question is what will happen in the beauty B and B_s cases. Combining the running Belle-II, the predictions for the branching fractions of various channels are important for our experimental colleagues. The future measurements will provide valuable information on the form factors as well as the structure of the

² We also note that the knowledge of the two-photon couplings to the scalars is helpful to understand their structures [23,24].

³ The proton–antiproton FSI has been elaborately examined in various decay channels [25–29].

Table 1 The total numbers of D^+D^- , $D^0\bar{D}^0$, $D_s^+D_s^-$ pairs from BESIII collaboration, where in the data-taking plan the future data samples will be 6 times as large as the current ones. The number of $D\bar{D}$ pair is from Ref. [42]

	Current	Planned
D^+D^-	$(8.296 \pm 0.031 \pm 0.064) \times 10^6$	$\sim 5 \times 10^7$
$D^0\bar{D}^0$	$(10.597 \pm 0.028 \pm 0.087) \times 10^6$	$\sim 6.4 \times 10^7$
$D_s^+\bar{D}_s^-$	$\sim 3.3 \times 10^6$	$\sim 2 \times 10^7$

Table 2 The total numbers of $B\bar{B}$ and $B_s^+B_s^-$ pairs from Belle collaboration, while BelleII will have the data samples of 50 times as large as Belle by the mid of next decade. The number of $B\bar{B}$ and $B_s\bar{B}_s$ pairs for Belle collaboration are from Refs. [30,31]

	Belle	BelleII
$B\bar{B}$	$(7.72 \pm 0.11) \times 10^8$	$\sim 3.9 \times 10^{10}$
$B_s\bar{B}_s$	$(6.53 \pm 0.66) \times 10^6$	$\sim 3.3 \times 10^8$

axial-vector mesons and scalar mesons, as already mentioned in the Introduction.

Here we discuss the difference and merit of measuring B meson decays comparing to D decays. The mass of B meson is heavy enough that the methods of Perturbative QCD and Soft-Collinear Effective Theory are available, while there are little reliable theoretical tools to treat the corresponding D decays. Therefore, some B and B_s decay channels considered by us in the manuscript have also been calculated in such approaches, as shown in e.g., Refs. [9–13]. We compared our results with them and a nice agreement is achieved. Consequently, we show the experimental values or the ones reported by PDG in the tables. On the other hand, the D meson has smaller phase space resulting in small branching fractions for decaying into $f_0(1310)$, $f_0(1500)$, $f_0(1710)$, e.g., $\mathcal{B}(D^+ \rightarrow f_0(1710)e^+\nu_e) \sim 10^{-9}$ [41] can not be measured at the BESIII factory due to limited statistics, but for the B meson case, the corresponding branching fraction is at the order of 10^{-6} which is in the scope of Belle and Belle-II detectors.

Besides the larger phase space available for the B decay, the experimental situation is also much better. There are much more B data samples than the ones of D , and especially when considering the running of the Belle-II, as the upgrade of Belle detectors. This constitutes one of our direct motivations to reconsider the B and B_s decay. The total event numbers of D/D_s and B/B_s pairs are listed in Tables 1 and 2, respectively. Clearly, we can see the difference of the order in magnitude. For example, the branching fractions $\mathcal{B}(D^+ \rightarrow f_0(1500)e^+\nu_e) \sim 1 \times 10^{-6}$ and $\mathcal{B}(D^+ \rightarrow f_0(1710)e^+\nu_e) \sim 5 \times 10^{-9}$ [41] are hard to be detected by BESIII collaboration, while Belle-II is capable of measuring

those channels due to $\mathcal{B}(B \rightarrow f_0(1500)\ell\nu_\ell) \sim 8 \times 10^{-6}$ and $\mathcal{B}(B^+ \rightarrow f_0(1710)\ell\nu_\ell) \sim 2 \times 10^{-6}$.

We finally make two supplemental remarks: (1) in Refs. [39,40] all the form factors correspond to the $V - A$ current, and the “x” in Fig.1b therein [39,40] denotes the insertion of W boson, thus we will not consider the $b \rightarrow s$ transition, penguin operators etc. Or stated differently, transitions like $B \rightarrow K$, K^* , $K_1(1270)$, $K_1(1400)$ or $B_s \rightarrow \phi$ will not be considered in the current framework. (2) In general, for the transitions involving the (axial-) vector meson, the form factors receive the contribution of additional B functions, and for our current case, those have been confirmed to be negligibly small by numerical calculations [39]. The issues concerning the self-consistency and covariance of light-front quark models are discussed in Ref. [43].

3 Results and discussion

In this part, we report our theoretically predicted branching fractions for various semileptonic decay channels considered by us, and discuss some details and implications.

In Table 3, we list out the mesons that we considered in the B and B_s semileptonic decays:

- P denotes the pseudoscalar bosons containing π , K , η , η' , D^0 , D_s ,
- V contains the vector mesons ρ , ω , ϕ , K^* , D^* , D_s^* .

S contains the heavy scalar nonet $a_0(1450)$, $f_0(1370)$, $f_0(1500)/f_0(1710)$, $K^*(1430)$ suggested by $q\bar{q}$ quark model [44], and the charmed mesons $D_0^*(2400)$, $D_{s0}^*(2317)$, i.e., the calculation and results are based on assuming them as the conventional $q\bar{q}$ meson. In fact, except for the structures of $a_0(1450)$ and $K_0^*(1430)$ which are less controversial, those of others still need to be ascertained. Especially, a common viewpoint is to interpret $D_{s0}^*(2317)$ as a DK molecular or a tetraquark state, see recent reviews in Refs. [45–48]. The $D_0^*(2400)$ ⁴ is the excited state of D meson and can be understood from the heavy-quark spin symmetry, where the light system has $j = s_q + L$, with s_q denoting the spin of light quark and L the orbital angular momentum, and thus there are two doublets with J^P as:

$$j = 1/2, \quad \begin{pmatrix} D_0^* = D_0^*(2400) & 0^+ \\ D_1' = D_1'(2430) & 1^+ \end{pmatrix} \quad (1)$$

and

$$j = 3/2, \quad \begin{pmatrix} D_1 = D_1(2420) & 1^+ \\ D_2^* = D_2^*(2460) & 2^+ \end{pmatrix}. \quad (2)$$

⁴ The near mass degeneracy of $D_0^*(2400)$ and $D_{s0}^*(2317)$ is explained by the hadronic loop effects using the heavy meson chiral perturbation theory [49].

Table 3 The involved mesons in the semileptonic B and B_s decays, where P , V , S , A denotes the pseudoscalar, vector, scalar (mass above 1 GeV), and axial-vector mesons, respectively. In the axial-vector

P	π	K	η	η'	D^0	D_s	
V	ρ	ω	K^*	ϕ	D^*	D_s^*	
S	$a_0(1450)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$K_0^*(1430)$	$D_0^*(2400)$	$D_{s0}^*(2317)$
A	1^{++}	$a_1(1260)$	$f_1(1285)$	$f_1(1420)$	$K_1(1270)$	$K_1(1400)$	
	1^{+-}	$b_1(1235)$	$h_1(1170)$	$h_1(1380)$			

Table 4 The form factors for $B \rightarrow f_{0q}$ and $B_s \rightarrow f_{0s}$ transitions Ref. [40]

F	$F(0)$	a	b
$F_1^{Bf_{0q}}$	0.25 ± 0.03	1.53 ± 0.04	$0.64_{-0.09}^{+0.12}$
$F_0^{Bf_{0q}}$	0.25 ± 0.03	0.54 ± 0.07	$0.01_{-0.01}^{+0.02}$
$F_1^{B_s f_{0s}}$	0.28 ± 0.01	1.64 ± 0.04	1.07 ± 0.12
$F_0^{B_s f_{0s}}$	0.28 ± 0.01	0.52 ± 0.04	0.20 ± 0.03

However, other interpretations also exist, e.g., $D_0^*(2400)$ shows two-pole structure and the lower pole associated with $D_{s0}^*(2317)$ forms an SU(3) multiplet [50]. Concerning the f_0 states, it is generally argued that they are the $q\bar{q}$ meson mixed by glueball contents, but differing in which state is dominated by glueball component in literature.⁵ Considering the available lattice and experimental information, the authors of Refs. [53,54] have done a careful analysis: under the assumption of the exact SU(3) symmetry, $f_0(1500)$ is an SU(3) isosinglet octet state and is degenerate with $a_0(1450)$; in the absence of glueball-quarkonium mixing, $f_0(1710)$ would be a pure glueball and $f_0(1370)$ a pure SU(3) singlet; when the glueball-quarkonium mixing is turned on, there will be additional mixing between the glueball and the SU(3)-singlet, and then

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.78(2) & 0.52(3) & -0.36(1) \\ -0.55(3) & 0.84(2) & 0.03(2) \\ 0.31(1) & 0.17(1) & 0.934(4) \end{pmatrix} \begin{pmatrix} f_{0q} \\ f_{0s} \\ G \end{pmatrix} \quad (3)$$

where the number in the parenthesis indicates the uncertainty for the last digit of the central value; the scalar f_{0q} (f_{0s}) is the pure $q\bar{q}$ ($s\bar{s}$) states with the spin-parity $J^P = 0^+$, whose mass is 1.474 GeV (1.5 GeV), while the glueball (G) is 1.7 GeV [53,54]. Clearly, $f_0(1710)$ contains mainly glueball and $f_0(1500)$ has the flavor octet structure. To be specific, we show the corresponding $B \rightarrow f_{0q}$ and $B_s \rightarrow f_{0s}$ transition form factors [40] in Table 4.

⁵ Again, there are also other interpretations, e.g., $f_0(1370)$ and $f_0(1710)$ as bound state of two vector mesons [51,52].

mesons $K_1(1270)$ and $K_1(1400)$ have no definite C -parity and do mix with each other

In fact, we wish to stress that the proposed measurements of the semileptonic $B(B_s)$ decays to f_0 states will be a powerful test for their inner structure due to the absence of the final-state interaction between f_0 and the lepton pair.

The axial-vector mesons A has two kinds of 1^{++} and 1^{+-} , where the former contains $a_1(1260)$, $f_1(1285)$, $f_1(1420)$ and the latter contains $b_1(1235)$, $h_1(1170)$, $h_1(1380)$, while $K_1(1270)$ and $K_1(1400)$ (with $d\bar{s}$ quark components) mix since they do not have the definite C -parity.

We should consider mixing angle:

$$\begin{aligned} \eta &= \eta_q \cos \phi - \eta_s \sin \phi, \\ \eta' &= \eta_q \sin \phi + \eta_s \cos \phi, \end{aligned} \quad (4)$$

with $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$, and $\phi = 39.3^\circ \pm 1.0^\circ$ extracted from Refs. [55,56] is consistent with the recent result $\phi = 42^\circ \pm 2.8^\circ$ from the analysis of the CLEO data [57].

Let A_L be the light axial vector, and A_H the heavier one.

$$\begin{aligned} A_L &= \sin \alpha_A A_q + \cos \alpha_A A_s, \\ A_H &= \cos \alpha_A A_q - \sin \alpha_A A_s, \end{aligned} \quad (5)$$

where A_q and A_s denotes the corresponding components $(u\bar{u} + d\bar{d})/\sqrt{2}$ (note the factor of 1/2 for calculating the branching fraction) and $s\bar{s}$ in the wave functions. Following the strategy in Refs. [20,21,41] we will take the values $\alpha_{f_1} = 69.7^\circ$, $\alpha_{h_1} = 86.7^\circ$. Recently, $h_1(1380)$ has been confirmed by the BES-III collaboration [58] in the decay channel $J/\psi \rightarrow \eta' K \bar{K} \pi$, where its mass and width, and the product branching fraction have been measured. Also, the mixing angle is determined to be $90.6^\circ \pm 2.6^\circ$ [58] based on the mixing angle $\theta_{K_1} = 34^\circ$ and the masses of the axial-vector mesons. This is consistent with the value that is adopted by us above. Clearly, the quark contents of $h_1(1170)$ is dominated by h_{1q} , while the $h_1(1380)$ mainly consists of $s\bar{s}$. Note that in the literature, e.g., Ref. [59], the mixing angle θ is often referred to the singlet-octet one, and $\alpha = \theta + 54.7^\circ$. An ideal mixing is defined as $\tan \theta = 1/\sqrt{2}$, i.e., $\theta = 35.3^\circ$.

The physical mass eigenstates $K_1(1270)$ and $K_1(1400)$ are the mixture of the 1P_1 state K_{1B} and 3P_1 state K_{1A} [22],

$$K_1(1270) = K_{1A} \sin \theta_{K_1} + K_{1B} \cos \theta_{K_1},$$

Table 5 Our theoretical predictions for the branching fractions of semileptonic B^+ decays, $B^+ \rightarrow (P, V, S, A)e^+\nu_e$, confronting with the PDG values [6] if available. Units are shown in the parentheses. The branching fraction $(250 \pm 50) \times 10^{-5}$ corresponds to the joint decay $B^+ \rightarrow \bar{D}_0^*(2400)\ell\nu_\ell$, $\bar{D}_0^*(2400) \rightarrow D^-\pi^+$. For the axial-vector mesons, we vary the mixing angle within the errors, and show the resulting regions in the brackets

Channel (10^{-5})	$B \rightarrow \pi^0$	$B \rightarrow \eta$	$B \rightarrow \eta'$	$B \rightarrow \bar{D}^0$
Theory	7.66 ± 1.69	5.27 ± 1.16	2.56 ± 0.56	2608 ± 287
PDG	7.80 ± 0.27	3.9 ± 0.5	2.3 ± 0.8	2200 ± 100
Channel (10^{-4})	$B \rightarrow \rho^0$	$B \rightarrow \omega$	$B \rightarrow \bar{D}^{*0}$	
Theory	2.13 ± 0.47	2.0 ± 0.44	671 ± 74	
PDG	1.58 ± 0.11	1.19 ± 0.09	488 ± 10	
Channel (10^{-5})	$B \rightarrow a_0(1450)$	$B \rightarrow f_0(1500)$	$B \rightarrow f_0(1710)$	$B \rightarrow \bar{D}_0^*$
Theory	2.72 ± 0.60	0.77 ± 0.17	0.21 ± 0.05	231 ± 25
PDG				250 ± 50
Channel (10^{-5})	$B \rightarrow a_1(1260)$	$B \rightarrow f_1(1285)$	$B \rightarrow f_1(1420)$	
Theory	6.3 ± 1.4	{3.8, 7.3}	{0.17, 1.34}	
Channel (10^{-5})	$B \rightarrow b_1(1235)$	$B \rightarrow h_1(1170)$	$B \rightarrow h_1(1380)$	
Theory	7.7 ± 1.7	{6.7, 10.8}	{0, 0.16}	

Table 6 Same as Table 5, but for the tau lepton modes

Channel (10^{-5})	$B \rightarrow \pi^0$	$B \rightarrow \eta$	$B \rightarrow \eta'$	$B \rightarrow \bar{D}^0$
Theory	5.21 ± 1.15	3.23 ± 0.71	1.36 ± 0.30	783 ± 86
PDG				770 ± 25
Channel (10^{-4})	$B \rightarrow \rho^0$	$B \rightarrow \omega$	$B \rightarrow \bar{D}^{*0}$	
Theory	1.16 ± 0.26	1.07 ± 0.24	166.5 ± 18.3	
PDG			188 ± 20	
Channel (10^{-5})	$B \rightarrow a_0(1450)$	$B \rightarrow f_0(1500)$	$B \rightarrow f_0(1710)$	$B \rightarrow \bar{D}_0^*$
Theory	1.04 ± 0.23	0.29 ± 0.06	0.07 ± 0.02	29.7 ± 3.3
Channel (10^{-5})	$B \rightarrow a_1(1260)$	$B \rightarrow f_1(1285)$	$B \rightarrow f_1(1420)$	
Theory	2.68 ± 0.59	{1.56, 3.05}	{0.06, 0.52}	
[1.5ex] Channel (10^{-5})	$B \rightarrow b_1(1235)$	$B \rightarrow h_1(1170)$	$B \rightarrow h_1(1380)$	
Theory	3.0 ± 0.7	{2.57, 4.12}	{0, 0.06}	

$$K_1(1400) = K_{1A} \cos \theta_{K_1} - K_{1B} \sin \theta_{K_1}, \quad (6)$$

and we will take $\theta_{K_1} = 33^\circ$ from the analysis of Refs. [20, 21].

As mentioned in Sect. 2, the form factors and the formula for calculating the branching fractions can be found in Refs. [39–41], respectively. Only one point is needed to be notified: generally, the form factor is expressed by

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}, \quad (7)$$

with the parameters $F(0)$, a , b given in Ref. [40]. As discussed in [39], the form factor $V_2(q^2)$ for $B(B_s) \rightarrow A(1^{+-})$ transition approaches zero at very large $-|q^2|$ where the three-parameter parametrization, Eq. (7), becomes questionable. Instead, a variant has been exploited,

$$V_2(q^2) = \frac{V_2(0)}{(1 - q^2/m_B^2)[1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2]}. \quad (8)$$

The form factor with the expression of Eq. (8) is applied to the 1P_1 case, i.e., b_1 , h_1 and K_{1B} . One may consider to replace

m_B by m_{B_s} in Eqs. (7) and (8) for B_s decays, however, such difference is negligible, in practice. We are now in position to provide the values for the branching fractions, which are listed in Tables 5 and 6 for B decays and Table 7 for B_s decays. Generally, the small masses of the electron and muon compared to the one for B meson does not make visible difference for the corresponding branching fractions, as also stated in PDG “ ℓ denotes e or μ ”.

Several remarks are in order:

- All the $B^+ \rightarrow P(V)e^+\nu_e$ modes have been measured by experiment and our values agree very well with the values reported by PDG within one standard deviation around. Certainly, our results may even better match some specific measurements, e.g., $\mathcal{B}(B^+ \rightarrow \bar{D}^0 e^+ \nu_e) = (2.29 \pm 0.08 \pm 0.09)\%$ by the BaBar collaboration [61], $\mathcal{B}(B^+ \rightarrow \bar{D}^{*0} e^+ \nu_e) = (6.50 \pm 0.20 \pm 0.43)\%$ by CLEO [62], and $\mathcal{B}(B^+ \rightarrow \omega e^+ \nu_e) = (1.35 \pm 0.21 \pm 0.11) \times 10^{-4}$ by BaBar [63]. The experimental results are not yet available for $B^+ \rightarrow S(A)e^+\nu_e$ modes. The branching fractions for semileptonic $B^+ \rightarrow a_0(1450)$, $a_1(1260)$, $b_1(1235)$, $f_1(1285)$, $h_1(1170)$ tran-

Table 7 Same as Table 5 but for the B_s decays, with the numbers in the parentheses indicate the branching fractions for the corresponding τ modes

Channel (10^{-4})	$B_s \rightarrow K$	$B_s \rightarrow D_s$
Theory	1.0 ± 0.22 (0.68 ± 0.15)	245 ± 27 (73.3 ± 8.1)
Channel (10^{-4})	$B_s \rightarrow K^*$	$B_s \rightarrow D_s^*$
Theory	3.3 ± 0.73 (1.72 ± 0.38)	605 ± 67 (151 ± 17)
Channel (10^{-5})	$B_s \rightarrow K_0^*(1430)$	$B_s \rightarrow D_{s0}^{*-}$
Theory	6.05 ± 1.33 (2.55 ± 0.56)	307 ± 34 (44.4 ± 4.9)
Channel (10^{-4})	$B_s \rightarrow K_1(1270)$	$B_s \rightarrow K_1(1400)$
Theory	2.47 ± 0.57 (0.98 ± 0.23)	0.21 ± 0.05 (0.09 ± 0.02)

sitions are predicted to be at the order of 10^{-5} , while those of other transitions are at the order of 10^{-6} . Considering the secondary decays $\mathcal{B}(a_0(1450) \rightarrow \pi\eta) = 0.093 \pm 0.020$, $\mathcal{B}(a_0(1450) \rightarrow \pi\eta') = 0.033 \pm 0.017$ and $\mathcal{B}(a_0(1450) \rightarrow K\bar{K}) = 0.082 \pm 0.028$, and $\mathcal{B}(f_1(1285) \rightarrow 4\pi) = (33.5^{+2.0}_{-1.8})\%$ [6], the statistics for Belle and Belle-II should be enough for measuring the transition $B^+ \rightarrow a_0(1450), f_1(1285)e^+\nu_e$. The precise determination of the pole position (mass and width) for $f_0(1370)$ is still challenging. PDG [6] shows that its pole position is at $(1200 - 1500) - i(150 - 250)$ MeV, and the Breit-Wigner or K-matrix mass and width at $(1200 - 1500) - i(200 - 500)$ MeV, suffering from large uncertainty. We thus refrain from showing the branching fractions involving $f_0(1370)$.

- BaBar [64] and Belle [65] have measured the four semileptonic decay modes involving the P-wave charmed mesons, cf. Eqs. (1) and (2), which of course, includes $D_0^*(2400)$. The PDG average value of $(2.5 \pm 0.5) \times 10^{-3}$ means the joint branching fraction $\mathcal{B}(B^+ \rightarrow \bar{D}_0^*(2400)\ell\nu_\ell, \bar{D}_0^*(2400) \rightarrow D^-\pi^+)$. Comparing with our theoretical value for $B^+ \rightarrow D_0^*(2400)e^+\nu_e$ in Table 5, we expect the mode of $D_0^*(2400) \rightarrow D\pi$ is the dominant one in $D_0^*(2400)$ decays. In fact, the semileptonic decay $B \rightarrow D_0^*(2400)$, as a background contributing to one of the leading sources of the systematical uncertainty for the extraction of $|V_{cb}|$ from $B \rightarrow D^*\ell\nu_\ell$, is still poorly known, see the review in Ref. [66]. The Belle-II and LHCb detectors will provide the opportunity for the precision measurements. We also display the differential decay rate for $B \rightarrow D_0^*(2400)$ as well as $B \rightarrow D_{s0}^*(2317) + \text{lepton pairs}$ in Fig. 1 for convenience of comparison with the future experiments.
- Due to the factor of $|V_{cb}/V_{ub}|^2 \approx 115$, the branching fraction of $b \rightarrow c$ decay is generally enhanced by two orders compared to $b \rightarrow u$ decay, as can be seen in Tables 5, 6 and 7 (Note that the additional factor of $\sin^2\phi$ or $\cos^2\phi$ appears in the processes of $B^+ \rightarrow \eta, \eta'$ to calculate the branching fractions). $\mathcal{B}(B_s \rightarrow D_s^- + X) = (93 \pm 25)\%$ [6] again shows the dominance of $b \rightarrow c$ transition. In Table 7, the B_s decay branching fraction is at

Fig. 1 The differential decay rate $d\Gamma/dq^2$ as a function of momentum transfer q^2 . In the upper plot, the solid and dashed line indicate $B^+ \rightarrow D_0^*(2400)e^+\nu_e$ and $B_s \rightarrow D_{s0}^*(2317)e^+\nu_e$, respectively, while the lower one shows the corresponding tau lepton modes

the order of 10^{-4} for $b \rightarrow u$ and 10^{-2} for $b \rightarrow c$. Unfortunately, there is scarce experimental information on the semileptonic B_s decay except for the inclusive semileptonic decay $\mathcal{B}(B_s \rightarrow X\ell\nu_\ell) = (9.6 \pm 0.8)\%$ [6]. As can be clearly seen, the sum of the branching fractions for the channels considered in Table 7 does not exceed this limit. The theoretical predictions for $\mathcal{B}(B_s \rightarrow D_s\ell\nu_\ell)$ vary from 1.0% to 3.2% and for $\mathcal{B}(B_s \rightarrow D_s^*\ell\nu_\ell)$ vary from 4.3% to 7.6% [67], see e.g., Refs. [68–71]; $\mathcal{B}(B_s \rightarrow D_{s0}^*(2317)\ell\nu_\ell) \sim 0.20\% - 0.57\%$ [71–73].

Regarding $D_{s0}^*(2317)$ as a DK molecular state, the authors of Ref. [74] predict $\mathcal{B}(B_s \rightarrow D_{s0}^*(2317)\ell\nu_\ell) = 0.13\%$. The process $B_s \rightarrow K^-\ell\nu_\ell$ has been calculated in Refs. [71, 75] and also examined in lattice QCD [76, 77]. The $B_s \rightarrow K_j^*\ell\nu_\ell$ decay is investigated in Ref. [60], and our result for $B_s \rightarrow K_0^{*-}(1430)\ell\nu_\ell$ agrees very well with theirs, but not for $K_1(1270)$ and $K_1(1400)$ sector. Adopting their values of the mixing angles as input still does not remedy such discrepancy, so we will regard such discrepancy as the different predictions from the two models. Given the branching fractions of $\mathcal{B}(K_1(1270) \rightarrow K\rho) = (42 \pm 6)\%$ and $\mathcal{B}(K_1(1400) \rightarrow K^*(892)\pi) = (94 \pm 6)\%$, the $B_s \rightarrow K_1$ transitions could be measured with the current statistics in Belle/Belle-II and LHCb. Overall speaking, little has been known for the experimental information on the exclusive semileptonic B_s decay, while plentiful theoretical predictions have been done. This situation highly calls for the true experimental measurements, and this can be realized with Belle/Belle-II and LHCb detectors.

- We wish to comment that even-parity light mesons, including the axial-vector meson, the scalar meson above 1 GeV, and the P -wave charmed meson, can be also studied via hadronic two-body B decays within the factorization scheme [78–81]. The semileptonic decay modes investigated here will provide a much cleaner environment to explore the nature of these mesons owing to the absence of the strong hadronic final-state interactions manifested in the two-body hadronic decay. At least, the investigation of such semileptonic modes could serve as a supplement to the hadronic two-body decay.
- The CKM matrix element $|V_{ub}|$ suffers from large uncertainty around 19%, while $|V_{cb}|$ has been determined better with the uncertainty of 4%. Roughly assigning 10% error induced by form factors, we have the combined uncertainty of 22% and 11% for the processes $b \rightarrow u$ and $b \rightarrow c$, respectively. Additionally, the uncertainty induced by the mixing angle needs more care. Guided by Ref. [20, 21] we allow the variations of α_{f_1} , α_{h_1} , θ_{K_1} within 8° , 6° , 4° , in order,⁶ which may produce the (very) asymmetry error. In such cases, we show in the brackets the resulting allowed regions for the branching fractions. The branching fractions for K_1 case is not sensitive to the mixing angle θ_{K_1} . The mixing angle α_{h_1} for $h_1(1170)$ and $h_1(1380)$ states crosses 90° , where $h_1(1170)$ purely consists of $q\bar{q}$ and $h_1(1380)$ purely $s\bar{s}$. This shows the origin of vanishing branching fractions of $B \rightarrow h_1(1380)\ell\nu_\ell$ in Tables 5 and 6. In some cases, the branching fractions are sensitive to the mixing angles,

e.g., for $B \rightarrow f_1$, h_1 . From this point of view, it should be understood that the future measurements on these channels will be highly meaningful for a “precise” determination of the mixing angles, as also mentioned in the Introduction.

- There are several recent tests of the lepton universality via the semileptonic decay modes, e.g., $\mathcal{B}(D \rightarrow \pi\mu\nu_\mu)/\mathcal{B}(D \rightarrow \pi e\nu_e)$ done by the BES-III collaboration [82], and $\mathcal{B}(B \rightarrow D^*\tau\nu_\tau)/\mathcal{B}(B \rightarrow D^*\mu\nu_\mu)$ done by the LHCb collaboration [83]. Motivated by this, we also calculate the branching fractions of the semileptonic decays involving the τ lepton mode. Our results agree very well with the experimental values $\mathcal{B}(B^+ \rightarrow \bar{D}^0\tau\nu_\tau) = (7.7 \pm 2.5) \times 10^{-3}$ and $B^+ \rightarrow \bar{D}^{*0}\tau\nu_\tau = (1.88 \pm 0.20)\%$ [6]. That is, what we compared is the direct number of the branching fraction of the τ mode, but not the ratio between the τ and electron one. The latter is related to the recently well-known R_D or R_{D^*} puzzle [84]. In fact, we do not touch this issue since we are working in the framework of Standard Model (keeping the lepton universality) and also there is the tricky estimate of uncertainties. However, as a passing comment, we want to remind the importance of the precise determination of the uncertainties. Starting from the decay $B \rightarrow D^*\ell\bar{\nu}_\ell$, the authors of Ref. [85] also discussed the corresponding strange quark partner, $B_s \rightarrow D_s^*\tau\nu_\tau$, which is also investigated by us. Very recently, $B^* \rightarrow (D, D_s, \pi, K)\ell\nu_\ell$ is also discussed for probing the New Physics effects [86]. On the experimental aspect, the electron mode is usually the easiest one to be measured, while one may encounter the large misidentification between μ and π .⁷ For the τ case, the experimental error will be even larger: the two largest decay channels of τ are [6] $\mathcal{B}(\tau \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau) = (17.39 \pm 0.04)\%$ and $\mathcal{B}(\tau \rightarrow e^-\bar{\nu}_e\nu_\tau) = (17.82 \pm 0.04)\%$; both of them contain two neutrinos, which hinders the full construction resulting in large background, and also there is no way to use the recoiling information due to the existence of multi-neutrinos.

4 Conclusion

Based on the analysis of the form factors from the covariant light-front quark model [39, 40], we provide the branching fractions for $B \rightarrow (P, V, S, A)\ell\bar{\nu}_\ell$ with P, V, S, A denoting the corresponding pseudoscalar, vector, the scalar mesons with mass above 1 GeV, and the axial-vector mesons, respectively. Those mesons are listed in Table 3. Under the framework of the lepton flavor universality, the branching

⁶ These uncertainties are also used in Ref. [41]. We notice that the error for α_{h_1} agrees very well with the very recent determination of 7.2° by the BESIII collaboration [58].

⁷ The decaying of muon to electron occurs outside the detector and thus muon can be regarded as a stable particle inside the detector.

fractions for the semileptonic decay involving the τ mode are also provided. The predicted branching fractions are typically in the range of $10^{-6} \sim 10^{-4}$. On the experimental side, $(772 \pm 11) \times 10^6 B\bar{B}$ and $(6.53 \pm 0.66) \times 10^6 B_s\bar{B}_s$ pairs have already been collected by the Belle detector, and Belle-II will have a larger statistics with 40 times more than Belle. Those decay modes can be accessed by the Belle, Belle-II and LHCb data samples, which renders the test of theoretical calculation, and more importantly, provides the valuable information on the structure of the scalar and axial-vector meson, e.g., the weights of quark-antiquark components in the $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ states (cf. Eq. (3)), the mixing angles for $f_1(1285) - f_1(1420)$ and $h_1(1170) - h_1(1380)$ states.

Assuming $D_0^*(2400)$ and $D_{s0}^*(2317)$ as the conventional quark-antiquark mesons, we predict the branching fractions of $\mathcal{B}(B \rightarrow D_0^*(2400)\ell\nu_\ell) = (2.31 \pm 0.25) \times 10^{-3}$ and $\mathcal{B}(B_s \rightarrow D_{s0}^*(2317)\ell\nu_\ell) = (3.07 \pm 0.34) \times 10^{-3}$. Confronting these values with future experimental results will provide a further scrutiny for the possible assignment of $q\bar{q}$ interpretation.

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